統計學習初論（106-2）

作業三

作業設計：盧信銘
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截止時間：2018 年 4 月 10 日上午 9 點

第一題請至 RSAND 上批改，範例命令：sl_check_hw3q1 ./your_program。第二題請使用 RStudio 中的 R Notebook (File->New->R Notebook) 功能製作你的回答。R Notebook 可以讓你方便的將你的回答、程式碼、圖表等整合至一個文件中。完成之後將 HTML 檔案 (xxx.nb.html)上傳至 Ceiba 作業區繳交。如果你的回答中有公式或數學符號，可以使用 Latex 語法製作。如果你對 R Notebook 不熟悉，可以參考線上文件: https://rmarkdown.rstudio.com/r_notebooks.html

作業自己做。嚴禁抄襲。不接受紙本繳交，不接受遲交。請以英文或中文作答。

第一題

(50 points) We are going to practice evidence approximation in this question. I have created a video that explain the derivation of evidence approximation: https://www.youtube.com/playlist?list=PLvVPTibZrkBqfya8MCijxsPl2IgkVtR0G

You should watch this video first before doing this homework.

Write a function named lm_evmax that takes the outcome matrix *y* and a feature matrix *xmat*, and perform evidence maximization. The matrix *y* should be a matrix of one column, and the first column of the *xmat* matrix should be all ones. Follow the following procedure closely to implement lm_evmax. We should start with an initial estimation that is not too far from the optimal solution. To do this, use the regularized least square solution in Section 3.1.14. Set $\lambda = 0.001\ N$, where N is the number of observation in $xmat$. We can now estimate the initial value for m_N by $w = (\lambda I + xmat^T xmat)^{-1} xmat^T y$. The initial value for β is: $\beta = \frac{N}{e_0^T e_0}$, where $e_0 = y - xmat\ w$. The initial α, following our discussion in class, is $\alpha = \lambda\ \beta$.

With all the initial values, we can now start to iterate by computing

$A = \alpha I + \beta xmat^T xmat,$

$m_N = \beta A^{-1} xmat^T y,$

$\gamma = \sum_{i=1}^{M} \frac{\lambda_i}{\alpha + \lambda_i}.$

The new α is $\alpha_{new} = \frac{\gamma}{m_N^T m_N}$, and the new β is $\frac{1}{\beta_{new}} = \frac{e_1^T e_1}{N-\gamma}$, where $e_1 = y - xmat\ m_N$.

Iterate until the $(|\alpha - \alpha_{new}| + |\beta - \beta_{new}|) < 10^{-5}$.

Your function should output a list that store m_N under the name of mN, the square root of the diagonal elements of A^{-1} under the name of mNsd, α under the name of alpha, and β under the name of beta.

Sample input and output 1:

```
> setwd('your_path_to_data')
> load(file='rtb2_train.rdata')
>
> nfeat=20
> rtb3 = rtb2_train[1:(nfeat+1)]
> y=as.matrix(rtb3[,1])
> xmat = model.matrix(paying_price~., data=rtb3)
> lmev1 = lm_evmax(y, xmat)
> lmev1
$mN
                  [,1]
(Intercept)  98.941001
agent_1      43.453947
agent_2     114.196607
agent_3       6.545915
agent_4     -15.585473
agent_5       1.861077
agent_6       6.271456
agent_7      17.880470
agent_8       5.386568
agent_9     -12.751414
agent_10     -3.897748
agent_11     -7.807772
agent_12     -2.450647
agent_13     -4.958611
agent_14      5.725306
agent_15      8.330833
agent_16      9.684011
agent_17      1.678518
agent_18     -5.641692
agent_19     -9.100067
agent_20     -7.277845

$mNsd
(Intercept)     agent_1     agent_2     agent_3     agent_4
  3.0052509   5.6445189   4.3651917   5.3672246   3.7451977
    agent_5     agent_6     agent_7     agent_8     agent_9
  3.2022098   7.7406664   7.5503850   6.3772277   3.9300660
   agent_10    agent_11    agent_12    agent_13    agent_14
  0.6215702   1.0467525   2.9594740  24.2552267  24.7024972
   agent_15    agent_16    agent_17    agent_18    agent_19
 11.4676217   5.3772346   3.8749929   3.6493229   2.4588253
   agent_20
  3.0145566

$alpha
[1] 0.0007585334

$beta
[1] 0.0002538865
```

