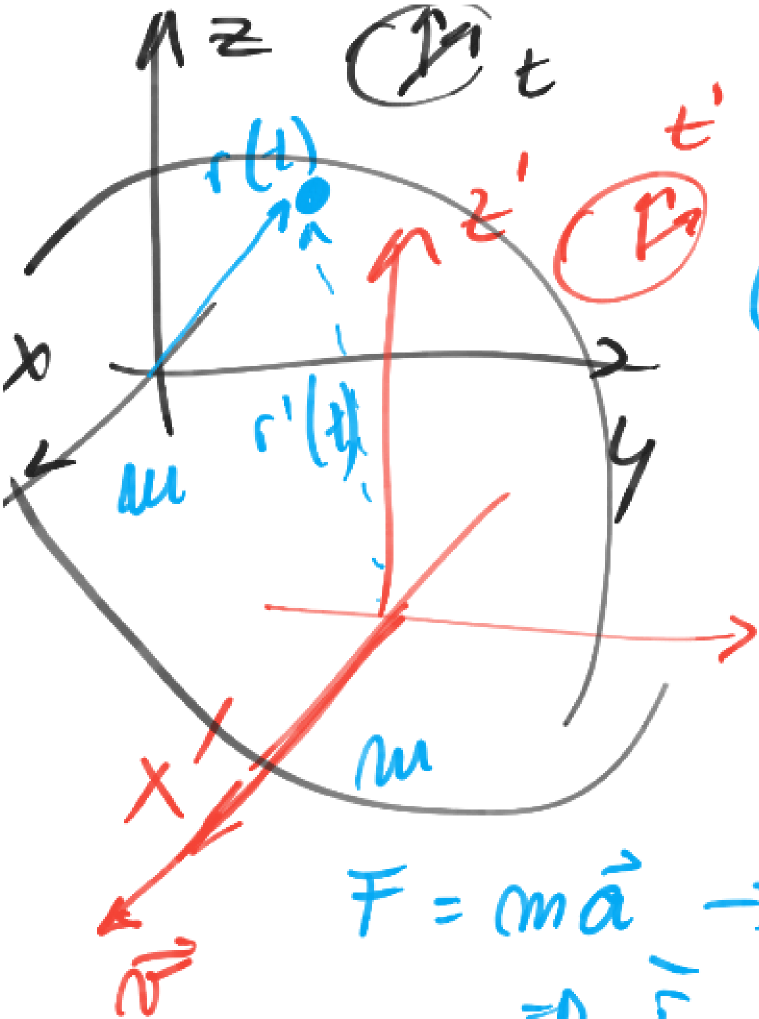
ipac-2019-U01C03-2808-dinamica-relativista



$$t = t'$$

$$\vec{r}'(t) = r(t) - \vec{v}\cdot t$$

$$dt = dt'$$

$$\boxed{\vec{u}'(t) = u(t) - \vec{v}}$$

$$a'(t) = a(t)$$

$$F = m\vec{a} \rightarrow F' = m a'$$

$$\Rightarrow F = F'$$

Para observadores inerciales.

transformaciones de Lorentz.

mecanica

~~transformaciones de Galileo~~ $(\vec{v}, \vec{u}) \ll c$

EMagnetismo.

"Leyes"

1° Postulado.

Principio de invariancia de Lorentz.

2° postulado. $c = c'$

$$c = \frac{dr}{dt} \qquad c' = \frac{dr'}{dt'}$$

$$\cancel{c' = c - v}$$

$$c' = c \Rightarrow dt \neq dt'$$

$$\Downarrow$$

$$\boxed{t \neq t'}$$

$$\begin{cases} t' = \gamma\left(t - \dfrac{v x}{c^2}\right) \; ? \\ x' = \gamma\left(x - v t\right) \end{cases}$$

$$1 \leq \gamma < +\infty \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\beta = v/c \quad \text{ó} \quad \beta = u/c$$

$$\Delta t' = \gamma \, \Delta t$$

Dilatación temporal

$$dt \neq dt'$$

$$\boxed{u' = \frac{u - v}{1 - \dfrac{u v}{c^2}}}$$

$$\Delta x' = \frac{\Delta x}{\gamma}$$

Contracción de Lorentz.

