NEW ZEALAND
MARITIME SCHOOL

STCW 1978 III/2
(as amended in 2010)

MECHANICAL TECHNOLOGY 1

COURSE CODE: 942. 594

ELEMENT 2: KINEMATICS; VELOCITY & ACCELERATION

Chapter 3

Kinematics.

This is the study of motion: speed, velocity, etc, when no mass, weight or forces are involved.

Velocity and Acceleration.

3.1 Linear Motion

Initial concepts we need to understand;

- **Speed;** rate at which a body moves through space; ie expressed as distance in given time;
- Usually expressed as km/hr or knots (**one knot – 1.852km/hr)**
- Determined by dividing distance by time
- Constant speed; when the distance travelled is the same in successive intervals of time
- Average speed is the result of dividing the total distance travelled by the time it took to do the journey;

$$V_{\text{average Velocity}} = \frac{\text{distancetravelled}}{\text{timetaken}} \left(\frac{\text{metres}}{\text{seconds}}\right) = \text{m/s}$$

Velocity; indicates speed in a given specified direction and therefore represents two facts about a body

Speed and direction; therefore it is what is called a **vector quantity** and can be illustrated by scale drawings where the length of the scale drawing represents the speed of the body and an arrow is used to identify direction

Uniform velocity is motion along a straight line at constant speed.

Average Velocity. When a body starts from rest and reaches a final velocity after a time has elapsed the average velocity will be the sum of the two velocities divided by 2.

Therefore the average velocity of a body starting from rest and reaching a speed of

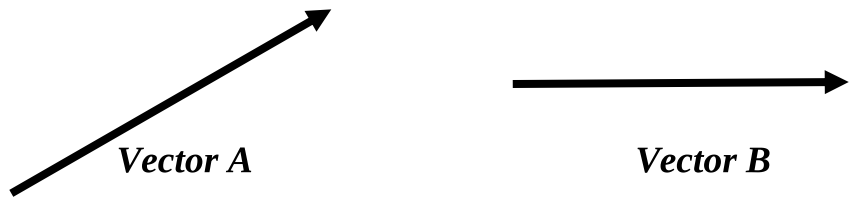
$$\textbf{20m/s} = \frac{0+20}{2} = \textbf{10m/s.}$$

We use the letter **"U"** to signify an initial velocity and the letter **"V"** to signify a final velocity.

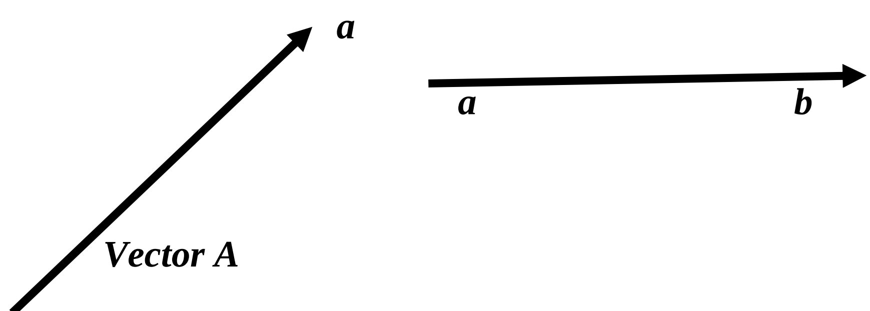
Distance and Displacement; distance travelled in a particular direction and will often be the 'resultant' displacement of a journey which includes travel in two different concurrent directions. Displacement must be stated relative to some point in space

- **Distance travelled = average velocity x time**

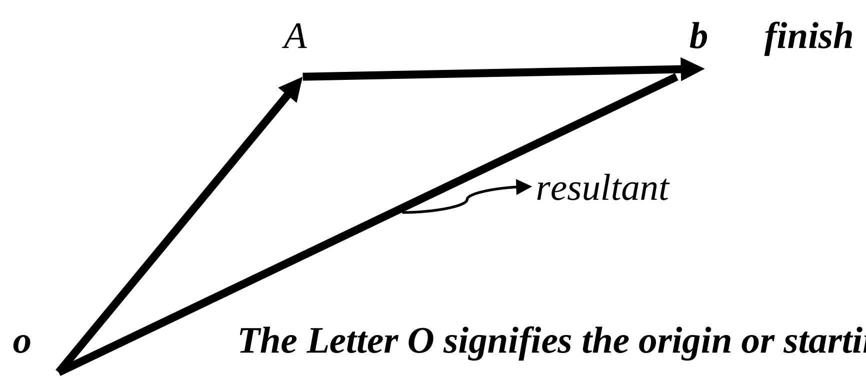
Adding Vectors; Scalar quantities (single dimension) can be added or subtracted using simple arithmetic, but vectors must be added or subtracted graphically (by drawing them), in order to take account of the direction.



The two vectors shown above can be added graphically to give the resultant vector C;



O ***The letter O signifies the origin or starting point.***

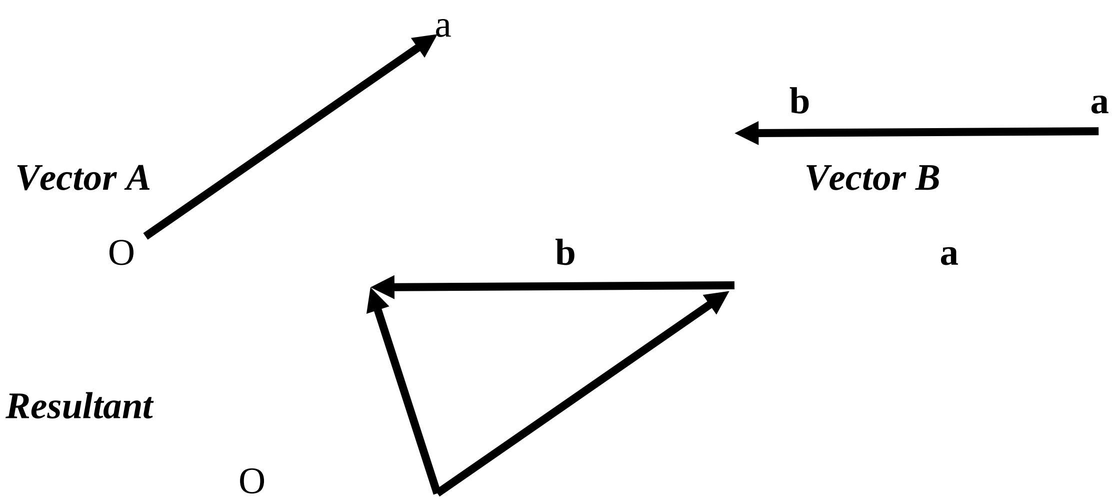


The Letter O signifies the origin or starting point

SubtractingVectors

Vectors may also been subtracted. To do this, we change the vector we are going to subtract into a negative quantity, simply by reversing the direction of the arrow, and then add them together. Adding a negative quantity to a positive quantity is the same as subtracting the **A+ (-B) = (A-B)**

