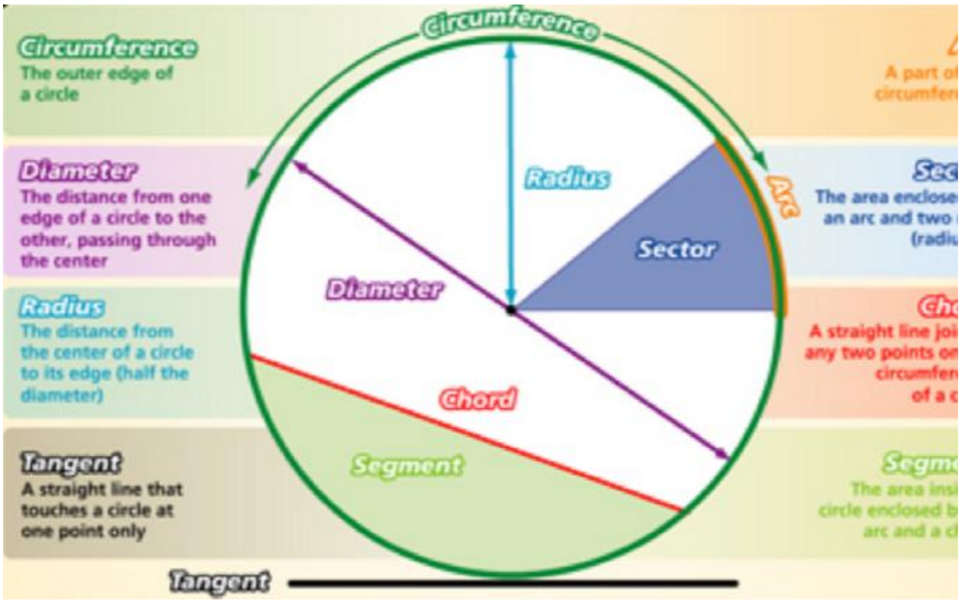
MODULE 4 : MEASUREMENT/GEOMETRY

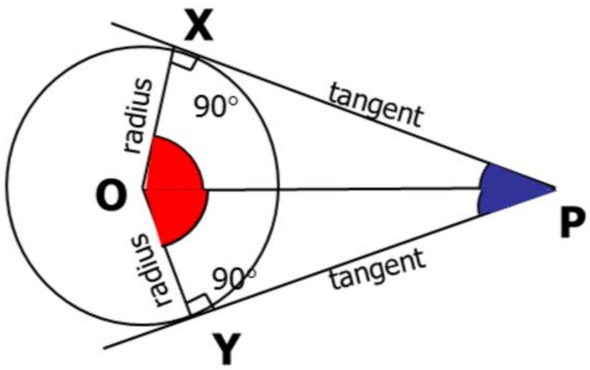
Properties of Circles

Terms

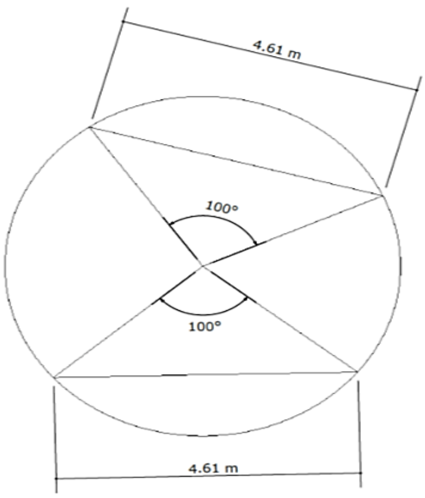
- Circumference
- Arc
- Radius, r
- Diameter, $d = 2r$
- Chord
- Sector
- Segment
- Tangent



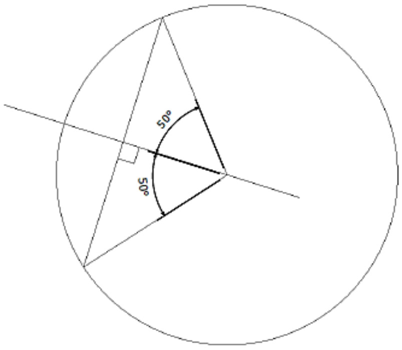
- Tangent- is at right angles to a radius from the tangent point of contact to the circle centre.
- Two tangents from a point to a circle are equal in length. i.e. length PX=PY



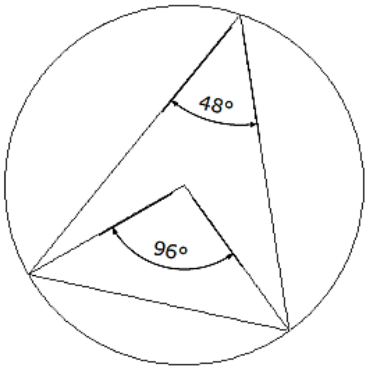
- Equal chords or arcs subtend equal angles at the centre of the same or equal circles.