$$\Delta t' = \gamma \Delta t$$

$$c = \frac{dr}{dt} = \frac{dr'}{dt'} \rightarrow \Delta$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r' = \sqrt{x'^2 + y'^2 + z'^2}$$

$$c \Delta t = \Delta r \Rightarrow c^2 \Delta t^2 = \Delta r^2$$

$$c^2 \Delta t^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$$

$$c^2 \Delta t'^2 = \Delta x'^2 + \Delta y'^2 + \Delta z'^2$$

c^2

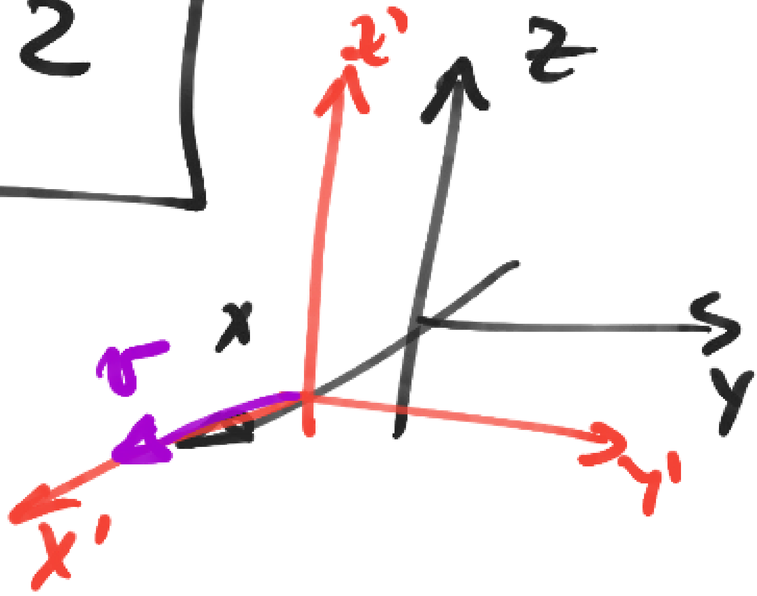
S intervalo invariante.

$$s^2 \overset{def}{=} \underbrace{c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2}_{s^2}$$

$$s'^2 \overset{def}{=} c^2 \Delta t'^2 - \Delta x'^2 - \Delta y'^2 - \Delta z'^2$$

