EXPERIMENT NO: 4

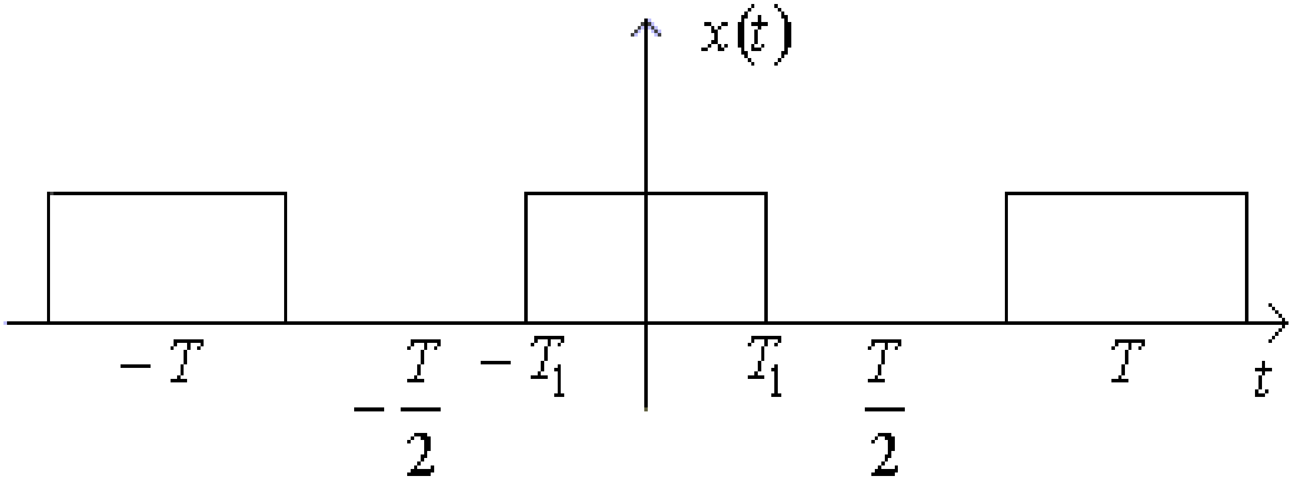
Continuous Time Fourier series and Fourier Transform

Objective:

A. To study and analyze the continuous time Fourier series (CTFS) for below figure(a).
B. To study and analyze the continuous time Fourier transform (CTFT) for below figure(b).

Theory:

Continuous time Fourier series (CTFS) is the way to represent a continuous time function as the sum of sine and cosine waves. It is used to determine the frequency for a continuous periodic signal. This CTFS exists at particular conditions only. Those conditions are defined as the function must be absolutely integrable, function must have finite maxima and minima in a time interval, and finite number of discontinuities in a time interval. These conditions are called as Dirichlet's conditions. CTFS is expressed in terms of "trigonometric", "exponential", and "polar" forms. Let $x(t)$ is an input signal or function.



` Fig. a) Periodic Pulse Signal

Trigonometric CTFS is expressed as:

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nw_0 t) + b_n \sin(nw_0 t) \ \text{ Volts}$$

$$a_0 = \left(\frac{1}{T}\right) \int_0^T x(t)\, dt$$

$$a_n = \left(\frac{2}{T}\right) \int_0^T x(t)\, \cos(nw_0 t)\, dt$$

$$b_n = \left(\frac{2}{T}\right) \int_0^T x(t)\, \sin(nw_0 t)\, dt$$

Where "T" is the time period of an input signal, "w_0" is the fundamental frequency, "nw_0" is the "n^{th}" harmonic frequency. The amplitude value of an input signal $x(t)$ at a time "t" is an exactly equal to at a time "$t + T$".

Exponential CTFS is expressed as:

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jnw_0 t} \text{ Volts}, \quad w_0 = \left(\frac{2\pi}{T}\right) \text{ Radians}$$

$$C_n = \left(\frac{1}{T}\right) \int_0^T x(t)\, e^{-jnw_0 t} dt$$

Polar CTFS is expressed as:

$$x(t) = \sum_{n=1}^{\infty} A_n \cos(nw_0 t + \theta_n) \text{ Volts}$$

Here
$$A_n = \sqrt{{a_n}^2 + {b_n}^2}$$

$$\theta_n = -tan^{-1}\left(\frac{b_n}{a_n}\right) \text{ Radians}$$

Continuous time Fourier transform (CTFT) is used to determine the frequency for a continuous periodic and aperiodic signal. CTFT exists at particular conditions only. Those conditions are defined as the function must be absolutely integrable, function must have finite maxima and minima in a time interval, and finite number of discontinuities in a time interval. These conditions are called as Dirichlet's conditions. CTFT is expressed in terms of "exponential" form. Let $x(t)$ is an input signal or function.

