Frekans Uzayında Kirchhoff Kanunları

Kirchhoff Voltaj Kanunu: Kapalı bir yoldaki voltajların cebirsel toplamı 0'dır

$$v_1 + v_2 + \cdots + v_n = 0,$$

$$\mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n = 0,$$

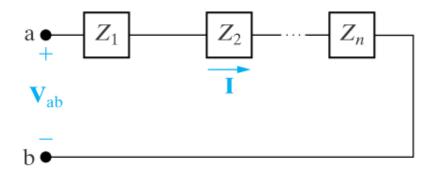
Kirchhoff Akım Kanunu: Bir düğümdeki akımların cebirsel toplamı 0'dır

$$i_1 + i_2 + \cdots + i_n = 0,$$

$$\mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_n = 0,$$

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Seri, Paralel ve Δ - Y Sadeleştirmeleri

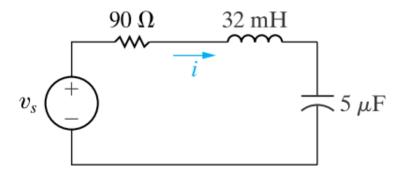


$$\mathbf{V}_{ab} = Z_1 \mathbf{I} + Z_2 \mathbf{I} + \cdots + Z_n \mathbf{I}$$
$$= (Z_1 + Z_2 + \cdots + Z_n) \mathbf{I}.$$

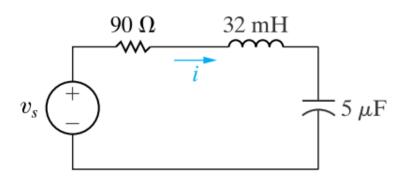
$$Z_{ab} = \frac{\mathbf{V}_{ab}}{\mathbf{I}} = Z_1 + Z_2 + \cdots + Z_n.$$

Soru: Verilen devrede $v_s = 750\cos(5000t + 30)$ ise

- a) Frekans uzayı eşdeğer devreyi elde ediniz.
- b) Kararlı durum i(t) akımını bulunuz.



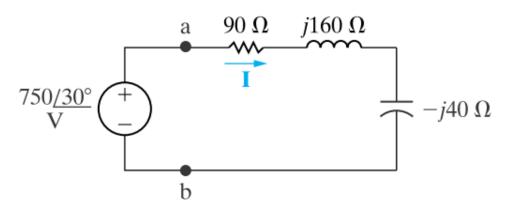
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$$\omega = 5000 \text{ rad/s}.$$

$$Z_L = j\omega L = j(5000)(32 \times 10^{-3}) = j160 \Omega,$$

$$Z_C = j \frac{-1}{\omega C} = -j \frac{10^6}{(5000)(5)} = -j40 \ \Omega.$$

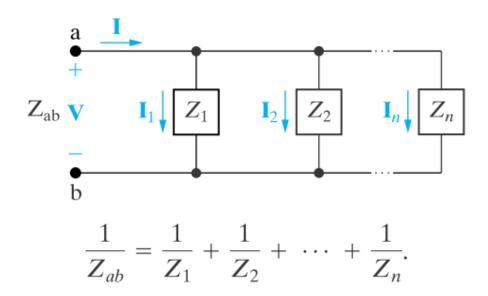


$$Z_{ab} = 90 + j160 - j40$$

= $90 + j120 = 150/53.13^{\circ} \Omega$.

$$\mathbf{I} = \frac{750 \angle 30^{\circ}}{150 \angle 53.13^{\circ}} = 5 \angle -23.13^{\circ} \text{ A.} \quad i = 5 \cos (5000t - 23.13^{\circ}) \text{ A.}$$

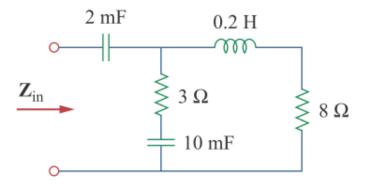
Seri, Paralel ve Δ - Y Sadeleştirmeleri



Paralel iki empedans için:
$$Z_{ab}=rac{Z_1Z_2}{Z_1+Z_2}.$$
 $Y_{ab}=Y_1+Y_2+\,\cdots\,+\,Y_n.$

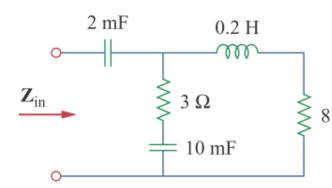
-/ --

Soru: $\omega = 50 \text{ rad/s}$ ise şekildeki devrenin input empedansını bulunuz.



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Seri, Paralel ve Δ - Y Sadeleştirmeleri



 $\mathbf{Z_1}$: 2mF kapasitör olsun,

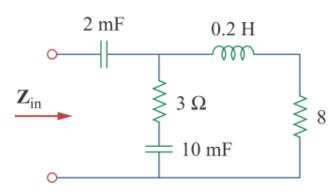
 $\mathbf{Z_2}$: Seri 3 Ω ve 10 mF kapasitör olsun.

 $\mathbf{Z_3}$: Seri 8 Ω ve 0.2 H bobin olsun.

$$\mathbf{Z}_{1} = \frac{1}{j\omega C} = \frac{1}{j50 \times 2 \times 10^{-3}} = -j10 \,\Omega$$

$$\mathbf{Z}_{2} = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j50 \times 10 \times 10^{-3}} = (3 - j2) \,\Omega$$

$$\mathbf{Z}_{3} = 8 + j\omega L = 8 + j50 \times 0.2 = (8 + j10) \,\Omega$$



 $\mathbf{Z_1}$: 2mF kapasitör olsun,

 $\mathbf{Z_2}$: Seri 3 Ω ve 10 mF kapasitör olsun.

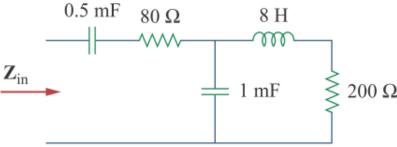
 Z_2 : Seri 3 Ω ve 10 Z_3 : Seri 8 Ω ve 0.2 H bobin olsun.

$$\mathbf{Z}_{\text{in}} = \mathbf{Z}_1 + \mathbf{Z}_2 \| \mathbf{Z}_3 = -j10 + \frac{(3-j2)(8+j10)}{11+j8}$$
$$= -j10 + \frac{(44+j14)(11-j8)}{11^2+8^2} = -j10 + 3.22 - j1.07 \Omega$$

