

## Chapter 11

### Hydrostatics and Hydraulics

#### 11.0 Hydrostatics:

**Fluid statics** or **hydrostatics** is the branch of *fluid mechanics* that studies *fluids* at rest. It embraces the study of the conditions under which fluids are at rest in *stable equilibrium*; and is contrasted with *fluid dynamics*, the study of fluids in motion.

Hydrostatics is fundamental to *hydraulics*, the *engineering* of equipment for storing, transporting and using fluids.

Hydrostatics offers physical explanations for many phenomena of everyday life, such as why *atmospheric pressure* changes with *altitude*, why wood and oil float on water, and why the surface of water is always flat and horizontal whatever the shape of its container

#### 11.1 Definition:

Hydrostatic pressure is what is exerted by a liquid when it is at rest. The height of a liquid column of uniform density is directly proportional to the hydrostatic pressure. The hydrostatic properties of a liquid are not constant and the main factors influencing it are the *density* of the liquid and the *local gravity*. Both of these quantities need to be known in order to determine the hydrostatic pressure of a particular liquid.

**The formula for calculating the hydrostatic pressure of a column of liquid in SI units is:**

**Hydrostatic Pressure (Pa or N/m<sup>2</sup>) = Height (m) x Density (kg/m<sup>3</sup>) Gravity (m/s<sup>2</sup>)**

The density of a liquid will vary with changes in temperature so this is often quoted alongside hydrostatic pressure units while the local gravity depends on latitudinal position and height above sea level.

For convenience the most common standard for hydrostatic pressure is metres of water or feet of water at 4 deg C (39.2 deg F) with a standard gravity of 9.81 m/s<sup>2</sup>.

The density of pure water at 4 deg C is very close to 1000 kg/m<sup>3</sup> and therefore this has been adopted as the standard density of water. Another reason for the significance of choosing 4 deg C is that it is very close to the temperature that water reaches its maximum density. In practical terms hydrostatic pressure units are rarely absolutely precise because the temperature of any liquid is not always going to be 4 deg C. In summary hydrostatic pressure units are a very

convenient method for relating pressure to a height of fluid but they are not absolute pressure units and it is not always clear what density/temperature has been assumed in their derivation,

## 11.2 Density of liquids and Specific gravity of materials.

Explaining the terms 'specific weight' and 'relative density'

a) **Specific weight** of a liquid is the density of a liquid multiplied by the acceleration due to gravity (i.e. 9.81)

b) **Relative density** of a liquid is the mass of the liquid per unit volume compared to that of fresh water

**Question:** What is Density?

A material's density is defined as its mass per unit volume. It is, essentially, a measurement of how tightly matter is crammed together. The principle of density was discovered by the Greek scientist Archimedes. Densities of liquids are measured by a Hydrometer and the unit is  $kg/m^3$ . To calculate the density (usually represented by the Greek letter "*rho*") of an object, take the mass (*m*) and divide by the volume (*v*):

**Mass = volume x density**

**Question:** explain specific gravities of liquids? What is it?

The specific gravity ("Sp.G.") of a liquid tells you how much more or less dense the liquid is than water. Water has a specific gravity of 1.000 (near 4°C). If a liquid is more dense than water, then its specific gravity is greater than 1. If it is less dense than water, then the specific gravity is less than 1. To calculate the specific gravity of a liquid, you have to know its density. Take the density of the liquid, divided by the density of water ( $1 \text{ gm/cm}^3$ ), and you will get the specific gravity of the liquid:

$$\text{Sp.G.} = (\text{density of liquid}) / (1 \text{ gm/cm}^3)$$

Specific gravity is also related to buoyancy. The denser a liquid is, the greater the force of buoyancy will be on an object floating in/on it (the higher it will float). You can calculate the magnitude of the buoyancy force like this:

$$F_{\text{buoyancy}} = (\text{density of liquid}) \times (\text{volume of object}) \times (\text{acceleration of gravity})$$

To summarize the above in calculations:

$$\text{Specific gravity} = \frac{\text{weight in air}}{\text{wt.in air} - \text{wt.in water}}$$

**Example:** *A piece of brass is suspended by a wire from a spring balance and it registers a weight of 7 N in the air. When the brass is lowered into fresh water until it is submerged the balance shows a weight of 6.17 N. Find the specific gravity of this brass.*

**Solution:**

$$\begin{aligned}\text{Specific gravity} &= \frac{\text{wt.in air}}{\text{wt.in air} - \text{wt.in water}} \\ &= \frac{7}{7 - 6.17} \\ &= \frac{7}{0.83} \\ &= 8.433 \text{ Ans ;}\end{aligned}$$

### **11.3 Floating Bodies:**

Bodies that float in liquid have to be in equilibrium; therefore its downward force weight must be equal to the upward force to create buoyancy. Thus the upward thrust is equal to the weight of liquid displaced; hence a *floating body will displace an amount of liquid equal to its own weight.*

The law of equilibrium for this body will be:

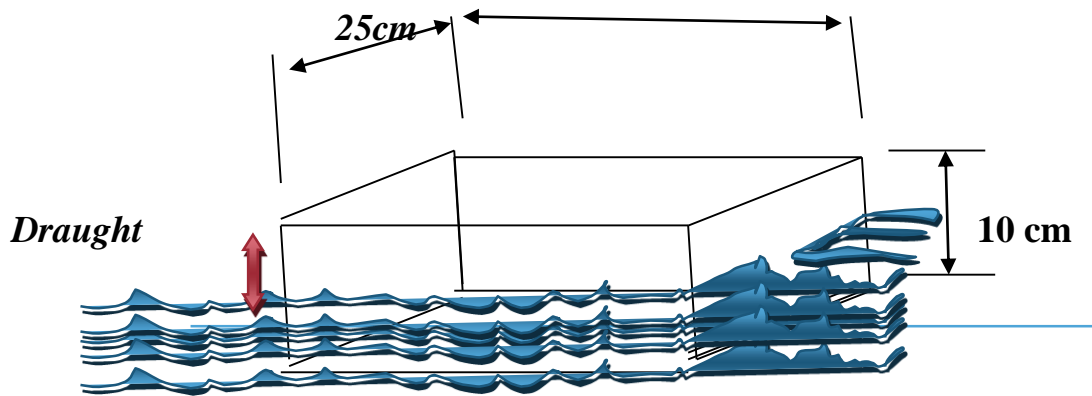
$$\text{Total downward forces} = \text{Total upward forces.}$$

*The points to understand during these calculations will be*

- 1) *Weight of the object. (that is the square area to be taken into account )*
- 2) *Density of the material that the object is made from.*
- 3) *If the object is carry a mass, and then the mass has to be added to the object to total one weight.*
- 4) *As the object has a specific gravity of 9.81 been pushed down?*
- 5) *The weight of the water displaced will be area of object  $\times$  the draught  $\times$  9.81.*

**Example 1:** A rectangular block of wood 375mm long by 250mm broad by 100mm deep, floats in fresh water. If the density of the wood is  $0.75 \text{ kg/m}^3$ , find the draught which it floats?

37.5 cm



**Solution.**

Let  $d$  = draught in cm.

Downward forces = upward forces.