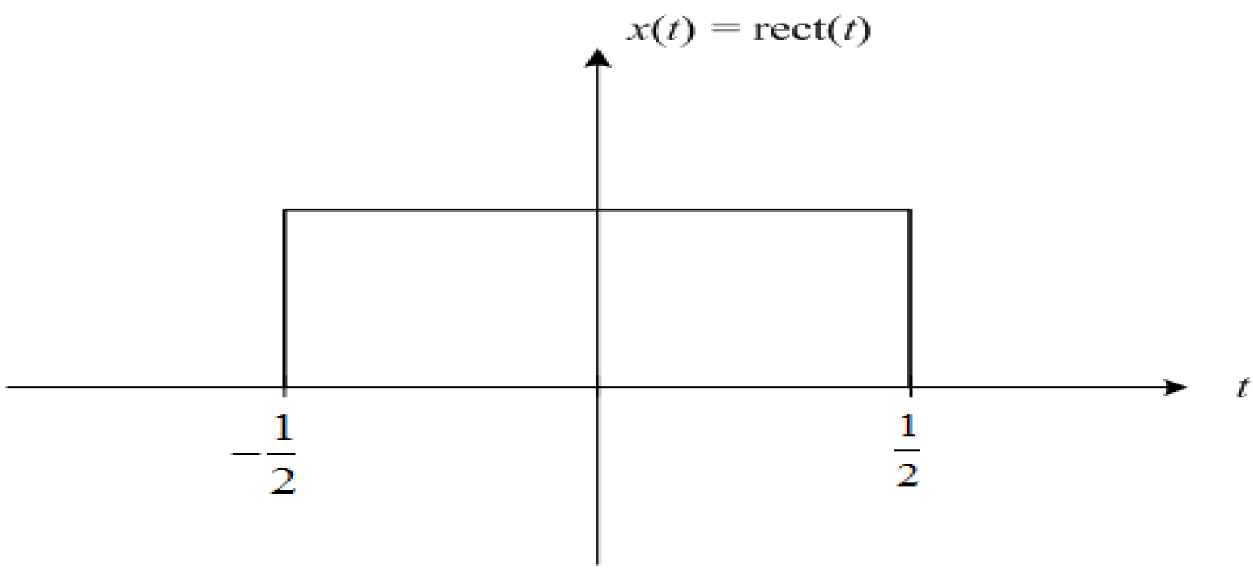


Fig. b) Aperiodic Rectangular Pulse Signal

Exponential CTFT is expressed as:

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}\, dt \text{ Volts/Hz}$$

Exponential Inverse-CTFT is expressed as:

$$x(t) = (\frac{1}{2\pi}) \int_{-\infty}^{\infty} X(jw)e^{jwt}\, dw \text{ Volts}$$

Waveforms to be observed:

Cases:

a) $T_1 = 0.5\, ms\ and\ T = 5\, ms.$
b) $T_1 = 0.5\, ms\ and\ T = 10\, ms.$
c) $T_1 = 0.5\, ms\ and\ T = 20\, ms.$

In each of the above cases, plot the CTFS coefficients with the following forms.

i) Using Exponential form
ii) Using Trigonometric form
iii) Using Polar form

In each of the above cases, plot the Exponential-CTFS coefficients ($with\ C_n$) for the following signals.

a) $x(t-8)$
b) $3x(t+9) - 5x(t)$
c) $x(-t)$
d) $x(-t/9)$
e) $x^*(t)$
f) $\frac{d}{dt}(x(t))$
g) $\int x(t)$
h) $x\left(\frac{7t}{12}\right)x(t-\frac{8}{3})$
i) $\frac{d}{dt}(x(t))$
j) $\int x(t)$
k) $e^{j(6\pi/T)t}x(t)$

Plot the CTFT ($with\ X(jw)$) for the following signals.

a) $x(t)$
b) $x(t+8)$
c) $e^{j4t}x(t)$
d) $4x(t-7) - 2x(t)$
e) $x(-t)$
f) $x(-6t)$
g) $x^*(t)$

h) $\frac{d}{dt}(x(t))$
i) $\int x(t)$
j) $t^5 x(t)$
k) $X(t)$
l) $x\left(\frac{4t}{3}\right) * x(-t + \frac{5}{6})$
m) $x\left(\frac{4t}{3}\right) x(-t + \frac{5}{6})$

Results:

Continuous Time Fourier Series (CTFS):

In each of the above signal,

i) Fundamental frequency (f_0)
ii) Power (P_x)
iii) Absence of higher order harmonics (nf_0)

Continuous Time Fourier Transform (CTFT):

In each of the above signal,

i) Power (P_x)

MATLAB Program:

The student must write the code from .mfile and associated functions used in this section.

Results:

Using an example, the student must clearly mention input arguments used and corresponding output obtained (in the form of numerical values / graphs) in this section.

Comments on the results:

The student must describe in short the inferences drawn from the experiment and observations from the results obtained in his/ her own words