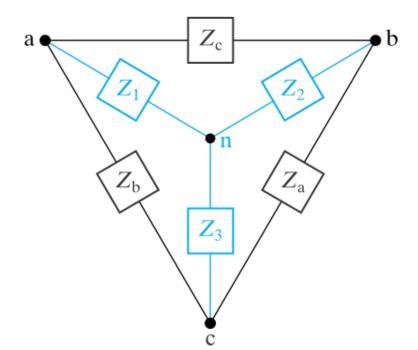
$$\mathbf{Z}_{\rm in} = 3.22 - j11.07 \,\Omega$$

Seri, Paralel ve Δ - Y Sadeleştirmeleri

Ödev: $\omega = 10$ rad/s ise şekildeki devrenin input empedansını bulunuz.



$$(129.52 - j295)$$

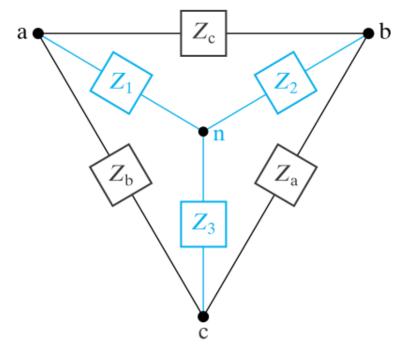


$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c},$$

$$Z_2 = \frac{Z_{\rm c}Z_{\rm a}}{Z_{\rm a} + Z_{\rm b} + Z_{\rm c}},$$

$$Z_3 = \frac{Z_{\rm a} Z_{\rm b}}{Z_{\rm a} + Z_{\rm b} + Z_{\rm c}}.$$

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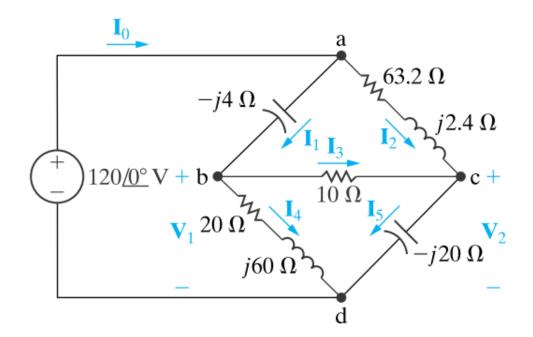


$$Z_{\rm a} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1}$$

$$Z_{b} = \frac{Z_{1}Z_{2} + Z_{2}Z_{3} + Z_{3}Z_{1}}{Z_{2}}$$

$$Z_{\rm c} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3}$$

Soru: Δ -Y dönüşümünü kullanarak verilen devrede I_0 , I_1 , I_2 , I_3 , I_4 , I_5 , V_1 , V_2 , değerlerini bulunuz.



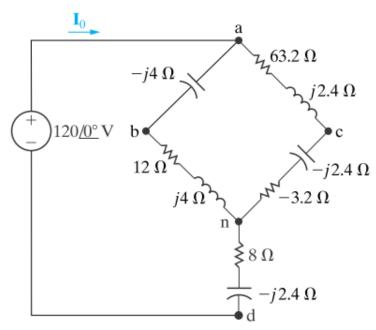
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$$Z_{1} = \frac{(20 + j60)(10)}{30 + j40} = 12 + j4\Omega$$

$$Z_{1} = \frac{(20 + j60)(10)}{30 + j40} = 12 + j4\Omega$$

$$Z_{2} = \frac{10(-j20)}{30 + j40} = -3.2 - j2.4 \Omega$$

$$Z_3 = \frac{(20 + j60)(-j20)}{30 + j40} = 8 - j24 \Omega.$$



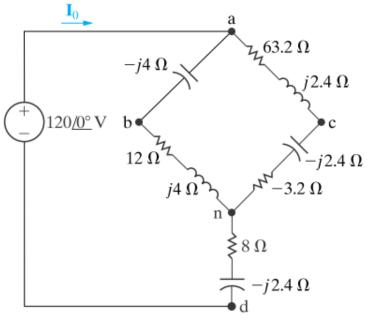
$$Z_{abn} = 12 + j4 - j4 = 12 \Omega,$$

$$Z_{acn} = 63.2 + j2.4 - j2.4 - 3.2$$

$$= 60 \Omega$$

$$Z_{an} = \frac{(60)(12)}{72} = 10 \Omega.$$

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$$\mathbf{I}_{0} = \frac{120 \cancel{0^{\circ}}}{18 - j24} = 4 \cancel{53.13^{\circ}}$$

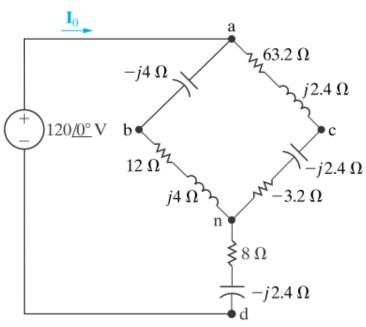
$$= 2.4 + j3.2 \text{ A}.$$

$$\mathbf{V}_{nd} = (8 - j24)\mathbf{I}_{0} = 96 - j32 \text{ V}$$

$$\mathbf{V} = \mathbf{V}_{an} + \mathbf{V}_{nd}$$

$$\mathbf{V}_{an} = 120 - 96 + j32$$

$$= 24 + j32 \text{ V}$$



$$\mathbf{I}_{abn} = \frac{24 + j32}{12} = 2 + j\frac{8}{3} \mathbf{A},$$

$$\mathbf{I}_{\text{acn}} = \frac{24 + j32}{60} = \frac{4}{10} + j\frac{8}{15} \text{ A}.$$

$$\mathbf{I}_1 = \mathbf{I}_{abn}$$

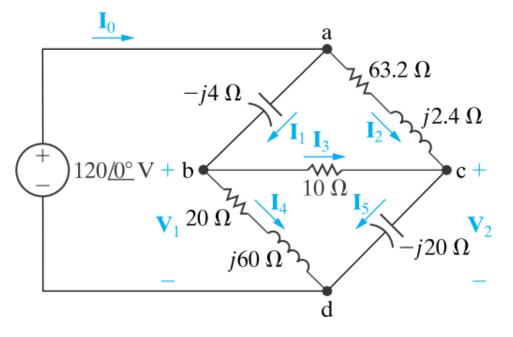
$$\mathbf{I}_2 = \mathbf{I}_{\mathrm{acn}}$$

