Graph algorithmic approach

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Feasibility and optimality

Simple instance

```
4          (time slots)
2          (types)
0 1 0 0    (demand)
0 0 1 1    (demand)
3          (costs)
0 4        (costs)
5 0        (costs)
```

- Feasible schedule:
  - demand of each type is fulfilled
  - at each time slot at most one type is produced
- Optimal schedule:
  - Lowest cost (inventory + changeover) among all feasible schedules

Feasible schedules

```
0 1 0 0   (demand type 0)
0 0 1 1   (demand type 1)
```

- 6 feasible schedules:
  a) [-1 0 1 1]
  b) [0 -1 1 1]
  c) [0 1 -1 1]
  d) [1 0 -1 1]
  e) [0 1 1 -1]
  f) [1 0 1 -1]

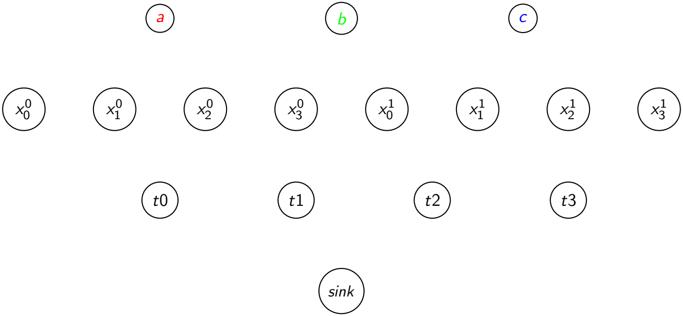
Variables

- consider $x_j^i \in \{0, 1\}$ with
  $x_j^i = 1 \;\Leftrightarrow$ type i is produced at time slot j

Graph instance

0 1 0 0 (demand type 0)

0 0 1 1 (demand type 1)

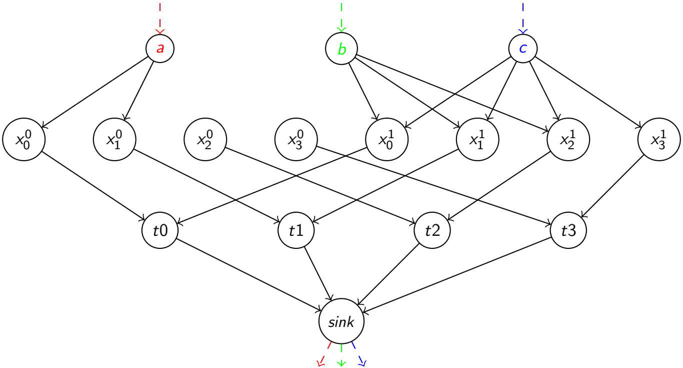


- Number of vertices: $|\text{items}| + (|\text{time\_slots}| + 1) \cdot |\text{types}| + 1$

Graph instance

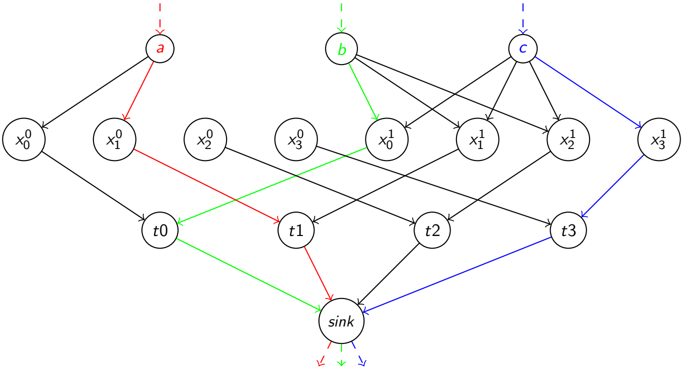
0 1 0 0 (demand type 0)

0 0 1 1 (demand type 1)



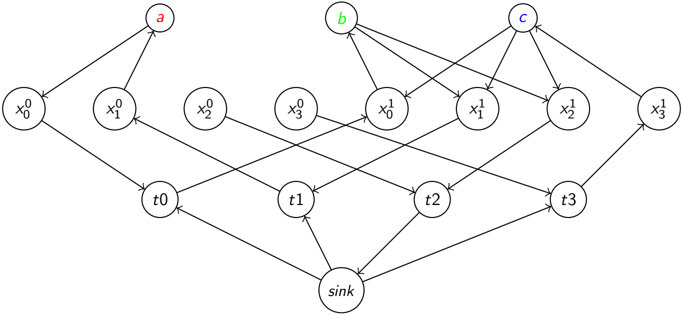
- Unit capacity on each edge

Feasible flow corresponds to schedule

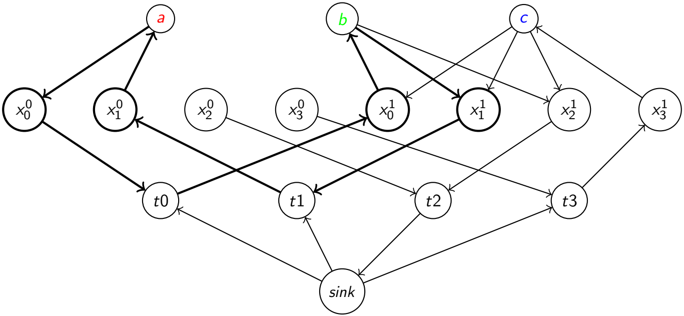


- corresponding schedule: [1 0 -1 1]

Consider directed cycles in residual graph

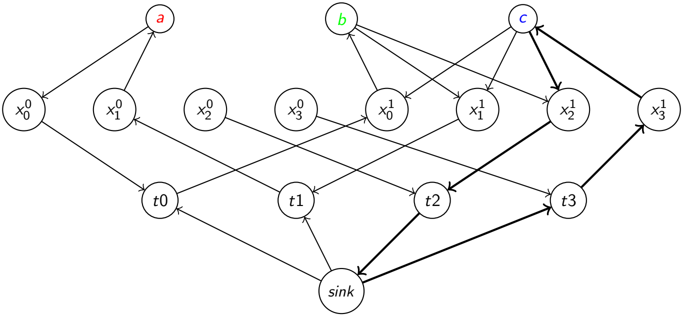


Simple directed cycle yielding different schedule



- schedule: [1 0 -1 1] $\Rightarrow$ [0 1 -1 1]

Another simple directed cycle yielding another different schedule



- schedule: [1 0 -1 1] $\Rightarrow$ [1 0 1 -1]