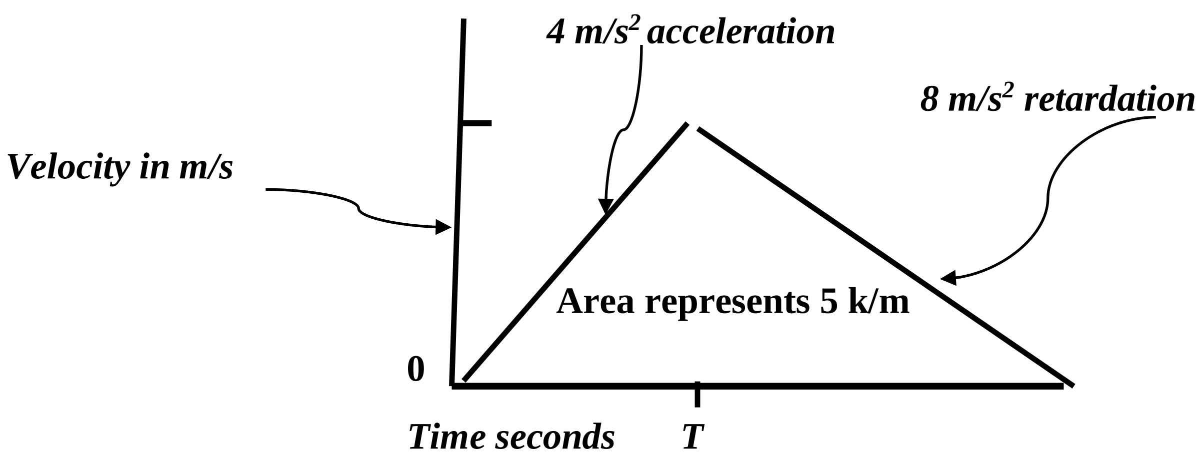
We then draw the solution making sure we follow the direction of the arrows;



Take care when using vectors; it is easy to an arrow or direction wrong.

3.2 Velocity Time Graphs

A typical speed –time graph is depicted below for a body accelerating from rest with uniform acceleration to a maximum speed which is maintained at a constant rate for a time and then decelerates uniformly back to rest. Provided the acceleration is uniform, the **v-t** graph consists of straight lines and the area under the graph represents the total distance travelled whilst the incline or slope of the graph will show the rate of acceleration.



Let *T* = total time in seconds

t = time to accelerate

***T* –*t* =** time to retard

V = maximum velocity, m / s

Initial velocity is zero and acceleration is uniform, hence maximum velocity is given by

$V = a\, t$

$= 4\, t$……………………………... *(i)*

Similarly, for the retardation

$V = 8\,(T - t)$

Equating, $8\,(T - t) = 4\, t$

$T = \frac{2}{3}\, t$

Hence from (i) $V = 4 \times \frac{2}{3}\, t$

$V = \frac{8}{3}\, T$…………………... *(ii)*

Area of triangle represents Distance travelled:

$\frac{1}{2}\ T\, v = 5000$

$T\, v = 10\,000$

Substituting from (ii)

$V = \frac{8}{3}\, T = 10\,000$

$T^{2} = 3750$

T = 61 seconds Ans.

i.e. The shortest time is 61 seconds

3.3 Acceleration;

The average acceleration of a body moving in a straight line is the rate of change of velocity and is therefore a derivative of displacement,

Acceleration occurs when the velocity changes in direction as well as magnitude and is expressed as change of velocity in a given time. If a body is increasing in velocity it is accelerating and if its velocity is reducing it is retarding or has negative acceleration.

Linear acceleration usually represented by either a or ∫

Acceleration $= \frac{\textbf{Increase in velocity}}{\textbf{Time to change}}$

Average Acceleration $= \frac{\textbf{change in velocity}}{\textbf{time taken}}$

Where; **v =** is final velocity

U= is initial velocity

t = is time taken

If the velocity increases by equal amounts in equal intervals of time the acceleration is said to be constant. From above;

Av Acceleration $\boldsymbol{a_{av}} = \frac{\boldsymbol{v-u}}{\boldsymbol{t}}$

Can be rewritten**; v = u + at**

Example 1; if a cyclist increases his speed uniformly from 2 meters per second to 14 meters per second in 6 seconds, his total increase in velocity is 14 -2 = 12 meters per sec. If he takes 6 seconds to increase his velocity by 12m/s, then his increase in each of these 6 secs must have been 12 ÷6= 2m/s. His acceleration is therefore 2m/s every second which is written as 2 meters per sec per sec and usually written as 2m/sec2

Example 2;

A train, starting from rest, increases its speed uniformly for 2 minutes and attains a speed of 108 km/hr in that time its acceleration could be stated as?

Ans; 108km/hr in 2 min or 54km/hr per min

Now we need to convert to compatible units

Acceleration= 108 km/h in 2 min

= 108000 m/h in 2 min

=108000/3600 m/s in 2 min

= 108000/3600/2 m/s in 1 min

=108000/3600/2/60 m/s in 1 sec

Or rewritten is; Acceleration = Increase in Velocity/time

Example;

A Ships engines are stopped when she is travelling at a speed of 16 knots and the ship comes to rest after 20 mins. Assuming uniform retardation, find the retardation in meters per second per second and the distance travelled in nautical miles in that time.

Solution:

Retardation = 16knots in 20min

Need to find; change in velocity (m/s)/time in seconds

$$= \frac{16\times1.852\times10^3}{3600}\textit{m/s in 20 min}$$

$$= \frac{16\times1.852\times10^3}{3600\times20\times60}\textit{m/s in 1sec}$$

$$= \mathbf{6.859 \times 10^{-3}} \textbf{ m/s/s}$$

To find distance travelled;

Distance = Average velocity x time

$$= \frac{16+0}{2}\mathbf{x}\frac{20}{60}$$

= **2.666 nautical miles**

Always remember to take care with units

Example;

A locomotive starting from rest attains a speed of 48km/hr over a distance of 150m. The rate of increase in velocity is uniform, find the acceleration of the locomotive in meters per sec per sec.

Max Velocity = $\frac{48 \times10^3}{3600}$ **m/sec**

= 13.33 m/s

Average Velocity = $\frac{(0+13.33)}{2}$ = 6.66 m/s

Distance (m) = av. velocity x time

$$150 = 6.66 \times t$$

$$t = \frac{\mathbf{150}}{\mathbf{6.66}} \text{ secs}$$

t = 22.52 secs

$$\text{Acceleration} = \frac{\textbf{max velocity}}{\textbf{time}}$$

$$\text{Acceleration} = \frac{\mathbf{13.33}}{\mathbf{22.52}} \frac{\boldsymbol{m}}{\boldsymbol{sec^2}}$$

= 0.59 m/s/s

3.4 Gravitational Acceleration

Free Falling Bodies

Galileo proved that all bodies dropped at the same place fell to earth with the same acceleration (g) due to the force of gravity .Therefore if a body falls to earth from a height which is small compared with the radius of the earth, it is found to increase its velocity by an equal amount each second, ie its acceleration downwards is uniform. Similarly, if a body is projected upwards, its deceleration upwards is uniform and equal to g.