Sağlama:

$$\mathbf{I}_1 + \mathbf{I}_2 = 2.4 + j3.2 = \mathbf{I}_0.$$

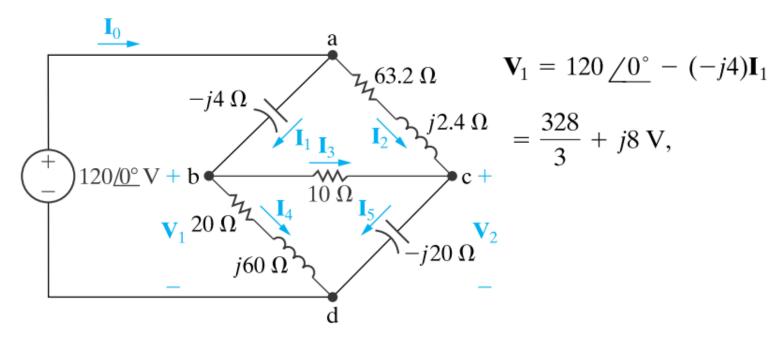
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Seri, Paralel ve Δ - Y Sadeleştirmeleri

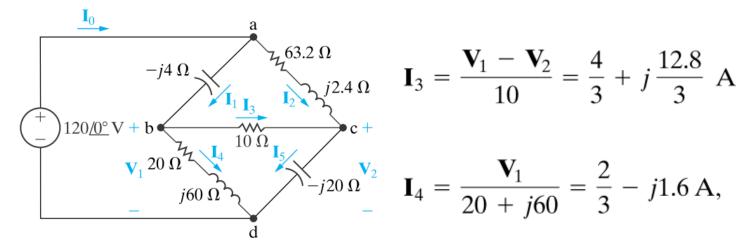


 V_1 : b noktasındaki voltaj

 V_2 : c noktasındaki voltaj



$$\mathbf{V}_2 = 120 \underline{/0^{\circ}} - (63.2 + j2.4)\mathbf{I}_2 = 96 - j \frac{104}{3} \mathbf{V}.$$

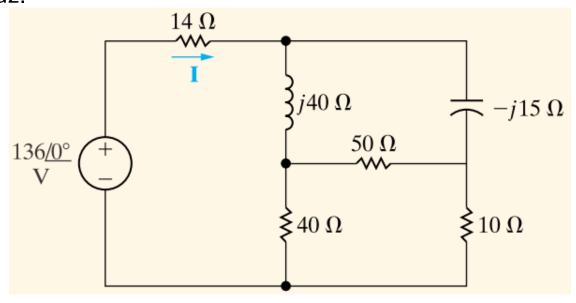


$$\mathbf{I}_3 = \frac{\mathbf{V}_1 - \mathbf{V}_2}{10} = \frac{4}{3} + j \frac{12.8}{3} \mathbf{A}$$

$$\mathbf{I}_4 = \frac{\mathbf{V}_1}{20 + j60} = \frac{2}{3} - j1.6 \,\mathrm{A},$$

$$\mathbf{I}_5 = \frac{\mathbf{V}_2}{-j20} = \frac{26}{15} + j4.8 \,\mathrm{A}.$$

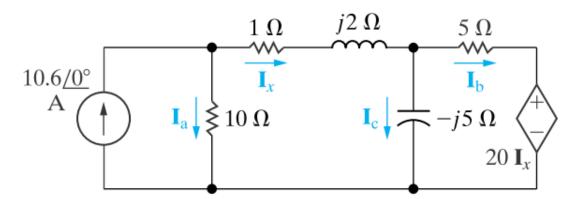
Ödev: Δ -Y dönüşümünü kullanarak verilen devrede ${f I}$ akımını bulunuz.

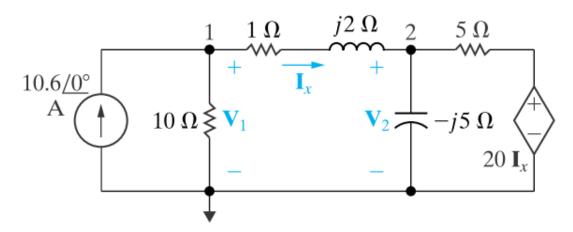


$$I = 4 / 28.07^{\circ} A.$$

Düğüm Voltaj Yöntemi

Soru: Verilen devrede düğüm voltaj yöntemini kullanarak \mathbf{I}_a , \mathbf{I}_b , \mathbf{I}_c akımlarını bulunuz.

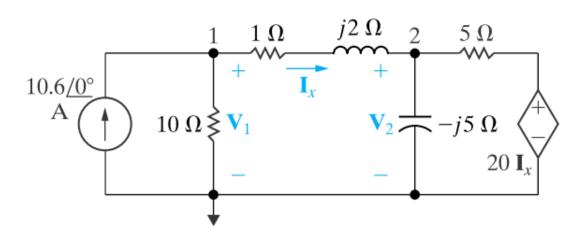




$$-10.6 + \frac{\mathbf{V}_1}{10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{1 + j2} = 0.$$

$$\mathbf{V}_1(1.1 + j0.2) - \mathbf{V}_2 = 10.6 + j21.2.$$

Düğüm Voltaj Yöntemi



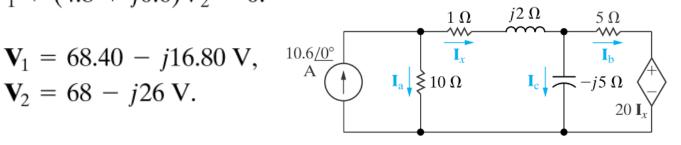
$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{1 + j2} + \frac{\mathbf{V}_2}{-j5} + \frac{\mathbf{V}_2 - 20\mathbf{I}_x}{5} = 0. \qquad \mathbf{I}_x = \frac{\mathbf{V}_1 - \mathbf{V}_2}{1 + j2}.$$
$$-5\mathbf{V}_1 + (4.8 + j0.6)\mathbf{V}_2 = 0.$$

$$\mathbf{V}_1(1.1 + j0.2) - \mathbf{V}_2 = 10.6 + j21.2.$$

$$-5\mathbf{V}_1 + (4.8 + j0.6)\mathbf{V}_2 = 0.$$

$$\mathbf{V}_1 = 68.40 - j16.80 \,\mathrm{V},$$

 $\mathbf{V}_2 = 68 - j26 \,\mathrm{V}.$



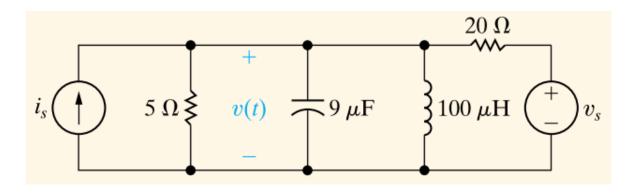
$$\mathbf{I}_{a} = \frac{\mathbf{V}_{1}}{10} = 6.84 - j1.68 \,\mathrm{A}, \quad \mathbf{I}_{b} = \frac{\mathbf{V}_{2} - 20\mathbf{I}_{x}}{5} = -1.44 - j11.92 \,\mathrm{A}$$

$$\mathbf{I}_x = \frac{\mathbf{V}_1 - \mathbf{V}_2}{1 + j2} = 3.76 + j1.68 \text{ A}, \qquad \mathbf{I}_c = \frac{\mathbf{V}_2}{-j5} = 5.2 + j13.6 \text{ A}.$$

