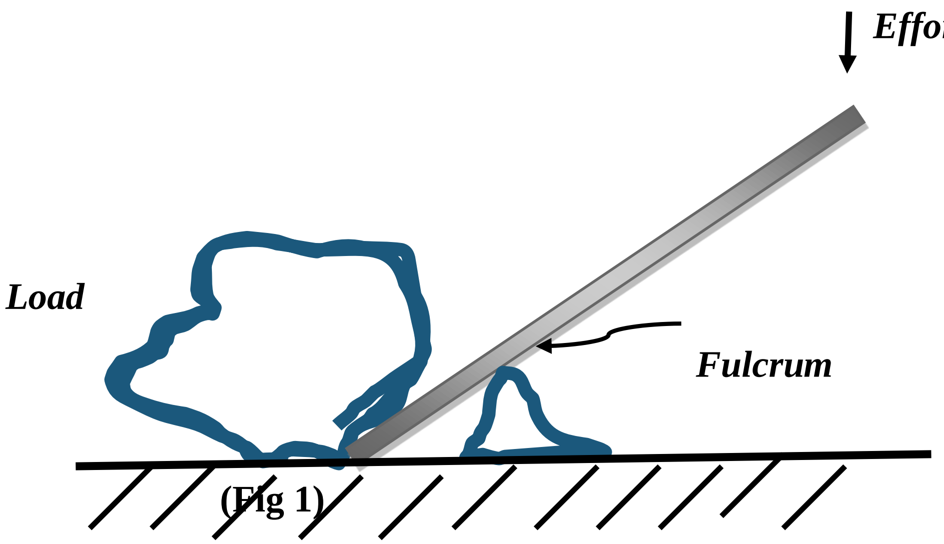
Chapter 5

Moments

5.1 Introduction:

A machine is a device where force is applied at one end in order to obtain force at another point. Machines normally make work easier. The force applied is usually called the ***effort*** and the force overcome by the effort, is called the ***load***. The effort overcomes the load as soon as their moments are equal.

A lever, which consists of a rigid bar that can be turned around a fixed point, is simplest of machines. The fixed point, about which the lever rotates, is called the ***fulcrum*** or turning point. (See (fig 1) below.)



(Fig 1)

Assume that a rock can be partially lifted by applying a force to the one end of a crowbar as shown in figure (1) as above. If the same force is applied closer to the fulcrum, the rock seems to be heavier because it does not move; a greater force will now be needed to lift the rock.

1.2 Moment of a force

The moment of a force about a point can be calculated as follows:

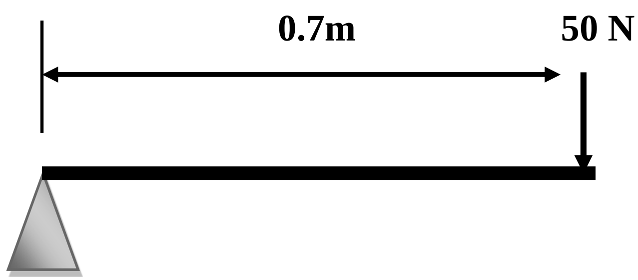
Moment of a force = ***force (in Newton's) × perpendicular Distance from fulcrum to the line of action of the force (in meters)***

The unit for the moment is the ***Newton meters (N.m),*** therefore the force must be in Newton's and the distance in meters.

Example:

A force of 50 N is applied perpendicularly to the horizontal spanner,700 mm from the fulcrum. Calculate the moment of the force.

Solution:



(fig 3)

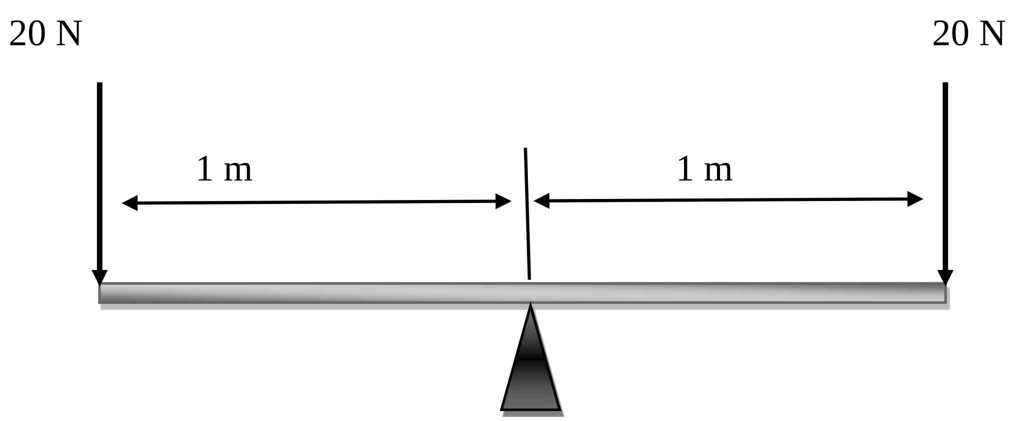
Moment = force x perpendicular distance (in Meters)

= 50 x 0,7

= 35 N.m

5.3 The law of moments;

A uniform lever is used to carry two equal loads: each load is exactly 1m from the fulcrum (support) (see fig 4)



In perfect conditions it may happen that the lever will be in equilibrium, because:

Anti- clockwise moments) = **(clockwise moments about**

About fulcrum. **same fulcrum.)**

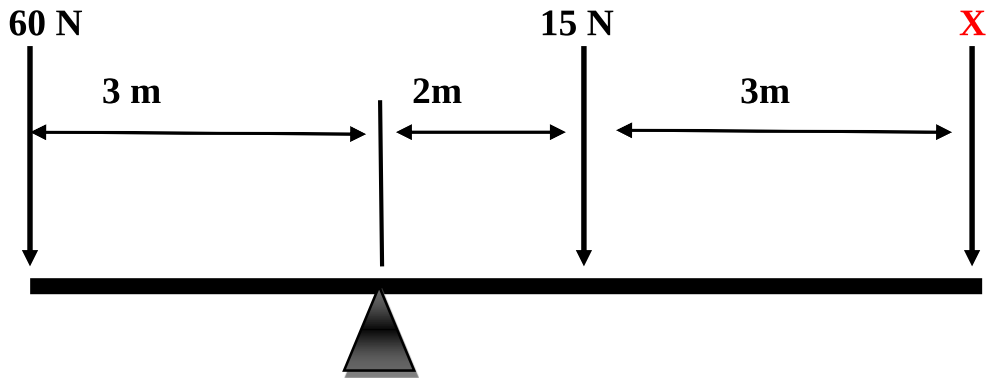
20 n x 1 m = 20 N x 1 m

20 Nm = 20 Nm

5.3.1 Definition of the law of moments.

If a system of forces is in equilibrium, the sum of the clockwise moments about any turning point equals the sum of the anti- clockwise moments about the same turning point.

Example:



Clock wise Moments = Anti- clockwise moments

About the support about the support

$$(X \times 5) + (15 \times 2) = 60 \times 3$$

$$5x + 30 = 180$$

$$5x = 180 - 30$$

$$X = \frac{150}{5}$$

$$x = 30\ N$$

Unknown force = X = 30 N

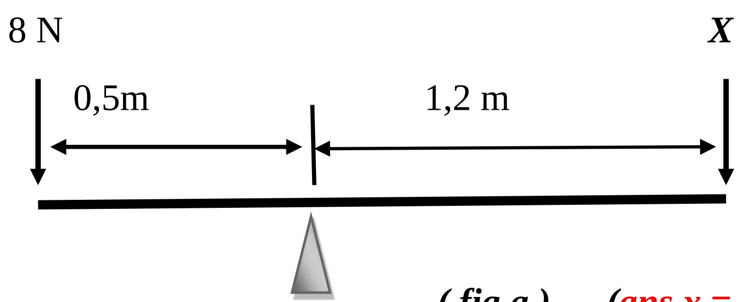
Load on support =total of all the downward forces

= 60 + 15 + 30

= ***105 N...... Ans***

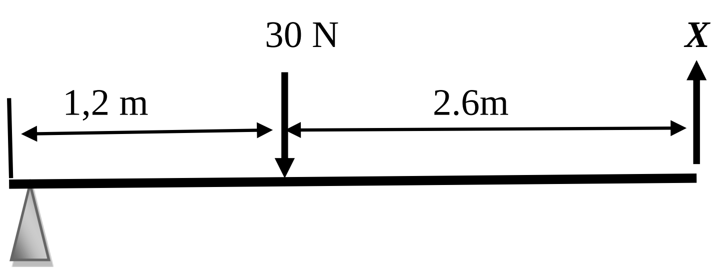
Exercise 5:

1).**Calculate the magnitude of the unknown force** ***X in*** **Fig (a)**



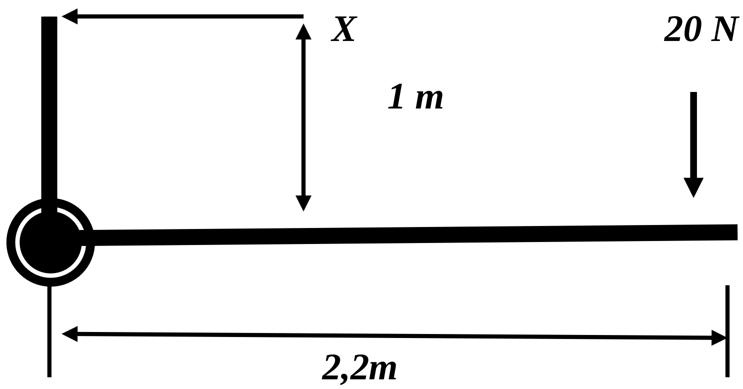
(fig a)......(ans x = 3,33 N).

