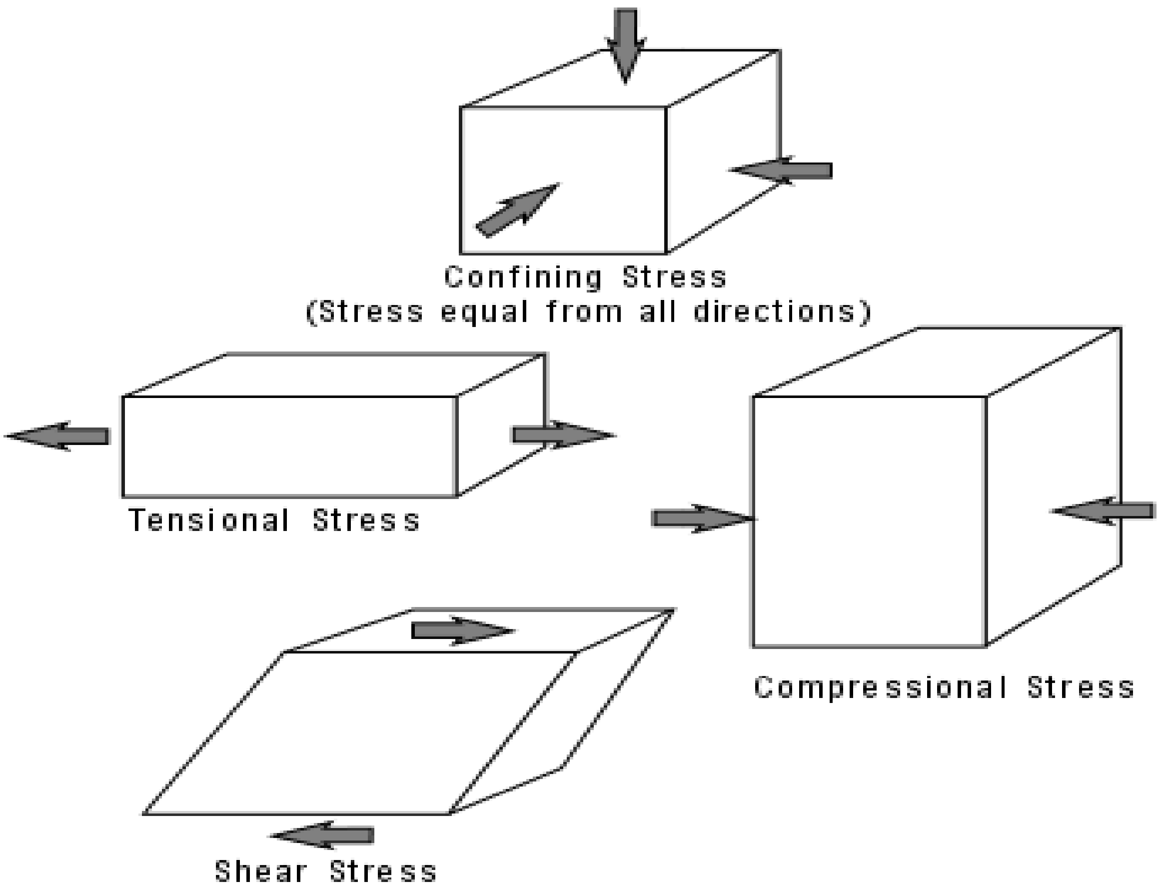
Strength of Materials.

9.1 Introduction to forces acting on a Body.

When an external force is applied to a body, the body resists or tries to resist this external force. The shape of the body may even be changed in this process.



(1) Tensile Stress:

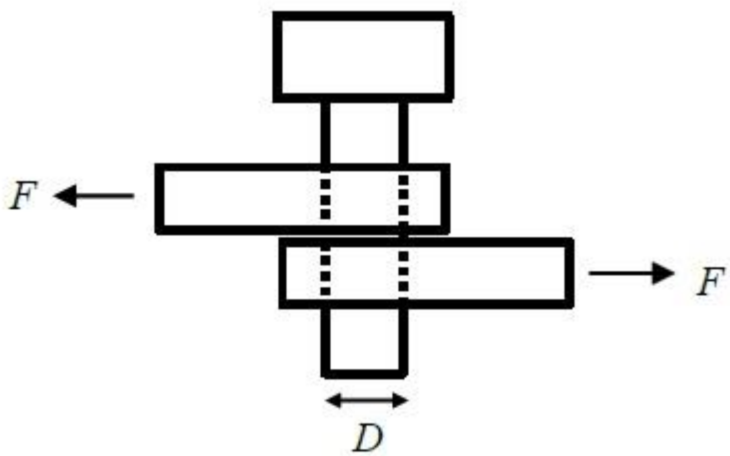
Tensile stress is caused if a tensile force, or force, are applied to an object.

(2) Compressive stress:

Compressive stress is caused if a compressive force tends to compress an object.

(3) Shear Stress: the below drawing shows two plates bolted together and two tensile forces applied to the plates .A shear stress is caused in the bolt as the shank of the bolt prevents the two plates from sliding over each other.

$$\tau = \frac{F}{A} \qquad \tau = \frac{F}{\pi(r)^2} \qquad \tau = \frac{4F}{\pi(D)^2}$$



9.2Stress:

The ability of an object to resist external forces depends on the size of the area, i.e. the effective cross-sectional area, subject to these external forces. Thus each unit of area of the section carries an equal share of the load and this is called the intensity of stress. Usually the intensity of stress referred to as simply “the stress “

Stress is obtained by dividing the load (force) acting on the object by the effective cross – sectional cross- sectional area of the object**.**

$$\textbf{Stress} = \frac{\boldsymbol{Load}\ (\boldsymbol{N})}{\boldsymbol{Area}\ (\boldsymbol{m})}$$

The unit for stress is the **N/m**. Since the load has to be in Newton’s and the area in Square meters**, N/$\boldsymbol{m^2}$**.