Düğüm Voltaj Yöntemi

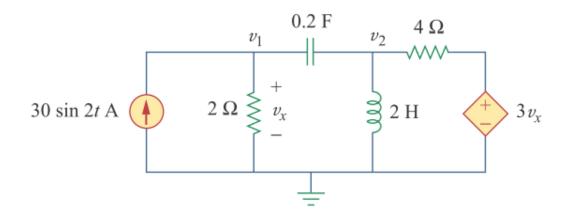
Ödev: Verilen devrede düğüm voltaj yöntemini kullanarak v(t)gerilim ifadesini bulunuz.

$$i_s = 10\cos\omega t \text{ A}$$
 $v_s = 100\sin\omega t \text{ V}$ $\omega = 50 \text{ krad/s}$



$$v(t) = 31.62\cos(50,000t - 71.57^{\circ}) \text{ V}.$$

Ödev: Verilen devrede düğüm voltaj yöntemini kullanarak v_1 ve v_2 gerilim ifadelerini bulunuz.

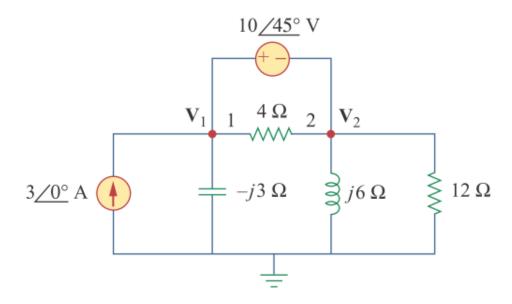


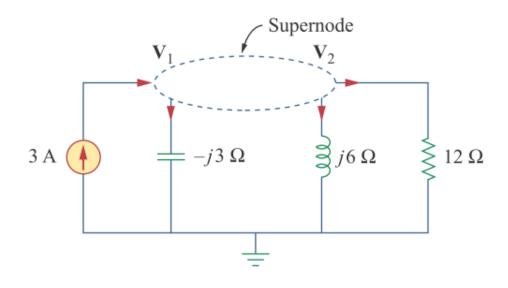
$$v_1(t) = 33.96 \sin(2t + 60.01^\circ) \text{ V},$$

$$v_2(t) = 99.06 \sin(2t + 57.12^\circ) \text{ V}.$$

Düğüm Voltaj Yöntemi

Soru: Verilen devrede düğüm voltaj yöntemini kullanarak \mathbf{V}_1 ve \mathbf{V}_2 değerlerini hesağlayınız. (Süper düğüm)





$$3 = \frac{\mathbf{V}_1}{-j3} + \frac{\mathbf{V}_2}{j6} + \frac{\mathbf{V}_2}{12} \qquad \qquad \mathbf{V}_1 = \mathbf{V}_2 + 10/45^\circ$$

$$\mathbf{V}_1 = \mathbf{V}_2 + 10 / 45^{\circ}$$

Düğüm Voltaj Yöntemi

$$3 = \frac{\mathbf{V}_1}{-i3} + \frac{\mathbf{V}_2}{i6} + \frac{\mathbf{V}_2}{12}$$

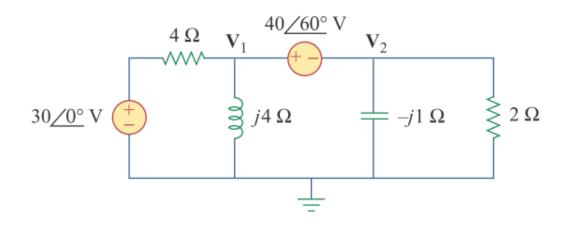
$$\mathbf{V}_1 = \mathbf{V}_2 + 10 / 45^{\circ}$$

$$36 = j4\mathbf{V}_1 + (1 - j2)\mathbf{V}_2$$

$$36 - 40/135^{\circ} = (1 + j2)\mathbf{V}_2 \implies \mathbf{V}_2 = 31.41/-87.18^{\circ} \,\mathrm{V}$$

$$\mathbf{V}_1 = \mathbf{V}_2 + 10/45^{\circ} = 25.78/-70.48^{\circ} \,\mathrm{V}$$

Ödev: Verilen devrede düğüm voltaj yöntemini kullanarak \mathbf{V}_1 ve \mathbf{V}_2 değerlerini hesağlayınız. (Süper düğüm)

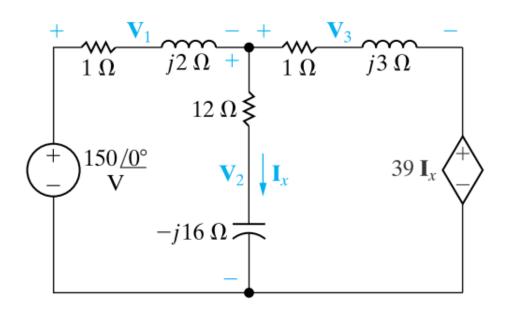


$$\mathbf{V}_1 = 38.72 / 69.67^{\circ} \,\mathrm{V}, \,\mathbf{V}_2 = 6.752 / 165.7^{\circ} \,\mathrm{V}.$$

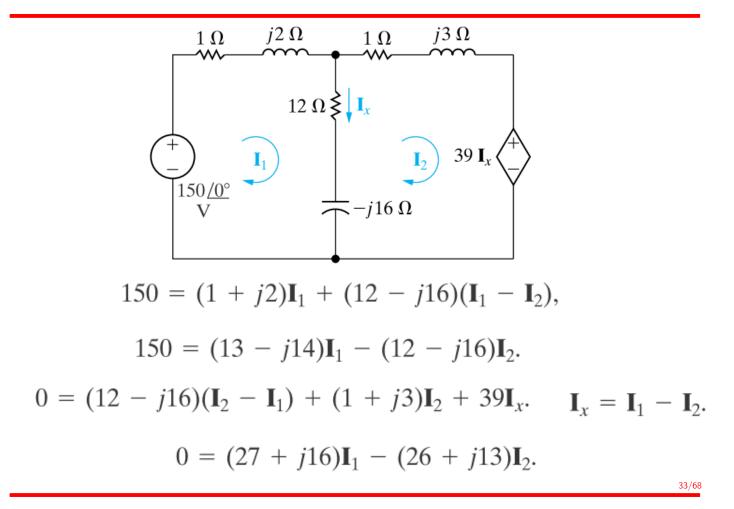
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Ağ Akım Yöntemi

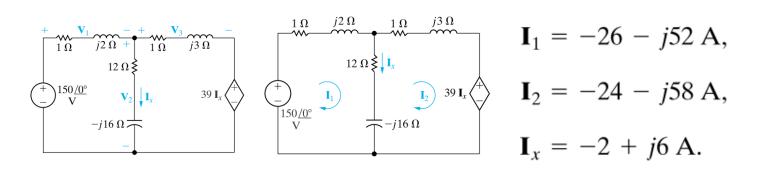
Soru: Verilen devrede ağ akım yöntemini kullanarak ${f V}_1$, ${f V}_2$ ve ${f V}_3$ değerlerini hesağlayınız.