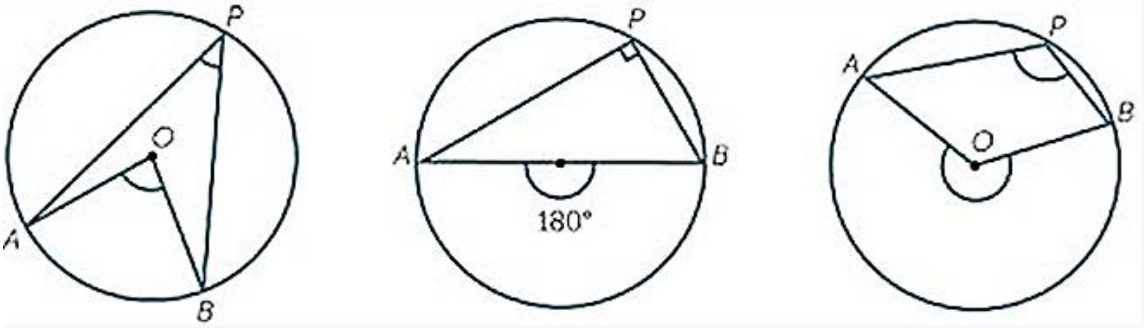


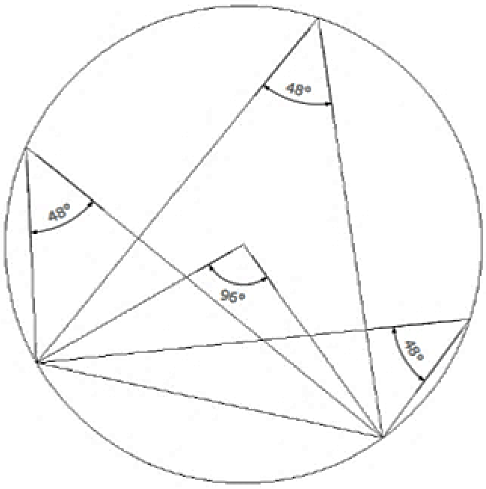
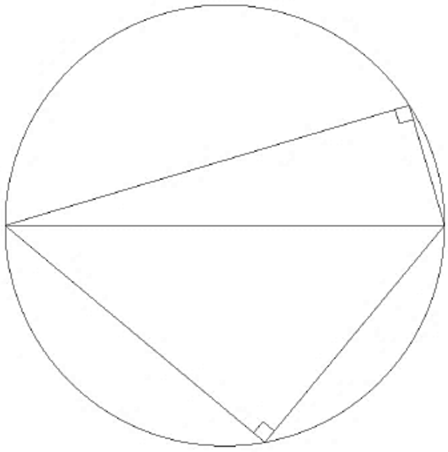
- The perpendicular bisector of a chord passes through the centre of the circle and bisects the angle at the centre.



- The **angle at the centre is twice the angle at the circumference** of a circle. i.e. If an angle at the circumference of a circle and an angle at the centre are subtended by the same arc (or chord), then the angle at the centre is twice the angle of the circumference.





• All angles in the same arc (or segment) of a circle are equal.	
The angle in a semicircle is a right angle.	

Circle Circumference

The circumference (or perimeter) of a circle is found using formula:

$2\pi r$ or πD where r is the radius and D is the diameter

Circle Area

The area of a circle is found using formula:

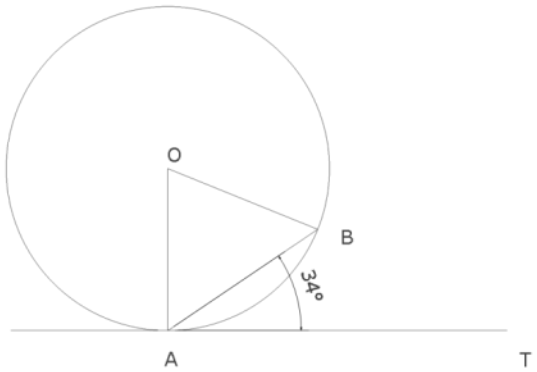
πr^2 or $\frac{\pi D^2}{4}$ where r is the radius and D is the diameter

Exercise 4.1: Simple Circle Geometry, Circumference, Area Problems

1. AT is a tangent to the circle centre 0.

   Angle BAT is 34°,

   Find angle AOB.

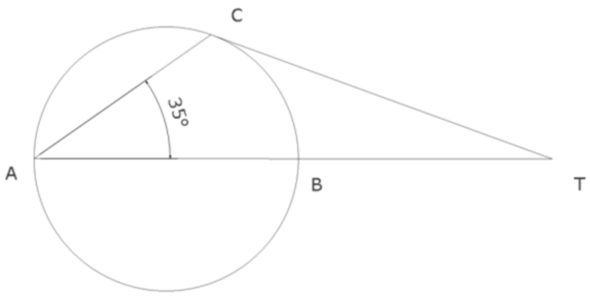


1. AB is a diameter of the circle,

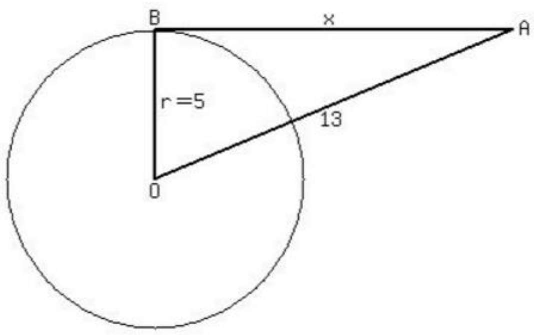
   CT is a tangent.

   Angle BAC is 35°.

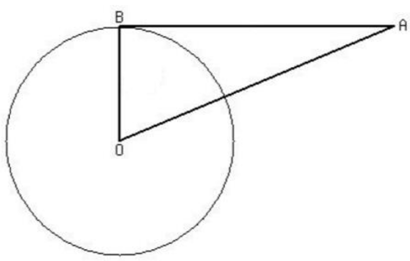
   Find angle CTB.



