



# Universidad Nacional de Río Negro

## Int Partículas, Astrofísica & Cosmología - 2021

- **Unidad** 02-Astrofísica, estrellas y planetas
- **Clase** UO2 C02 - 6/16
- **Fecha** 08 Sep 2021
- **Cont** Modelos estelares
- **Cátedra** Asorey



# Contenidos: un viaje en el tiempo y el espacio

## HOW DID OUR UNIVERSE BEGIN?

Some 13.8 billion years ago our entire visible universe was contained in an unimaginably hot, dense point, a billionth the size of a nuclear particle. Since then it has expanded—a lot—fighting gravity all the way.

**Inflation**  
In less than a nanosecond a massive energy field inflates space 10<sup>26</sup> times, stretching it with a soup of subatomic particles called quarks.

**Age:** 10<sup>-2</sup> milliseconds  
**Size:** Infinitesimal to golf ball

**Early building blocks**  
The universe expands, cools. Quarks clump into protons and neutrons; then strings of atomic nuclei. Perhaps dark matter forms.

**Age:** .01 milliseconds  
**Size:** 1-billionth present size

**First nuclei**  
As the universe continues to cool, the lightest nuclei of hydrogen will form, then helium, then other elements. This light is as far back as our instruments can see.

**Age:** .01 to 200 seconds  
**Size:** .0009 present size

**First atoms, first light**  
As electrons begin orbiting nuclei, creating atoms, the glow from their infalling orbits is unveiled. This light is as far back as our instruments can see.

**Age:** 380,000 years  
**Size:** .0009 present size

**The “dark ages”**  
For 300 million years this cosmic fog of neutral hydrogen is the only light. Clumps of matter that will become galaxies glow brightest.

**Age:** 380,000 to 300 million years  
**Size:** .0009 to 0.1 present size

**Gravity wins: first stars**  
Dense gas clouds collapse under their own gravity and of dark matter to eventually form galaxies and stars. Nuclei fusion lights up the stars.

**Age:** 300 million years  
**Size:** 0.1 present size

**Antigravity wins**  
After being slowed for billions of years, dark energy speeds up again. The culprit: dark energy. Its nature: unclear.

**Age:** 10 billion years  
**Size:** .77 present size

**Today**  
The universe continues to expand, becoming ever less dense. As a result, fewer new stars and galaxies are forming.

**Age:** 13.8 billion years  
**Size:** Present size

## COSMIC QUESTIONS

In the 20th century the universe became a story—a scientific one. It had always been seen as static and eternal. Then astronomers observed other galaxies flying away from ours, and Einstein's general relativity theory implied space itself was expanding—which meant the universe had once been denser. What had seemed eternal now had a beginning and an end. But what beginning? What end? Those questions are still open.

## WHAT IS OUR UNIVERSE MADE OF?

Stars, dust and gas—the stuff we can discern—make up less than 5 percent of the universe. Their gravity can't account for how galaxies hold together. Scientists figure about 24 percent of the universe is a mysterious dark matter—perhaps exotic particles formed right after inflation. The rest is dark energy, an unknown energy field or property of space that counters gravity, providing an explanation for observations that the expansion of space is accelerating.



## WHAT IS THE SHAPE OF OUR UNIVERSE?

Einstein discovered that a star's gravity curves space around it. But is the whole universe curved? Might space close up on itself like a sphere or curve the other way, opening out like a saddle? By studying cosmic background radiation, scientists have found that the universe is poised between the two: just dense enough with just enough gravity to be almost perfectly flat, at least the part we can see. What lies beyond we can't know.

**Observable Universe**  
The universe began 13.8 billion years ago. Because it has been expanding ever since, the farthest observable edge is now 47 billion light-years.

**The Unknown Beyond**  
What we can't see. The possible shapes are:



## Unidad 2

## Astrofísica, escalas medias

### DO WE LIVE IN A MULTIVERSE?

What came before the big bang? Maybe other big bangs. The uncertainty principle holds that even the vacuum of space has淡淡微光。 Inflationary inflation theory says our universe exploded from such a fluctuation—a random event that, odds are, had happened many times before. Our cosmos may be one in a sea of others just like ours—or nothing like ours. These other cosmos will very likely remain forever inaccessible to observation, their possibilities limited only by our imagination.

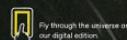
## HOW WILL IT END?

Which will win in the end, gravity or antigravity? Is the density of matter enough for gravity to halt or even reverse cosmic expansion, leading to a big crunch? It seems unlikely—especially given the power of dark energy, a kind of antigravity. Perhaps the acceleration in expansion caused by dark energy will trigger a big rip that shreds everything, from galaxies to atoms. If not, the universe may expand for hundreds of billions of years, long after all stars have died.



## Unidad 1

## Partículas, lo más pequeño



By through the universe on  
our digital edition  
LONDON PHOTOS: ANDREW FERGUSON; GENEVA PHOTOS: ART MONTAGNA DESIGN;  
SOURCES: CHARLES BENNETT, JOHN HESTER, ANDREW LINSLEY, ANDREW LINSLEY, UNIVERSITY OF CHICAGO, CERN, 2012 NATIONAL GEOGRAPHIC SOCIETY



# U2: Astrofísica, escalas intermedias

## 5 encuentros, del 01/Sep al 29/Sep

- **Estrellas.**
  - **Modelo politrópico.** La fusión nuclear estelar.
  - **Clasificación estelar.** Diagrama H-R.
  - **Evolución estelar.** Nebulosas.
- **Planetas**
  - **El Sistema Solar**
  - **Exoplanetas**
  - **Vida en el Universo: Astrobiología.**
- **Trabajo unidad → fecha máxima de entrega 12/Nov**



# Estrellas



Objeto astronómico que consiste en un esferoide luminoso y autogravitante luminoso de plasma



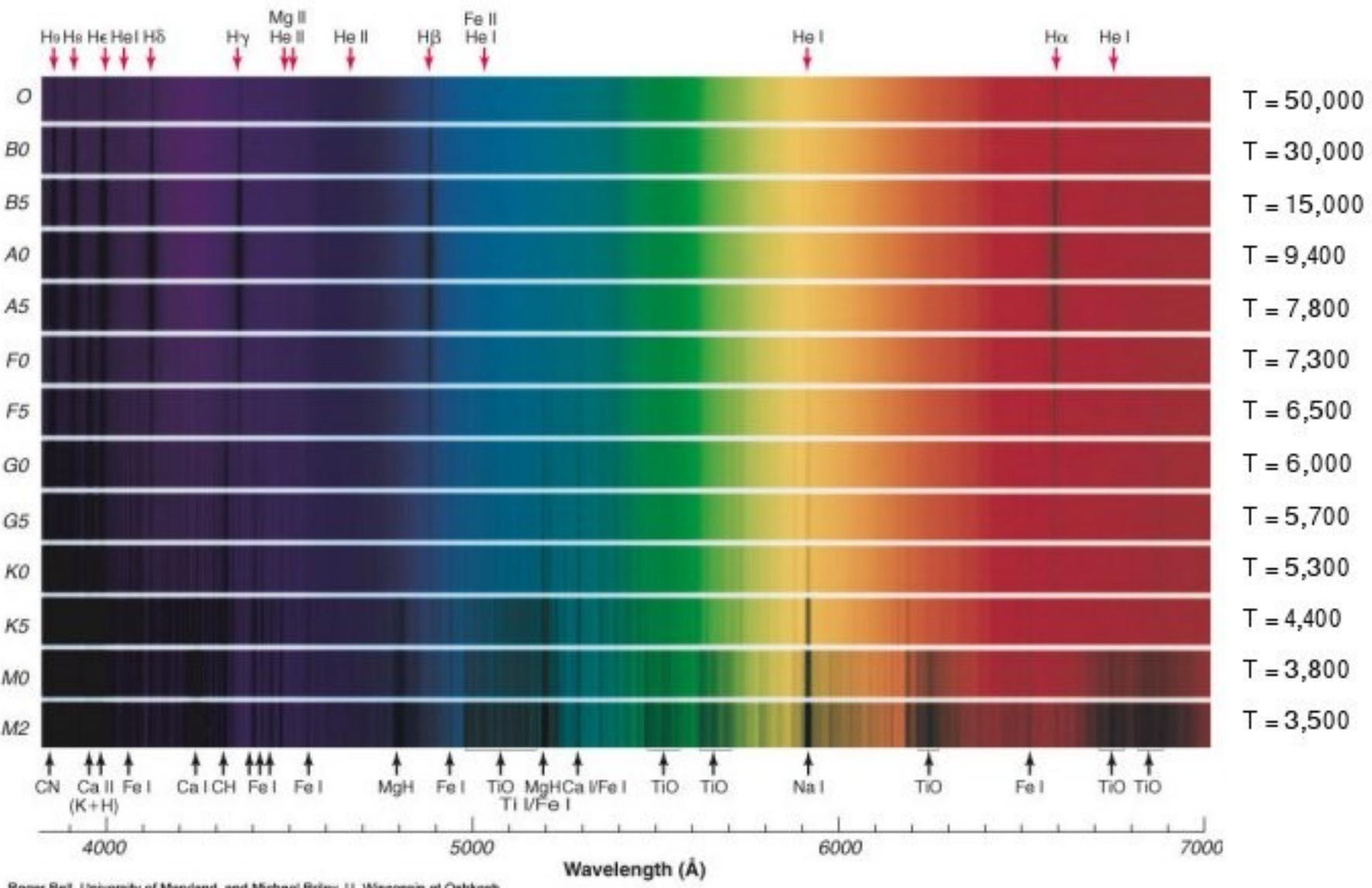
# No todas son iguales

- Estado de evolución (edad)
- Temperatura
- Constitución inicial
- Masa
- **Energía emitida por unidad de tiempo: Luminosidad**
- Y además...

$$L = \frac{\Delta E}{\Delta t}$$

**No todas están a la misma distancia**

# Clasificación espectral



Roger Bell, University of Maryland, and Michael Briley, U. Wisconsin at Oshkosh



# ¿Qué define todo?

- **Relaciones entre parámetros:**

- Luminosidad (L)
- Masa (M)
- Temperatura (T)
- Radio (R)