For practical engineering purposes g may be taken as 9.81m/s^2 although for certain purposes a standard international value is required which is 9.80665 m/s^2. This value needs to be further modified when dealing with high altitudes but that is beyond our concerns.

The value of ***g*** is found to be independent of the weight, size and shape of the body provided the air resistance is neglected.

Therefore if a body falls from rest, its velocity increases by 9.81 m/s every second it is falling.

At the end of its first second, its velocity will be 9.81m/s

At the end of its second its velocity will be 19.62m/s

At the end of its third second its velocity will be 29.43m/s

The formulae for uniformly accelerated motion in a straight line apply to the motion of freely falling bodies or bodies moving freely upwards provided that the acceleration ***g*** replaces the acceleration ***a***.

For motion downwards; a = g (acceleration)

For motion upwards;   a = g (deceleration)

Acceleration ***g*** may be regarded as a vector directed towards the centre of the earth; ie vertically downwards

Example 1 ; ***A body is allowed to fall from rest. Find the velocity after falling for 6 seconds and the distance fallen in that time.***

Increase in velocity = 9.81m/s in each second it is falling

= 9.81 x 6 m/s in 6 secs.

= 58.86 m/s

Initial velocity = 0 m/s

Final velocity = 0 + 58.86 m/s

= 58.86 m/s

Distance travelled = av. velocity x time

$= \frac{0+58.86}{2} \times 6$

= 176.58 m

Example 2:

A projectile is fired vertically upwards with an initial velocity of 420 m/s. Find;

a) ***Its velocity after 15 secs***
b) ***The height it attains after 15 secs***
c) ***How long it takes to reach the height found in(b).***
d) ***The maximum height attained***
e) ***The total elapsed time from firing to returning to the ground***

Retardation = 9.81 m/s

a) Loss of velocity after 15 secs = 9.81 x 15 m/s

= 9.81 x 15 m/s

= 147.15

Velocity after 15 seconds = initial velocity –loss of velocity

= 420 – 147.15 m/s

= 272.85 m/s **ans (a)**

(b) Distance = average velocity x time

Height after 15 secs = $\frac{\mathbf{420+272.85}}{\mathbf{2}}$ x 15 m

= 5,196.4m ans (b)

(c) Max height occurs when the projectile loses all its upward velocity and this occurs at the rate of 9.81m/s^2Given its initial velocity was 420 m/s this is the total velocity it must lose.

Therefore time to reach maximum height = $\frac{\mathbf{420}}{\mathbf{9.81}}$

= 42.813 sec ans (c)

(d) When the projectile reaches maximum height it has slowed to a velocity where v = 0 m/s. Therefore for upward flight;

Average velocity = $\frac{\mathbf{420+0}}{\mathbf{2}}$

Therefore max height = average velocity x time

= $\frac{\mathbf{420+0}}{\mathbf{2}}$ x 42.58 m

= 8941.8m ans (d)

e) The time taken to fall will be the same as the time taken to reach its maximum height;

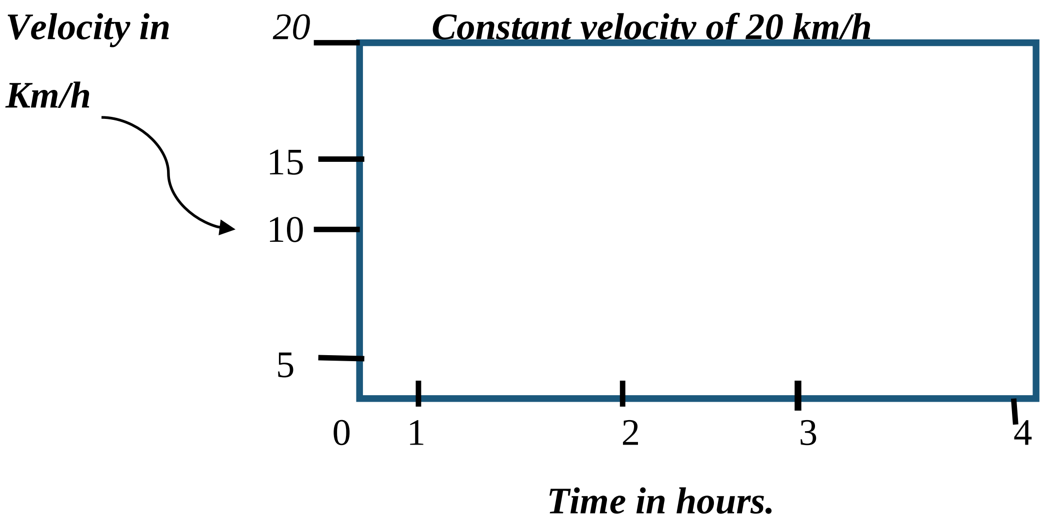
therefore time to max height = 42.813 sec

therefore total time elapsed = 42.813 x 2

= 85.63 sec/s ans (e)

3.5 Velocity Time Graphs& Acceleration

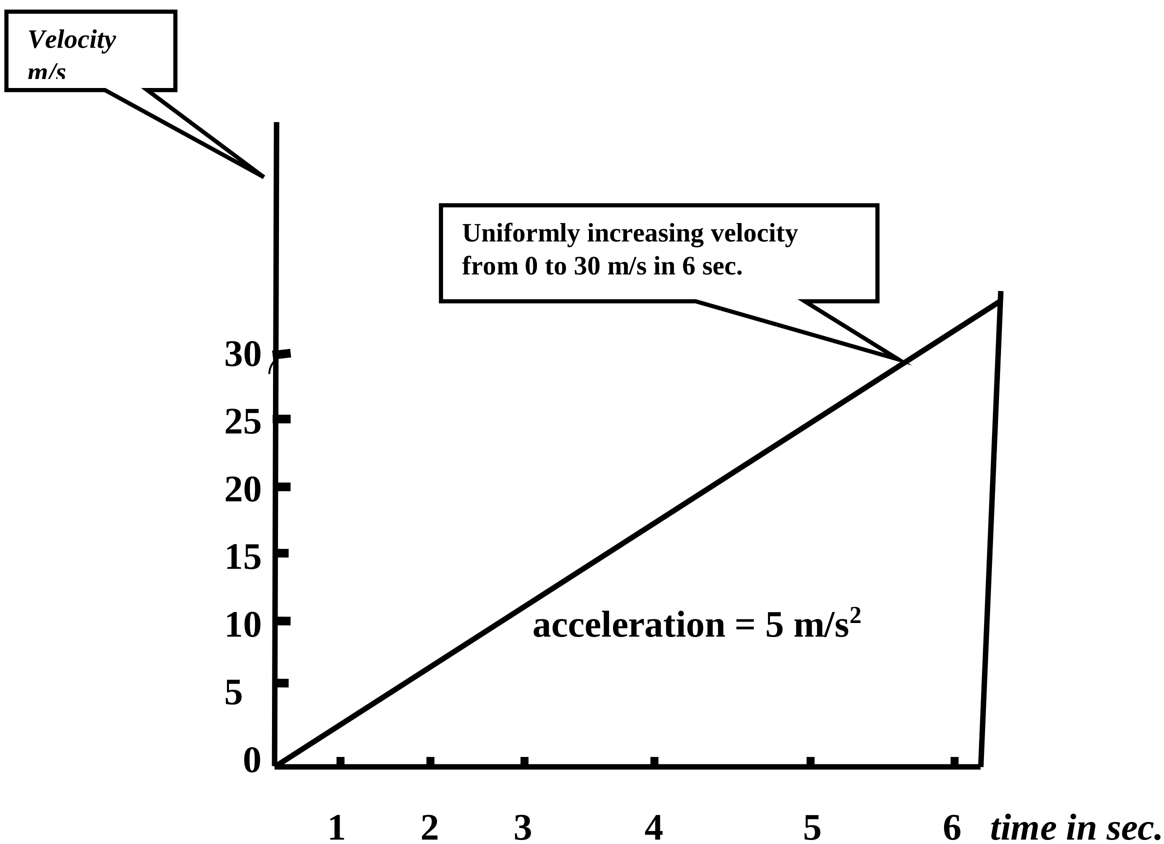
A typical speed –time graph is depicted below for a body accelerating from rest with uniform acceleration to a maximum speed which is maintained at a constant rate for a time and then decelerates uniformly back to rest. Provided the acceleration is uniform, the ***v-t graph*** consists of straight lines and the area under the graph represents the ***total distance travelled*** whilst the incline or slope of the graph will show the rate of acceleration.