2). Calculate the magnitude of the unknown force X **in Fig** ***(b)***



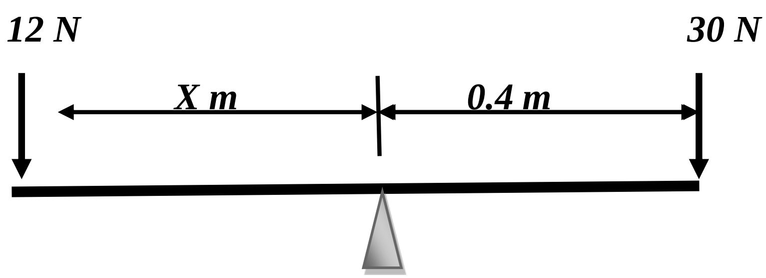
(fig. b**).....(ans *X* = 9,278N)**

3). *Calculate the magnitude of the unknown force X in fig.(c)*



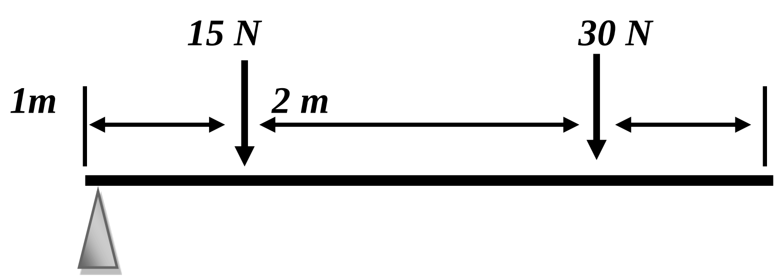
(Fig c.)……...(ans X = 44N)

4). Calculate the distance of the unknown force X in (fig.d)



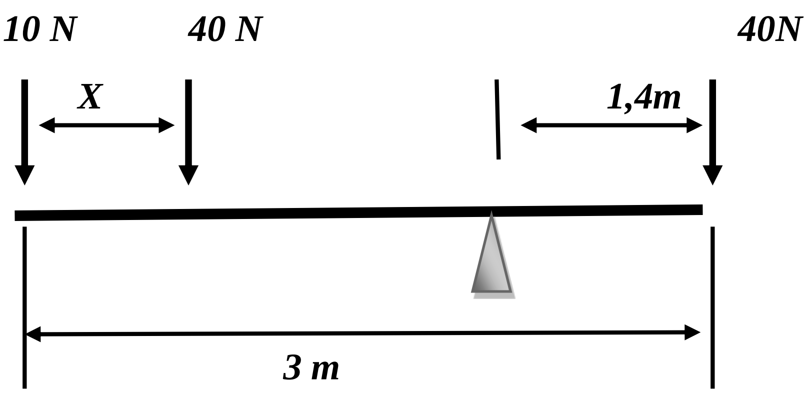
(fig.d) ….. (ans X = 1 m)

5).Calculate the magnitude of force X in (fig e)



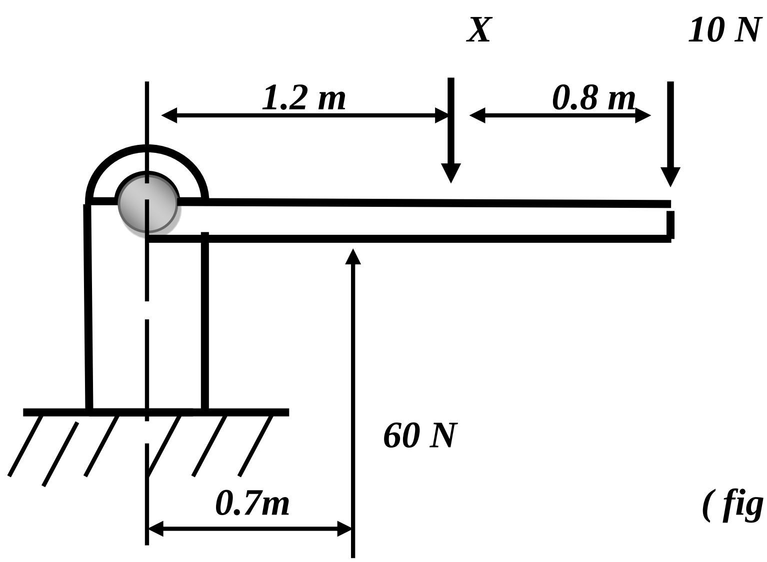
fig, e)……...(ans X =26,25 N

6).Calculate the distance X in (fig f)



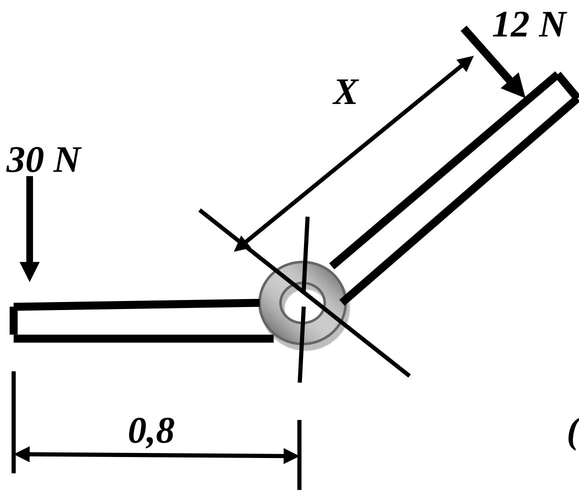
(fig f.......(ans X = 0.6 m)

7). Calculate the magnitude of force X to obtain equilibrium in (Fig G)



(fig G........(ans X = 18,33)

8). Calculate the length of the inclined portion of the lever in (fig h.)



(fig h... (ans X = 2m)

9). A light bar is supported as shown in figure (i) Find the magnitude of X for equilibrium. Also find the pulling force in the rope.

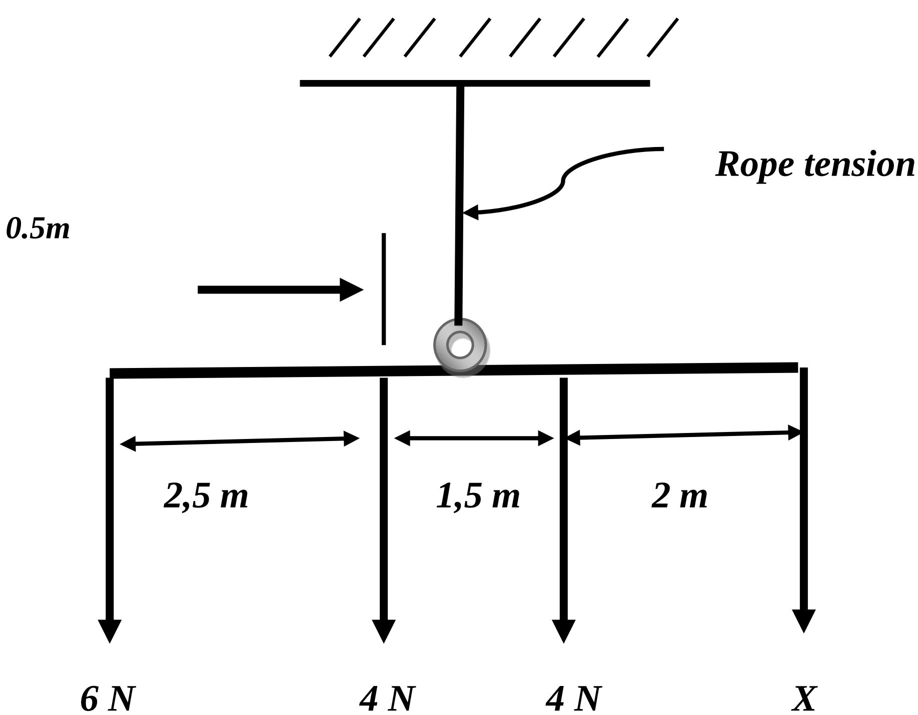
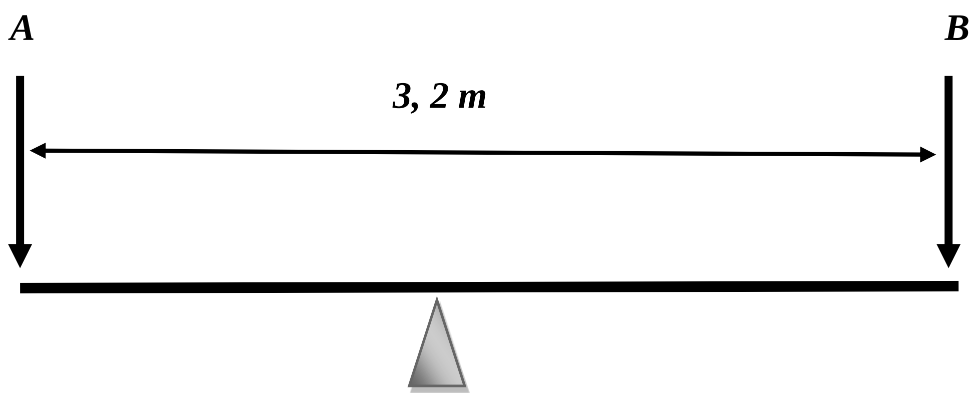


Fig (i).....ans X = 5.33 N

Rope tension 19.33 N

10). A lever that can turn freely about the turning point is shown in the (fig j.). Calculate what distance from load A to the turning point must be when A = 3B



(fig j)........ans 0,8 m

5.4 Beams and the reaction of the supports.

Beams are based on moments of forces; the moment of a force about a point can be calculated as follows:

Moment of a force = force (in N)× perpendicular distance from the fulcrum to the action of the force (in m)

Example:

A force of 50 N is applied perpendicularly to the horizontal spanner, 700mm from the fulcrum. Calculate the moment.