2. Two circles, centred at C and D, overlap so that their perimeters meet at A and B. DA and DB are tangents to the circle C. If AB = AC = 50 mm, find the radius of circle D.
4. Find the circumference of a circle of radius 12mm
5. If the diameter of a circle is 7.5cm, calculate its circumference.
6. Determine the radius of a circle of circumference 102cm
7. Determine the radius of a circle of Area $452cm^2$
8. Determine length, x, of this triangle



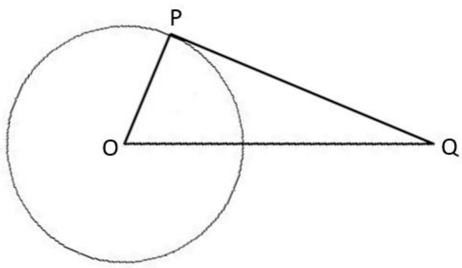
9. AB is a tangent to the circle at B. If the circle radius is 40mm and length AB=150mm, calculate length AO.



10. Find the area of a circular metal plate having diameter of 40mm

11. Find the area of a circle having a circumference of 50mm

12. A crank mechanism is shown in this diagram. PQ is a tangent to the circle at point P. If the circle radius OP is 10cm and length OQ is 40cm, determine the length of the connecting rod PQ



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13. Estimate the diameter of the earth (to the nearest 10km), given that the earth's circumference at the equator is 40,000 km.

Circumference and Area Problems

Sometimes problems involve portions (parts) of circles and may be solved using proportion.

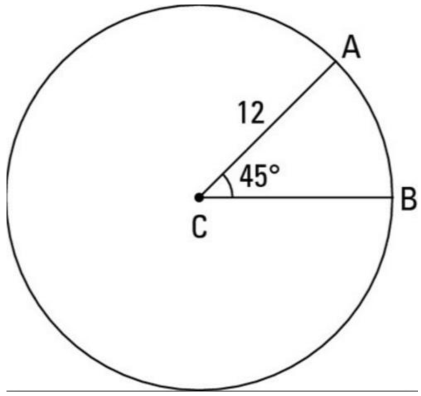
Worked Example 1

Find the length of the arc AB.

Step 1: Determine the proportion of the whole circle circumference represented by the arc. i.e. $\frac{arc\ angle}{360^\circ} = \frac{45^\circ}{360^\circ} = \frac{1}{8}$

Step 2: Apply the proportion calculated above to the full circle circumference.

i.e. $\frac{45}{360} \times 2 \times \pi \times 12 = 9.42$



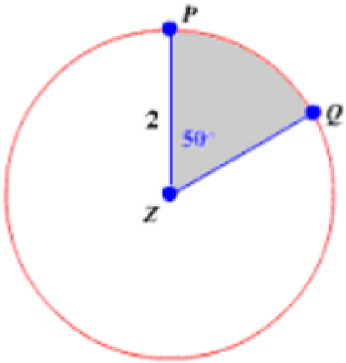
Worked Example 2

Calculate the area of this sector :

Sector Area $= \frac{50}{360} \times \pi r^2$

$= \frac{50}{360} \times \pi \times 2^2$

$= 1.75$ (2dp)

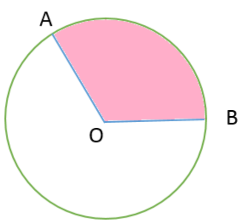


Exercise 4.2: Circumference and Area Problems

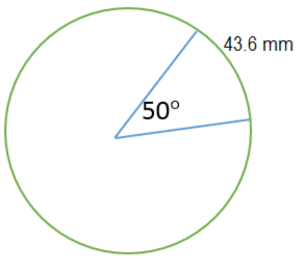
Length OA is 10cm. Angle AOB is 110°.

1. Find the length of the minor (shorter) arc AB.

2. Calculate the area of sector AOB

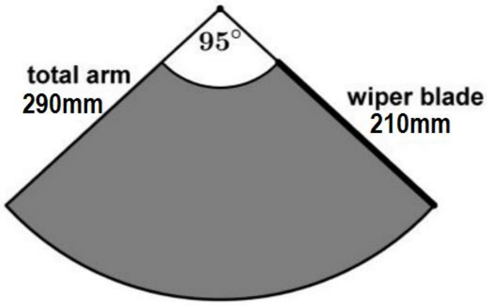


3. Determine the diameter and circumference of a circle if an arc of length 43.6cm subtends and angle of 50°

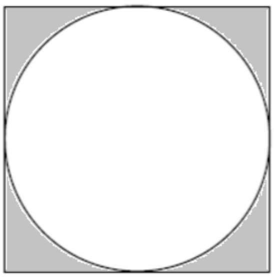


4. A football stadium floodlight can spread its illumination over an angle of $55°$ to a distance of 60 metres. Determine the maximum area that is floodlit.

5. Calculate the shaded area swiped by the windscreen wiper.



6. A circle fits exactly inside a square. The sides of the square are 120mm. Calculate the shaded area.



7. A circle, centre 0, has a chord AB which is 100mm in length. If the area of the triangle AOB is 1500 mm^2, find the area of the circle.

8. A right–angled isosceles triangle fits exactly inside a circle. If the two equal sides of the triangle are 40 mm in length, find the area of the circle.

