PG4200: Algorithms And Data Structures

Lesson 09: Graphs

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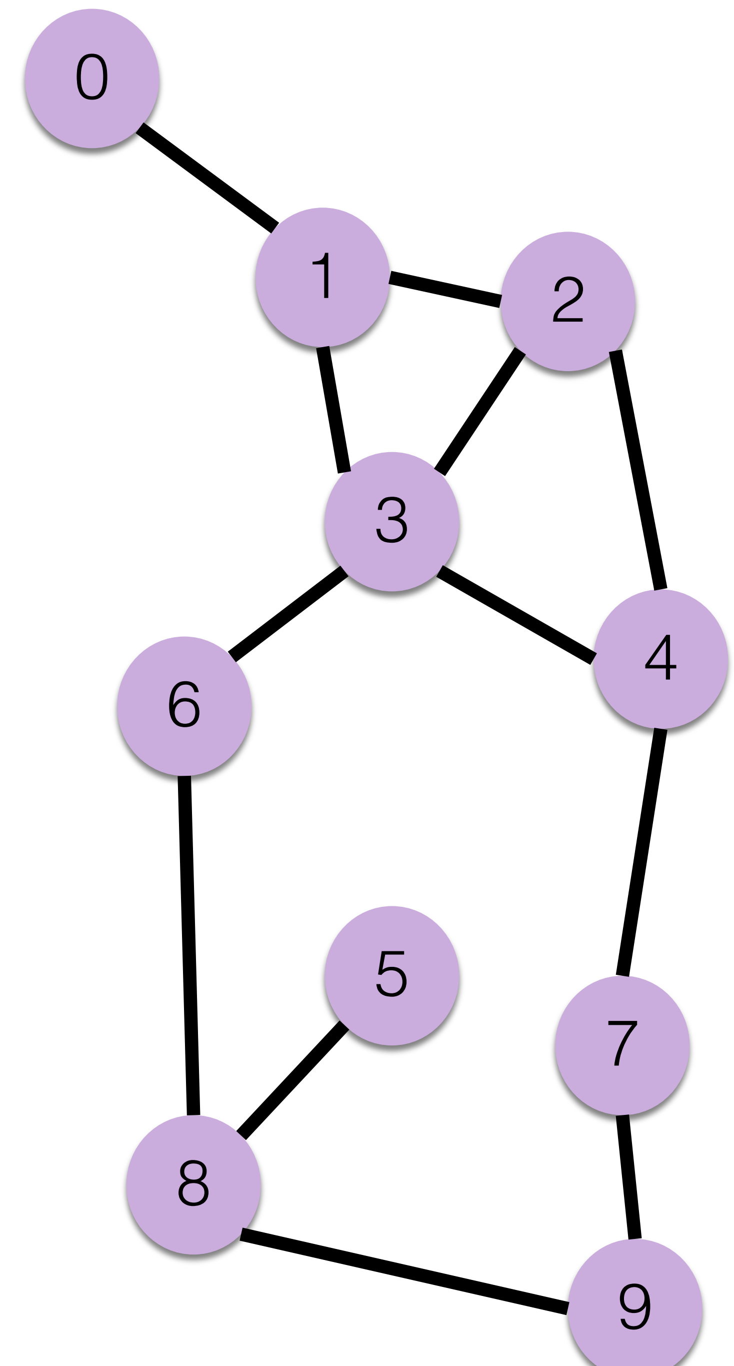
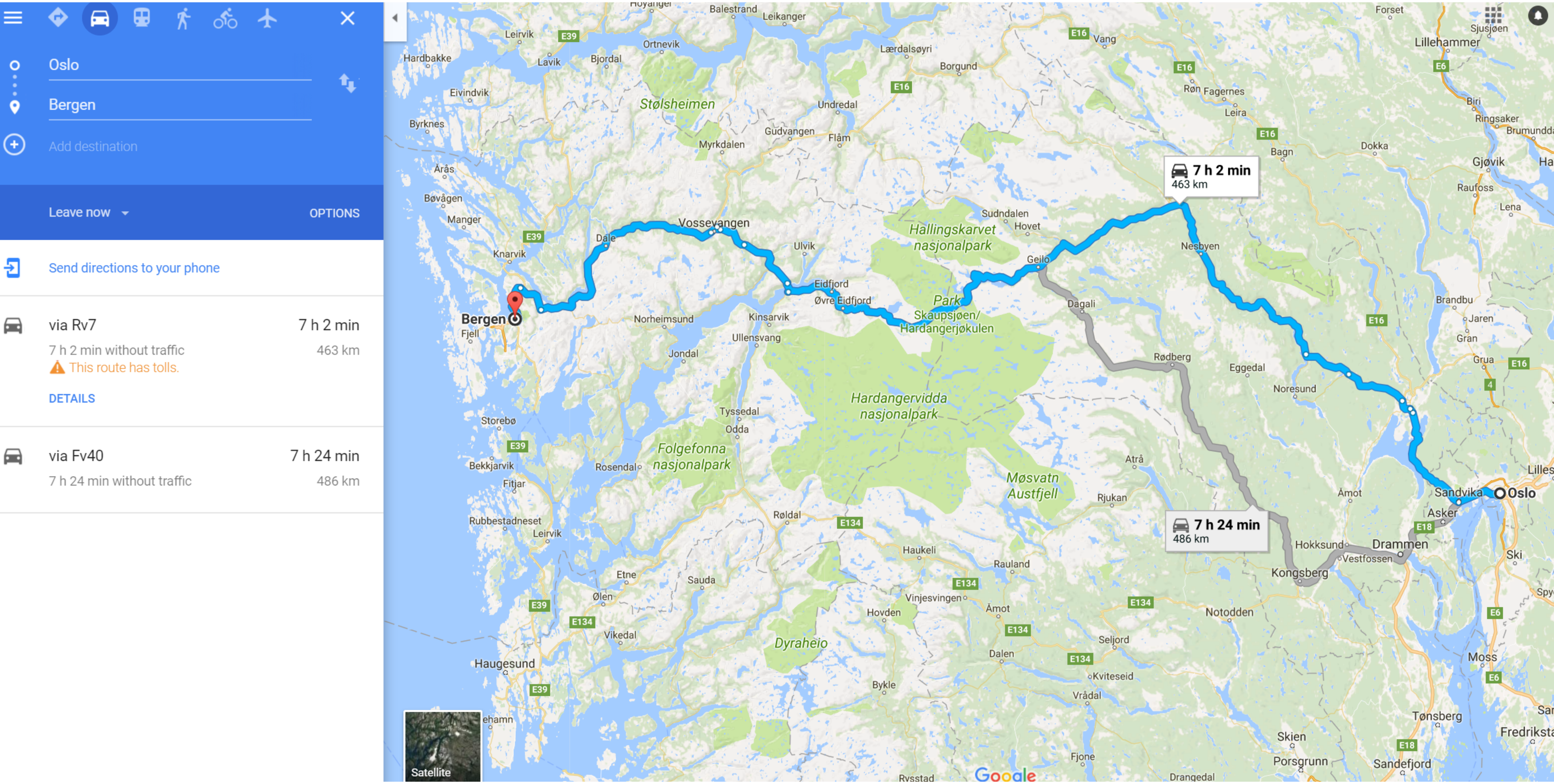
Graphs

- A set of **vertices** connected by **edges**
- **Directed** or **Undirected** graphs
    - If edge from X to Y, implies edge from Y to X?
- Many different problems can be represented with graphs
- Many different algorithms specialized for graphs
- Here just having a very high level view

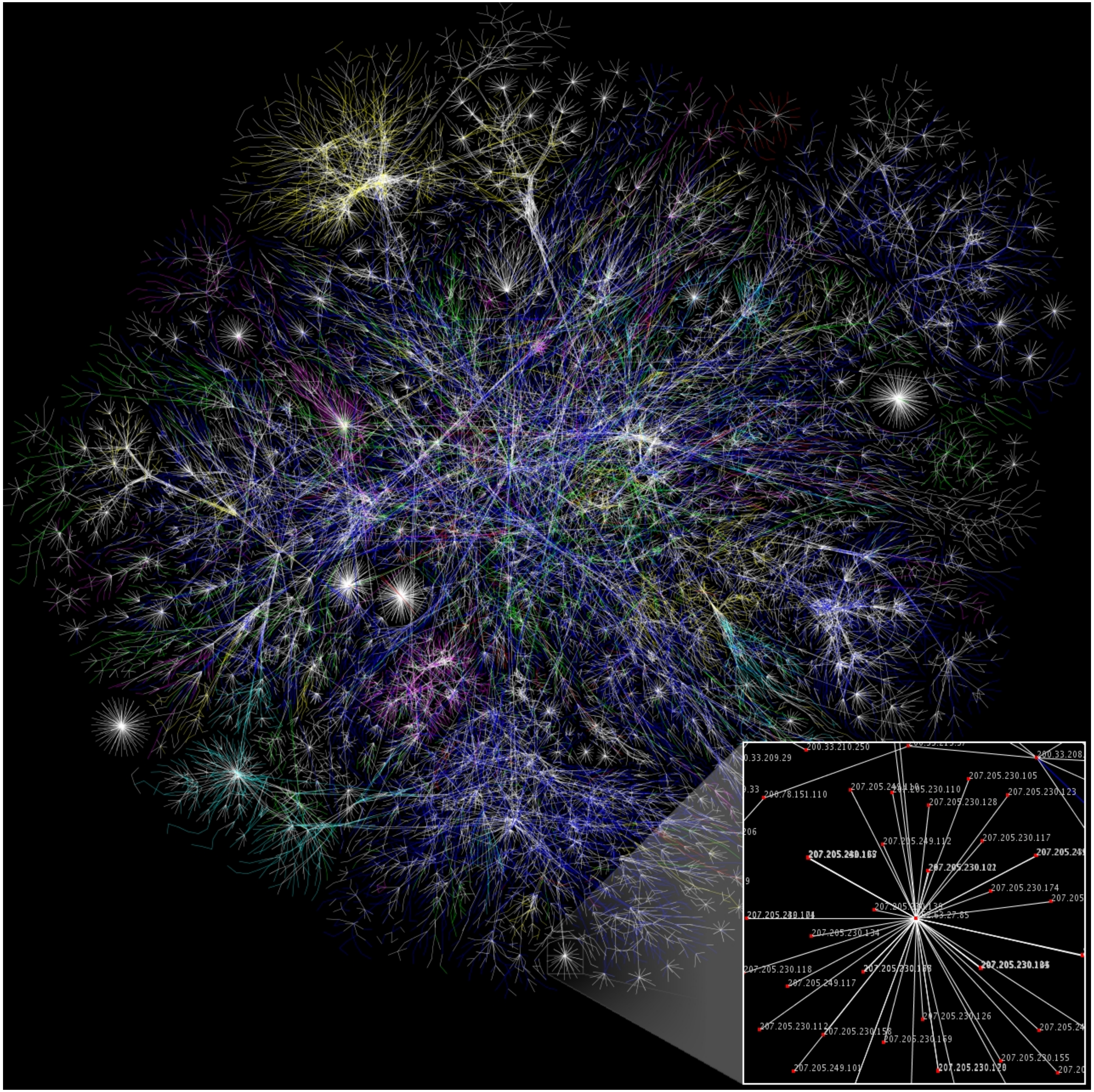


Oslo Metro
1
Frognerseteren
Voksenkollen
Lillevann
Skogen
Voksenlia
Holmenkollen
Besserud
Midtskogen
Skådalen
Vettakollen
Gulleråsen
Gråkammen
Slemdal
Ris
Gaustad
Vinderen
Steinerud
Frøen
2
Østerås
Lijordet
Eiksmarka
Ekraveien
Røa
Hovseter
Holmen
Makrellbekken
3
Sognsvann
Kringsjå
Holstein
Østhorn
Tåsen
Berg
Ullevål stadion
Forskningsparken
Blindern
4
5
6
Nydalen
Storo
Sinsen
5
Vestli
Stovner
Rommen
Romsås
Grorud
Ammerud
Kalbakken
Rødtveit
Veitveit
Linderud
Vollebekk
Risløkka
Økern
Hasle
Carl Berners plass
1
2
Ellingsrudåsen
Furuset
Lindeberg
Trosterud
Haugerud
Tveita
Ensjø
Helsfyr
Brynseng
Hellerud
Smestad
Borgen
Montebello
Ullernåsen
Åsjordet
Bjørnsletta
Jar
6
Ringstabekk
Bekkestua
Gjønnes
Haslum
Avløs
Valler
Gjettum
Hauger
Kolsås
Majorstuen
Nationaltheatret
Stortinget
Jernbanetorget
Grønland
Tøyen
Høyenhall
Manglerud
Ryen
Brattlikollen
Karlsrud
Lambertseter
Munkelia
Bergkrystallen
4
Godila
Skøyenåsen
Oppsal
Ulsrud
Bøler
Bogerud
Skullerud
Mortensrud
3
1
Line and terminus
Station
Closed for renovation
Tansfer to tram
Tansfer to train