Ağ Akım Yöntemi



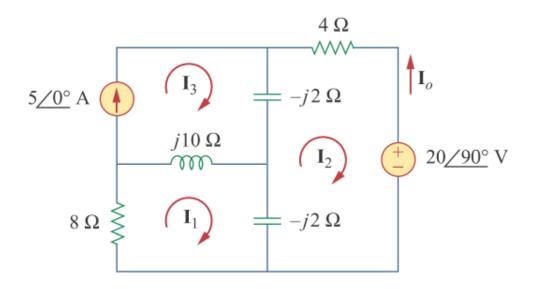
Ağ Akım Yöntemi



$$\mathbf{V}_1 = (1 + j2)\mathbf{I}_1 = 78 - j104 \,\mathrm{V},$$

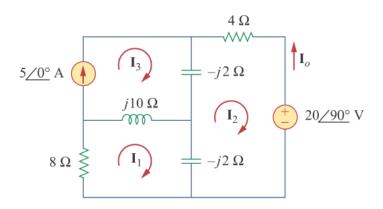
 $\mathbf{V}_2 = (12 - j16)\mathbf{I}_x = 72 + j104 \,\mathrm{V},$
 $\mathbf{V}_3 = (1 + j3)\mathbf{I}_2 = 150 - j130 \,\mathrm{V}.$

Soru: Verilen devrede ${f I}_0$ akımını bulunuz.



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Ağ Akım Yöntemi



$$(8 + j10 - j2)\mathbf{I}_{1} - (-j2)\mathbf{I}_{2} - j10\mathbf{I}_{3} = 0$$

$$(4 - j2 - j2)\mathbf{I}_{2} - (-j2)\mathbf{I}_{1} - (-j2)\mathbf{I}_{3} + 20/90^{\circ} = 0$$

$$\mathbf{I}_{3} = 5.$$

$$\begin{aligned}
(8+j8)\mathbf{I}_1 + j2\mathbf{I}_2 &= j50 \\
j2\mathbf{I}_1 + (4-j4)\mathbf{I}_2 &= -j20 - j10
\end{aligned}
\begin{bmatrix}
8+j8 & j2 \\
j2 & 4-j4
\end{bmatrix}
\begin{bmatrix}
\mathbf{I}_1 \\
\mathbf{I}_2
\end{bmatrix} = \begin{bmatrix}
j50 \\
-j30
\end{bmatrix}$$

Ağ Akım Yöntemi

$$\begin{bmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{vmatrix} = 32(1+j)(1-j) + 4 = 68$$

$$\Delta_2 = \begin{vmatrix} 8+j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340 - j240 = 416.17 / -35.22^{\circ}$$

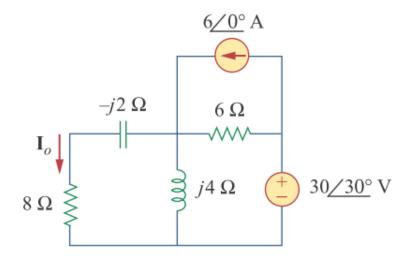
$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17 / -35.22^{\circ}}{68} = 6.12 / -35.22^{\circ} \text{ A}$$

$$\mathbf{I}_o = -\mathbf{I}_2 = 6.12 / 144.78^{\circ} \text{ A}$$

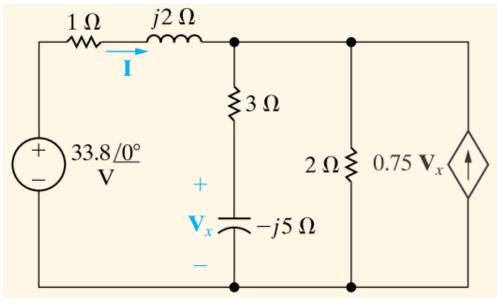
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Ağ Akım Yöntemi

Ödev: Verilen devrede \mathbf{I}_o fazör akımını bulunuz.



 $\ddot{\text{O}}$ dev: Verilen devrede \mathbf{I} fazör akımını bulunuz.

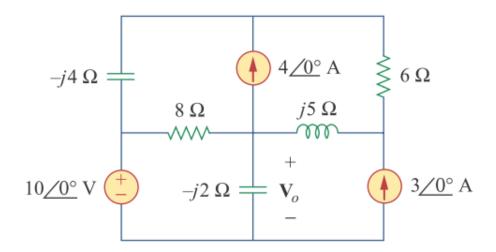


$$I = 29 + j2 = 29.07 / 3.95^{\circ} A.$$

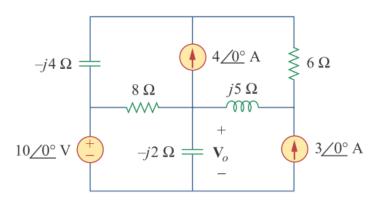
39/0

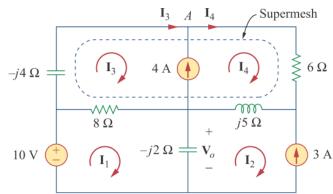
Ağ Akım Yöntemi

Soru: Verilen devrede \mathbf{V}_o fazör gerilimini bulunuz.