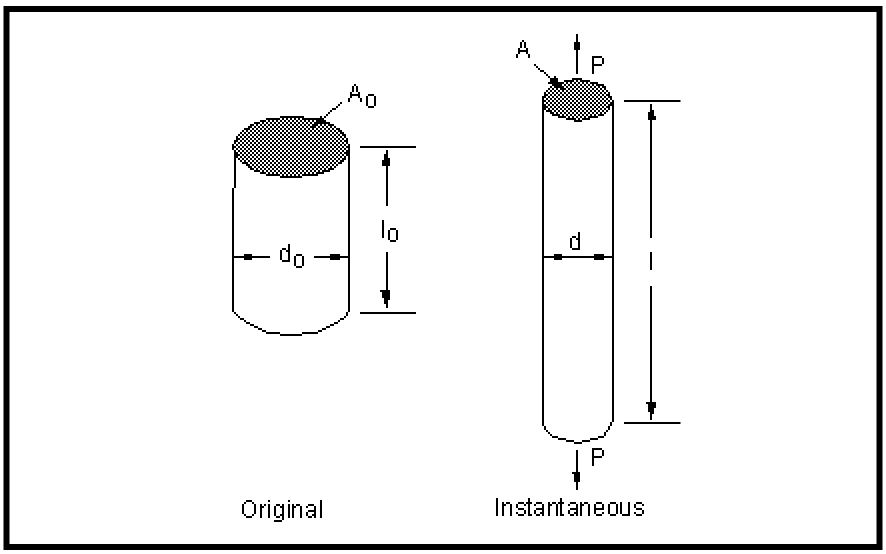
9.3 Strain:

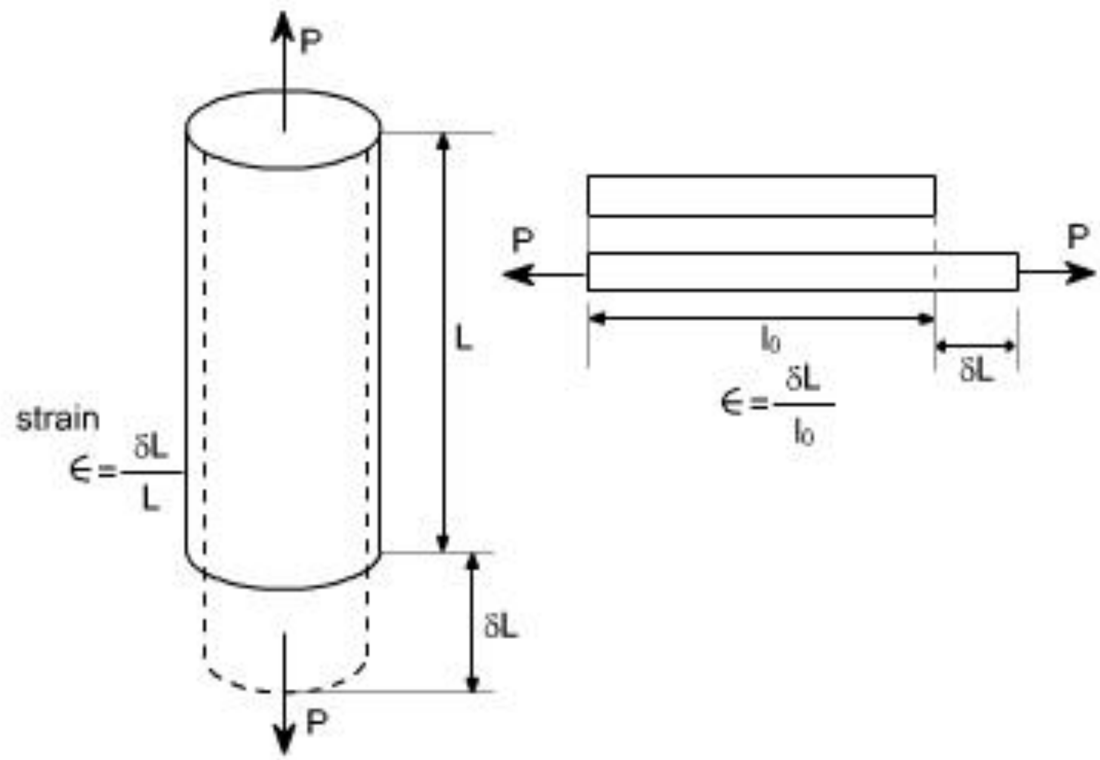
Values for strain are considered on that Strain is dimensionless since it represents length divided by its length. Dynamic strain associated with the passage of seismic waves in the far field are typically less than 10^{-6}. When the material of an object is in a state of stress, deformation may take place and the length, or the shape, of the object may change. The manner of deformation will depend on the type of load acting on the object.

A tensile stress can increase the original length, $l_\delta$of an object by a distance or X as seen from the insert drawing.

A compressive stress can reduce the original length $\boldsymbol{l_\delta}$, of an object by a distance of X.

$\therefore$ Strain $= \dfrac{\boldsymbol{change\ in\ length\ (m)}}{\boldsymbol{original\ length\ \ (m)}}$