Plane Shapes and Properties of Polygons

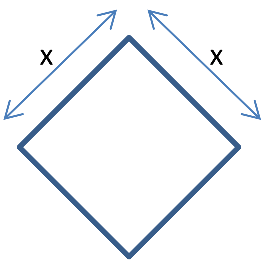
Definition of Centroid

The Centroid of a plane shape is its centre of mass, or balancing point (or the average position of all the points in the shape).

Plane Shapes

Square

Area = x^2

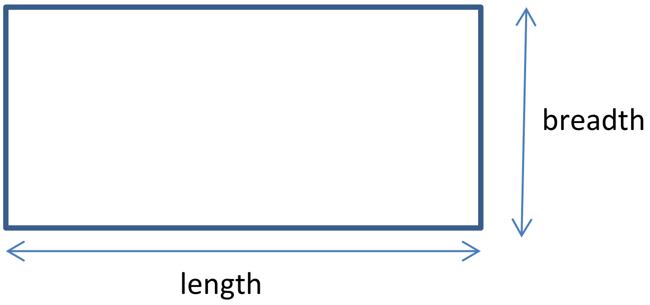


Centroid is at intersection of diagonals

Rectangle

Area = length x breadth

Centroid is at intersection of diagonals



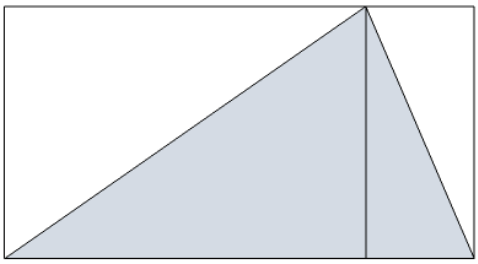
Triangle

Area of a triangle:

Area = ½ base x (vertical) height

Area = $\frac{1}{2}absinC$ or $\frac{1}{2}acsinB$ or $\frac{1}{2}bcsinA$
(if 2 sides and the included angle are known)

Area = $\sqrt{[s(s-a)(s-b)(s-c)]}\ where\ s = \frac{a+b+c}{2}$



Centroid is at the intersection of the triangle medians (a median is a line segment joining a vertex to the midpoint of the opposing side).

Parallelogram

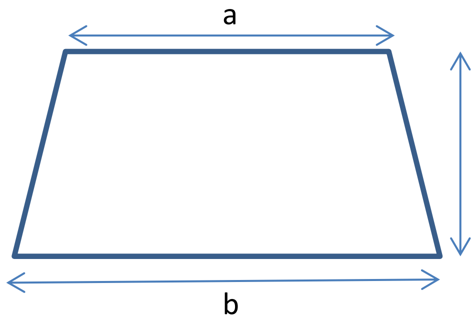
A quadrilateral (four-sided) figure in which opposite sides are parallel but adjacent sides are not at right angles as in the rectangle.

Area = base x (vertical) height

Trapezium

A quadrilateral [four-sided] figure in which two sides are parallel, but the other two are not.

Area = average of 2 parallel sides x height
Area= ½ (a + b) x vertical height.



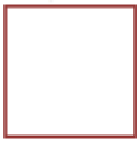
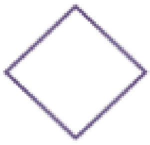
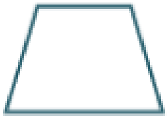
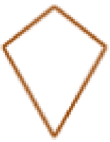
Moment of Area

When considering ship stability, a term in the calculation will be the moment of area about a certain axis, often the water level.

Moment of Area= Area x distance from the axis to the centroid. (Units m^3).

Properties of Quadrilaterals

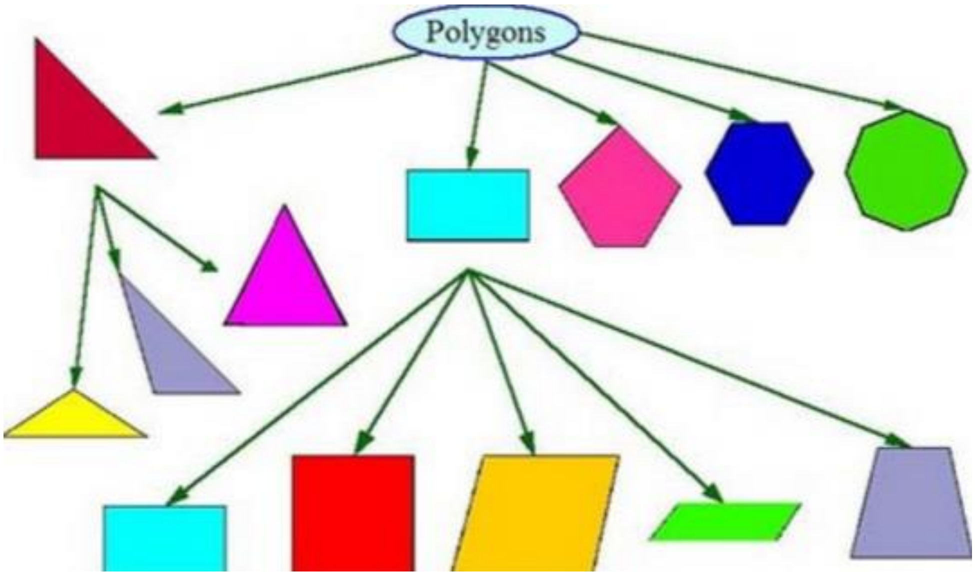
The term <u>Quadrilateral</u> describes '4-sided shapes'.