Weight of wood (N) = Weight of water displaced (N)

$$37.5 \times 25 \times 10 \times 0.75 \times 9.81 = 37.5 \times 25 \times d \times 1 \times 9.81$$

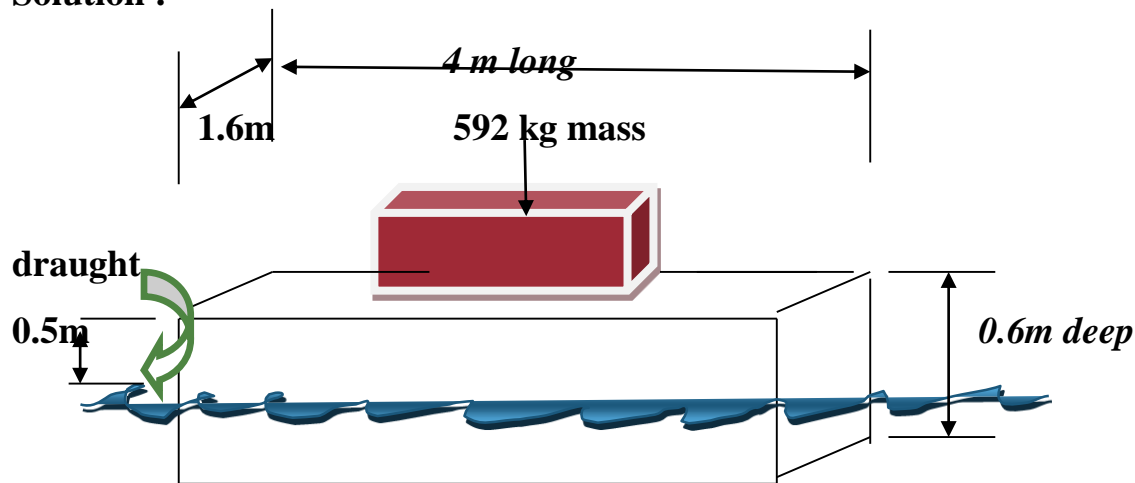
$$10 \times 0.75 = d$$

$$d = 7.5 \text{ cm Ans.}$$

**Example 2** (to consider a mass been carried on the object or floating body)

A solid wood raft 4m long by 1.6m broad by 0.6m deep, floats at a draught of 0.5m, in sea water when carrying a mass of 592 kg on top of the raft. Find the density of the wood, taking the density of the sea water as 1.025 g/ml

**Solution :**



Let  $\rho$  (the letter rho) = density of wood, in  $\text{kg/m}^3$

Density of seawater =  $1.025 \text{ g/ml} = 1.025 \times 10^3 \text{ kg/m}^3$

Downward forces = Upward forces

Wt. of raft [N] + Wt. of mass [N] = Wt. of water Displaced.

*{Note: Wt of raft is the area to be taken into account}*

$$(4 \times 1.6 \times 0.6 \times \rho \times 9.81) + (592 \times 9.81) = 4 \times 1.6 \times 0.5 \times 1.025 \times 10^3 \times 9.81$$

$$1.84 \times \rho + 592 = 328$$

$$3.84 \rho = 2688$$

$$\rho = 700 \text{ kg/m}^3 \text{ Ans:}$$

## 11.4 Mixing of liquids with different Densities & Specific Gravity.

### Mixing of Liquids of Different Densities

When mixing liquids of different densities it is assumed that their volumes and masses are not affected and the final mass of the whole mixture is equal to the sum of the masses of each constituent before mixing. **Solution;**

$$\text{Total mass of mixture} = 2+4 = 6 \text{ grams} \quad (\text{i})$$

As the densities of the oils are 0.8 and 0.9 g/ml respectively, and

Volume = mass  $\div$  density, then,

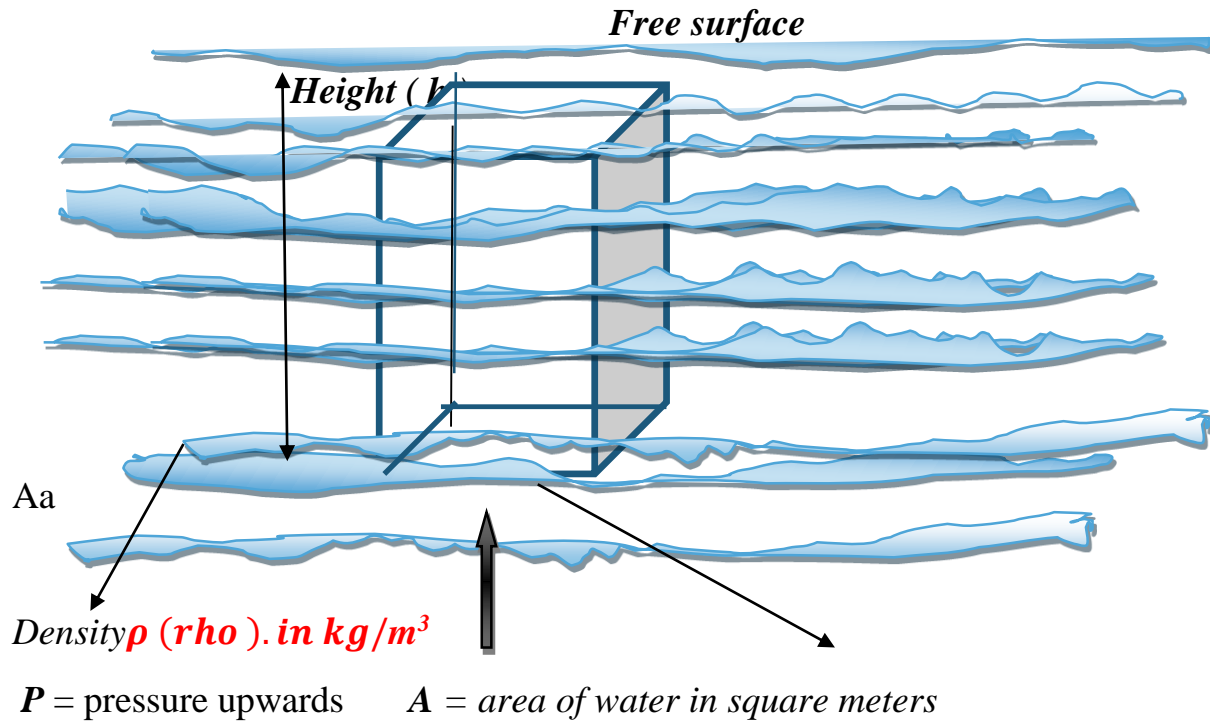
$$\begin{aligned} \text{Total volume} &= \frac{2}{0.8} + \frac{4}{0.9} \text{ millilitres} \\ &= \frac{1.8+3.2}{0.72} \\ &= \frac{5}{0.72} \text{ml} \dots\dots\dots (\text{ii}) \end{aligned}$$

$$\begin{aligned} \text{Density of mixtures} &= \frac{\text{total mass}}{\text{Total volume}} \\ &= \frac{6 \times 0.72}{5} \\ &= 0.864 \text{ g/ml} \end{aligned}$$

$\therefore$  Specific gravity = 0.864 Ans.

## 11.5 Pressure Heads and pumping against pressure

Consider the equilibrium of a vertical column of liquid of height  $h$  meters. And the cross sectional area been  $A$  square meters. And of density  $\rho$  ( $\rho$ ). in  $\text{kg}/\text{m}^3$  as in illustrated diagram



The drawing illustrates that the column is at rest, and the total force acting on it in one direction must be equal to the force acting on it in the opposite direction

Therefore: Mass of the column of liquid =  $h \times A \times \rho$  ( $\rho$ )  $\text{kg}/\text{m}^3$

(i) Down ward force of gravity acting on the column =  $h \times A \times \rho$  ( $\rho$ )  $\times 9.81$  [ N ]

(ii) Let  $P$  = to the intensity of pressure acting on the base in [  $\text{N}/\text{m}^2$  ]

Then to summaries the above;

Upward forces = downward forces

$$P \times A = h \times A \times \rho$$

$$A = h \times \rho \times 9.81$$

The weight of the liquid is (density)  $\rho$  ( $\rho$ )  $\times 9.81$ .

Let  $W$  represent the specific weight of the liquid in Newton's per cubic meter  $[N/m^3]$

**Then**  $P = hw$

From the above notes we have established that the density of pure water is  $10^3 \text{ kg/m}^3$ , hence  $W$  is  $10^3 \times 9.81 \text{ N/m}^2$ .