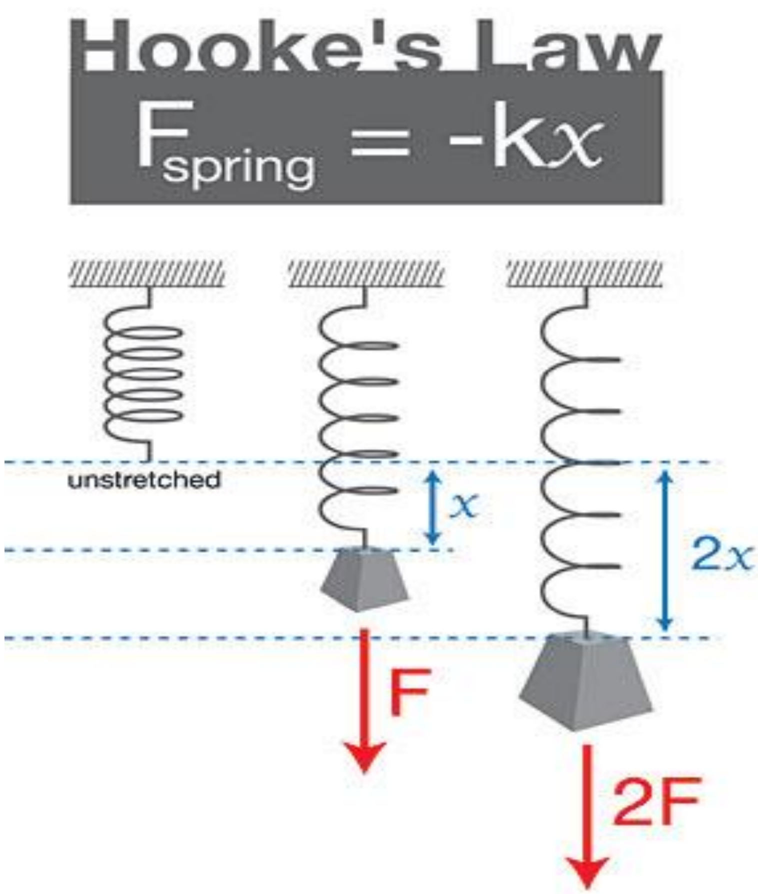


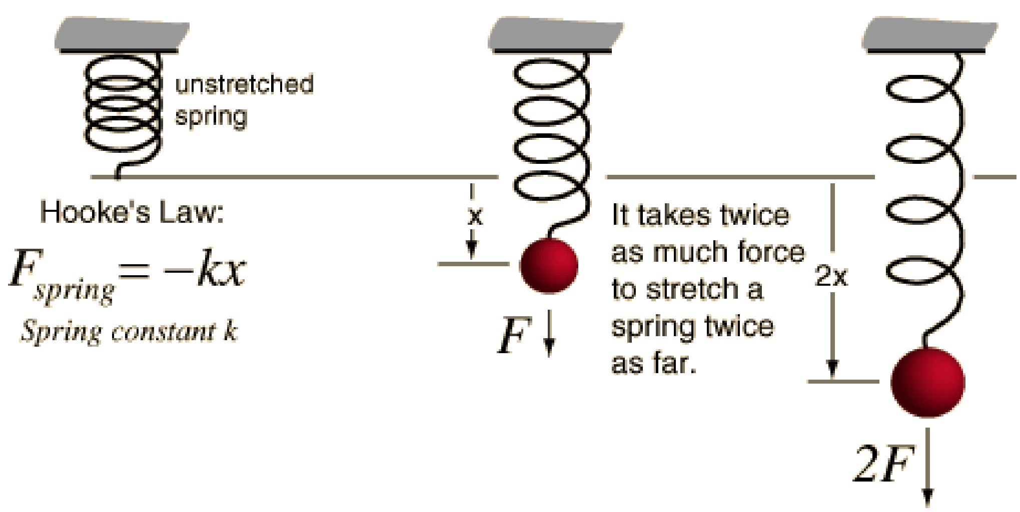


"Failure at any point in a body subjected to a state of stress begins only when the energy density absorbed at that point is equal to the energy density absorbed by the material when Subjected to elastic limit in a uniaxial stress state"

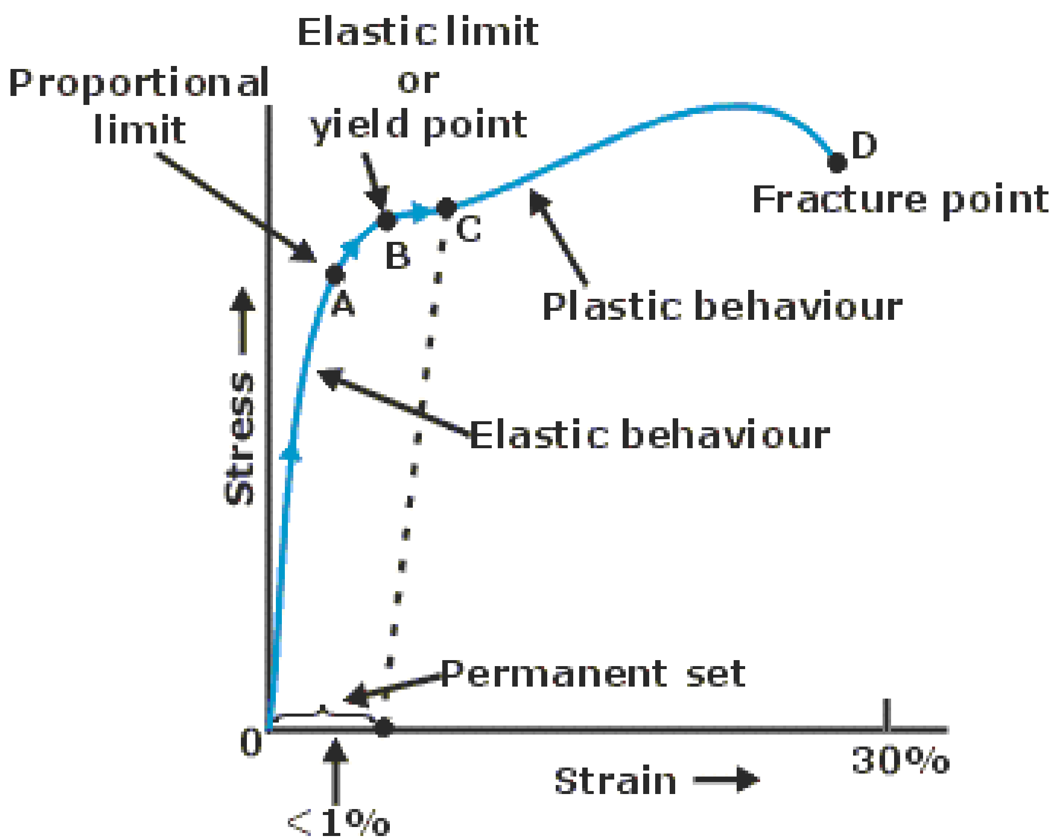
9.4 Definition of Hooke's Law.

For an Elastic object the strain is proportional to the stress producing it.





Hooke performed numerous experiments on various materials and discovered that the deformation, caused by the application of a tensile or compressive force on the object,was propotional to the load.He also discovered that the object returns to its original length (shape). If the load is removed and the elastic limit is not exceeded.The elastic limit is exceeded if a certain limit of stress is exceeded for an object, but if the limit is exceeded,the strain ceases to be proportional to the stress and when the stress is removed, it can happen that the object does not return to its original Length (shape). Thus the object has taken a ***" permanent set "***



A typical stress-strain curve for a ductile metal

9.5Elasticity:

Most materials used in engineering possess property of elasticity. Elasticity is the property which allows an object to regain its original shape and size when the load (force) producing a state of strain, is removed**.**

9.6 **Young's Modulus of Elasticity.**

Because strain is proportional to the stress which produces it, thus: $\frac{\textbf{Stress}}{\textbf{Strain}}$ = **a constant for each type of material.**

This constant is called Young's modulus of elasticity (or Young's modulus) and thus:

Young's modulus of elasticity, $\boldsymbol{E} = \frac{\boldsymbol{stress}}{\boldsymbol{Strain}}$

Unit for Young's modulus of elasticity is the ***N/mm²***, because as strain is only a ratio without a unit, the unit for Young's modulus of elasticity will be the same as the unit for stress.

Example:

A tensile force of 10 KN is applied to a round bar with a diameter of 20 mm and length of 2m.Youngs modulus of elasticity for this steel is 200 GPa.

Calculate: (i) the stress and

(ii) the extension due to the tensile force.

Solution:

Given: load (force) = 10 KN = 10 X 10^3 N

L_0 = 2m

D = *20* mm = 20 X 10^{-3} m

E = 200 GPa = 200 X 10^9 Pa

(I) **Stress** = $\frac{\boldsymbol{Load}}{\boldsymbol{area}}$

$$= \frac{\boldsymbol{load}}{\frac{\boldsymbol{\pi D^2}}{\boldsymbol{4}}}$$

$$= \frac{10 \times 10^3 \times 4}{\pi \times (20 \times 10^{-3})^2}$$

$$= \frac{10 \times 10^3 \times 4 \times 10^3 \times 10^3}{\pi \times 20 \times 20}$$

= 31, 8309 X 10^6 Pa

= 31, 8309MPa

(ii) $\boldsymbol{E} = \frac{\boldsymbol{Stress}}{\boldsymbol{Strain}}$

Strain $= \frac{\boldsymbol{changeinlength}}{\boldsymbol{originallength}} = \frac{\boldsymbol{Stress}}{\boldsymbol{E}}$

Change in length (or extension) $= \frac{\boldsymbol{stress}}{\boldsymbol{E}} \times \boldsymbol{originallength}$

$= \frac{31{,}8309 \times 10^6}{200 \times 10^9} \times 2$

$=$ 0.318 309 $\times$ 10^{-3}m (0,000 318 309 m)

= 0.318 309 m. *Ans*

Example:

a*) Describe the terms;*

i) ***limit of proportionality***
ii) ***factor of safety***
iii) ***ductility***

b) A mass of 400 Kg rests on top of a 2 m long vertical pipe. The pipe has an outside diameter of 100 mm and an inside diameter of 90 mm.

i) ***Calculate the load pressing down on the end of the pipe.***

ii) ***Calculate the stress in the material of the pipe in kN/ m^2. State whether this stress is tensile, compressive or shear.***

If the steel has a Young's Modulus of 200GN/$\mathbf{m}^2$calculates the strain in the pipe.