Area of enclosed by graph: (rectangle)

= 20 × 4

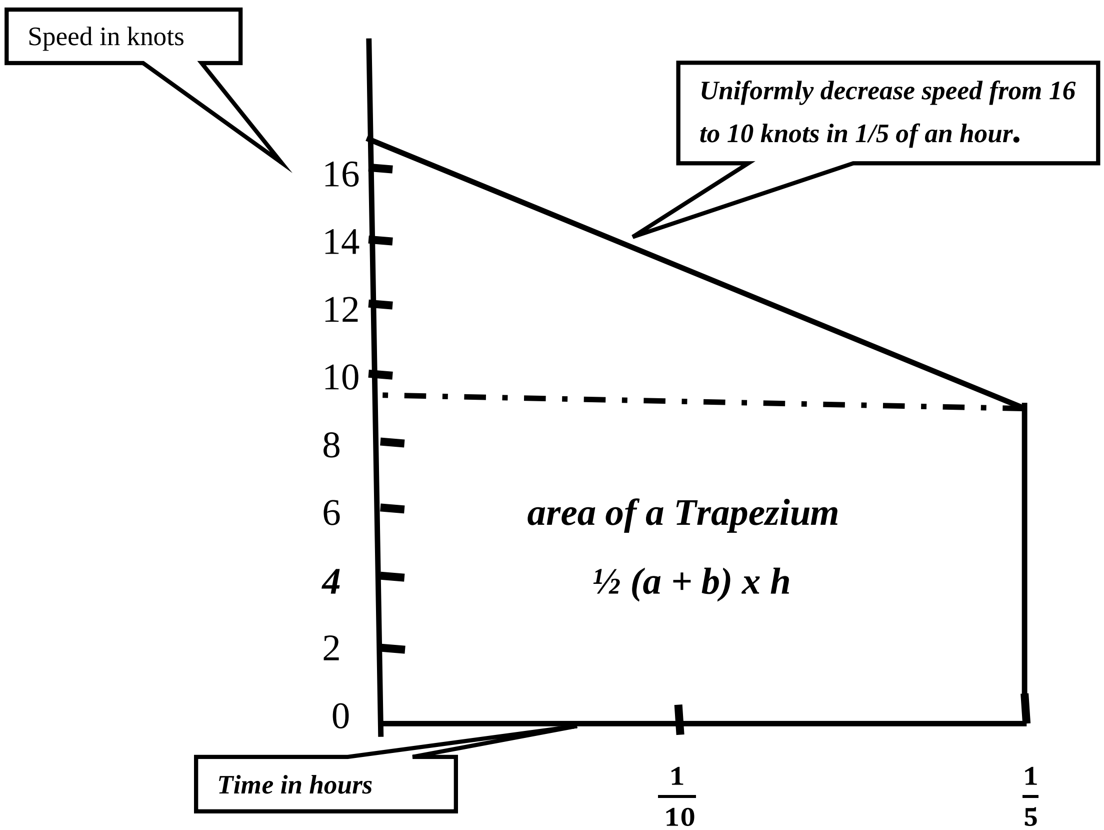
= 80 km



Area = distance travelled (***area of a triangle***)

$$= \frac{30 \times 6}{2}$$

= **90 m**



Area = Distance travelled (total area of a trapezium)

$$= \frac{16+10}{2} \times \frac{1}{5}$$

= **2.6 nautical miles.**

Example;

The graph attached shows a body starting from rest and reaching a velocity of 30 m/s in 6 secs, the rate of increase of velocity (its acceleration) being uniform.

Area of graph = area of triangle

= 0.5 x 30 x 6

= 90

Also, Distance = average speed x time

= 0.5(0+30) x 6

= 90m ans

Practice Exercise:

A locomotive starts from rest and reaches a speed of 90 km/h in 25 seconds, it runs at this speed for 1.$\frac{1}{2}$ minutes and then reduces speed to come to rest in 20 seconds. Assume acceleration and retardation to be uniform, draw a speed – time graph, find the total distance travelled and express the acceleration and retardation in m/s²

3.6 Derivation of Formulae:

There are ***four formulae***that are commonly used to solve velocity and acceleration problems but only when acceleration is constant (although the first equation may be used if average acceleration is known. These are solved below using the symbols;

u **=** initial velocity in m/s

v **= final velocity in m/s**

a **= acceleration in m/s^2**

t **= time in seconds**

s **= displacement or distance in meters**

given; therefore;

a= increase in velocity for each second

at= increase in velocity in t seconds

Final velocity = initial velocity + increase in velocity

therefore; $v = u + at$------------------------ (1)

Distance travelled = average velocity x time

Therefore**;** $s = (\frac{u+v}{2}) \times t$------------------------ (2)

Substituting value of v from (1) into (2

$$S = (\frac{u+U+at}{2})\times t = (\frac{2u+at}{2})\times t$$

$$S = (u + \frac{1}{2}at) \times t$$

$$S = ut + \frac{1}{2}at^2 \text{------------------- (3)}$$

Transposing (2) to make t the subject;

$$S = (\frac{u+v}{2}) \times t$$

$$t = \frac{2s}{u+v}$$

Substituting this value of *t* into (1);

$$V = u + at$$

$$V = u + \frac{2as}{u+v}$$

Multiplying throughout by (u+v);

$$Uv + v^2 = u^2 + uv + 2as$$

$$v^2 = u^2 + 2as \text{ ----------------------- (4)}$$

These formulae can be written with the sign as a positive or negative to deal with both acceleration and retardation. = **18 .6 km/hr**

3.7 Angular Motion

Angular motion is concerned with the motion of shafts, discs, motors and wheels which may spin but do not move bodily. The same terms of velocity, displacement and acceleration still apply. ***Angular velocity*** is the rate of change of angular displacement and is expressed in ***radians per second.***

One radian is the **angle subtended by a circular arc of length equal to the radius** and since the **circumference of a circle is 2π × radius** there are 2π radians in a circle. Usually no symbol is used to denote radians although we use ω for rad/s.

