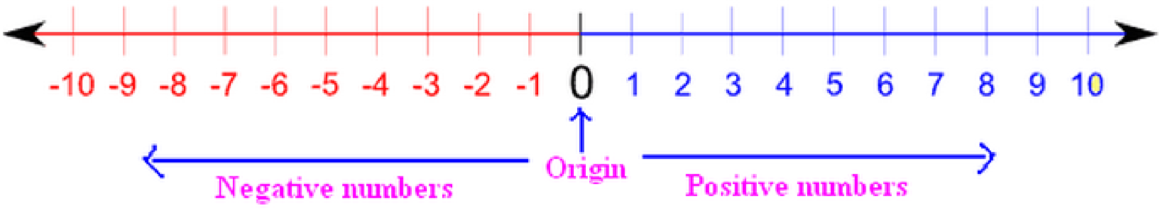
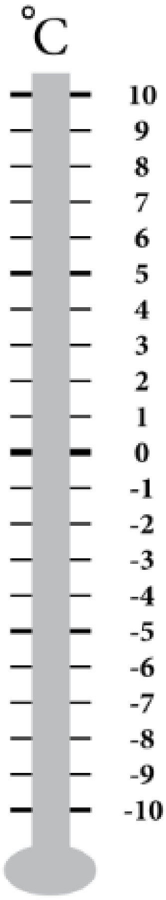
MODULE 1: BASIC NUMBER SKILLS

Basic Number Operations and Calculations

Positive and Negative Numbers

Positive numbers are above zero (e.g. +3 or +7). Negative numbers are below zero (e.g. -5 or -1). When adding or subtracting, simply start from the first number in the equation (on the LHS, left hand side), and move right (along a number line) if adding the next number, or left if subtracting the next number





Example:

4 – 7 = -3 (start at 4, move left to subtract 7)

-8 + 12 = 4 (start at -8, move right adding 12)

-5 - 3 = -8 (start at -5, move left subtracting 3)

However, note that a double negative between the numbers in a calculation (in the gap between the numbers) is replaced by a positive.

Example:

1 – (-7) = 1 + 7 = 8

-10-(-2) = -10 + 2 = -8

When multiplying or dividing two numbers, numbers of the same sign, whether positive or negative, give a positive answer.

Example:

14 x 53 = 742

(-14) x (-53) = 742

(-8) ÷ (-2) = 4

Numbers of opposite sign give a negative answer.

Example:

14 x (-53) = -742

8 ÷ (-2) = -4

Note: when multiplying or dividing more than two numbers, if the total number of like signs is even, (whether positive or negative), the answer will be positive.

Example:

2 x (-3) x (-5) = 30

$2 \times (-3) \times 5 = -30$

Order of Operations

Calculations involving a number of operations are always evaluated in a specific order as follows (known as BEDMAS or BIDMAS or BODMAS):

Brackets
Exponents (**I**ndices , P**o**wers)
Division
Multiplication
Addition
Subtraction

Example: Evaluate $5 - 3 \times 4 + 24 \div (3+5) - 4^2$
$= 5 - 3 \times 4 + 24 \div 8 - 4^2$ (evaluating the **B**racket and replacing (3+5) with 8)
$= 5 - 3 \times 4 + 24 \div 8 - 16$ (evaluating the **E**xponent term and replacing 4^2 with 16)
$= 5 - 12 + 3 - 16$ (evaluating the **M**ultiplication and **D**ivision terms starting from LHS)
$= -20$ (evaluating the **A**ddition and **S**ubtraction terms starting from LHS)

Rules of Brackets and Operators

The order of numbers when adding does not matter. The order of numbers when multiplying does not matter (Commutative Law)	2+3=3+2=5 2x3=3x2=6
The use of brackets when adding does not affect the result. The use of brackets when multiplying does not affect the result. (Associative Law)	2+(3+4)=(2+3)+4 =9 2x(3x4)=(2x3)x4 =24
A number placed outside of a bracket indicates the whole contents of the bracket must be multiplied by the number. (Distributive Law)	2x(3+4)=2(3+4) =2x3+2x4 =6+8 =14
Adjacent brackets indicate multiplication.	(2+3)x(4+5)=5x9 =45
When an expression contains nested brackets (inner and outer brackets), the inner brackets are removed (evaluated) first.	

Number Exercise 1.1

Evaluate the following:

1. -6-6+2
2. 5+3-4-5
3. -8+55.1-23.8
4. -32.7+6.89
5. –(-52)
6. -12.5-(-5.21)
7. -5×-23
8. $-3 \div -9$
9. $-72 \times -4.44 \div 2.054$
10. $3.51 \times -2.08 \div -0.99$
11. 22+5(6-22)
12. 13-(-12.5+5.21)$\times$ 7
13. 106.43-(-1.05-0.02)$\times$ 9.9
14. 0.6-8.85$\times$ $(13.08 + 0.64) \div 0.2$
15. 2.82+-5.54(6.8-13.2)
16. $23 + 6 \times (0.8 - 21.8 - 0.22) + 0.54$
17. $\frac{3+\sqrt{(5^2-3^2)+2^3}}{1+(4\times6)\div(3\times4)}$

Fractions

A mark out of 24 in an exam may be written as $\frac{17}{24}$ or 17 parts out of 24. This is an example of a fraction, and consists of a denominator (number below the line) and numerator (the number above the line). The line is a fraction bar (or division sign ÷).

When the value of the numerator is less than the value of the denominator the fraction is called a **proper fraction**. e.g. $\frac{17}{24}$

When the value of the numerator is greater than the value of the denominator, the fraction is called an **improper fraction**. e.g. $\frac{7}{4}$ (is an example of an improper fraction).

A **mixed number** is a combination of a whole number and a fraction. e.g. $1\frac{3}{4}$

Note: $mixed\ number\ 1\ \frac{3}{4}\ is\ equivalent\ to\ \frac{7}{4} and\ is\ equivalent\ to \frac{4}{4} + \frac{3}{4}$

Ensure you are familiar with using the fraction key(s) on your calculator as many calculations can be evaluated directly using this functionality. Fraction keys vary from calculator to calculator. e.g. Casio fx-82AU…..  

e.g. Casio fx-82MS… 

Fractions representing the same quantity are called **equivalent fractions.** e.g. $\frac{3}{4} = \frac{6}{8} = \frac{24}{32}$

Equivalent fractions may be formed by dividing the numerator and denominator by the same number.

e.g. the numerator and denominator of $\frac{24}{32}$ can both divided by 8, so $\frac{24}{32} = \frac{3}{4}$

Simplifying a fraction means cancelling it to its smallest possible equivalent form.

e.g. the **simplest form** of fraction $\frac{24}{32}$ is $\frac{3}{4}$

Multiplying Fractions

To multiply two or more fractions, multiply the numerators together to give a new single numerator, and multiple the denominators together to give a new single denominator.

Example: $\frac{1}{2} \times \frac{2}{3} \times \frac{4}{5}$

$$= \frac{1 \times 2 \times 4}{2 \times 3 \times 5}$$

$$= \frac{8}{30} = \frac{4}{15}$$

Dividing Fractions

To divide one fraction by another, invert the second fraction and multiply by the first.

Example 1: $\frac{1}{2} \div \frac{4}{5}$ $= \frac{1}{2} \times \frac{5}{4}$ $= \frac{1 \times 5}{2 \times 4} = \frac{5}{8}$	Example 2: $3\frac{1}{2} \times \frac{4}{5} \div 2\frac{3}{4}$ $= \frac{7}{2} \times \frac{4}{5} \div \frac{11}{4}$ $= \frac{7}{2} \times \frac{4}{5} \times \frac{4}{11}$ $= \frac{7 \times 4 \times 4}{2 \times 5 \times 11}$ $= \frac{112}{110} = \frac{56}{55}\, or\ 1\frac{1}{55}$

Adding and Subtracting Fractions

When adding (or subtracting) fractions with the same denominators the numerators are added (or subtracted) and the denominator of the answer is the same as the original denominators.

Example: $\frac{2}{5}+\frac{1}{5}=\frac{3}{5}$

If the denominators of the fractions are different, then the fractions must first be changed into equivalent fractions with the same denominator. Then proceed as before by adding numerators and keeping common denominator in the answer.