Solution:

Taking moments about the support and applying a force of 50 N at a distance of 0.7m

5.4.1 Concentrated loads and mass of the beam:

Two fulcrums, called supports are normally used in the application of the beams. One of these supports is used as a turning point to calculate the moments of the applied forces. The reactions of these supports are normally unknown and must be calculated. One unknown support is eliminated by using a support as a fulcrum (turning point,)since the moment of the fulcrum support is equal to nil ; moment is equal to force (i.e. the fulcrum support) times the distance. Since the support is also the turning point (fulcrum), the support and the fulcrum coincide and the distance is nil. The moment will therefore be nil.

To ***calculate the reactions of the supports*** we equate the clockwise moments to the anti-clockwise moments.

To ***calculate Uniform beams*** it can therefore be accepted that the weight of the beam can be represented by a single down ward force at the centre of the beam.

If the ***system of forces is in equilibrium***, then the sum of the parallel forces acting in on direction is equal to the sum of the parallel forces acting in the opposite direction.

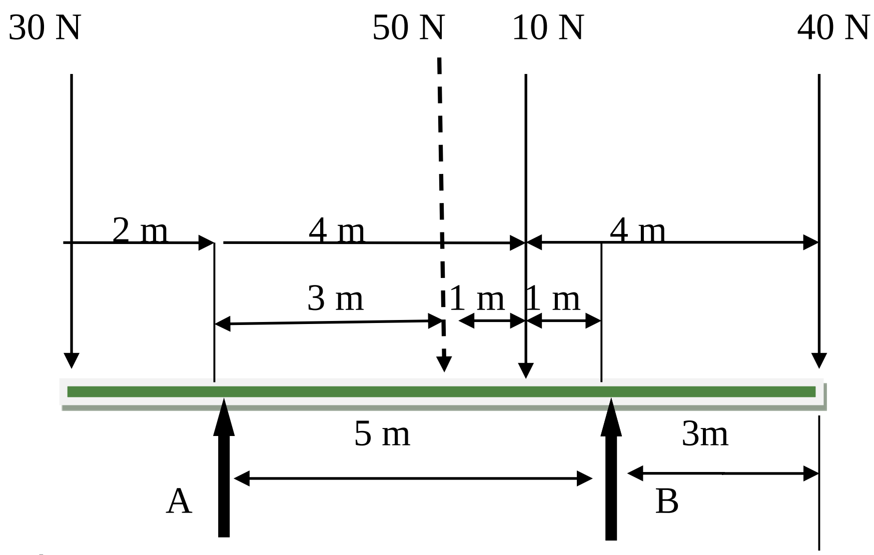
Simplified: ***sum of the upward forces = sum of the downward forces.***

Example:

A beam with a length of 10 m and a weight of 50 N is supported by two supports. One support is 2m from the left end, the other 3 m from the right end. The concentrated load is as follows; 30 N on the left end:40N at the right end; and a further 10 N at 4m from the rightend. Calculate the supports A and B.

Solution:

Draw a line to indicate the beam details. A dashed line may be used to indicate the weight of the beam; then there is a distinctive difference between the applied loads and the weight. Number the supports, e.g. ***A*** *and* ***B.*** *or* $\mathbf{R_1}$ *and* $\mathbf{R_2}$



*Take moments about **A:***

Anti clockwise moments = Clockwise moments.

$$(B \times 5) + (30 \times 2) = (50 \times 3) + (10 \times 4) + (40 \times 8)$$

$$5B + 60 = 150 + 40 + 320$$

$$B = \frac{450}{5}$$

$$B = 90\ N$$

*Take moments about **B:***

Clockwise moments = anti clockwise moments

$$(A \times 5) + (40 \times 3) = (10 \times 1) + (50 \times 2) + (30 \times 7)$$

$$5A + 120 = 320$$

$$5A = 320 - 120$$

$$A = \frac{200}{5}$$

$$= 40\ N$$

Test if your answer is correct for Equilibrium:

Upward Forces = Downward Forces

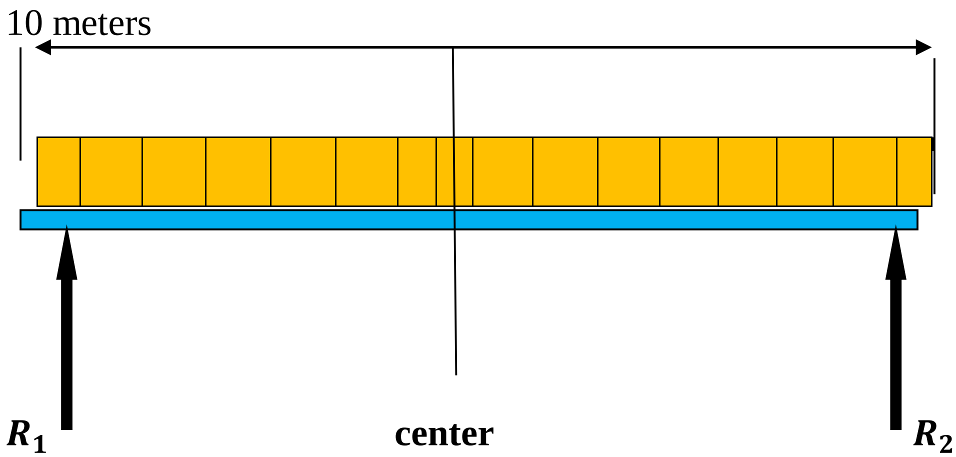
$$A + B = 30 + 50 + 10 + 40$$

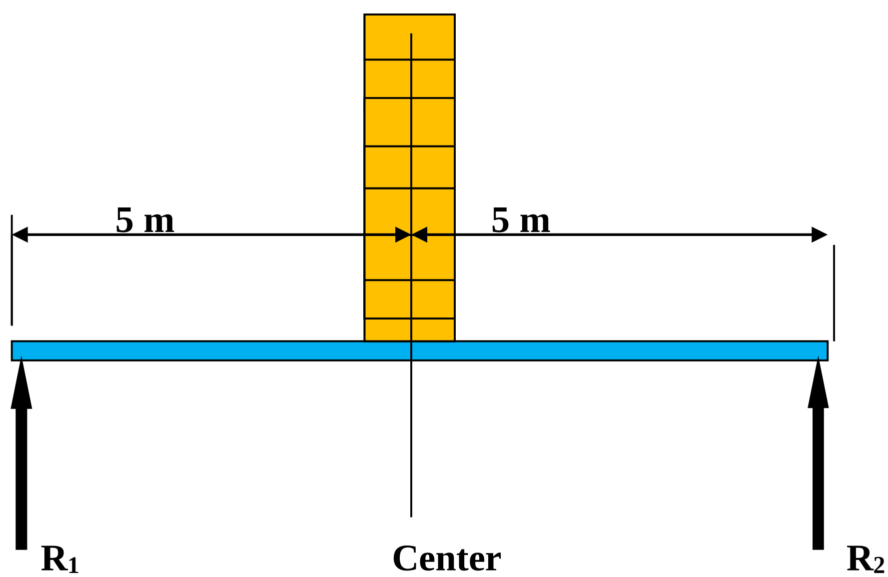
$$90 + 40 = 130$$

130 N ***= 130 N***

5.4.2 Uniform distributed Loads (UDL)

A uniformly distributed load is that load that is uniformly distributed over the length of the beam, or over a section (part)of the beam, Example ,take a number of bricks and place them side by side to cover the entire beam of example of 10 mtrs.This will carry a load evenly over the entire beam. But if the same amount of bricks were to be placed on top of each other in the centre of the beam then this will have the same load effect.