**Therefore the head of pure water to exert a pressure of one bar is ;**

$$H = \frac{P}{W} = \frac{10^5}{10^3 \times 9.81} \quad \text{note 1 bar is } = 10^5 \\ = 10.19 \text{ meters.}$$

Therefore 1 bar pressure will push a column of water up to 10.19 meters

Thus, pumping against a pressure can be considered as lifting the liquid to an equivalent height and the work done or power exerted can be calculated by this method.

**Example:**

An engine developing 3730 kw uses 7.25 kg of steam per kw per hour. If the boiler pressure is 17 bar ( $= 17 \times 10^5 \text{ N/m}^2$ ), calculate the output power of the feed pump.

**Solution:**

Equivalent of pure water head of 17 bars,

$$= 17 \times 10.19 = 173.2 \text{ meters}$$

Mass of water pumped into the boiler every second

$$= \frac{7.25 \times 3730}{3600} = 7.511 \text{ kg.}$$

Force to lift against gravity

$$= 7.511 \times 9.81 = 73.69 \text{ N}$$

$$\text{Power [ W ] } = \text{work done per second [ Nm/s = J/s ]}$$

$$= \text{Force} \times \text{height, per second.}$$

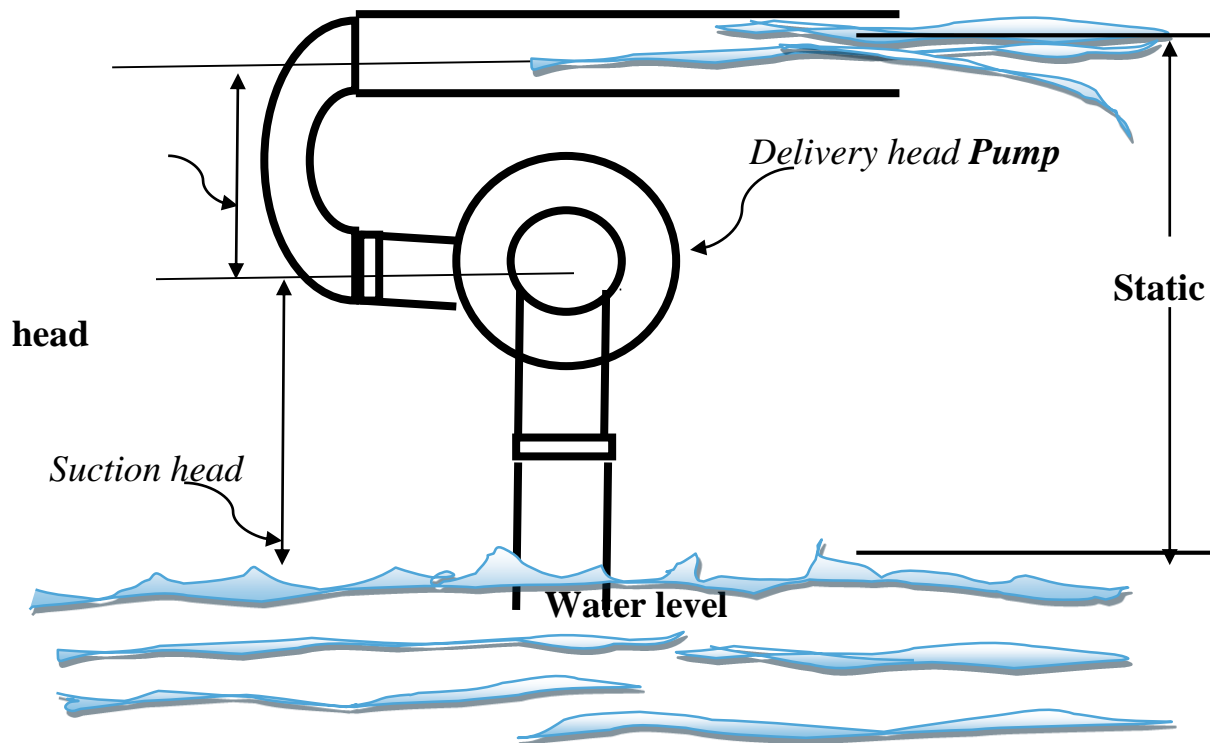
$$= 73.69 \times 173.2$$

$$= 1.277 \times 10^4 \text{ W } = 12.77 \text{ kW, Ans.}$$



## 11.6 Work done to deliver water against a head

*Force is also needed to pump water in a case where water is to be delivered at a certain height.*



The drawing shows a centrifugal pump installation where water is pumped from a storage tank or dam to a higher level. The distance from the water level to the centre of the shaft of the pump is called the suction head. The distance from the centre to the outlet of the delivery pipe is called the delivery head. General use on board ship will be used as a fire pump, but the suction will come from the sea suction situated in the engine room.

**Static head = suction head + delivery head.**

**Work done = force in Newtons x distance moved in meters.**

*Force is also needed to pump water in a case where water is to be delivered at a certain height. The water has a known mass or volume and a downward force is exerted by the water, as in the case of a solid object, due to the gravitational force of the earth on the water.*

*In the case where water is pumped vertically upwards, the water in the pipe could be taken as a stationary amount of water for a second; this column of water exerts a downward pressure. An upward force must be applied in order to obtain motion and this upward force must be at least equal to the downward force.*

*The distance moved in this case is taken as the vertical height that the water is to be delivered above the water level, that is the head. The mass of the water in the pipe, or the volume of the water in the pipe, is to be converted in a force unit.*

***Thus we can say;***

**Mass of water = volume of water x density of water.**

**The mass of 1 liter pure water = 1kg.**

**1 m<sup>3</sup> water = 1 k l water**

**= 1000 kg water ( or 1 ton )**

**The density ( $\rho$  (rho)) of pure water =  $\frac{\text{mass of water}}{\text{volume of water}}$**   
**=  $\frac{1\,000\,kg}{1\,m^3}$**   
**= 1 000 kg/ m<sup>3</sup>**

**To calculate the work done and all other loses, etc, will not be taken into account, we use a simple formula:**

**Work done = force in (N) x distance moved (in meters).**

**= (mass x gravitational acceleration) x head**

**=  $m \times g \times \text{head}$ .**

**OR: Work done = (volume x density x g) x head**

**= (mass x g) x head**

**=  $m \times g \times \text{head}$ .**

***To Summaries where:***

***M = mass of water in the pipe in kg***

***.g = gravitational acceleration = 10 m/s<sup>2</sup>***

**Volume = volume of water in the pipe in  $m^3$**

**density = density of the water in  $kg/m^3$**

**head = vertical height in *meters***

**Example:**

*Given: Inside diameter of the water pipe is = 50 mm*

*Effective head = 30 m*

*Density of water = 1 000  $kg/m^3$*

*Calculate the work done to deliver the water at this height (head ).*

**Solution:**

**Given:  $d = 50mm = 0.05\ m$**

**$Head = 30\ m$**

**$\rho = 1\ 000\ kg/m^3$**

**$Volume\ of\ water\ in\ the\ water\ pipe = inside\ area\ x\ head$**

$$= \frac{\pi d^2}{4} x\ head$$

$$= \frac{\pi x (0,05)^2}{4} x\ 30$$

$$= 0,058\ 9\ m^3$$

**$Work\ done = force\ x\ distance\ moved$**

$$= (mass\ x\ g)\ x\ head$$

$$= (volume\ x\ density\ x\ g)\ x\ head$$

$$= (0,058\ 9\ x\ 1\ 000\ x\ 10)\ x\ 30$$

$$= 17\ 670\ J$$

***Or = 17.67 kJ Ans.***

## 11.7 Work done to deliver water against a pressure.

(Basically the same principals applies to deliver water against a head)