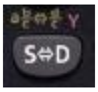
Example 1: $\frac{2}{3}+\frac{1}{6}$ $=\frac{4}{6}+\frac{1}{6}$ $=\frac{5}{6}$ (note: lowest common multiple LCM of denominators is 6)	Example 2: $\frac{3}{4}-\frac{5}{12}$ $=\frac{9}{12}-\frac{5}{12}$ $=\frac{4}{12}=\frac{1}{3}$ (always simplify answer)	Example 3: $\frac{6}{27}-\frac{2}{3}+\frac{5}{9}$ $=\frac{6}{27}-\frac{18}{27}+\frac{15}{27}$ $=\frac{(6-18+15)}{27}$ $=\frac{3}{27}=\frac{1}{9}$

Number Exercise 1.2 Fractions

1. $\frac{7}{8}\times\frac{3}{4}$
2. $\frac{2}{3}\div\frac{9}{12}$
3. $\frac{3}{8}\times\frac{3}{7}\times\frac{11}{12}$
4. $\frac{2}{7}\times\frac{3}{8}\times\frac{1}{4}$
5. $\frac{9}{12}\times\frac{3}{9}\times\frac{5}{12}$
6. $\frac{4}{9}\times\frac{3}{8}\div\frac{3}{7}$
7. $\frac{9}{6}\div\frac{7}{9}\times\frac{1}{2}$
8. $\frac{5}{9}\times\frac{7}{12}\div\frac{3}{4}$
9. $\frac{12}{6}\div\frac{81}{18}$
10. $\frac{18}{42}\div\frac{16}{20}$
11. $\frac{2}{6}\times\frac{108}{81}$
12. $\frac{26}{52}\div\frac{13}{104}$
13. $\frac{4}{9}+\frac{2}{3}$
14. $\frac{8}{11}+\frac{3}{4}$
15. $\frac{6}{7}-\frac{2}{3}$
16. $\frac{7}{10}+\frac{7}{9}$
17. $5\frac{2}{3}+7\frac{5}{12}$
18. $2\frac{8}{9}-2\frac{7}{12}$
19. $6\frac{2}{7}+3\frac{5}{10}$
20. $8\frac{2}{5}\times2\frac{7}{8}$

Converting between Fractions and Decimals

Recall that every fraction can also be expressed in decimal form. Ensure that you are familiar with the calculator key used for converting between fraction and decimal form.

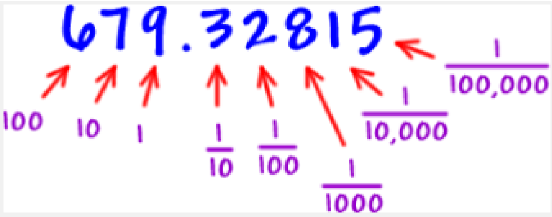
Use the 

key on Casio fx-82AU to convert between fraction and decimal form.

Use 

key on Casio fx-82MU to convert between fraction and decimal form.

When converting a fraction to a decimal manually, recall the fraction must be in a form equivalent to decimal place values. i.e. denominators much be tenths, hundredths, thousandths etc.



For example, when converting $\frac{1}{8}$ to decimal form, firstly write an equivalent fraction with denominator thousandths (since $\frac{1}{8}$ cannot be written with denominator tenths or hundredths). i.e. $\frac{1}{8} = \frac{125}{1000} \Rightarrow$ decimal form 0.125

Example: Express $\frac{3}{20}$ in decimal form.

Percentages

Percentages are used to give a common standard. Percent mean 'per one hundred' i.e. fractions out of 100. For example $\frac{40}{100}$ is written as 40%

To convert a decimal to a percentage

A decimal is converted to a percentage by multiplying by 100 (or moving the decimal point 2 places *to the right*).

Example: Express 0.015 as a percentage

0.015=0.015 x 100%= 1.5%

To convert a percentage to a decimal

A percentage is converted to a decimal by dividing by 100 (or moving the decimal point 2 places *to the left*).

Example: Express 6.5% as a decimal

$$6.5\% = \frac{6.5}{100} = 0.065$$

To convert a fraction to a percentage

A fraction is converted to a percentage by multiplying by 100.

Example: Express $\frac{3}{20}$ as a percentage.

$$\frac{3}{20} = \frac{3}{20} \times 100\% = \frac{300}{20}\% = 15\%$$

To convert a percentage to a fraction

A percentage is converted to a decimal by dividing by 100 and reducing the fraction to its simplest form.

Example: Express 15% as a fraction

$$15\% = \frac{15}{100} = \frac{3}{20}$$

Example: Express 1.5% as a fraction

$$1.5\% = \frac{1.5}{100} = \frac{15}{1000} = \frac{3}{200}$$

Number Exercise 1.3 Converting Percentages, Decimals, Fractions

Complete the following table of fractions, decimals and percentages.

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50%
$\frac{}{}$	0.75	
$\frac{}{}$		80%
$\frac{2}{5}$		
$\frac{}{}$		5%
$\frac{1}{3}$		
$\frac{}{}$	1.25	

Finding a percentage of a quantity

To find a percentage of a quantity convert the percentage to a fraction (or decimal) and multiply by the quantity (recall that 'of' means multiply).

Example: Find 16% of \$60

Using fractions $\frac{16}{100} \times 60 = \$9.60$ or using decimals $0.16 \times 60 = \$9.60$

Example: Find 2% of 100g

$$0.02 \times 100 = 2g$$

Example: Find 115% of \$60

$$1.15 \times 60 = \$69$$

Example: Find 88% of \$500

$$0.88 \times 500 = \$440$$

Example: Find 600% of 12ml

$$6.00 \times 12 = 72ml$$

Expressing one quantity as a percentage of another

To express one quantity as a percentage of another, divide the first quantity by the second and multiply by 100. (Remember the 'compared to' quantity is the denominator of the calculation).

Example: Express 2.4cm as a percentage of 12cm

$$\frac{2.4}{12} \times 100 = 20\%$$

Example: Express 45 minutes as a percentage of 3 hours

$\frac{45}{180} \times 100 = 25\%$ (use the same units by converting 3 hours to 180 minutes)

Example: Express 25 litres as a percentage of 1000 litres

Example: Express 600mm as a percentage of 3m.

Example: Express test score $\frac{18}{40}$ as a percentage.

Percentage increase and decrease

New value = $\frac{(100+\%\ \text{increase})}{100} \times$ original value

Example: The price of a fully installed combination condensing boiler is increased by 8%. It originally cost \$5000. What is the new price?

$$\frac{100+8}{100} \times 5000$$

$$= 1.08 \times 5000 = \$5400$$

Example: The retail price of a mobile phone is \$450. It is currently on sale for 20% off the retail price. Determine is the sale price?

$$\frac{100-20}{100} \times 450$$

$$= 0.80 \times 450 = \$360$$

Percentage Change

Percentage change is given by $\frac{(\text{new value}-\text{original value})}{\text{original value}} \times 100\%$

Example: A box of resistors increases in price from \$90 to \$104. Calculate the % change in cost to 2 decimal places.

$$\frac{(104-90)}{90} \times 100$$

$= \frac{14}{90} \times 100 = 15.56\%$ (2dp)

Example: A car is purchased for \$12,000. It is sold a year later for \$9,900. Calculate the % decrease in value (compared to the original cost).

$$\frac{(9900-12000)}{12000} \times 100$$

$$= \frac{-2100}{12000} \times 100 = -17.5\%\ or\ 17.5\%\ decrease\ in\ value$$

Percentage Error

Percentage error = $\frac{\text{(error)}}{\text{correct value}} \times 100\%$

Example: The length of a component is measured incorrectly as 64.5mm. The actual length is 63.5mm. What is the percentage error in the measurement?

$$\frac{(64.5-63.5)}{63.5} \times 100 = 1.57\%$$

Finding an Original Value

Original value = $\frac{\text{new value}}{100 \pm \%\ \text{change}} \times 100\%$

Example: A man paid $333.50 for a mobile phone including GST of 15%. Determine the price excluding GST?

$$\frac{333.50}{(100+15)\%} \times 100$$

$$= \frac{333.5}{1.15} = \$290$$

Example: Jack sells his car for $5000 making a 22% loss on his purchase price. Determine the original purchase price rounded to the nearest dollar?

$$\frac{5000}{(100-22)\%} \times 100$$

$$= \frac{5000}{0.78} = \$6410$$

Number Exercise 1.4 Percentage Calculations

1. Find 75% of 36kg
2. A test was marked out of 80. Jon scored 85% of the marks. How many marks did he score?
3. Find 55% of 1 day and give your answer in minutes.
4. David pays a deposit of 15% on a car which costs $8200. (a) How much deposit does he pay? (b) What percentage of the cost has he still to pay?
5. The number of people employed by a company increased by 16% from 650. How many people are employed by the company now?
6. The normal price of a netbook is $550. In a sale, normal prices are reduced by 15%. Find the sale price of the computer.
7. A year ago, Richard bought a new car for $12,680. Since then, it has lost 12% of its value. Find his car's current value.
8. Tony buys a camera for $130 and later sells it, making a loss of 12.5%. Find the selling price.
9. The surface area of the Earth is 510 066 000 square kilometres. 361 743 000 square kilometres of this area is made up of water. What percentage of the Earth's surface is water?
10. The population of a town fell from 23 650 to 20 812. Find the percentage decrease.
11. Helen's annual salary was increased from $45 000 to $46 800. Find her percentage increase.
12. The original price of a camera was $184. In a sale, its price was $161. Find the percentage reduction.
13. Bill put $5500 in a bank account. A year later, interest was added and the amount in his account increased to $5885. Find the rate of interest.
14. Kim bought a computer for $750 and later sold it for $495. Find her percentage loss.
15. An art dealer buys a painting for $3500 and sells it for $4480. Find his percentage profit.
16. A computer costs $1650.25 including GST (15%). Find its price excluding GST.
17. A car is purchased and later sold for $3510, making a 35% loss on original purchase price. What was the original purchase price?
18. The uniform speed of a ship’s propeller through the water is 16 knots, but slip causes the vessel to cover only 345 nautical miles in 24 hours. Determine the percentage slip. (Slip is considered as the difference between the speed of the engine and actual speed of the ship, and percentage slip is the slip as a percentage of engine speed).