Oslo
Bergen
Add destination
Leave now
OPTIONS
Send directions to your phone
via Rv7
7 h 2 min
7 h 2 min without traffic
463 km
This route has tolls.
DETAILS
via Fv40
7 h 24 min
7 h 24 min without traffic
486 km
7 h 2 min
463 km
7 h 24 min
486 km
Bergen
Oslo
Vossevangen
Dale
Eidfjord
Øvre Eidfjord
Geilo
Nesbyen
Sandvika
Asker
Drammen
Kongsberg
Hokksund
Vestfossen
Dagali
Rødberg
Hardangervidda nasjonalpark
Hallingskarvet nasjonalpark
Folgefonna nasjonalpark
Stølsheimen
Haugesund
Lillehammer
Gjøvik
Tønsberg
Fredrikstad
Sandefjord
Porsgrunn
Skien
Notodden
Satellite

Friends in a social network

powered by TouchGraph

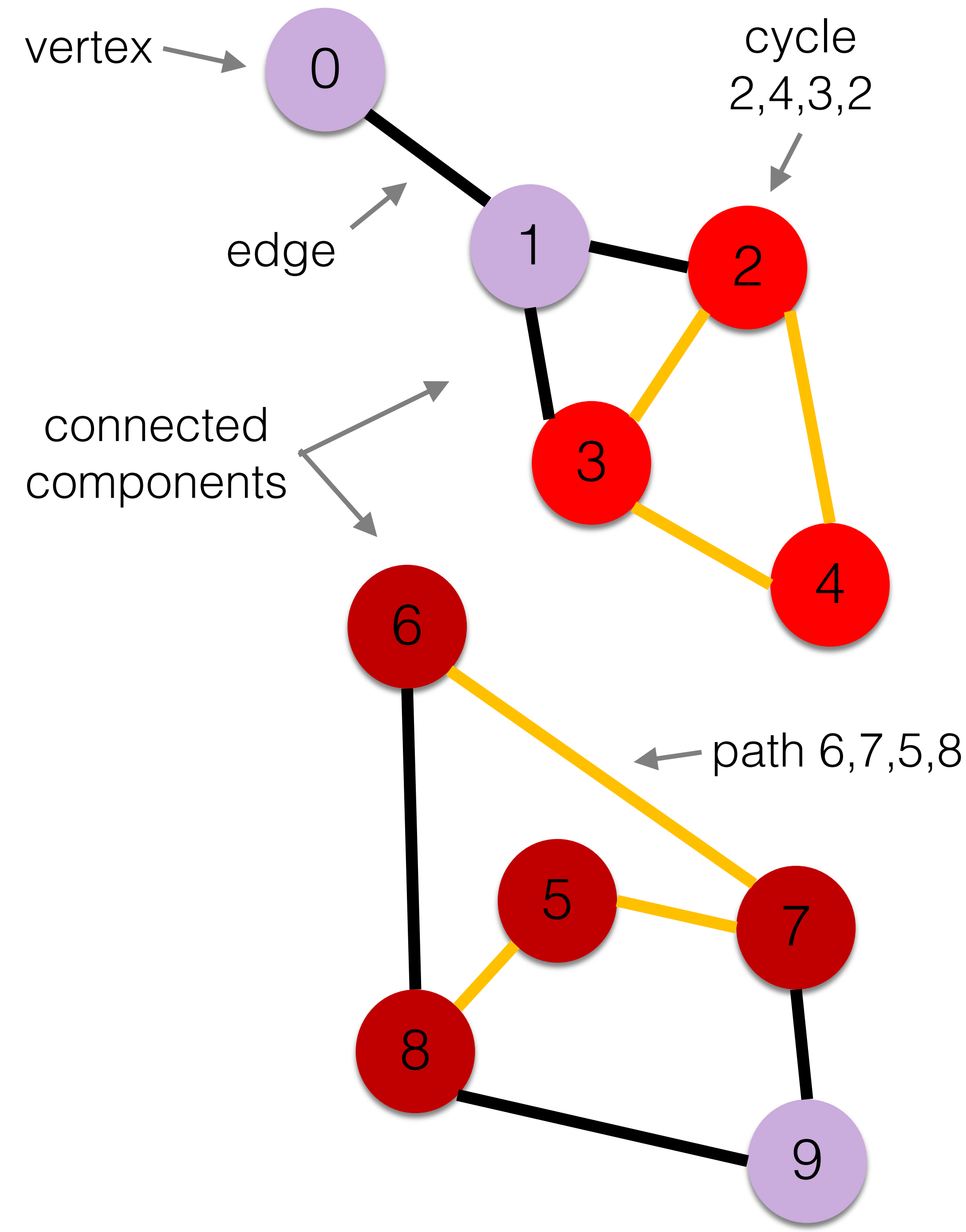


Machines connected on internet

Note: the picture only show a tiny subset of the whole internet graph…

Terminology

- **Vertex**: a node, for which can use a label to identify it
- **Edge**: connection between 2 nodes
- **Path**: a sequence of connected nodes
- **Cycle**: a path starting and ending on the same node
- Note: in a graph, not all nodes are necessarily connected



How to represent a graph?

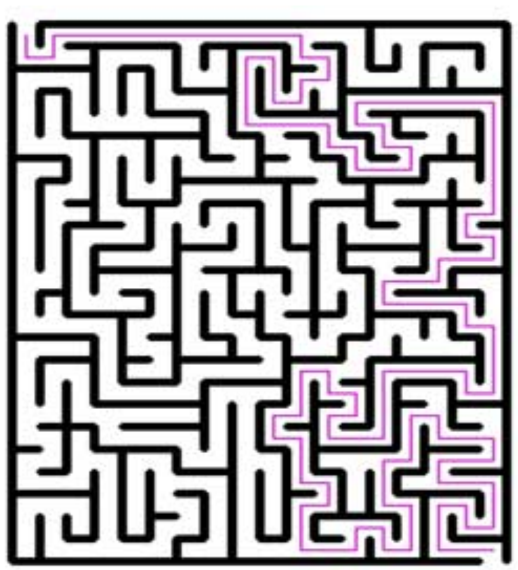
- Vertices can be object with state
  - Eg, name of station, city, friend, IP address
- “Map” from vertex X (key) to a collection of vertices (value) reachable from X
- Note: in this way, do not need objects to explicitly represent edges

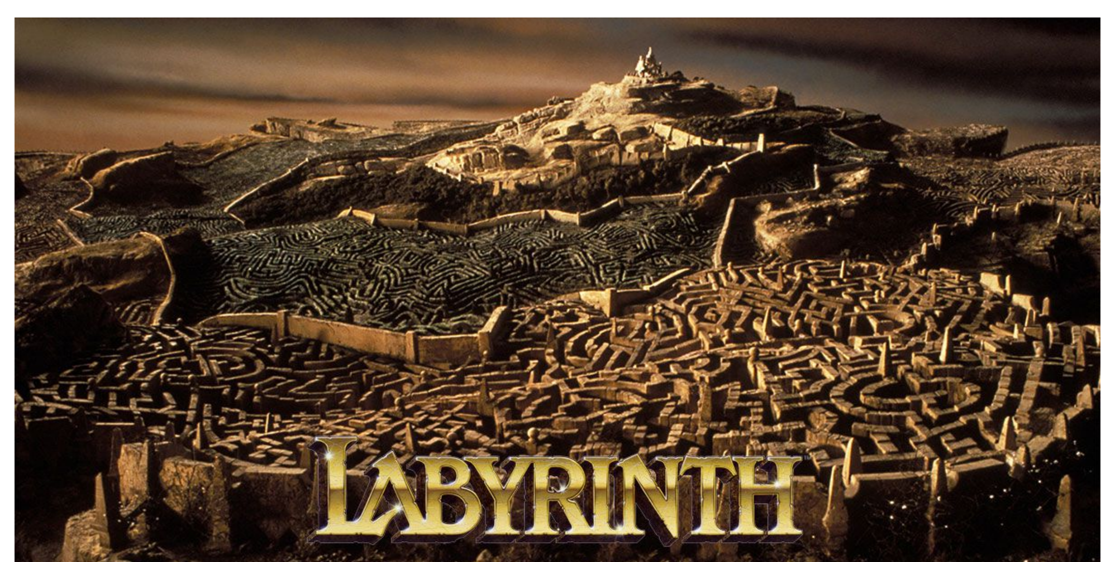
Operations

- Given an existing graph, there can be different operations we might need to do
- Path Finding: given two vertices (eg, 2 cities), find a path connecting them, and avoiding cycles

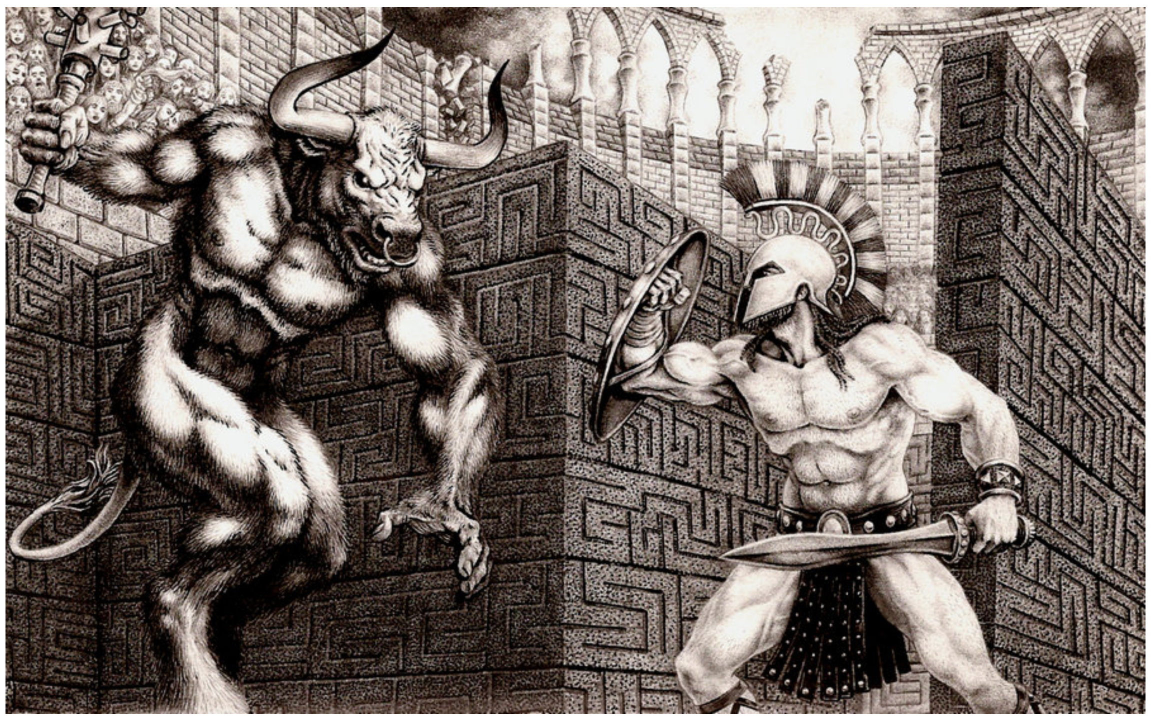
Maze / Labyrinth

- Mazes can be represented with a graph
- Vertices: intersections
- Edges: passages between two intersections
- Find path from starting vertex to the vertex of the exit