Type	Properties
Parallelogram 	• Opposite sides are equal and parallel • Opposite angles are equal
Rectangle 	• Opposite sides are equal and parallel • All angles are right angles (90°)
Square 	• Opposite sides are parallel • All sides are equal • All angles are right angles (90°)
Rhombus 	• Opposite sides are parallel • All sides are equal • Opposite angles are equal • Diagonals bisect each other at right angles (90°)
Trapezoid 	• One pair of opposite sides is parallel
Kite 	• Two pairs of adjacent sides are equal • One pair of opposite sides are equal • One diagonal bisects the other • Diagonals intersect at right angle (90°)

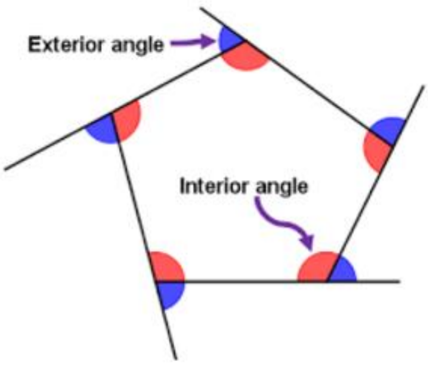
Properties of Polygons

The term Polygon describes 'many sided shapes'.

Polygons include the triangle, quadrilateral (e.g. rectangle, square, rhombus, kite, parallelogram, trapezium), pentagon, hexagon, heptagon (septagon), octagon, decagon.



1. The number of interior angles in a polygon is equal to the number of sides

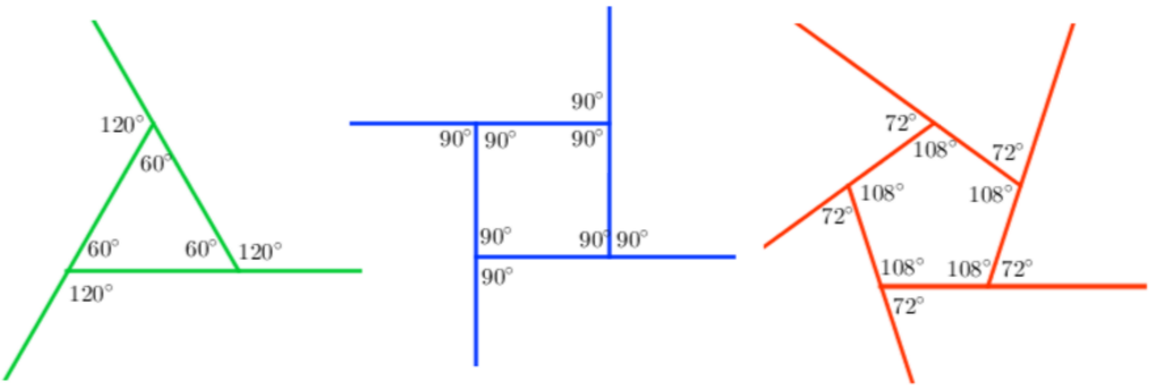


- The sum of the interior angles is found by subtracting 2 from the number of sides and multiplying by 180.i.e $(n-2)\times 180^\circ$

For example, the sum of the interior angles of a triangle is $(3-2)\times 180^\circ = 180^\circ$

the sum of the interior angles of an octagon is $(8-2)\times 180^\circ = 1080^\circ$

- The sum of the exterior angles of any polygon is 360°



Exercise 4.3: Plane Shape Area Problems

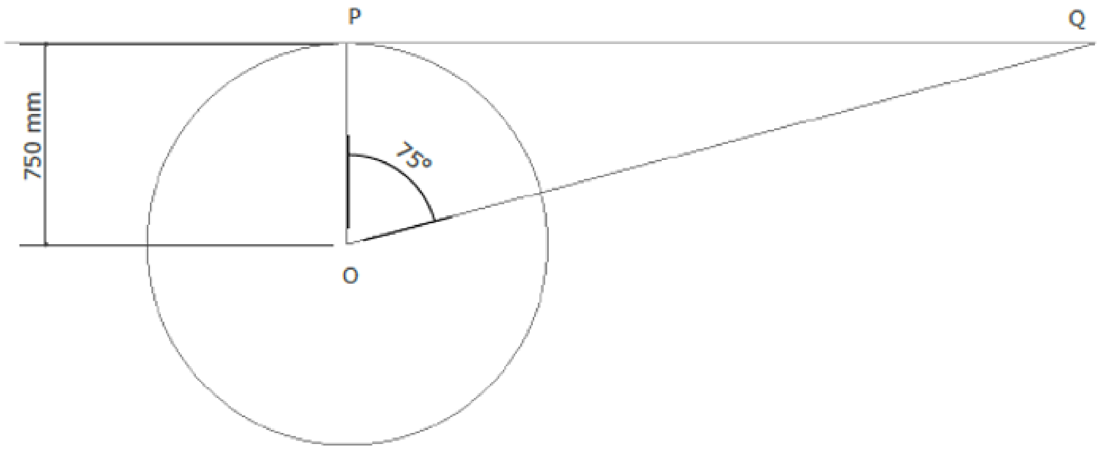
1. A circle has an equal area to a triangle which measures 6 cm by 7 cm by 7 cm. What is the radius of the circle?