Decimal Rounding and Rounding Error

Recall rounding involves the following steps:

1. Identify the number of decimals places being rounded e.g. 27.3849 rounded to 3dp
2. Check the digit to the right of the last required decimal place e.g. 27.374 | 9
3. If the digit is 4 or less, do not round the number up.
4. If the digit is 5 or more, round the number up. e.g. 27.374 | 9 becomes 27.375 (3dp)

Rounding an answer too soon in calculations can cause significant rounding error. Where possible, round toward the end of a calculation. Compare the examples below:

Rounding early (every number rounded 2 dp from first step in calculation)	**Rounding later** (every number rounded 4dp from first step in calculation, final answer rounded 2 dp)	**Rounding more dp later** (every number rounded 6 dp from first step in calculation, final answer rounded 2 dp)
$\frac{23.29\times\pi}{0.15^2} = \frac{23.29\times 3.14}{0.02} = 3656.53$	$\frac{23.29\times\pi}{0.15^2} = \frac{23.29\times 3.1416}{0.0225} = 3251.91$	$\frac{23.29\times\pi}{0.15^2} = \frac{23.29\times 3.141593}{0.0225} = 3251.90$

Number Exercise 1.45 Rounding

1. Round 2.756983 to
   a) 1 d.p.
   b) 2 d.p.
   c) 3 d.p.
   d) 4 d.p.
   e) 5 d.p.

2. Evaluate $\frac{7.6\times\pi}{\sqrt{42+17^2}}$ Round your answer to
   a) 4 d.p.
   b) 0 d.p.

Ratio and Proportion

Ratio

A **ratio** compares amounts and shows how much bigger one thing is than the other. Practical examples include mixing paint, alloy constituents, fuel, and gears. Ratios are shown as numbers separated by a colon (:) so, for example, the ratio of ‘3 to 1’ is shown as 3:1.

e.g. mixing 1 part cement to 4 parts sand could be represented by the ratio 1:4. In total there are 5 parts, $\frac{1}{5}$ of the total quantity is cement and $\frac{4}{5}$ of the total quantity is sand.

Example 1: A gear wheel, with 128 teeth is in mesh with a 48 tooth gear. What is the gear ratio?

Gear ratio is 128 : 48

Example 2: Brass is made up of copper and zinc in a ratio of 7 : 3. How many kg of copper and zinc are there in 15kg of Brass?

<u>Step 1:</u> calculate the number of parts altogether by adding the ratio values. i.e. there are 7+3=10 parts in all.

<u>Step 2:</u> calculate the quantity of each component using the ratio value as a fraction of the total number of parts, i.e. $\frac{ratio\ value}{(sum\ of\ all\ ratio\ values)} \times total\ quantity$

$\frac{7}{10}$ of the total 15kg will be copper, i.e. $\frac{7}{10} \times 15 = 10.5$kg of copper

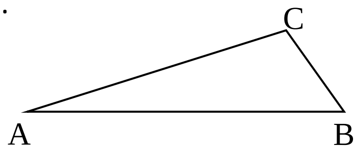
$\frac{3}{10}$ of the total 15kg will be zinc, i.e. $\frac{3}{10} \times 15 = 4.5$kg of zinc

[another way of approaching the problem is to consider the total quantity (15kg) being divided into the total number of parts being shared (in this case, 10) i.e. $\frac{15}{10} = 1.5$*. Then allocate each component its share of these parts. i.e. copper comprises 7 of these parts,* $7 \times 1.5 = 10.5$*kg and zinc comprises 3 of these parts,* $3 \times 1.5 = 4.5$*kg]*

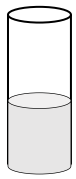
Number Exercise 1.5 Ratio

1. Divide 72 in the ratio of 2 : 3 : 4

2. A piece of brass contains 7 parts by weight of copper and 3 parts by weight of zinc. How much copper and how much zinc is required to make a 500kg brass casting?

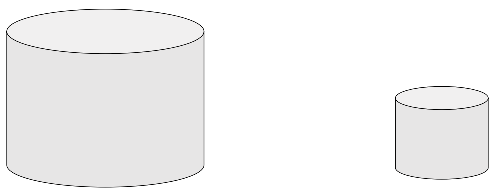
3. 

Angles A, B and C in this triangle are divided in ratio 3:4:8.
Calculate the size of each angle.

4. 

This glass contains 240ml of water. The glass is $\frac{2}{5}$ full.
How many ml would the glass contain if it was full?

5. Two full shore-side tanks contain a mixture of oil and water pumped from a ship's tanks. Tank 1 contains a mixture of 9 parts oil and 2 parts water, and is 4 times larger than tank 2. The ratio in tank 2 is 8 parts oil to 3 parts water. Prior to final processing, the contents of both tanks is transferred into a third tank. Calculate the oil : water ratio of the combined mixture in the third tank.



6. Find two numbers which add to 91 and have a ratio of 6 : 7.

7. An alloy contains a mixture of 55% copper, 27% zinc, and 18% nickel. What mass of each constituent is required to make a 2000 kg casting?

8. A certain alloy contains 70% copper, 18% zinc, and 12% tin by mass.
   a) Calculate the mass of tin in 30 kg of the alloy.
   Extension Question: The purchaser then decides they want more tin added to bring the tin percentage up to 20% of the total new alloy quantity.
   b) What mass of tin must be added to the 30 kg of alloy?
   c) Calculate the percentages of each metal in the new alloy.

Direct Proportion (Direct Variation)

Two quantities are in **direct proportion** when they increase or decrease in the same ratio. i.e. any change in one quantity produces a corresponding change in the other. For example,

the weight of 12 cans of beer is 4 kg,
the weight of 24 cans of beer is 8 kg,
the weight of 36 cans of beer is 12 kg, etc.

In this case, the number of cans is proportional to weight i.e. #cans α weight

Any two quantities which vary directly to each other can be divided by each other and their answer will be constant. Using the above example,

$$\frac{\#\ cans_1}{weight_1} = \frac{\#\ cans_2}{weight_2} = \frac{\#\ cans_3}{weight_3}$$

$$\frac{12}{4} = \frac{24}{8} = \frac{36}{12} = \frac{3}{1}$$

One way of writing and solving direct proportional relationships between quantities is to calculate the **'constant of proportionality'** This is a constant (a number) denoted by the letter k, which always relates the two proportional quantities. In the above example, if 12 cans have a weight of 4kg, and number of cans is proportional to the weight of cans, then the weight of other quantities of cans may be found by

1. finding the weight of a single can, (or constant of proportionality, often denoted by the letter *k*);
2. using this constant of proportionality (or unit ratio) to find the weight of other quantities of cans.

   i.e. $\frac{\text{weight}}{\#\ \text{cans}} = \frac{4}{12}$ (divide the known weight by known # cans to find weight of a single can, or constant of proportionality)

   ∴ 24 cans would weigh $\frac{4}{12} \times 24 = 8$kg

Three examples of engineering laws involving direct proportion:

1. **Charles's Law** states that , for a given mass of gas at constant pressure, the volume V is directly proportional to its thermodynamic temperature, T. i.e. V α T
2. **Hooke's Law** states that, within the elastic limit of a material, the strain is directly proportional to the stress. i.e. strain α stress
3. **Ohm's Law** states that the current, *I* flowing through a fixed resistance is directly proportional to the applied voltage, *V*. i.e. *I α V*

Example 1: If 45 litres of petrol costs $87.75, calculate the cost of 20 litres.