$$L = \frac{\Delta E}{\Delta t} = 4\pi\sigma R^2 T^4$$

$$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

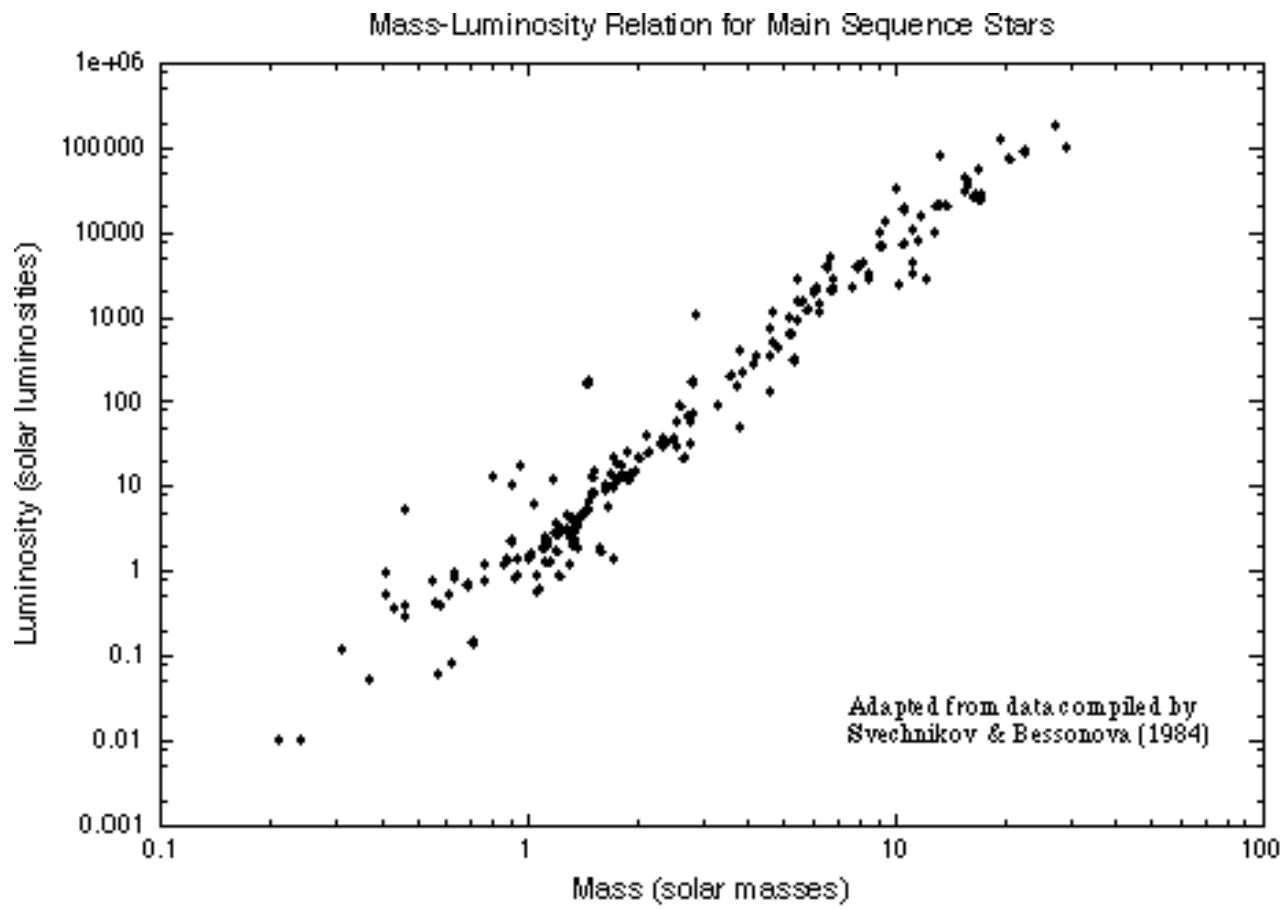
- **¿Cuál es el más importante en condiciones normales?**

- **Cantidad de materia → Masa**
- Está fijada por condiciones externas → Nacimiento

# Luminosidad → Masa

- Si:  $(0.1 < \text{Masa Estelar} < 50)$  masas solares:  
**L es proporcional a la  $M^4$**
- Nota: En general,  $M^a$ , con a entre 3 y 4 (~ masa)

$$\left( \frac{L_{\text{Estrella}}}{L_{\text{Sol}}} \right) = \left( \frac{M_{\text{Estrella}}}{M_{\text{Sol}}} \right)^4$$



Luminosidad → Masa

## Mass-Luminosity Relationship

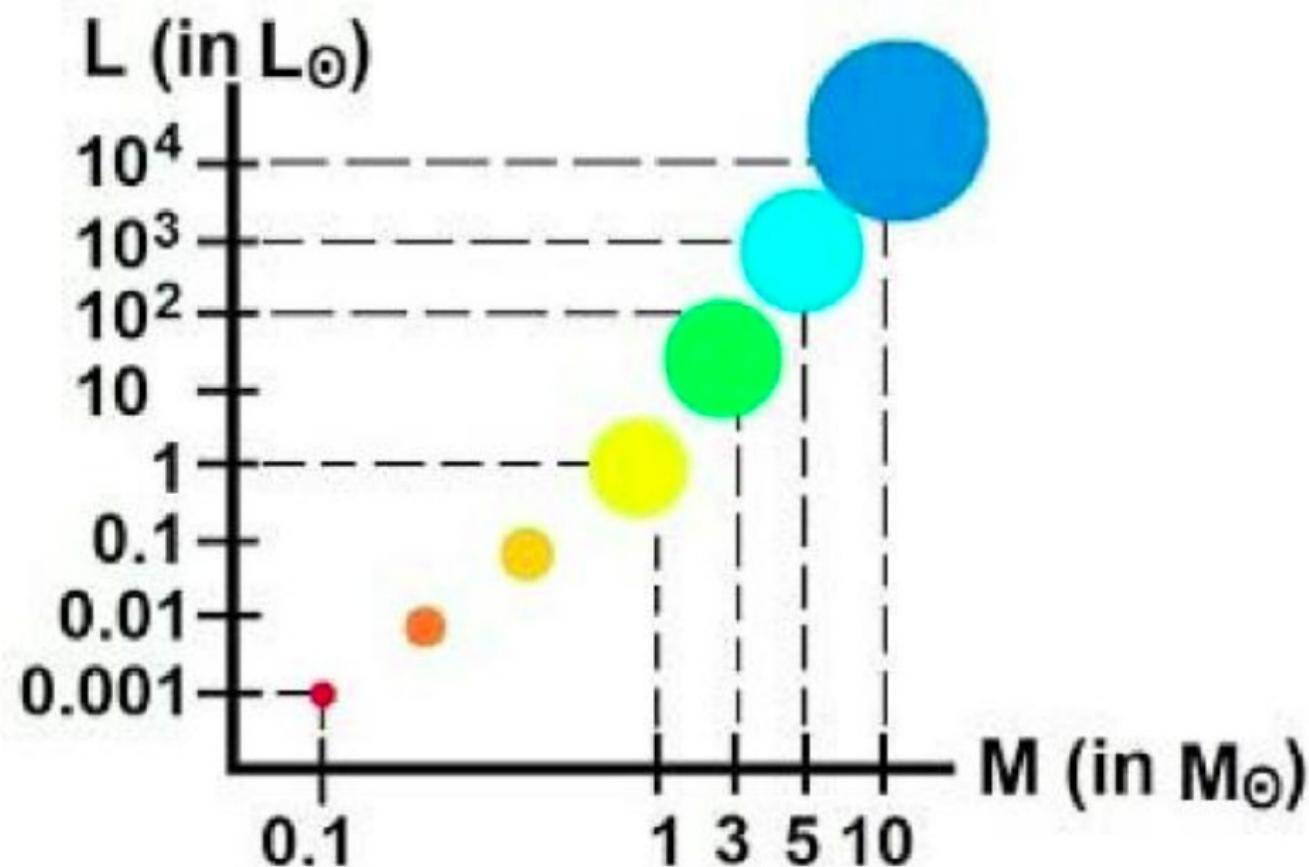
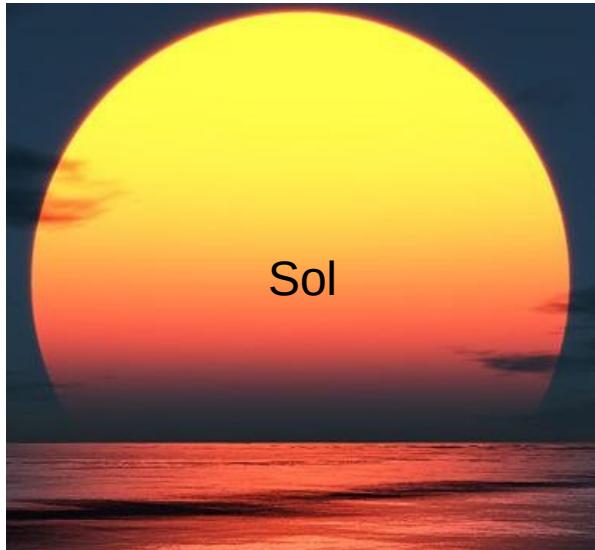


Fig. 2 Schematic representation of the mass-luminosity relation. Source: Astronomy – Ch. 17: The Nature of Stars: Mass-Luminosity Relationship

# Y: (Luminosidad, Temperatura) → Radio



- $T = 5700 \text{ K}$  (Amarilla)
- $T = 3400 \text{ K}$  (Roja)

$$L = (4\pi\sigma) R^2 T^4$$

¿Cómo será la luminosidad de Betelgeuse respecto al Sol?

# Y: (Luminosidad, Temperatura) → Radio



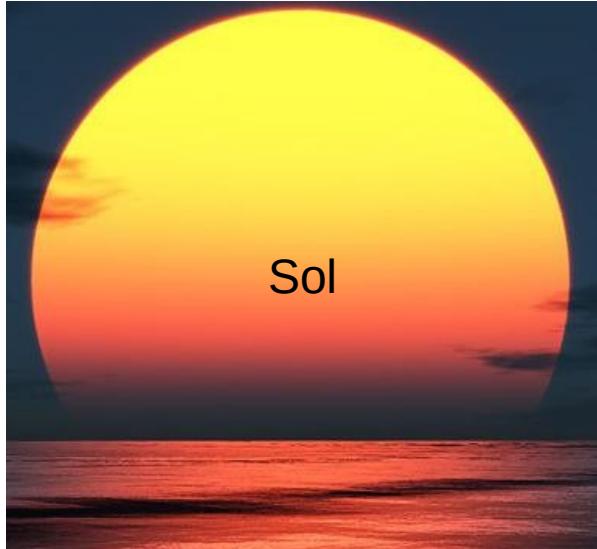
- $T = 5700 \text{ K}$  (Amarilla)
- $L = L_S$

Menor temperatura,  
menos emisión  
Pero, la emisión es  
mucho mayor

- $T = 3400 \text{ K}$  (Roja)
- $L = 135000 L_S$

$$L = (4\pi\sigma) R^2 T^4$$

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mucho mayor

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- $L = 135000 L_s$



**Betelgeuse debe  
ser mucho más  
grande que el  
Sol**



# Radio Estelar

- Veamos...  $(T_B/T_S)^4$ 
  - $(3400 / 5700)^4 \sim = 1/7.8$
  - Si dependiera sólo de T, el Sol sería 7.8 veces más luminoso que Betelgueuse
- Pero...

$$L_B / L_S = [(4\pi R_B^2) \sigma T_B^4] / [(4\pi R_S^2) \sigma T_S^4]$$

$$135000 = (R_B/R_S)^2 (T_B/T_S)^4$$

$$135000 = (R_B/R_S)^2 / 7.8$$

$$1.053 \times 10^6 = (R_B/R_S)^2$$

- Veamos...  $(T_B/T_S)^4$

$$R_B = 1026 R_{\text{Sol}}$$

**Betelgeuse es una supergigante roja**

mas luminoso que Betelgeuse

- Pero...