Thesus vs. the Minotaur



- Thesus slays the Minotaur at the center of the labyrinth

- Used a thread to trace back the exit

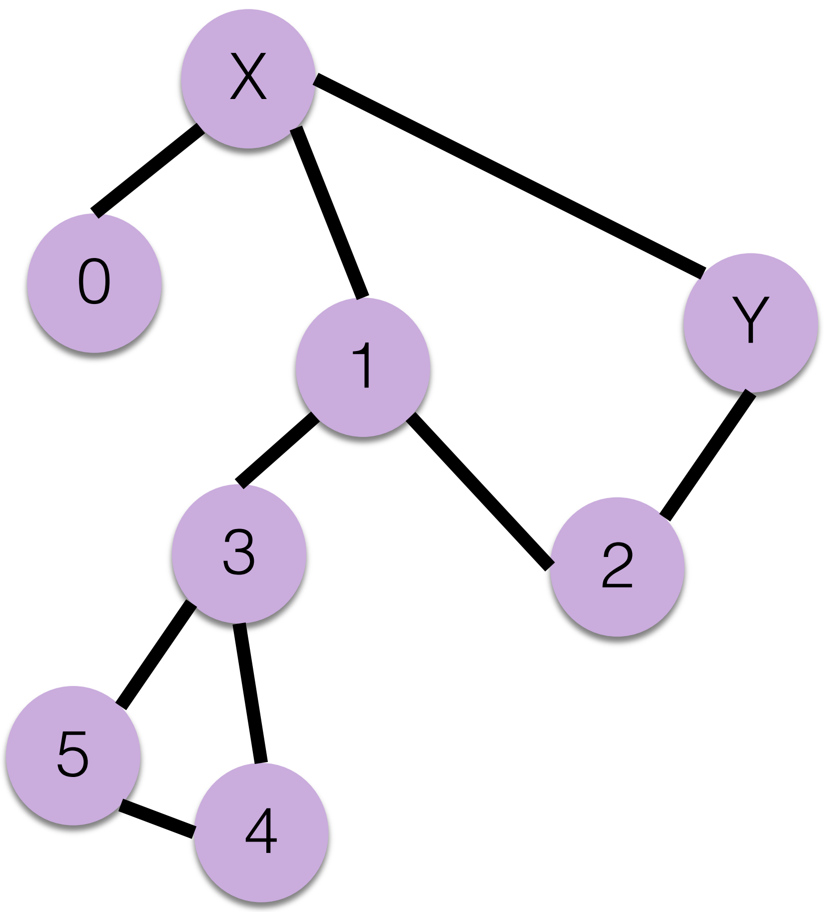
Trémaux's Algorithm

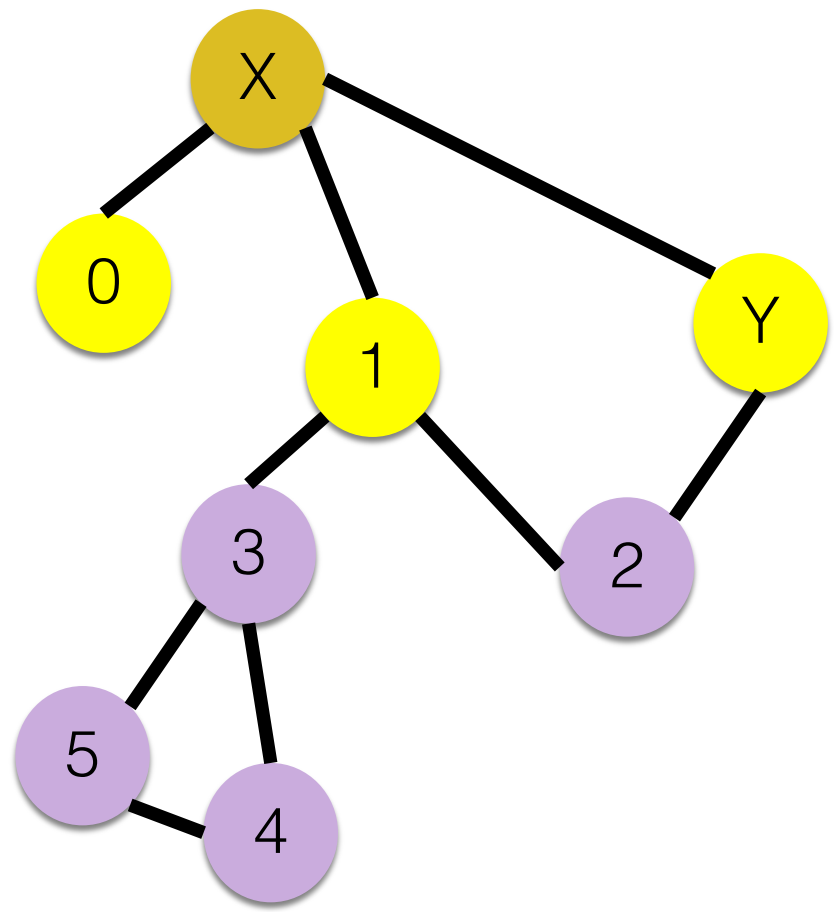
- Charles Pierre Trémaux, 19th-century
- Method that guarantees to find an exit in a maze
  - Note: talking about actual mazes, not computers…
- Trémaux's Algorithm is an instance of what we now call *Depth-First Search* in graphs

Depth-First Search (DFS)

- Try to find a path from vertex X to Y
- Mark current vertex as visited
- *Recursively* look at each connected vertex from the current
  - But skip already marked vertices (eg, by using a set)
- Use stack to represent path from X toward current vertex
  - Push when recursively evaluate a connected vertex
  - Pop when backtrack out of a recursive call