Solution:

Limit of Proportionality: *is the stress above which the material no longer obeys Hooke's Law; i.e the stress is no longer proportional to strain.*

Factor of Safety: is a value that reduces the Ultimate Tensile Stress to an allowable working stress. i.e***. working stress is the ultimate tensile stress divided by a factor of safety.***

Ductility: is the property that allows a metal to be drawn out easily into a wire form

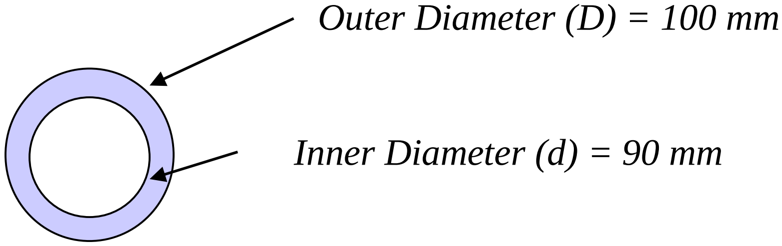
b) i) *Mass = 400 kg and g = 9.81 m/ s*^{2}

Load pressing down on the pipe (Force) = M x g

= 400 x 9.81 N

Load = 3.924 x 10^3 *N or 3.924 kN*

ii) *Cross Sectional Area = Outer Area – Inner Area*



$$C.S.A. = \frac{\pi D^2}{4} - \frac{\pi d^2}{4}$$

$$= \frac{\pi}{4}\left(100^2 - 90^2\right)x10^{-6}\, m^2$$

Area = 14.928 x 10^{-4} *m*^{2}

Stress = Load / C.S.A.

$$= \frac{3.924x10^3}{14.928x10^{-4}}\, N/m^2$$

$= 2628.6\ x\ 10^3\ N/m^2$ or $2628.6\ KN/m^2$

It is a Compressive Stress

iii) $E = \frac{stress}{strain}$

$$200\ \ 10^9 = \frac{2628.6\ x\ 10^3}{strain}$$

$$\text{‘} \quad strain = \frac{2628.6\ x\ 10^3}{200\ x\ 10^9}$$

$$= 1.3143\ x 10^{-5} \quad \textbf{Ans.}$$

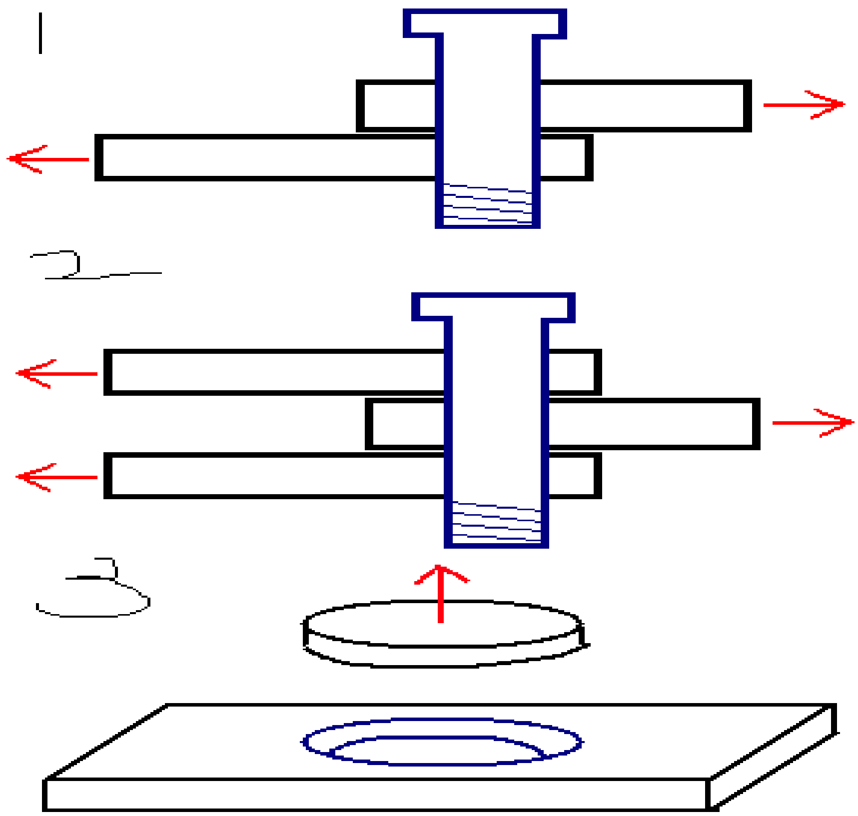
Self Exercises:

1. *A tensile force of 50 KN is applied to a round bar. The diameter of the bar is 50 mm and the original length is 3m.*
   Calculate: (i) the stress in the material;
   (ii) the strain if the final length of the bar is 3,005 m. (Ans. (i) 25,464 7 MPa; (ii) 1,666 666 $X10^{-3}$)
2. *A square steel section, with sides 100mm long, is subjected to a compressive force of 80 kn;*
   Calculate: (i) The stress in the material

(ii)The strain if young's modulus of elasticity is 200 GPa for the steel. (Ans. (i) 8 Mpa (ii) 1,666 666 $X10^{-3}$)

