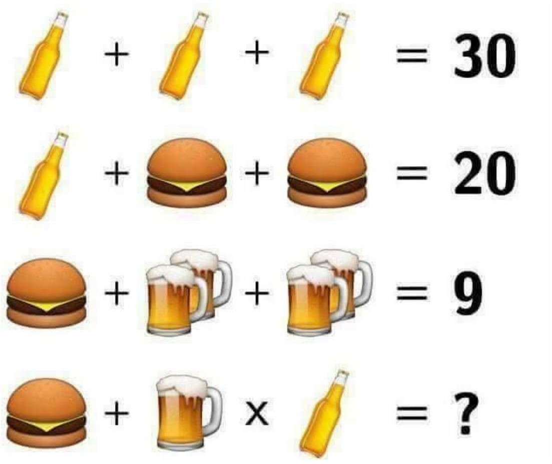
MODULE 2: ALGEBRA

Algebra uses numbers, letters and symbols to represent objects and concepts just as a language uses words for the same purpose..

For example:

Words representing a concept	**Algebraic equations representing a concept**
The area of a rectangle is found by multiplying its length by its width.	$A = l \times w$ If $l = 120$ metres and $w = 40$ metres, then the Area $A = 120 \times 40 = 480\ m^2$
The total resistance R_T, of resistors R_1, R_2, and R_3 connected in series.	$R_T = R_1 + R_2 + R_3$ If $R_1 = 6.2\ k\Omega$, $R_2 = 2.3k\Omega$, and R_3=8.4$k\Omega$, then $R_T = 6.2 + 2.3 + 8.4 = 16.9k\Omega$
The cost of two bolts and five nuts and ten washers	$C = 2b + 5n + 10w$ If the cost of a bolt, b=$2, the cost of a nut, n=$1 and the cost of a washer, w=50 cents, then the total cost of these tools is $2 \times 2 + 5 \times 1 + 10 \times 0.5 = \$14$



The cost of 3 bottles of lager.		3L = 30
The cost of 2 hamburgers and 1 bottle of lager.		L + 2H = 20
The cost of 1 hamburger and 2 pints of beer.		H + 4B = 9
The cost of 1 hamburger, 1 pint of beer and 1 bottle of larger.		(H + B + L = ?).

Terms

An **expression** consists of multiple **terms** separated by operators ($+, -, \times, \div$) but no equal $=$ sign.

Terms can be **constant** (a number) e.g. 5; or a **variable** (letter) e.g. x; or a **variable with a co-efficient** (number $\times$ letter) e.g. $5x$

'Like terms' are terms having the same variable with the same exponent (power).Only like terms can be added or subtracted. e.g. $x$and $5x$ are both 'x

e.g. $x + y + 3$ is an expression

e.g. $x + y + 3 + 2x - 4y$ is an expression; $x$and $2x$ are like terms; y and $-4y$ are also like terms.

Expressions and/or terms linked by an = sign form an **Equation**

e.g. an equation could look like:

$$2x^2 + 3y^2 + 3x^2 = 7$$

In the above example, identify:
a coefficient, a variable, a term, like terms, a constant, an expression

Language Used in Mathematical Problems:

Simplify	means 'make small', by various methods that you will learn, keeping the value of the expression unchanged.
Solve	means find the value of the unknown/s.
Substitute	means if we know the numerical value of a variable we can replacethat variable in an equation or expression by its numerical value.
Evaluate	means substituting the given values for all variables in an expression and then working out the value of the whole expression.
Expand	means open up the brackets.
Factorise	means take out the common factors and form brackets. It is the opposite of expanding.

Basic Rules

Rules of Zero

- Any term multiplied by zero is zero.
- Zero divided by any term is zero.
- A term divided by zero is not defined in mathematics but is considered to be infinity.

Rules of One

- The number one is often hidden. A variable without a coefficient actually has one as a hidden coefficient e.g. x means $1 \times x$

- Any term can be converted into a fraction by dividing by one.. e.g. y can be written as $\frac{y}{1}$ and e.g. 25 can be written as $\frac{25}{1}$.
- A variable without an exponent actually has an exponent of one e.g. x is the same as x^1 and 25 is the same as 25^1

Some Rules of Algebra

There are certain rules governing the manipulation of algebraic expressions, and these are shown below.

$4a$	means a + a + a + a (or $4 \times a$)
$4a + 3a$	equals 7a
$5a - 7a$	equals –2a
ab	means $a \times b$
$3pq$	means $3 \times p \times q$
y^3	means $y \times y \times y$
$(3a)^2$	means $3a \times 3a$
$4a \times 3a$	equals $12a^2$

It is important to carry out operations in a certain order. This order is to work the brackets first, then carry out the multiplication and division and end with addition and subtraction (BEDMAS).

There are 3 laws that must be understood with respect to algebra, and these are shown below:

Reminder : Rules of signs

The product (multiplying) of two terms with the same sign (+, +) or (-, -) is always positive. The product of two terms with different signs (+, -) or (-, +) is always negative.

