EXPERIMENT NO: 3

Convolution and correlation for continuous and discrete time signals

Objective:

A. To demonstrate convolution in continuous time domain.

B. To demonstrate convolution in discrete time domain.

C. To demonstrate correlation in continuous time domain.

D. To demonstrate correlation in discrete time domain.

Theory:

Convolution is an operation between two signals, resulting in a third signal, can be used to determine a linear time invariant system's output from knowledge of the input and the impulse response.

Continuous time convolution is an operation on two continuous time signals defined by the integral.

$$f(t) * g(t) = \int_{-\infty}^{+\infty} f(\tau)g(t-\tau)d\tau$$

Convolution is a linear operator and, therefore, has a number of important properties including the commutative, associative, and distributive properties.

In continuous time domain,

The commutative property of the convolution operator states that

$$x(t) * h(t) = h(t) * x(t)$$

The associative property of the convolution operator states that

$$[x(t) * y(t)] * z(t) = x(t) * [y(t) * z(t)]$$

The distributive property of the convolution operator states that

$$x(t) * [y(t)] + z(t)] = x(t) * y(t) + x(t) * z(t)$$

Autocorrelation, also known as serial correlation, is the correlation of a signal with a delayed copy of itself as a function of delay. It is a mathematical tool for finding repeating patterns, such as the presence of a periodic signal obscured by noise.

Given a signal $f(t)$ then the continuous autocorrelation $R_{ff}(\tau)$ is most often defined as the continuous cross-correlation integral of $f(t)$ with itself, at lag τ.

$$R_{ff}(\tau) \ = \int_{-\infty}^{\infty} f(u+\tau)f(u)du$$

The autocorrelation is an even function, so that $R(\tau) = R(-\tau)$.

The continuous autocorrelation function reaches its peak at the origin, where it takes a real value, i.e. for any delay $\tau,\ R(-\tau)| \leq R(0)$.

In signal processing cross-correlation is a measure of similarity of two series as a function of the displacement of one relative to the other. This is also known as a “sliding dot product”.

For continuous functions f and g, the **cross-correlation** is defined as:

$$R_{fg}(\tau) \;=\; \int_{-\infty}^{+\infty} f(t)g(t+\tau)\,dt$$

Discrete time convolution is an operation on two discrete time signals defined by the integral:

$$f[n] * g[n] = \sum_{k=-\infty}^{k=+\infty} f[k]g[n-k]$$

In discrete time domain,

The commutative property of the convolution operator states that

$$x[n] * h[n] = h[n] * x[n]$$

The associative property of the convolution operator states that

$$[x[n] * y[n]] * z[n] = x[n] * [y[n] * z[n]]$$

The distributive property of the convolution operator states that

$$x[n] * [y[n]] + z[n]] = x[n] * y[n] + x[n] * z[n]$$

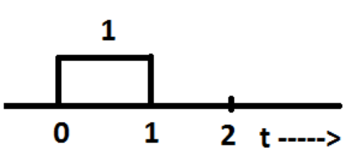
For discrete time signals, the **cross-correlation** is defined as:

$$R_{fg}(\tau) \;= \sum_{m=-\infty}^{m=+\infty} f[m]g[m+n]$$

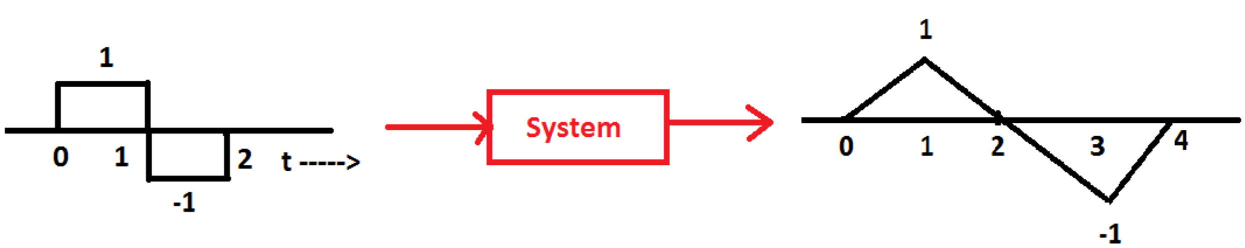
Waveforms to be observed:

Part A:

1. Determine the output $y(t)$ if the input and impulse response of a linear time invariant system are given as $x(t) = e^{-t}u(t)$ and $h(t) = \; e^{-2t}u(t)$.
2. Calculate $y(t) = x(t) * h(t)$ where $x(t) = \sin(\pi t)(u(t) - u(t-1))$ and $h(t) =$ $1.5\left(u(t) - u(t-1.5)\right) - u(t-2) + u(t-2.5)$.
3. What is the output of LTI system when the following input is given?



Given that:



Part B:

Perform convolution operation for following sets of input signals:

1. $x[n] = u[n] - u[n-8]$ $h[n] = sin(\frac{2\pi n}{8})(u[n] - u[n-8])$

2. $x[n] = (0.8)^n u[n]$ $h[n] = (0.3)^n u[n]$

3. $x[n] = (e)^{-n} u[n]$ $h[n] = (2)^{-n} u[n]$

Part C:

a. Perform cross correlation of following signals.

1. $x(t) = rect(t - 0.5)$ $h(t) = t\, rect(t - 1/2)$

2. $x(t) = e^{-t} u(t)$ $h(t) = e^{-2t} u(t)$

b. Find auto correlation of $x(t) = rect(t - 0.5)$

Part D:

a. Perform cross correlation for following sets of input signals:

1. $x[n] = u[n] - u[n-8]$ $h[n] = sin(\frac{2\pi n}{8})(u[n] - u[n-8])$

2. $x[n] = (0.2)^n u[n]$ $h[n] = (0.4)^n u[n]$

b. What is the auto correlation of $x[n] = nu[n]$?

MATLAB Program:

The student must write the code from .mfile and associated functions used in this section.

Results:

Using an example, the student must clearly mention input arguments used and corresponding output obtained (in the form of numerical values / graphs) in this section and compare results with inbuilt MATLAB commands.

Comments on the results:

The student must describe in short the inferences drawn from the experiment and observations from the results obtained in his/ her own words.