9.7 Double Shear:

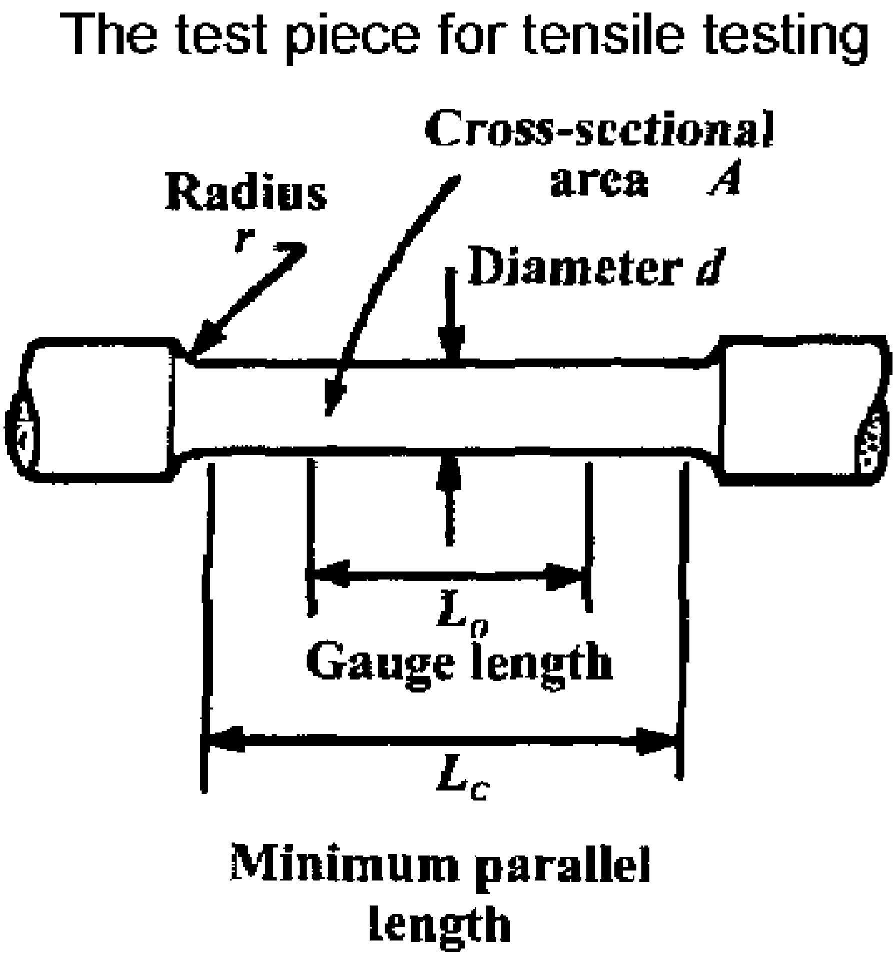
Double Shear is when two planes of area resist the shearing force as example on drawing.



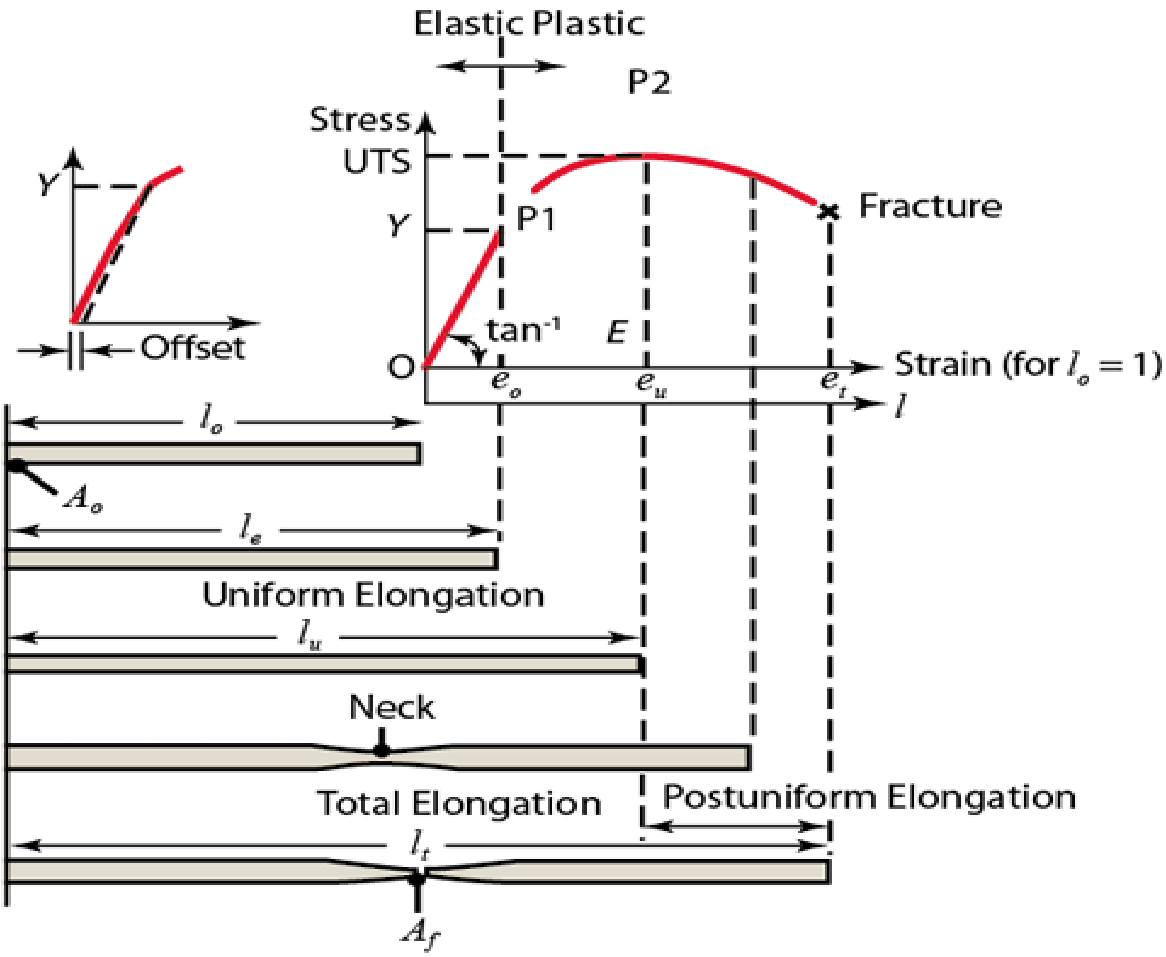
9.8Tensile Test to Destruction.

Destructive Testing

To find out how strong, resilient, flexible, or long-lived a material is often requires the ultimate sacrifice: the destruction of the sample by equipment and instruments designed to precisely measure its performance in the face of an overwhelming force.



Whether a sample is slowly pulled by tensile testing or subjected to sudden catastrophic impact, destructive tests can be the most economical and decisive ways to predict a material's capabilities and lifespan.



9.9 Ultimate tensile strength (UTS),

Often shortened to **tensile strength** (**TS**) or **ultimate strength**, is the maximum stress that a material can withstand while being stretched or pulled before failing or breaking. Tensile strength is the opposite of compressive strength and the values can be quite different.

Some materials will break sharply, without deforming, in what is called a brittle failure. Others, which are more ductile, including most metals, will stretch some - and for rods or bars, shrink or neck at the point of maximum stress as that area is stretched out.

The UTS is usually found by performing a tensile test and recording the stress versus strain; the highest point of the stress-strain curve is the UTS. It is an intensive property; therefore its value does not depend on the size of the test specimen. However, it is dependent on other factors, such as the preparation of the specimen, the presence or otherwise of surface defects, and the temperature of the test environment and material.

$$\textbf{U.T.S} = \frac{\textbf{Maximum breaking load}}{\textbf{Original cross-sect. area}}$$

Example:

The test piece of a diameter 10 mm and gauge length 50 mm, was made from the deposited metal of a welded joint. The tensile test on this specimen produced the following result.