Examples: Solve the following expressions:

$2 \times (-3) = -6$

$9 \div (-3) =$ ______

$(-3) \times (-4) =$ _______

$(-8) \div (-2) =$ _______

$(-4)^2 = (-4) \times (-4) = 16$

$(-5)^3 = (-5) \times (-5) \times (-5) = -125$

$14 + (-6) = -8$

$-14 + (-6) =$ ______

Associative Law	$a + (b + c) = (a + b) + c$	Numerical Example: $1 + (2 - 3) = (1 + 2) - 3$
Commutative Law	$\boldsymbol{a + b = b + a}$ $\boldsymbol{ab = ba}$	Numerical Example: $1 + 2 = 2 + 1$ Numerical Example: $1 \times 2 = 2 \times 1$
Distributive Law	$\boldsymbol{a\,(b + c) = ab + ac}$ $\boldsymbol{\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}}$	Numerical Example: $1 \times (2 - 3) = 1 \times 2 - 1 \times 3$ Numerical Example: $\frac{1+2}{3} = \frac{1}{3} + \frac{2}{3}$ Algebraic Example: $\frac{1+y}{(x+3)} = \frac{1}{x+3} + \frac{y}{x+3}$

Laws of indices in Algebra

Recall: when multiplying, the indices are added.

$x^m \times x^n = x^{m+n}$	Example: $p^2 \times p^3 = (p \times p) \times (p \times p \times p) = p^5$

Recall: when dividing, the index in the denominator is subtracted from the index in the numerator.

$\frac{x^m}{x^n} = x^{m-n} = \frac{1}{x^{n-m}}$	Example: $\frac{y^2}{y^5} = y^{(2-5)} = y^{-3} = \frac{1}{y^3}$ Example: $\frac{q^5}{q^2} = q^{(5-2)} = q^3$

Recall: Powers (indices) of powers (indices) - i.e. Indices and brackets. Every number and variable in the bracket is raised to the index outside the bracket i.e. the indices are multiplied.

$(x^m)^n = x^{mn}$ $(xy)^m = x^m y^m$	Example: $(x^5)^2 = x^{5\times2} = x^{10}$ Example: $(2\mathrm{y}^5)^2 = 2^2 \times \mathrm{y}^{5\times2} = 4\mathrm{y}^{10}$

When a variable has an index of zero, its value is 1

$x^0 = 1$	Example: $5x^0 = 5$

A variable raised to a negative power is the reciprocal of that variable raised to a positive power. i.e. 'flip' the variable

$x^{-n} = \frac{1}{x^n}$ or $\frac{1}{x^{-n}} = x^n$	Example: $p^{-3} = \frac{1}{p^3}$ Example: $\frac{1}{q^{-3}} = q^3$

When a variable is raised to a fractional power the denominator of the fraction is the root of the variable, and the numerator is the power of the variable.

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$	Example: $x^{\frac{1}{3}} = \sqrt[3]{x}$ Example: $x^{\frac{2}{3}} = \sqrt[3]{x^2}$ Example: $x^{\frac{-2}{3}} = \frac{1}{\sqrt[3]{x^2}}$ Example: $\frac{1}{x^{\frac{-2}{3}}} = x^{\frac{2}{3}} = \sqrt[3]{x^2}$

Simplifying Algebraic Expressions

Addition and subtraction

Only like terms can be added or subtracted. Like terms have the same letters and powers but may have different coefficients.

Examples: Simplify the following expressions.

$4x + 3x = 7x$ (both x type terms can simplify by adding the coefficients)

$4x + 3x + 2y = 7x + 2y$ (only x terms simply)

$4x + 3x + 2y - y =$ ________________

$5a + 2c + 4b - 4a - 6c + 6a =$ ________________________

$3ab - a^2 + 4b^2 - 2ab - a^2 - b^2 = ab - 2a^2 + 3b^2$

$5ab - 6a^2 + 4b^2 - 8ab - 2a^2 + 3b^2 =$ _____________________

$p^2 + pq - 3q^3 + 4p^2 - 3pq + 2q^3 - 3p^2 + 5 =$ ___________________

$5x + 7 - (3x - 4) = 5x + 7 - 3x + 4 =$ _________________________

$9 - 2x + 3x^2 + 6 - 10x - (x^2 + 2) =$ _______________________________

$6x^3 + 2x - x^2 + x^3 - 3x^2 =$ _____________________________

Multiplication and division

These operations can be done in any order. Combine numbers and letters separately taking account of any powers that may be present. Common factors may be cancelled.

Examples: Simplify the following expressions.

