# Chapter 6:

# Dynamics,

## 6.1 Work Power and Energy.

### Work:

**Work is said to be done if we apply force on a body and the body moves in the direction of force applied. The work can be calculated by the product of the force and the distance through which a body moves.**

i.e. Work done = Force (F) X distance (d)

The work done is expressed in *joule*. Which is defined as the work done when a force of 1 Newton is applied on a body through a distance of 1 metre?

**Power: The rate of doing work is known as power.**

i.e. Power = Work done/Time taken

The power is expressed in joule/sec. or watt (1 watt = 1 Newton metre/second)

**Power: The rate of doing work is known as power.**

The bigger unit of power such as Horse power and Kilowatt are also used. Where 1 Horse power = 746 watts and 1 Kilowatt = 1000 watts.

### Energy:

**Capacity to do work is called energy. It has the same unit as that of work i.e. joule. It can also be expressed in calories**.

1 Calorie = 4.18 joules

The energy can be in transition or is stored in the system boundaries.

***At the end of this section you should be able to:***

- describe potential energy as energy due to position and derive potential energy as *mgh*

- describe kinetic energy as energy due to motion and derive kinetic energy as $mv^2/2$
- state conservation of energy laws and solve problems where energy is conserved
- define power as rate of energy transfer
- define *couple*, *torque* and calculate work done by variable force or torque
- solve problems where energy is lost due to friction

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## 6.2 Work Done:

Work is done when an object is for example picked up at a certain place, carried to another place and put down at that place, Work is also done when a force of a body causes it to move and is measured by the product of the force and the distance through which the force moves.

*As mentioned before the* ***unit of work is joule [ J ]***

***Definition of the Joule ( J )***

The joule is the work done when the point of application of a force of 1 N is displaced through a distance of 1 m in the direction of the force.

***Unit of work done***

***The joule ( J ) is the unit for work done e.g.***

***Work done = force ( in newtons ) x distance moved ( in meters )***

***= 100 x 15 ( Nm )***

*= 1500 J*

*Or = 1.5 KJ.*

***Definition of work done.***

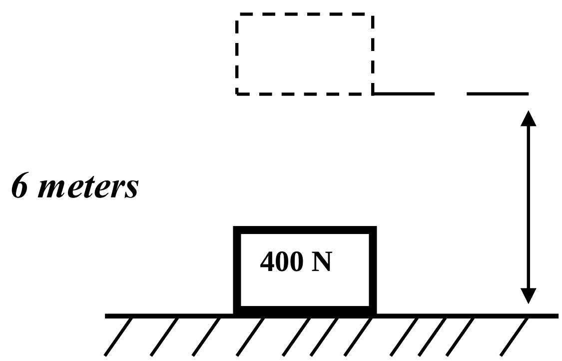
If the point of application of a force moves in the direction in which the force acts, then work is done. The amount of work done is the product of the force and the distance moved by the force.

***Example:***

***Determine the work done in each of the following cases:***

1. ***A load of 400 N is lifted vertically for 6 m***
2. ***A wagon with a resistance to motion of 42 N is pushed for 200m.***

***Solutions: (a)***



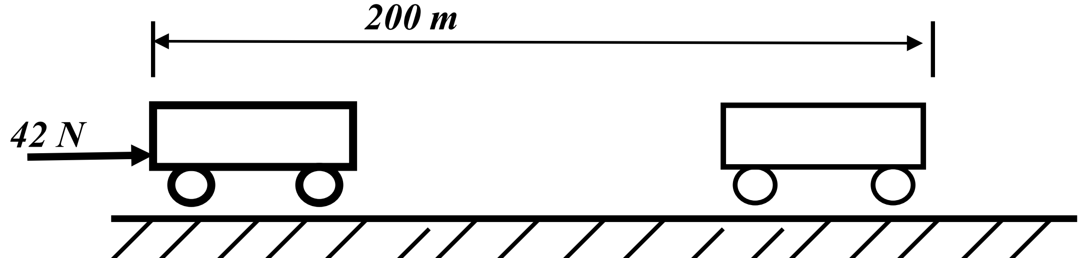

***Work done = force x distance***

***= 400 N x 6 m***

***= 2 400 joule***

***= 2,4kj***

***Solution: (b)***



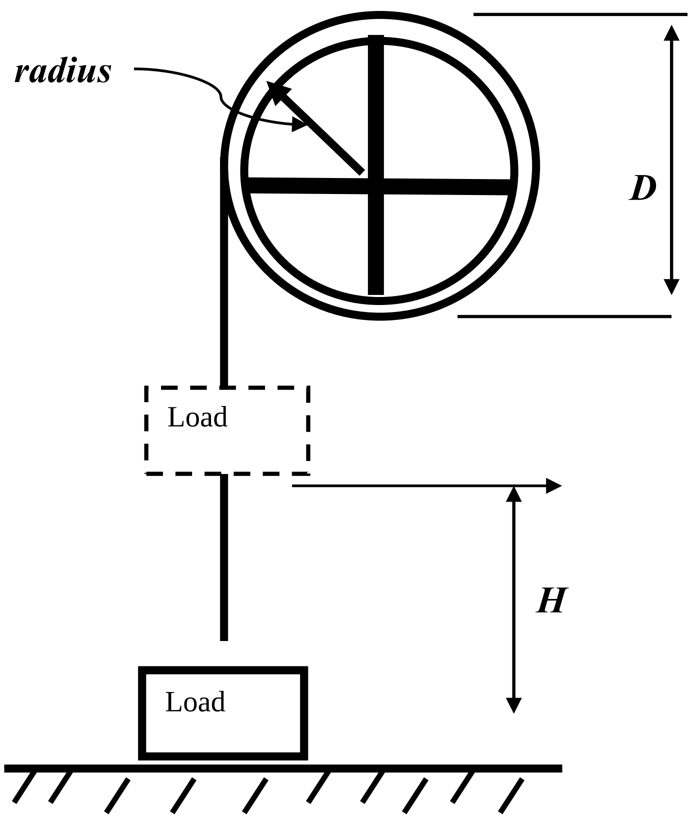
***Work done = force x distance moved***