Cost for 1 litre $= \frac{87.75}{45} = \$1.95$,

Cost for 20 litres $= 20 \times 1.95 = \$39$

Example 2: Hooke's law states that stress σ, is directly proportional to strain ϵ, within the elastic limit of a material. When, for mild steel, the stress is 63MPa, the strain is 0.0003.

Determine (a) the value of the strain when stress is 42MPa, and (b) the value of stress when strain is 0.00072.

(a) When stress is 63MPa, strain is 0.0003

$\therefore$ a stress of 1MPa corresponds to a strain of $\frac{0.0003}{63}$

and $\therefore$ strain when stress is 42 is $\frac{0.0003}{63} \times 42 = 0.0002$

(b) When strain is 0.0003, stress is 63MPa

$\therefore$ strain of 1 corresponds to a stress of $\frac{63}{0.0003}$ MPa

and $\therefore$ stress when strain is 0.00072 is $\frac{63}{0.0003} \times 0.00072 = 151.2$ MPa

Inverse (indirect) Proportion

Two quantities are said to be **inversely (or indirectly) proportional** to each other if one quantity increases as the other decreases (and vice versa). For example, the time for a journey is inversely proportional to the speed of travel.

Example 1: If 4 people take 3 hours to dig a hole, how long will 2 people take to dig the hole working at the same rate?

$a\ hole\ takes\ 4 \times 3 = 12\ hours\ to\ dig$

$$\therefore 2\ people\ take\ \frac{4 \times 3}{12} = 6\ hours\ to\ dig\ a\ hole$$

Note: mathematically, two variables, x and y are inversely proportional if y is proportional to $\frac{1}{x}$ i.e. $k = xy$ where k is the constant of proportionality.

Example 2: The electrical resistance R of a piece of wire is <u>inversely proportional</u> to the cross-sectional area A. When A = 5 mm^2 , R = 7.2 ohms. Determine (a) the constant of proportionality, and (b) the cross-sectional area when resistance is 4 ohms.

(a) for inverse/indirect proportional relationships, the constant of proportionality is found by multiplying the resistance and the cross-sectional area. i.e.
$5 \times 7.2 = 36\Omega mm^2$

(b) $\frac{36}{4} = 9mm^2$

Example 3: Converting from ratio to proportion. In a two cylinder engine, the power developed in No. 1 cylinder is 20% more than the power developed in No. 2 cylinder. What percentage of total power is developed in each cylinder?

Ratio of No.1 to No. 2 $= 120: 100\ or\ simplified\ 1.2: 1$

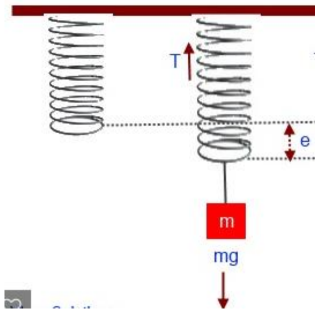
Sum of ratio $= 1.2 + 1 = 2.2$

$$\%\ power\ developed\ by\ No. 1\ cylinder = \frac{1.2}{2.2} \times 100 = 54.55\%$$

$$\%\ power\ developed\ by\ No.2\ cylinder = \frac{1}{2.2} \times 100 = 45.45\%$$

Number Exercise 1.6 Direct and Inverse Proportion

1. The tension, T, of a stretched spring is directly proportional to its extension, e. If the spring has an un-stretched length of 20 mm, and is extended 5mm by a load of 8 N, calculate the length under a load of 12 N.



2. The time of swing of a pendulum is directly proportional to the square root of its length. If the time of swing of a 40 cm pendulum is 2.2 seconds, calculate the time of swing of a 90 cm pendulum.

3. A pump delivers 4800 litres of water in 60 minutes.
   a) How many litres would be delivered in 85 minutes?
   b) How long would it take to deliver 7200 litres?

4. Boyle's Law states that, at constant temperature, the volume of a fixed mass of gas is inversely proportional to its absolute pressure. Given that 0.7m^3 of the gas has an absolute pressure of 178 kPa, calculate :-
   a) The constant of proportionality.
   b) The volume of gas if pressure is raised to 182.5 kPa.

5. It takes 3 fitters 11 hour to assemble 4 engines.
   a) How many man-hours does it take to assemble a single engine?
   b) How long will it take 9 fitters to assemble 6 engines?

6. The resistance of a wire is directly proportional to length L and inversely proportional to cross-sectional area A. If the resistance of 1000 m of a particular wire of cross-sectional area 0.2 square metres is 18 ohms, calculate the resistance of a similar wire of length 100 m, and cross-sectional area 0.001m^2 .

7. Hook's law states that, within the elastic limit of a mater, stress is directly proportional to strain. For brass, stress is 21MPa when strain is 0.00025. Determine the stress when strain is 0.00035.

8. A car travelling at 50km/hr makes a journey in 70 minutes. How long will the journey take at 70km/hr? *(Hint: convert minutes to hours)*

9. Within a certain power range, the mass of fuel consumed per hour is directly proportional to the power developed in an engine. When an engine uses 180kg/h of fuel, the power developed is 800kW. What will be the mass of fuel consumed when the power developed is 900kW?

10. A simple machine has an effort: load ratio of 4:37. Determine the effort, in kg to lift a load of 5.6 kN.

11. A general service pump can empty a tank in 8 hours and the ballast pump can empty the same tank in 4 hours. If both pumps are working together how long will it take to empty the tank?

Powers, Indices, Exponents and Roots

Powers and Roots

The **square** of a number is found by multiplying that number by itself.

For example, $3^2 = 3 \text{ x } 3 = 9$

In a similar way, numbers can be raised to higher powers.

For example, $3^4 = 3 \text{ x } 3 \text{ x } 3 \text{ x } 3 = 81$

In general, $x^n = x \times x \times x \times \ldots x$ (x multiplied by itself n times)

Note: special case $x^0 = 1$ and $x^1 = x$

When written as 3^4, 3 is called the **base** and the 4 is the **index** (or **power** or **exponent**). We say ‘three to the power of four’.

Example: Evaluate $3^4 \times 2^3 = 3 \times 3 \times 3 \times 3 \times 2 \times 2 \times 2 = 648$

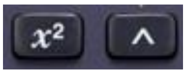
The **root** of a number is the inverse of raising a number to a power.

For example, $\sqrt{9}$ is the number which squares to make 9, i.e. 3

For example, $\sqrt[4]{81}$ is the number which, when multiplied by itself 4 times, makes 81,

i.e. $\sqrt[4]{81} = 3$ since $3 \text{ x } 3 \text{ x } 3 \text{ x } 3 = 81$

Ensure you understand how to operate roots and power keys on your calculator.

Power keys may look like  (casio fx82AU) or  (casio fx82MU).

Root keys may look like  (casio fx82AU) or  (casio fx82MU).

Roots of numbers written with the $\sqrt{\ }$ sign are said to be written in **surd form**. Roots can also be written as fractional powers. This is called **index form.** In index form, the denominator of the fraction is the root. (see Laws of Indices section below for general explanation of fractional powers).

Example: $\sqrt{9}$ can also be written as $9^{\frac{1}{2}}$ (square root of nine) and $\sqrt[4]{81}$ can be written as $81^{\frac{1}{4}}$ (fourth root of eighty-one).

Laws of Indices

1. When multiplying two (or more) numbers having the same base, the indices are added.

$x^m \times x^n = x^{m+n}$	Example: $4^2 \times 4^3 = (4 \times 4) \times (4 \times 4 \times 4) = 4^5 = 1024$

2. When dividing two numbers having the same base, the index in the denominator is subtracted from the index in the numerator.

$\frac{x^m}{x^n} = x^{m-n}$ $= \frac{1}{x^{n-m}}$	Example: $\frac{4^5}{4^2} = 4^{(5-2)} = 4^3 = 64$ Example: $\frac{3^2}{3^5} = 3^{(2-5)} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

3. Powers of powers. When a number which is raised to a power is raised to a further power, the indices are multiplied.

$(x^m)^n = x^{mn}$ $(xy)^m = x^m y^n$	Example: $(3^5)^2 = 3^{5\times2} = 3^{10}$ Example: $(2 \times 3^5)^2 = 2^2 \times 3^{5\times2} = 4 \times 3^{10}$

4. When a number has an index of zero, its value is 1

$x^0 = 1$	Example: $5^0 = 1$ or $27^0 = 1$

5. A number raised to a negative power is the reciprocal of that number raised to a positive power. i.e. 'flip' the

$x^{-n} = \frac{1}{x^n}$ or $\frac{1}{x^{-n}} = x^n$	Example: $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$ Example: $\frac{1}{2^{-3}} = 2^3 = 8$

6. When a number is raised to a fractional power the denominator of the fraction is the root of the number, and the numerator is the power of the number.