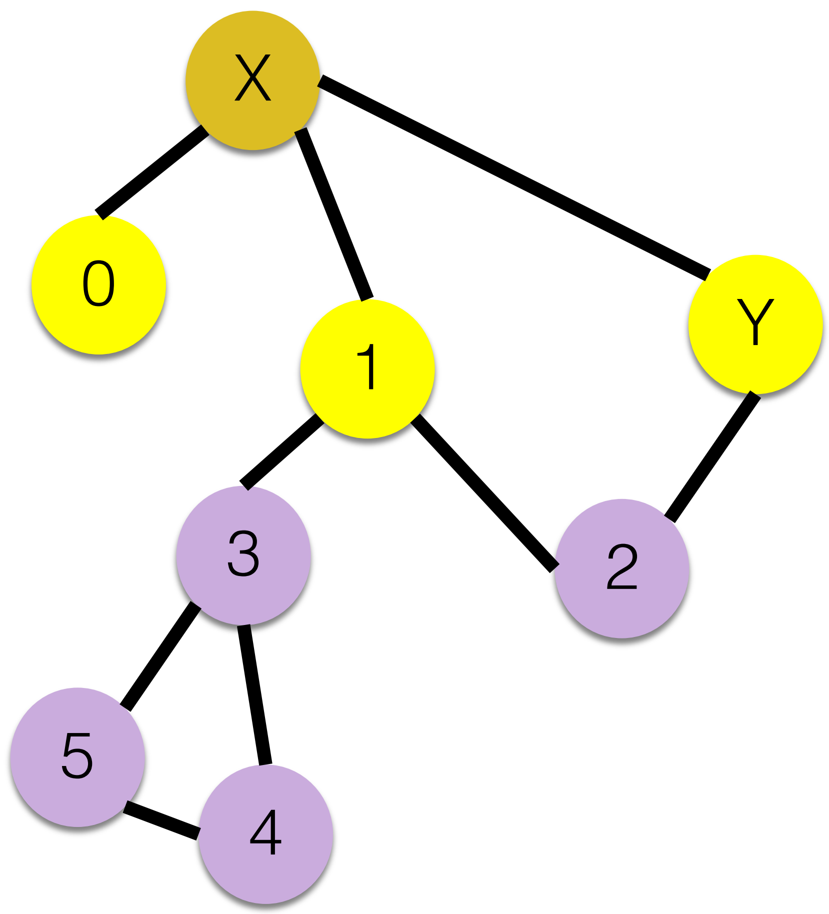
Example

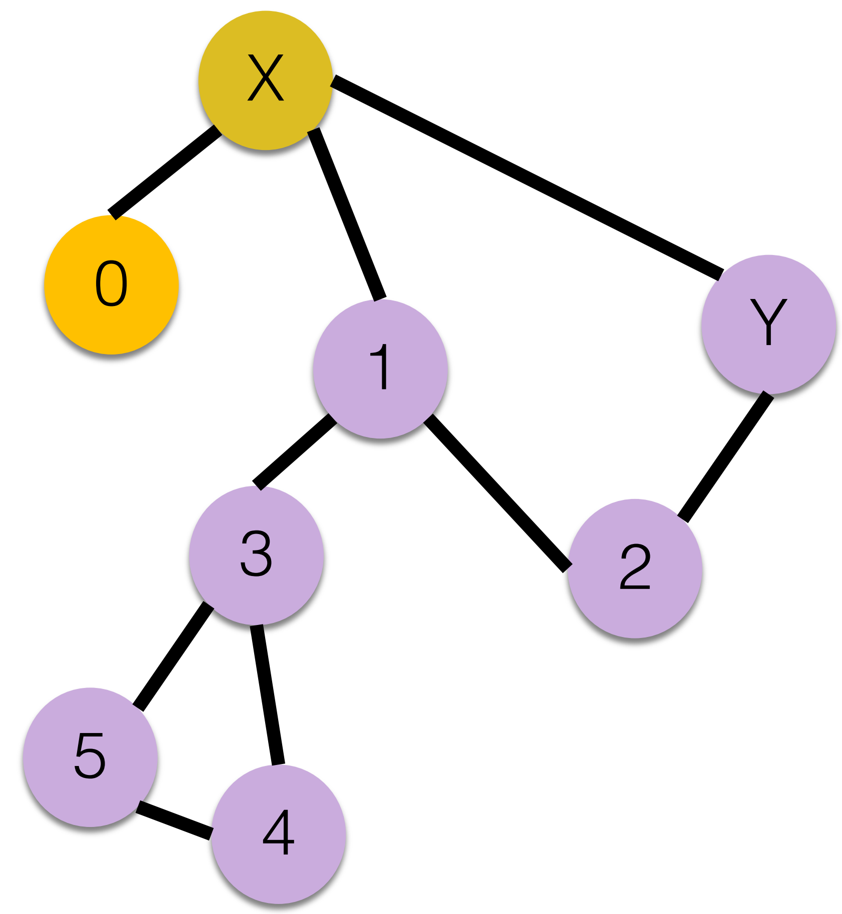




Visited: X
Stack: X
Connected: 0, 1, Y

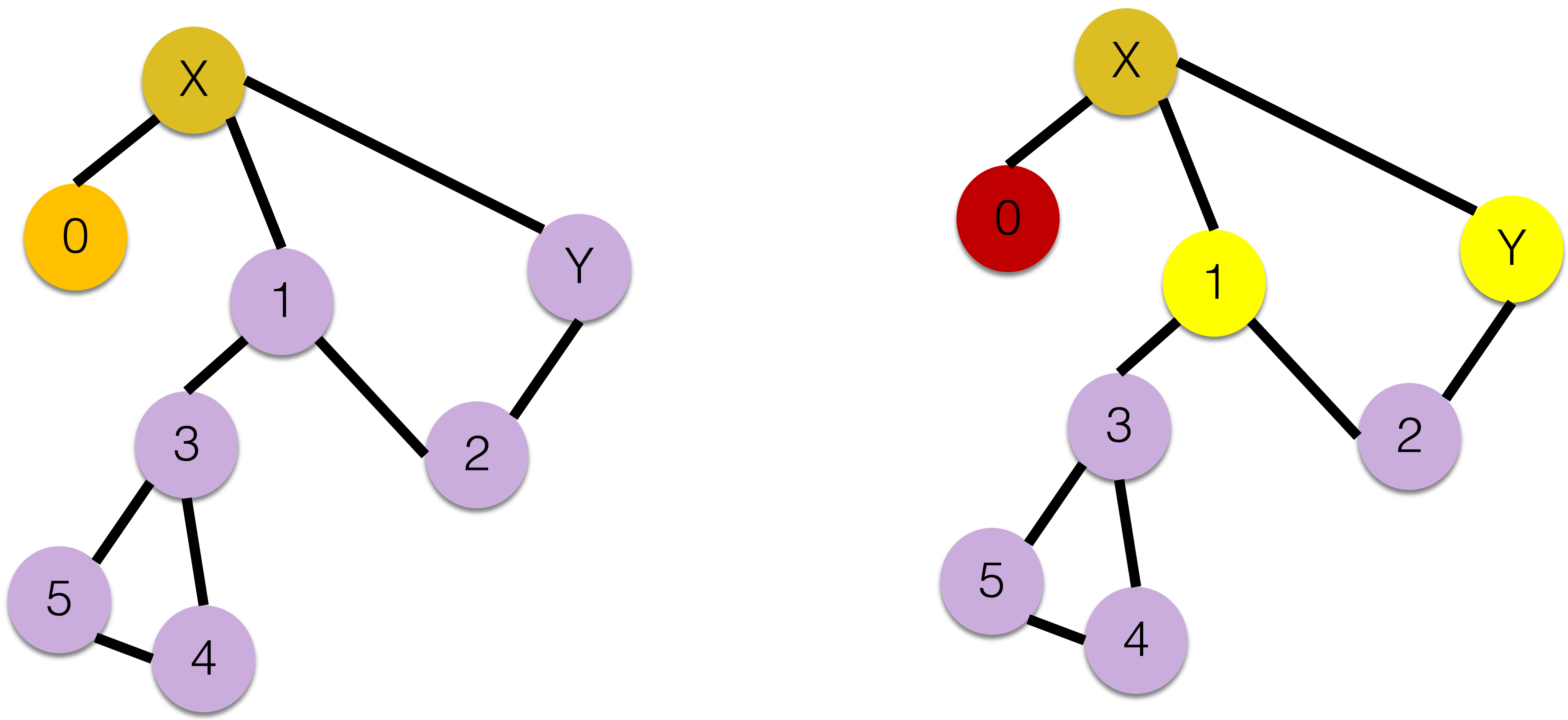
Recursion on 0





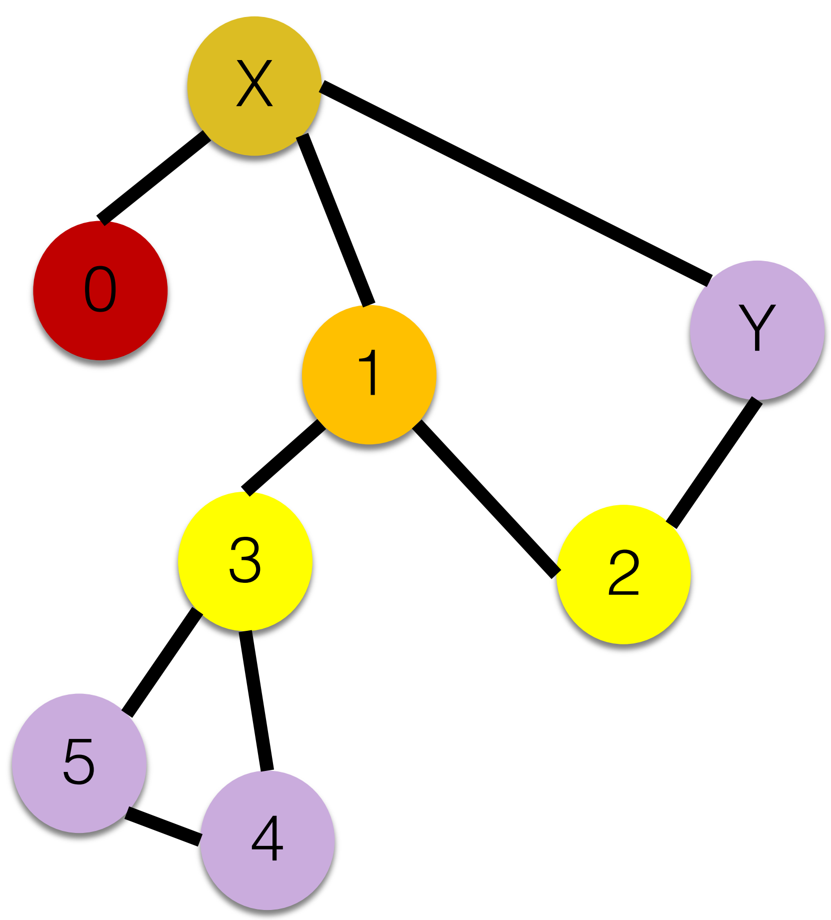
Visited: X, 0
Stack: X, 0
Connected: (X)