***= 42 N x 200m***

***= 8.400 joule***

***= 8.4 k j.***

**Work Done by Torque.**




*The above illustrates the drum of a lifting machine; a rope is wound around the drum for hoisting the load. For one revolution of the drum the load will move upwards and the distance moved up wards **H**,is equal to the circumference of the drum.*

***We can calculate that for one revolution:***

***Distance load moved upwards = circumference of drum***

$H = \pi D = 2 \pi R$

***But work done = force x distance moved, and since the upward force in the rope is equal to the load then:***

***Work done in one revolution = force (in rope) X vertical distance of load.***

***= force x H***

***= force x $\pi$ D***

***Work done during 'n' revolution = force x 2 $\pi$ R n***

***Also: Torque, T = force x radius***

***T = Force x R***

***Thus, work done may also be formulated as follows:***

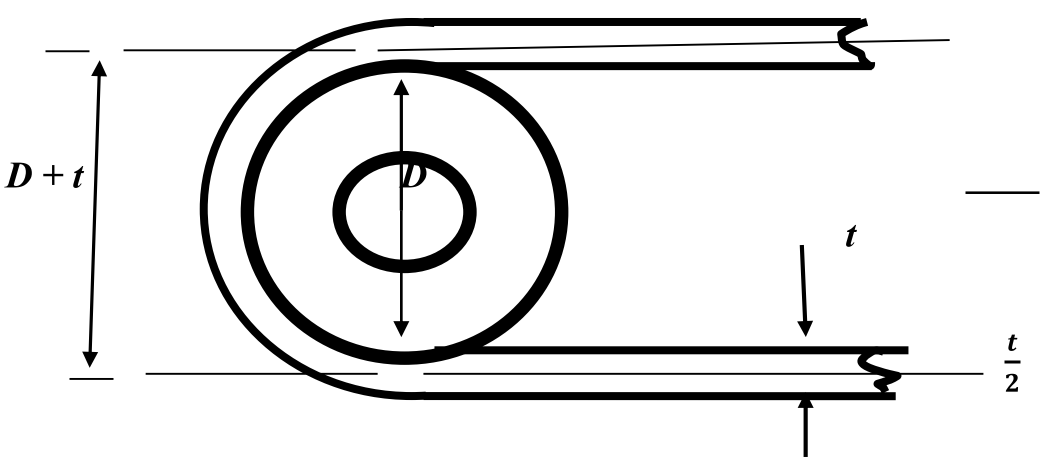
***Work done during 'n' revolutions = T 2 $\pi$ n***

***Where T = torque in N.m.***

***= force x radius***

***N = revolutions of the drum.***

***If the Thickness of the rope is given then the Diameter of the drum will have to include the rope and the following calculation is worked out as:***



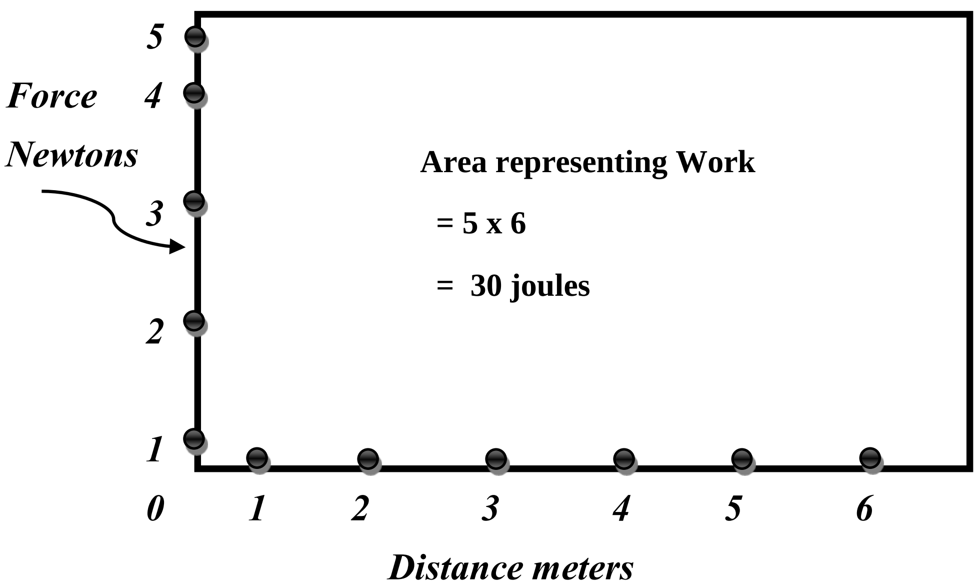
***Work done by a constant force***

When the point at which a force acts moves, the force is said to have done work.

When the force is constant, the work done is defined as the product of the force and distance moved.