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$	Example: $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$ Example: $27^{\frac{2}{3}} = \sqrt[3]{27^2} = 9$ Example: $27^{\frac{-2}{3}} = \frac{1}{\sqrt[3]{27^2}} = \frac{1}{9}$ Example: $\frac{1}{27^{\frac{-2}{3}}} = 27^{\frac{2}{3}} = \sqrt[3]{27^2} = 9$

Number Exercise 1.7 Powers, Roots and Laws of Indices

1. Evaluate the following :
   a) 4^2
   b) $(-5)^4$
   c) 2^6
   d) 6^{-1}
   e) 5^{-1}
   f) $(0.9)^0$
   g) $25^{0.5}$
   h) $112^{1/2}$
   i) $(5^2)^3$
   j) $6^{3.5}$
   k) $\sqrt{6^7}$
   l) $(2^3 + 2^4)^{-1}$
   m) $125^{1/3}$
   n) $81^{-1/2}$
   o) $16^{-3/4}$
   p) $64^{-5/6}$
   q) Express $\frac{1}{16}$ as a power of 4
   r) Express 625 as a power of 5
   s) Express 32 as a power of 2

2. Write the following in surd form :
   a) $3^{\frac{1}{2}}$
   b) 4^{-3}
   c) $3^{-0.5}$

3. Find the values of the following without the aid of a calculator:
   a) $10000^{0.5}$
   b) $10000^{-0.5}$
   c) $(10000^{-1})^{0.5}$
   d) $\left(\frac{1}{10000}\right)^{-0.5}$

4. Write the following in surd form and also evaluate:
   a) $8^{\frac{3}{4}}$
   b) $\left(4^{\frac{2}{5}}\right)^{0.5}$

Logarithms and Logarithmic Scales

A logarithm is the power to which a base must be raised to give the required number.

'Common Logs' use the base of 10 so the log of 100 is 2, 1000 is 3 etc.

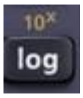
The log of 500 is 2.699 because $10^{2.699}$= 500. The log of 0.5 is -0.301 because 10^{-301}=0.5

Natural or Napierian Logs use the base of 2.71828, this number being known as 'e'. This is a 'natural' number found by a mathematical series and is used for engineering calculations.

Your calculator can work in both types of logs, the buttons being marked 'log' for common logs and 'ln' for natural logs.

Before calculators, logs were used all the time to multiply large numbers together. Using the law of indices (whereby indices of the same base e.g. base 10, are added when numbers are multiplied), complex multiplication could be carried out using log tables and anti-log tables to convert the logs back to actual numbers.

Example: Using logs, calculate 254.3 x 576

log(254.3)=2.4053 (since $10^{2.4053}$=254.3 using  or  button on your calculator) and log(576.01)=2.7604 (since $10^{2.7604}$=576)

so 254.3 x 576 = log(254.3) + log(576)= 2.4053+2.7604=5.1657

then anti-log (5.1657) to determine final answer using the '<shift> log' key on a calculator. (i.e. $10^{5.1657}$= 146,454).

Since a logarithm is the same as the power of a particular base number, we can write equivalent expressions in **logarithmic** and **index** form.

If $\boldsymbol{log_b y = x}$ **then** $\boldsymbol{b^x = y}$ *log form* *index form*	Example: $log_5 25=2$ since $5^2=25$ Example: write $log_{10}1000000=6$ in index form *Answer:* $10^6=1000000$ Example: write $log_4 64=3$ in index form *Answer:* $4^3=64$ Example: write $3^4=81$ in log form *Answer:* $log_3 81=4$ Example: write $6^4=1296$ in log form *Answer:* $log_6 1296=4$

Laws of Logarithms

There are three laws of logarithms which apply to any base:

To multiply two numbers, x and y: $logx + logy = log(xy)$	Example: log (3x2)=log3+log2 Example: log5+log2=log(5x2)=log(10)
To divide two numbers, x and y: $logx - logy = log\left(\frac{x}{y}\right)$	Example: log$\left(\frac{6}{3}\right)$= log6-log3 Example: log12-log4=log$\left(\frac{12}{4}\right)$=log3
To raise a number, x to a power, n : $logx^n = nlogx$	Example: 2log3=log(3^2)=log9 Example: log(81)=log(3^4)=4log3

Number Exercise 1.8 Laws of Logarithms

Write the following expressions as logarithms of a single number (do not evaluate).

1. log4+log3=log(4x3)=log12
2. log4+log3-log2
3. 2log2+3log5
4. 2log2+log5-log10

Write the following logarithmic expressions in index form and then solve.

5. $log_3 81 = x$
6. $log_x 8 = 3$
7. $log_5 x = 3$

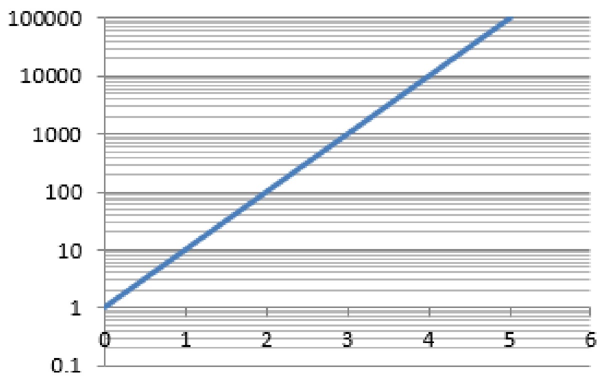
Relationship between logarithms of different bases:

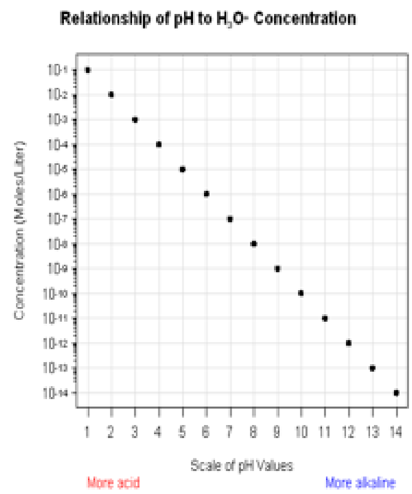
$log_b(N) = \frac{log_a(N)}{log_a(b)}$	Example : Find $log_2(8)$ using common logs ('log' key on the calculator). $log_2(8) = \frac{log(8)}{log(2)} = \frac{0.9030}{0.3010} = 3$ Example : Find $log_3(81)$ using common logs ('log' key on the calculator). $log_3(81) = \frac{log(81)}{log(3)} = \frac{1.9085}{0.4771} = 4$

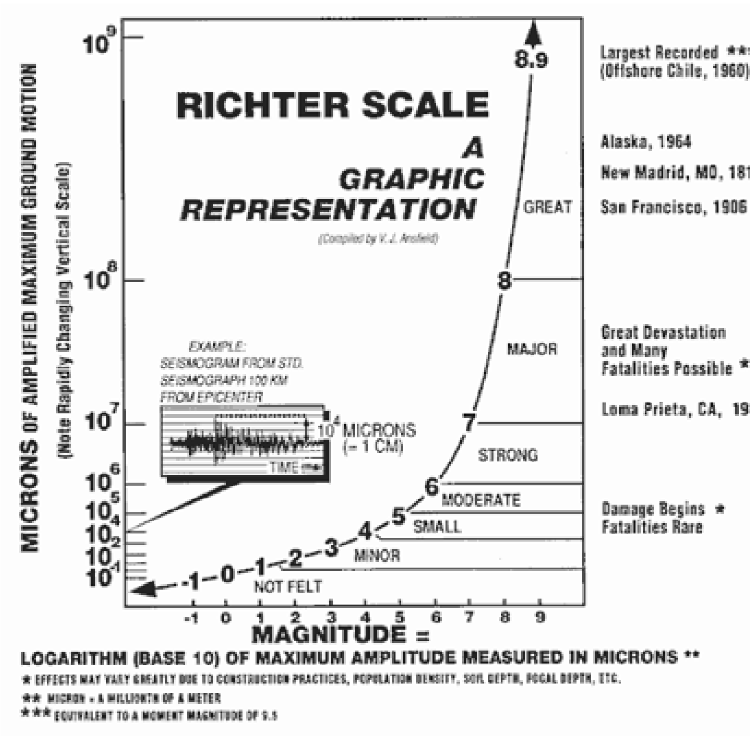
Logarithmic scales:

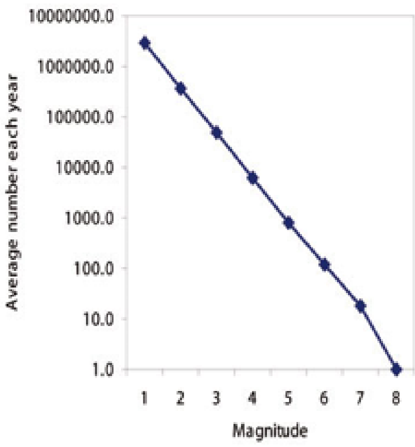
Where the value of y ranges from very small to very large on a graph, the log of the number is often plotted. The scale of the y axis therefore varies, the distance from 0 to 10 being the same as from 10 to 100 and from 100 to 1000.