5.4.3 Shearing Force and Bending Moment Calculations & Diagrams

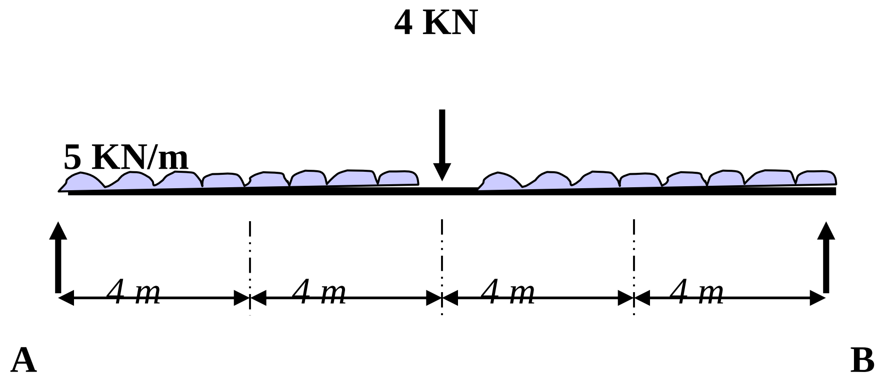
Example :

A 16 m long beam with UDL 5 kN/m is simply supported at both ends and carries a load of 4kN at its mid point. Draw the diagram: And

a) Calculate the reaction at the supports.

b) Draw the shear force diagram

c) Calculate and draw the bending moments at 4 m intervals and draw the Bending Moment's diagram

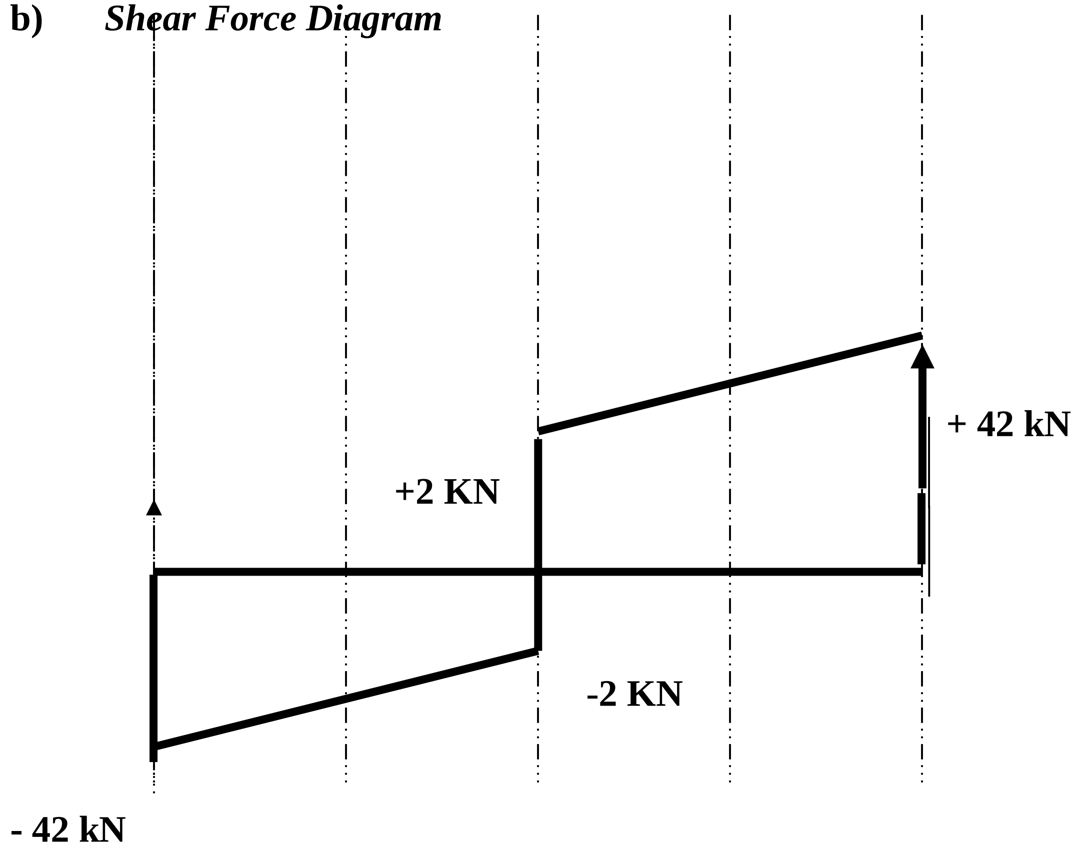


Weight of the Beam $= 5\ KN/m \times 16\ m = 80\ KN$

Total force down $= 80 + 4\ KN = 84\ KN$

Reactions at A and B $= 84/2 = 42\ KN$ ***Each.***

b) ***Shear Force Diagram***



(c) ***UDL = 5 kN/m***

Taking moments about the left side:

@ A = 0

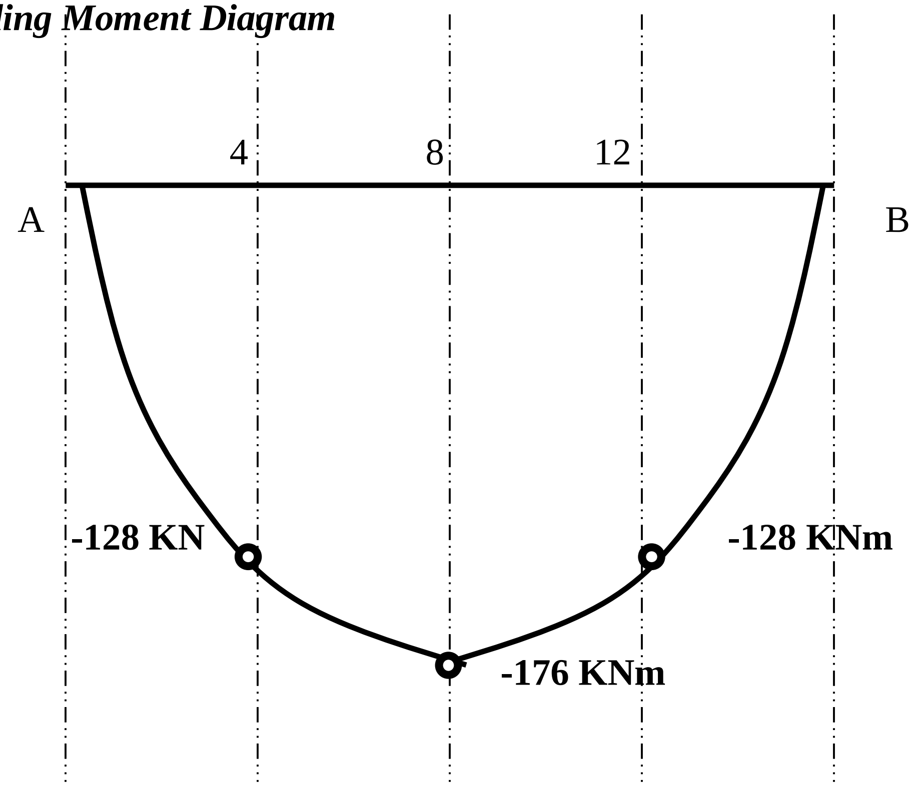
@ 4 m = (-42x4) + (4x5x2) = -168 + 40 = -128 kNm

@ 8 m = (-42x8) + (8x5x4) = -336 + 160= -176 kNm

@ 12m = (-42x12) + (12x5x6) + (8x2) = - 504 + 360 + 16 = -128 kNm

@ 16m = (-42x16) + (16x5x8) + (8x4) = - 672 + 640 + 32 = 0 kNm

Bending Moment Diagram



Exercise 1

A16 m long beam with UDL 5 kN/m is simply supported at both ends and carries a load of 4kN at its mid-point.

(a) Draw the diagram:

(b) And calculate the reaction at the supports.

(c)Draw the shear force diagram

(d) Calculate and draw the bending moments at 4 m intervals and draw the Bending Moments diagram.

Exercise 2

A 10 m long beam of negligible weight is simply supported at each end (A and B). At 2 m from A, it carries a load of 20 kN and at 4 m from B, it carries a load of 30 kN.

(a). Draw the diagram.

(b). Calculate the reaction at the supports

(c). Draw the Shear Force diagram

(d). Calculate and draw the Bending Moment diagram, indicating the value of the maximum B.M.

Exercise 3

A simply supported beam ABCD is shown in Fig. Q 4. It carries loads of 8.5 kN at B and 5.5 kN at C.

(a)Calculate the values of shear force and bending moment at the points A, B, C and D.

(b) Draw the shear force and bending moment diagrams to a suitable scale.

(c) Give the values of shear force and bending moment at the mid-length of the beam

Exercise4

(***1) Explain the terms 'shear force' and 'bending moment'.***

(2) A horizontal beam, simply supported at its ends, is 8 metres long. It carries a uniformly distributed load of 2 kN/m over its entire length together with a concentrated load of 4 kN acting at mid-length.

(a) Calculate the reactions.

(b) Calculate the values of shear force at 2 m intervals along the beam using these values to draw the shear force diagram.

(c)Calculate the values of bending moment at 2 m intervals using these values to draw the bending moment diagram.

(d) State the magnitudes and positions of the maximum shear force and maximum bending moment.