***Work done = force x distance moved in direction of force***

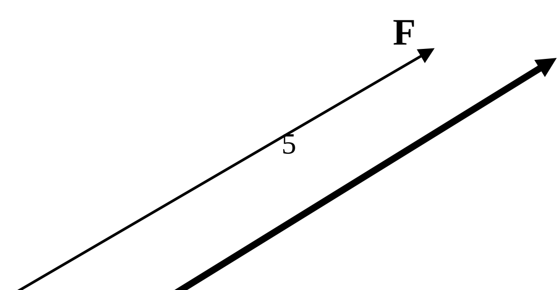
**Constance Force**

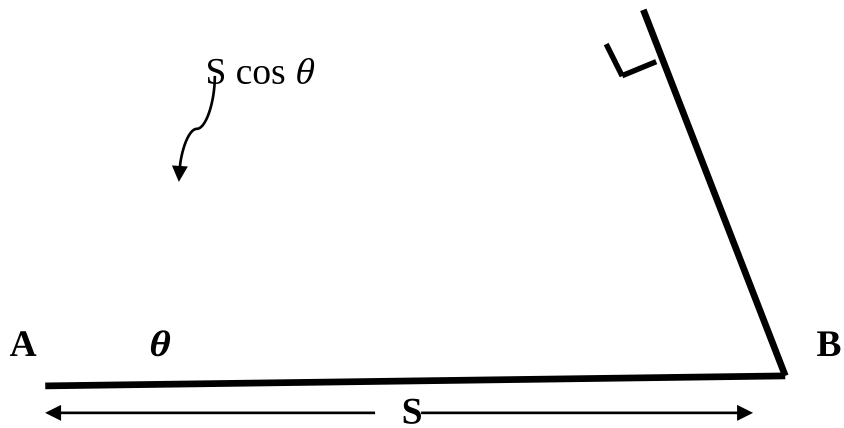



***A simple graphical representation of Work:***

*The graph above represents a constant force of 5 Newton acting through a distance of 6 Meters, the* ***work done is 5 [ n ] x 6 [ m] = 30 [ j ].***

Consider the example in a force F acting at the angle moves a body from point A to point B.






**Notation for work done by a force**

The distance moved in the direction of the force is given by

***Distance in direction of force = s cos θ***

So the work done by the force F is

***Work done = F s cos θ***

If the body moves in the same direction as the force the angle is 0.0 so

Work done = *Fs*

When the angle is 90 then the work done is zero.

The SI units for work are Joules *J* (with force, *F*, in Newton's *N* and distance, *s*, in metres *m*).

***Worked Example***

How much work is done when a force of 5 kN moves its point of application 600mm in the direction of the force.

**Solution**

Work done = $(\mathbf{5 \times 10^{3}}) \times (\mathbf{600 \times 10^{-3}})$

= *3000 J*

= *3 kj*

***Worked Example***

Find the work done in raising 100 kg of water through a vertical distance of 3m.

*Solution*

The force is the weight of the water, so

$$\text{Work done} = (\ 100 \times 9.81\ ) \times 3$$
$$= 2943\ J$$

## 6.3 Energy

A body which has the capacity to do work is said to possess energy.

For example, water in a reservoir is said to possess energy as it could be used to drive a turbine lower down the valley. There are many forms of energy e.g. electrical, chemical heat, nuclear, mechanical etc.

The SI units are the same as those for work, Joules *J*.

In this module only purely mechanical energy will be considered. This may be of two kinds, **potential** and **kinetic**

### 6.3.1 Potential Energy

There are different forms of potential energy two examples are:

(i) A pile driver raised ready to fall on to its target possesses gravitational potential energy while

(ii) A coiled spring which is compressed possesses an internal potential energy.

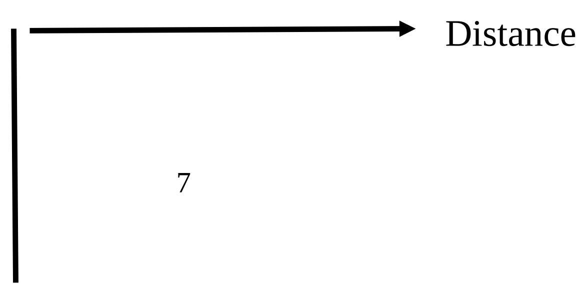
Only gravitational potential energy will be considered here. It may be described as energy due to position relative to a standard position (normally chosen to be the earth's surface.)

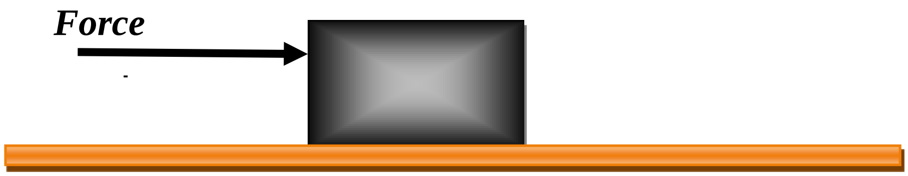
The potential energy of a body may be defined as the amount of work it would do if it were to move from the current position to the standard position

Energy is defined in physics as the ability to do work. The interesting part is what does it mean to do work?

If we push a block from point A to point B we did work in moving that block. While the block was in motion it was said to have had Kinetic energy which you can think of as the energy of motion.