$3a^2b \times 6ab^3 = 18a^3b^4$ *(multiply the numbers, and multiply the letters, remember multiplication signs are assumed between letters and numbers)*

$-2a^3b^2 \times 3ab^3 \times 5ab =$ ____________________

$$4xy^3 \div 6xy = \frac{4xy^3}{6xy} = \frac{2y^2}{3}$$

$12xy^2 \div 24x^2y =$ ___________ $=$ _______________

$36pq^3r^2 \times 2r^3 \div 8q^3 =$

Algebra Exercise 2.1 Simplifying Algebraic Expressions

1. Simplify by combining like terms, where possible.

a) $10x^2 - 3x^2$ b) $2xy - 4y^2 + 3xy + 5y^2 - 7xy$

c) $5a + 4 - 9a + 3$ d) $4a^2b - 6ab^2 + 3ab^2 - 7ab^2$

e) $4a - 2a^2$ f) $5x^2y - 3xy^2 + 2x^2y - 4xy^2$

g) $2y - 4x - 5y + 3x - 10x$ h) $6x - 8x + 4xy + 2x - 3xy$

2. Simplify the following

a) $5a \times (-4a)$ b) $(-3x)^3$

c) $-4 \times \sqrt{xy} \times 3x^3y \times \sqrt{xy}$ d) $(-3x)^2$

e) $3x^2y \div (-18xy^2)$ f) $-3a^3b \div 6ab^2$

g) $-3a^2b \times 6ac \div 2abc$ h) $-20xy^2 \times (2x)^3 \div (-4x^2)$

3. Simplify the following giving the answers in positive index form

(a) $(16x^6)^{½}$

(b) $(2x^2)^{-3}$

(c) $(9x^2)^{3/2}$

(d) $\left(\frac{3}{x^3}\right)^{-2}$

(e) $(16x^2)^{\frac{3}{4}}$

(f) $(2x)^3 \times (4x^2y)^{-2}$

(g) $\frac{6m^5}{\sqrt{m^{16}}}$

(h) $(2xy^2)^{-3}$

(i) $\frac{(3xy)^2}{3x^{-1}}$

(j) $\frac{2y^{\frac{3}{2}}}{(4y)^{\frac{1}{2}}}$

(k) $\frac{(2x)^{-3} \times 4x^2y}{y}$

Expanding Expressions (removing brackets)

The opposite of factorising : removing brackets from an expression.

The Distributive Law is used for expanding brackets. All terms inside the brackets are multiplied by the terms outside the brackets

Expanding single brackets

Worked Example 1

$$3(2a - b^2 + 3c) = 3 \times 2a - 3 \times b^2 + 3 \times 3c = 6a - 3b^2 + 9c$$

A negative sign in front of a bracket means that the signs of all the terms inside the bracket should be changed when the bracket is removed.

Worked Example 2

$$-(4c - a + 3b) = -1 \times 4c - 1 \times -a - 1 \times 3b = -4c + a - 3b$$

Expanding multiple brackets

Worked Example 3

$$(2a - 3)(a + 4) = 2a \times a + 2a \times 4 - 3 \times a - 3 \times 4$$

$$= 2a^2 + 8a - 3a - 12$$

$$= 2a^2 + 5a - 12$$

Each term in the first bracket is multiplied by each term in the second bracket. Then like terms are combined to simplify the result.

Worked Example 4: Expand $(2a + 3b)^2$

$$(2a + 3b)(2a + 3b) = 4a^2 + 6ab + 6ab + 9b^2$$

$$= 4a^2 + 12ab + 9b^2$$

Worked Example 5: Expand $(x - 2y)^2$

$$(x - 2y)(x - 2y) = x^2 - 2xy - 2xy + 4y^2$$

$$= x^2 - 4xy + 4y^2$$

Algebra Exercise 2.2 Expanding

1. Expand the brackets and combine like terms.

a)	$5x - 7 - (3x - 7)$	b)	$3x^2 - 4x + 1 - 2(x^2 - x + 1)$
c)	$3a\,(a + b) + 2b\,(a + b)$	d)	$4x - 4y + 1 - 3(x + y - 2)$
e)	$a^2 + 3b - 5b(2b - 4)$	f)	$z^2 + zx + 6z(x - z)$

2. Expand the following expressions and collect like terms.

a)	$(x - 3)\,(x + 6)$	b)	$(x + 2y)\,(2x + y)$
c)	$(2x - 7)\,(4x + 5)$	d)	$(2x + y)^2$
e)	$(3a - 4b)^2$	f)	$(2x - 3y)\,(2x + 3y)$
g)	$(2 - x)\,(x + 5)$		

Factorising (writing with brackets)

Where two or more terms in an algebraic expression have a common factor then it is possible to show this factor outside a bracket. These common factors will normally be identified by inspection as shown below.

Worked Example 1

Factorise: $xa + xb$

<u>Step 1:</u> $x($ *Look for factors common to each term i.e. x. Write this factor followed by an open bracket*

<u>Step 2:</u> $x(a + b)$ *Add the remaining terms inside the bracket. Note that you can always check your answer by expanding the brackets to get back to the original expression.*

Worked Example 2

Factorise $3a^2 + 6ac$

<u>Step 1:</u> $3a($ *Look for factors common to each term i.e. 3 and a. Write these 2 factors down followed by an open bracket*

<u>Step 2:</u> $3a(a + 2c)$ *Add the remaining terms inside the bracket. Note that you can always check your answer by expanding the brackets to get back to the original expression.*

Worked Example 3

Simplify the following expression: $\frac{a+3a^2}{2a}$

<u>Step 1:</u> $\frac{a(1+3a)}{2a}$ *This is a fraction so firstly, try and factorise the numerator and denominator*

<u>Step2:</u> $\frac{\cancel{a}(1+3a)}{2\cancel{a}}$ *Look for possible numbers or letters to cancel from both the numerator and denominator. Either whole brackets or numbers or variable outside the brackets.*

<u>Step3:</u> $\frac{1+3a}{2}$

Factorising double brackets

Given that $(x + a)(x + b) = x^2 + x(a + b) + ab$,
note that the constant (number) term, ab is the product of a and b,
and the coefficient of x is $a + b$.