Also since there are 360 degrees in a circle (or complete revolution) 2π radians must equal 360 degrees.

Therefore; **1 degree =** $\frac{2\pi}{360} = \frac{\pi}{180}$ **radians**

And; **1 radian** $= \frac{180}{\pi}$ degrees **= 57.3 degrees**

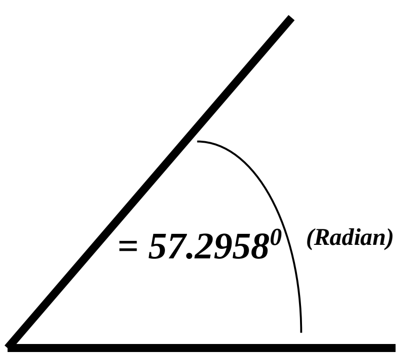
Note;

$\frac{\pi}{2}$ radians = 90 deg

π **radians** = 180 deg

Simple Explanation of the above are:

Radians: We can measure Angles in Radians.

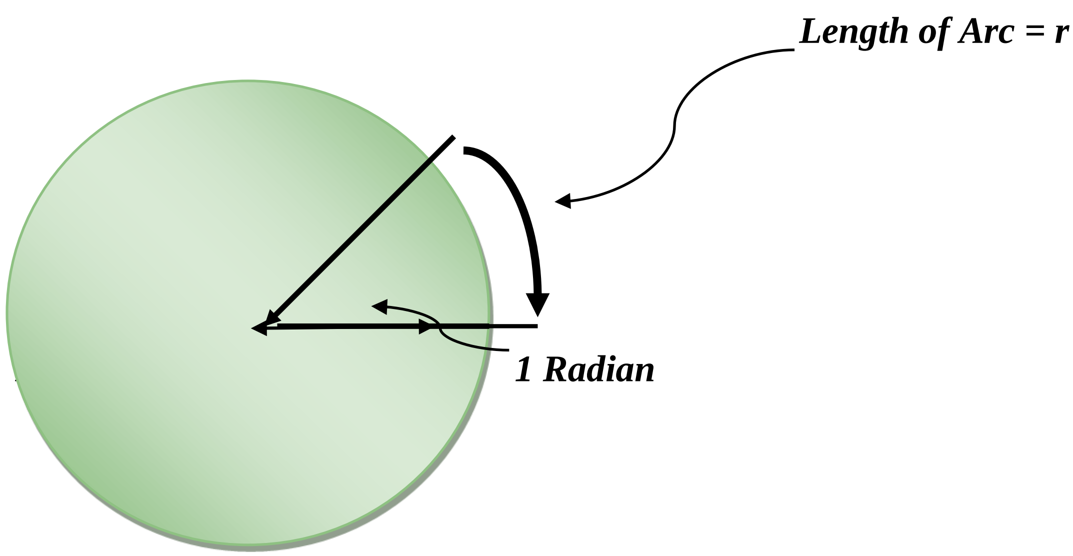


1 Radian is about 57.2958 degrees.

Does 57.2958... Degrees seem a strange value?
Maybe degrees are strange!

The Radian is a pure measure based on the Radius of the circle:

Radian: the angle made by taking the radius
and wrapping it along the edge of a circle:



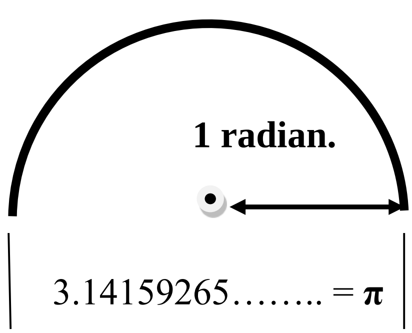
So, a Radian "cuts out" a length of a circle's circumference equal to the radius.

As shown in the animation above:

- **There are π radians in a half circle**
- **And also 180° in a half circle**

So π radians = 180°

So 1 radian = 180°/π = 57.2958° (approximately)



Degrees	*Radians (exact)*	*Radians (approx)*
30^0	**$\pi/6$**	**0.524**
45^0	**$\pi/4$**	**0.785**
60^0	**$\pi/3$**	**1.047**
90^0	**$\pi/2$**	**1.571**
180^0	**π**	**3.142**
270^0	**$3\pi/2$**	**4.712**
360^0	**2π**	**6.283**

Example:

How many radians in a full Circle? In other words, if you cut up pieces of string exactly the length from the centre of a circle to its edge, how many pieces would you need to go around the edge of the circle?

Answer: 2π**,** or about 6.283 pieces of string**.**

3.8 Angular Acceleration

We often use radians in problems with rotational speeds of engines and we are given revs/min and need to convert it to rad/s. In this case we multiply by 2π to convert to rad/min and then divide by 60 to get rad/s.

$$\boldsymbol{\alpha} \ \frac{\mathbf{2\pi \times rev/\,min}}{\mathbf{60}} = \ \text{rad/s}$$

Angular acceleration is the rate of change of angular velocity and is usually expressed in ***radians per second per second***, the abbreviation for these units is rad/s^2 and is usually represented by the symbol $\boldsymbol{\alpha}$ **(alpha)**

Example; ***A flywheel is increased in speed from 180 to 390 revs/min in 45 secs. Find the acceleration in rad/s and the number of revolutions turned during this time.***

Solution:

Acceleration = (390-180) rev/min in 45 secs

$$= \frac{\mathbf{210 \times 2\pi}}{\mathbf{60}} \ \text{rad/s in 45 secs}$$

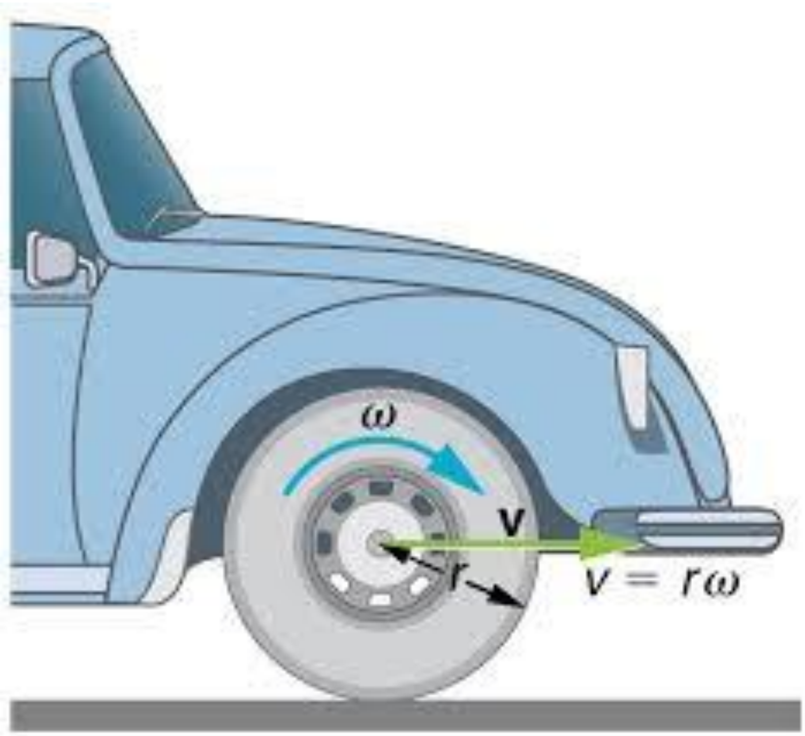
$$= \frac{\mathbf{210 \times 2\pi}}{\mathbf{60\ x\ 45}} \ \text{rad/s}$$