Examples of logarithmic scales in common use are the Richter Scale for earthquakes and the Decibel scale for sound. This explains why an earthquake of force 7 is greatly more powerful than one of 6.5.









Application of natural logarithms

Heat exchange calculations in thermodynamics use natural logarithms. When checking the performance of heat exchangers the temperature difference used in the calculation must be the Logarithmic Mean Temperature Difference, not the simple temperature difference.

$$LMTD = \frac{(T_1 - t_2) - (T_2 - t_1)}{ln\left(\frac{(T_1 - t_2)}{(T_2 - t_1)}\right)}$$

where	T_1	=	hot stream inlet temperature
	T_2	=	hot stream outlet temperature
	t_1	=	cold stream inlet temperature
	t_2	=	cold stream outlet temperature

Example: Calculate the logarithmic mean temperature difference for a heat exchanger where the thermometers the following:

Hot stream inlet 85 °C (T_1)

Hot stream outlet 30 °C (T_2)

Cold stream inlet 17 °C (t_1)

Cold stream outlet 40 °C (t_2)

$$LMTD = \frac{[(85 - 40) - (30 - 17)]}{\left[ln\frac{(85 - 40)}{(30 - 17)}\right]}$$

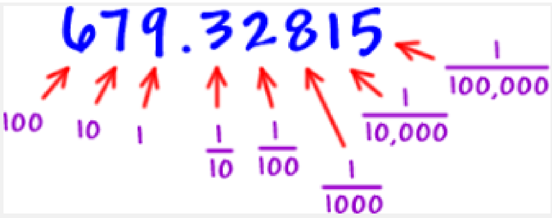
$$= \frac{[45 - 13]}{\left[ln\left(\frac{45}{13}\right)\right]}$$

$$= \frac{32}{1.2417}$$

$$= 25.77 \text{ °C}$$

Standard Form, Engineering Form, Unit Prefixes

Recall, numbers can be written in terms of powers of 10. For example:
100 could be written as 10^2
10 could be written as 10^1
1 could be written as 10^0
0.1 $\left(\text{or } \frac{1}{10}\right)$ could be written as 10^{-1}
0.01 $\left(\text{or } \frac{1}{100}\right)$ could be written as 10^{-2}
0.1 $\left(\text{or } \frac{1}{10}\right)$ could be written as 10^{-3}



It follows that the number 600 could be written as 6×10^2 and the number 0.008 could be written as 8×10^{-3}.

Standard Form and Engineering Notation

A number written with one digit to the left of the decimal point and multiplied by powers of 10 is said to be written in **standard form**.
Example: $21400 = 2.14 \times 10^4$ in standard form
Example: $0.0365 = 3.65 \times 10^{-2}$

Engineering notation is similar to standard form except the power of 10 is always a multiple of 3. This form of notation is useful when converting between units such as mm to km.
Example: $21400 = 21.4 \times 10^3$ in engineering notation
Example: $0.0365 = 36.5 \times 10^{-3}$ in engineering notation

Make sure you understand how to use the $\times \mathbf{10^x}$ key 

or EXP key 

on your calculator for calculations involving standard and engineering form. Know how to interpret numbers displayed in exponent form on your calculator.
Example: 1.47E+2 on a calculator display is actually the number $1.47\text{x}10^2$ or 147
Example: 2.068E-3 on a calculator display is actually the number $2.068\text{x}10^{-3}$ or 0.002068

Multiplication and Division of numbers in Standard and Engineering Form

Powers of 10 obey the rules of indices so when multiplying or dividing numbers in standard form, indices can simply be added or subtracted.
Example: $(21.4 \times 10^3) \times (2 \times 10^4) = (21.4 \times 2) \times 10^{(3+4)} = 42.8 \times 10^7 \; or \; 4.28 \times 10^8$
Example: $(21.4 \times 10^3) \div (2 \times 10^2) = (21.4 \div 2) \times 10^{(3-2)} = 10.7 \times 10^1 or \; 1.07 \times 10^2$

Addition and Subtraction of numbers in Standard and Engineering Form

If exponents are the same for all numbers in an addition or subtraction calculation then there is no problem
Example: $(3 \times 10^6) + (2 \times 10^6) = (3 + 2) \times 10^6 = 5 \times 10^6$

Example: $(3 \times 10^3) - (2 \times 10^3) = (3 - 2) \times 10^3 = 1 \times 10^3$

However, if exponents are different to begin with, then they must be made equal before beginning the calculation.

Example: $(3 \times 10^6) + (2 \times 10^4) = (3 \times 10^2 \times 10^4) + (2 \times 10^4)$

$$= (300 \times 10^4) + (2 \times 10^4) = 302 \times 10^4 or\ 3.02 \times 10^6$$

Number Exercise 1.9 Standard Form

Write these decimal numbers in standard form:

1. $7{,}600{,}000{,}000$
2. 0.0000000000176

Calculate the following, writing your answer in standard form to the required level of decimal place rounding:

3. $0.00000054876 \times 0.000054821$ (round to 4 d.p.)

4. $9483.365487 \div 5.3876698 \times 10^{-5}$ (round to 6 d.p.)

SI Unit Prefixes

Common prefixes used for multiples of SI units are shown below. Note whether the prefix is written as UPPER or lower case. For example, Mega is 'M', milli is 'm'.

Prefix	Name	Meaning
G	giga	multiply by 10^9 i.e. x 1 000 000 000
M	mega	multiply by 10^6 i.e. x 1 000 000
k	kilo	multiply by 10^3 i.e. x 1 000
m	milli	multiply by 10^{-3} i.e. $\div 1000$, $\text{i.e.} \times \frac{1}{10^3}$, $\text{i.e.} \times \frac{1}{1000}$, $\text{i.e.} \times 0.001$
μ	micro	multiply by 10^{-6} i.e. $\div 1\,000\,000$, $\text{i.e.} \times \frac{1}{1000000}$, $\text{i.e.} \times 0.000001$
n	nano	multiply by 10^{-9} i.e. $\div 1\,000\,000\,000$, $\text{i.e.} \times \frac{1}{1000000000}$, $\text{i.e.} \times 0.000000001$
p	pico	multiply by 10^{-12} i.e. $\div 1\,000\,000\,000\,000$, $\text{i.e.} \times \frac{1}{1000000000000}$, $\text{i.e.} \times 0.000000000001$

Lengths (such as perimeter) are measured in linear units e.g. mm, cm, m

Areas (and **Surface Areas**) are measured in square units e.g. m^2

Volumes are measured in cubic units e.g. mm^3, m^3

Unit Conversions

When converting units during calculations, ensure consistency. That is, if one component of the problem is given in millimetres (mm) and another in metres (m), ensure you convert one of these units so the whole problem is solved in either millimetres or metres. It is often useful to check the units required in your final answer and work in that unit from the beginning of the problem if possible.

Example: A triangle has sides measuring 120mm, 0.05m and 13cm. Calculate it's perimeter in mm.

Converting all lengths into mm: sides are 120mm, 50mm and 130mm.

Perimeter = 120 + 50 + 130 = 300mm

Example: A small cube has side length of 5mm. What is the volume of the cube in cubic metres?