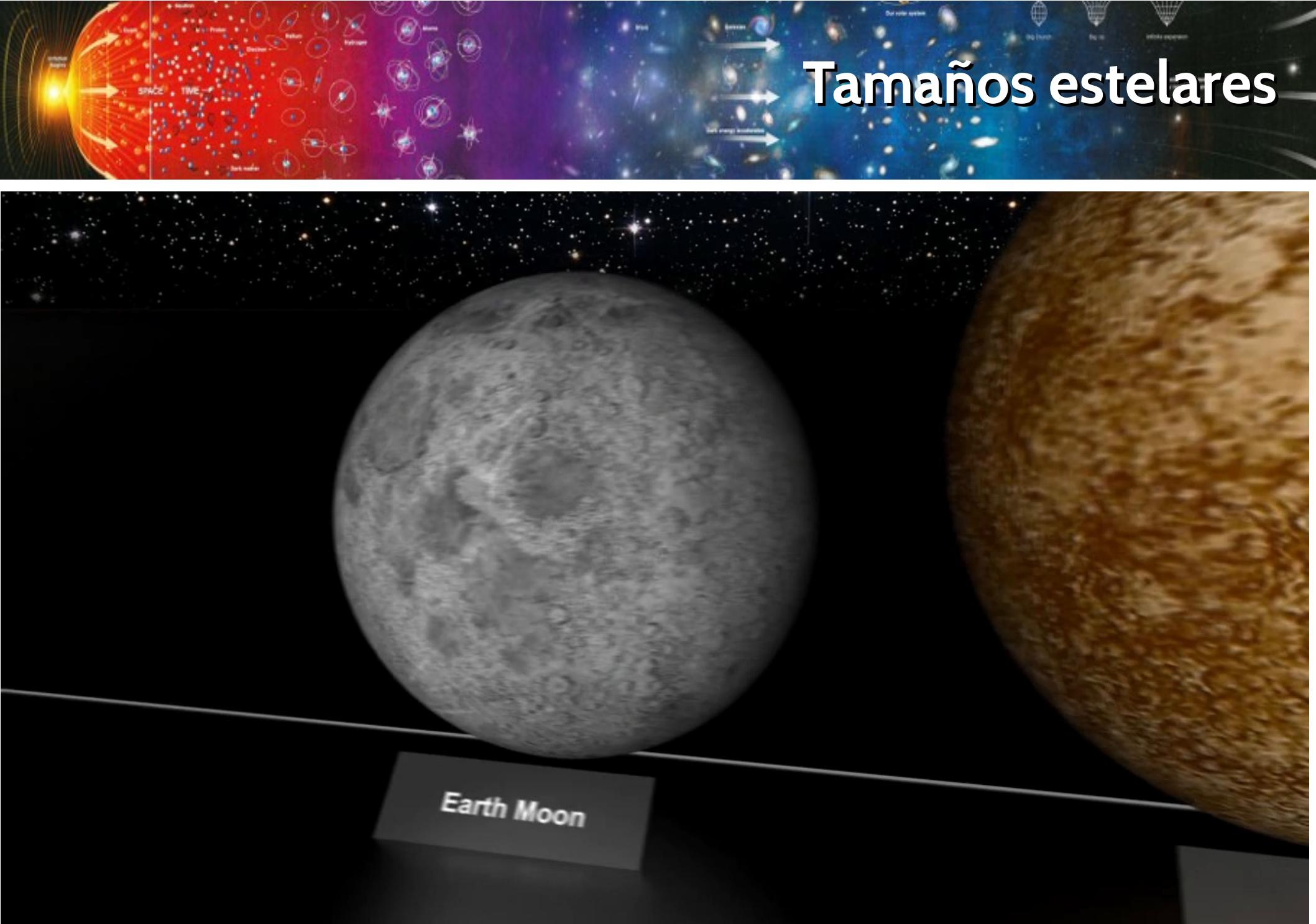
$$\frac{L_B}{L_S} = \frac{[(4\pi R_B^2) \sigma T_B^4]}{[(4\pi R_S^2) \sigma T_S^4]}$$

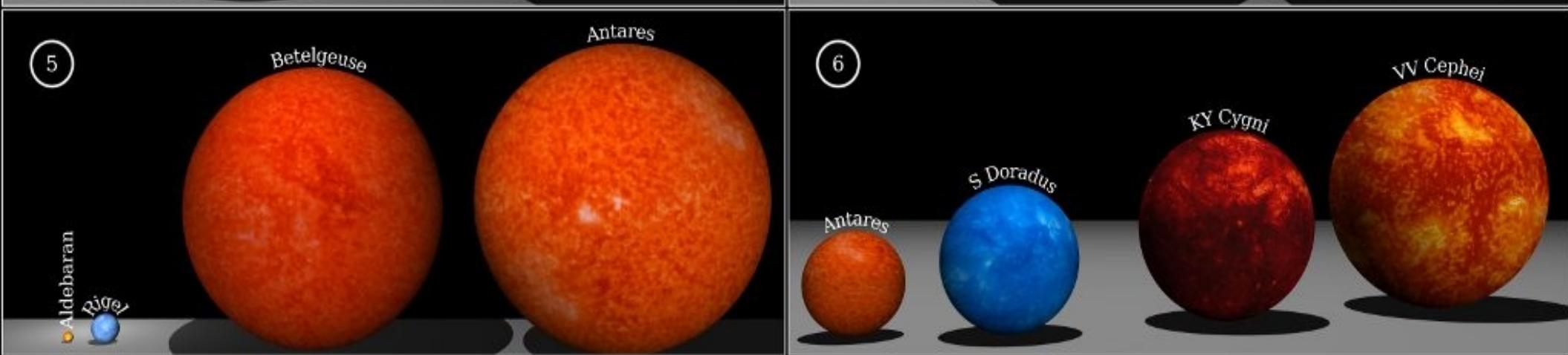
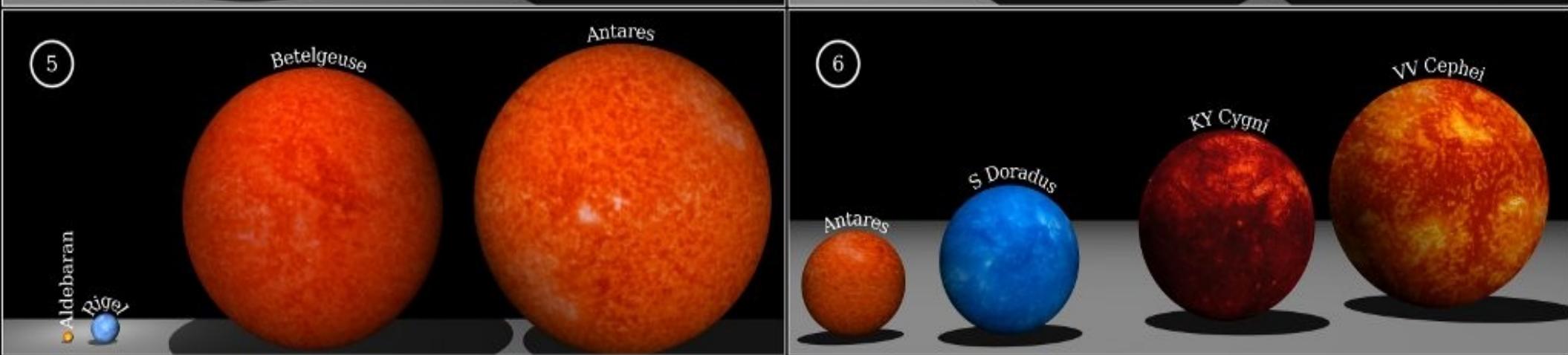
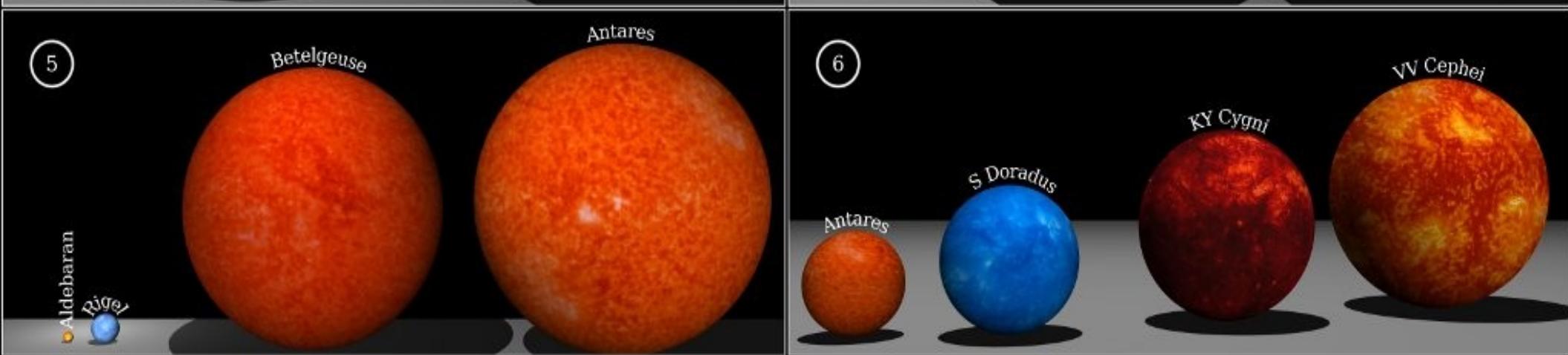
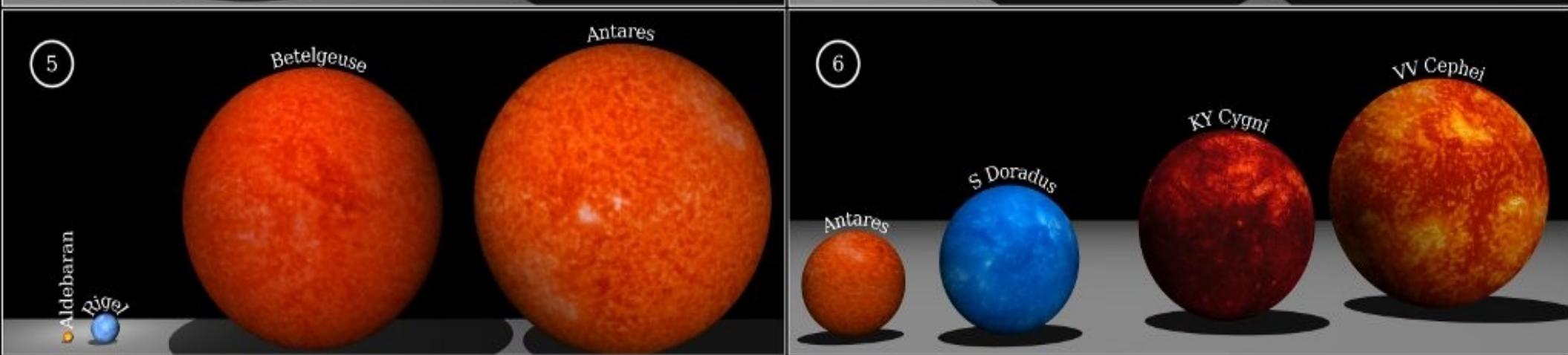
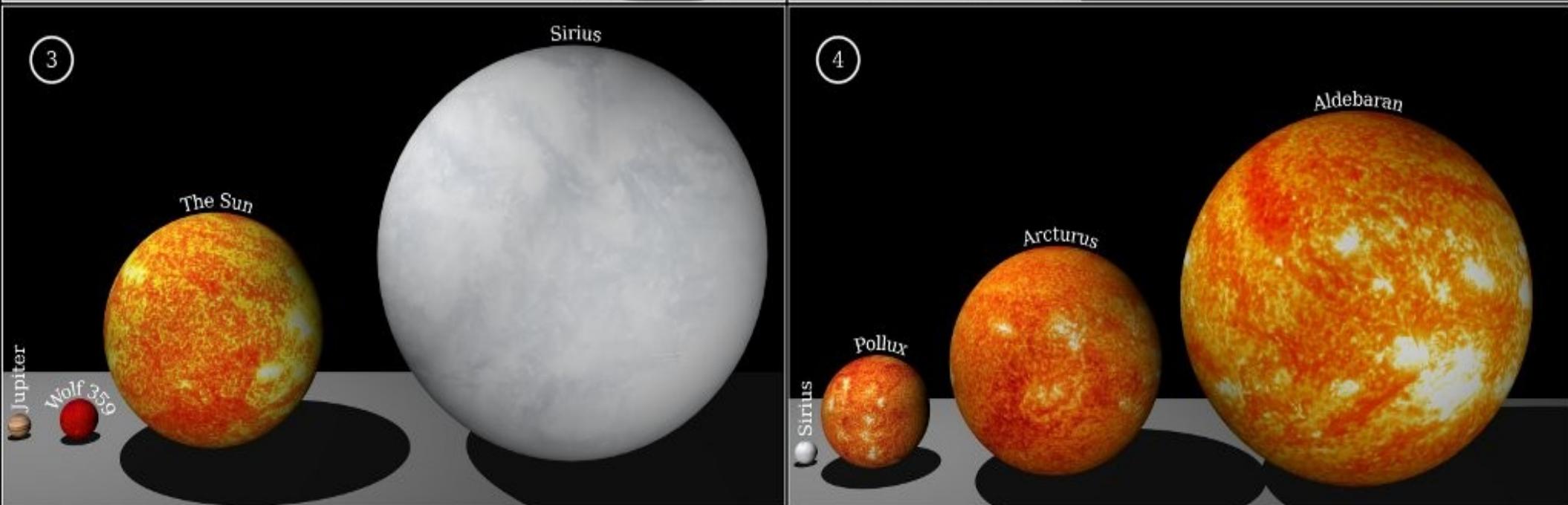
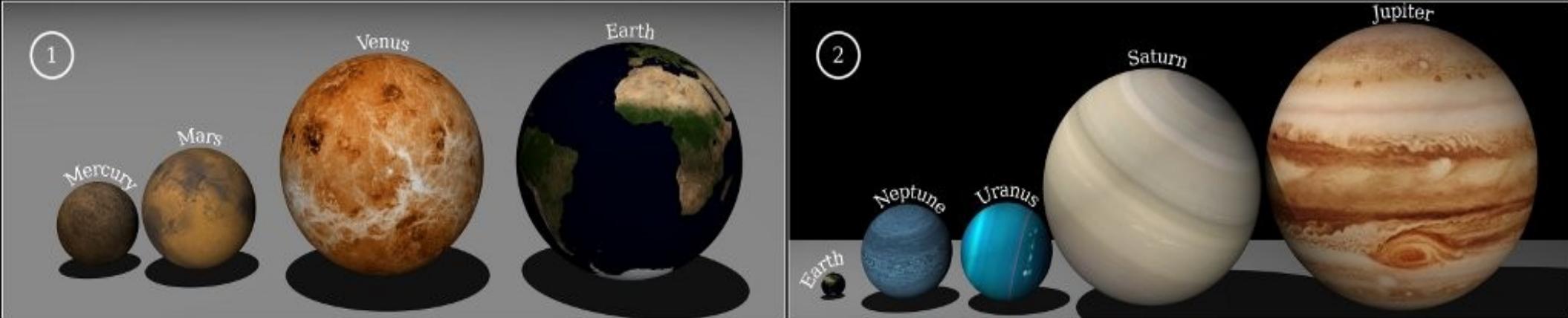
$$135000 = (R_B/R_S)^2 (T_B/T_S)^4$$