Worked Example 1:

Factorise $x^2 + 7x + 12$

+	X
+7	+12
	Factors
	1,12
	2,6
	3,4

Step 1: Use this calculation working panel to help discern factors which both
i) multiply to the constant term +12, and
ii) also add to the 'x coefficient' of +7

Step 2: $(x + 3)(x + 4)$ *3 and 4 are the two factors of +12 which add to +7. Note it does not matter which order the brackets are written.*

> *Note: When the 'co-efficient of x' (the number with the x term) is positive then both factors must either be positive as shown above, or both must be negative.*

Partially worked Example 2:

Factorise $x^2 - 7x + 12$

+	X
-7	+12
	Factors
	1,12
	2,6
	-3,-4

Step 1: Look for factors which both
i) multiply to the constant term +12, and
ii) also add to the 'x coefficient' of – 7

Partially worked Example 3:

Factorise $x^2 - 7x - 60$

+	X
-7	-60
	1,60
	2,30
	3,20
	4,15
	+5,-12
	6,10

Step 1: Look for factors which both
i) multiply to the constant term -60, and
ii) also add to the 'x coefficient' of – 7

Special Case – The difference of 2 Squares

$x^2 - y^2 = (x + y)(x - y)$

Example: $x^2 - 25 = (x + 5)(x - 5)$

Example: $4x^2 - 9y^2 = (2x + 3y)(2x - 3y)$

Special Case - Perfect Square (Squaring a binomial)

$x^2 + 2xy + y^2 = (x + y)^2$

$x^2 - 2xy + y^2 = (x - y)^2$

Factorising binomials when the co-efficient of x term is >1 *(not assessed)*

Up until now we have not really considered the factors of the x^2 term, but this was because in each case so far the coefficients were 1 (unity).

When the coefficient of the x^2 term is not unity then it is necessary to establish the two pairs of factors as follows in this worked example:

Example: Factorise $10x^2 - 19x + 6.$

Step 1	Multiply the coefficient of the x^2 term by the constant.	10 x 6 =60
Step 2	Determine 2 factors of this result which will add to the coeeficient of the *x* term	Factors of +60 which add to -19: 1,60 2,30 3,20, 4,15 5,12 -4 and -15 will multiply to 60 and add to -19
Step 3	Rewrite expression with two *x* terms using the factors found in step 2	$10x^2 - 4x - 15x + 6$
Step 4	Group the expression into two groups, the first two terms and the last two terms.	$\underline{10x^2 - 4x}$ $\underline{-15x + 6}$
Step 5	Factorise each group in such a way that the bracketed terms in each group are the same.	$2x(5x-2) \; -3(5x-2)$
Step 6	Note the bracket is now a common factor	Common factor is $(5x-2)$
Step 7	Factorise using the common factor.	$(5x-2)(2x-3)$

Algebra Exercise 2.3

1. Factorise the following expressions, looking for common factors first.

a) $7a + 28ac$
b) $4x^2y - 8xy^2$
c) $5xy^2 - 20yz$
d) $\pi R^2h + 2\pi R$
e) $\sin^2\beta - 3\sin\beta\cos\beta$
f) $x(x-4) + 5(x-4)$
g) $2a^2 - 8b^2$
h) $x^2 + 6x + 8$
i) $a^2 - 19a + 18$
j) $x^2 + x - 20$
k) $8a^2 - 200b^2$
l) $x^2 - 2x - 3$
m) $x^2 + 11x - 60$
n) $x^2 + 15x + 36$

Algebraic Fractions

The rules of fractions in algebra are the same as those in arithmetic.

Adding and subtracting Algebraic Fractions

To add and subtract fractions it is necessary to express the fractions with a common denominator by multiplying the numerator and denominator of each fraction by the appropriate factor. Use brackets where possible to ensure that the signs are correct.