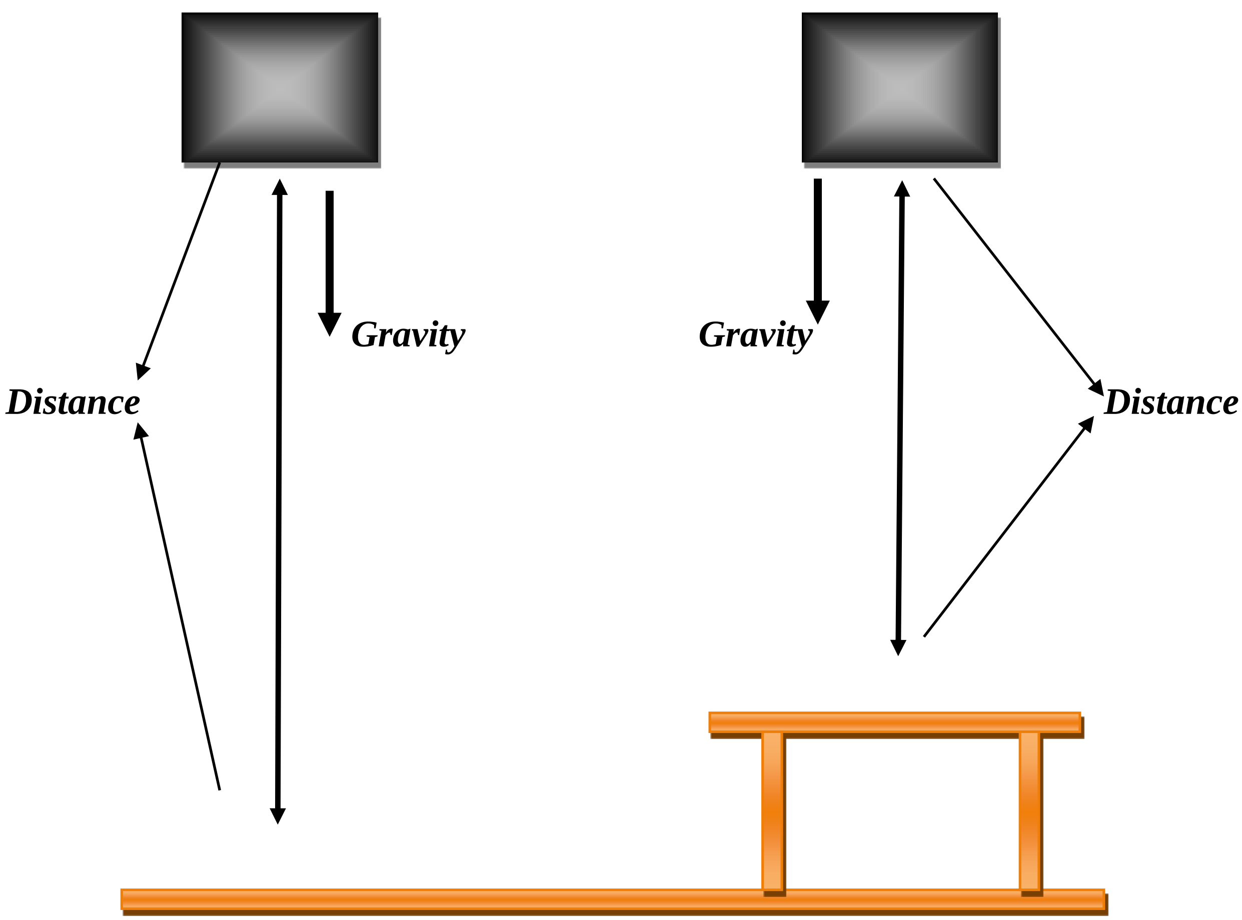
**Kinetic Energy**






If we lift that same block from the ground to a point above our head, we did work to get it there and it now said to have ***potential energy***. We use the term potential energy, because it has the potential to do work, by falling to the ground or to be more accurate being acted upon by gravity through a distance. It has this energy because of its position and the distance that it can fall. If we release it, all of the potential energy will convert to kinetic energy as the block goes flying towards the ground. Half way to the ground the energy is 50% potential and 50% kinetic and just as it hits the ground the energy is 100% kinetic. When it hits the ground all of the kinetic energy is transformed into deforming the ground and the block. The amount of potential energy that the block has is relative to its position. If we move the block so that it is positioned above a table, it doesn't have nearly as far to fall and therefore, relative to the table top, it has less potential energy.

**Potential Energy**




## 6.3.2 Formulae for gravitational potential energy

A body is at rest on the earth's surface. It is then raised a vertical distance *h* above the surface. The work required to do this is the force required times the distance *h*.

Since the force required is its weight, and weight,

**W = mg, then the work required is *m g h.***

The body now possesses this amount of energy - stored as potential energy - it has the capacity to do this amount of work, and would do so if allowed to fall to earth.

Potential energy is thus given by:

$$\textit{Potential energy} = mgh$$

Where ***h*** is the height above the earth's surface.

***<u>Worked example</u>*** what is the potential energy of a 10kg mass?

a. 100m above the surface of the earth
b. at the bottom of a vertical mine shaft 1000m deep.

**Solution**

a) potential energy = ***mgh***

$= 10 \times 9.81 \times 100$

$= 9801\ J$

$= 9.81\ kJ$

b) Potential energy = *mgh*

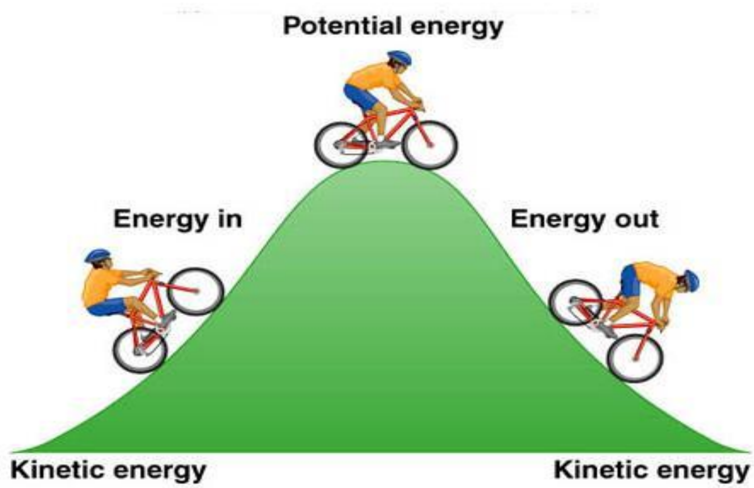
$= 10 \times 9.81 \times (-1000)$

$= -98100\ J$

$= -98.1\ KJ$

## 6.3.3 Kinetic energy

A cyclist and his bicycle has a mass of energy. After he reaches the top of a hill, with slope 1 in 20 measured along the slope, at a speed of 2 m/s. He then free wheels to the bottom of the hill where his speed has increased. From the illustration given we can see the kinetic energy.




Kinetic energy may be described as energy due to motion.

The kinetic energy of a body may be defined as the amount of work it can do before being brought to rest.