Exercise 5

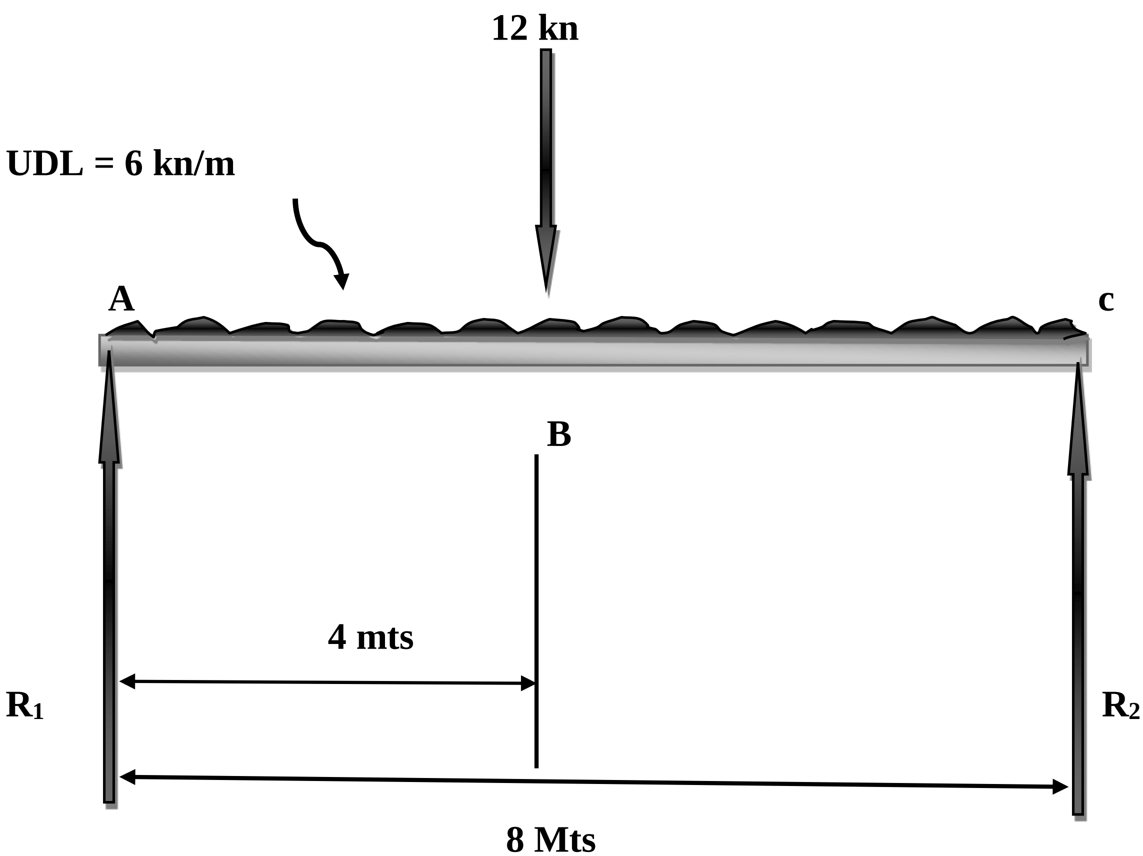
A loaded simply supported beam ABC is shown in

Calculate the reaction forces R_1 and R_2.

Calculate sufficient values for shear force and bending moment at points along the length of the beam and use these values to plot;

- ***(i) The shear force diagram to a scale of 1 cm = 10 kN***
- ***(ii) The bending moment diagram to a scale of 1 cm = 20 kNm***

State the magnitude and position of the maximum shear force and the maximum bending moment.



Exercise 6

A16 m long beam with UDL 5 kN/m is simply supported at both ends and carries a load of 4kN at its midpoint. Draw the diagram: And (a) Calculate the reaction at the supports. (b) Draw the shear force diagram

Exercise 7

A beam of length 14.5m and of mass 3 tonnes is simply supported at its ends. The beam is loaded as follows:-

4 kN at 3 m from the left end support,

6 kN at 2 m to the right of the centre of the beam,

1.5 kN at 1 m from the right hand end support.

Solution:

Draw to scale the shear force and bending moment diagrams.

From your diagram, state the magnitude and position from the left hand end of the highest bending moment

Exercise 2

A 10 m long beam of negligible weight is simply supported at each end (A and B). At 2 m from A, it carries a load of 20 kN and at 4 m from B, it carries a load of 30 kN.

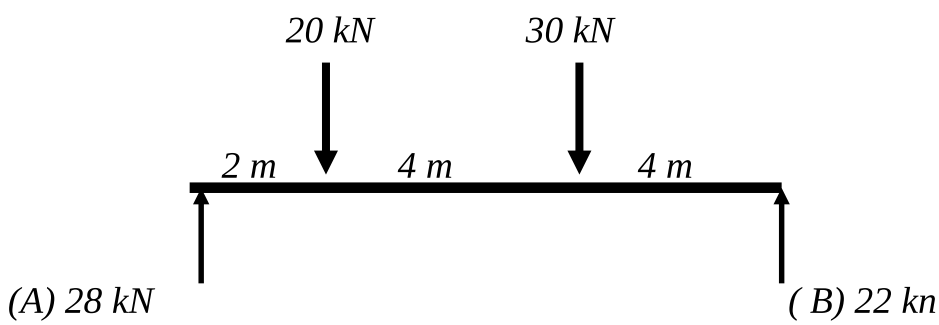
(a). Draw the diagram.

(b). Calculate the reaction at the supports

(c). Draw the Shear Force diagram

(d). Calculate and draw the Bending Moment diagram, indicating the value of the maximum B.M.

(a). Draw the diagram.



(b). Calculate the reaction at the supports

Clockwise moments = Anti-clockwise moments

Taking moments about A:

(20x2) + (30x6) = 10RB

10RB = 40 + 180

RB = 22 kN

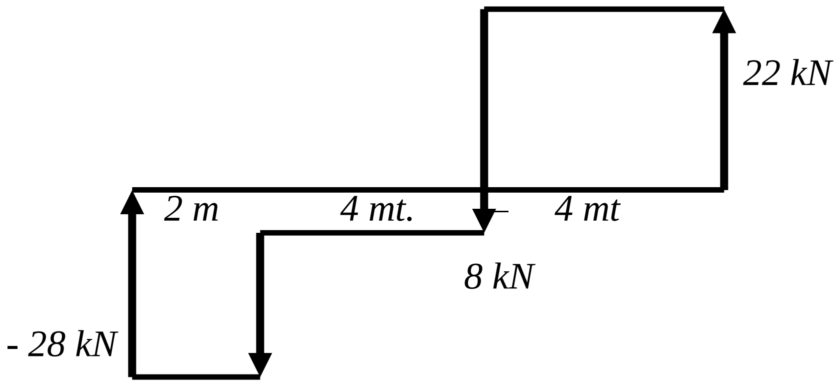
Total downward force *= 20 + 30 kN = 50 kN*

$$RA + RB = 50\ kN$$

$$RA = 50\ kN - RB = 50 - 22\ kN$$

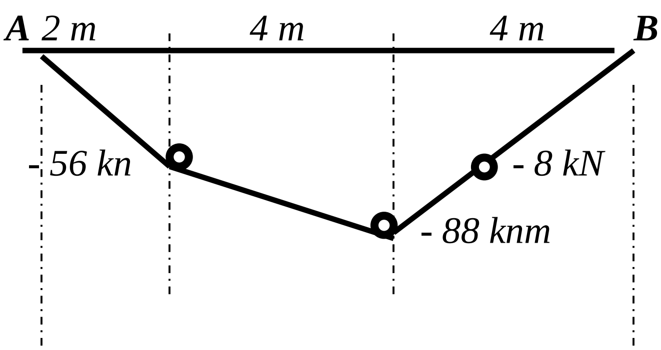
$$RA = 28\ kN$$

(c). Draw the Shear Force diagram



(d). Calculate and draw the Bending Moment diagram, indicating the value of the maximum

B.M.



5.5 Centre of Gravity

Centre of gravity, in physics, imaginary point in a body of matter where, for convenience in certain calculations, the total weight of the body may be thought to be concentrated. The concept is sometimes useful in designing static structures (e.g., buildings and bridges) or in predicting the behavior of a moving body when it is acted on by gravity.

5.5.1 Definition of Centre of Gravity

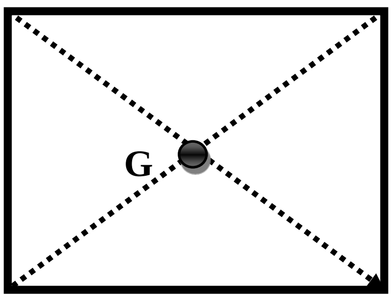
(1) center of mass

(2) the point at which the entire weight of a body may be considered as concentrated so that if supported at this point the body would remain in equilibrium in any position

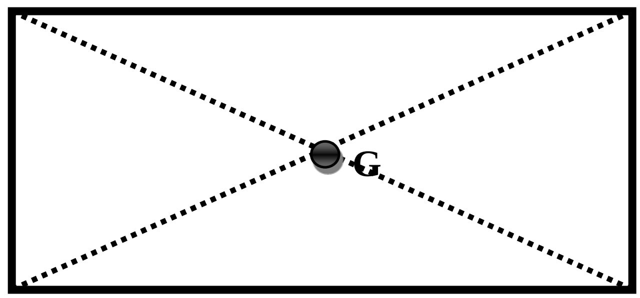
Overview

The centroid of an area is situated at its geometrical centre. In each of the following figures G represents the centroid, and if each area was suspended from this point it would balance.