$$= 0.488 \ \text{rad/}s^2 \quad \text{ans}$$

Distance = average velocity x time

$$= \frac{\mathbf{180+390}}{\mathbf{2}} \ \text{(rpm)} \ \text{x}\frac{\mathbf{45}}{\mathbf{60}} \ \text{mins}$$

= **213.75 Revs ans**



Angular Velocity

To convert linear values to angular equivalents the conversion rules are;

Linear distance (s) = angular distance × radius **(θ r)**

Linear velocity (v) = angular velocity ×radius **(ω r)**

Linear acceleration (a) = angular acceleration × radius **(α r)**

Example;

A flywheel of 1.2m dia. rotates at 300 rev/min. Calculate the velocity of a point on its rim.

Solution; Rotation speed = 300 rev/min

$\boldsymbol{\omega} = 2\pi N$ rad/min

$\boldsymbol{\omega} = \frac{\mathbf{2\pi x300}}{\mathbf{60}}$ rad/sec

therefore = 31.42 rad/sec

now $\boldsymbol{v = \omega r}$

therefore; $\boldsymbol{v}$ = 31.42 x 0.6

$\boldsymbol{v}$ **= 18.85 m/s Ans**

3.9 Self Exercise:

1).A flywheel is increased in speed from 150 to 350 revolutions per minute in half - a- minute. Express the acceleration in radians per second per second and calculate the number of revolutions turned during that time.

2) A motor car increases speed from 20 to 74 km/h in 12 seconds. Find the acceleration in meters per second per second, and the distance travelled in meters during that time.

3) The speed of a generator is increased from 1000 to 1400 revolutions per minute in 8 seconds. Find the angular acceleration in radians per second per second and the number of revolutions turned in during that time.

4) A turbine is increased in speed from 2000 to 3600 rev/min while it turns 5500 revolutions. Assuming that the rate of increase of speed is uniform, find the acceleration in rad/s^2.

3.10 Linear and Angular Motion.

Consider a point moved around on a circle path, if **θ**(**theta**) represents the angular displacement in radians, ***r*** **the radius,** and ***s*** the length of the arc or linear distance moved, then:

(i) $\boldsymbol{S\ (distance\ ) = \theta \times r}$

Similarly, if a point is travelling in a circular path, the linear distance travelled in one second is the number of radians moved through in one second multiplied by the radius. Let ***v*** represent the **linear velocity**,and **ω(omega)**the **angular velocity** in radians per second, and ***r*** **the radius**, then:

(ii) $\boldsymbol{V\ (velocity) = \omega \times r}$

Further, if the point is accelerating at the rate of **α (alpha)** radians per second per second and the linear acceleration is represented by ***a*,** then:

$$\boldsymbol{a\ (acceleration) = \alpha \times r}$$

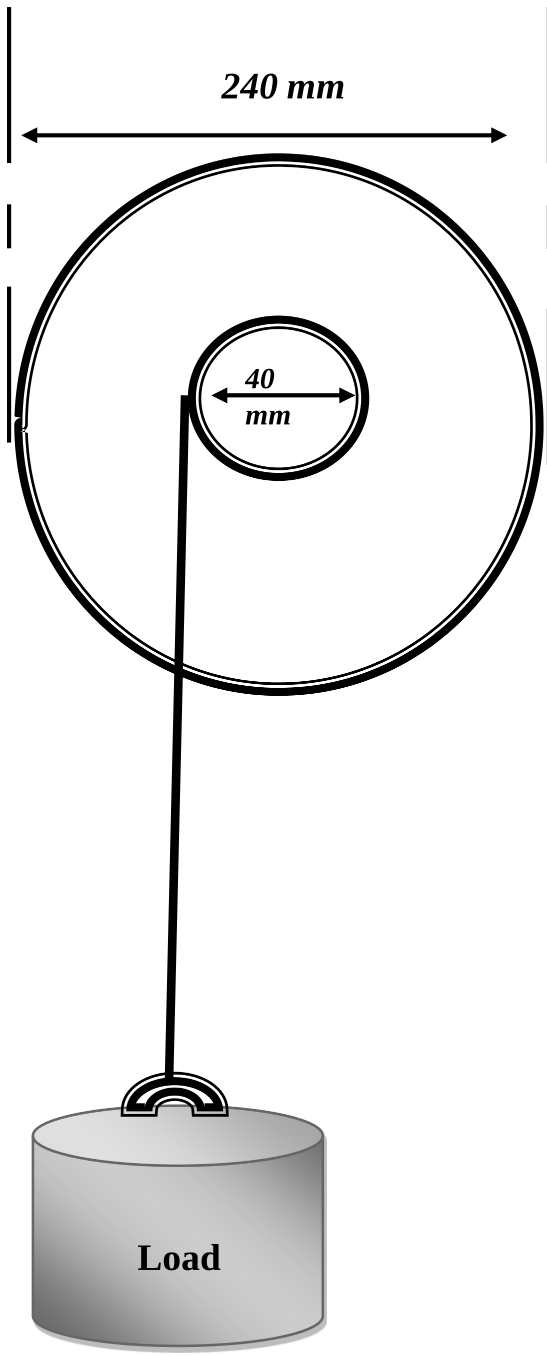
In Words the above conversion rules are:

Linear distance = angular distance × radius.

Linear velocity = angular velocity × radius.

Linear acceleration = angular acceleration × radius.

In conclusion we find that in all cases it is simply multiplying the angular quantity by the radius, to obtain the corresponding linear quantity.