Backtrack to X

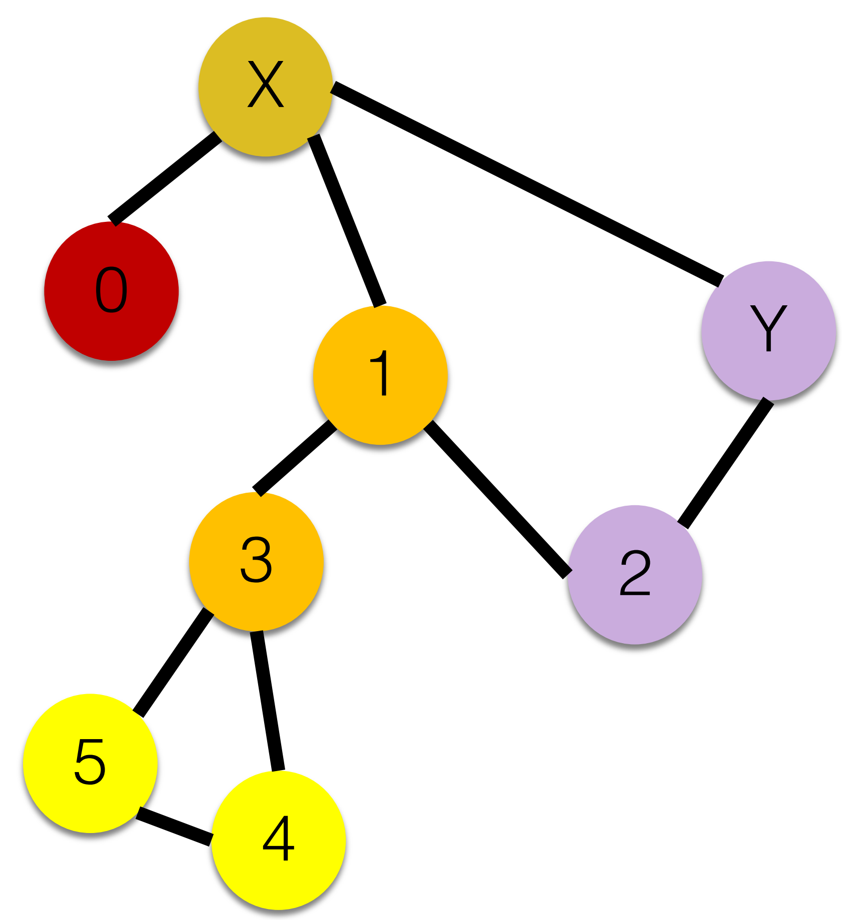


Visited: X, 0
Stack: X
Connected: (0), 1, Y

Evaluating 1 and then 3

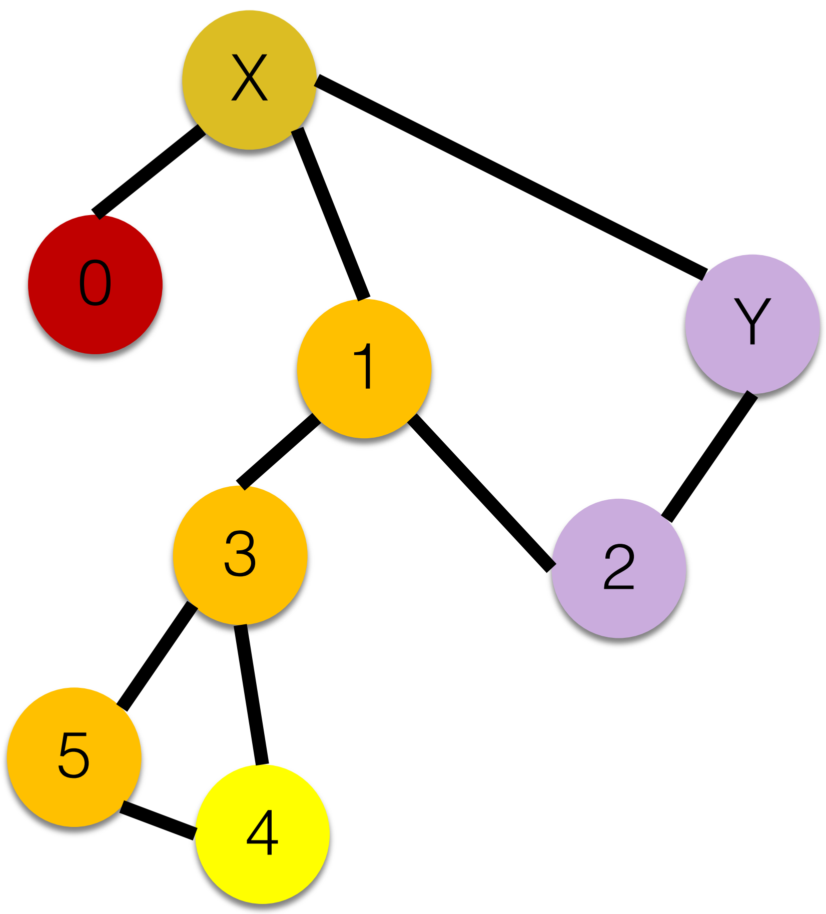


Visited: X, 0, 1
Stack: X, 1
Connected: (X),3, 2

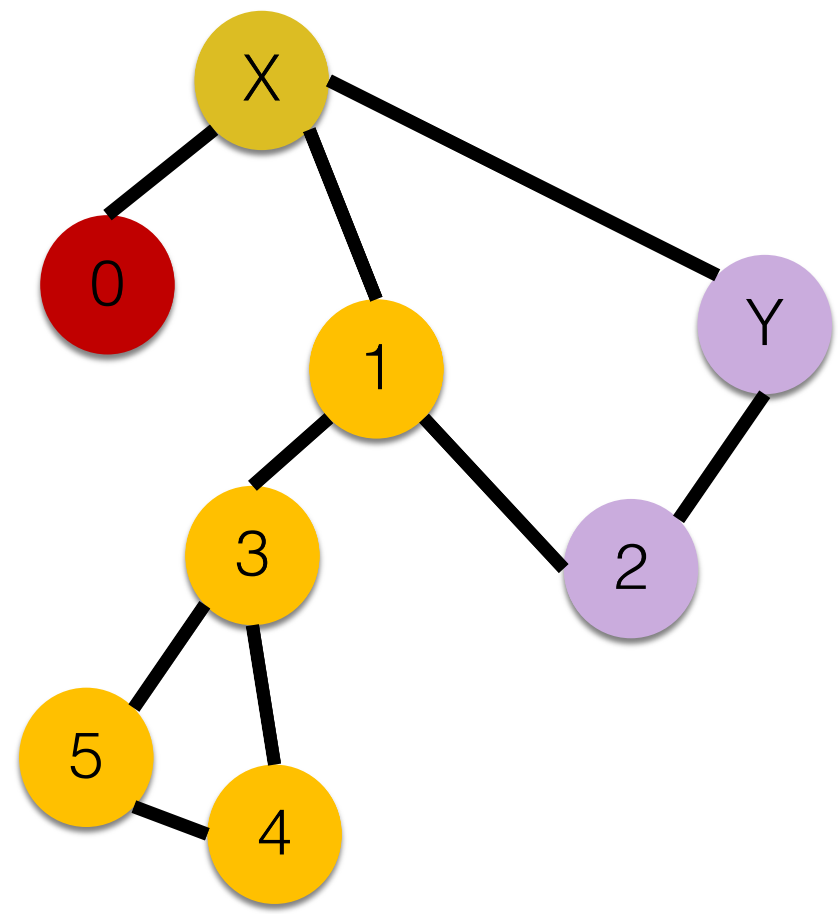


Visited: X, 0, 1, 3
Stack: X, 1, 3
Connected: (1), 4, 5

Evaluating 5 and then 4

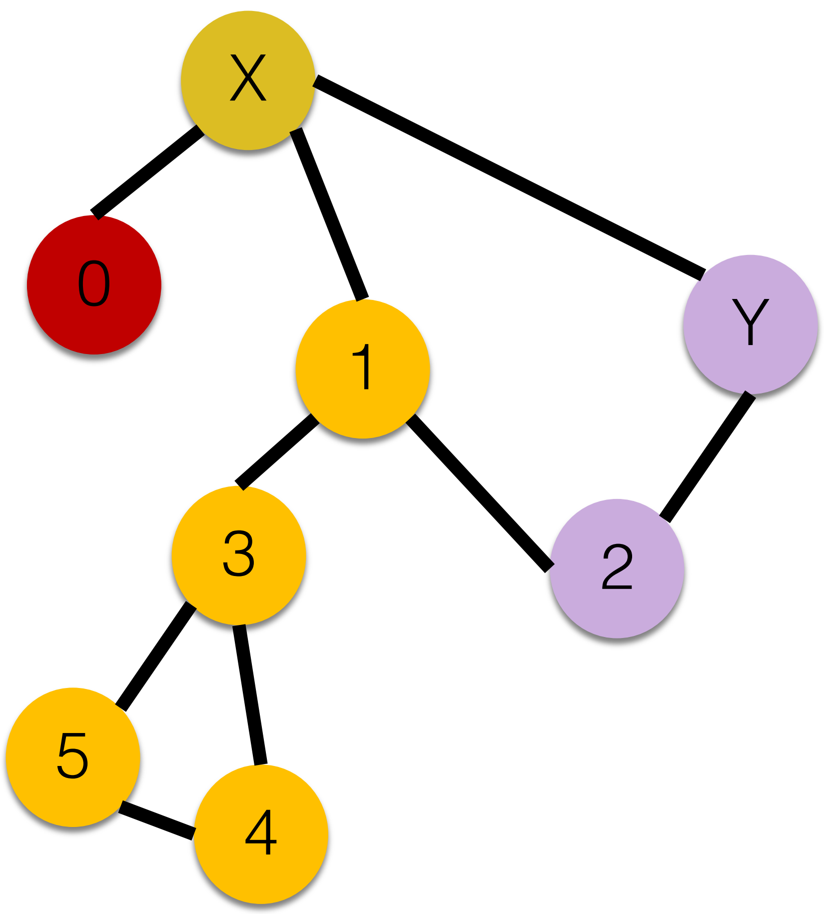


Visited: X, 0, 1, 3, 5
Stack: X, 1, 3, 5
Connected: (3), 4

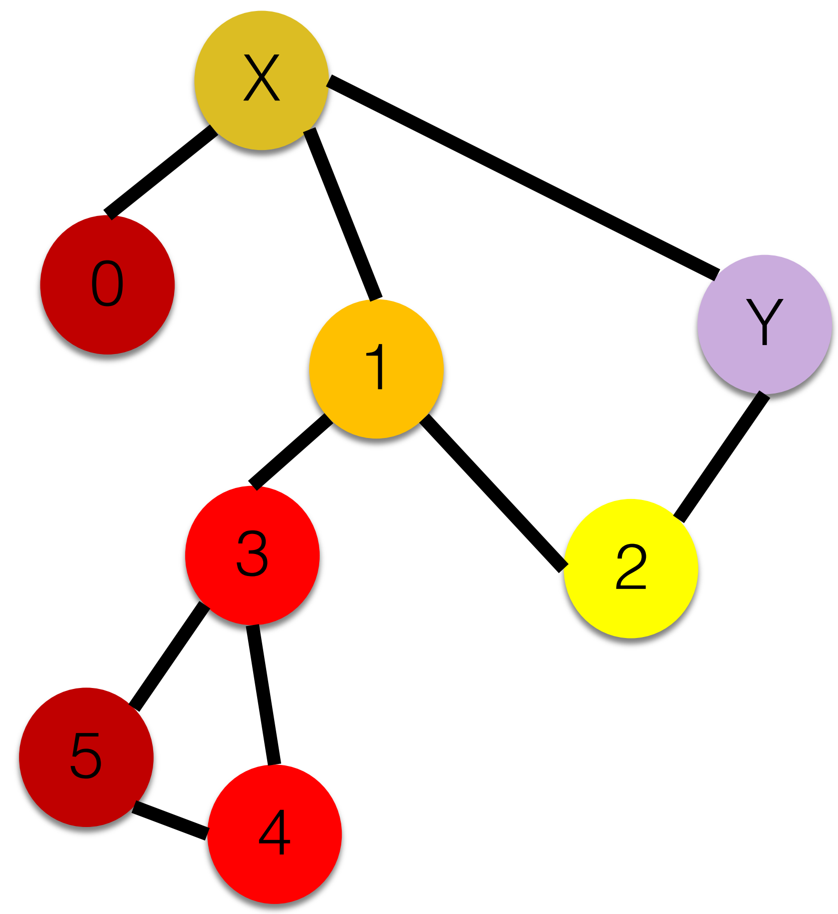


Visited: X, 0, 1, 3, 5, 4
Stack: X, 1, 3, 5, 4
Connected: (3), (5)