2. A square has an area of $30m^2$. A circle has a circumference equal to the perimeter of the square. Find the length of the sides of the square, the perimeter of the square and the radius of the circle.

3. A ship has a radar set with a 6 mile range scale and a 12 mile range scale. What is the difference in the areas covered by these two range scales?

4. The diagonals of a quadrilateral intersect at right angles, and are 10cm and 12cm long respectively. Find the area of the quadrilateral.

5. In the diagram radius OP = 750 mm. Angle POQ= 75°. Calculate the area of the triangle that lies outside of the circle.



6. Find the area of triangle ABC if AB = 45 cm, BC = 60 cm and angle ABC = 65°.

Solid Shapes

Definition of Surface Area

The surface area of a shape is simply the sum of all the areas which make up the total surface of that shape. (Consider all surfaces that might be painted).

Definition of Volume

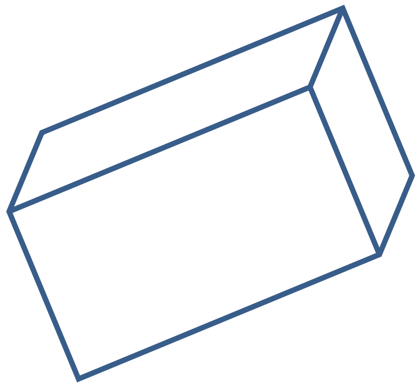
The volume of a shape can be thought of as the space it takes up. For regular prism shapes (e.g. cylinders, rectangular prisms, cuboids), the volume is the product of the area of a face and length.

Cuboid

Area of one face = b x h

Volume = b x h x l   where l = length.

Centre of volume/gravity is the point at which all the mass of an object can be considered to act. It is the point of intersection of diagonals running from a top corner to the opposite bottom corner. Alternatively, the centre of area of one face, projected along to mid-length.



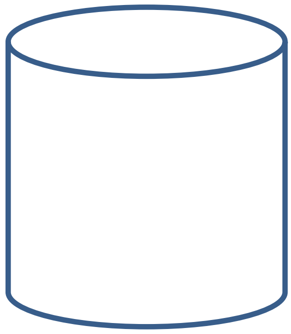
Cylinder

Area of circular face = πr^2

Volume = πr^2 x l   where l = length.

Curved surface area = 2πr x l

The Centre of volume is the centre of the circular face, projected along to mid-length.

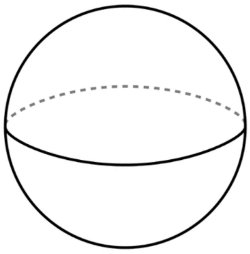


Sphere

Surface Area= $4\pi r^2$

Volume = $\frac{4}{3}\pi r^3$

The Centre of volume is the centre of the sphere.



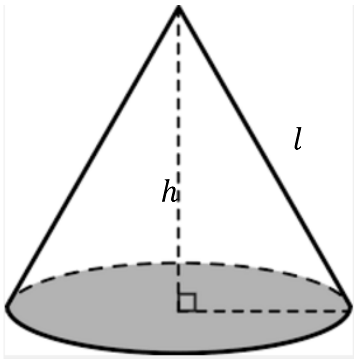
Cone

A cone is a specific case of a pyramid with a circular base.

Curved surface area of a cone = $\pi r l$ where l is the slant length, (NOT the vertical height).

Volume=$\frac{1}{3} \times base\ area \times vertical\ height$

i.e. Volume=$\frac{1}{3}\pi r^2 h$ where h is the perpendicular height.



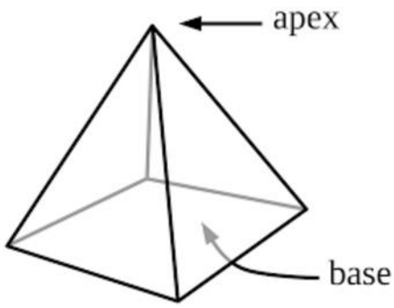
Pyramid (regular square pyramid)

A pyramid can be considered as a set of triangles joined together at their apex.

Surface Area=sum of the area of all triangles plus area of base.

Volume=$\frac{1}{3} \times base\ area \times vertical\ height$

The Centre of volume is one-third of vertical height from the centre of the base.



Exercise 4.4

1. A tank 4 metres long, 3 metres wide and 3 metres deep is filled to the top with water. A solid sphere 2 metres in diameter, is then totally immersed in the water causing some of the water to spill. How many cubic metres of water remain in the tank?
2. A ship's day shapes for the aground signal (three black balls) are to be painted. If the balls each have a diameter of 0.6 metres, what is the total surface area to be painted?

3. A cone has a volume of 300 cubic metres and a height of 10 metres. Find its surface area.
4. A cylindrical tank, diameter 10 metres and height 10 metres, is full of water. If this water is then transferred to a new rectangular prismatic tank measuring 10 metres by 10 metres by 10 metres, what percentage of the new tank remains unfilled?
5. A pyramid has a square base, each side of which is 6 metres. If the total surface area of the pyramid is 80 square metres, what is the slant height, measured from the apex to the centre of the side of the base?
6. A conical buoy and a cylindrical buoy both have bases of radius 2 metres. If they are both to have the same surface area, and the cylindrical buoy is 2 metres high, what is the slant height of the conical buoy?
7. A cone has a volume of 48m3. If its perpendicular height is 7 metres, what is the radius of its base?
8. A cylindrical tank is 10 metres high. What is its diameter if a box–shaped tank of the same capacity measures 7 metres by 6 metres by 3 metres?