Sample input and output 2:

```
> setwd('your_path_to_data')
> load(file='rtb2_train.rdata')
>
> nfeat=seq(1, length(rtb2_train), by = 50)
> rtb3 = rtb2_train[1:10000,nfeat]
> y=as.matrix(rtb3[,1])
> xmat = model.matrix(paying_price~., data=rtb3)
> lmev1 = lm_evmax(y, xmat)
> lmev1
$mN
                    [,1]
(Intercept)    100.4745627
agent_50        24.0762994
agent_103       10.1641802
agent_158        4.0172520
user_10057      -8.2734590
url_16          93.3343427
url_66          81.9263975
url_116         31.3215269
url_166         -0.8606502
url_216        -34.1168234
url_266         -2.4165514
url_316         -9.4242748
url_366          0.0000000
url_416        -39.4635232
url_466         12.1393132
url_516          0.0000000
url_566        -51.2127917
url_616         38.1233404
url_666         15.6048899
url_716          0.0000000
url_766        -25.9820298
url_816        -12.8898514
url_866          0.0000000
url_916         13.3998047
url_966          0.0000000
url_1016         0.0000000
url_1066        17.9632028
url_1116         0.0000000
ad_slot_id_1   -41.9625073
ad_slot_id_51   13.9047828
ad_slot_id_101 -18.7191215
ad_slot_id_151  48.2599954
ad_slot_id_201  51.0284580
ad_slot_id_251  36.3351298
ad_slot_id_301 -26.3513146
ad_slot_id_351  49.0202346
ad_slot_id_401   7.1334803
ad_slot_id_451  41.0191186
ad_slot_id_501   1.1261907
ad_slot_id_551   0.0000000
ad_slot_id_601  11.9589780
ad_slot_id_651  14.1927424
ad_slot_id_701   0.0000000
ad_slot_id_751  -6.0329717
ad_slot_id_801   0.0000000
domain_46       94.1003037
domain_96        0.0000000
domain_146     -21.9265105
domain_196       0.0000000
domain_246     -35.7886961
domain_296      37.1806638
domain_346      12.5243773
domain_396       0.0000000
```

```
domain_446       0.0000000
domain_496     -45.8577932
domain_546      23.9770084
domain_596      -6.0329717

$mNsd
   (Intercept)        agent_50      agent_103      agent_158     user_10057
     0.7379481     36.9458157     15.2760156     14.2058923      1.5374625
        url_16         url_66        url_116        url_166        url_216
    10.0060958     20.3319885     23.1867582     15.6205344     17.4960124
       url_266        url_316        url_366        url_416        url_466
    20.3139447     36.9458157     47.1126442     31.3945727     36.9458157
       url_516        url_566        url_616        url_666        url_716
    47.1126442     27.7725249     25.1708054     36.9458157     47.1126442
       url_766        url_816        url_866        url_916        url_966
    36.9458157     36.9458157     47.1126442     36.9490225     47.1126442
      url_1016       url_1066       url_1116   ad_slot_id_1  ad_slot_id_51
    47.1126442     27.7725249     47.1126442      1.4056166      9.3709884
ad_slot_id_101 ad_slot_id_151 ad_slot_id_201 ad_slot_id_251 ad_slot_id_301
     8.4810507     27.7725819     27.7725249     37.4095318     16.7858621
ad_slot_id_351 ad_slot_id_401 ad_slot_id_451 ad_slot_id_501 ad_slot_id_551
    31.3955976     36.9458157     36.9458157     31.3945727     47.1126442
ad_slot_id_601 ad_slot_id_651 ad_slot_id_701 ad_slot_id_751 ad_slot_id_801
    18.2999500     31.3945727     47.1126442     35.9801340     47.1126442
     domain_46      domain_96     domain_146     domain_196     domain_246
    27.7725249     47.1126442     20.3160959     47.1126442     14.2068564
    domain_296     domain_346     domain_396     domain_446     domain_496
    18.2988486     36.9458157     47.1126442     47.1126442     31.3945727
    domain_546     domain_596
    21.6084555     35.9801340

$alpha
[1] 0.0004505314

$beta
[1] 0.0002821163
```

Sample input and output 3:

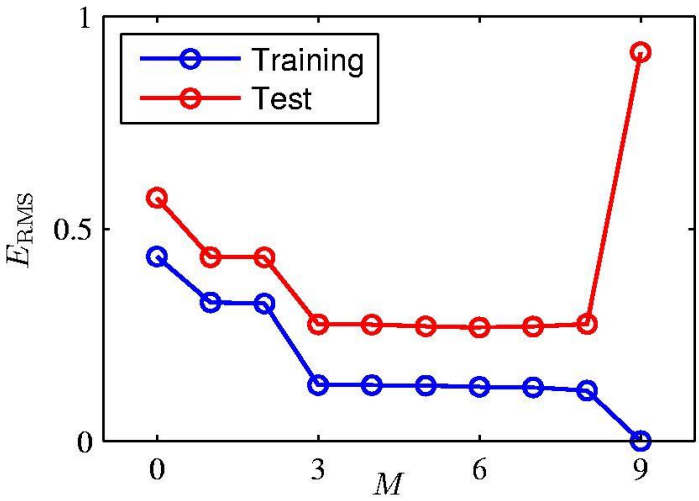
```
> setwd('your_path_to_data')
> load(file='rtb2_train.rdata')
>
> nfeat=seq(1, length(rtb2_train), by = 40)
> rtb3 = rtb2_train[nfeat]
> y=as.matrix(rtb3[,1])
> xmat = model.matrix(paying_price~., data=rtb3)
> lmev1 = lm_evmax(y, xmat)
> lmev1
$mN
                    [,1]
(Intercept)    101.8302808
agent_40         5.5742646
agent_83         5.5949491
agent_125       10.6679739
agent_169       10.3619323
user_10057      -8.1791695
url_6          -66.2237601
url_46           1.5576434
url_86           0.2777827
url_126        -41.9386318
url_166          0.5658397
url_206         76.2007520
url_246        -63.4182569
url_286         -2.1062712
url_326        -66.8628019
url_366        -50.9903056
url_406         21.0375925
url_446         23.7072943
url_486        -53.5287665
url_526        -54.9506555
url_566        -35.3568148
url_606         29.4711094
url_646         21.9153860
url_686         26.7922792
url_726        -39.7309547
url_766        -41.2635100
url_806         16.6446985
url_846         -0.9691111
url_886        -37.7676340
url_926         13.2095923
url_966         -0.4014126
url_1006        11.8079327
url_1046        -1.1496182
url_1086        16.3418655
url_1126       -25.9745522
ad_slot_id_1   -43.5619818
ad_slot_id_41   53.8200654
ad_slot_id_81   26.2876093
ad_slot_id_121  24.4952044
ad_slot_id_161  45.3289173
ad_slot_id_201  63.1118155
ad_slot_id_241  43.7379560
ad_slot_id_281 -67.4577704
ad_slot_id_321  18.5629974
ad_slot_id_361 -53.4090614
ad_slot_id_401  39.0564170
ad_slot_id_441  33.9566347
ad_slot_id_481  32.2257312
ad_slot_id_521  33.7058323
ad_slot_id_561  28.1896232
ad_slot_id_601   4.2483136
ad_slot_id_641  12.1009326
ad_slot_id_681 -15.3877926
ad_slot_id_721 -31.5576145
ad_slot_id_761 -26.1128588
```

```
ad_slot_id_801  12.8207545
domain_36       34.3964054
domain_76       76.3594959
domain_116      45.1899791
domain_156      11.7679577
domain_196      70.1342099
domain_236      59.9155922
domain_276      49.2458507
domain_316      52.1008217
domain_356      36.4211597
domain_396      24.2672065
domain_436      28.5159067
domain_476      21.2111630
domain_516      24.9081412
domain_556       1.9285828
domain_596       4.5992392

$mNsd
   (Intercept)       agent_40       agent_83      agent_125      agent_169
     0.2209299      2.4069776      4.5390369      8.6546355     13.7431687
    user_10057          url_6         url_46         url_86        url_126
     0.4512137      2.2399299      2.8632804      4.3313212      4.9832576
       url_166        url_206        url_246        url_286        url_326
     6.5114389     22.5081167     14.5044244      9.3616472     19.8816980
       url_366        url_406        url_446        url_486        url_526
    14.0979968     10.9340741     13.0577709     18.8719809     22.5081819
       url_566        url_606        url_646        url_686        url_726
    11.5266057     19.8817496     15.4348582     21.0731725     18.8719003
       url_766        url_806        url_846        url_886        url_926
    22.5081167     16.5709957     18.8734339     22.5081819     16.5709957
       url_966       url_1006       url_1046       url_1086       url_1126
    21.0743198     17.2420457     22.5091551     22.5081167     13.7242217
  ad_slot_id_1  ad_slot_id_41  ad_slot_id_81 ad_slot_id_121 ad_slot_id_161
     0.4248913     10.5791740      3.6283353      4.8996879      9.9839483
ad_slot_id_201 ad_slot_id_241 ad_slot_id_281 ad_slot_id_321 ad_slot_id_361
    15.4354625     12.4800296     15.4348138      8.1562509     13.7242217
ad_slot_id_401 ad_slot_id_441 ad_slot_id_481 ad_slot_id_521 ad_slot_id_561
    18.0019511     17.2420457     18.0018544     21.0731725     18.8719003
ad_slot_id_601 ad_slot_id_641 ad_slot_id_681 ad_slot_id_721 ad_slot_id_761
     7.7390422     13.7433142      2.3760928     15.9727699     13.7240643
ad_slot_id_801      domain_36      domain_76     domain_116     domain_156
    22.5081167     12.1422525     10.4297786      8.7433101      4.2342114
    domain_196     domain_236     domain_276     domain_316     domain_356
    18.8719809     18.0019363     17.2420457     21.0731725     17.2421446
    domain_396     domain_436     domain_476     domain_516     domain_556
    13.3783913     18.0019511     16.5710958     22.5081819      8.5612977
    domain_596
    11.9723083

$alpha
[1] 0.0005838557

$beta
[1] 0.0002780223
```