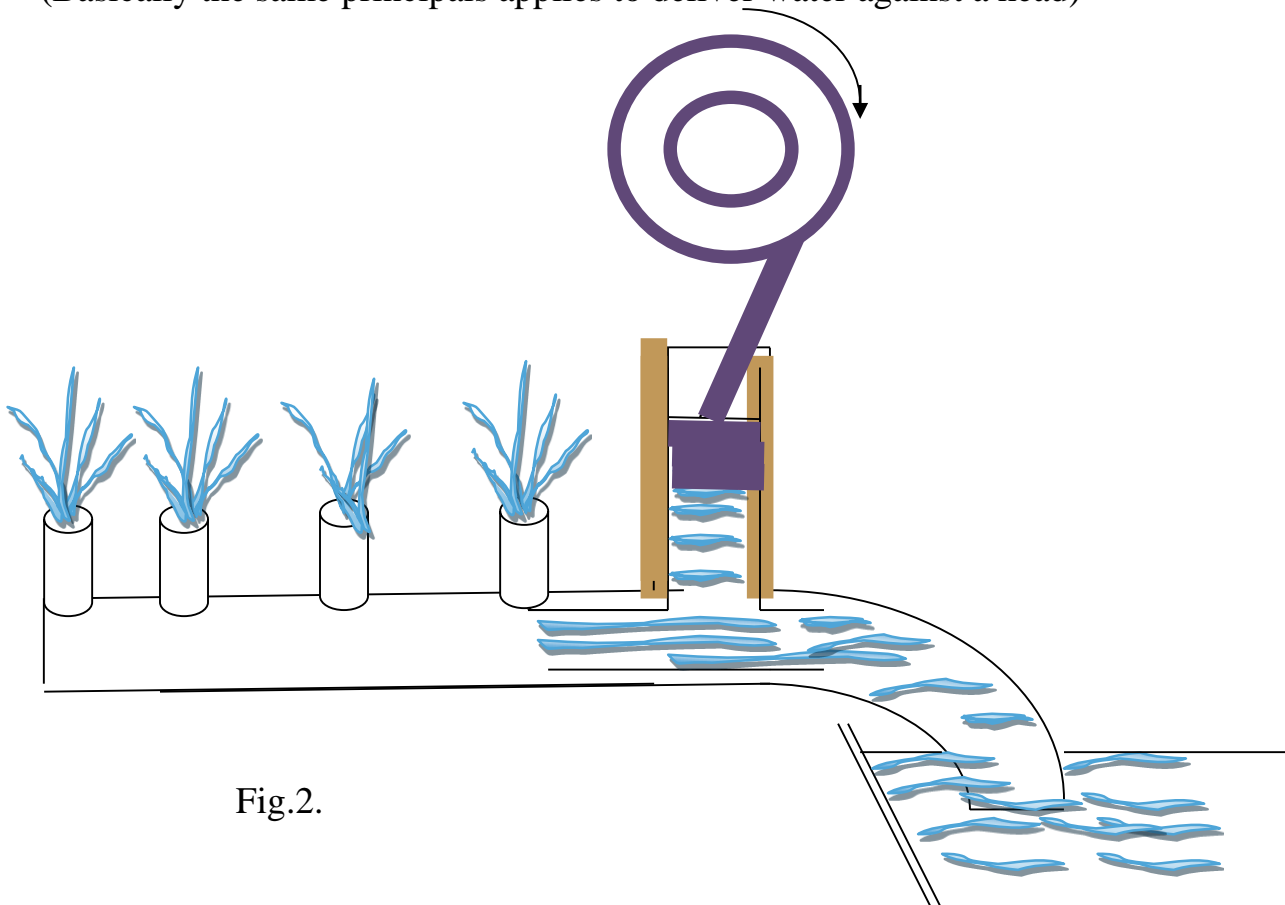


Fig.2.

**Example:** *a Single Acting Piston Pump used as a Fire Pump:*

**Example** suction from sea water is forced through fire hydrants on the fire main line .

A high pressure is exerted on the line only during the stroke when the water is forced through the line. This could be called the delivery stroke.

The pressure in the delivery pipe could be attributed to the force exerted by the piston on the surface of the water if all losses, etc. is ignored. Then:

$$\text{Pressure exerted} = \frac{\text{force applied on liquid}}{\text{area of liquid}}$$

**Force applied = pressure in liquid X area.**

(The piston moves through a distance called the stroke. The work done for the delivery stroke of a single-acting piston pump could be given as follows if losses, etc. is ignored):

Work done per delivery stroke = force X distance

= ( pressure X area ) X stroke

= pressure X volume.

Where **Pressure** = pressure in the liquid in **Pa**

**Area** = area of piston in **m<sup>2</sup>**

**Volume** = volume of liquid displaced by piston in **m<sup>3</sup>**

**Stroke** = length of stroke in **m**

**Example:**

*Calculate the work done per delivery stroke of a single- acting piston pump with a piston diameter of 100 mm and a stroke of 150 mm. A pressure gauge in the delivery pipe shows a pressure of 200 kpa during the delivery stroke.*

**Solution:**

**Given** d = 100 mm = 0,1 m

**Stroke** = 150 mm = 0,15 m

**Pressure** = 200 kpa = 200 X 10<sup>3</sup> Pa.

**Therefore: Work done per delivery stroke = force X distance**

**= (pressure X area) X stroke**

**= pressure X  $\frac{\pi d^2}{4}$  x stroke**

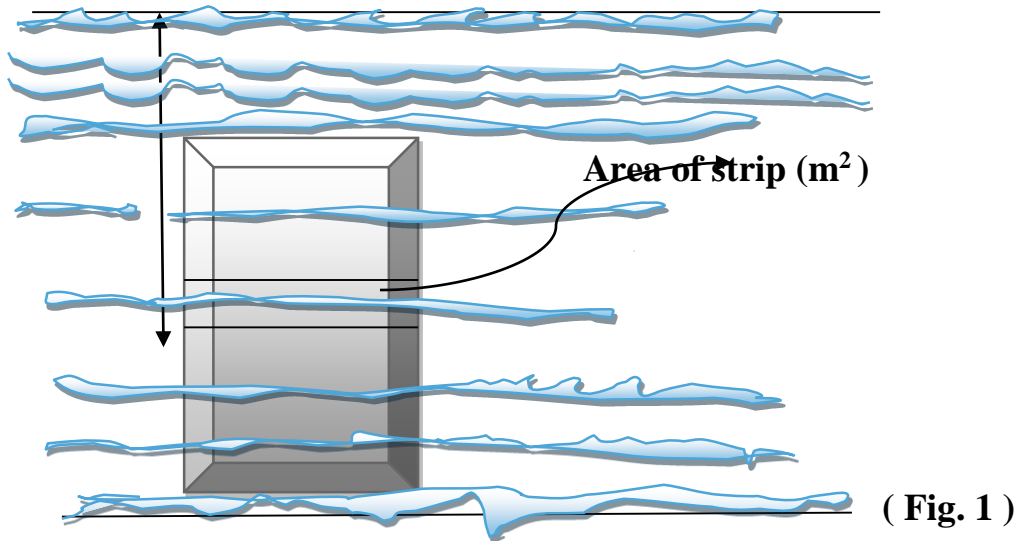
**= 200 X 10<sup>3</sup> X  $\frac{\pi \times (0,1)^2}{4}$  X 0,15**

**= 235,619 J Ans (ii )**

## 11.8 Pressure of Immersed Surfaces...

Consider a plate immersed in liquid as seen in drawing below:

$hm$  (Vertical distance from free surface)



Let  $a$  = area of narrow strip across the plate, in  $m^2$

$h$  = vertical distance from free surface of liquid to the strip of area, in meters,

$W$  = specific weight of the liquid, in  $N/m^3$

Then, Intensity of pressure on strip =  $hw(N/m^2)$

Total pressure on strip =  $h w a$  (N)