$$135000 = (R_B/R_S)^2 / 7.8$$

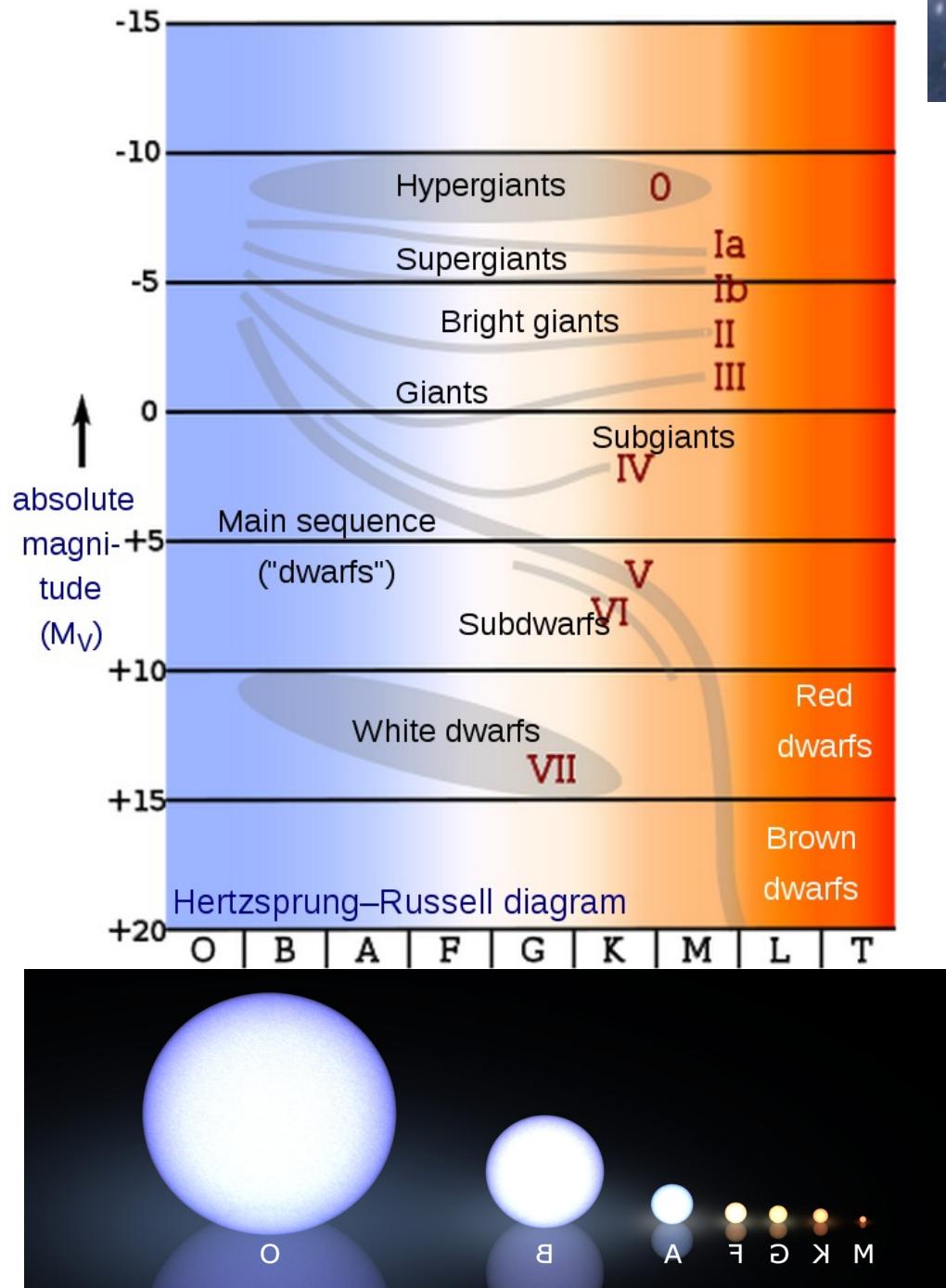
$$1.053 \times 10^6 = (R_B/R_S)^2$$

# Tamaños estelares





# Dijimos que la masa define todo

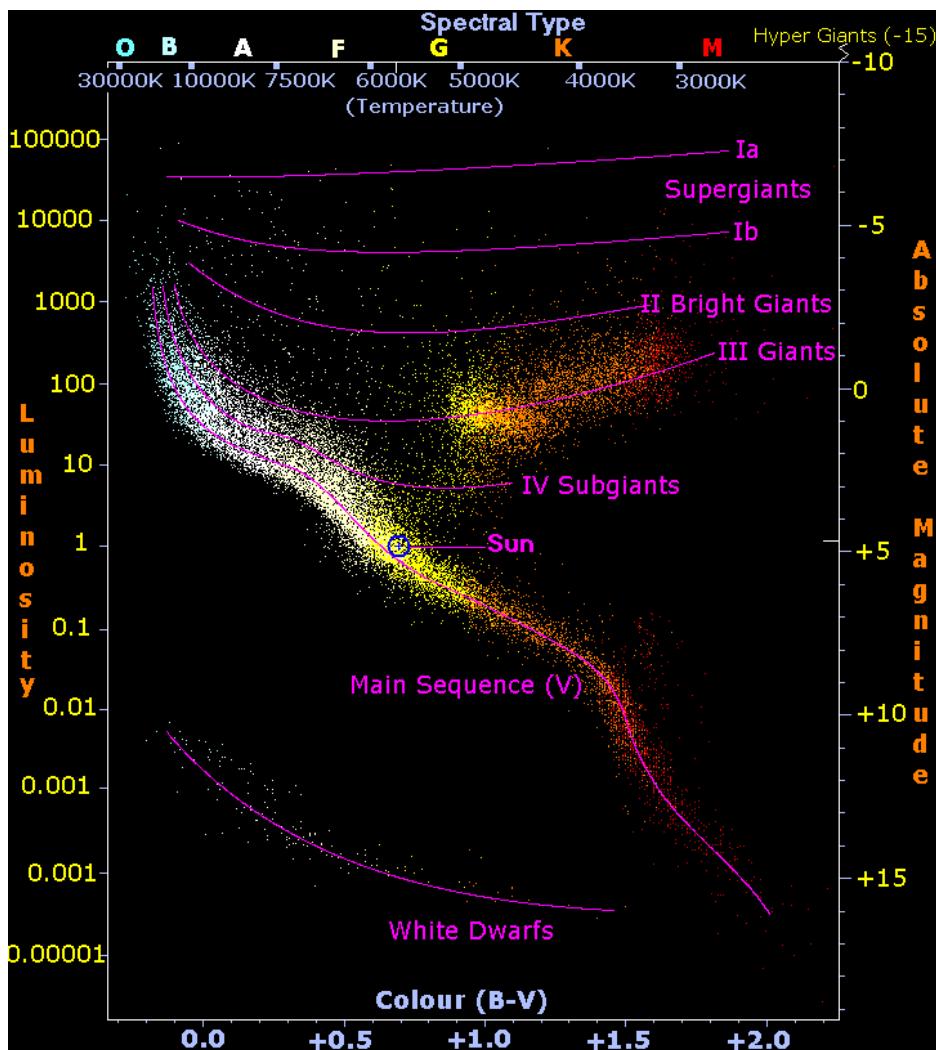


<https://www.nasa.gov/subject/6892/stars/>

Surface temperature ranges for different stellar classes<sup>[134]</sup>

Class	Temperature	Sample star
O	33,000 K or more	Zeta Ophiuchi
B	10,500–30,000 K	Rigel
A	7,500–10,000 K	Altair
F	6,000–7,200 K	Procyon A
G	5,500–6,000 K	Sun
K	4,000–5,250 K	Epsilon Indi
M	2,600–3,850 K	Proxima Centauri