$$\boxed{s^2 = s'^2}$$

$$\begin{cases} \Delta y' = \Delta y \\ \Delta z' = \Delta z \end{cases}$$

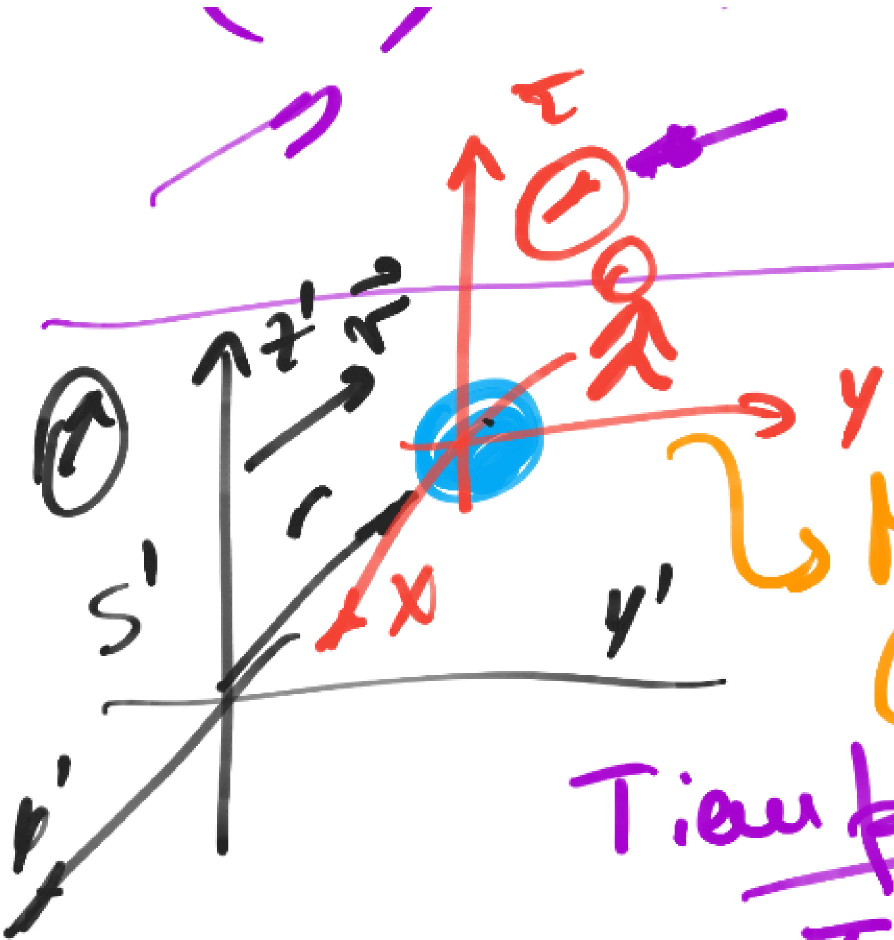


Convención Métrica.

$(+, - - -)$ Convención HEP Física Partícula

$(-, + + +)$ Convención

Reloj Universal General.



Marco Comóvil

Tiempo propio

τ

dt

$d\tau$

$$S^2 = S'^2$$

$$\underbrace{c\Delta t^2 - \Delta r^2}_{\text{tiempo propio.}} = c\Delta t'^2 - \Delta r'^2$$

$$\underbrace{c\Delta \tau^2 - \cancel{\Delta r^2}^{\to 0}} = c\Delta t'^2 - \Delta r'^2$$

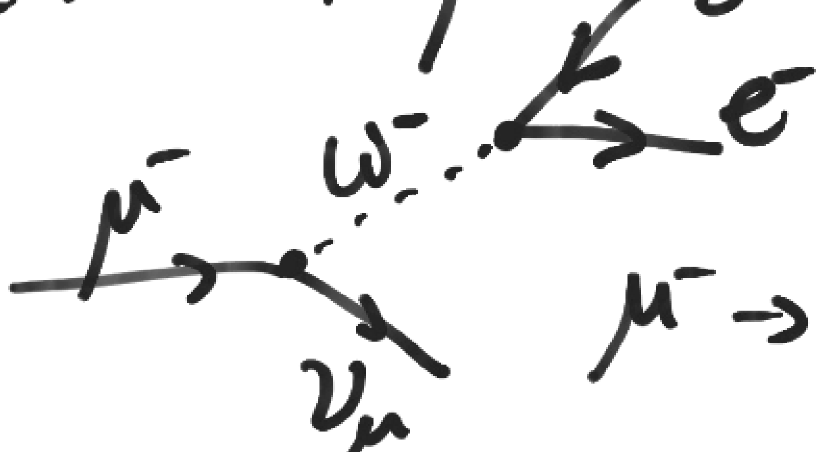
comóvil

$$c\Delta\tau^2 = c\Delta t'^2 - \Delta r'^2$$

Es el tiempo "más rápido" posible entre todos los obs. inerciales.