Backtracking Till 1

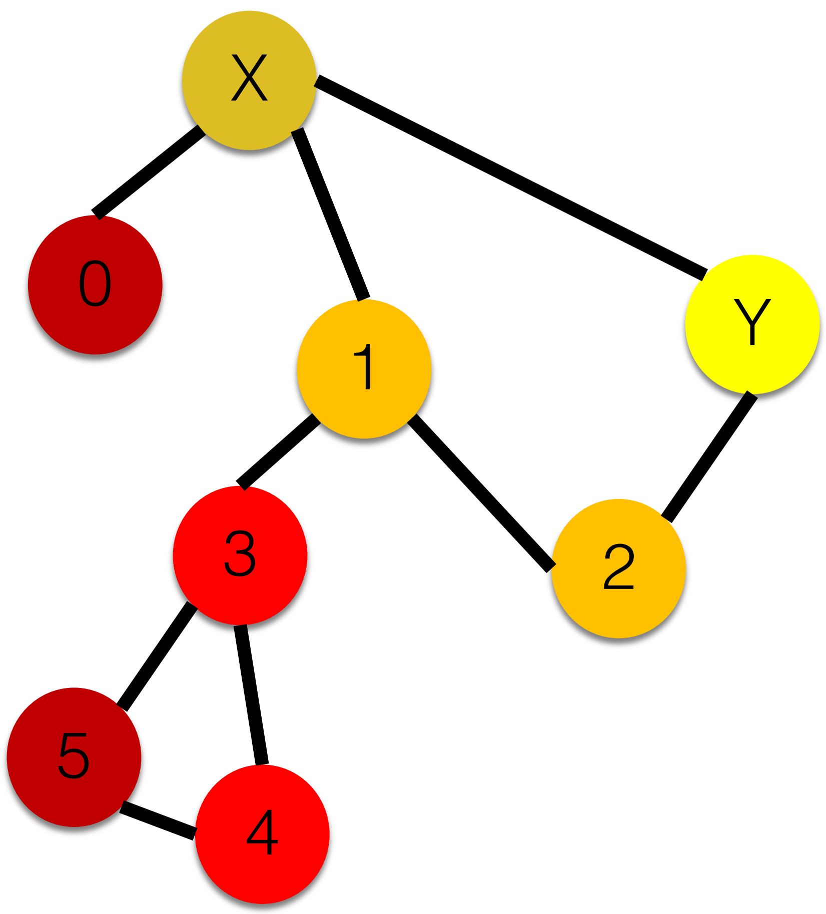


Visited: X, 0, 1, **3, 5**, 4
Stack: X, 1, 3, 5, 4
Connected: **(3), (5)**

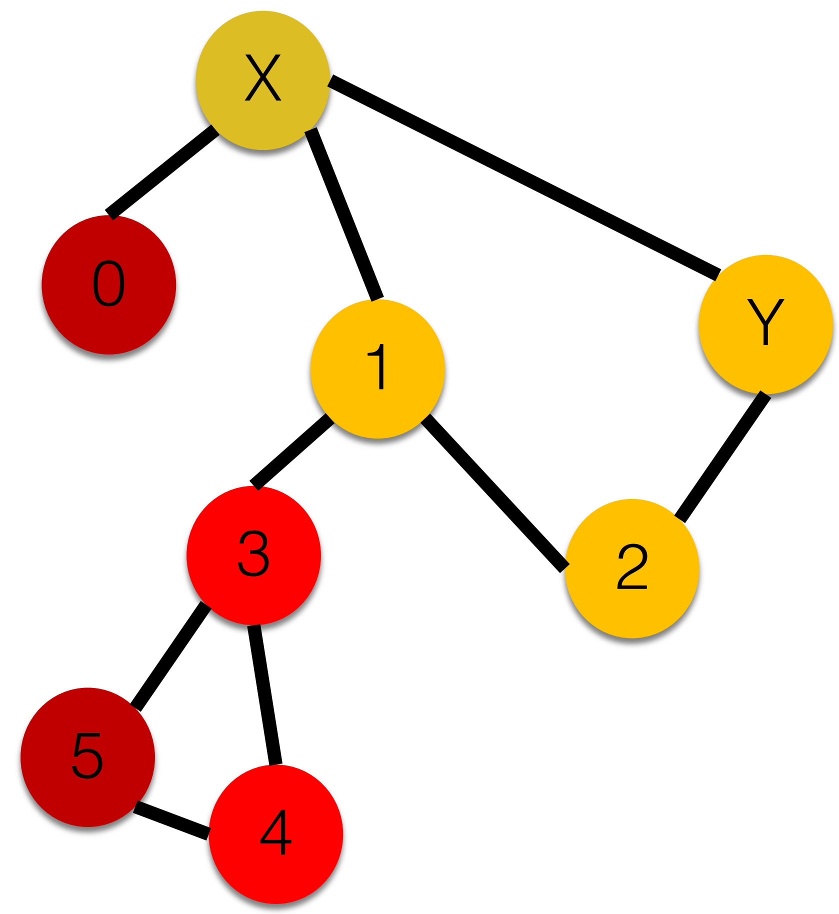


Visited: X, 0, 1, 3, 5, 4
Stack: X, 1
Connected: (X), (3), **2**

Evaluating 2 and 1

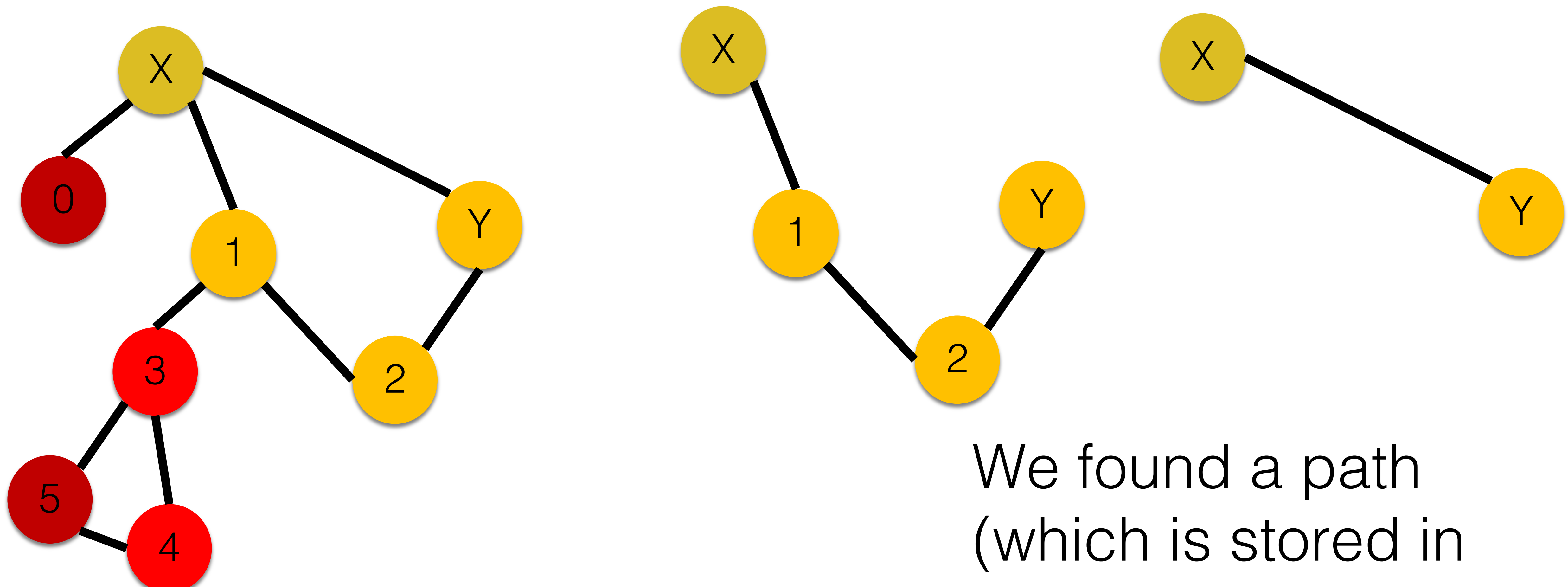


Visited: X, 0, 1, 3, 5, 4, 2
Stack: X, 1, 2
Connected: (1), Y



Visited: X, 0, 1, 3, 5, 4, 2, Y
Stack: X, 1, 2, Y
Connected: (X), (2)

Final Path X, 1, 2, Y



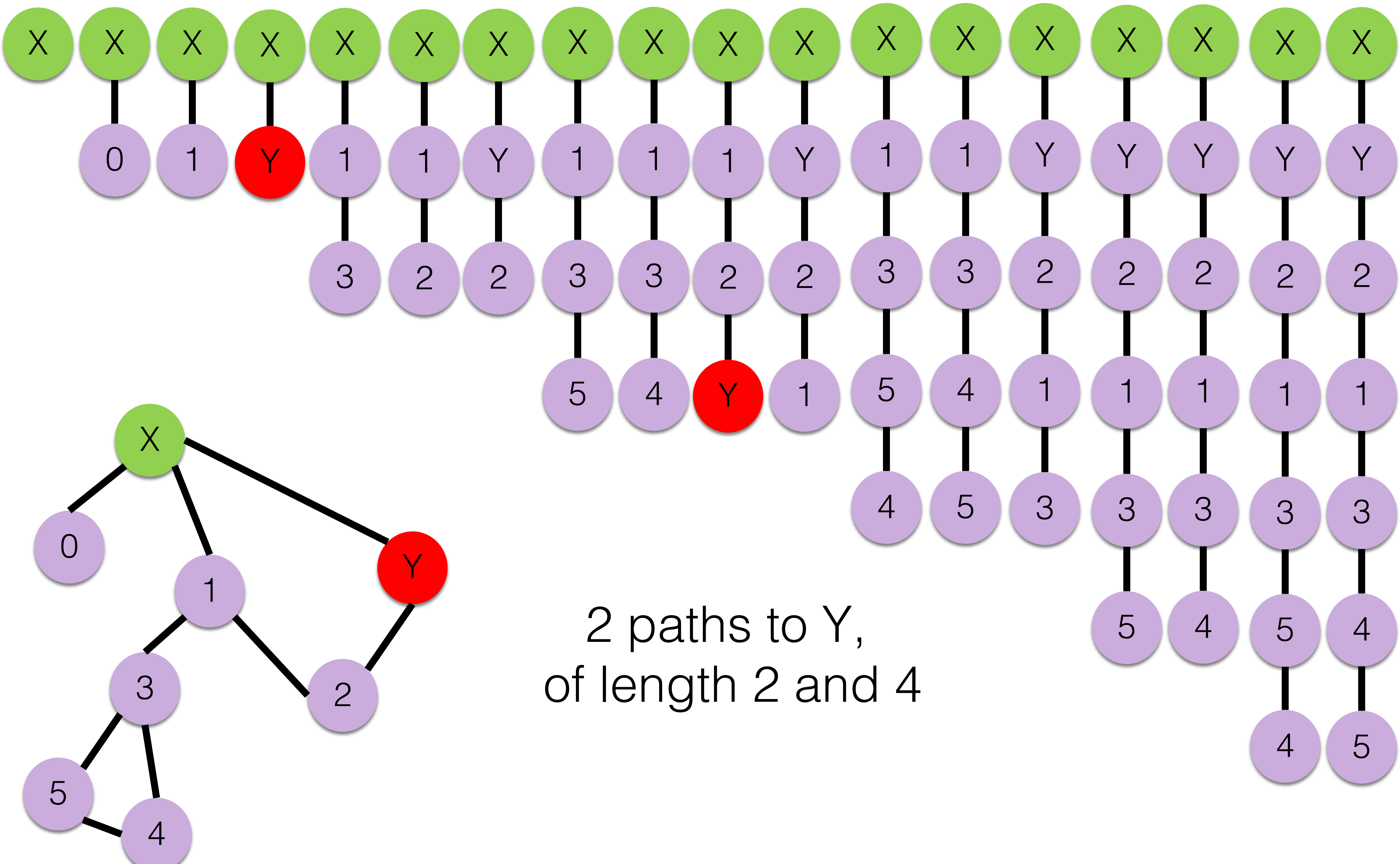
Visited: X, 0, 1, 3, 5, 4, 2, Y
Stack: X, 1, 2, Y
Connected: (X), (2)

We found a path (which is stored in the stack), but it is not necessarily the shortest (which would be X-Y)

Breadth-First Search (BFS)

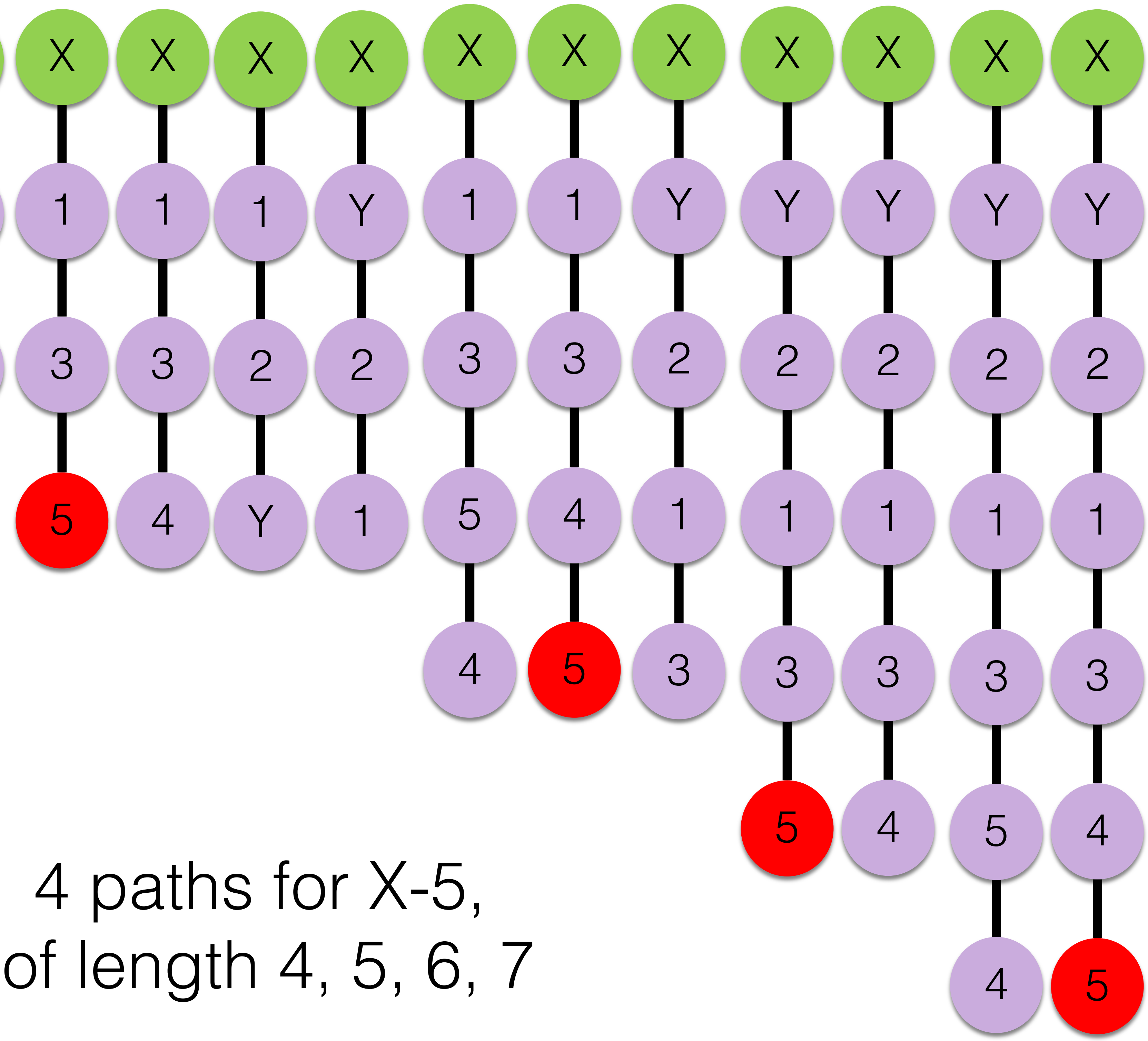
- DFS no guarantee to find shortest path, whereas BFS does
- From starting point X, look at all paths of length 1, then all paths of length 2, then 3, … then N, until found Y or visited whole graph
- Considering paths without cycles