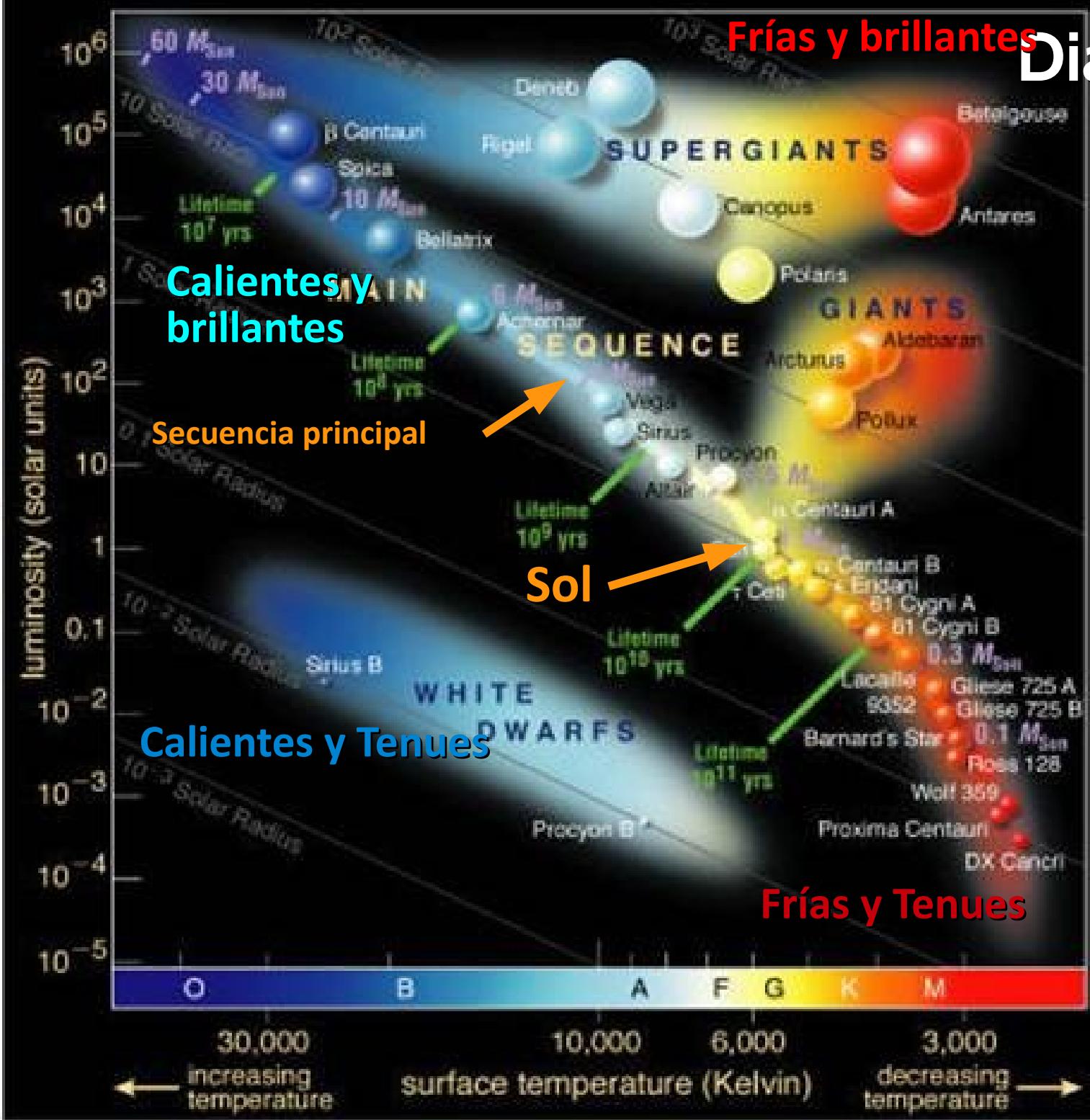
# Secuencia principal



- Estrellas que están en su fase normal de quema de combustible:  
 $H \rightarrow He$
- 90% del tiempo de vida las estrellas permanecen en este estadio
- Metalicidad: contenido de elementos más masivos que el Helio
- OBAFGKM

Frías y brillantes

# Diagrama H-R



$$T^4$$

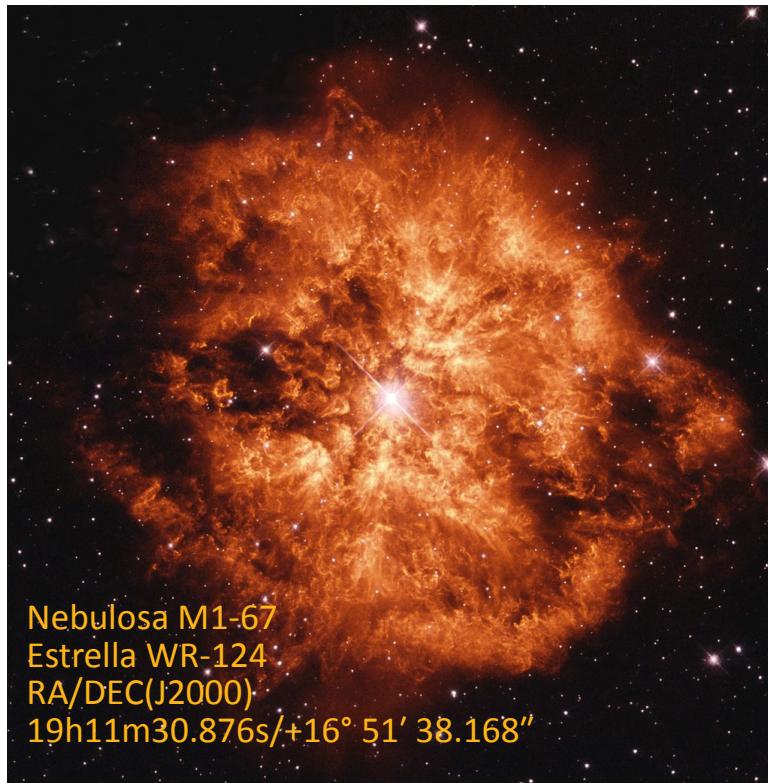
$$R^2$$

$$L$$

# Otros tipos de estrellas

- **W: Wolf-Rayet**

- **Estrellas masivas ( $>20M_s$ )**
- **Excesivamente calientes**



- **Los nuevos: L, T, Y**

- **L: Enanas frías o subestrellas, temperaturas  $1300K < T < 2400K$**
- **T: Enanas marrones (subestrellas, sin fusión H) con prominencia de metano y  $500K < T < 1300K$**
- **Y: enanas marrones ultra frías (superplanetas?) c/amoníaco y  $T < 600K$ , y  $10 < M/M_{Júp} < 90$**

# Estrellas supermasivas

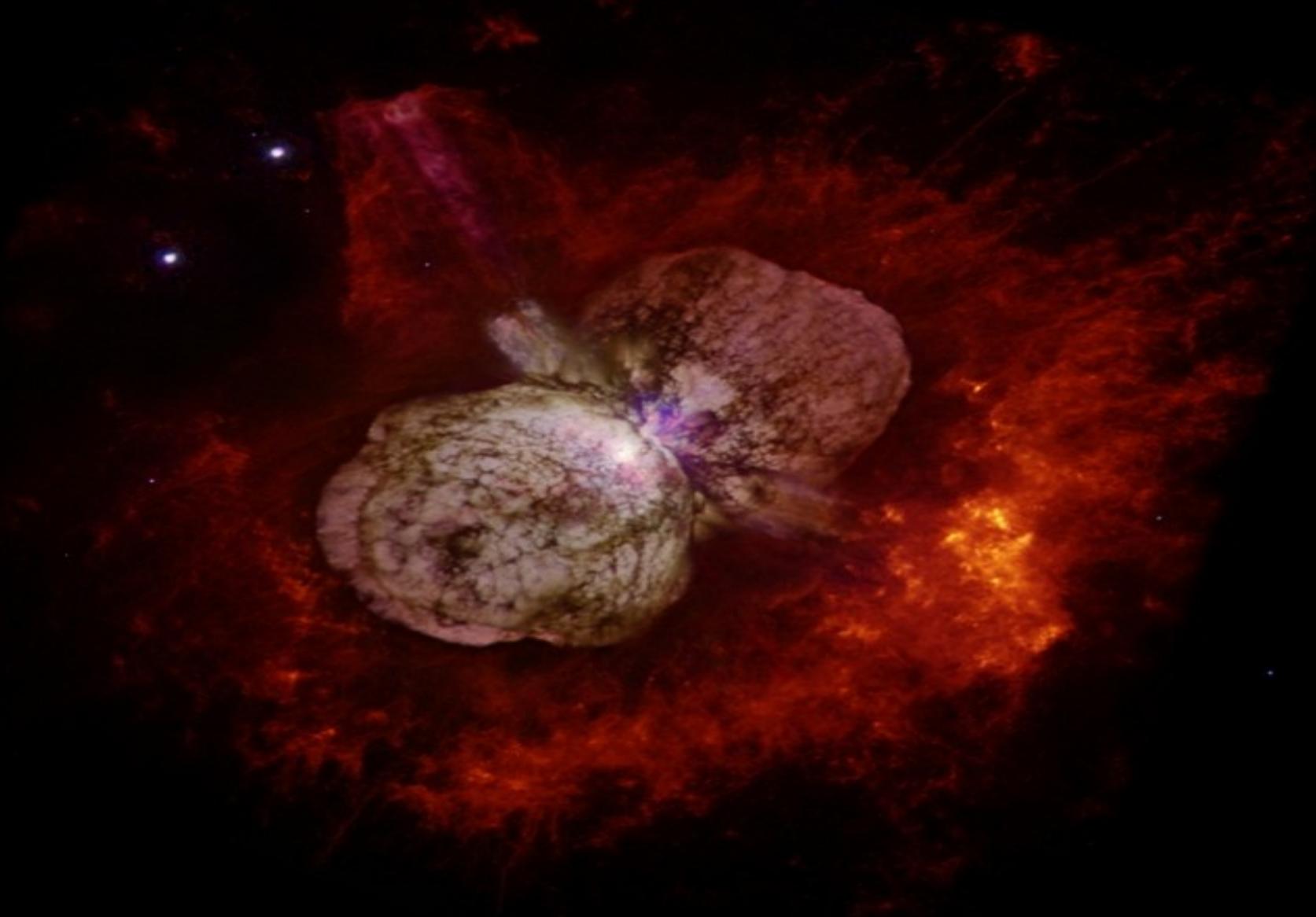
NGC3372 – Nebulosa Carina (Carina)

# Estrellas supermasivas



NGC3372 – Nebulosa Carina (Carina)

# $\eta$ Carinae: Una binaria a punto caramelo



Nebulosa Homúnculo en la Nebulosa Eta Carina (en la Nebulosa Carina)



# ¿por qué brillan?

- Hipótesis de Kelvin → Energía del colapso gravitatorio de la protoestrella.
- Suponiendo colapso desde el infinito hasta una esfera de masa  $R$ :

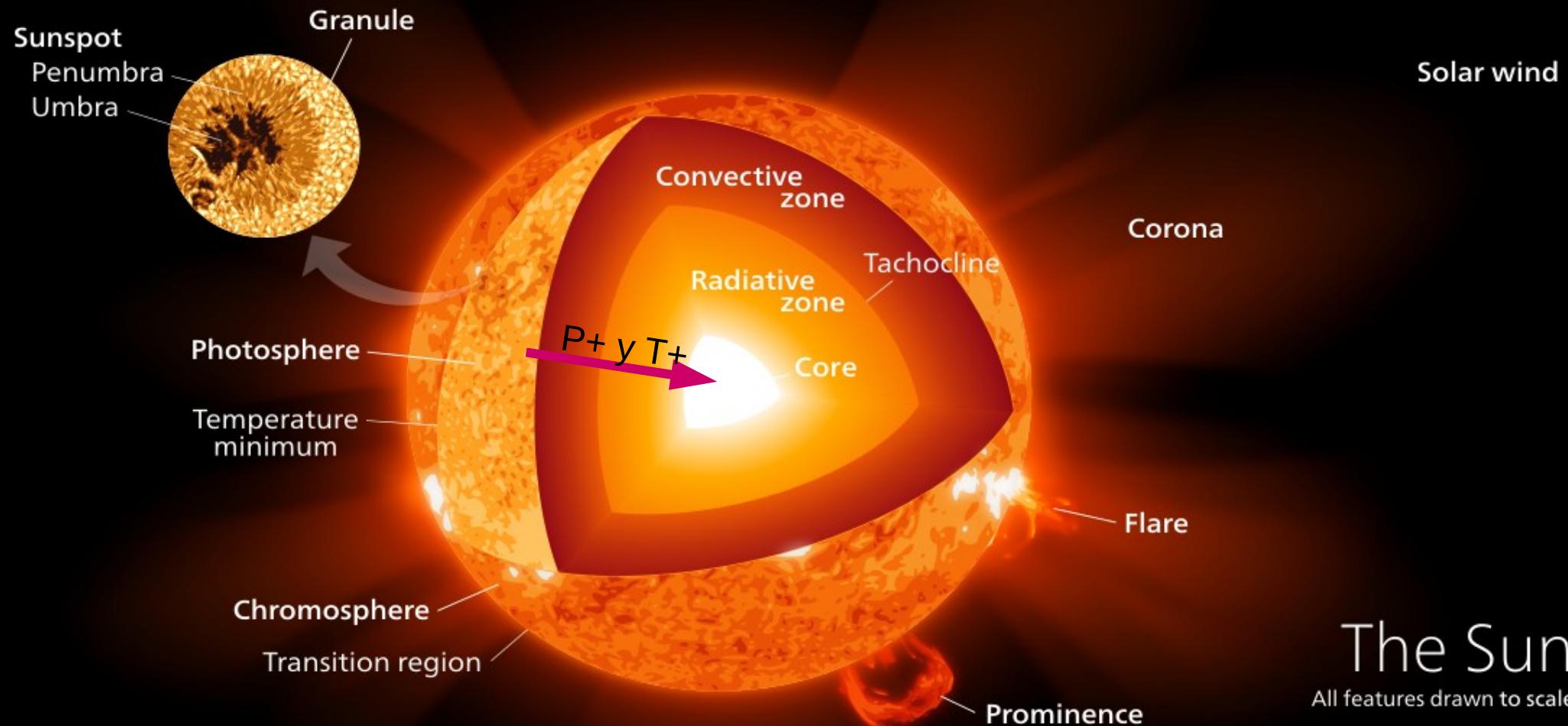
$$E_G = \frac{GM^2}{R}, \text{ y para el Sol: } E_G = \frac{GM_\odot^2}{R_\odot} \rightarrow E_G \approx 3,8 \times 10^{41} \text{ J}$$

- Luego, la vida del Sol será

$$\tau_\odot = \frac{E_G}{L_\odot} \rightarrow \tau_\odot \approx 9,9 \times 10^{14} \text{ s} \sim 3 \times 10^7 \text{ años}$$

- Factor  $\sim 10^{-3}$ , gracias Kelvin, seguí participando...