$$\Delta t' = \gamma\,\Delta\tau$$

$$\langle t \rangle = 2{,}2\ \mu s$$



$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

$\beta = 0{,}999999\ldots$ $\qquad \gamma_\mu = 10$

$$\Delta t = \gamma\,\Delta\tau$$

$\Delta t = 10\ 212\ \mu s$

$\Delta t \approx 22\,\mu s$

C $c\,\Delta\tau \simeq 660\ m$

$c\,\Delta t = 6600\ m$

$\Delta x = \frac{\Delta x}{\gamma} = 660\ m$

$\vec{F} = m\vec{a}$

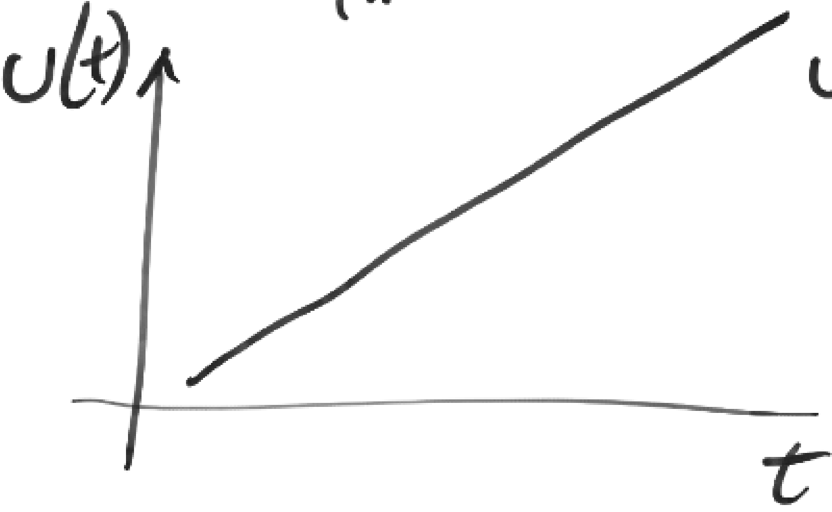
$\downarrow$

$\vec{F} = \frac{d\vec{p}}{dt}$ $\quad m = cte$

$$\bar F = m \frac{dv}{dt} \rightarrow dv = \frac{F}{m}\, dt$$

$$\vec v(t) = \frac{\vec F}{m}\Delta t \qquad \int dv = \frac{F}{m}\int_0^t dt$$

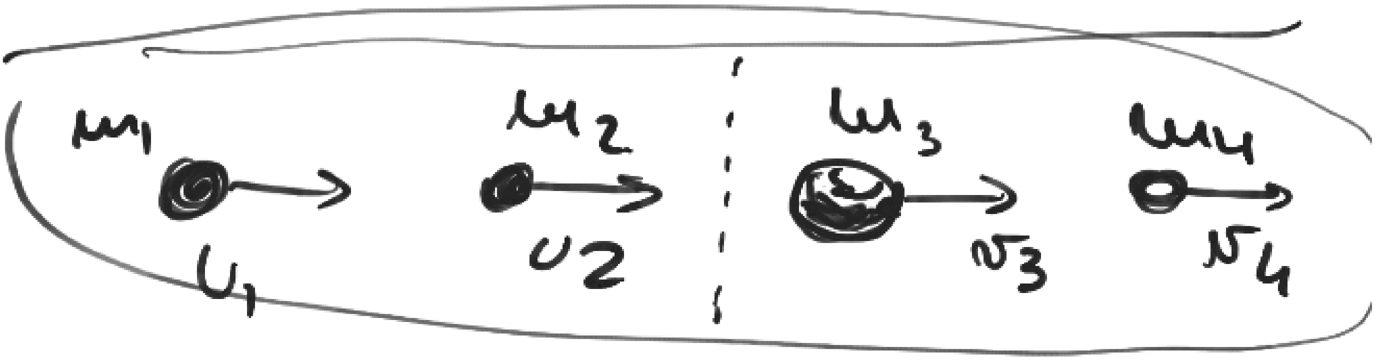
$$v(t) = \underbrace{\frac{F}{m}}_{a}\, t$$



$\cancel{F' = F}$

$$F' = \frac{dp'}{dt'}$$

$$F = \frac{dp}{dt}$$



$u \rightarrow$ vel inicial

$v \rightarrow$ vel final

V vel cte

> 90

En el S: $p = mv$ $\quad p_i = p_f$

$m_1 v_1 + m_2 v_2 = m_3 v_3 + m_4 v_4$

En S'