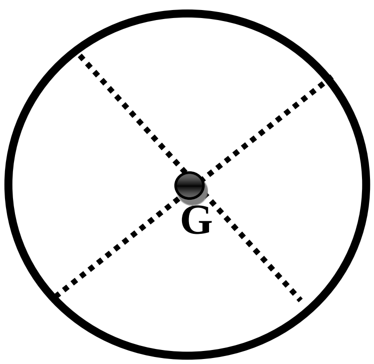
Square



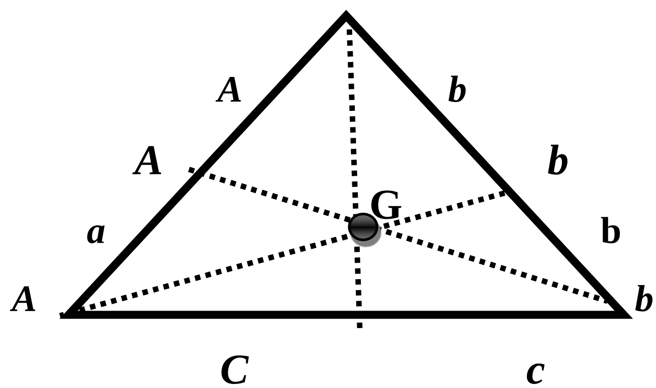
Rectangle



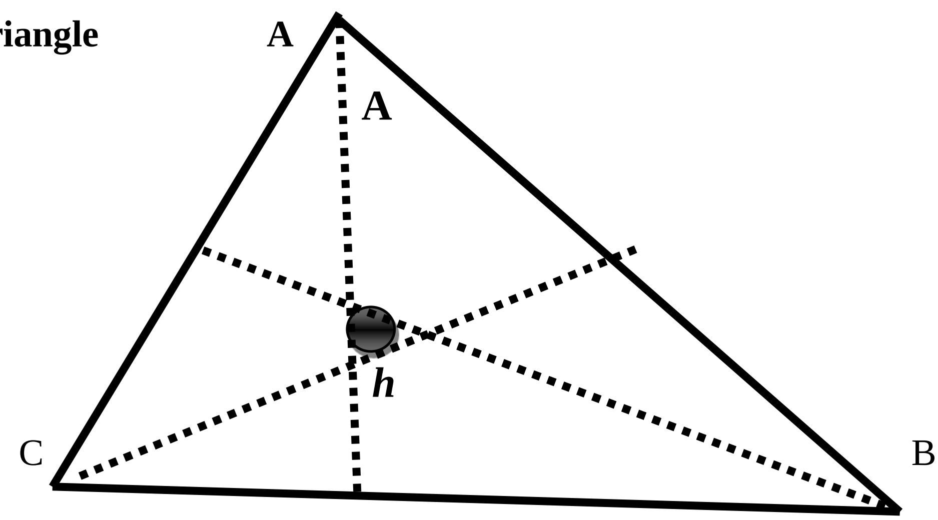
Circle

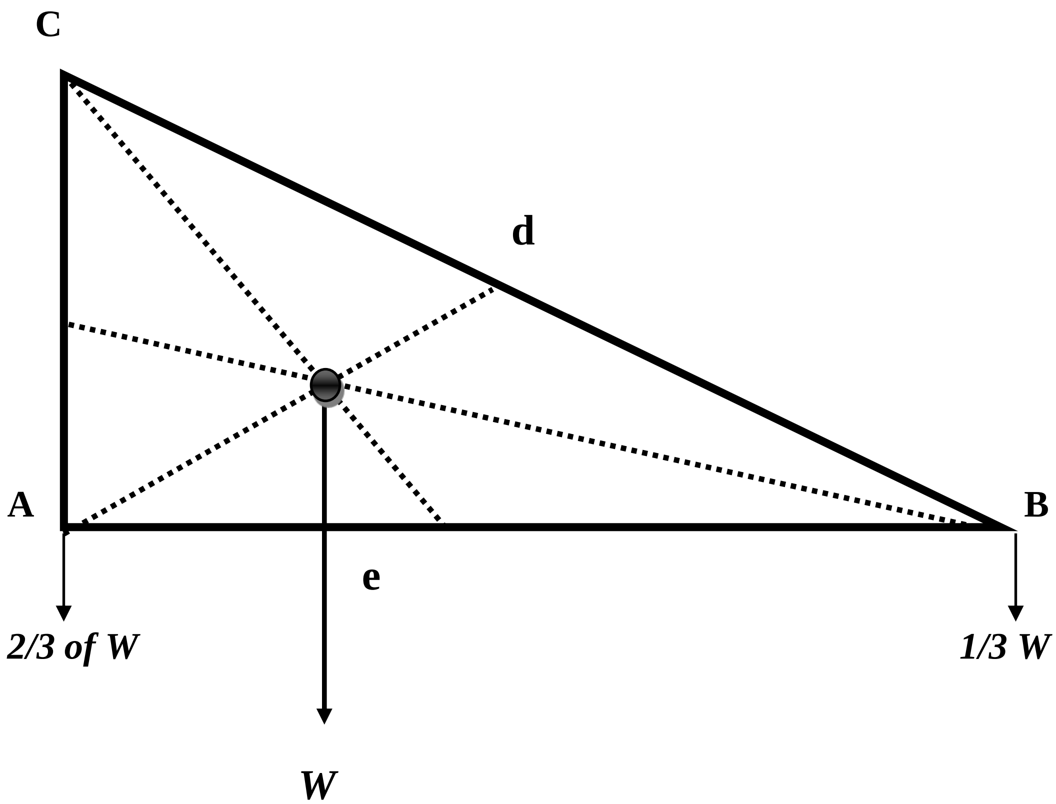


Triangle

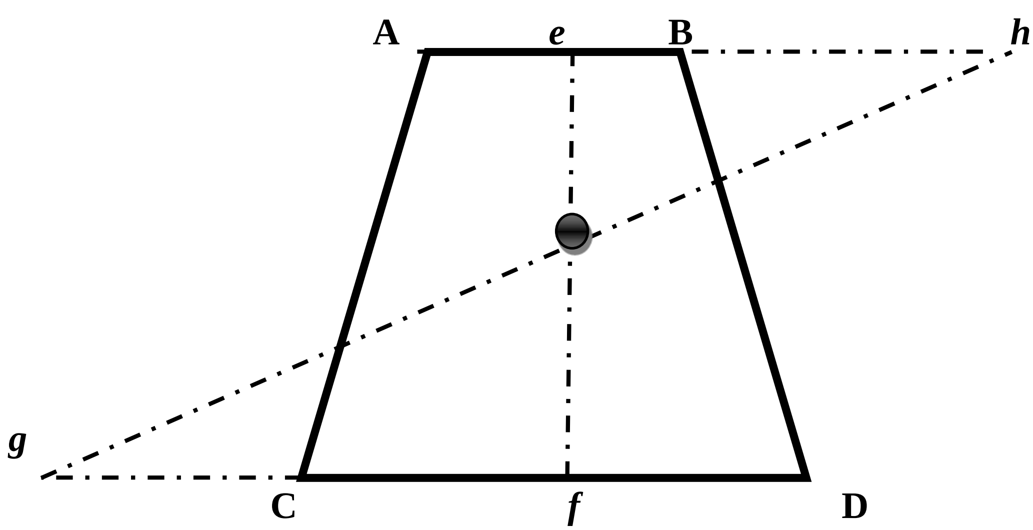


Scalene Triangle





TRAPEZIUM

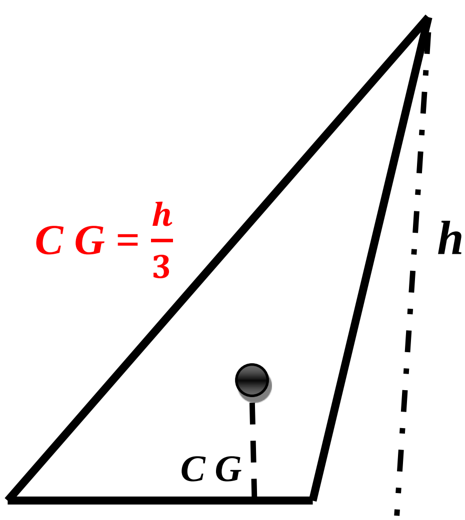


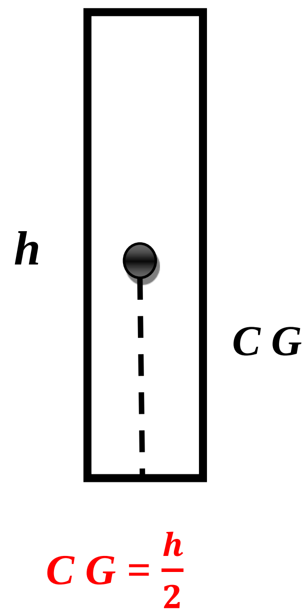
1) To find the centroid, bisect AB and CD at E and F, Join E and F,

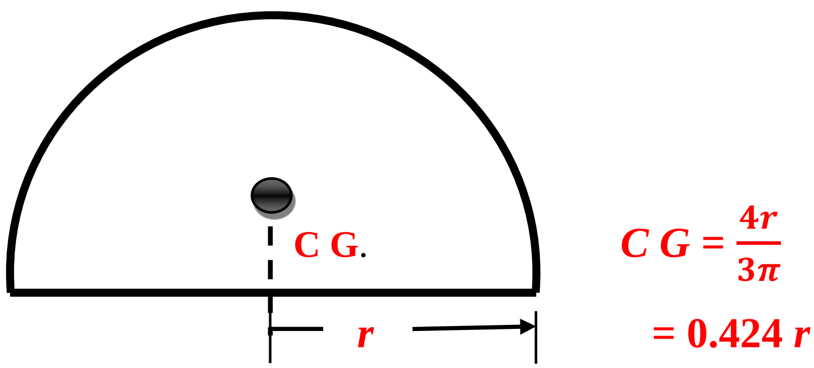
2) Extend DC to G such that CG equals DC.