The whole area is made up of small strips  $a_1 a_2 a_3$  etc., at the distance of  $h_1 h_2 h_3$  etc., from the free surface, therefore,

Total Pressure on whole surface

$$= h_1 w a_1 + h_2 w a_2 + h_3 w a_3 + \text{etc.}$$

$$= w (h_1 a_1 + h_2 a_2 + h_3 a_3 + \text{etc.,})$$

*In the brackets we have the summation of the each area and its vertical distance from the free surface of the liquid. This is the first moment of the whole area about the free surface. This is also the product of the total area and the distance of its centre of gravity from the point about which moments are taken, therefore,*

**Total pressure =  $w \times$  moment of area about free surface.**

**Let.  $H$  = distance of centre of gravity of the area of the immersed plate from the**

**free surface of the liquid, (in meters. ).**

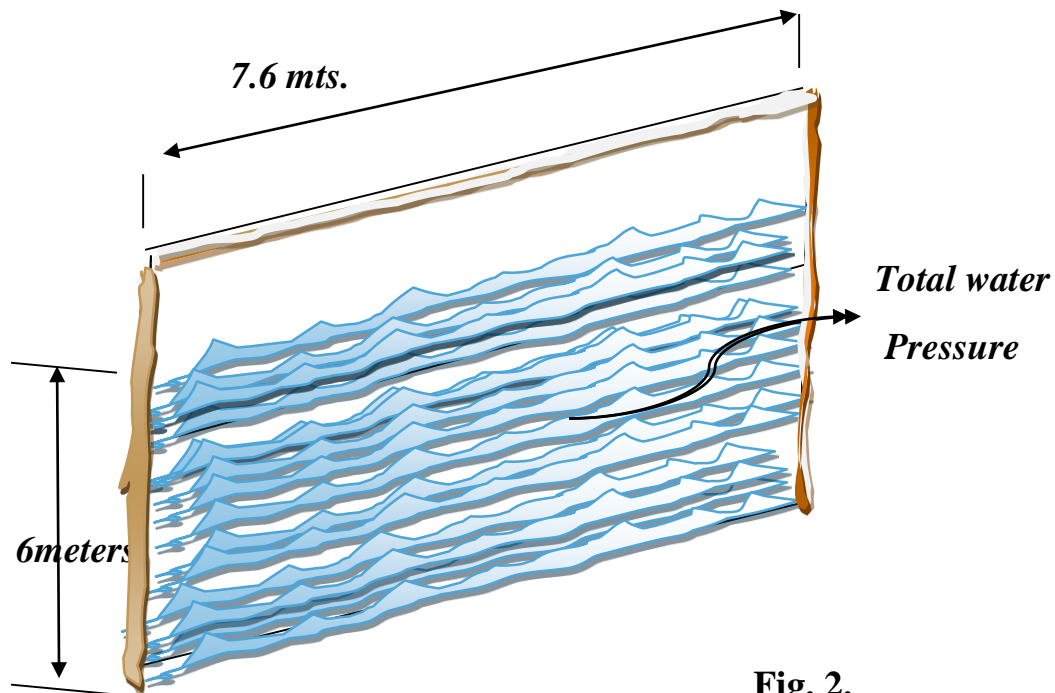
**$A$  = area of immersed plate, (in  $m^2$ ).**

**$W$  = specific weight of the liquid, in (  $N/m^3$ ).**

**$P$  = total pressure on the plate, in (Newtons ).**

**Example:**

*A vertical rectangular bulkhead is 7.6m wide and has fresh water to a height of 6 m on one side. Find the total pressure of the water on the bulkhead.*



**Solution:**

*Being rectangular, the center of gravity of the wetted part of the bulkhead is at half-depth of the water which is 3m below the free surface. The area of the wetted part is  $7.6 \times 6 \text{ m}^2$ , and the density of the fresh water can be taken as  $10^3 \text{ kg/m}^3$ , hence,*

$$\begin{aligned}
 P &= H A w \\
 &= 3 \times 7.6 \times 6 \times 10^3 \times 9.81 \\
 &= 1.342 \times 10^6 \text{ (N)}. \\
 &= \text{or } 1.32 \text{ Mn Ans.}
 \end{aligned}$$

The centre of gravity of a triangle being **two- thirds** of the distance from the apex then the centre of pressure is at two- thirds the depth from the free surface.

**Example:**

*A vertical bulkhead 9.6 m wide and 6 m deep separates two tanks .One tank contains oil of specific gravity 0.85 to a depth of 5.4 m, and the other tank is empty. Find the total pressure on the bulkhead in MN and state the position of the centre of pressure.*

**Solution:**

Centre of gravity of wetted plate below free surface

$$= \frac{1}{2} \times 5.4 = 2.7 \text{ m}$$

$$\text{Area of wetted plate} = 9.6 \times 5.4 \text{ m}^2$$

$$\text{Density of oil} = 0.85 \times 10^3 \text{ kg/m}^3$$

$$\text{Total pressure } P = H A w$$

$$= 2.7 \times 9.6 \times 5.4 \times 0.85 \times 10^3 \times 9.81$$

$$= 1.168 \times 10^6$$

$$= 1.168 \text{ MN Ans (i)}$$



Centre of pressure on a rectangular area is at the one- third of the height from the base;

$$\text{Centre of pressure} = \frac{1}{3} \times 5.4$$

= 1.8 m from the bottom. Ans (ii)

*For a triangle with an apex at the bottom, which is very common description for a fore-peak bulkhead, the centre of pressure will be at half –depth.*

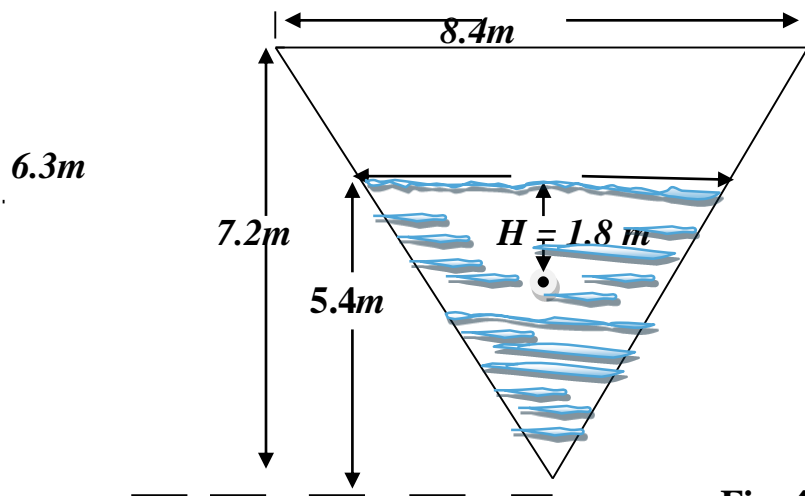


Fig. 4.