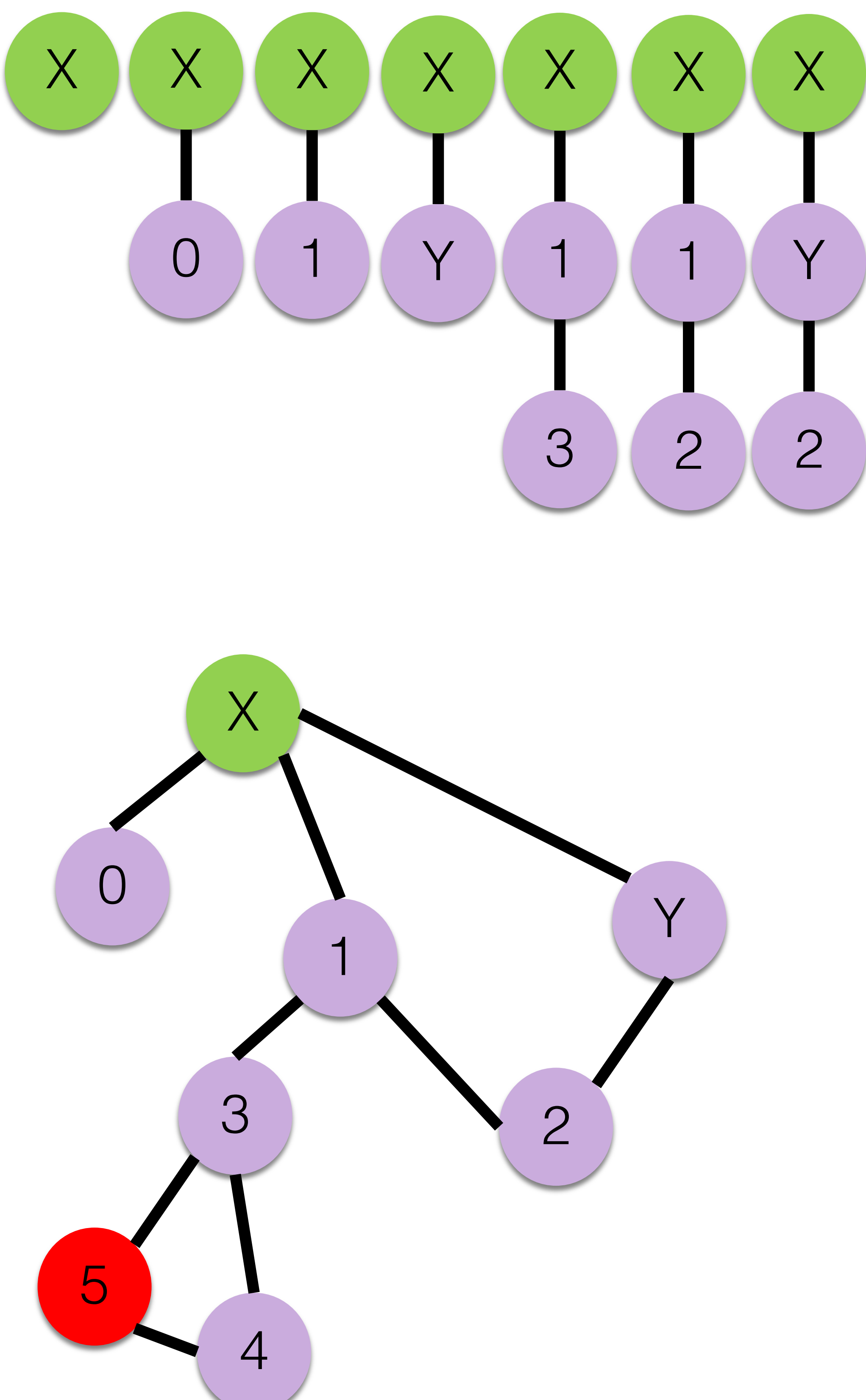
Paths Starting From X



2 paths to Y,
of length 2 and 4

From X to 5



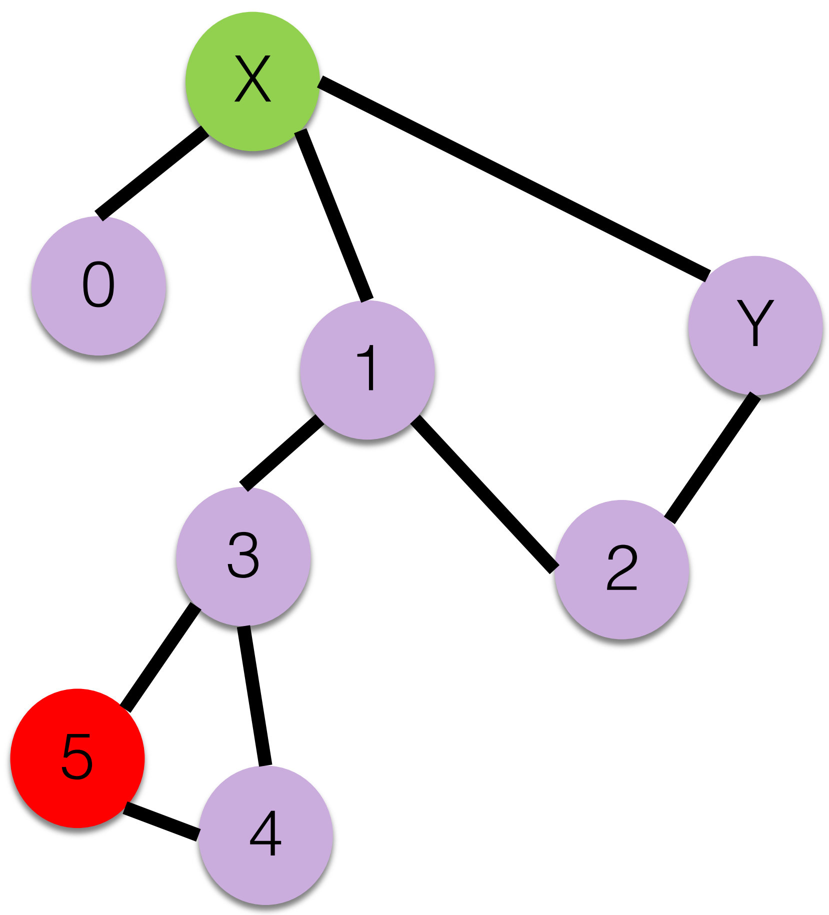


4 paths for X-5,
of length 4, 5, 6, 7

BFS Details

- Need special algorithm to keep track and visit all paths of length N in increasing order
- BFS: use a *queue* of yet to visit vertices
- Pull vertex from queue, add connected vertices to back, if not already visited
- Keep track of which pulled vertex X added a connected vertex Y

Example

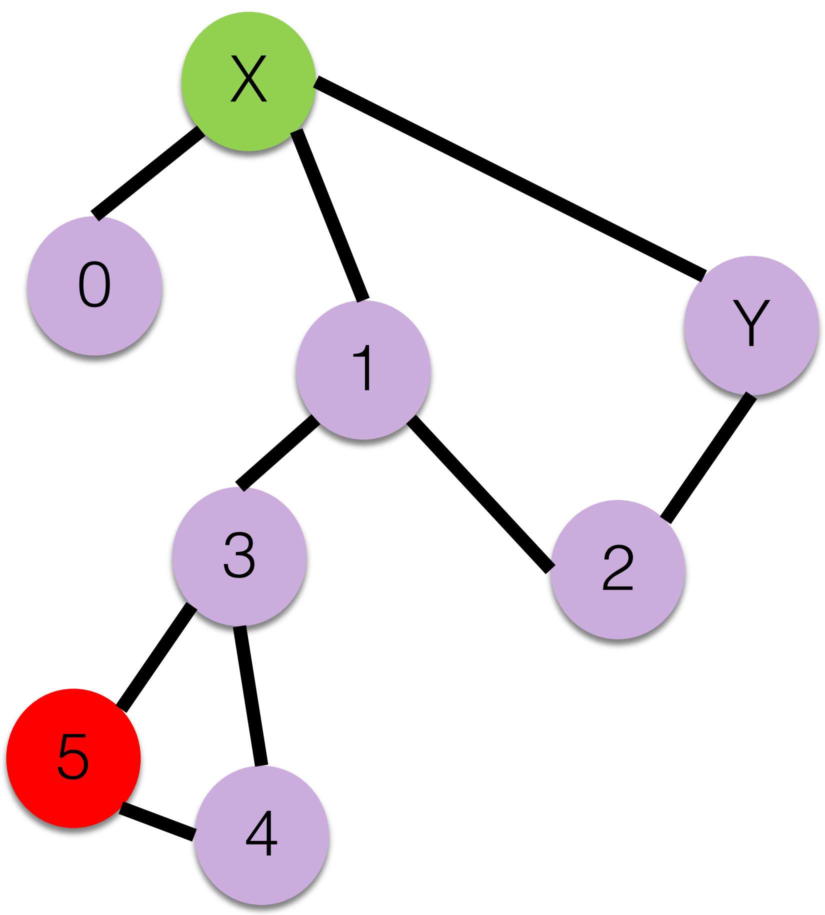


Queue: X

Map:

X	root
Y	null
0	null
1	null
2	null
3	null
4	null
5	null

Cont.



Queue:

0	1	Y

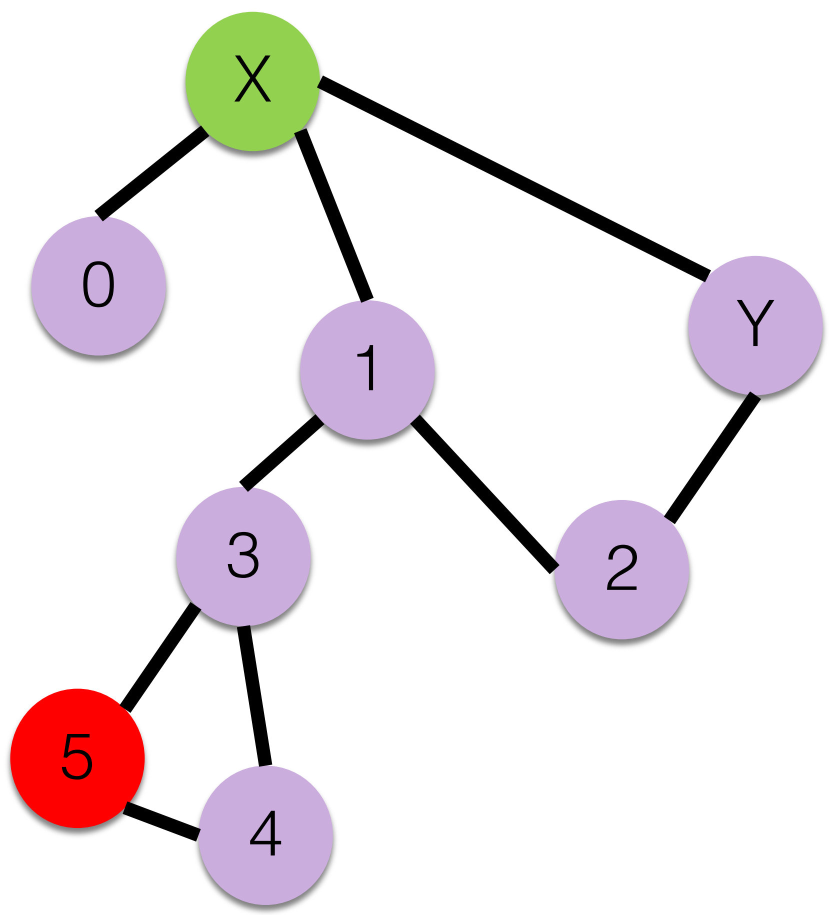
Map:

X	root
Y	X
0	X
1	X
2	null
3	null
4	null
5	null

Pulled X, added 0, 1, and Y.

Those represent all paths of length 2

Cont.