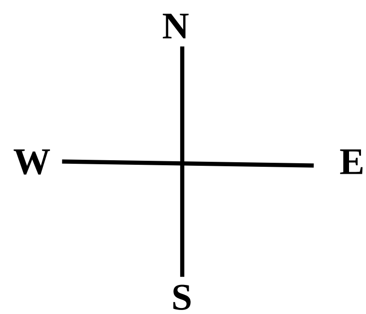


Example: ***A flywheel 240 mm dia. Is keyed to a shaft 40 mm dia. Mounted in a bearing which carry the shaft horizontally. A cord is wrapped around the shaft, one end of the cord being fixed to the shaft and the other end carrying a load. When the load is allowed to fall from rest, it falls a distance of 2 m in 5 seconds. Neglecting the thickness of the cord, find***

(i) ***The linear velocity of the load after 5 seconds,***
(ii) ***The angular velocity of the wheel and shaft after 5 seconds.***
(iii) ***The linear velocity of the rim of the wheel after 5 seconds.***

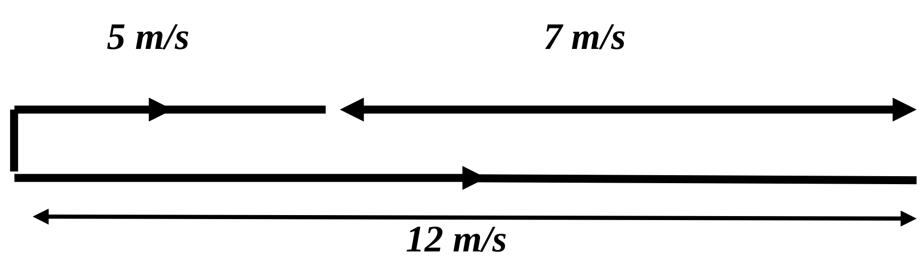
3.11 Change of Velocity due to change of direction.

Velocity is a vector quantity representing speed and direction and therefore a change of velocity takes place if the speed changes without any change of direction, or if the direction changes while the speed remains the same, or if a change in both speed and direction.



Example A:

Space diagram

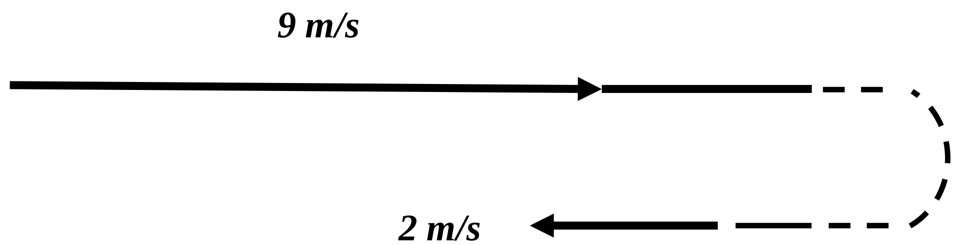


Vector Diagram

Solution of Example A *represents a body which was moving at* ***5m/s due East****, having its velocity changed to* ***12 m/s due East,*** *Here the vectors of each velocity is drawn from a common point, the difference between the free ends of the vectors is the change of velocity, in this case it is* ***7 m/s.***

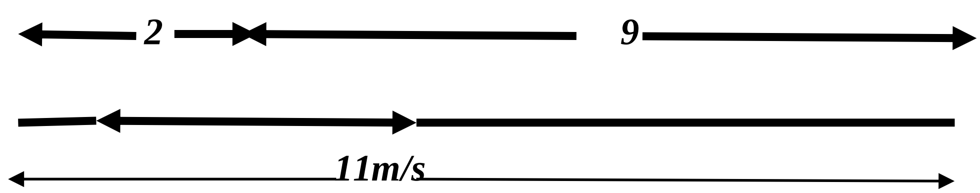
Example B

(a)



Space Diagram:

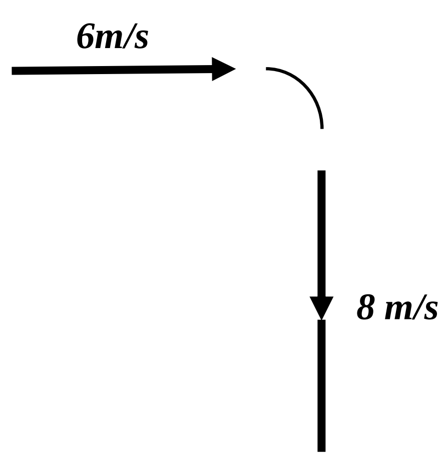
(b)

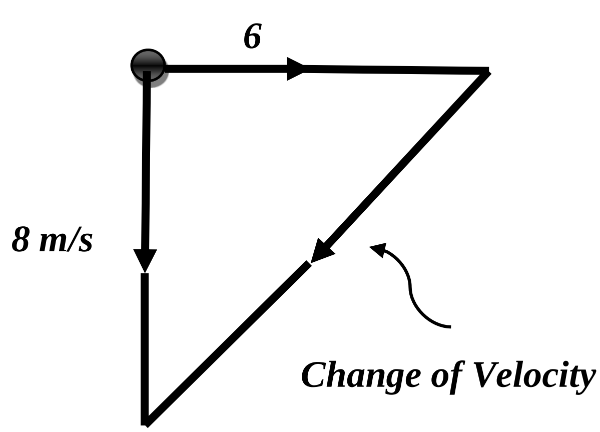


Vector Diagram:

Solution of Example B *represents a body with an initial velocity of* ***9 m/s due East,*** *being changed to* ***2 m/s due West,*** *the vector diagram shows the vector of each velocity drawn from a common point, the difference between their free ends is the change of velocity which is* ***11 m/s.***

Example C :





Solution of Example C *is that of a body with an initial velocity of* ***6 m/s due East*** *changed to* ***8 m/s due South.*** *The vector diagram is constructed on the same principal of the two vectors drawn from a common point. The change of velocity is, as always, the difference between the free ends of the two vectors, this is ;*