Load at the yield point = 25.3 kn.

Maximum breaking load = 37.95 kn.

Length between gauge points after fracture = 63.5 mm

Diameter at the neck of waist after fracture = 7.5 mm

Calculate: (i) the stress at the yield point, (ii) ultimate tensile strength, (iii) percentage elongation, (iv) percentage contraction area.

Solution:

Original cross –sectional area = 0.7854 x 10^2

= 78.54 mm^2 = 78.54 x $10^{-6}m^2$

Yield stress $[N/m^2]$ = $\frac{\textbf{yield load [n]}}{\textbf{cross-sect.area } [\mathbf{m^2}]}$

$$= \frac{\mathbf{25.3 \times 10^3}}{\mathbf{78.54 \times 10^{-6}}}$$

$= 3.221 \times 10^8 \; N/m^2$

= 322.1 MN/m^2 **Ans (i)**

$$\textbf{U.T.S} = \frac{\textbf{Maximum breaking load}}{\textbf{Original cross-sect. area}}$$

$= \frac{\mathbf{37.95 \times 10^3}}{\mathbf{78.54 \times 10^{-6}}}$ $\quad = 4.832 \times 10^8$ N/m^2

= 483.2 MN/m^2 **Ans (ii)**

$$\% \text{ Elongation} = \frac{\text{Elongation}}{\text{original length}} \times 100$$

$$= \frac{\mathbf{63.5 - 50}}{\mathbf{50}} \times 100$$

= 27 %**Ans. (iii)**

$$\% \text{ Reduction in area} = \frac{\textbf{reduction in area}}{\textbf{original area}} \times 100$$

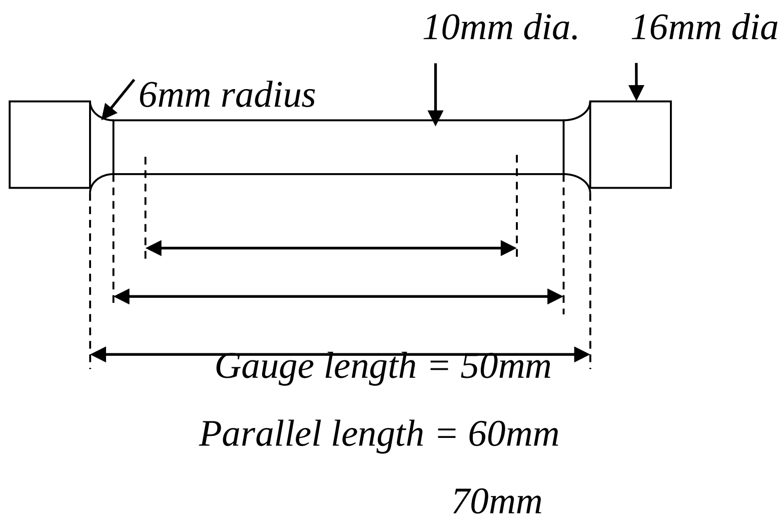
$$= \frac{\mathbf{0.7854 \times 10^2 - 0.7854 \times 7.5^2}}{\mathbf{0.7854 \times 10^2}}$$

$$= \frac{\mathbf{10^2 - 7.5^2}}{\mathbf{10^2}} = 100 = 43.75\ \%$$ **Ans.(iv)**

Self Exercises:

1 Example:

A test specimen of specific dimensions is placed between the jaws of a hydraulically operated testing machine. This specimen is then stretched and the load and extension are monitored till the specimen crosses the elastic limit and then continues through the plastic stage till it finally breaks.

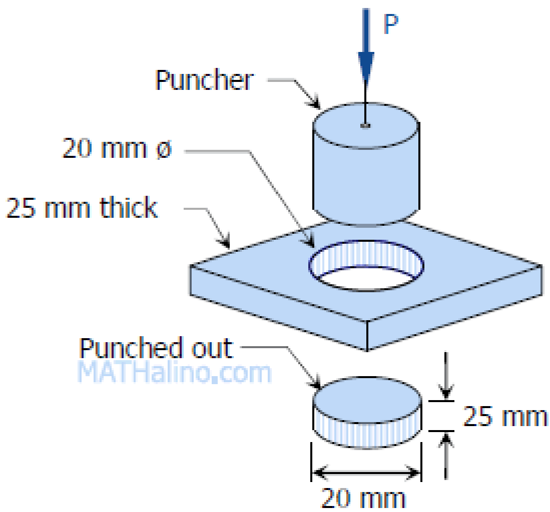


The test specimen shown above, is subjected to a destructive test. The yield point load was measured at 28kN. What is the yield stress?

Example 2:

A specimen of mild steel, 20mm diameter, is tested in a testing machine and snapped when the maximum pull on the specimen was 154 kn. Calculate the tensile strength of this material. (Ans. 490.1 MN/m^2)

3. Example:



A hole 20mm diameter is to be punched through a plate 25 mm thick. If the shear strength of the material is 300 N/mm², find the load required on the punch. **Solution:**

The curved surface area of a cylinder is circumference X height, therefore:

Area resisting load = circum.of plug x thickness

$$= \pi \times 20 \times 25\ mm^2$$

$$\text{Shear Strength} = \frac{\textbf{maximum shear load}}{\textbf{area resisting load}}$$