For example when a hammer is used to knock in a nail, work is done on the nail by the hammer and hence the hammer must have possessed energy.

Only linear motion will be considered here.

### 6.3.4 Formulae for kinetic energy

Let a body of mass *m* moving with speed *v* be brought to rest with uniform deceleration by a constant force *F* over a distance *s*.

Using Equation 1.4 $\quad \boldsymbol{v^2 = u^2 + 2as}$

$$\boldsymbol{O = u^2 + 2as}$$

$$\boldsymbol{S = \frac{v^2}{2a}}$$

And work done is given by

$$\text{Work done} = \text{force x distance}$$
$$= \boldsymbol{Fs}$$
$$= \boldsymbol{F}\,\frac{\boldsymbol{v^2}}{\boldsymbol{2\,a}}$$

The force is $F = ma$ so

$$\textbf{Work done} \text{ during Kinetic energy} = ma\frac{v^2}{2a}$$
$$= \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{mv^2}$$

Thus the kinetic energy is given by

$$\textbf{\textit{Kinetic energy}} = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{mv^2}$$

You can use physics to calculate the kinetic energy of an object. When you start pushing or pulling a stationary object with a constant force, it starts to move if the force you exert is greater than the net forces resisting the movement, such as friction and gravity. If the object starts to move at some speed, it will acquire kinetic energy. ***Kinetic energy*** is the energy an object has because of its motion. ***Energy*** is the ability to do work.

So how do you calculate kinetic energy?

Force acting on an object that undergoes a displacement does work on the object. If this force is a ***net*** force that accelerates the object (according to Newton's second law), then the velocity changes due to the acceleration. The change in velocity means that there is a change in the kinetic energy of the object.

The change in kinetic energy of the object is equal to the work done by the net force acting on it. This is a very important principle called the ***work-energy theorem***. After you know how work relates to kinetic energy, you're ready to take a look at how kinetic energy relates to the speed and mass of the object. The equation to find kinetic energy, ***KE***, is the following, where ***m*** is mass and ***v*** is velocity:

$$\boldsymbol{K\,E} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{m\ v^2}$$

This is just the work-energy theorem stated as an equation.

You normally use the kinetic energy equation to find the kinetic energy of an object when you know its mass and velocity. Say, for example, that you're at a firing range and you fire *a 10-gram bullet with a velocity of 600 meters/second at a target. What's the bullet's kinetic energy?*

Using the equation to find kinetic energy, you simply plug in the numbers, remembering to convert from grams to kilograms first to keep the system of units consistent throughout the equation:

$$\boldsymbol{KE = \frac{1}{2} m\, v^2}$$

$$= \frac{1}{2}(\,0.010\ kg\,)\,(\,600\ m/s\,)^2 = 1{,}800\ J$$

The bullet has 1,800 joules of energy, which is a lot of energy to pack into a 10-gram bullet

## 6.3.5 Kinetic energy and work done

When a body with mass $m$ has its speed increased from $u$ to $v$ in a distance $s$ by a constant force $F$ which produces an acceleration $a$, then from Equation 1.3 we know

$$\boldsymbol{v^2 = u^2 + 2as}$$

$$\boldsymbol{\frac{1}{2}v^2 - \frac{1}{2}u^2 = as}$$

Multiplying this by $m$ give an expression of the increase in kinetic energy (the difference in kinetic energy at the end and the start)

$$\boldsymbol{\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas}$$

Thus since $F = ma$

$$\textit{Increase in kinetic energy} = Fs$$

but also we know

$Fs$ = *work done*

So the relationship between kinetic energy can be summed up as

Work done by forces acting on a body = change of kinetic energy in the body

This is sometimes known as the work-energy theorem.

***Worked example***

A car of mass 1000 kg travelling at 30m/s has its speed reduced to 10m/s by a constant breaking force over a distance of 75m.

Find:

a. The cars initial kinetic energy
b. The final kinetic energy
c. The breaking force

***Solution***

a) Initial kinetic energy = $\boldsymbol{\frac{1}{2}mv^2}$

$= 500 \text{ x } 30^2$

$= 450000\ J$

**= 450 *KJ***

b) Final Kinetic energy = $\frac{1}{2}\ mv^2$

$= 500 \text{ x } 10^2$

*= 50000 J*

***= 50 KJ***

c) Change in kinetic energy = 400 *kJ*

By Equation 3.5 work done = change in kinetic energy so

*Fs = change in kinetic energy*

*Breaking force x 75 = 400 000*

***Breaking force = 5333 N***

## 6.3.6 Conservation of energy

***The principle of conservation of energy state that the total energy of a system remains constant. Energy cannot be created or destroyed but may be converted from one form to another.***

Take the case of a crate on a slope. Initially it is at rest, all its energy is potential energy. As it accelerates, some of it potential energy is converted into kinetic energy and some used to overcome friction. This energy used to overcome friction is not lost but converted into heat. At

the bottom of the slope the energy will be purely kinetic (assuming the datum for potential energy is the bottom of the slope.)
If we consider a body falling freely in air, neglecting air resistance, then **mechanical energy is conserved**, as potential energy is lost and equal amount of kinetic energy is gained as speed increases.

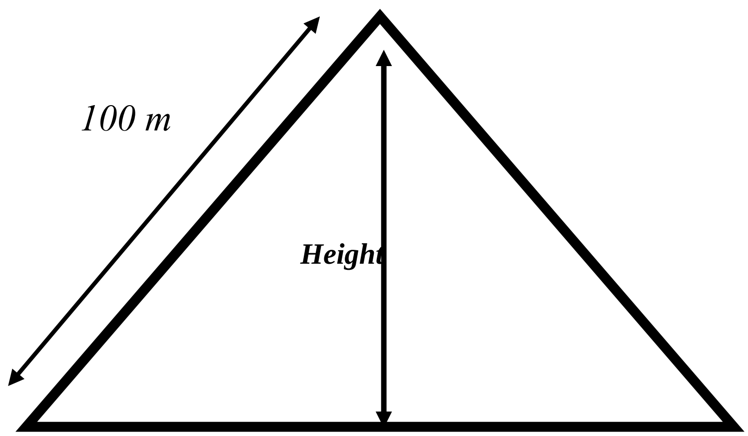
If the motion involves friction or collisions then the principle of conservation of energy is true, but conservation of mechanical energy is not applicable as some energy is converted to heat and perhaps sound.