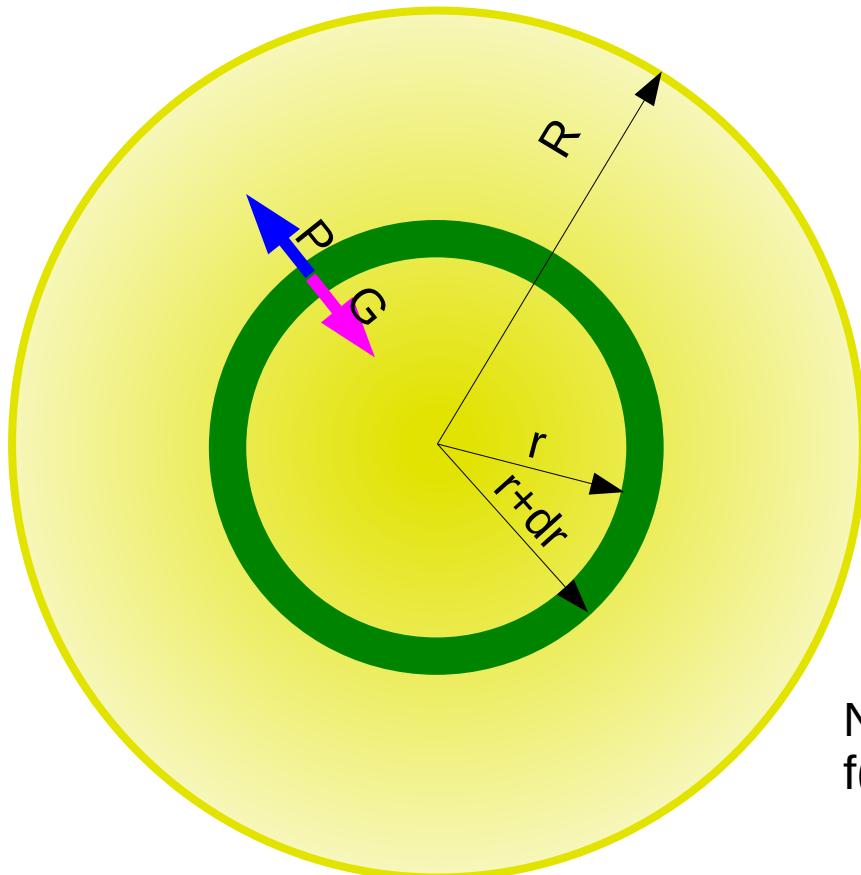
# Estructura de una estrella típica (Sol)



The Sun

All features drawn to scale

# Estructura estelar



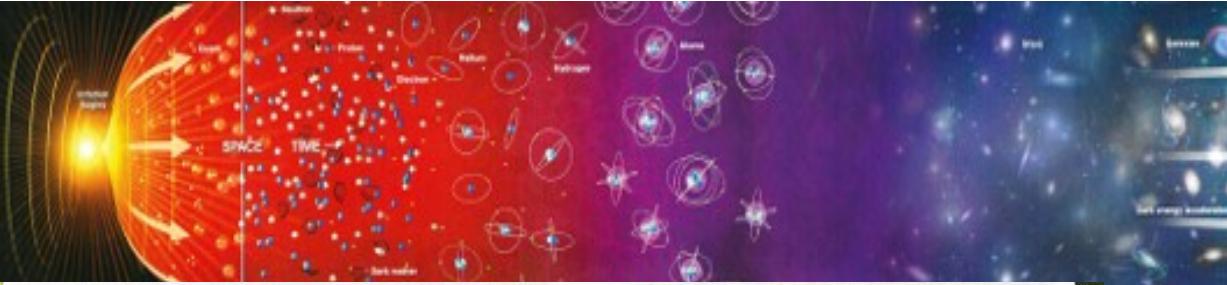
Notación:  
 $f(r) \rightarrow f_r$

- Suponemos esfera “perfecta”
- Parámetros estelares:  $M_s, R_s, T_s, L_s$
- De adentro hacia afuera ( $r \rightarrow$ )
  - Temperatura  $T$  disminuye
  - Densidad  $\rho$  disminuye
  - Gravedad  $G$  aumenta
  - Presión  $P$  disminuye
  - Emisión energía  $l$  aumenta
- Estado estacionario y dos parámetros  $\varepsilon(r)$  y  $\kappa(r)$ :

$\kappa(r) \equiv \kappa_r \propto \frac{1}{\rho(r) dr}$ : coeficiente específico de absorción

$\epsilon(r) \equiv \epsilon_r = \frac{dE}{dm dt}$ : producción de energía por unidad de masa y tiempo

# Modelo estelar, 1



## ① EC de continuidad.

En una estrella con densidad  $\rho(r)$  la gravedad en  $r$  es producida por los masas contenidas entre los radios  $r$ :

$$\vec{F}_G = -\frac{GM(r)m}{r^2} \hat{r}$$

Sea una capa de espesor  $dr \Rightarrow m \rightarrow dm = 4\pi r^2 dr \rho(r) \rightarrow$

Conservación de Masa  $\boxed{dm = 4\pi r^2 \rho(r) dr} \quad (1)$

Luego  $\vec{F}_G = -\frac{GM(r)dm}{r^2} \hat{r} \rightarrow d\vec{F}_G = -\frac{GM(r)}{r^2} \cdot 4\pi r^2 \rho(r) dr \hat{r}$

Este capa no colapsa porque se sostiene por la fuerza de presión de las capas interiores:

$$d\vec{F}_P = d\vec{p} \cdot A \Rightarrow d\vec{F}_P = 4\pi r^2 dp \hat{r}$$

Finalmente, equilibrio hidrostático:  $d\vec{F}_G(r) = d\vec{F}_P(r)$ . Módulo:  $y signo$

$$\Rightarrow 4\pi r^2 dp = -\frac{GM(r)}{r^2} \rho(r) dr$$

$$\Rightarrow \boxed{dp = -\frac{GM(r)\rho(r)}{4\pi r^3} dr} \quad (2)$$

Equilibrio hidrostático

## ② Conservación de la energía

La capa de espesor  $dr$  a prof.  $r$  produce  $E(r)$  juntas por unidad de masa y tiempo:  $E = \frac{dE}{dm dt} \Rightarrow E \cdot dm = \frac{dE}{dt} = dE(r)$

Donde  $E(r)$  es la luminosidad de la estrella en profundidad  $r$ . ( $E(r=R)=L$ ) .

$$\Rightarrow dE = 4\pi r^2 dr \rho(r) \cdot E(r)$$

Balance y Producción de Energía  $\Rightarrow \boxed{dE = 4\pi r^2 \rho(r) E(r) r^2 dr} \quad (3)$

$$\boxed{E(r+dr) = E(r) + dE}$$

Conservación de la Energía.

③ Sea  $q(r)$  el flujo de energía por unidad de radio de la profundidad  $r$ .

$$E(r) = 4\pi r^2 q(r)$$

Puesto que se emite una en un campo negro en temperaturas  $T \Rightarrow E(r) = \sigma T^4$  ( $A$  es el área) luego

$$q(r) = \sigma T(r)^4 \quad \text{es el flujo} \quad (\Delta E / \Delta A, \Delta t)$$

Debido a que  $T$  varía con la profundidad  $\Rightarrow$

$$\begin{aligned} dq &= \sigma dT^4 = 4\sigma T^3 dT \\ \Rightarrow dq &= 4\sigma T(r)^3 dT \end{aligned}$$

B+

# Modelo estelar, 2

Dado que la energía se conserva, los cambios de flujo es producido por la absorción en la capa  $\rightarrow$  se pierde.

El flujo se absorbe  $\Rightarrow d\dot{q} \leftarrow$

$$d\dot{q} = -K_f q \rho dr$$

mesa por  
medio de  
superficie.

Coef de  
Absorción.

Flujo de energía.

Luego, por conservación, la variación del flujo es igual a la de  $(y es la responsabilidad del gradiente de T)$ :

$$4\pi r^3 dT = -K_f \rho dr$$

Luego,  $q(r)$ :

$$4\pi r^3 dT = -K_f \rho dr \Rightarrow$$

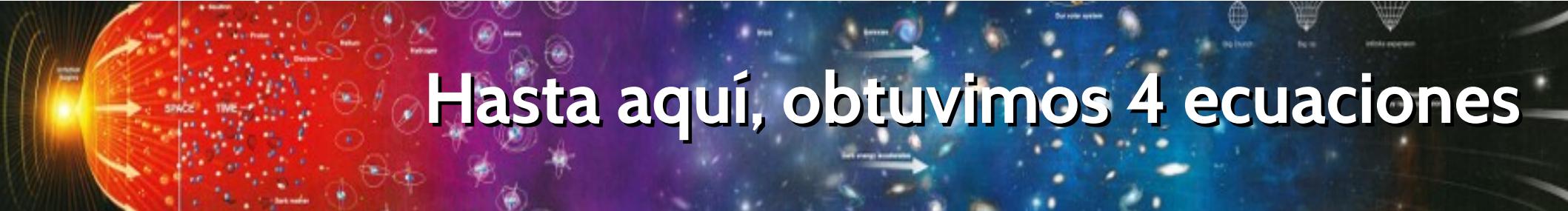
$$q = -\frac{4\pi r^3}{K_f \rho} \frac{dT}{dr}$$

y entonces  $\ell(r)$ :  $\ell(r) = 4\pi r^2 q \Rightarrow$

Ecu de transporte radiativo.