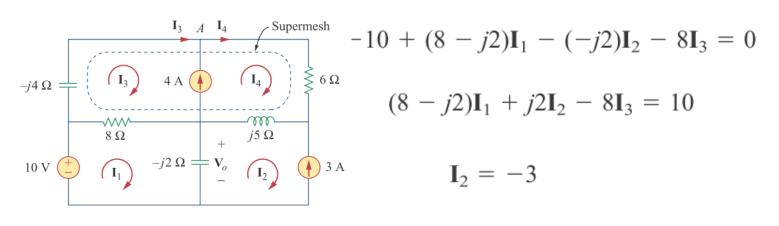
Ağ Akım Yöntemi





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Ağ Akım Yöntemi



$$(8 - j4)\mathbf{I}_3 - 8\mathbf{I}_1 + (6 + j5)\mathbf{I}_4 - j5\mathbf{I}_2 = 0$$

$$\mathbf{I}_4 = \mathbf{I}_3 + 4$$

$$(8 - j4)\mathbf{I}_3 - 8\mathbf{I}_1 + (6 + j5)\mathbf{I}_4 - j5\mathbf{I}_2 = 0$$
$$\mathbf{I}_4 = \mathbf{I}_3 + 4$$
$$-8\mathbf{I}_1 + (14 + j)\mathbf{I}_3 = -24 - j35$$

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Ağ Akım Yöntemi

$$-10 + (8 - j2)\mathbf{I}_{1} - (-j2)\mathbf{I}_{2} - 8\mathbf{I}_{3} = 0$$

$$(8 - j2)\mathbf{I}_{1} + j2\mathbf{I}_{2} - 8\mathbf{I}_{3} = 10$$

$$\mathbf{I}_{2} = -3$$

$$(8 - j2)\mathbf{I}_{1} - 8\mathbf{I}_{3} = 10 + j6$$

$$-8\mathbf{I}_{1} + (14 + j)\mathbf{I}_{3} = -24 - j35$$

$$\begin{bmatrix} 8 - j2 & -8 \\ -8 & 14 + j \end{bmatrix} \begin{bmatrix} \mathbf{I}_{1} \\ \mathbf{I}_{3} \end{bmatrix} = \begin{bmatrix} 10 + j6 \\ -24 - j35 \end{bmatrix}$$

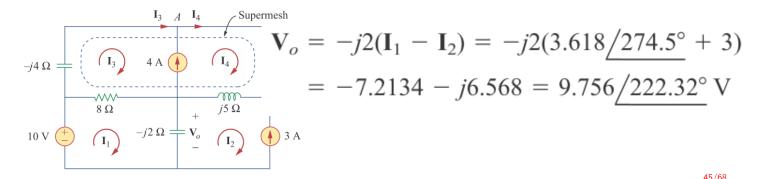
Ağ Akım Yöntemi

$$\Delta = \begin{vmatrix} 8 - j2 & -8 \\ -8 & 14 + j \end{vmatrix} = 112 + j8 - j28 + 2 - 64 = 50 - j20$$

$$\Delta_1 = \begin{vmatrix} 10 + j6 & -8 \\ -24 - j35 & 14 + j \end{vmatrix} = 140 + j10 + j84 - 6 - 192 - j280$$

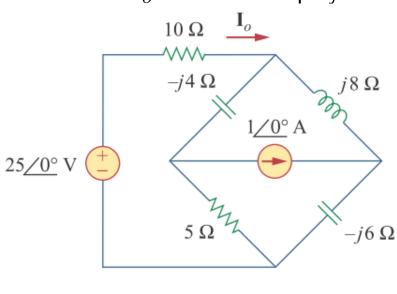
$$= -58 - j186$$

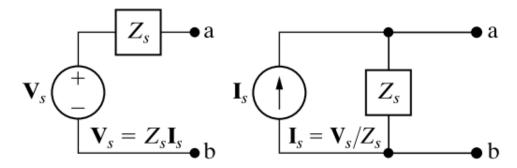
$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{-58 - j186}{50 - j20} = 3.618 / 274.5^{\circ} \,\text{A}$$



Ağ Akım Yöntemi

Odev: Verilen devrede \mathbf{I}_o akımını hesaplayınız.



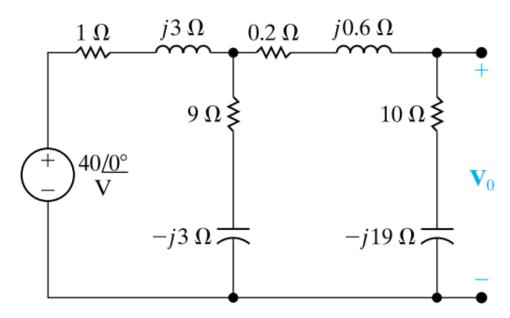


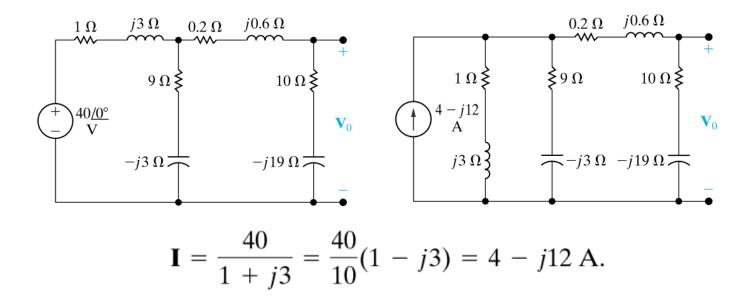
Frekans uzayında kaynak dönüşümü

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Kaynak Dönüşümü

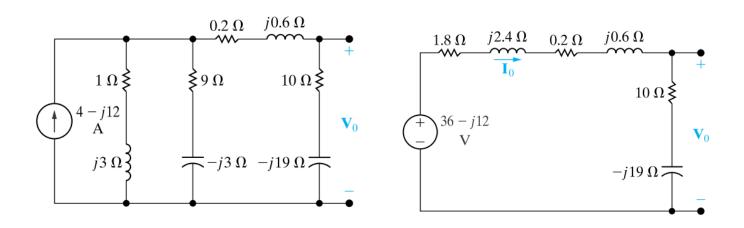
Soru: Verilen devrede kaynak dönüşümü yöntentemini kullanarak V_0 fazör voltajını bulunuz.





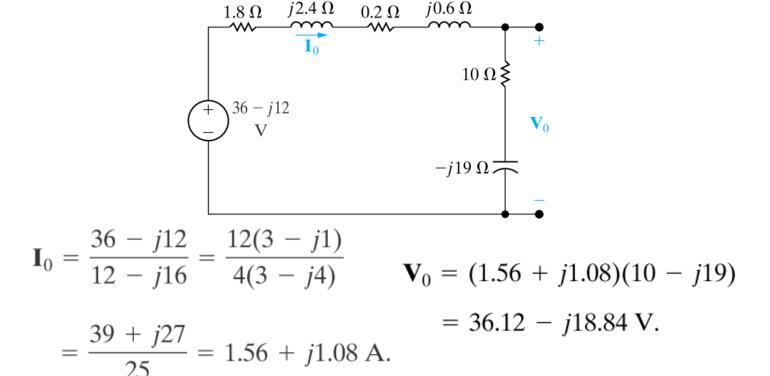
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Kaynak Dönüşümü