Convert the side measurements into metres before solving the problem (since the answer is required in cubic metres)

$$sides\ are\ 0.005\ m\ long$$

$$\therefore vol = 0.005 \times 0.005 \times 0.005 = 0.000000125\ m^3\ or\ 1.25 \times 10^{-7}\ m^3$$

Linear unit	Area Unit	Volume Unit
1 cm=10 mm	1 cm^2 =1 cm×1 cm =10 mm×10 mm =100 mm^2	1 cm^3 =1cm×1cm×1cm =10mm×10mm×10mm =1000 mm^3
1 m=100 cm	1 m^2=100 cm×100 cm =10 000 =1x10^4 cm^2	1 m^3 =1m×1m×1m =100cm×100cm×100cm =1x10^6 mm^3
1 m=1000 mm	1 m^2=1000 mm×1000 mm =1 000 000=1x10^6 mm^2	1 m^3 =1000mm×1000mm×1000mm =1x10^9 mm^3
1 km=1000 m	1 km^2=1000 m×1000 m =1 000 000 1x10^6 m^2	1 km^3 =1000m×1000m×1000m =1x10^9 m^3

Volume and capacity $1000\ cm^3 = 1$ litre

$1\ cm^3 = 1$ millilitre (ml)

$1\ m^3 = 1000$ litre

Mass conversions 1 kg= 1 litre = $1000\ cm^3$ (water)

1000kg = 1 tonne

Density Density (ρ) is a measure of the mass per unit volume

mass=volume x density

m=Vρ

Units for density:

kg/m^3 or g/cm^3 or $tonne/m^3$ for solid materials

g/ml for liquids

g/l or kg/l for gases

The density of pure water is 1g/ml or $1000kg/m^3$ or $1tonne/m^3$ or 1kg/l

Number Exercise 1.10 Unit conversions

1. $20g + 1600mg =$..................g
2. $500\ cm \times 500\ cm \times 200\ cm =$ ....m^3
3. $15.3\ m^3 =$ .... $litres$
4. A small cube has side of length 4 cm. What is the volume of the cube in m^3?
5. There are approximately $1.5x10^{24}$ molecules in 5 grams of hydrogen gas. Calculate the number of molecules in 6 kg of hydrogen gas.
6. What does the prefix μ mean?
7. Rewrite 3.375 kg in grams
8. Rewrite 0.058 mA in μA
9. The length of a room is 5.3m. The width of the room is 3300mm. Calculate the area of the room in m^2.

Evaluation of Formulae

Evaluation is the process of substituting the numerical values of algebraic symbols and working out the value of the whole expression.

Example:
$V = IR$ (Ohms Law) is a formula for V (voltage) in terms of I (current) and R(resistance). Find the voltage When I=5 Amps and R=15.8Ω

$V = IR$

$V = 5 \times 15.8$

$V = 79V$

Number Exercise 1.11 Evaluation of Formulae

1. The surface area A of a hollow cone is given by $A = \pi rl$. Determine (correct to 1dp), the surface area when r=3cm and l=8.5cm.
2. Velocity is given by $v = u + at$. If u=9.86m/s, a=4.25m/s^2, t=6.84s, find v (correct to 1dp).
3. Force, F newtons is given by the formula $F = \frac{Gm_1m_2}{d^2}$ where m_1 and m_2 are masses, d their distance apart and G is a constant. Find the value of the force given that $G = 6.67 \times 10^{-11}$, $m_1 = 7.36$, $m_2 = 15.5$ and $d = 22.6$

Linear Interpolation from Tables of Data

(Additional content with application in Ship's Stability)

Linear Interpolation is a technique sometimes used to estimate the value of a quantity which lies between two other known (or published) quantities. For example, the table below comes from part of a published table of hydrostatic data used for ship stability calculations. Quantities lying between published values can be estimated using linear interpolation.

MV Maritime Liberty			
Displacement	Draught	TPC	VOL
6403.48	8.00	24.44	6247.30
6525.65	3.45	24.60	6366.49
6648.64	3.50	24.76	6486.48
6772.44	3.55	24.92	6607.26
6897.05	3.60	25.08	6728.83
7022.48	3.65	25.25	6851.20
7148.71	3.70	25.41	6974.35
7275.76	3.75	25.57	7098.30
7403.62	3.80	25.73	7223.04
7532.29	3.85	25.90	7348.57
7661.77	3.90	26.06	7474.90
7792.06	3.95	26.22	7602.01
7923.17	4.00	26.38	7729.92
8055.09	4.05	26.55	7858.62
8187.81	4.10	26.71	7988.11
8321.36	4.15	26.87	8118.40
8455.71	4.20	27.03	8249.47
8590.87	4.25	27.20	8381.34
8726.85	4.30	27.36	8514.00
8863.64	4.35	27.52	8647.45
9001.24	4.40	27.68	8781.70

Example: Using the hydrostatic data above, find displacement and volume for a ship's draught of 3.62 metres.

Displacement	Draught	TPC	VOL
6403.48	8.00	24.44	6247.30
6525.65	3.45	24.60	6366.49
6648.64	3.50	24.76	6486.48
6772.44	3.55	24.92	6607.26
6897.05	3.60	25.08	6728.83
7022.48	3.65	25.25	6851.20

By observation, we expect answers to lie between the two published draught values of 3.60 and 3.65.

By proportion, 3.62 lies $\frac{(3.62-3.60)}{(3.65-3.60)} = \frac{2}{5} = 0.4$ of the way along the line between draughts 3.60 and 3.65.

Interpolating Displacement: Using the proportion of 0.4 calculated for draught, we can estimate the displacement (associated with a draught of 3.62) to be 0.4 of the way along the line between displacement values 6897.05 and 7022.48.

i.e. $0.4 \times (7022.48 - 6897.05)$ $\boldsymbol{more\ than}$ 6897.05

$= 50.172 + 6897.05$

$= 6947.22\ m$

Interpolating Volume: Using the proportion of 0.4, we can estimate the Volume (associated with a draught of 3.62) to be 0.4 of the way along the line between Volume values 6728.83 and 6851.20.

i.e. $0.4 \times (6851.20 - 6728.83)\ more\ than\ 6727.83$

$= 48.948 + 6728.83$

$= 6777.78\ m$

Exercise 1.12: Linear Interpolation of Data Tables

1. Using the table on the right, find Displacement and Volume for a ship's draught of 4.13 m.

2. Using the Hydrostatic Data table on the right, find Displacement and TPC immersion (tonnes per cm immersion) for a ship's draught of 3.888 m.

3. Using the Saturated water-Temperature table, find the saturation pressure of water at $33°C$?

4. Using the Saturated water-Temperature table, find the saturation pressure of water at $71°C$?

MV Maritime Liberty

Displacement	Draught	TPC	VOL
6403.48	8.00	24.44	6247.30
6525.65	3.45	24.60	6366.49
6648.64	3.50	24.76	6486.48
6772.44	3.55	24.92	6607.26
6897.05	3.60	25.08	6728.83
7022.48	3.65	25.25	6851.20
7148.71	3.70	25.41	6974.35
7275.76	3.75	25.57	7098.30
7403.62	3.80	25.73	7223.04
7532.29	3.85	25.90	7348.57
7661.77	3.90	26.06	7474.90
7792.06	3.95	26.22	7602.01
7923.17	4.00	26.38	7729.92
8055.09	4.05	26.55	7858.62
8187.81	4.10	26.71	7988.11
8321.36	4.15	26.87	8118.40
8455.71	4.20	27.03	8249.47
8590.87	4.25	27.20	8381.34
8726.85	4.30	27.36	8514.00
8863.64	4.35	27.52	8647.45
9001.24	4.40	27.68	8781.70

Saturated Water Temperature Table

Saturation Temperature (°C)	Saturation Pressure (MPa)	Specific Volume (m^3/kg) Saturated Liquid	Saturated Vapor
0.01	0.00061165	0.0010002	205.99
5	0.00087258	0.0010001	147.01
10	0.0012282	0.0010003	106.30
15	0.0017058	0.0010009	77.875
20	0.0023393	0.0010018	57.757
25	0.0031699	0.0010030	43.337
30	**0.004247**	0.0010044	32.878
35	**0.005629**	0.0010060	25.205
40	0.0073849	0.0010079	19.515
45	0.0095950	0.0010099	15.252
50	0.012352	0.0010121	12.027
55	0.015762	0.0010146	9.5643
60	0.019946	0.0010171	7.6672
65	0.025042	0.0010199	6.1935
70	0.031201	0.0010228	5.0395
75	0.038595	0.0010258	4.1289
80	0.047414	0.0010291	3.4052
85	0.057867	0.0010324	2.8258
90	0.070182	0.0010360	2.3591
95	0.084608	0.0010396	1.9806
100	0.10142	0.0010435	1.6718

ANSWERS TO EXERCISES

Number Exercise 1.1

1. -10
2. -1
3. 23.3
4. -25.81
5. 52
6. -7.29
7. 115
8. 0.333
9. 155.638
10. 7.375
11. -58
12. 64.03
13. 117.023
14. -606.51
15. 38.276
16. -103.7817
17. 2.633