$\ell(r) = -\frac{16\pi r}{K_f \rho} \cdot r^2 T^3 \frac{dT}{dr}$	(4)
---	-----

Dado que  $dT/dr < 0 \Rightarrow \ell(r) > 0$ .  $\therefore$



# Hasta aquí, obtuvimos 4 ecuaciones

- Todas son ecuaciones diferenciales ordinarias y dependen de la coordenada radial  $r$ .
  - Balance de masa:
  - Equilibrio hidrostático:
  - Balance de energía:
  - Transporte radiativo:
- Y tenemos 5 variables:  $M_r$ ,  $P_r$ ,  $\rho_r$ ,  $l_r$  y  $T_r$ , y 2 parámetros  $\epsilon_r$ ,  $\kappa_r$ .

$$\frac{dM_r}{dr} = 4\pi \rho_r r^2$$

$$\frac{dP_r}{dr} = -\frac{GM_r \rho_r}{r^2}$$

$$\frac{dl_r}{dr} = 4\pi \rho_r \epsilon_r r^2$$

$$\frac{dT}{dr} = -\frac{l_r \kappa_r \rho_r}{16\pi \sigma r^2 T^3}$$



# Ecuación de estado $\rightarrow P=P(\rho, T)$

- $\varepsilon_r$ ,  $\kappa_r$  surgen de las condiciones locales:

$$\epsilon_{pp}(r) \propto T^4 \text{ y } \epsilon_{CNO}(r) \propto T^{20} \text{ y } \kappa(r) = \text{cte}$$

- Por ejemplo, ec. de estado de mezcla gases ideales:

$$P = \frac{N}{V} k T \rightarrow P = \frac{\langle m \rangle N}{\langle m \rangle V} k T \rightarrow P = \left( \frac{k}{\langle m \rangle} \right) \rho T \rightarrow P = \alpha \rho T \simeq (\alpha T) \rho^1$$

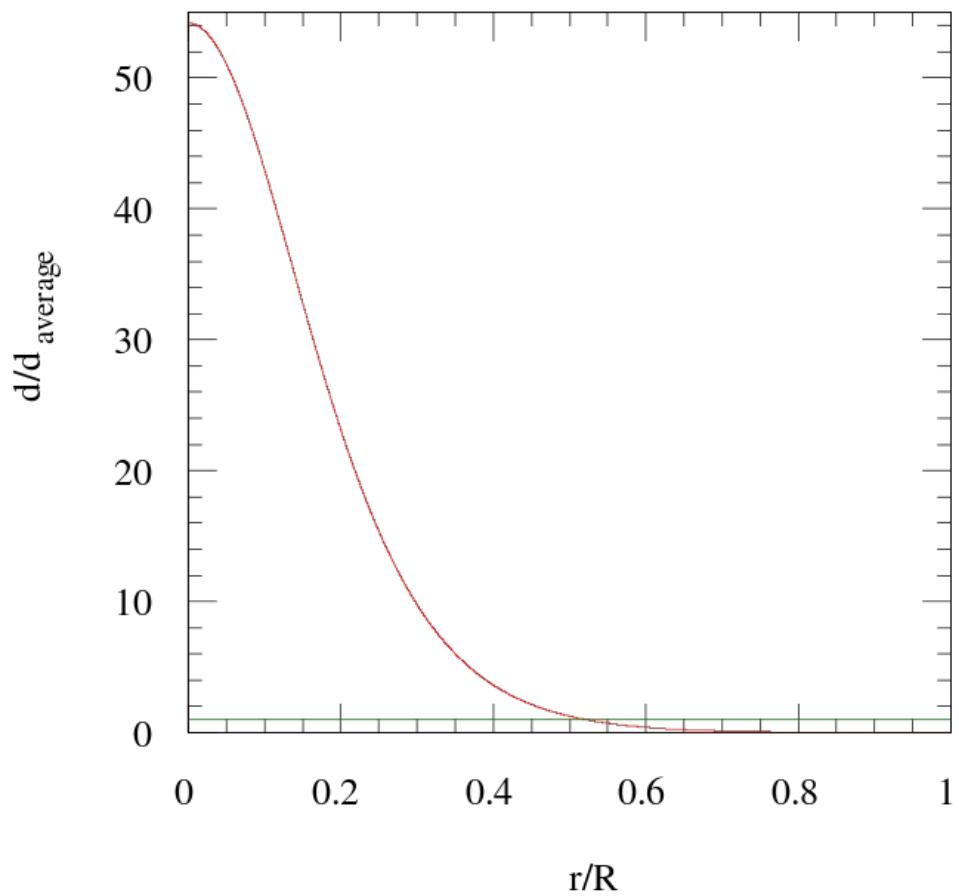
$$\text{Estrella gas ideal} \rightarrow P_r = \alpha T_r \rho_r^1$$

- Con la ecuación de estado tenemos un sistema cerrado

podemos calcular todas las variables  
→  $P(r); r(r); T(r); l(r); M(r)$  ←

# Politropos

Polytrope with index n=3



- En general, se denomina politropo a un fluido tal que:
- $$P = K \rho^{(n+1)/n}$$
- $n$  es el índice politrópico:
    - $\frac{1}{2} < n < 1 \rightarrow$  estrellas neutrones
    - $n = 3/2 \rightarrow (\gamma = 5/3)$ : planetas gaseosos, estrella gas ideal, enanas blancas no relativ.
    - $n = 3 \rightarrow$  estrellas secuencia principal ( $\gamma = 4/3$ ) y enanas blancas



# Tenemos 5 ecuaciones (4 e.d.o.) acopladas

- Balance de masa:

$$\frac{dM_r}{dr} = 4\pi \rho_r r^2 \quad (1)$$

- Equilibrio hidrostático:

$$\frac{dP_r}{dr} = -\frac{GM_r \rho_r}{r^2} \quad (2)$$

- Balance de energía:

$$\frac{dl_r}{dr} = 4\pi \rho_r \epsilon_r r^2 \quad (3)$$

- Transporte radiativo:

$$\frac{dT}{dr} = -\frac{l_r \kappa_r \rho_r}{16\pi \sigma r^2 T^3} \quad (4)$$

- Ecuación de estado:

$$P_r = \frac{k}{\langle m \rangle} T_r \rho_r \quad (5)$$



# Con estas condiciones de borde

Centro, $r=0$	Superficie, $r=R_s$
$M(0) = M_0 = 0$	$M(R_s) = M_s$
$l(0) = l_0 = 0$	$l(R_s) = L_s$
$P(0) = P_0$	$P(R_s) = 0$
$T(0) = T_0$	$T(R_s) = T_s \sim 0$
$\rho(0) = \rho_0$	$\rho(R_s) = 0$
$\varepsilon(0) = \varepsilon_0$	$\varepsilon(R_s) = 0$
$\kappa = \text{cte}$	$\kappa = \text{cte}$

# Integración numérica a mano de dos pasos

- Consideramos dos puntos, en el centro ( $r=0$ ) y en la superficie externa ( $r=R_s$ ). Es burdo pero efectivo →

- Diferenciales a diferencias finitas:

$$dM_r \rightarrow \Delta M_r = M_{R_s} - M_0 = M_s - 0 = M_s$$

- Evaluaciones a valores promedios

$$\rho_r \rightarrow \rho_r = \frac{\rho_{R_s} + \rho_0}{2} = \frac{0 + \rho_0}{2} = \frac{\rho_0}{2}$$

$$r^2 \rightarrow r^2 = \left( \frac{r_{R_s} + r_0}{2} \right)^2 = \left( \frac{R_s + 0}{2} \right)^2 = \frac{R_s^2}{4}$$

Usando los valores en los bordes y de (2):

$$dM = 4\pi \rho_r r^2 dr \rightarrow \Delta M = 4\pi \rho_r r^2 4r$$

Tomar valores medios para las variables  $\Rightarrow r \rightarrow r/2$  y

$$\rho_{r/2} \rightarrow \rho_{r/2} = \frac{\rho_0 + \rho_s}{2} = \frac{\rho_0}{2}$$

$$r^2 \rightarrow r^2 = \left(\frac{R_s + 0}{2}\right)^2 \Rightarrow r^2 = R_s^2/4.$$

$$\text{y } \Delta M = M_{r=R_s} - M_{r=0} \Rightarrow \Delta M = M_s \quad \text{y } \Delta P = R_s - 0 = R_s.$$

$$\Rightarrow M_s = 4\pi \frac{\rho_0}{2} \cdot \frac{R_s^2}{4} \cdot R_s = 0 \quad \boxed{M_s = \frac{\pi}{2} \rho_0 R_s^3} \quad (6)$$

El mismo proceso en (2)  $\Rightarrow$

$$dP = \frac{G M_r \rho_r}{r^2} dr \rightarrow \Delta P = \frac{G M_r \rho_r}{r^2} \Delta r.$$

$$(P_{R_s} - \rho_0) = - \frac{G M_r \rho_r}{r^2} (R_s - 0). \quad \text{y } M_r = \frac{M_{R_s} - M_0}{2} = M_s/2$$

$$\Rightarrow \rho_0 = + \frac{G M_s}{2} \frac{\rho_0}{3} \frac{R_s}{R_s^2/4}$$

$$\Rightarrow \boxed{\rho_0 = \frac{G M_s \rho_0}{R_s}} \quad (7)$$

Para (3) y tomando  $\epsilon_r = (\epsilon_0 - 0)/2 \Rightarrow$

$$dE = 4\pi \rho_r \epsilon_r r^2 dr \rightarrow \Delta E = 4\pi \rho_r \epsilon_r r^2 \Delta r =$$

$$L_s - 0 = 4\pi \frac{\rho_0}{2} \cdot \frac{\epsilon_0}{2} \cdot \frac{R_s^2}{4} \cdot (R_s - 0) \Rightarrow \boxed{L_s = \frac{\pi \rho_0 \epsilon_0 R_s^3}{4}} \quad (8)$$

Para la ec. de transferencia (4) tenemos:

$$l_r = \frac{-16\pi G}{\kappa \rho r} \cdot r^2 T_r^3 \frac{\Delta T}{dr} = 0.$$

$$l_r = \frac{-16\pi G}{\kappa \rho r} \cdot r^2 T_r^3 \frac{\Delta T}{\Delta r} = 0$$

$$\frac{L_s - 0}{2} = \frac{-16\pi G}{\kappa \frac{\rho_0}{2}} \cdot \frac{R_s^2}{4} \left(\frac{T_0}{2}\right)^3 \frac{(0 - T_s)}{(R_s - 0)}$$

$$\frac{L_s}{2} = + \frac{16\pi G \cdot Z}{\kappa \rho_0} \cdot \frac{R_s^2 T_0^3}{A \cdot B} \cdot \frac{(T_s)}{R_s}.$$

$$\boxed{L_s = \frac{2 \pi G}{\kappa \rho_0} \cdot R_s T_0^4} \quad (9)$$

Finalmente en la ec. de estados. (5)

$$P_r = \frac{k_B}{\text{cm}^3} T_r \rho_r = 0 \quad \frac{\rho_0 - 0}{2} = \frac{k_B}{\text{cm}^3} \cdot \frac{T_0}{2} \cdot \frac{\rho_0}{2}$$

$$\Rightarrow \boxed{P_0 = \frac{\rho_0}{2} \cdot k_B T_0.} \quad (10)$$

Despejando  $\rho_0$  en (6) y (7) e igualando:

$$\rho_0 = \frac{2 M_s}{\pi R_s^3} \quad \text{y} \quad \frac{R_s \rho_0}{G M_s} = \rho_0 = 0 \quad \frac{2 M_s}{\pi R_s^3} = \frac{R_s \rho_0}{G M_s}$$

$$\Rightarrow \boxed{\rho_0 = \frac{2}{\pi} \frac{G M_s^2}{R_s^4}} \quad \text{La presión en el auto}$$



# Entonces, poniendo números para el Sol

$$P_0 = \frac{2}{\pi} G \frac{M_S^2}{R_S^4} \approx 8 \times 10^{14} \text{ Pa} \approx \frac{P_\odot}{35} \rightarrow P_0 \propto M_S^2$$

$$\rho_0 = \frac{2}{\pi} \frac{M_S}{R_S^3} \approx 4 \text{ g/cm}^3 \approx \frac{\rho_\odot}{35} \rightarrow \rho_0 \propto M_S$$

$$T_0 = \frac{G \langle m \rangle}{k_B} M_S R_S \approx 2,3 \times 10^7 \text{ K} \approx 1,7 T_\odot \rightarrow R_S \propto M_S$$

$$L = \left( \frac{\pi^2 \sigma}{\kappa} \right) \left( \frac{G \langle m \rangle}{k_B} \right)^4 M_S^3 \rightarrow L \propto M_S^3 . \text{ En general: } L \propto M_S^x ; \quad 3 < x < 5$$

Todas las variables estelares pueden ser relacionadas con la masa estelar

# Tres datos fundamentales

- Relación luminosidad masa:

$$L \propto M^3 \text{ (masivas } L \propto M^4\text{)} \rightarrow \left( \frac{L}{L_{\odot}} \right) = \left( \frac{M_s}{M_{\odot}} \right)^x ; \quad 3 < x < 4$$

- Tiempo de vida de la estrella  $\rightarrow E = mc^2 \rightarrow$

$$\tau_s \propto \frac{M_s}{L} \propto \frac{M_s}{M_s^x} \propto M_s^{1-x} \quad \tau_s \propto M_s^{1-x}$$

- Finalmente la temperatura en el interior:

$$T_0 \approx 10^7 - 10^8 \text{ K}$$



¿Qué pasa en los extremos?