Queue:

Y	3	2

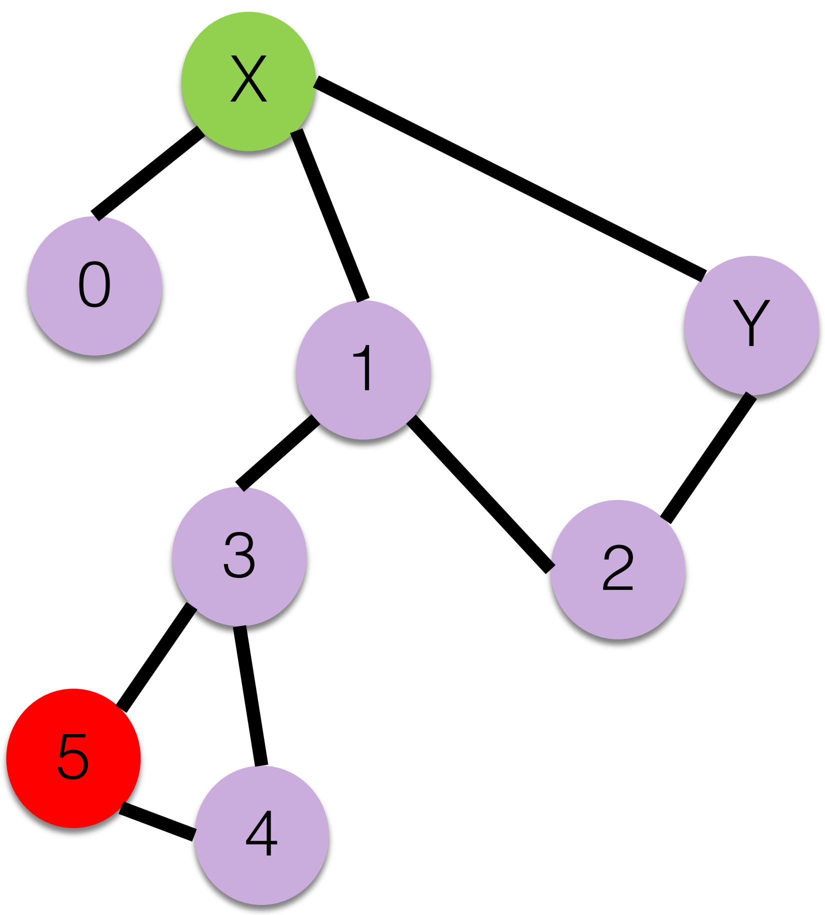
Map:

X	root
Y	X
0	X
1	X
2	1
3	1
4	null
5	null

Pulling 0 has no effect, as not adding X back

Pulling 1 results in adding 3 and 2

Cont.



Queue:

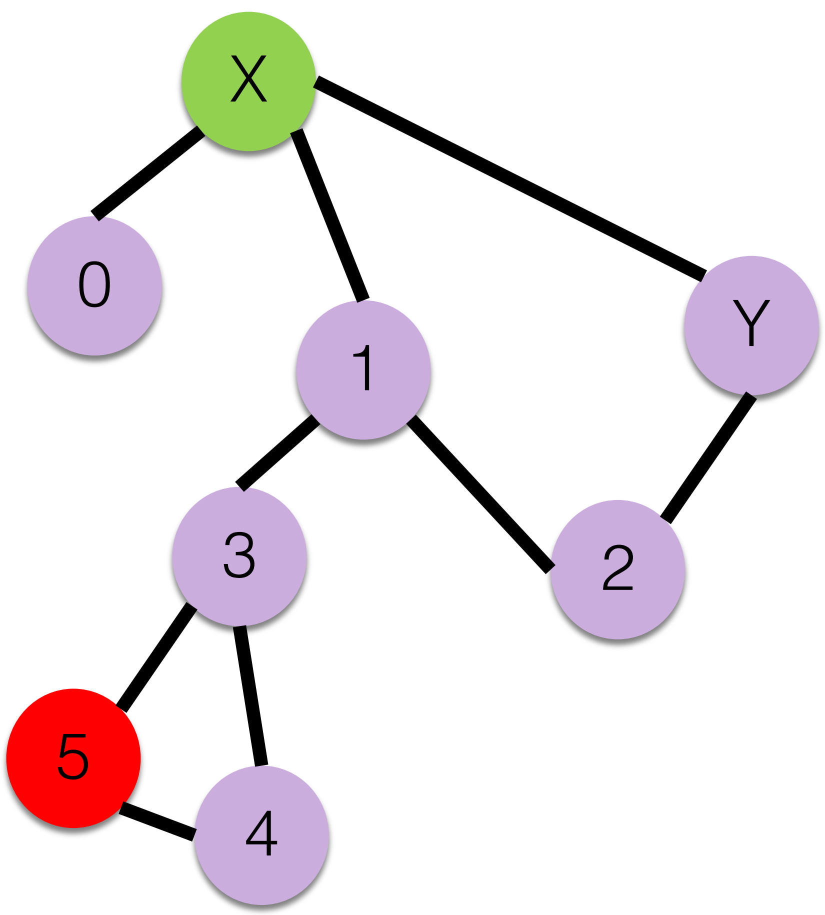
2	3

Map:

X	root
Y	X
0	X
1	X
2	1
3	1
4	null
5	null

Pulling Y has no effect, as connected 2 and X have already been handled

Cont.



Queue: | 2 | 3 |

Queue: | 3 |

Queue: | 4 | 5 |

Map:

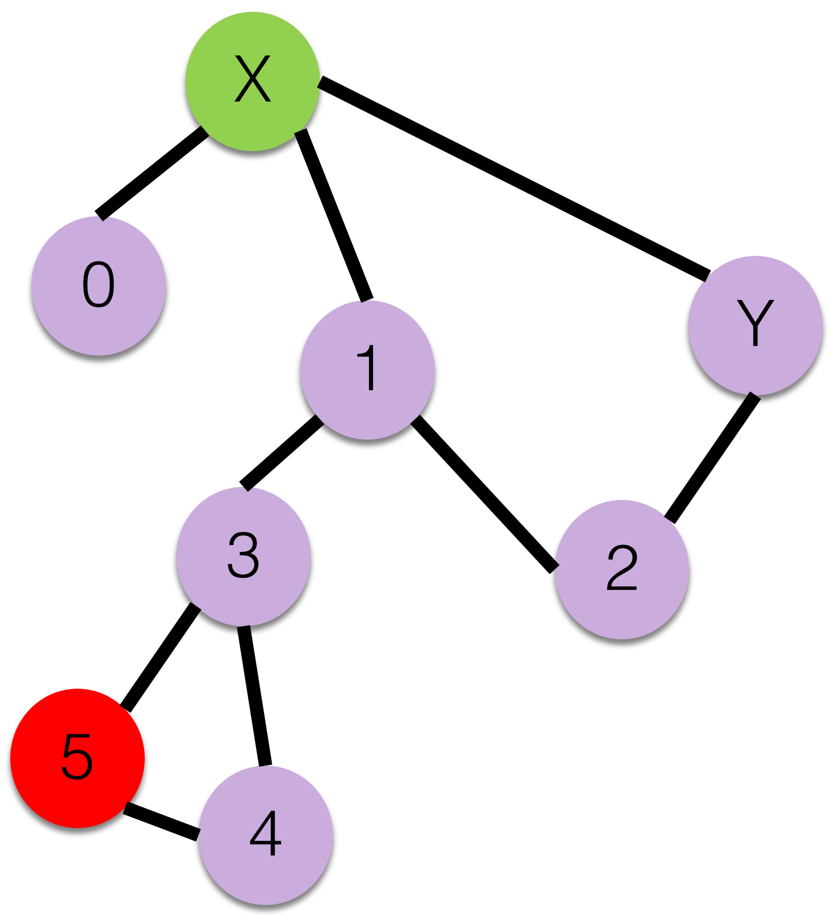
X	root
Y	X
0	X
1	X
2	1
3	1
4	3
5	3

Pulling 2 has no effect (1 and Y already handled).

Pulling 3 leads to push 4 and 5 (but not 1 that has already been handled)

5 is our target

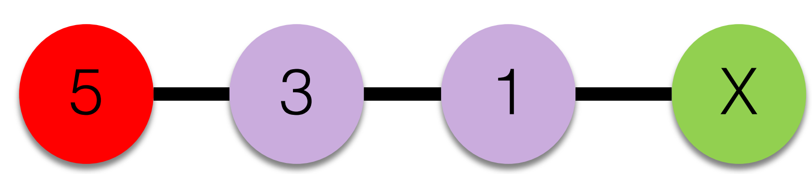
Retrieve Path



Map:

X	root
Y	X
0	X
1	X
2	1
3	1
4	3
5	3

From 5, follow links backward in the map till X



DFS or BFS?

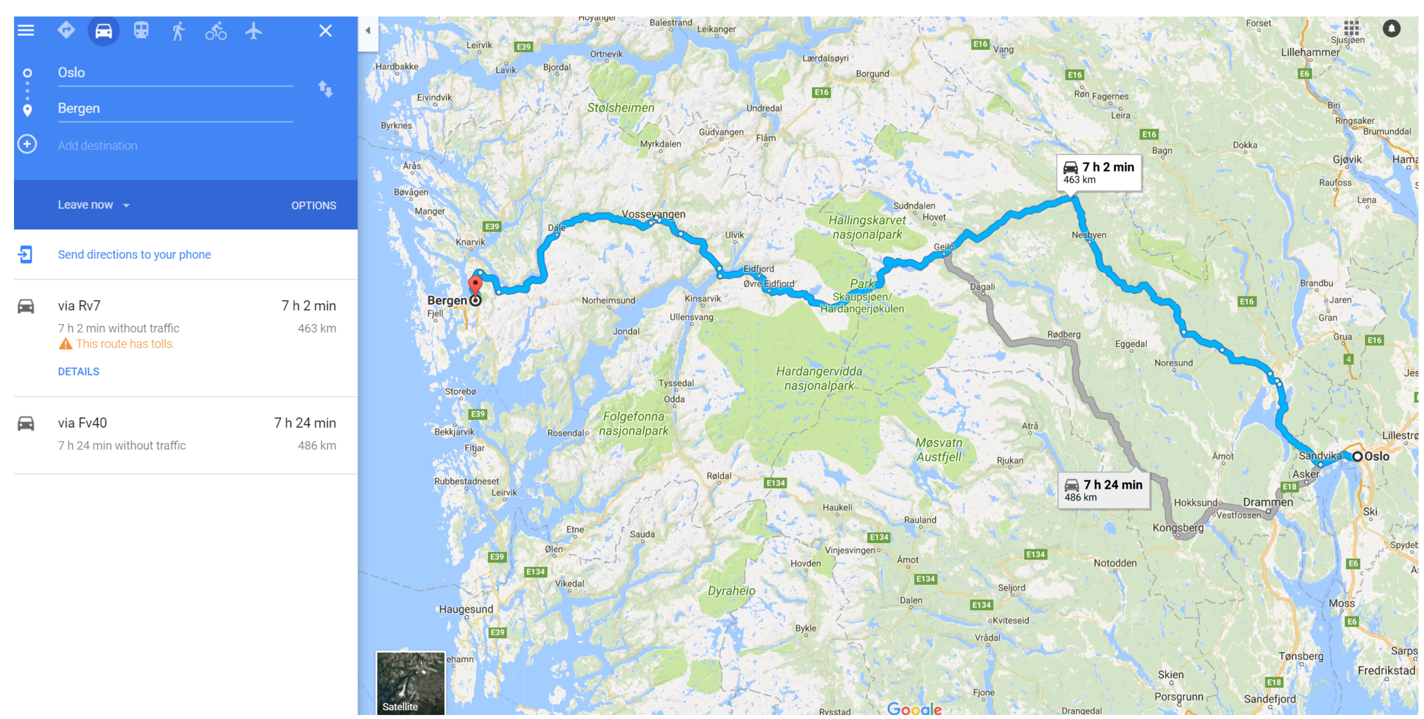
- BFS guarantees to find minimum path, whereas DFS does not

- But BFS is "usually" more expensive, both in terms of time and memory
  - given |A| the average arity of the node connections, it needs to compute an exponential number $|A|^N$ of paths, if the target optimal path has length N.
  - recall that exponential cost is really bad… for already not so large N, its cost can become prohibitive

Weighted Graphs

- You can have graphs where edges have weights
  - Eg, distance between two cities, road tolls, etc.

- Find paths with shortest weight/cost on the traversed edges, even if traversing more vertices



Homework

- Study Book Chapter 4.1
- Study code in the *org.pg4200.les08* package
- Do exercises in *exercises/ex08*
- Extra: do exercises in the book