Worked Example 1:

$$\frac{x}{2}+\frac{x}{5}=\frac{5x+2x}{2\times 5}=\frac{7x}{10}$$

Worked Example 2:

$$\frac{2}{x+1}+\frac{3}{x-2}=\frac{2(x-2)+3(x+1)}{(x+1)(x-2)}$$

$$=\frac{2x-4+3x+3}{(x+1)(x-2)}$$

$$=\frac{5x-1}{(x+1)(x-2)}$$

Worked Example 3:

$$\frac{3}{x-3}-\frac{2}{x-1}=\frac{3(x-1)-2(x-3)}{(x-3)(x-1)}$$

$$=\frac{3x-3-2x+6}{(x-3)(x-1)}$$

$$=\frac{x+3}{(x-3)(x-1)}$$

Algebra Exercise 2.4

1. Write the following as single fractions.

a) $\frac{2}{5}+\frac{4}{3}$ b) $\frac{a}{3}-\frac{2a}{15}$ c) $\frac{2c}{3}+\frac{c}{2}-\frac{5c}{12}$

d) $\frac{5}{2a}-\frac{2}{3a}$ e) $\frac{3}{2a}+\frac{4}{a}$ f) $\frac{5}{2a}-\frac{3}{2b}$

g) $\frac{1}{R_1}+\frac{1}{R_2}$ h) $\frac{x}{x+1}-\frac{x}{x-1}$ I) $x+6-\frac{x^2}{x-6}$

Multiplying and Dividing Algebraic Fractions

When multiplying algebraic fractions, simply multiply the numerators, and multiply the denominators. Simplify your answer if possible.

When dividing fractions, invert the second fraction and multiply.

Worked Example 1:

$$\frac{8x^2}{5y} \times \frac{10y^3}{2x} = \frac{80x^2y^3}{10xy}$$
$$= 8xy^2$$

Worked Example 2:

$$\frac{\frac{4x}{3y}}{\frac{4x^2}{15y^2}} = \frac{4x}{3y} \times \frac{15y^2}{4x^2}$$
$$= \frac{60xy^2}{12yx^2}$$
$$= \frac{5y}{x}$$

Simplifying algebraic fractions

When simplifying fractions, only factors common to both the numerator and the denominator can be cancelled.

For example, the expression $\frac{x+y}{2x}$ cannot be further simplified as x is not a common factor to both terms in the numerator.

In some cases, algebraic fractions can be simplified by factorising and cancelling. To fully simplify an algebraic fractions, check if the numerator or denominator can be factorised.

Worked Example 1:

Simplify $\frac{x^2 - a^2}{x+a}$

$= \frac{\cancel{(x+a)}(x-a)}{\cancel{(x+a)}}$ *Step 1: Try factorise the numerator and denominator and see if any brackets cancel)*

$= x - a$

Worked Example 2:

Simplify $\frac{9}{(x^2-9)} \div \frac{3}{(x-3)}$

$= \frac{9}{(x^2-9)} \times \frac{(x-3)}{3}$ *Step 1: Flip second fraction and multiply*

$= \frac{9(x-3)}{3(x+3)(x-3)} = \frac{3}{(x+3)}$ *Step 2: Factorise the numerator and denominator and see if any brackets or terms cancel)*

Worked Example 3:

Factorise and simplify the following expression: $\frac{x^2+6x+9}{x+3}$

$= \frac{(x+3)(x+3)}{x+3}$ Step 1: *This is a fraction so firstly, try and factorise the numerator and denominator. Notice the numerator quadratic (x^2) so may be factorised into two brackets.*

$= \frac{\cancel{(x+3)}(x+3)}{\cancel{x+3}}$ Step2: *Cancel brackets from numerator and denominator*

$= x + 3$

Worked Example 4:

Factorise and simplify the following expression: $\frac{x-2}{x^2-4}$

$= \frac{x-2}{(x+2)(x-2)}$ Step 1: *This is a fraction so firstly, try and factorise the numerator and denominator. Notice the denominator is quadratic (x^2) so may be factorised into two brackets.*

$= \frac{\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}$ Step2: *Cancel brackets from numerator and denominator*

$= \frac{1}{(x+2)}$

Algebra Exercise 2.5

1. Simplify the following fractions.

a) $\frac{6x}{4x}$

b) $\frac{30}{6x^2}$

c) $\frac{20-5x^2}{5x^2}$

d) $\frac{2p^3}{q} - \frac{q}{30p} - \frac{12}{p^2q}$

e) $\frac{a+2x}{a-2x} \times \frac{a-2x}{a+2x}$

f) $\frac{x^2-y^2}{x+y}$

g) $\frac{a^2-2ab}{2ab}$

h) $\frac{x^2y+xy^2}{xy}$

i) $\sqrt{\frac{a+b}{a^2-b^2} \times \frac{1}{a-b}}$

j) $\frac{a+b}{a^2-b^2}$

k) $\frac{1}{x+4} - \frac{1}{x+2}$

2. Factorise and simplify the following fractions.

a) $\frac{2x+2}{x+1}$

b) $\frac{\pi r^2 h}{rh}$

c) $\frac{x^2-2x-3}{x+1}$

d) $\frac{x^2-25}{x-5}$

e) $\frac{x^2+8x}{2x}$

f) $\frac{x^2+12x}{x^2-144}$

g) $\frac{2x^2+12x}{2x}$

Types of Algebraic Equations

An equation with highest power of the unknown variable of 1 is called a linear equation.
e.g. $2x + 3 = 15$ *(recall x^1 is the same as writing x, the power is not written))*

An equation with highest power of the unknown variable of 2 is called a quadratic equation.
e.g. $2x^2+3x=10$

An equation with highest power of the unknown variable of 3 is called a cubic equation.
e.g. $x^3+4x^2+3x=1$

This course covers solving linear and quadrative equations only.