# ¿Qué pasa en los extremos?

Recordem:  $L^* \propto T^4$ ;  $3 < x < 4$ . Considerem un estrella  $M_s \sim 50 M_\odot \Rightarrow$

$$\frac{L_s}{L_\odot} = \left( \frac{M_s}{M_\odot} \right)^x \Rightarrow L_s = \left( \frac{M_s}{M_\odot} \right)^4 \cdot L_\odot \quad \text{si } M_s = 50 M_\odot \Rightarrow \left( \frac{M_s}{M_\odot} \right)^4 = 50^4 = 6,25 \times 10^6$$

$$\Rightarrow L_s = 6,25 \times 10^6 L_\odot \quad \text{o bien } L_s \approx 2,4 \times 10^{33} \text{ J/s}$$

Dijo tardado por absorción

Recordando Co de la anterior en un volumen  $\frac{4\pi r^3}{3} dr$  tenemos:  $dq = -k_r q_r S_{rdr} \Rightarrow q_{rdr} < q_r$

Wegs puntos que es la fljgo de emisión en la forma de fotones  $\Rightarrow E_\gamma = h\nu$  y  $p_\gamma = h\nu/c$   
 $\frac{dq}{dt} = E_\gamma \frac{dr}{dt} \Rightarrow \frac{dq}{dt} = -k_r q_r p_r \Rightarrow \frac{dr}{dt} = -\frac{k_r q_r p_r}{E_\gamma} = -\frac{k_r q_r p_r}{h\nu/c} \cdot \frac{dr}{dt} = -\frac{k_r p_r}{c} \frac{dr}{dt}$

$$\frac{d(N_{ph})}{dt} = -\frac{k_r p_r}{c} \frac{dr}{dt} \Rightarrow \frac{d}{dt}(N_{ph}) = \boxed{F_y = -\frac{k_r p_r}{c} \frac{dr}{dt}} \quad \begin{array}{l} \text{fuerza ejercida por ionización-} \\ \text{Hacia Afuera.} \end{array}$$

Recordando:  $F_G = -\frac{GM_r p_r}{r^2} dr$  Fuerza de Gravedad (Hacia Adentro)

$$F_y = F_G \Rightarrow -\frac{GM_r p_r}{r^2} dr = -\frac{k_r p_r}{c} \frac{dr}{dt} \quad \text{y para } r = r_s \Rightarrow \frac{GM_s^2}{R_s^2} = \frac{k_r}{c} \quad \text{y } p_r = \frac{L}{A} = \frac{C}{4\pi r^2}$$

$$\Rightarrow \frac{GM_s^2}{R_s^2} = \frac{KL}{4\pi c R_s^2} \Rightarrow \boxed{M_s^2 = \frac{KL}{4\pi c G}} \quad \text{Límite de Eddington fm. Si } M_s > M_s^+ \text{ la Rod. se funde la Gravedad}$$

De igual forma:  $L^* = \frac{4\pi GM_s c}{K} \cdot \frac{1}{r} \cdot \frac{1}{m_p} \Rightarrow K = C_f / m_p = \frac{8\pi r_c^2}{3} / m_p \Rightarrow$

# Límite de Eddington

$$\Rightarrow L = \frac{4\pi G c M_S m_p}{\sigma_T} \approx 3,2 \times 10^4 \left(\frac{M}{M_\odot}\right) L_\odot$$

Finalmente, si  $\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^x \Rightarrow L = \left(\frac{M}{M_\odot}\right)^x L_\odot \Rightarrow M_S = \frac{\kappa M_S^x L_\odot}{4\pi G c M_\odot^x}$

$$\Rightarrow \frac{M_S}{M_S^x} = \frac{\kappa}{4\pi G c} \frac{L_\odot}{M_\odot^x} \Rightarrow M_S^{1-x} = \frac{\kappa}{4\pi G c} \frac{L_\odot}{M_\odot M_\odot^{x-1}} \Rightarrow \frac{M_\odot^{x-1}}{M_S^{x-1}} = \frac{\kappa}{4\pi G c} \frac{L_\odot}{M_\odot} \quad \text{Invertiendo:}$$

$$\frac{(M_S)^{x-1}}{(M_\odot)} = \frac{4\pi G c}{\kappa} \left(\frac{M_\odot}{L_\odot}\right) \quad \text{Si } x=4 \Rightarrow \frac{M_S^3}{M_\odot^3} = \frac{4\pi G c}{\kappa} \frac{L_\odot}{L_\odot} = \frac{M_S^3}{4\pi G c L_\odot} M_\odot$$

$$\Rightarrow M_S^x \approx M_\odot M_\odot \quad \text{Límite de Eddington}$$

Star name	Mass ( $M_\odot$ , Sun = 1)	Distance from Earth (ly)	Method used to estimate mass
R126a1	315	163,000	Evolutionary model
R126c	230	163,000	Evolutionary model
BAT99-98	228	165,000	Luminosity/atmosphere model
R126a2	195	163,000	Evolutionary model
	199	163,000	Luminosity/atmosphere model

$$\left(\frac{M_S}{M_\odot}\right)^{x-1} = \frac{4\pi G c}{\kappa} \left(\frac{M_\odot}{L}\right) \rightarrow M_S \approx 110 M_\odot$$

④ Asorey - Física IV B

Límite de Eddington

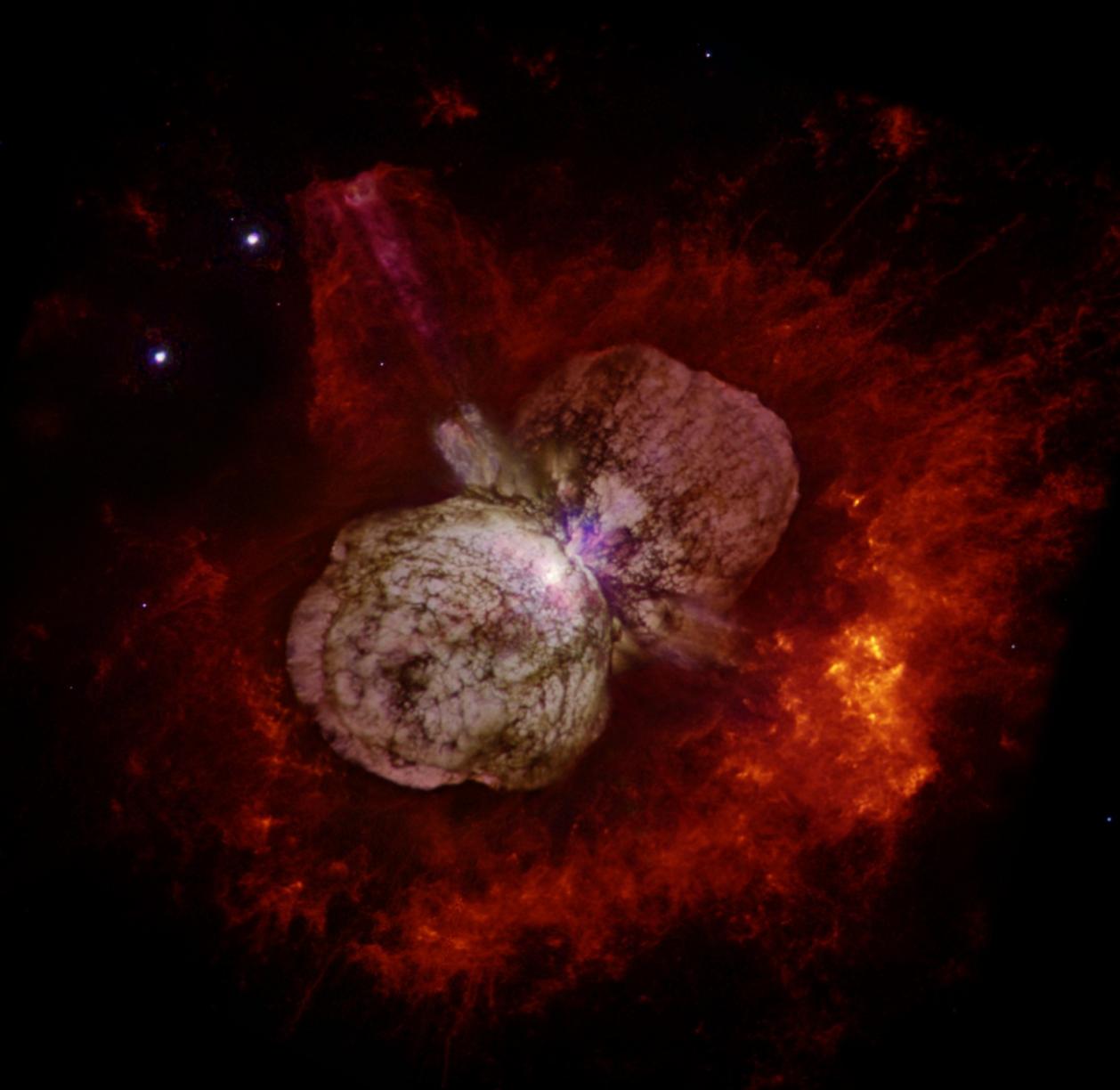
# Estrellas más masivas



Stars  $80 M_{\odot}$  or greater

Star name	Mass ( $M_{\odot}$ , Sun = 1)	Distance from Earth (ly)	Method used to estimate mass
R136a1	315	163,000	Evolutionary model
R136c	230	163,000	Evolutionary model
BAT99-98	226	165,000	Luminosity/atmosphere model
R136a2	195	163,000	Evolutionary model
Melnick 42	189	163,000	Luminosity/atmosphere model
R136a3	180	163,000	Evolutionary model
HD 15558 A	$>152 \pm 51$	24,400	Binary
VFTS 682	150	164,000	Luminosity/atmosphere model
R136a6	150	157,000	Evolutionary model
Melnick 34 A	147	163,000	Luminosity/atmosphere model
LH 10-3209 A	140	160,000 <sup>[10]</sup>	
Melnick 34 B	136	163,000	Luminosity/atmosphere model
NGC 3603-B	$132 \pm 13$	24,700	Luminosity/atmosphere model
HD 269810	130	163,000	Luminosity/atmosphere model
P871	130		?
WR 42e	$130 \pm 5$	25,000	Ejection in triple system
R136a4	124	157,000	Evolutionary model
Arches-F9	$121 \pm 10$	25,000	Luminosity/atmosphere model
NGC 3603-A1a	120	24,700	Eclipsing binary
LSS 4067	120	9,500–12,700	Evolutionary model
NGC 3603-C	$113 \pm 10$	22,500	Luminosity/atmosphere model
Cygnus OB2-12	110	5,220	Luminosity/atmosphere model
WR 25	110	10,500	Binary?
HD 93129 A	110	7,500	Luminosity/atmosphere model

$\eta$ Carinae (O, $\zeta$ WR?),  $M \sim 100-200 M_{\odot}$ ,  $L \sim 5 \times 10^6 L_{\odot}$ .



# R136 Cluster, el Himalaya estelar