*A fore-peak Bulkhead.*

**Example:** *A fore peak bulkhead is an isosceles triangle 7.2m vertical height and 8.4m wide at the top. If the peak contains fresh water to a height of 5.4m, find the total pressure on the bulkhead in kn. and give the position of the centre of pressure from the bottom.*

**Solution:** By similar triangles,

$$\text{Breath of water level} = \frac{5.4}{7.2} \times 8.4 = 6.3 \text{ m}$$

$$\text{Area of wetted surface} = \frac{1}{2} \times 6.3 \times 5.4 = 17.01 \text{ m}^2$$

Distance from free surface to centre of gravity of wetted plate,

$$H = \frac{1}{3} \times 5.4 = 1.8 \text{ m}$$

$$P = H A w$$

$$= 1.8 \times 17.01 \times 10^3 \times 9.81$$

$$= 300.4 \times 10^3 \text{ N}$$

$$= 300.4 \text{ kn Ans (i)}$$

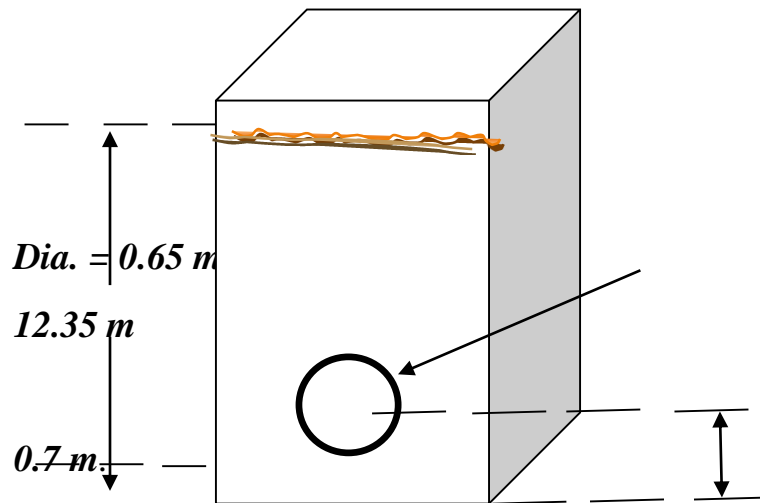
*Centre of pressure for a triangular plate with apex at the bottom is at half the depth,*

$$= 2.7 \text{ m from bottom. Ans (ii)}$$

### Exercise

*A circular inspection plate in the side on an oil tank has a diameter of 65 cm exposed to the pressure of the oil in the tank. The centre of the plate is 70 cm above the bottom of the tank and the depth of the oil is 12.35 m. If the relative density of the oil is 0.88, what is the total load on the inspection plate?*

**Solution:**



*Dia of plate = 650 mm = 0.65 m*

$$\text{Area of circular plate } \frac{\pi D^2}{4} = \frac{\pi 0.65^2}{4} = 33.196 \times 10^{-2} \text{ m}^2$$

*Height of Plate Centroid from liquid surface (h) = 12.35 - 0.7 m*

*Height (h) = 11.65 m*

*Relative Density = 0.88 so Density of liquid = 0.88 × 1000 Kg / m<sup>3</sup>*

$$\rho = 880 \text{ Kg / m}^3$$

***Pressure of liquid column at centroid (P) =  $\rho \cdot g \cdot h$***

$$P = 880 \times 9.81 \times 11.65 \text{ N/m}^2$$

***Pressure =  $100.572 \times 10^3 \text{ N/m}^2$  or  $100.572 \text{ KN/m}^2$***

***Load acting on the plate = Pressure at Centroid x Area of plate.***

$$\text{Load} = 100.572 \times 10^3 \times 33.196 \times 10^{-2} \text{ N}$$

***Load is  $33.385 \times 10^3 \text{ N}$  or  $33.385 \text{ KN}$***

**Exercise:**

- . A tank 12 meters deep is 92% full with HFO of relative density 0.975. There is a circular manhole fitted to one side of the tank of 650 mm diameter and with its lower lip 750 mm above tank bottom.***

***(a) Calculate the load against the manhole cover.***

***(b) If the tank was to be overfilled so that there 3 meters of fuel oil up the vent pipe, calculate the pressure acting on the tank top.***

***(c) Explain the term “Relative Density”***

**SOLUTION:**

a) Tank 12m deep, 92% full

Therefore level of oil in tank =  $12 \times 0.92$

$$= 11.04\text{m}$$

Manhole dia = 0.65m

$$\begin{aligned}\text{Centre of manhole above tank bottom} &= 0.75 + \frac{0.65}{2} \\ &= 1.075\text{m}\end{aligned}$$

$$\text{Head of oil above centre of manhole} = 11.04 - 1.075$$

$$= 9.965\text{m}$$

$$\text{Pressure} = \rho \times g \times h$$

$$= 0.975 \times 1000 \times 9.81 \times 9.965$$

$$= 95.313 \text{ kN/m}^2$$

$$\text{Area of cover} = 0.7854 \times 0.65^2$$

$$= 0.3318 \text{ m}^2$$

$$\text{Load on cover} = 95.313 \times 0.3318$$

$$= \mathbf{31.63 \text{ kN} \quad \text{ANS}}$$

b) At 3m up the sounding pipe pressure =  $\rho \times g \times h$

Where  $h = 3\text{m}$  above tank top

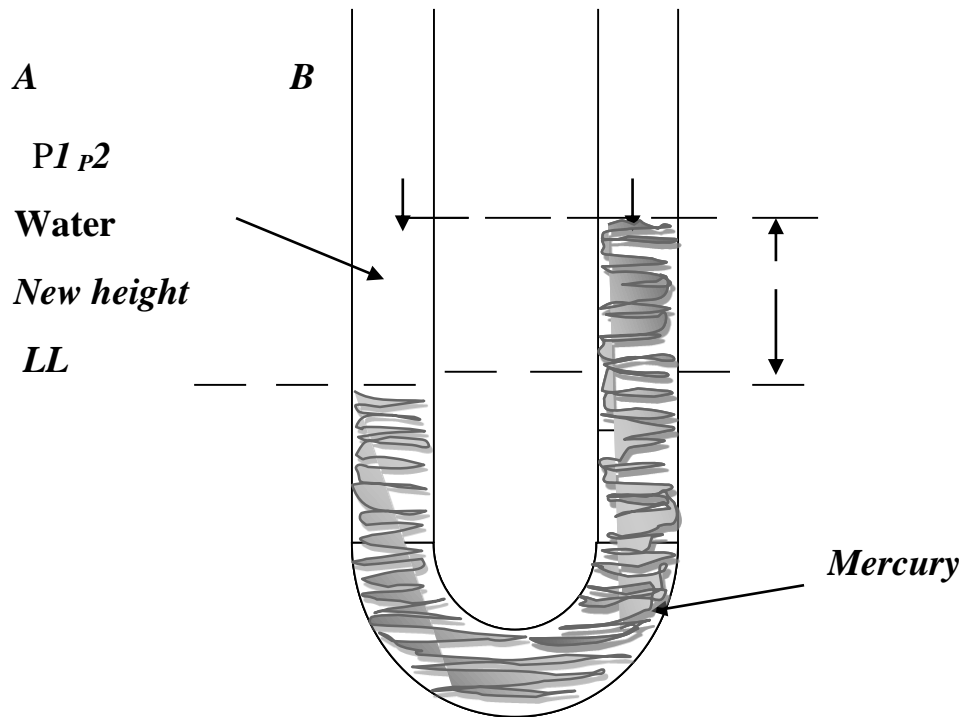
$$\text{Therefore pressure} = 0.975 \times 1000 \times 9.81 \times 3$$

$$= \mathbf{28.694 \text{ kN/m}^2 \quad \text{ANS}}$$

**Relative density** of a liquid is the mass per unit volume of the liquid compared to that of pure water. Thus R.D. is a number, there are no units

## 11.9 Manometers and relation to measure of small pressures.

Pressure measuring devices using liquid columns in vertical or inclined tubes are called manometers. One of the most common is the water filled u-tube manometer used to measure pressure difference in pitot or orifices located in the airflow in air handling or ventilation system.