Solving Linear (Simple) Equations

An equation is a statement that two quantities are equal, and has an equals sign dividing the equation into the left hand side (LHS) and the right hand side (RHS).

As described above, a linear equation is one where the variable has an exponent of 1. Linear equations are also called simple equations. If the equation of a linear equation is drawn out as a graph, the graph works out to be a straight line.

The general method of solving linear equations is to get the unknown quantity or variable on the LHS and the constants on the RHS. The way that the variable is taken to the LHS may be using some or all of the following

Remove fractions by multiplying with the denominator of each fraction in turn, or multiplying by the lowest common denominator (LCD) of all the fractions.

Remove brackets

Collect like terms

Add a number to each side (transposing)

Subtract a number from each side (transposing)

Multiply or divide each side by a number (cross multiplying)

A number of examples will now be shown to illustrate the above actions.

The most important thing to remember is that you must always carry out the same operation to both sides of the equation.

Worked Example 1:

$5\mathrm{x} + 8 = 2\mathrm{x} + 13$

$5\mathrm{x} - 2\mathrm{x} + 8 = \cancel{2\mathrm{x} - 2\mathrm{x}} + 13$ *Subtract 2x from both side to eliminate it on the RHS*

$3\mathrm{x} \cancel{+ 8 - 8} = +13 - 8$ *Subtract 8 from both side to eliminate it on the LHS*

$3\mathrm{x} = 5$

$\frac{3\mathrm{x}}{3} = \frac{5}{3}$ *Divide both side by 3 to cancel the 3 from the 3x on the LHS*

$\mathrm{x} = \frac{5}{3}$

Worked Example 2:

$6(\mathrm{a} - 2) = 2\mathrm{a}$

$6\mathrm{a} - 12 = 2\mathrm{a}$ *Expand LHS (remove brackets)*

$6\mathrm{a} - 2\mathrm{a} - 12 = \cancel{2\mathrm{a} - 2\mathrm{a}}$ *Subtract 2a from both side to eliminate it on the RHS*

$4\mathrm{a} - 12 + 12 = 0 + 12$ *Add 12 to both side to eliminate it on the LHS*

$4\mathrm{a} = 12$

$\frac{4\mathrm{a}}{4} = \frac{12}{4}$ *Divide both side by 4 to cancel the 4 from the 4a on the LHS*

$\mathrm{a} = 3$

Worked Example 3:

$\frac{3x - 2}{4} = \frac{5x - 7}{6}$

$6(3x - 2) = 4(5x - 7)$ *Multiply both side by 6 and 4 to eliminate the denominators on both sides. This is often referred to as cross multiplying*

$18x - 12 = 20x - 28$ *Expand (remove brackets)*

$18x - 20x = -28 + 12$ *Gather x terms on one side or equation and numbers on the other*

$-2x = -16$

$\frac{-2x}{-2} = \frac{-16}{-2}$ *Divide both side by -2 to cancel the -2 from the -2x on the LHS*

$x = 8$

Algebra Exercise 2.6

1. Solve the following equations.

a)	$2x = 18$	b)	$x + 3 = 7$
c)	$y - 3 = 5$	d)	$3p = 21$
e)	$a + 5 = 11$	f)	$\frac{x}{4} = 5$
g)	$\frac{y}{3} = 9$	h)	$b - 7 = 2$
i)	$2 - 4y = 30$	j)	$3x - 5 = 22$
k)	$2x + 12 = -30$	l)	$4 - 3x = -5$
m)	$5a + 7 = 7a - 21$	n)	$4(2x - 3) = 12$
o)	$3(4x - 3) - 2(5x - 1) = 1$	p)	$\frac{7}{2x} = 42$
q)	$\frac{x-2}{x-3} = 4$	r)	$\frac{2x-1}{3x+1} = \frac{6x-1}{9x-3}$
s)	$\frac{2x-1}{4x+1} = \frac{6x-1}{12x+11}$	t)	$\frac{2x}{x+1} - \frac{x-1}{x+3} = 1$
u)	$\frac{x-1}{x+2} - \frac{x-3}{2-x} = 2$	v)	$\frac{3}{x+4} - \frac{2}{x+3} - \frac{1}{x+1} = 0$

Transposition/Rearranging of Formulae *(Key Course Learning Outcome)*

A formula is a collection of algebraic symbols which express a relationship. For example, $V = IR$ is a formula representing the relationship between voltage, current and resistance. It is sometimes necessary to manipulate and change the subject of a formula. For example, in the formula $V = IR$, V is the subject. However, if the voltage and resistance are already known, we may wish to calculate current. In this case, we need to re-arrange (transpose) the formula to make current, I the subject. To do this we need to use inverse operations. A list of mathematical operations and inverse (opposite) operations are shown below:

Mathematical Operation	**Inverse Operation**
Addition	Subtraction
Multiplication	Division
Square	Square root
Cube	Cube root
x^n	$\sqrt[n]{x}$
Make negative (change sign)	Make positive (change sign)
Invert (flip upside down)	Invert

To make I the subject of formula $V = IR$ we need to isolate I on one side of the equation leaving all other symbols on the other side. To do this we need to examine the symbols which are on same side of the equation as I and find a way to transpose them away from I whilst keeping the equation in balance.

e.g. Use the following steps when transposing $V = IR$ to make I the subject of the formula:

$V = IR$ *By examining the RHS, note that I is <u>multiplied</u> by R.*

$\frac{V}{R} = \frac{I\cancel{R}}{\cancel{R}}$ *To transpose the R away from the RHS we must use the inverse operation of multiplying which is <u>dividing</u>.*

$\frac{V}{R} = I$ *By dividing both sides by R note that the R symbols on the RHS will cancel out.*

Worked Examples of Transposition/Rearranging of Formulae

1. Formulae involving addition and subtraction	
Transpose $p = q + r + s + t$, *making* r *the subject of the formula.* $p - q - s - t = \cancel{q} + r \cancel{+ s + t - q - s - t}$ $p - q - s - t = r$	*Note* q *and* s *and* t *are all being* *added* *to* r *Subtracting* q, s *and* t *from both sides will leave* r *as the subject*
Transpose $a + b = w - x + y$, *making* x *the subject of the formula.* $x + a + b = w \cancel{- x + x} + y$ $x \cancel{+ a + b - a - b} = w + y - a - b$ $x = w + y - a - b$	*Note:* x *is* *negative* *on the RHS. The subject of a formula should not be negative.* *Adding* x *to both sides will transpose* x *leaving it* *positive* *on the LHS* *Note* a *and* b *are being* *added* *to* x *Subtracting* a *and* b *from both sides will leave* x *as the subject*
2. Formulae involving multiplication	
Transpose $(x + y)^2 = 2w$ *making* w *the subject* $\frac{(x+y)^2}{2} = \frac{\cancel{2}w}{\cancel{2}}$	*Note* w *is currently being* *multiplied* *by 2.* *Dividing* *all terms on both sides by 2 will leave* w *as the subject*
Transpose $C = 2\pi r$ *to make* r *the subject* $\frac{C}{2\pi} = \frac{\cancel{2\pi}r}{\cancel{2\pi}}$	*Note* r *is currently being* *multiplied* *by* 2π. *Dividing* *all terms both sides by* 2 *and* π *will leave* r *as the subject*
3. Formulae involving division (fractions)	
Transpose $I = \frac{V}{R}$ *to make* V *the subject* $I \times R = \frac{V \times \cancel{R}}{\cancel{R}}$ $IR = V$	*Note* V *is currently being* *divided* *by* R *Multiplying* *all terms on both sides by* R *will leave* V *as the subject*
Transpose $I = \frac{V}{R}$ *to make* R *the subject* $I \times R = \frac{V \times \cancel{R}}{\cancel{R}}$	*Note* R *is currently the denominator of the fraction on the RHS. The subject of a formula should not remain the denominator (unless we have been asked to make* $\frac{1}{R}$ *the subject).* *Multiplying both sides by* R *will remove it from the denominator on the RHS*

$\frac{\cancel{I}R}{\cancel{I}} = \frac{V}{I}$ $R = \frac{V}{I}$	*Dividing both sides by I will leave R as the subject.*
4. Formulae involving multiplication and addition/subtraction	
Transpose $F = kx + a$ *making* x *the subject of the formula* $F - a = kx \cancel{+ a - a}$ $\frac{F - a}{k} = \frac{\cancel{k}x}{\cancel{k}}$	*Aim to isolate the term containing* x *i.e. the* $'kx'$ *on one side of the equation by subtracting* a *from both sides.* *Divide both sides by* k *to cancel the* k *from the* kx *on the RHS and leave* x *as the subject.*
Tranpose $s = ut + \frac{at^2}{2}$ *making* u *the subject* $s - \frac{at^2}{2} = ut \cancel{+ \frac{at^2}{2} - \frac{at^2}{2}}$ $\frac{s}{t} - \frac{at^2}{2t} = \frac{ut}{t}$ $\frac{s}{t} - \frac{at^{\cancel{2}}}{2\cancel{t}} = \frac{u\cancel{t}}{\cancel{t}}$ $\frac{s}{t} - \frac{at}{2} = u,\ \ or\ \ \frac{2s - at^2}{2t} = u$	*Aim to isolate the term containing* u *i.e.the* $'ut'$ *on one side of the equation by subtracting* $\frac{at^2}{2}$ *from both sides.* *Divide every term on both sides by* t *to cancel the* t *from the* $'ut'$ *on the RHS leaving* u *as the subject. (Note that every term on the LHS must be divided by* t*)* *Cancel terms. Note: there may be multiple ways of expressing the answer correctly (e.g. as a single fraction or multiple terms)*
Transpose $s = ut + \frac{at^2}{2}$ *making* a *the subject* $s - ut = \cancel{ut - ut} + \frac{at^2}{2}$ $\frac{2(s - ut)}{t^2} = \cancel{ut - ut} + \frac{2at^2}{2t^2}$ $\frac{2(s - ut)}{t^2} = a,\ \ \ \ or\ \ \frac{2s - 2ut}{t^2} = a$ $or\ \frac{2s}{t^2} - \frac{2u}{t} = a$	*Aim to isolate the terms containing* a *i.e.* $\frac{at^2}{2}$ *on one side of the equation by subtracting* ut *from both sides.* *Note a is currently being <u>multiplied</u> by t^2 and <u>divided</u> by 2* *Therefore, <u>Divide</u> all terms on both sides by t^2 and <u>multiply</u> all terms by 2* *Cancel terms. Note: there may be multiple ways of expressing the answer correctly (e.g. as a single fraction or multiple expanded terms)*