$m_1 v_1' + m_2 v_2' = m_3 v_3' + m_4 v_4'$

$v_1' = v_1 - V \qquad v_3' = v_3 - V$

$$m_1 (v_1 - V) + m_2 (v_2 - V) = m_3 (v_3 - V) + m_4 (v_4 - V)$$

p_i

$$m_1 v_1 + m_2 v_2 \cancel{- V (m_1 + m_2)} =$$

p_f

$$m_3 v_3 + m_4 v_4 \cancel{- V (m_3 + m_4)}$$

$m_1 + m_2 = m_3 + m_4$

Ley de conservación de la

mora.

Relativistic

$p \propto F_{H}$

F'

$\cancel{p = m\bar{v}}$

$\cancel{p' = m v'}$

$m = m'$

$m_1 u_1 + m_2 u_2 = m_3 v_3 + m_4 v_4$

$$u_1' = \frac{u_1 - V}{1 - \frac{u_1 V}{c^2}} *$$

$$m_1\left(\frac{u_1 - V}{1 - u_1 V/c^2}\right) + m_2\left(\frac{u_2 - V}{1 - u_2 V/c^2}\right) =$$

$$m_3\left(\frac{v_3 - V}{1 - \frac{v_3 V}{c^2}}\right) + m_4\left(\frac{v_4 - V}{1 - \frac{v_4 V}{c^2}}\right)$$

$$p = \frac{d(m\vec{r})}{dt} = m\frac{d\vec{r}}{dt}$$

$$p' = \frac{d(mr')}{dt'} = m\frac{dr'}{dt'}$$

$$\boxed{p = m\frac{dr}{d\tau}}$$

Cont. de Ecuac. relativista

$$\vec{p} = m\frac{d\vec{r}}{d\tau} \qquad dt = \gamma\, d\tau$$

$$\Rightarrow d\tau = dt/\gamma$$

$$\vec{p} = m\frac{d\vec{r}}{d\tau}\frac{dt}{dt} = m\left(\frac{d\vec{r}}{dt}\right)\left(\frac{dt}{d\tau}\right)$$

$$\underbrace{\frac{d\vec{r}}{dt}}_{v}$$

$$dt/d\tau = \gamma \Rightarrow \boxed{\vec{p} = m\vec{v}\gamma}$$

$$\gamma_1 = \frac{1}{\sqrt{1-(u_1/c)^2}} = \frac{1}{\sqrt{1-\beta_1^2}}$$

S:

$$m_1 v_1 \gamma_1 + m_2 v_2 \gamma_2 = m_3 v_3 \gamma_3 + m_4 v_4 \gamma_4$$

S':

$$m_1 v_1' \gamma_1' + m_2 v_2' \gamma_2' = m_3 v_3' \gamma_3' + m_4 v_4' \gamma_4'$$

Ocurre un milagro (Problema de la Guía)

$$\boxed{m_1 \gamma_1 + m_2 \gamma_2 = m_3 \gamma_3 + m_4 \gamma_4}$$

$m_i \gamma_i$ es una magnitud Conservada.

$$F = \frac{d}{dt}(mv)$$

γ es función de v

$$F = \frac{d}{dt}(m\gamma v) = \frac{d}{dt}(m\gamma)v$$

$$= m \frac{d}{dt}(\gamma v)$$

$$F = m\left(\frac{d\gamma}{dt} v + \gamma \overbrace{\frac{dv}{dt}}^{a}\right)$$

$$\Rightarrow F = m\gamma a + m v \frac{d\gamma}{dt}$$

$$\gamma = \frac{1}{\sqrt{\left(1 - v^2/c^2\right)}}$$

tutto come mec. Newtoniano
vale basta di
moltiplicare $m \to m\gamma$

---

m_0 = massa a riposo

$m_0\gamma \to$ massa relativistica

$\forall$

$m \quad m\gamma \quad$ si $v=0$

$\gamma = 1.$

$$\left(\sum m_i \gamma_i\right)_{\text{inicial}} = \left(\sum m_i \gamma_i\right)_{\text{fi}}$$