3) Join G to H

Where EF and GH intersect is the centroid







Semi- circular area

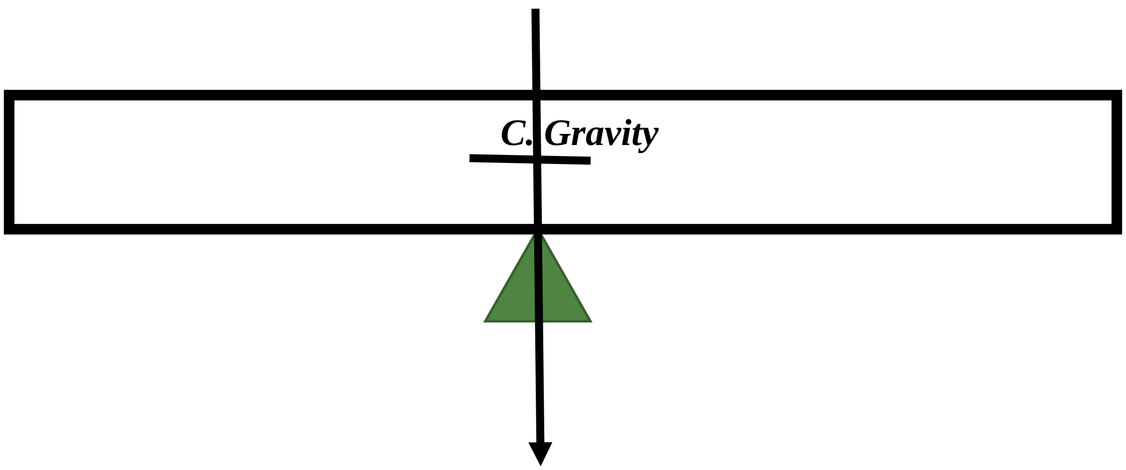
The centre of gravity of a body is the point at which all the mass of the body may be assumed to be concentrated and is the point through which the force of gravity is considered to act vertically downwards, with a force equal to the weight of the body. It is also the point about which the body would balance.

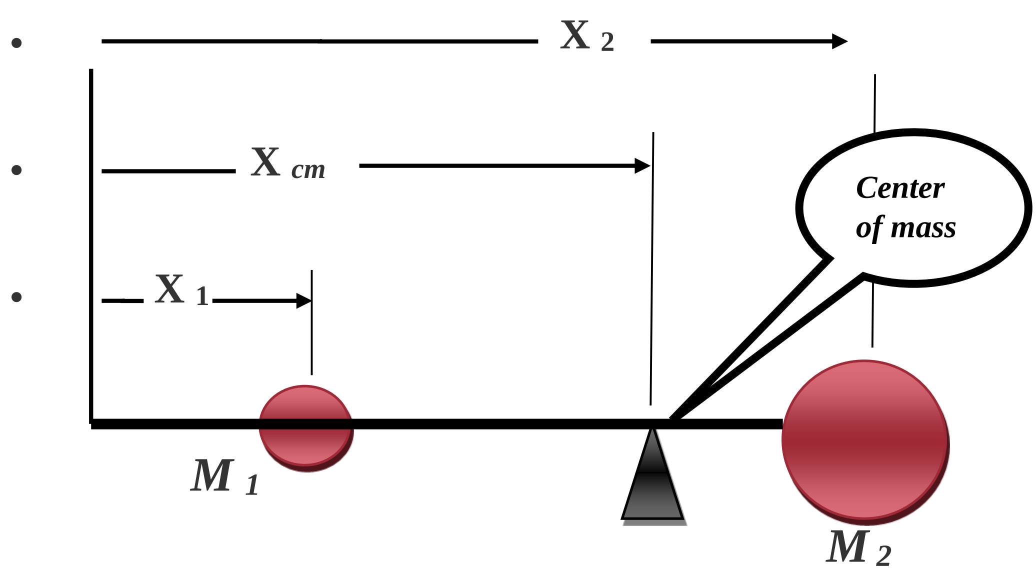
The centre of gravity of a homogeneous body is at its geometrical centre. Thus the centre of gravity of a homogeneous rectangular block is half-way along its length, half-way across its breadth and at half its depth.

Let us now consider the effect on the centre of gravity of a body when the distribution of mass within the body is changed.

5.5.2 Effect of Removing or Discharging Mass

Consider a rectangular plank of homogeneous wood. Its centre of gravity will be at its geometrical centre: i.e., half-way along its length, half-way across its breadth, and at half depth. Let the mass of the plank be W kg and let it be supported by means of a wedge placed under the centre of gravity as shown in Figure 2.2. The plank will balance.





- **For two masses:** $X_{cm} = \dfrac{m_1x_1 \times m_2x_2}{m_1+m_2}$

5.5.3 Centre of Mass for Particles

- The centre of mass is the point at which all the mass can be considered to be "concentrated"for the purpose of calculating the "**First Moment",**i.e, mass times distance. For two masses this distance is calculated from;

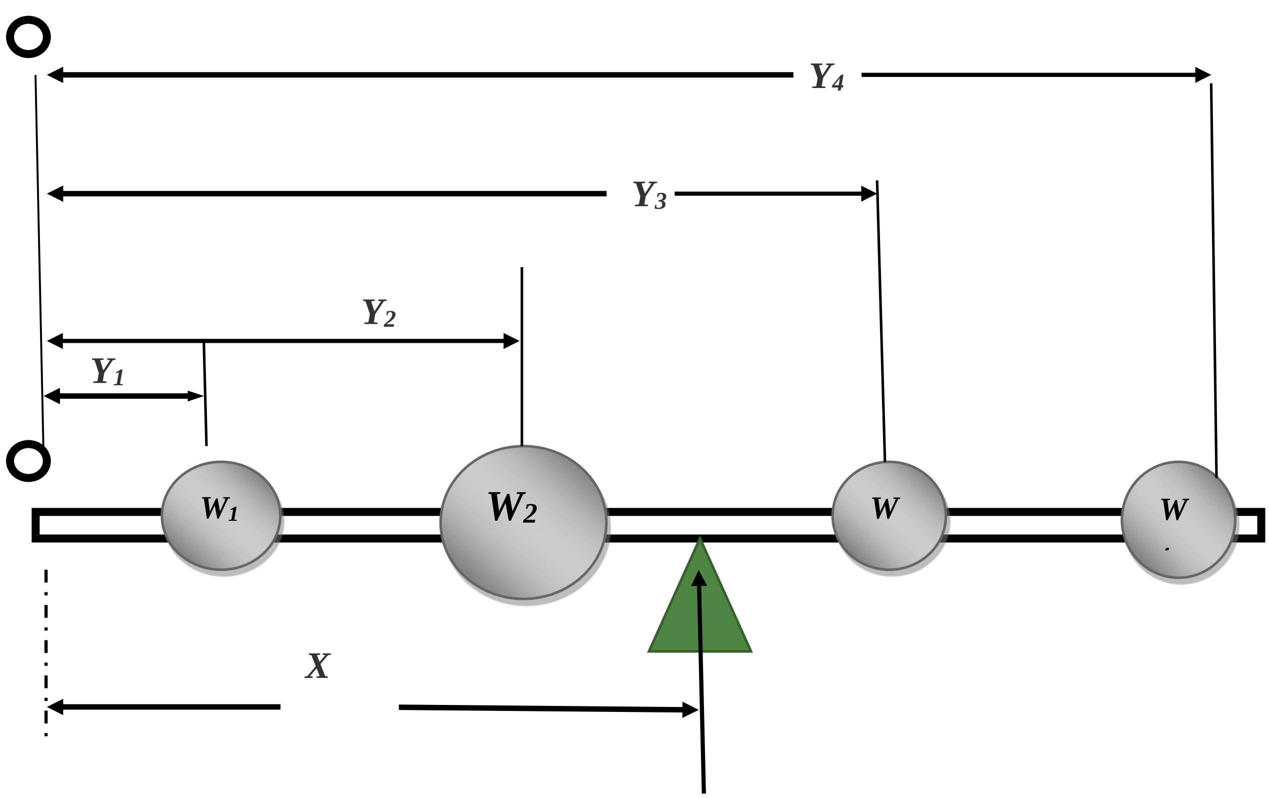
- $m_1 + m_2 \quad = \quad m_1x_1 + \underbrace{m_2x_2 \qquad\qquad}_{\text{Sum of the moments of Individual}}$