∴ Maximum load = shear strength x area

$$= 300 \times \pi \times 20 \times 18$$

$$\mathbf{= ?\ X\ 10^5\ N.}$$

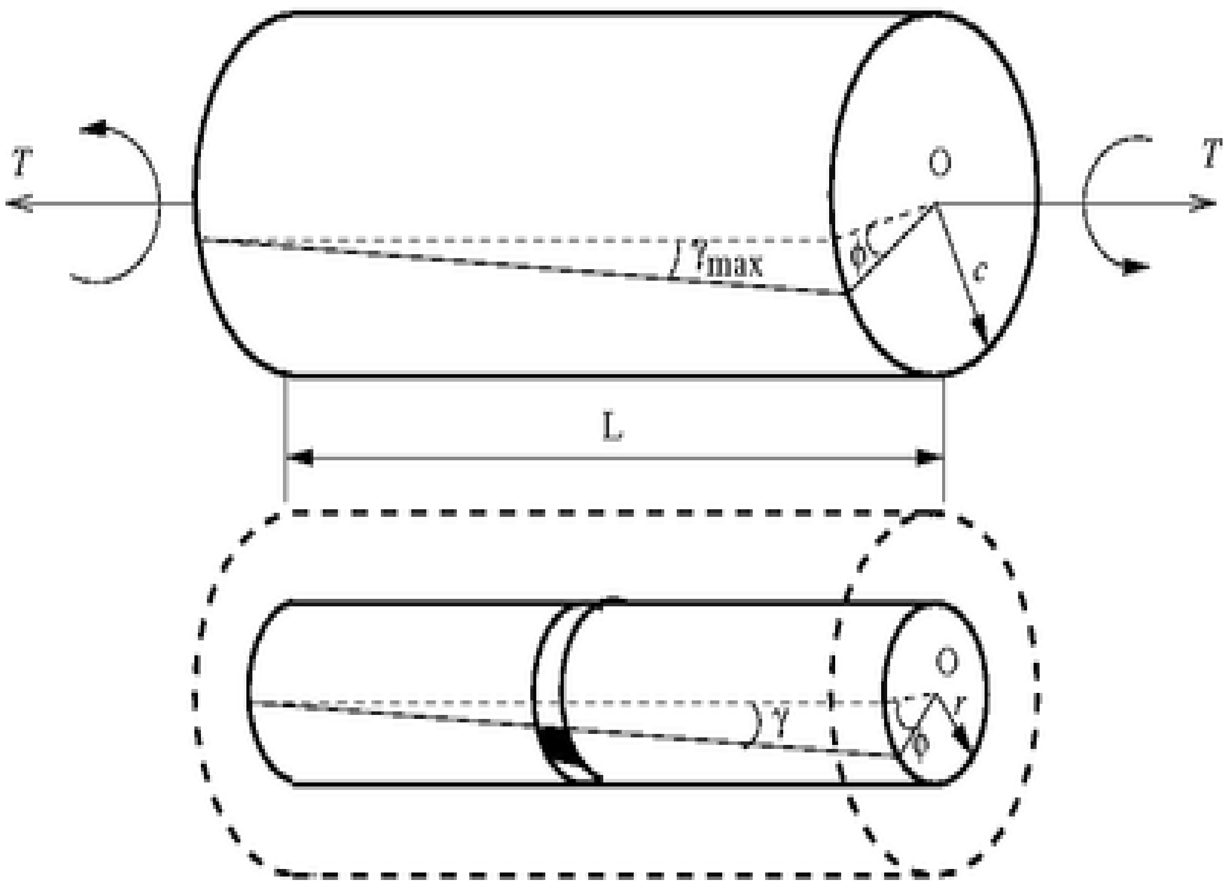
= ? Kn. Ans.

Chapter 10

Torsion

10.1 WHAT IS TORQUE?

***Torque* is a measure of how much a force acting on an object causes that object to rotate.** The object rotates about an axis, which we will call the **pivot point**, and will label 'O'. We will call the force '**F**'. The distance from the pivot point to the point where the force acts is called the **moment arm**, and is denoted by '**r**'. Note that this distance, '**r**', is also a vector, and points from the axis of rotation to the point where the force act



The fundamental Torsion Equation can be worked out by looking at the above drawing. Taking the values that are marked on the drawing we can consider a shaft of length ***L***twisted such as a result of the strain, a point on the circumference shifts from point 1 to point 2 when it is under twist. The amount of distortion is the arc length P_1 to P_2 and this depends on the length L under twist. As shear strain is expressed by the angle of distortion, in the drawing shown as ϒ or Ø, then:

Shear strain at outer fibers = $\frac{P_1 \; P_2}{L}$

Now; Let θ, = angle of twist at outer shaft, in radians

And r = radius from center to outer fibers

Then arc $P_1 P_2$ = angle in radians x radius = θr

Therefore, shear strain at outer fibers = $\frac{\theta \, r}{l}$

Note:

r is the radius to outer fibers. For a solid shaft this is simply the radius of the shaft and for a hollow shaft it is the *outer radius*, not the radius of the hole.

Also, θ is the angle of twist in *radians*, not in degrees?

To convert degrees to radians, divide by 57.3.

10.2 Relation between Torque and Stress

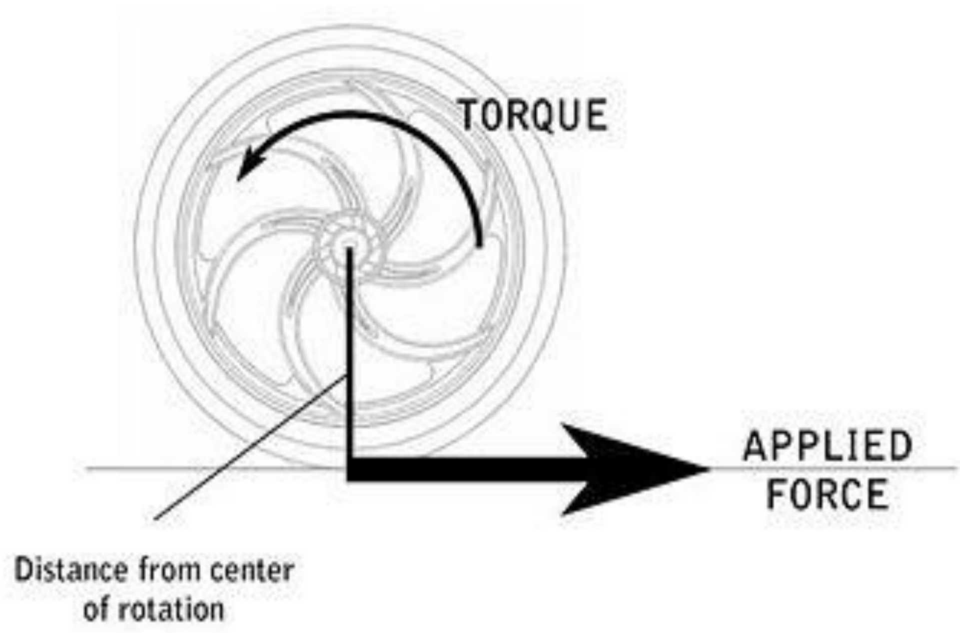
Torque for a Solid Shaft:

$$\boldsymbol{T = \frac{\pi \times D^3 \times q}{16}}$$

Torque for a hollow round Shaft,

$$\boldsymbol{T = \frac{\pi \; (D^4 - d^4) \, q}{16 \, D}}$$

10.3Relationship between Torque and Power.



Consider a steady force of ***F Newton*** applied at a leverage of ***L*** meters turning a shaft,

Work done to turn one revolution

$$= \text{force } [N] \times \text{distance } [m]$$

$$= F \times 2\pi L \text{ [Newton-meters = Joules]}$$

If the shaft is running at ***n*** revolutions per second then,

Work done per second = power

$$= F \times 2\pi L \times n \text{ [joules per second = watts]}$$

But, $F\ [N] \times L\ [m]$ = Torque [N m]

$$\therefore Power = 2\pi T n \quad [W]$$

When 2 πn is the angular velocity in radians per second = ω

$$\therefore Power\ [W] = Torque\ [Nm] \times \omega\ [rad/s]$$

$$Or,\ Power\ [kw] = Torque\ [knm] \times \omega$$

10.4 Maximum and Mean Torque

When transmitting power in a reciprocating engine the torque varies throughout the revolutions. This variation is based on the cycle operations within the cylinders and the number and arrangement of the crank. The shaft is designed to have sufficient strength to carry out the ***maximum*** torque. There for the ratio of Maximum and Mean Torque should be taken into account.