Considero el límite de $m\gamma$

$\rightarrow v \rightarrow 0$

$v=0 \Rightarrow \gamma = 1 \Rightarrow m\gamma \rightarrow m$

$$m\gamma = m \frac{1}{\sqrt{1-\beta^2}} = m\left(1-\beta^2\right)^{-1/2}$$

Serie de Taylor $(\epsilon \rightarrow 0)$

$$(1+\epsilon)^n = 1 + n\epsilon + \frac{1}{2}n(n-1)\epsilon^2 + \dots$$

$n = -1/2$

$\epsilon = -\beta^2$

$\downarrow \lim v \rightarrow 0$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 1 + \left(-\tfrac{1}{2}\right)\left(-\beta^2\right) + \frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)$$

$$0 \qquad -\beta^4$$

$$= 1 + \frac{1}{2}\beta^2 - \frac{3}{8}\cancel{\beta^4} + \cdots$$

$$\gamma m = m + \frac{1}{2} m \frac{v^2}{c^2} + \cdots$$

$$\gamma m c^2 = m c^2 + \frac{1}{2} m \frac{v^2}{\cancel{c^2}} \cancel{c^2}$$

$$\gamma m c^2 = m c^2 + \frac{1}{2} m v^2$$

En el limite $v \to 0$

$$\left(\gamma m c^2 - m c^2\right) = \frac{1}{2} m v^2 = E_K$$

$$\boxed{\gamma m c^2 = \text{Energía total del sistema de masa m}}$$

$$\boxed{\gamma mc^2 - mc^2 = E_k}$$

$E = mc^2$

Energia totale

Energia associata alla esistenza della massa.

relativistica

$$\boxed{E_k = mc^2(\gamma - 1)}$$

Def. relativistica de la Energia cinetica

$$E = m\gamma c^2$$

$$p = m\gamma v$$

$$E^2 = m^2\gamma^2 c^4$$

$$E^2 = m^2 \gamma^2 c^4$$

$$p^2 = m^2 \gamma^2 v^2$$

$$\downarrow$$

$$\times 1 \quad p^2 c^2 = m^2 \gamma^2 v^2 c^2$$

$$E^2 - p^2c^2 = m^2\gamma^2c^4 - m^2\gamma^2 v^2 c^2$$

$$= m^2\gamma^2c^4\left(1 - \frac{v^2}{c^2}\right)$$

$$E^2 - p^2c^2 = m^2\gamma^2c^4 \underbrace{\left(1 - \beta^2\right)}_{1/\gamma^2}$$

