# Graph algorithmic approach

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# Feasibility and optimality

Simple instance

```
4          (time slots)
2          (types)
0 1 0 0    (demand)
0 0 1 1    (demand)
3          (costs)
0 4        (costs)
5 0        (costs)
```

- Feasible schedule:
  - demand of each type is fulfilled
  - at each time slot at most one type is produced
- Optimal schedule:
  - Lowest cost (inventory + changeover) among all feasible schedules

# Feasible schedules

```
0 1 0 0   (demand type 0)
0 0 1 1   (demand type 1)
```

- 6 feasible schedules:
  a) [ -1 0 1 1 ]
  b) [ 0 -1 1 1 ]
  c) [ 0 1 -1 1 ]
  d) [ 1 0 -1 1 ]
  e) [ 0 1 1 -1 ]
  f) [ 1 0 1 -1 ]

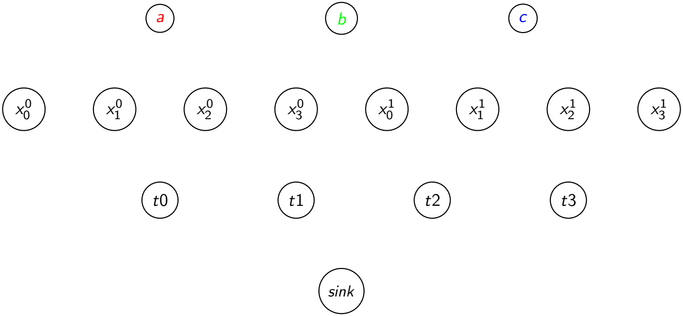
# Variables

- consider $x_j^i \in \{0, 1\}$ with
  $x_j^i = 1 \;\Leftrightarrow$ type $i$ is produced at time slot $j$

# Graph instance

0 1 0 0 (demand type 0)

0 0 1 1 (demand type 1)

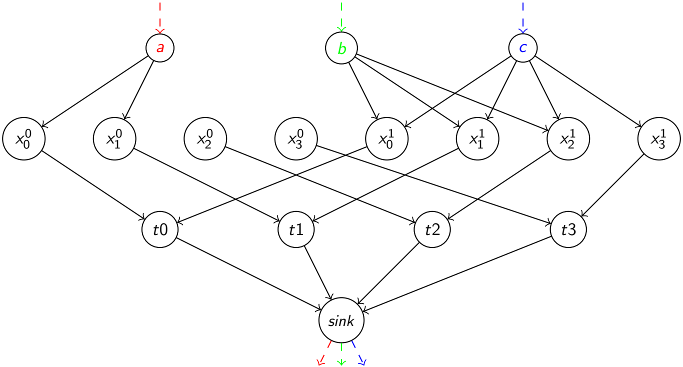



- Number of vertices: $|\text{items}| + (|\text{time\_slots}| + 1) \cdot |\text{types}| + 1$

# Graph instance

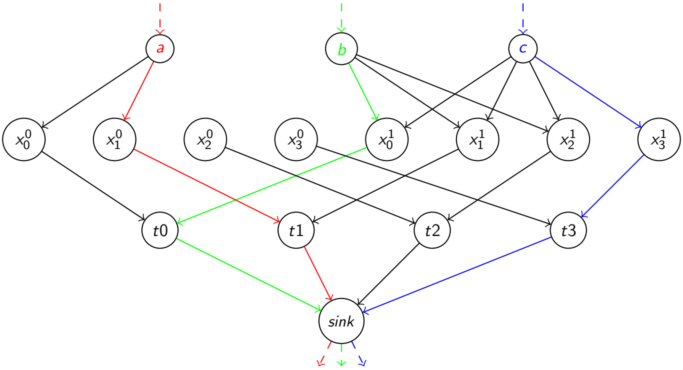
0 1 0 0 (demand type 0)

0 0 1 1 (demand type 1)



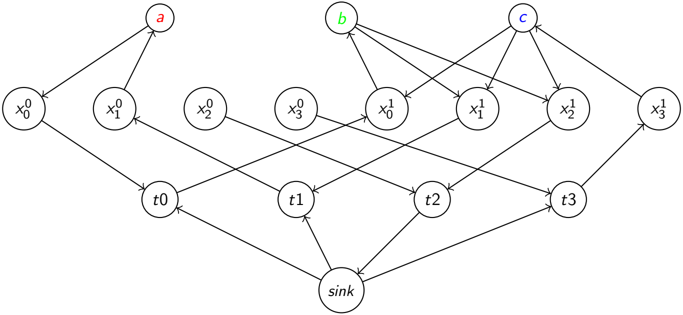

- Unit capacity on each edge

# Feasible flow corresponds to schedule

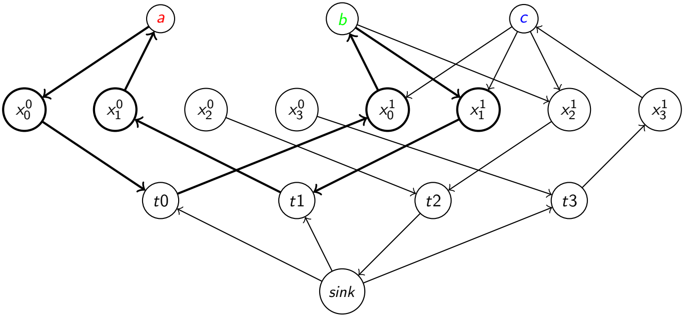



- corresponding schedule: [ 1 0 -1 1 ]

# Consider directed cycles in residual graph

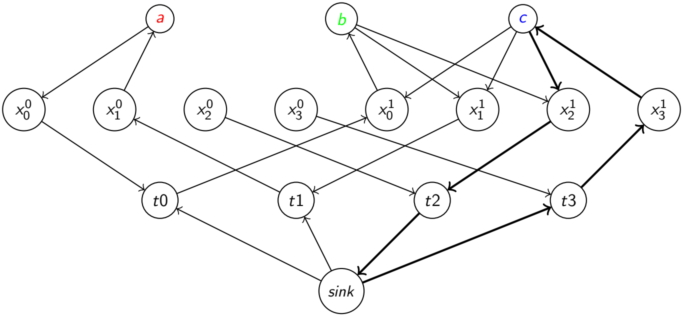


# Simple directed cycle yielding different schedule




- schedule: [ 1 0 -1 1 ] $\Rightarrow$ [ 0 1 -1 1 ]

# Another simple directed cycle yielding another different schedule




- schedule: [ 1 0 -1 1 ] $\Rightarrow$ [ 1 0 1 -1 ]