## Worked Example

***A cyclist and his bicycle has a mass of 80 kg. After 100m he reaches the top of a hill, with slope 1 in 20 measured along the slope, at a speed of 2 m/s. He then free wheels the 100m to the bottom of the hill where his speed has increased to 9m/s.***

***How much energy has he lost on the hill?***

## Solution:




***Dimensions of the hill in worked example***

If the hill is 100m long then the height is:

$$\boldsymbol{h = 100\frac{1}{20} = 5\ m}$$

So potential energy lost is

*Mgh = 80 x 9.81 x 5 =3924 J*

Increase in kinetic energy is

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}m(v^2 - u^2)$$

$$= 40\,(81 - 4)$$

$$= \boldsymbol{3080\ J}$$

By the principle of conservation of energy

*Initial energy = Final energy + Loss of energy (due to friction etc.)*

*Loss of energy (due to friction etc.) =* ***3924 -844 J***

## 6.4 Power

Power is the rate at which work is done, or the rate at which energy is used transferred.

Power has a specific duration as power refers to the quantity of work done per second. Power is the rate of doing work and is measured in **Watts ( W ).**

**Power is = work done per second**

**Power = force x distance moved per second**

**Power =** $\frac{force \times distance}{time\ in\ seconds}$

**Therefore we can say that** $Power = \frac{work\ done}{time\ taken}$

**Power = T 2** $\pi n$ **Watt**

where**T** = torque in N.m = force x radius

**N** = rotational frequency of object in r/s

The SI unit for power is the **watt *W*.**

A power of 1 $W$ means that work is being done at the rate of 1$J/s$.

Larger units for power are the kilowatt $kW$ ($1kW = 1000\ W = 10^3\ W$) and

the megawatt $MW$ ($1\ MW = 1{,}000{,}000\ W = 10^6\ W$).

If work is being done by a machine moving at **speed *v*** against a constant force, or resistance, $F$, then since work done is force x distance, work done per second is $Fv$, which is the same as power.

***Power = F v***

**Definition of the watt** ***The watt is the power which results in the production of energy at the rate of 1 J/s***

### 6.4.1 Mechanical Efficiency:

No machine is perfect and the power supplied to it some is used up in overcoming friction and other resistances, and the remainder is available for the useful work. The ratio of the power got out of the machine to the power put in is the ***mechanical efficiency*** of the machine

If the power of a machine is applicable, then the efficiency may be formulated as follows:

**Efficiency,** $= \frac{\textbf{power output}}{\textbf{power input}}\ \boldsymbol{x}\ \mathbf{100}\ \%$

## Example:

***A load of 2,5kn is hoisted by means of a rope and a lifting machine with a rotational frequency of 40 r/m and a drum diameter of 700mm. Calculate:***

- ***a). The torque,***
- ***b). The speed of the rope ( or load ) in m/s,***
- ***c) the work done per minute,***
- ***d) the power required for the load,***
- ***e). the efficiency of the lifting machine if an electric motor of 5 kw is required to drive the lifting machine.***

## Solution :

***Given : D = 700mm = 0,7 m*** ***Radius*** $= \frac{\mathbf{0{,}7}}{\mathbf{2}} =$ ***0.35 m***

***n = 40 r/min.***

***Load = force (tension) in rope = 2,5kn = 2500 N***

***(a ) for Torque = force x radius***

***= 2 500 x 0,35***

***= 875 N.m*** **Ans**

***(b) Speed of rope = speed of a point on the circumference of the drum***

$$= \frac{\boldsymbol{\pi D n}}{\mathbf{60}}$$

$$= \frac{\boldsymbol{\pi} \times \mathbf{0{,}7} \times \mathbf{40}}{\mathbf{60}}$$

**= 1,466 m/s ans.**

**(c ) Work done per minute = force x distance moved per minute**

**= force** $\times\ \boldsymbol{\pi\ D\ n}$

**= 2,500** $\times\ \boldsymbol{\pi}\ \times$ **0, 7** $\times$ **40**

$= 219\ 911$ **J**

$= 219.911$ **kj** **Ans**

(d ) Power required $= \frac{T\ 2\pi \times n}{60}$

$= \frac{875\ x\ 2 \times \pi \times 40}{60}$

**= 3 665 W**

**= 3.6 kw ans.**

**(e) Efficiency,** $= \frac{\text{power output}}{\text{power input}}$ **x 100**

$= \frac{3{,}665}{5} \times 100$

**= 73,3 % ans.**

## 6.4.2 Practice Exercise:

1. ***A pump lifts fresh water from one tank to another through an effective height of 12 m. If the mass flow of the water is 40 tonne /h, (i) find the output of the pump. (ii) if the input power of the pump was 1,75 kw then find the efficiency of the pump.***
2. ***A motor vehicle , with a tractive resistance of 800 N , travels at 50 km/h. Calculate :***

   ***(a) The power required to drive the vehicle at constant speed,***

   ***(b) The efficiency of the transmission of the vehicle if the engine power is 20 kw.***
3. ***The lifting drum with a diameter of 300 mm and a rotational frequency of 35 r/min is used to hoist a load of 12 kn. Calculate the power required.***
4. ***The rate of water supply to a hydraulic crane is 90 litres per minute at a steady pressure of 70 bar. Find the input power and its efficiency if the output is 7.5 k w.***

