EXPERIMENT NO: 5

To Study and Analyze DFT and IDFT Techniques

Objective:

A. To demonstrate the discrete Fourier transform (DFT) for discrete time domain signal.

B. To demonstrate the inverse discrete Fourier transform (IDFT) for discrete frequency domain signal.

Theory:

Discrete Fourier Transform (DFT):

The Discrete Fourier Transform (DFT) is the equivalent of Fourier Transform for signals known only at N instants separated by samples times T (i.e. finite sequence of data).

Let $f(t)$ be the continuous signal which is the source of the data. Let N samples be denoted $f[0], f[1], f[2], \ldots, f[k], \ldots, f[N-1]$.

The Fourier Transform of original signal, $f(t)$, would be

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

Here, each sample $f[k]$ can be regarded as an impulse having area $f[k]$. Then, since the integrand exists only at the sample points: $F(j\omega) = \int_{0}^{(N-1)T} f(t)e^{-j\omega t}dt$

$$F(j\omega) = f[0]e^{-j0} + f[1]e^{-j\omega T} + \cdots + f[k]e^{-j\omega kT} + \cdots f[N-1]e^{-j\omega(N-1)T}$$

$$\text{i.e } F(j\omega) = \sum_{n=0}^{N-1} f[n]e^{-j\omega nT}$$

We could evaluate this for any ω, but with only N data points to start with, only N final outputs will be significant.

The continuous Fourier Transform could be evaluated over a finite interval (usually the fundamental period T_0) rather than from $-\infty$ to $+\infty$ if the waveform was periodic. Similarly, since there are only a finite number of input data points, the DFT treats the data as if it were periodic (i.e. $f[N]$ to $f[2N-1]$ is the same as $f[0]$ to $f[N-1]$).

From above observation, we evaluate the DFT equation for the fundamental frequency (one cycle per sequence, $\frac{2\pi}{NT}$ rad/sec.) and its harmonics (not forgetting the d.c. component (or average) at $\omega = 0$).

$$\text{i.e. set } \omega = 0, \frac{2\pi}{NT}, \frac{2\pi}{NT} \times 2, \ldots \frac{2\pi}{NT} \times n, \ldots \frac{2\pi}{NT} \times (N-1)$$

Or, in general

$$F[K] = \sum_{n=0}^{N-1} f[n]e^{-j\frac{2\pi}{N}nK} \quad (K = 0 : N-1)$$

Here $F[K]$ is the Discrete Fourier Transform of the sequence $f[n]$.

Inverse Discrete Fourier Transform (IDFT):

The inverse discrete Fourier transform for F[K]:

$$f[n] = \frac{1}{N}\sum_{K=0}^{N-1} F[K]e^{+j\frac{2\pi}{N}nK}$$

i.e the inverse matrix is $\frac{1}{N}$ times the complex conjugate of the original (symmetric) matrix.

Note that the f[n] coefficients are complex. We can assume that the F[K] values are real.

Finally, the DFT pair:

$$F[K] = \sum_{n=0}^{N-1} f[n]e^{-j\frac{2\pi}{N}nK} \qquad \text{analysis}$$

$$f[n] = \frac{1}{N}\sum_{K=0}^{N-1} F[K]e^{+j\frac{2\pi}{N}nK} \qquad \text{synthesis}$$

Waveforms to be observed:

DFT:

Part-A:

i) $T_s \times \sin[20\pi nT_s]$ (Where N=50, f_s=100)

ii) $T_s \times e^{j2\pi\left(\frac{31}{3}\right)nT_s}$ (Where N=100, f_s=50)

iii) $3 \times 0.8^{|nT_s|}\cos[0.1\pi nT_s]$ (Where N=100, f_s=50)

IDFT:

Part-B:

i) $X[K] = \mathrm{tri}[2\pi(K-8)/N]$ (Where N=50, f_s=100)

ii) $X[K] = 1/(1 - 5 \times e^{-j12\pi\left(\frac{K}{N}\right)})$ (Where N=100, f_s=50)

MATLAB Program:

The student must write the code from .mfile and associated functions used in this section.

Results:

Using an example, the student must clearly mention input arguments used and corresponding output obtained (in the form of numerical values / graphs) in this section separately for **Part A** and **B**.

Comments on the results:

The student must describe in short the inferences drawn from the experiment and observations from the results obtained in his/ her own words.