(Extension optional questions)

9. a) Find the volume of a double bottom tank which is 20m long , 12 m wide and 1.2 metres deep.

   b) Calculate the tonnes of salt water (Density 1.025 t/m^3) if it is filled to a depth of 1.00 metres.

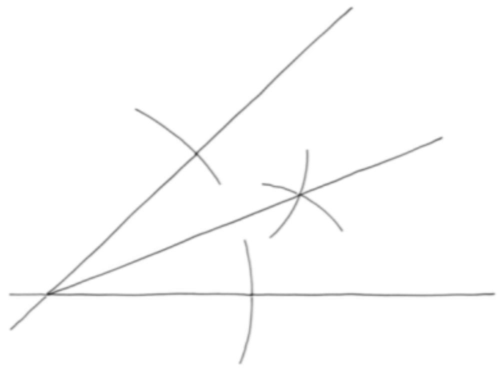
   c) Find the time to discharge it using a single piston pump (double acting) with piston diameter of 300 mm, stroke of 450 mm and running at 100 strokes per minute.
10. a) An oil drum has a capacity of 220 litres. If it has a diameter of 600 mm find its height.

    b) Find its weight if 45% full of oil of density 0.80 t/m^3
11. A triangular prism shaped piece of wood has a cross section of an equilateral triangle of sides 150 mm and is 1500 mm long. If its density is 0.45 t/m^3 find its mass in kg.

Constructions

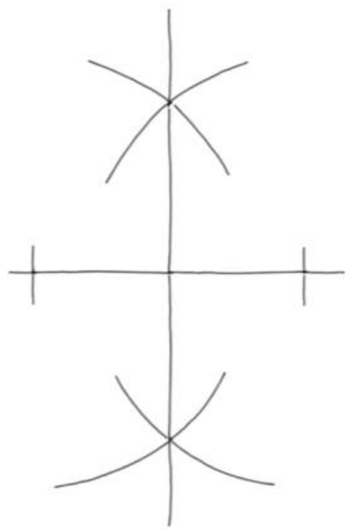
Bisecting a given angle

- Placing a compass at the apex, scribe two construction points on the two legs of the angle.
- From the two construction points, use the compass to scribe arcs to intersect.
- Draw a line from the apex through the intersection of the arcs.
- It is important to ensure that each pair of construction arcs are identical lengths.



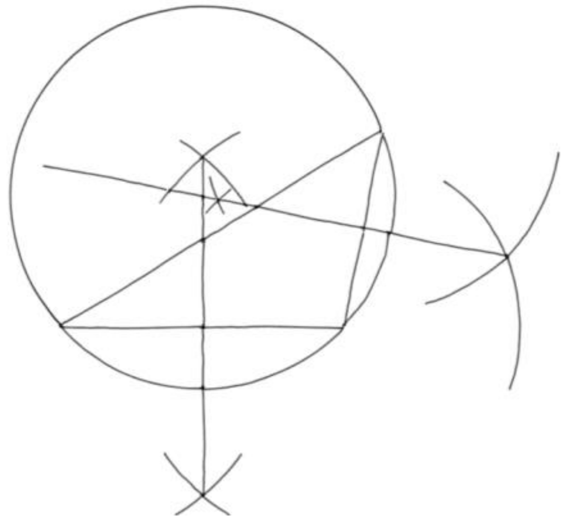
Bisect a given straight line

- From each end of the line, scribe two construction arcs above and below the line.
- Draw a line through both arc intersections.
- All four construction arcs must be the same length.



Drawing a circle through 3 given points
(circumcircle and triangle circumcentre)

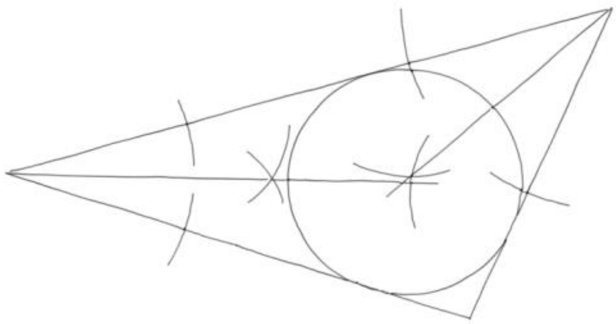
- Bisect any two sides of the triangle.
- The intersection of the two bisectors is the centre of a circle which will inscribe the three points of the triangle.



Inscribe a circle inside a triangle

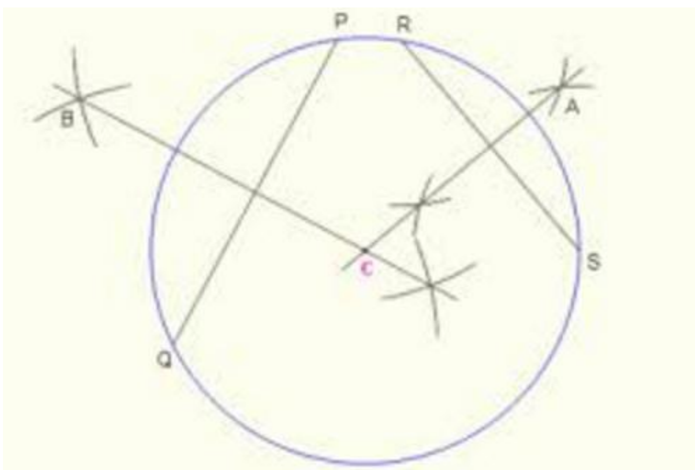
(in-circle and triangle in-centre)

- Bisect any two angles of the triangle.
- The intersection of the two angle bisectors is the centre of a circle which will touch the three sides tangentially.