Number Exercise 1.2

1. $\frac{21}{32}$
2. $\frac{8}{9}$
3. $\frac{33}{224}$
4. $\frac{3}{112}$
5. $\frac{5}{48}$
6. $\frac{7}{18}$
7. $\frac{27}{28}$
8. $\frac{35}{81}$
9. $\frac{4}{9}$
10. $\frac{15}{28}$
11. $\frac{4}{9}$
12. 4
13. $\frac{10}{9}$
14. $\frac{65}{44}$
15. $\frac{4}{21}$
16. $\frac{133}{90}$
17. $\frac{157}{12}$
18. $\frac{11}{36}$
19. $\frac{137}{14}$
20. $\frac{483}{20}$

Number Exercise 1.3

Fraction (simplest form)	Decimal	Percentage
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%
$\frac{4}{5}$	0.8	80%
$\frac{2}{5}$	0.4	40%
$\frac{1}{20}$	0.05	5%
$\frac{1}{3}$	0.3333	33.33% or $33\frac{1}{3}$%
$1\frac{1}{4}$	1.25	125%

Number Exercise 1.4

1. 27 kg
2. 68
3. 792 min
4. (a) \$1230 (b) 85%
5. 754
6. \$467.50
7. \$11,158.40
8. \$113.75
9. 71%
10. 12%
11. 4%
12. 12.5%
13. 7%
14. 34%
15. 28%
16. \$1435
17. \$5400
18. $ship\ speed = \frac{nautical\ miles}{hour} = \frac{345}{24} =$ 14.375 $knots$

    $slip = speed\ of\ engine - speed\ of\ ship$

    $slip = 16 - 14.375 = 1.625\ knots$

    $\therefore percentage\ slip = \frac{1.625}{16} \times 100$

    = 10.16% (2 d.p.)

Number Exercise 1.45

1. a) 2.8 b) 2.76 c) 2.757

d) 2.7570 e) 2.75698

2 a) 1.3123 b) 1

Number Exercise 1.5

1. 16 : 24 : 32
2. 350 kg Cu, 150 kg Zn
3. 36°:48°:96° angles A:B:C
4. 600ml
5. Tank 1 : $\frac{4}{5}$ $of\ the\ total\ mixture\ in\ Tank\ 3\ and\ is\ comprised\ \frac{9}{11}\ oil\ and\ \frac{2}{11}\ water$ 2

   Tank 2: : $\frac{1}{5}$ $of\ the\ total\ mixture\ in\ Tank\ 3\ and\ is\ comprised\ \frac{8}{11}\ oil\ and\ \frac{3}{11}\ water$

   $$total\ oil = \frac{9}{11} \times \frac{4}{5} + \frac{8}{11} \times \frac{1}{5} = \frac{36}{55} + \frac{8}{55} = \frac{44}{55}$$

   $$total\ water = \frac{2}{11} \times \frac{4}{5} + \frac{3}{11} \times \frac{1}{5} = \frac{8}{55} + \frac{3}{55} = \frac{11}{55}$$

   $$\therefore oil : water = \frac{44}{55} : \frac{11}{55} = 4 : 1$$
6. 42, 49
7. 1100 kg Cu, 540 kg Zn, 360 kg Ni.
8.
   a) 3.6kg tin in original quantity

   b) 3 kg tin added (33kg total of new alloy)

   c) 63.6% copper ($\frac{21}{33} \times 100$), 16.4% ($\frac{5.4}{33} \times 100$), zinc, 20%

Number Exercise 1.6

1. "Directly proportional" so $T = ke$

$$\frac{T}{e} = \frac{ke}{e}\ (dividing\ both\ sides\ of\ equation\ by\ e)$$

$$\frac{T}{e} = k$$

$$\frac{8}{5} = k\ or\ k = 1\frac{3}{5}\ or\ k = 1.6$$

Need to find e when T=12, recall $T = ke$

$$\frac{T}{k} = \frac{ke}{k}(dividing\ both\ sides\ of\ equation\ by\ k)$$

$$\frac{T}{k} = e$$

$$\frac{12}{1.6} = e$$

$$\frac{12}{1.6} = e$$

$$e = 7.5\ mm$$

$$\therefore length\ of\ spring\ under\ 12\ N\ load = 20 + 7.5 = 27.5\ mm$$

2. Direct proportion ⇒ $time = k \times square\ root\ of\ length$

$$t = k\sqrt{l}$$

$$\frac{t}{\sqrt{l}} = k$$

$$k = \frac{2.2}{\sqrt{40}} = 0.34785$$

$$when\ l = 90, t = k\sqrt{l}$$

$$t = 0.34785 \times \sqrt{90} = 3.3\ seconds$$

3. a) 6800 litres b) 90 minutes

4. Inverse proportion ⇒ $Volume = \frac{k}{Pressure}$

a) $V \times P = k$ $(by\ multiplying\ both\ sides\ of\ the\ equation\ by\ Pressure\ i.e.VP = \frac{kP}{P})$

$0.7 \times 178 = 124.6\ m^3kPa$

b) $V = \frac{k}{P}$

$$V = \frac{124.6\ m^3kPa}{182.5\ kPa} = 0.683\ m^3$$

5. a) 8.25 hours b) 5.5 hours

6. 360 ohms

7. 29.4MPa

8. 50 minutes or $(\frac{5}{6}hr)$

9. Direct proportion ⇒ $fuel\ per\ hour = k \times power$

$$\frac{fuel\ per\ hour}{power} = k$$

$$\frac{180}{800} = k$$

$$k = 0.225$$

$$when\ power = 900kW, fuel\ per\ hour = 0.225 \times 900 = 202.5\ kg/h$$

10. $\frac{effort_1}{load_1} = \frac{effort_2}{load_2}$

$\frac{4}{37} = \frac{effort_2}{5.6}$

$\frac{4 \times 5.6}{37} = effort_2$

$effort_2 = 0.605\ kg$

OR Alternative calculation

$since\ effort\ is\ directly\ proportional\ to\ load,$

$effort = k \times\ load$

$\frac{effort}{load} = k$

$k = \frac{4}{37} = 0.108$

$when\ load = 5.6kN\ \ effort = \ k \times\ load = 0.108 \times 5.6 = 0.605kg$

11. $\frac{1}{8}\ of\ tank\ is\ pumped\ by\ gen\ service\ pump\ in\ one\ hour$

$\frac{1}{4}\ of\ tank\ is\ pumped\ by\ ballast\ pump\ in\ one\ hour$

$\therefore total\ pumped\ in\ one\ hour\ by\ both\ pumps = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}\ of\ tank$

$\therefore time\ to\ pump\ tank = 1 \div \frac{3}{8} = \frac{8}{3}\ hours = 2\frac{2}{3} hours = 2\ hours\ 40\ mins$

Number Exercise 1.7

1.

(a)	16	(k)	529.09
(b)	625	(l)	0.0416
(c)	64	(m)	5
(d)	0.167	(n)	$\frac{1}{9}$
(e)	0.2	(o)	$\frac{1}{8}$
(f)	1	(p)	0.03125
(g)	5	(q)	4^{-2}
(h)	10.58	(r)	5^4
(i)	15625	(s)	2^5
(j)	529.09		

2. a) $\sqrt{3}$ b) $\frac{1}{\sqrt[3]{4}}$ c) $\frac{1}{\sqrt{3}}$

3. a) 100 b) $\frac{1}{100}$ c) $\frac{1}{100}$ d) 100

4. a) $\sqrt[4]{8^3} = 4.757$ b) $4^{\frac{1}{5}} = \sqrt[5]{4} = 1.3195$

Number Exercise 1.8

1. Log12
2. Log6
3. Log500
4. Log2
5. $x = 4$
6. $x = 2$
7. $x = 125$

Number Exercise 1.9

1. $7.6 \text{ X } 10^{9}$
2. $1.76 \text{ X } 10^{-11}$
3. $3.0084 \text{ X } 10^{-11}$
4. $1.760198 \text{ X } 10^{8}$

Number Exercise 1.10

1. 21.6 g
2. 50 m^3
3. 15300 litres
4. $6.4 \text{x} 10^{-5}$ m^3 or 0.000064 m^3
5. $1.8 \text{x} 10^{27}$
6. micro means divide by 1,000,000 (or multiply by 0.000001 or $\text{x}10^{-6}$)
7. 3375 g
8. 0.058mA = 0.058÷1000 = 0.000058A = 58μA
9. 17.49 m^3

Number Exercise 1.11

1. 80.1 cm^2
2. 38.9 ms^{-1}
3. $1.49 \text{x} 10^{-11}$ Newtons

Number Exercise 1.12

1. $Displ. = 8267.94\ m \quad Vol. = 8066.284\ m^3$
2. $Displ. = 7630.695\ m \quad TPC = 26.02\ Vol. = 7444.581\ m^3$
3. $0.005076\ MPa$
4. $0.032680\ MPa$