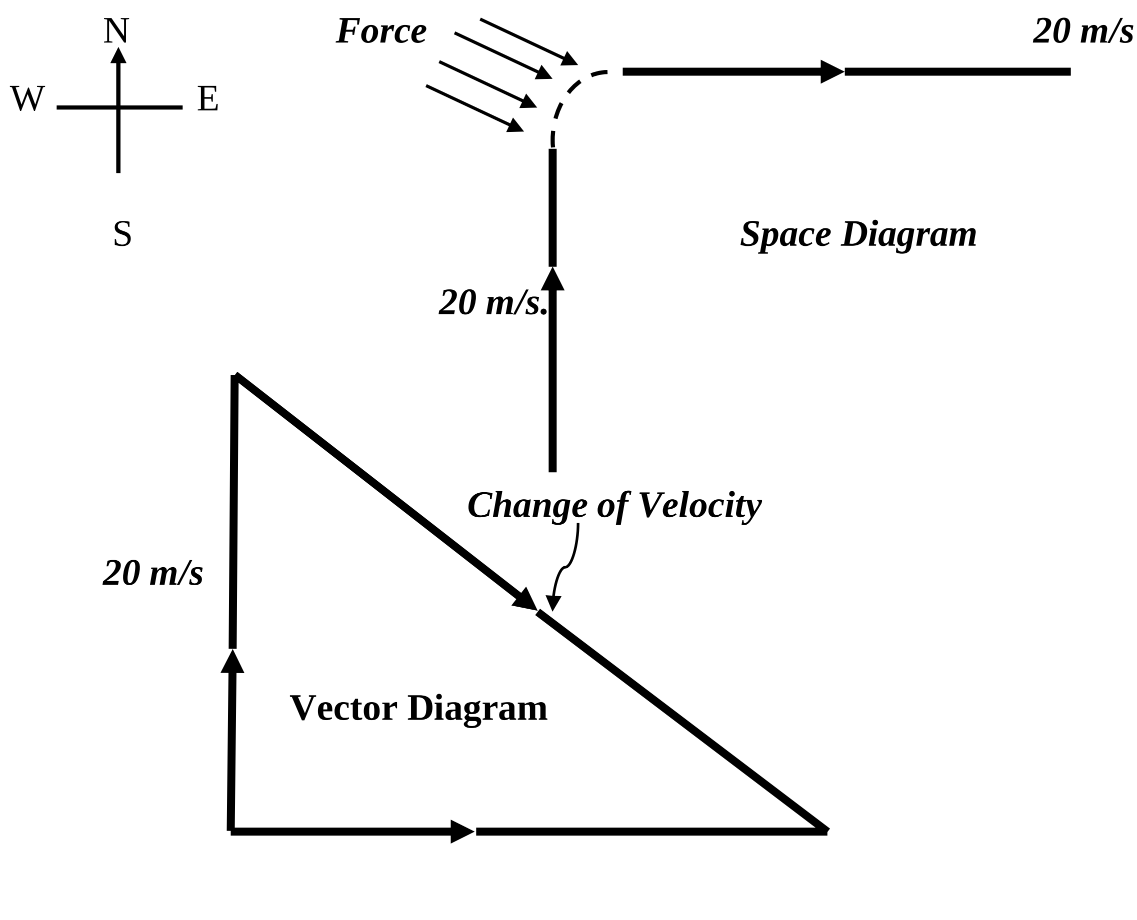
$\sqrt{8^2 + 6^2}$ ***= 10 m/s.***

*In New tons second law of motion it states that "**rate of change of momentum is proportional to the impressed force and takes place in the direction in which the force acts.",** hence if the mass of the body remains constant, this tell us that the change of velocity takes place in the direction of the applied force.*

Acceleration is the change of velocity, therefore in all of these cases the value of the acceleration can be obtained in the usual way by dividing change of velocity by time.

Work Example: (1)

A body moving at 20 m/s due North is acted upon by a force for 4 seconds which causes the velocity to change to 20m/s due East. Find the change of velocity and the average acceleration.



20 m/s

Solution:

Change of Velocity$=\sqrt{\mathbf{20^{2+}20^{2}}}$

= 28.28 m/s Ans (i)

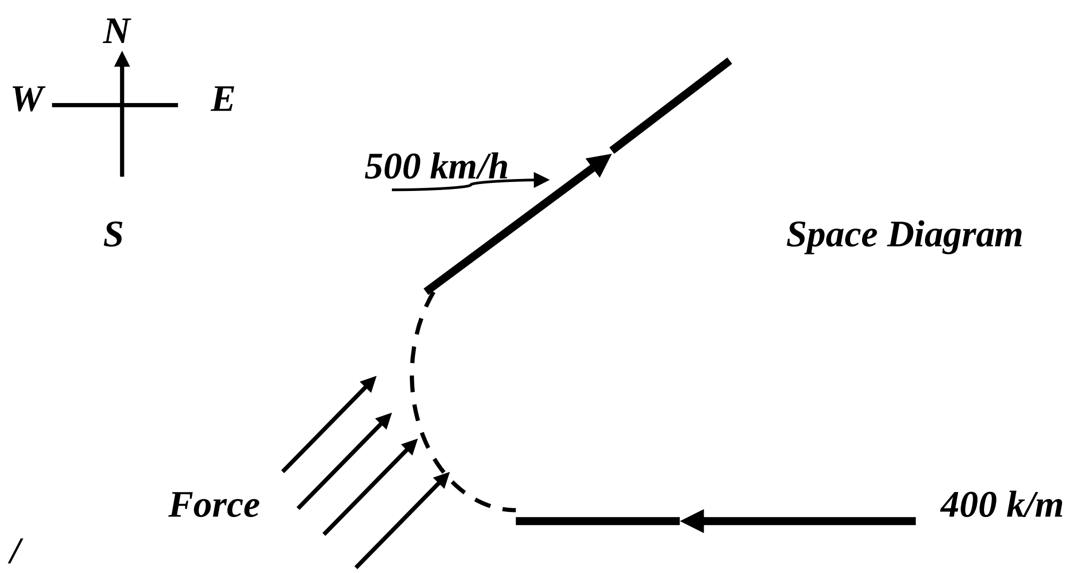
Acceleration $=\frac{\mathbf{ChangeofVelocity}}{\mathbf{Timetochange}}$

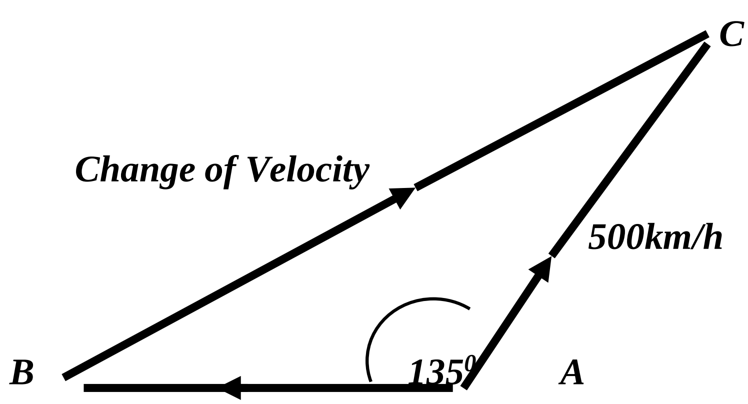
$=\frac{\mathbf{28.28}}{\mathbf{4}}$

= *7.07 m/s* ^{2} ***Ans. (ii)***

Example (2)

An aircraft changes velocity from 400 km/h due West to 500 km/h North East in $\mathbf{{}^{1}/_{2}}$ ***minute. Find the average acceleration in m/s²***.





400km/h

By cosine rule, working in 100 units to keep figures down,

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$= 5^2 + 4^2 - 2 \times 5 \times 4 \times \cos 135^0$$

$$= 25 + 16 + 28.28$$

$$a = \sqrt{69.28}$$

$$= 8.324$$

$\therefore$ **Change of Velocity = 832.4 km/h**

$$= \frac{832.4 \times 10^3}{3600} \text{ m/s}$$

$$\text{Acceleration} = \frac{\text{ChangeofVelocity}}{\text{Timetochange}}$$

$$= \frac{832.4 \times 10^3}{3600 \times 30}$$

=7.707 m/s^2 Ans.

3.12 Practice Exercise:

1) ***A motorist travels from one town to another, a distance of 135 km, in 3 hours. One the return journey the same distance iscovered in 2 $\frac{1}{2}$ hours. Find the average speed in each direction and the average speed for the double journey there and back.***
2) ***From a ship sailing due North, a light is sighted on the shore at a distance of 15 nautical miles in the direction of North 50^0 West, and 40 minutes later the light house is directly a beam. Find the speed of the ship.***
3) ***The speed and direction of a motor launch is changed from 9 knots due North to 11 knots due West in 30 seconds. Find the average acceleration in m/s$^{2.}$ One knot = 1.852 km/h.***