$$= m^2 \cancel{\gamma^2} c^4 / \cancel{\gamma^2}$$

$$E^2 - p^2c^2 = m^2c^4$$

$$\boxed{E^2 - p^2c^2 = (mc^2)^2}$$

$m\gamma c^2$

$E = m\gamma c^2$

$E' = m\gamma' c^2$

$\gamma' = 1$ $E = mc^2$ $\leftarrow m\gamma c^2$

$\underbrace{E^2 - p^2c^2}= (mc^2)^2$

$$\boxed{E^2 - p^2c^2 = E'^2 - p'^2c^2 = m^2c^4}$$

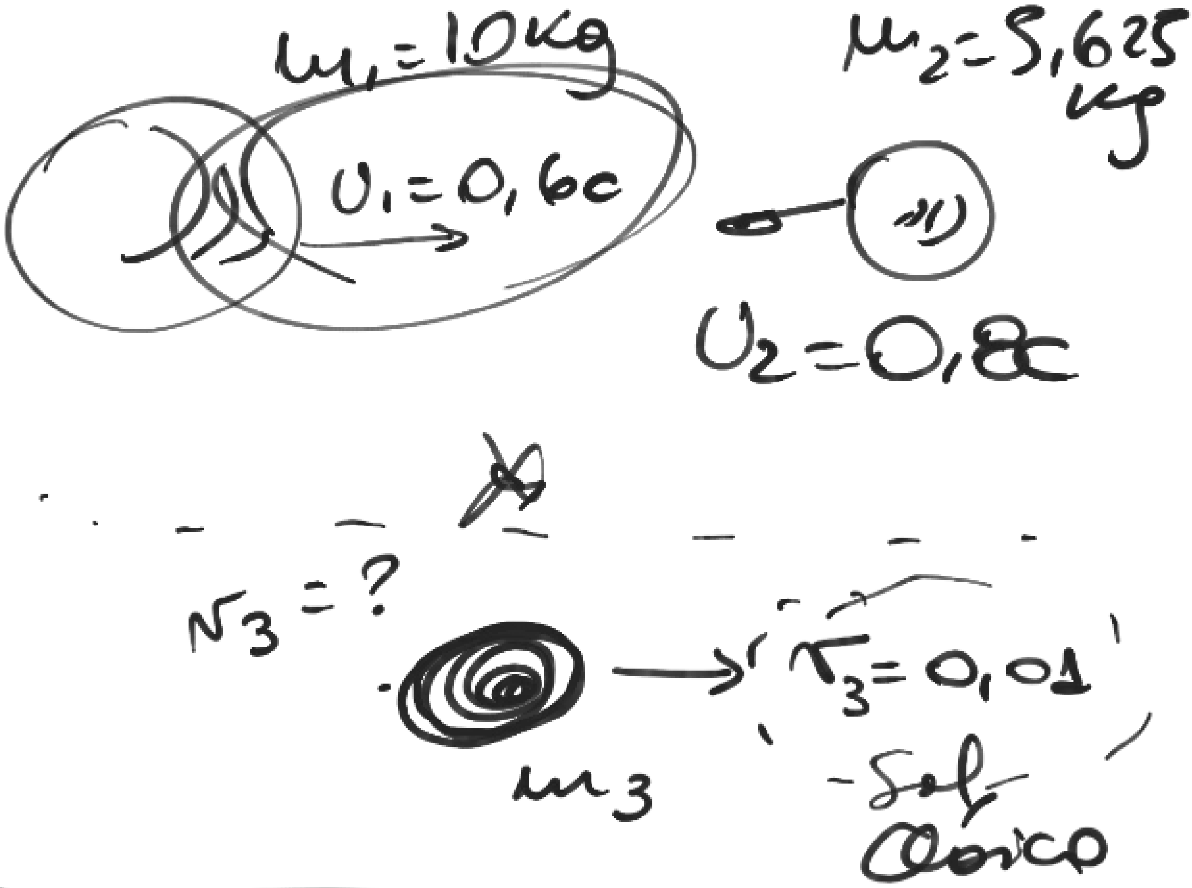
inv. relativista

$$\boxed{E^2 = p^2c^2 + m^2c^4}$$

$E = m\gamma c^2$

$\underline{m = 0}$ $\rightarrow$ $\underline{\text{fotón}}$

$$E^2 = p^2c^2 \Rightarrow p^2 = (E/c)^2$$



Solución clásica

$~~m_1 + m_2 = m_3 = 15,625~~$

$~~v_3 = +0,01 > 0~~$ No es correcto

Sol. relativista

$p_i = p_f$

$$m_1 v_1 \gamma_1 + m_2 v_2 \gamma_2 = m_3 \gamma_3 v_3$$