**Worked Example:**

***A constant force of 2kN pulls a crate along a level deck on board a ship for a distance of 10 m in 50s.***

***What is the power used?***

**Solution:**

***Work done = force x distance***

***= 2000 10***

***= 20 000 J***

***Power*** $= \frac{\text{work done}}{\text{time taken}}$

$$= \frac{\mathbf{20\,000}}{\mathbf{50}}$$

***= 400 w***

Alternatively we could have calculated the speed first

$$\boldsymbol{v = \frac{distance}{time}}$$

$$= \frac{\mathbf{10}}{\mathbf{50}} = \mathbf{0.2}\ \boldsymbol{m/s}$$

and then calculated power

*Power = Force x Speed*

*= Fv*

*= 2000 x 0.2*

= ***400W***

***Worked Example***

***A ships derrick hoist operated by an electric motor has a mass of 500 kg. It raises a gangway which has a load of 300 kg vertically at a steady speed of 0.2 m/s. Frictional resistance can be taken to be constant at 1200 N.***

***What is the power required?***

**Solution :**

*Total mass = m =800 kg*

*Weight = 800 x 9.81*

*= 7848 N*

*Total force = 7848 + 1200*

*= 9048 N*

From Equation

*Power = force x speed*

$= 9048 \times 0.2$

$= 1810\ W$

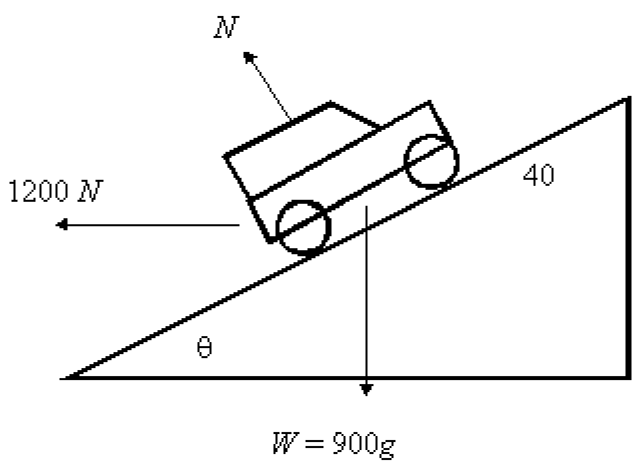
$= \mathbf{1.81\ kW}$

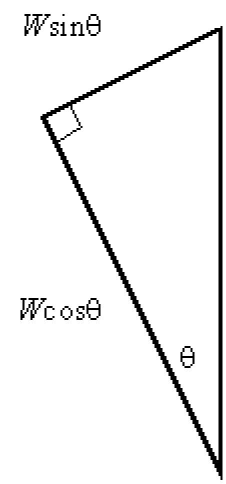
***<u>Worked example 3.8</u>***

***A car of mass 900 kg has an engine with power output of 42 kW. It can achieve a maximum speed of 120 km/h along the level.***

- ***a. What is the resistance to motion?***
- ***b. If the maximum power and the resistance remained the same what would be the maximum speed the car could achieve up an incline of 1 in 40 along the slope?***

**Solution:**




: Forces on the car on a slope in Worked Example

First get the information into the correct units:

$$120\ km/h = \frac{120 \times 1000}{3600}$$

$$= 33.33\ m/s$$

a) Calculate the resistance

*Power = Force x speed*

$$= Resistance \times speed$$

$$4200 \quad = Resistance \times 33.33$$

$$Resistance = \frac{42000}{33.33} = 1260\ N$$

b) Total force down the incline = friction force + component of weight down incline

$$= 1260 + mg \sin\theta$$

$$= 1260 + 900 \times 9.81 \frac{1}{40}$$

$$= 1260 + 221$$

$$= 1481\ N$$

$$Power \quad = Force \times speed$$

$$Speed \quad = \frac{Power}{force}$$

$$= \frac{42000}{1481}$$

$$= 28.4\ m/s$$

Or in km/h

$$Speed = 28.4 \times \frac{3600}{1000}$$

$$= \mathbf{102\ km/h}$$

### 6.4.3 Rotary Power :

*Consider a force F newtons at a radius of r meters on a rotation mechanism.*

*Work done in one revolution = force [ N ] × circumference [m]*

$$= \boldsymbol{F \times 2\pi\ r.}$$

*If it is running at n revolutions per second, [n for rev/sec.]*

$$\boldsymbol{Power\ [\ W] = F \times 2\pi\ r \times n}$$

*But we know that* $\boldsymbol{F \times r}$ *= torque [Nm] then let this be represented by* $\boldsymbol{T}$

***Let ω(omega) = speed of rotation in radians per second,***

***Then ω is the same as = 2π n***

***∴ Power [ W] = Torque [Nm] × ω [ rad/s]***

***Or Power [ kW] = Torque [kNm] × ω [ rad/s]***

**Example:**

***The mean torque in a propeller shaft is*** $\boldsymbol{2.26 \times 10^5}$ ***Nm when running at 140 rev/min. Find the power transmitted.***

**Solution:**

**Rotational speed = 140 rev/min**

$$= \frac{140}{60} \times 2\pi \ rad/s$$

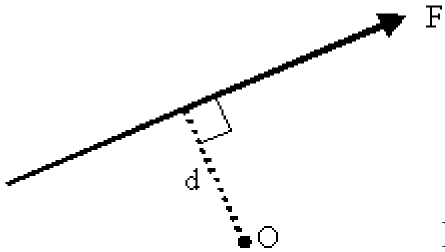
$$Power\ [kw] = 2.26 \times 10^2\ [\ kNm] \times \frac{140}{60} \times 2\pi\ rad/s.$$

$$= \ 3313\ k\ w\ Ans.$$

### 6.4.4 Moment, couple and torque

The moment of a force *F* about a point is its turning effect about the point.