Evaluation: All credits will be given based on the correctness of 10 testing cases. Correct output in a case is worth 5 points.

第二題

(50 points) We have discussed bias-variance tradeoff in class. In this homework, we are going to produce the bias-variance trade-off figure similar to the figure below:



In our figures, the X-axis will be the logarithm of regularization coefficient, and the Y-axis is training and testing RMSE. We are going to use the year prediction task of the Million Songs Dataset. Please use the data file (msong_slhw.rdata) that come with the homework instead of downloading the data from the Internet so that the teaching team can easily interpret your results. Use the load() function to load the rdata file.

Answer the following questions.

(1) (0%) List the summary statistics of all variables in the training (msong_train) and testing (msong_test) data frames. For each variable, you should provide number of data point (after removing NAs), mean, median, standard deviation, Q1 (first quantile), Q3 (third quantile), minimal and maximal values.

(2) (15%) Take the first 5000 rows in msong_train to form a new training dataset. Shift the outcome variable (year) to have mean zero by subtracting mean of year in the new training dataset. For the remaining feature values, standardize their values to have a mean zero and unit variance. Use the mean year in training data to shift outcome variables in msong_test. Use the mean and variance of features in the new training dataset to standardize corresponding feature values in msong_test.

Train ridge regression by minimizing

$$\frac{1}{2}\sum_{i=1}^{n}|t_i - w^T\phi_i(x)|^2 + \frac{\lambda}{2}w^Tw$$

Note that you should implement your own trainer instead of using existing trainer such as glmnet. Record the training and testing RMSE. Plot the result. We are expecting to see U-shape curves. Provide insightful comments on your results.

(3) (10%) Take the first 500 and 1000 rows in msong_train to generate two new training datasets. Repeat the procedure in the previous question. Compare results with different sample sizes. Provide insightful comments on your results.

(4) (10%) Take the first 1000 rows from msong_train. Shift year to have a mean zero in the new training and testing datasets as in (2). Do not standardize the feature values in this new setting. That is, use the original feature values for training and testing. Train ridge regression using different values of λ. Plot training and testing RMSE with respect to the logarithm of λ. Compare the results with those in (3)

with the same training data size. Provide insightful comments. Your comments, among others, should include a comparison of the best testing RMSE.

(5) (15%) Take the first 1000 rows from msong_train as the new training dataset. Do not adjust data values in the training and testing dataset in this setting. Plot the training and testing RMSE with respect to the logarithm of λ. Compare the results with those in (4) and (3). Provide insightful comments. Your comments, among other, should answer the question about feature pre-processing. Should we standardize feature values and shift the mean of outcome variables to zero? Why do you think this is the case?