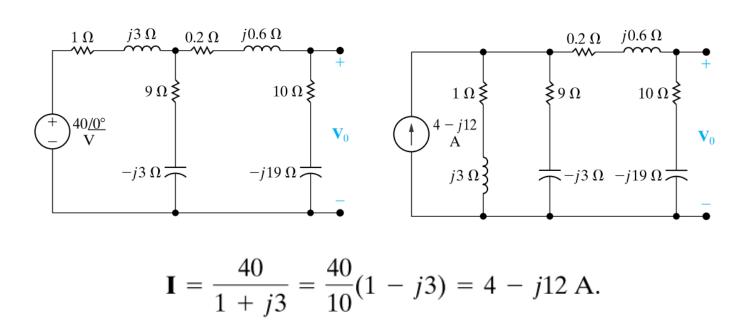
$$Z = \frac{(1+j3)(9-j3)}{10} = 1.8 + j2.4 \Omega,$$

$$\mathbf{V} = (4-j12)(1.8+j2.4) = 36 - j12 \text{ V}.$$

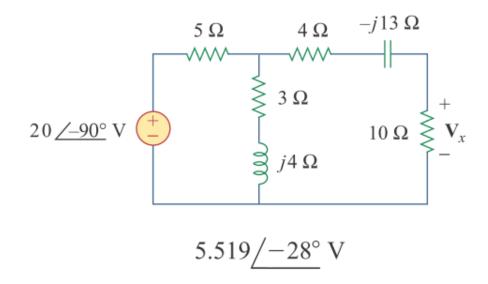


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Kaynak Dönüşümü



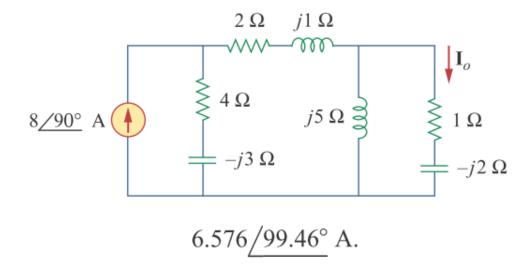
Ödev: Verilen devrede kaynak dönüşümü yöntentemini kullanarak V_x fazör voltajını bulunuz.

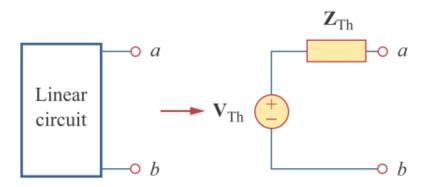


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Kaynak Dönüşümü

Ödev: Verilen devrede kaynak dönüşümü yöntentemini kullanarak I_o fazör akımını bulunuz.



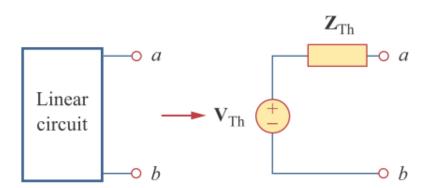


Genel Çözüm

- Thevenin eşlenik devre bulunmak istenilen terminal açık devre yapılır.
 - $V_{oc} = V_{Th}$ olur.
- Thevenin eşlenik devre bulunmak istenilen terminal kısa devre yapılır ve I_{sc} bulunur.
 - $V_{oc}/I_{sc}=Z_{Th}$ olur

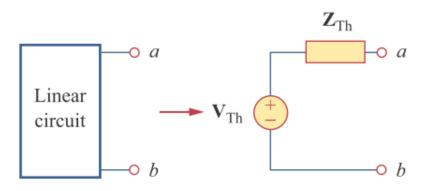
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Thevenin Eşlenik Devreleri



Bağımlı kaynak yok ise

- Thevenin eşlenik devre bulunmak istenilen terminal açık devre yapılır.
 - $V_{oc} = V_{Th}$ olur.
- Devredeki akım kaynakları açık devre, voltaj kaynakları kısa devre yapılır ve terminalden görünen empedans Z_{Th} olur

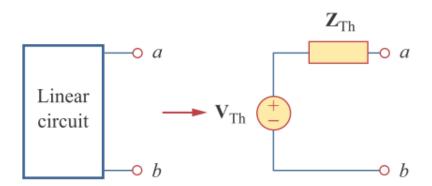


Bağımlı kaynak var ise

- Thevenin eşlenik devre bulunmak istenilen terminal açık devre yapılır.
 - $V_{oc} = V_{Th}$ olur.
- Thevenin eşlenik devre bulunmak istenilen terminale test kaynağı bağlanılır. Bu kaynağın terminalindeki voltaj ve akım değerleri kullanılıarak Z_{Th} bulunur. $V_t/I_t=Z_{Th}$

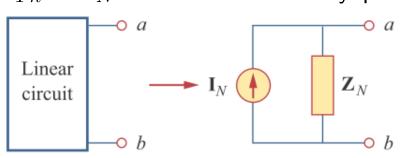
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Norton Eşlenik Devreleri

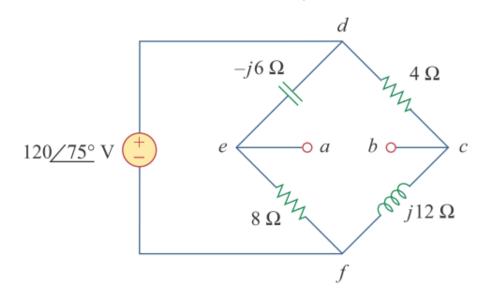


Thevenin Eşlenik bulunduktan sonra Norton Eşlenik devre kaynak çevrimi ile bulunabilir.

Veya $Z_N = Z_{Th}$ ve I_N terminal kısa devre yapılarak bulunur.

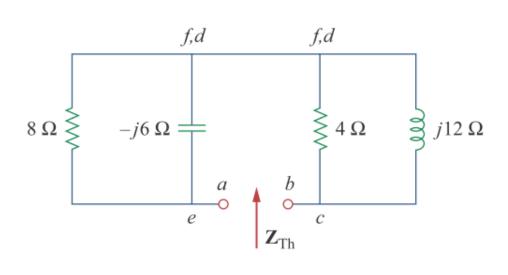


Soru: a-b terminalinden Thevenin eşlenik devresini bulunuz.