$$c^2 m_1 \gamma_1 + m_2 \gamma_2 c^2 = m_3 \gamma_3 c^2$$

$$\beta_1 = 0{,}6 \rightarrow \gamma_1 = 1{,}25$$

$$\beta_2 = 0{,}8 \rightarrow \gamma_2 = 5/3$$

$$p_i = 10\,kg\; 0{,}6c\; 1{,}25 +$$

$$+ 5{,}625\,kg\,(-0{,}8c)\; 5/3$$

$$= 7{,}5\,kg\,c - 7{,}5\,kg\,c$$

$$p_i = 0 = p_f \qquad v_3 = 0$$

$$m_1 \gamma_1 c^2 + m_2 \gamma_2 c^2 = m_3 \gamma_3 c^2$$

$$\gamma_3 = 1$$

$$m_1 \gamma_1 \cancel{c^2} + m_2 \gamma_2 \cancel{c^2} = m_3 \cancel{c^2}$$

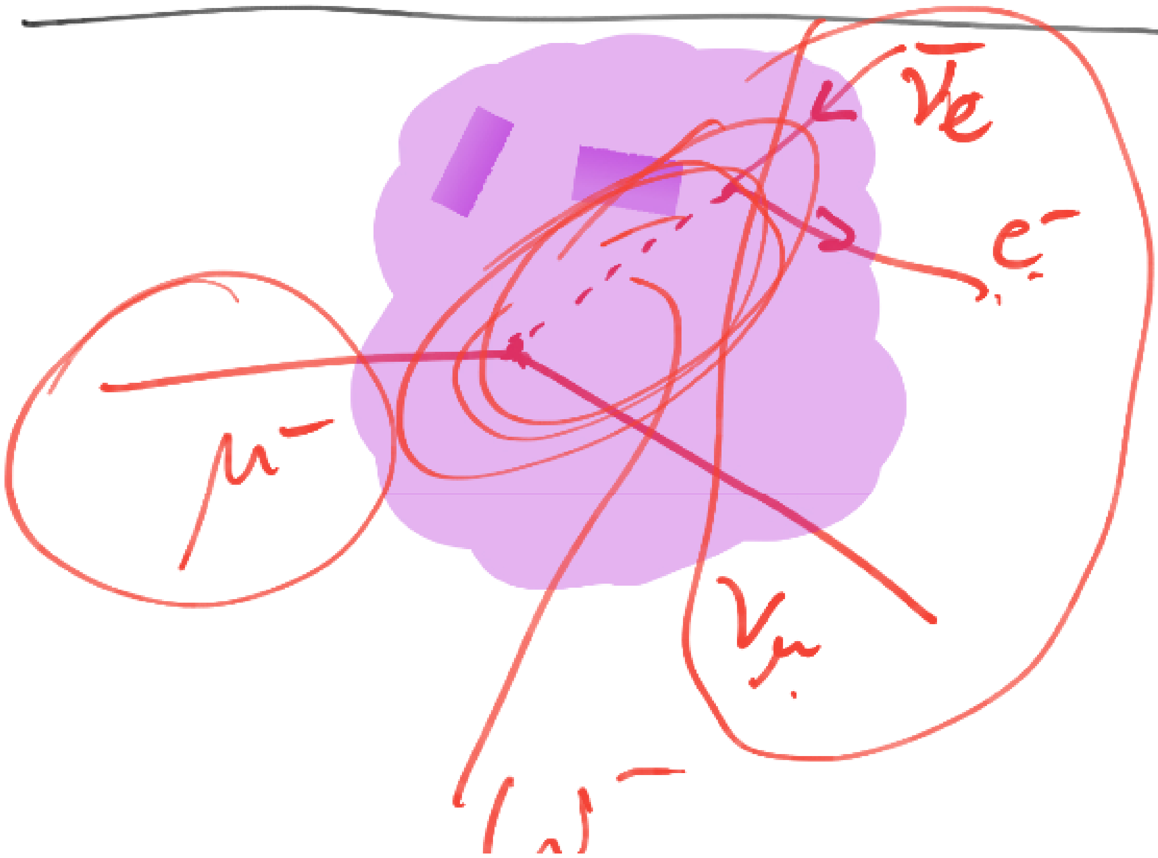
$$m_3 = m_1 v_1 + m_2 v_2$$

$$= 10\,\text{kg}\ 1{,}25 + 5{,}625\,\text{kg}\ 5/3 .$$

$$\boxed{m_3 = 21{,}875\ \text{kg}}$$

$$E^2 - p^2c^2 = m^2c^4$$

$$\left(E_1 - p_1 - m_1\right)$$



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