### 11.9 Vertical U-Tube Manometer

This consists of a glass U- tube containing water or mercury, depending upon the range of pressure to be measured, the difference in levels of the liquid in the legs registers the difference in pressure between the two points to which the ends of the legs are connected. The hydrostatic gauge which is commonly used for measuring the pressure of boiler draught, is a vertical U- Tube containing water., One leg is connected to the air pressure point and the other leg is open to the atmosphere. The greater pressure of the forced draught pushes the level of the water down in one leg with a consequent rise in the other which is subjected only to the atmospheric pressure.

$$\text{From } P [\text{N/m}^2] = h [\text{m}] \times w [\text{N/m}^2]$$

One millimeter difference in water level is equivalent to a pressure of

$$P = (1 \times 10^{-3}) \times (10^3 \times 9.81).$$

Pressures of small magnitude are often expressed in millibars [mbar].

$$\text{One mbar} = 10^2 \text{ N/m}^2$$

$$\therefore \text{one mm head of water} = 9.81 \text{ N/m}^2 = 0.0981 \text{ mbar.}$$

For one mbar of pressure, the head of water is

$$H = \frac{p}{w} = \frac{10^2 \times [N/m^2]}{10^3 \times 9.81 [N/m^2]} \text{ meter.}$$
$$= \frac{10^2 \times 10^3}{10^3 \times 9.81} \text{ mm} = 10.19 \text{ mm}$$

When slightly higher pressure is required to be measured between two points, mercury is used instead of water. If the specific gravity of mercury is taken as **13.6** then one mm difference in mercury level is **13.6 times the pressure equivalent of one mm of water.**

***One mm head of mercury [ mm Hg] is therefore equal to***

$$13.6 \times 9.81 \text{ N/m}^2 = 133.4 \text{ N/m}^2 = 1.334 \text{ mbar}$$

***Thus, when the mercury barometer stands at 760 mm, the atmospheric pressure is:***

$$760 \times 1.334 = 1014 \text{ mbar} = 1.014 \text{ bar.}$$

**The pressure difference in a vertical U-Tube manometer can be expressed as**

$$p_d = \gamma h$$

$$= \rho g h \text{ (1)}$$

***where***

$$p_d = \text{pressure}$$

$$\rho = (\text{rho}) \text{ density (kg/m}^3, \text{ )}$$

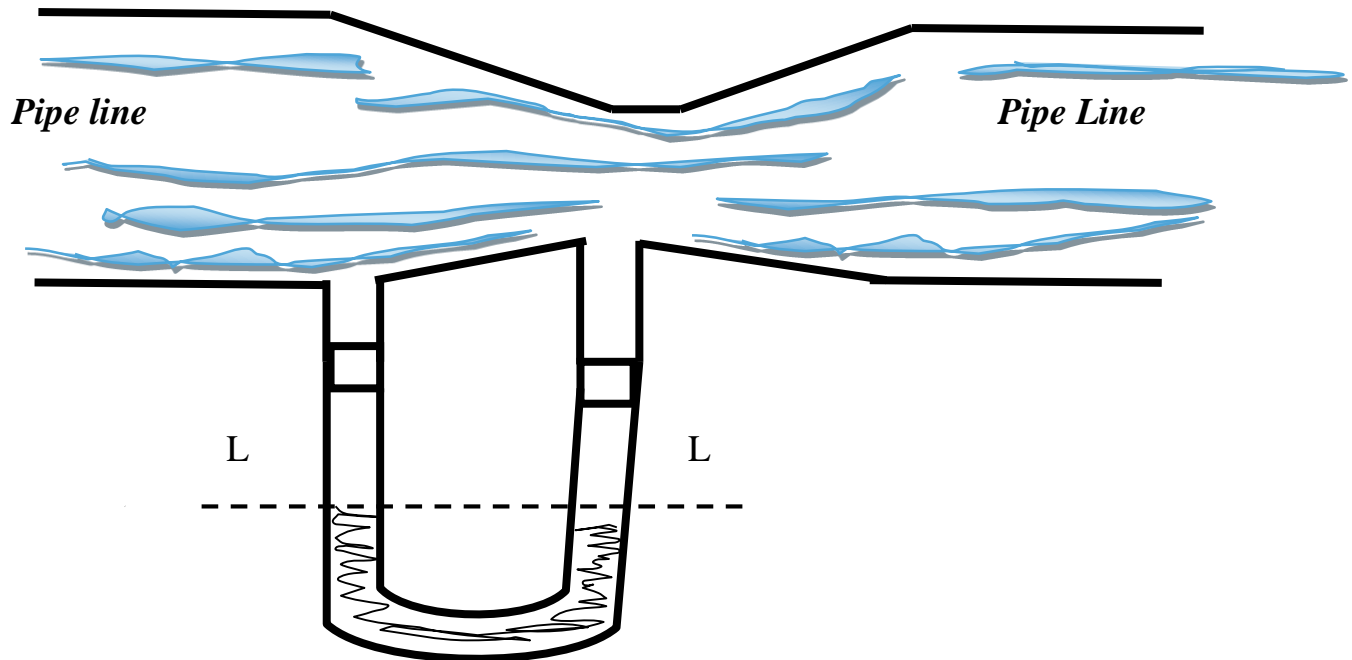
$$g = \text{acceleration of gravity (9.81 m/s}^2\text{, )}$$

$$h = \text{liquid height (m, ft)}$$

**The specific weight of water, which is the most commonly used fluid in u-tube manometers, is  $9.81 \text{ kN/m}^3$**

### 11.10 Example - Differential Pressure Measurement in an Orifice

When a mercury manometer is used for measuring the difference in pressure between two points in a venture meter, it is sometimes “submerged” as in diagram. Each leg of manometer is full of water above the mercury columns. For this particular case, referring to the first illustrated diagram and considering the pressure at level **L, L**, the mercury below this level is in equilibrium, therefore the pressure in one leg at level **L, L** is equal to the pressure in the other leg at the same level.



**A water manometer connects the upstream and downstream of an orifice located in an air flow. The difference height of the water column is *10 mm*.**

**The pressure difference head can then be expressed as:**



$$p_d = (9.8 \text{ kN/m}^3) (10^3 \text{ N/kN})$$

$$= \underline{98} \text{ N/m}^2 \text{ (Pa)}$$

*where*

*9.8 (kN/m<sup>3</sup>) is the specific weight of water in SI-units.*

## **11.11 Definition of Hydraulics.**

Hydraulics is that relating to or denoting the equilibrium of liquids and the pressure exerted by liquid at rest.

### **Introduction;**

Like gases, liquids also exert a pressure against the walls ( sides ) of a container. Pressure is exerted in all directions – upwards , downwards and sideward-at any point in a liquid. The pressure exerted by a liquid is due to the weight of the liquid and the fact that the molecules slip across each other easily.

The most important fact in connection with the pressure in liquids is as follows:

1. The pressure in a liquid increases with the depth and is directly proportional to the depth.
2. The pressure exerted by the liquid depends on the density of the liquid. The higher the density, the higher the pressure.
3. Pressure is exerted at any point in a liquid in all directions and is the same in magnitude in all directions.
4. The pressure at all points in the same horizontal plane of a vessel (container) is the same.
5. The pressure is independent of the size or shape of the vessel containing the liquid.
6. If an external force is applied to the surface of a liquid, then this force is transmitted with the same intensity through the liquid in all directions.

### **11.11.1 The measuring of pressure in a liquid.**

#### **11.11.2 Pressure at a point;**

The pressure at a point is the force exerted by the liquid due to its weight per unit area that includes the point.

Pressure at a point may also be described as the weight of a column of liquid on a unit area which includes that point. therefore ;

**Pressure at a point = Height X density X gravitational acceleration (g)**

### **11.11.3 Total Pressure;**

Total pressure on an area means the force, or weight, applied on the whole area.

To calculate the total pressure on a given area in a liquid, it is necessary to calculate the weight of the vertical column of the liquid resting on the area.

**Total pressure=area X height X density X gravitational acceleration. (g)**

### **11.11.4 Gauge Pressure:**

Gauge pressure may be described as the pressure indicated by a meter and it indicates the pressure above the atmospheric pressure i.e., the pressure of the atmosphere is not registered. These types of pressure gauges are so calibrated that they indicate the pressure of the atmosphere as zero and the atmospheric pressure at a certain place is taken as a standard i.e., taken as zero.

### **11.11.5 Absolute Pressure:**

Absolute pressure refers to the absolute zero. Thus, absolute pressure indicates the combined pressure exerted on the liquid.