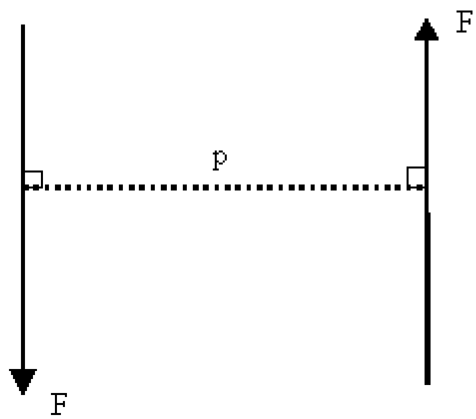
It is quantified as the product of the force and the perpendicular distance from the point to the line of action of the force.




Moment of a force

the moment of *F* about point *O* $moment = Fd$

Equation 3. A **couple** is a pair of equal and parallel but opposite forces as shown next.




A couple

The moment of a couple about any point in its plane is the product of one force and the perpendicular distance between them:

*Moment of couple = Fp*

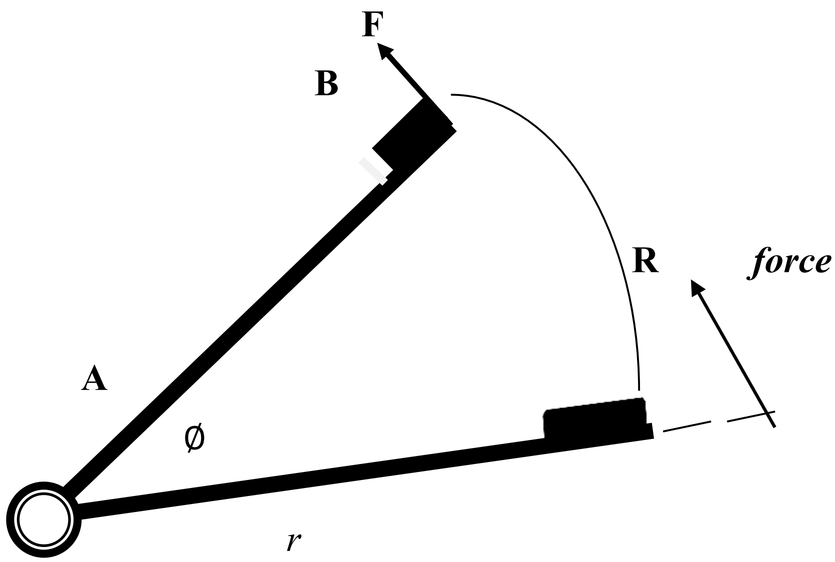
Examples of a couple include turning on/off a tap, or winding a clock.

The SI units for a moment or a couple are Newton metres, *Nm*.

In engineering the moment of a force or couple is known as **torque**. A spanner tightening a nut is said to exert a torque on the nut, similarly a belt turning a pulley exerts a torque on the pulley.

## 6.4.5 Work done by a constant torque

Let a force $F$ turn a light rod OA with length $r$ through an angle of $\theta$ to position OB, as shown




***Work done by a constant torque:***

The torque $T_Q$ exerted about O is force times perpendicular distance from O.

$$T_q = Fr$$

Now work done by $F$ is

*Work done* = ***F s***

***S*** is the arc of the circle, when **r** is measure in radians

$$s = r\theta$$

$$\text{work done} = Fr\theta$$

$$\text{work done} = T_q\theta$$

The work done by a constant torque $T_Q$ is thus the product of the torque and the angle through which it turns (where the angle is measured in radians.)

As the SI units for work is Joules, $T_Q$ must be in *Nm*

### 6.4.6 Power transmitted by a constant torque

Power is rate of doing work. It the rod in Figure 3.7 rotates at *n* revolutions per second, then in one second the angle turned through is

$$\boldsymbol{\theta = 2\,\pi\,n}$$

radians, and the work done per second will be, by Equation 3.11

work done per second = $\boldsymbol{T_q\,2\,\pi n}$

as angular speed is $\boldsymbol{\omega = 2\pi n}$

Then power = $\boldsymbol{2\,\pi n\,T_q}$

Power = $\boldsymbol{\omega\,T_q}$

The units of power are Watts, *W*, with *n* in *rev/s*, in *rad/s* and $T_Q$ in *Nm*.

***<u>Worked Example</u>***

***A spanner that is used to tighten a nut is 300mm long. The force exerted on the end of a spanner is 100 N.***

a. ***What is the torque exerted on the nut?***
b. ***What is the work done when the nut turns through $30^0$***

***Solution;***

$$\boldsymbol{Calculate\ the\ torque\ by\ Equation\ 3.10}$$

$$\begin{aligned} \boldsymbol{T_q} &= \boldsymbol{Fr} \\ &= \boldsymbol{100\,X\,(300 \times 10^{-3})} \\ &= \boldsymbol{30\ Nm} \end{aligned}$$

***<u>Worked Example</u>***

***An electric motor is rated at 400 W. If its efficiency is 80%, find the maximum torque which it can exert when running at 2850 rev/min.***

***Solution***

Calculate the speed in rev/s using

$$\text{Power} = 2\pi T_q$$

$$n = \frac{\boldsymbol{power}}{\boldsymbol{2\pi T_q}}$$

$$n = \frac{\mathbf{2850}}{\mathbf{60}} = 47.5\ rev/s$$

Calculate the power as the motor is 80% efficient

$$\text{Power} = 400 \text{ x } \frac{80}{100} = 320\ W$$

$$\text{Power} = 2\ \pi n T_q$$

$$\boldsymbol{T_q} = \frac{\boldsymbol{power}}{\boldsymbol{2\pi n}}$$

$$= \frac{\mathbf{320}}{\mathbf{2\pi\ 47.5}}$$

$$= 1.07\ Nm$$