Total mass

Effective distance for the total mass = distance to the centre of mass

The centre of gravity of a system of loads can be found by takingmoments. Consider a weightless bar carrying a number of loads,

W$_1$, **W**$_2$, **W**$_3$,etc, at **Y**$_1$, **Y**$_2$, **Y3,** respectively from one end, If the bar is to balance on one support, that support must be placed exactly under the position of the centre of gravity of the system



Taking Moments about **O – O:**

Clockwise moments = Anticlockwise Moments

$(w_1 \times y_1) + (w_2 \times y_2) + (w_3 \times y_3)$ etc. $= \boldsymbol{F} \times \boldsymbol{X}$

The magnitude of the upward support must be equal to the total downward weight, therefore,

$(w_1 \times y_1) + (w_2 \times y_2) + (w_3 \times y_3) = (w_1 + w_2 + w_3) \times \boldsymbol{X}$

$$\boldsymbol{X} = \frac{(\mathbf{w1} \times \mathbf{y1}) + (\mathbf{w2} \times \mathbf{y2}) + (\mathbf{w3} \times \mathbf{y3})}{(\mathbf{w1} + \mathbf{w2} + \mathbf{w3})}$$

Here we have the numerator of this fraction is the summation ($\sum$ $\boldsymbol{sigma}$)of the moments of weights, and the Denominator is the summation of the weights.

For Areas we use:

$$\mathbf{X} = \sum \frac{\boldsymbol{moments\ of\ volumes}}{\boldsymbol{volumes}}$$

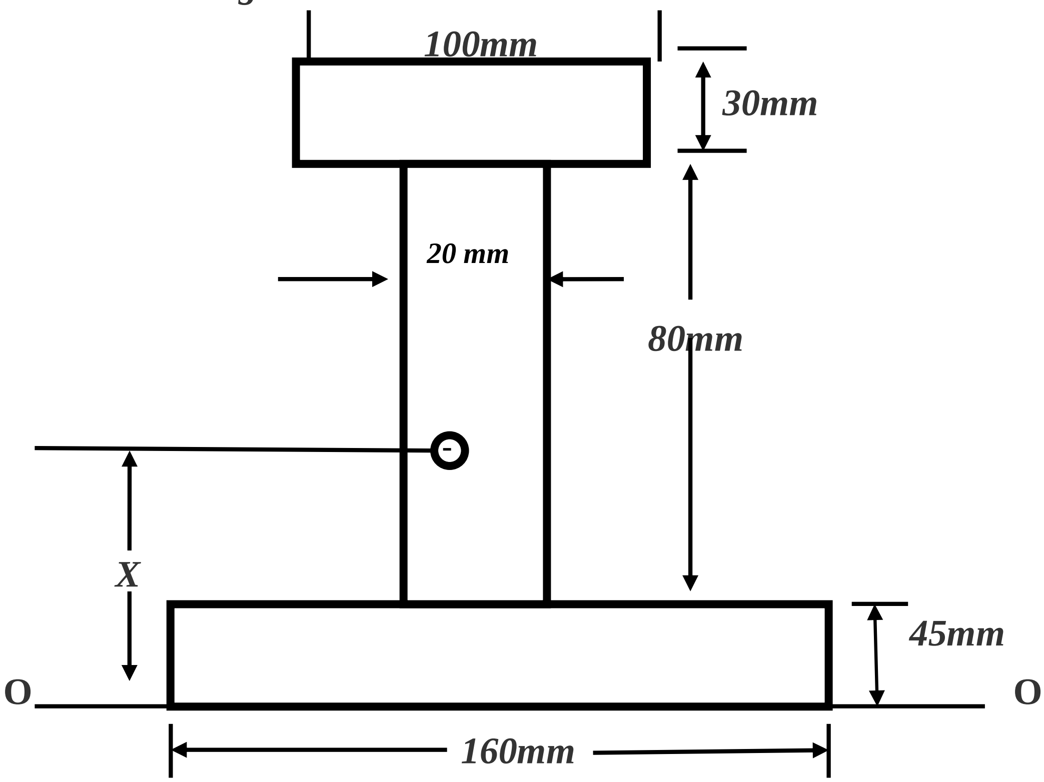
For Plates and sections where thickness is uniform throughout as well as the specific weight of the material, and since

Volume = area × thickness (we cancel thickness out of every term), then:

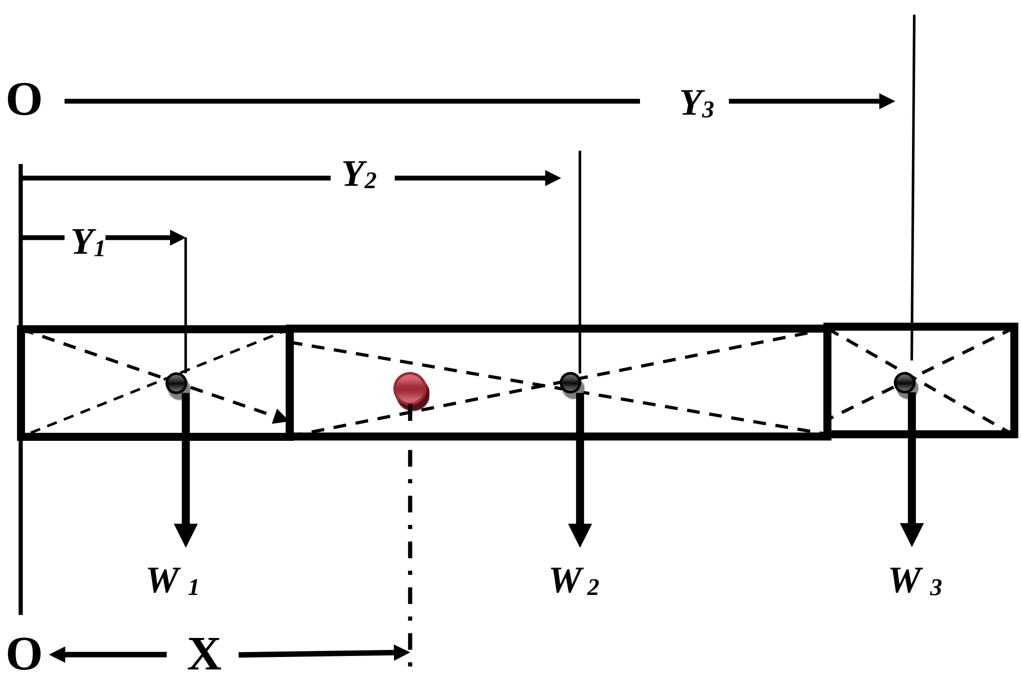
$$\mathbf{X} = \sum \frac{\boldsymbol{moments\ of\ areas}}{\boldsymbol{areas}}$$

Example:

A steel plate of uniform thickness is composed of three rectangles and its dimensions are given as in the drawing. Find the position of its centre of gravity from the bottom edge O - O.



Solution:



F

F = Fulcrum

Let $= W_1, W_2, W_3$, ***represent weights of the top Centre& bottom parts.***

$Y_1, Y_2\ Y3$ ***represent distance from O – O***

X represent position of Fulcrum

Moments about O-O

Clockwise moments = anticlockwise moments

$(W_1 , \times Y_1) + (W_2 , \times Y_2) + (W_3 \times Y_3) = F \times X$

Upward forces = downward forces

$\therefore F = W_1, + W_2, + W_3,$

$\therefore (W_1 , \times Y_1) + (W_2 , \times Y_2) + (W_3 \times Y_3) = (W_1, + W_2, + W_3,) \times X$

$$\therefore X = \frac{(W1 , \times Y1) + (W2 , \times Y2) + (W3 \times Y3)}{(W1 , + W2 , + W3,)}$$

Working in centimeters:

We first work out the area of each rectangle and there centre of gravity.

Area of bottom rectangle = 16 ×4.5 = 72 cm^2

C.G. " " from O-O = 2, 25 cm

Area of middle triangle = 8 × 2 = 16 cm ^{2}

C.G. " " from O-O = 8.5 cm

Area of top triangle = 10 × 3 = 30 cm ^{2}

C.G. “ “ **from O-O** = **14 cm** (*4.5* (*h*) *+8* (*h*)*+1.5* (*1/2*))

$$X = \sum \frac{moments\ of\ areas}{areas}$$

Taking the values worked from above:

$$X = \frac{72 \times 2.25 + 16 \times 8.5 + 30 \times 14}{72+16+30}$$

$$= \frac{718}{118} = 6.085 \text{ cm}$$

= **60.85 mm** from base**. Ans.**

Self Exercise:

a) Explain what a “First Moment” is

b) A steel plate of uniform thickness is cut to a shape as shown in the diagram below. Calculate the position of the Centre of Gravity of the plate from its bottom edge.