**Absolute pressure = gauge pressure + atmospheric pressure.**

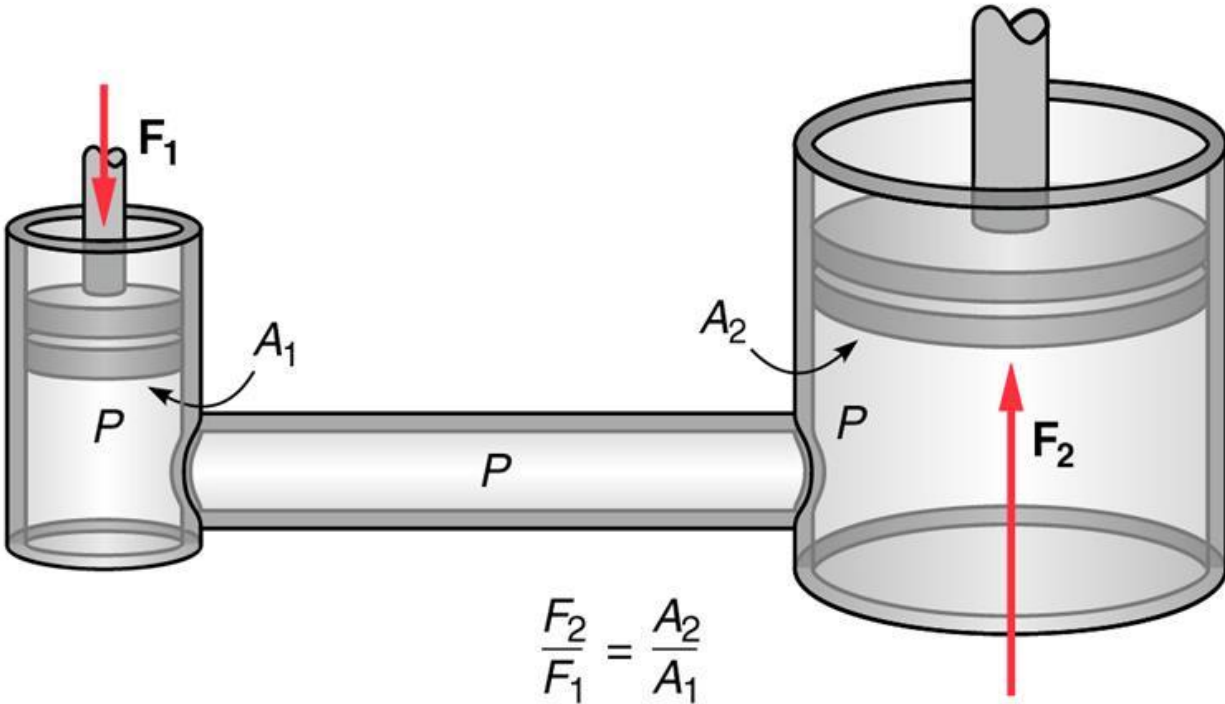
### **11.11.6 The Unit for pressure:**

**The unit for pressure is the Pascal (Pa).**

**Definition of the Pascal:** The Pascal is the pressure which results when a force of 1 N is applied evenly and perpendicularly to an area of 1 m<sup>2</sup>.

### **11.11.7 Pascal's Law**

The *definition of Pascal's law* is if a pressure is exerted on the surface of a liquid, then all this force is transmitted with the same intensity through the liquid in all directions.



The above apparatus could be used as an example of the application of Pascal's law in hydraulic equipment. Assume that a force of  $F$  newton is applied on the plunger piston (small piston), with a diameter of  $d$  meter :

$$\begin{aligned} \text{Pressure exerted by the plunger piston} &= \frac{\text{force in newtons}}{\text{area in m}^2} \\ &= \frac{F}{\frac{\pi d^2}{4}} \text{ Pa} \end{aligned}$$

$$\text{Simplified it is: pressure exerted} = \frac{\text{force}}{\text{area}}$$

**And the force applied = pressure exerted X area.**

The pressure being exerted by the plunger on the surface of the liquid is transmitted through the liquid so that a pressure with the same magnitude will work upwards against the load,  $W$

From the equation: force exerted X area, the following statement could be made.

***Force exerted by load = pressure exerted on load X area of load***

$$W = \frac{F}{\frac{\pi d^2}{4}} \times \frac{\pi D^2}{4}$$

Where  $\frac{F}{\frac{\pi d^2}{4}} = \text{pressure exerted by plunger on the liquid (or load pressure)}$

And  $\frac{\pi D^2}{4} = \text{area of ram's piston (large piston) in m}^2$ .

The above equation can be simplified further as follows:

$$W = \frac{F \times 4}{\pi d^2} \times \frac{\pi D^2}{4}$$

$$W = \frac{F}{d^2} \times D^2$$

$$\frac{W}{F} = \frac{D^2}{d^2}$$

Where  $W = \text{load in newtons}$

$F = \text{force applied on the plunger (small Piston) in N}$

$D = \text{diameter of Ram (Large Piston) in meters}$

$d = \text{diameter of plunger (Small piston) in meters}$

The following deduction could be made in terms of work done:

**Force on plunger X distance moved by plunger = force on ram X distance moved by ram.**

If friction and other losses are omitted then:

$$\text{Efficiency, } \eta = \frac{\text{work output}}{\text{work input}} \times 100 \%$$

**Example:**

*The ram of a single-acting hydraulic press has a diameter of 100mm and the plunger has a diameter of 20mm with a stroke of 40mm. The mechanical advantage is 30 on the plunger. Calculate:*

- (1) The effort applied on the handle to lift a load of 1.2 Mg if the efficiency is 80 % ;*
- (2) The number of strokes required by the plunger to lift the load 80 mm.*

**Solution:**

*( 1 ) ( Convert to meters)*

*Given:  $D = 100 \text{ mm} = 0,1 \text{ m}$*

*$d = 20 \text{ mm} = 0.02 \text{ m}$*

*Stroke of plunger = 40 mm = 0.04 m*

$$\eta = 80 \%$$

$$M.A. = 30$$

$$W = 1.2 \text{ Mg} = 1\,200 \text{ kg} = 1\,200 \times 10 \text{ N}$$

*( from formula )*

$$\frac{W}{F} = \frac{D^2}{d^2}$$

$$\text{And } \frac{F}{W} = \frac{d^2}{D^2}$$

$$\begin{aligned} \therefore F &= \frac{d^2 \times W}{D^2} \\ &= \frac{(0,02)^2}{(0,1)^2} \times 1\,200 \times 10 \end{aligned}$$

$$F = 480 \text{ N}$$

$$\text{Effort of handle} = \frac{F}{M.A.}$$

$$= \frac{480}{30}$$

$$= 16 \text{ N}$$

***But  $\eta = 80 \%$***

***Thus, the actual force required on the handle =  $\frac{16 \times 100}{80}$***   
***= 20 Newtons Ans (i )***

***(2 ). If the load is to be lifted 80 mm = 0,08mm, then all the required liquid is to be supplied by the plunger, thus:***

***Volume to be displaced = Volume of liquid required by ram to lift the required distance.***

***Volume per plunger stroke X number of strokes = volume of liquid required by ram***

$$\left( \frac{\pi d^2}{4} \times \text{stroke} \right) \times \text{number of strokes} = \frac{\pi D^2}{4} \times \text{height lifted}$$

$$d^2 \times \text{stroke} \times \text{number of strokes} = D^2 \times \text{height lifted}$$

$$\text{Number of strokes} = \frac{D^2 \times \text{height lifted}}{d^2 \times \text{stroke}}$$

$$= \frac{(0,1)^2 \times 0,08}{(0,02)^2 \times 0,04}$$

$$= 50 \text{ strokes Ans (ii )}$$