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Thevenin Eşlenik Devreleri



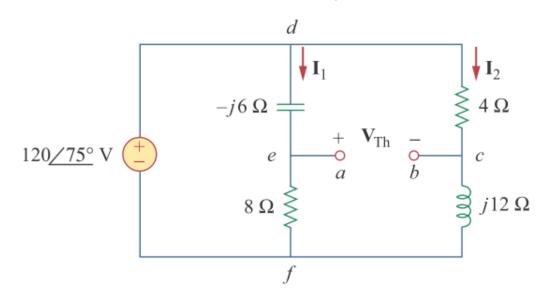
$$\mathbf{Z}_1 = -j6 \parallel 8 = \frac{-j6 \times 8}{8 - j6} = 2.88 - j3.84 \Omega$$

$$\mathbf{Z}_2 = 4 \parallel j12 = \frac{j12 \times 4}{4 + j12} = 3.6 + j1.2 \Omega$$

$$\mathbf{Z}_{\mathrm{Th}} = \mathbf{Z}_1 + \mathbf{Z}_2 =$$

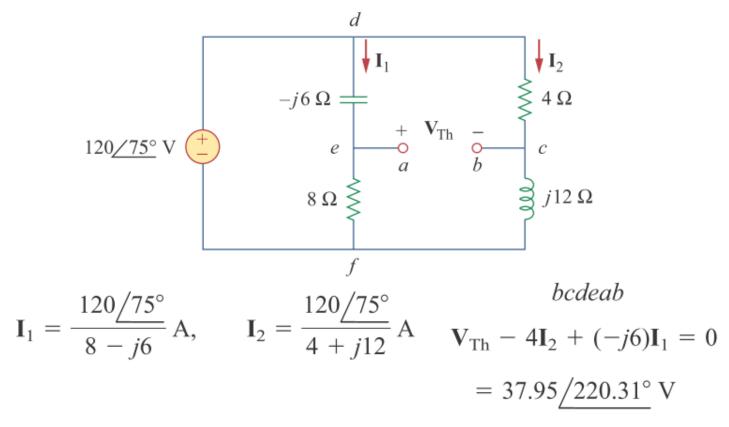
$$6.48 - j2.64 \Omega$$

Soru: a-b terminalinden Thevenin eşlenik devresini bulunuz.

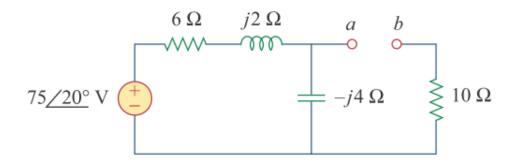


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Thevenin Eşlenik Devreleri



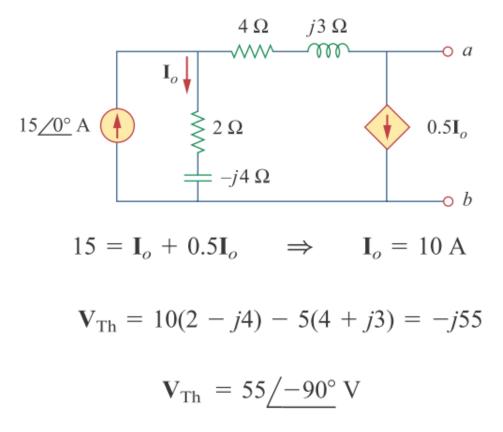
Ödev: a-b terminalinden Thevenin eşlenik devresini bulunuz.



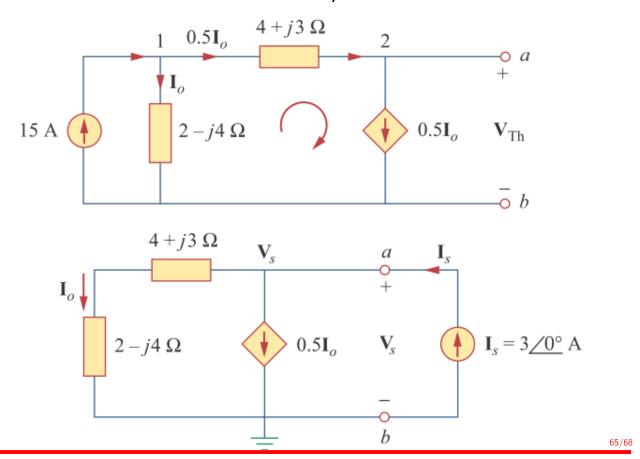
$$\mathbf{Z}_{\text{Th}} = 12.4 - j3.2 \ \Omega, \mathbf{V}_{\text{Th}} = 47.42 / -51.57^{\circ} \ V.$$

Thevenin Eşlenik Devreleri

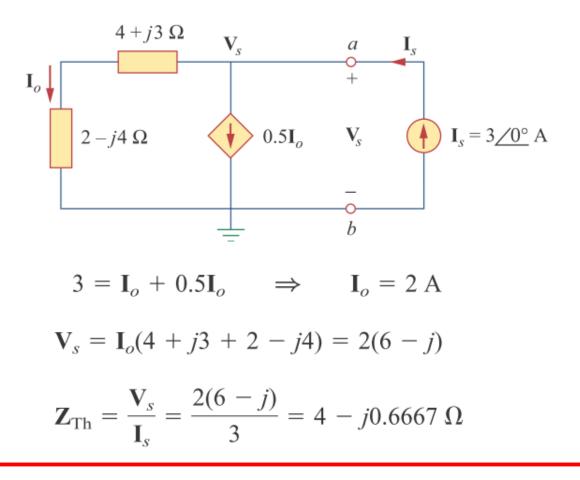
Soru: a-b terminalinden Thevenin eşlenik devresini bulunuz.



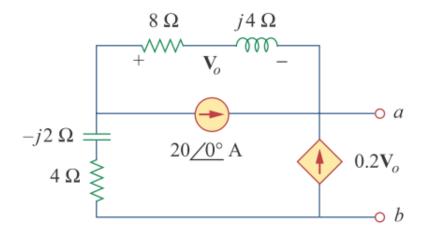
Soru: a-b terminalinden Thevenin eşlenik devresini bulunuz.



Thevenin Eşlenik Devreleri



Ödev: a-b terminalinden Thevenin eşlenik devresini bulunuz.

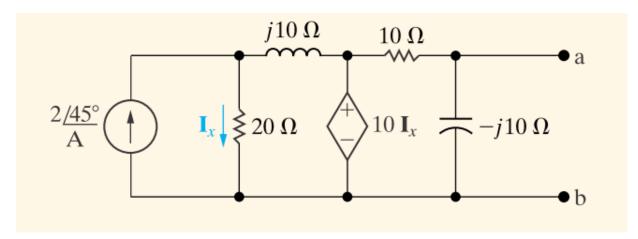


$$\mathbf{Z}_{Th} = 4.473 / -7.64^{\circ} \, \Omega, \, \mathbf{V}_{Th} = 29.4 / 72.9^{\circ} \, V.$$

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Thevenin Eşlenik Devreleri

Ödev: a-b terminalinden Thevenin eşlenik devresini bulunuz.



$$\mathbf{V}_{\mathrm{Th}} = \mathbf{V}_{\mathrm{ab}} = 10 \, \underline{/45^{\circ}} \, \mathrm{V}; \ Z_{\mathrm{Th}} = 5 - j5 \, \Omega.$$