The terms Maximum Torque for a Solid & Hollow Shaft and are still:

$$T = \frac{\pi \times D^3 \times q}{16} \quad \textit{for a solid Shaft}$$

$$T = \frac{\pi\ (D^4 - d^4)\ q}{16\ D} \quad \textit{for a hollow Shaft}$$

∴ The ***Mean Torque*** will be

$$= \frac{power}{\omega}$$

$$\text{And} \quad \omega = radians\ (i.e\ \frac{rev/min\ \times 2\ \pi}{60})$$

Example: ***A solid shaft from a turbine is to transmit 2000 kW at 3000 rev/min. calculate the diameter of the shaft allowing a stress of 35 MN/m^2***.

Solution:

$$\frac{3000\ rev/\min \text{ x } 2\ \pi}{60} = 100\pi \text{ rad/s}$$

$$\text{Torque} = \frac{\pi \times D^3 \times q}{16} = \frac{power}{\omega}$$

$$\frac{\pi \times D \times 10^6 \quad \times 35}{16} = \frac{200 \times 10^3}{100 \times \pi}$$

$$D = \sqrt[3]{\frac{200 \times 10^3 \times 16}{\pi \times 35 \times 10^6 \times 100 \times \pi}}$$

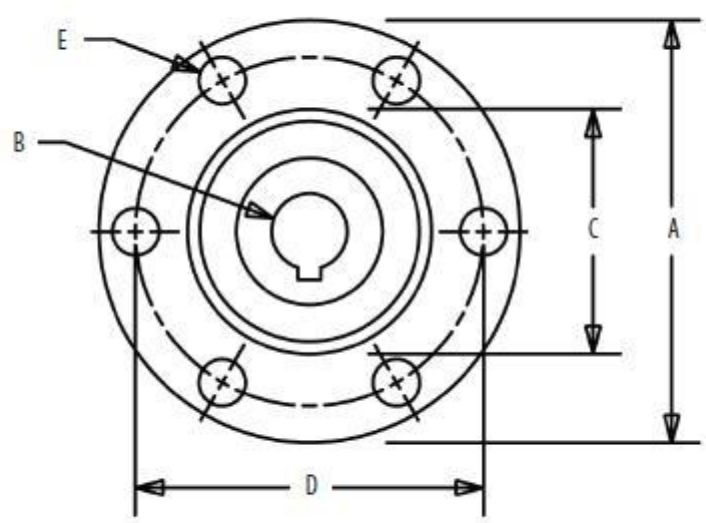
$$= 0.9747 \text{ m} = 97.47 \text{ mm } \textit{Ans.}$$

Self-Exercise:

1. *Calculate the maximum power that can be transmitted through a solid shaft 380 mm diameter when driven by a reciprocating engine at 110 rev/min if the stress is not to exceed 40 MN/m^2 and the ratio of maximum to mean torque is 1· 4 to 1.*(Ans. 3.547 MW or 3547 kW)
2. *When a solid shaft is transmitting 7.5 MW of power the stress in it is 50 MN/m^2 and the twisting moment is 750 knm. Calculate the diameter of the shaft and its speed in revolutions per minute. (Ans. 424.3mm(i) 95.48 rev/m(ii))*
3. *If a shaft 250 mm diameter can safely transmit 1860 kw of power when running at 125rev/min, calculate the power that can be safely transmitted by 375 mm diameter when running at 80 rev/min, assuming the shafts are of similar material. (ans. 4018 kW.)*

10.5 Transmission of Power through Coupling Bolts:

Bolts connecting shaft couplings carry the transmitted torque from one Shaft to the next. The shear stress induced in them will depend upon the magnitude of the stress and the sectional area of bolt material carrying the load and the radius of the circle on which the bolts are pitched.



Torque transmitted by each Bolt

= Load on one Bolt × Radius of the pitch circle

= *Stress in bolt × Cross-sect. area × radius of Pitch Circle.*

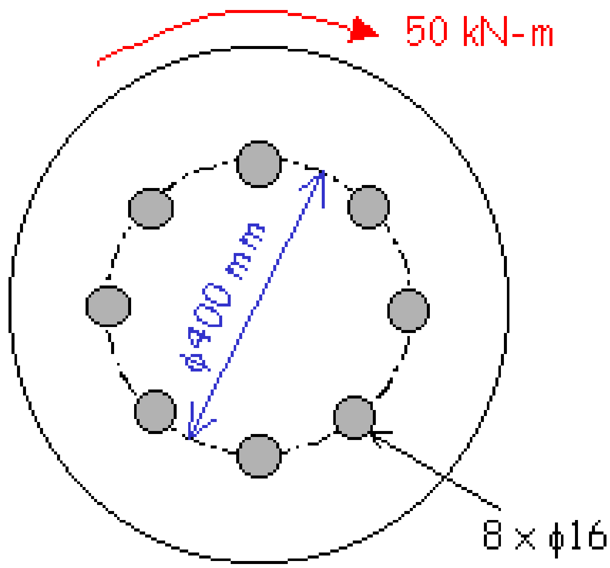
$$= \boldsymbol{f_s} \times \frac{\boldsymbol{\pi}}{\boldsymbol{4}} \boldsymbol{d^2} \times \boldsymbol{R} \times \boldsymbol{N} \ [\ \boldsymbol{Nm}]$$

Where: $\boldsymbol{f_s = shear\ stress\ in\ bolts}$

$$\frac{\boldsymbol{\pi}}{\boldsymbol{4}} \boldsymbol{d^2} = \boldsymbol{cross\ sect\ area\ of\ each\ bolt}$$

$\boldsymbol{R = radius\ of\ pitch\ circle}$ [meters]

$$\boldsymbol{N = number\ of\ bolts.}$$



Note :***Torque Transmitted by Bolts = Torque Transmitted by Shaft***

Example: ***The couplings of a 350 mm diameter solid shaft are 700 mm diameter and there are 8 bolts per coupling on a PCD of 525 mm. Under maximum working conditions the stress in the shaft is 42 MN/m^2. Find the diameter of the coupling bolts allowing a stress in them of 35 MN/m^2.***

Solution:

Torque transmitted by bolts = Torque Transmitted by Shaft.

$$\boldsymbol{f_s \times \frac{\pi}{4} d^2 \times R \times N \; [Nm] = \frac{\pi \times D^3 \times q}{16}}$$

$$35 \times 10^6 \times 0.2625 \times 8 = \frac{\pi}{16} \times 0.35^3 \times 42 \times 10^6$$

$$\therefore d = \sqrt{\frac{0.35^3 \times 42 \times 4}{35 \times 0.2625 \times 8 \times 16}}$$

= 0.07825 m = 78.25 mm. *Ans*

[Note: 0.2625 is the PCD dia. Of 525 and convert to Radius =0.2625]