Find the centre of a circle

- Draw two chords approximately at right angles (PQ and RS)
- Construct perpendicular bisectors for each chord.
- The cetre of the circle is at the intersection of the two perpendicular bisectors.



Exercise 4.5: Constructions Activity

1. Trace around a circular object (e.g. a cup) on a blank (unlined) sheet of paper. Use construction methods to determine the centre of this circle.
2. Draw a triangle on a blank (unlined) sheet of paper. Construct a median for each side of this triangle.
3. Draw a triangle on a blank (unlined) sheet of paper. Construct a perpendicular bisector for 2 sides of this triangle and determine the point of intersection of these.
4. Construct a triangle with sides 50mm, 60mm and 70mm.

ANSWERS TO EXERCISES

Exercise 4.1

1. 68°
2. 20°
3. 28.9 mm
4. 75.4 mm
5. 23.6 cm (1dp.)
6. 16.2 cm
7. 12 cm
8. 12 cm
9. 155.2 mm
10. 1256.7 mm^2
11. 198.9 mm
12. 38.7 cm
13. 12730 km

Exercise 4.2

1. 19.2 cm
2. $96.0cm^2$
3. Diameter=100mm; Circumference= 313.9mm
4. $1727.9m^2$
5. $64416\ mm^2\quad(0\ dp)$
6. $3090.3mm^2$
7. $10681.4mm^2$
8. $2513.2mm^2$

Exercise 4.3

1. $2.5cm$
2. square side = 5.48 cm; square perimeter= 21.91 cm; circle radius=3.49 cm
3. $339.3\ miles^2$
4. $60\ cm^2$
5. $area\ triangle = 1,049,639\ mm^2;$

   $area\ segment = 368,155\ mm^2;$

$$difference = 681,484\, mm^2$$

6. $Area = 1223.5cm^2$

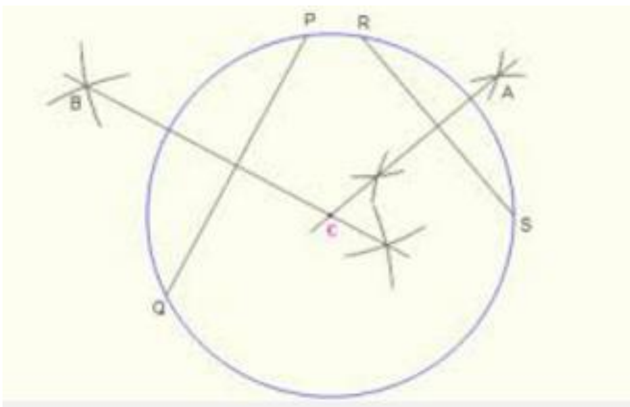
Exercise 4.4

1. 31.811m^3
2. 3.393m^2
3. 280.721m^2
4. 21.5%
5. 3.667m
6. 6m
7. 2.559m
8. 4.005m
9. a) 288m^3
   b) 246 tonnes
   c) Vol pump 0.0318m^3, Discharge 3.18 m^3/min, Time =75.48 nibs or 1h 15m 28s
10. a) 778mm
    b) 79.2 kg

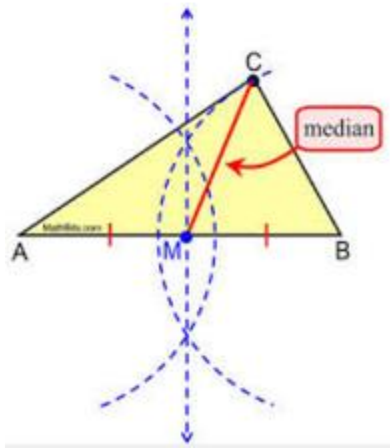
11. h = 0.13m,
Area end = 0.0098m^2
Vol = 0.0146 m^3
mass = 6.58 kg (weight 64.56kN)

Exercise 4.5

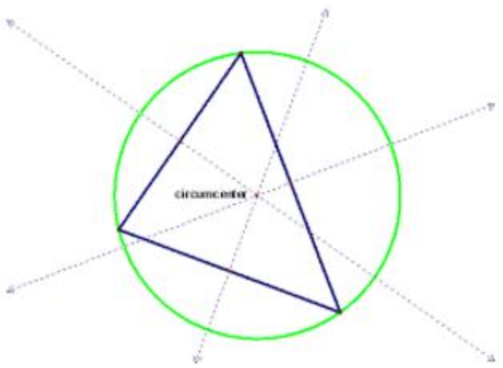
1.

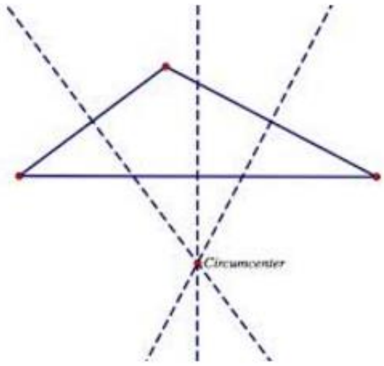


2.



3. Answers will vary. Examples may look like the following:


circumcenter


Circumcenter