Algebra Exercise 2.65

Make the symbol shown in brackets the subject of the formula

1. $a + b = c + d \quad (d)$
2. $a + b = c - d \quad (d)$
3. $Q = 6R \quad (R)$
4. $STW = \frac{R}{3} \quad (R)$
5. $STW = \frac{R}{3} \quad (T)$
6. $y = mx + c \quad (c)$
7. $y = mx + c \quad (x)$
8. $y = \frac{p}{1-q} \quad (p)$
9. $y = \frac{p}{1-q} \quad (q)$
10. $Q = \frac{2}{3}R+15 \quad$ (R)

Transposition Worked Examples (cont.)

5. Formulae with powers and roots	
Transpose $A = \pi r^2$ *making* r *the subject.* $\frac{A}{\pi} = \frac{\cancel{\pi} r^2}{\cancel{\pi}}$ $\sqrt{\frac{A}{\pi}} = \sqrt{r^2}$ $\sqrt{\frac{A}{\pi}} = r$	*Divide both sides by* π *Use the inverse operation of squaring to back-out the* r *from the* r^2 *i.e. square root both sides of the equation.*
Transpose $k = \frac{1}{2}mv^2$ *making* v *the subject.* $\frac{2k}{m} = \frac{\cancel{2m}v^2}{\cancel{2m}}$ $\sqrt{\frac{2k}{m}} = v$	*(perhaps rewrite question as a single fraction or using a decimal number i.e.* $k = \frac{mv^2}{2}$ *or* $k = 0.5mv^2$*)* *Divide both sides by* m *and multiply both sides by 2.* *Square root both sides leaving* v *as the subject*
Transpose $c^2 = a^2 + b^2$ *making* a *the subject.* $c^2 - b^2 = a^2$ $\sqrt{c^2 - b^2} = \sqrt{a^2}$ $\sqrt{c^2 - b^2} = a$	*Subtract* b^2 *from both sides* *Square root both sides leaving* a *as the subject.*
6. Formulae with the new subject in a bracket	
Transpose $s = \frac{(u+v)}{2}$ making v the subject. $2s = \frac{\cancel{2}(u+v)}{\cancel{2}}$ $2s - u = \cancel{u - u} + v$ $2s - u = v$	*Aim to isolate the bracket first.* *Multiply both sides by 2 leaving* v *as the subject* *Subtract* u *from both sides*
7. Formulae where the new subject appears in more than one place	
Transpose $3p = pq + 1$ making p the subject.	*Aim to rearrange the formula so that all the terms which contain the new subject* p *are on the same side.*

$3p - pq = \cancel{pq - pq} + 1$ $3p - pq = 1$ $p(3 - q) = 1$ $\frac{p\cancel{(3-q)}}{\cancel{(3-q)}} = \frac{1}{(3-q)}$ $p = \frac{1}{(3-q)}$	*Subtract pq from both sides* *Factorise using p as a common factor* *Divide both sides by the bracket leaving p as the subject.*
Transpose $x = \frac{(y+1)}{y}$ making y the subject. $xy = y + 1$ $xy - y = 1$ $y(x - 1) = 1$ $y = \frac{1}{(x-1)}$	***Remember you do not want the new subject in the denominator of a fraction*** *;* *first rearrange so that the new subject y is NOT in the denominator.* *Then further rearrange so all the terms which contain the subject y are on the same side.* *Then factorise using the new subject y as the common factor* *Divide by the bracket leaving y as the subject*

Algebra Exercise 2.66

1. Transpose the formulas making the letter in brackets the subject.

a)	$V = IR$	**(I)**	b)	$E = mc^2$	**(c)**
c)	$T = 2\pi\sqrt{l/g}$	**(l)**	d)	$L = L_0(1 + At)$	**(t)**
e)	$\omega = \frac{1}{\sqrt{LC}}$	**(L)**	f)	$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$	**(R_1)**
g)	$T = \frac{k(x - a)}{a}$	**(x)**	h)	$\frac{P}{Q} = \frac{x - y}{y}$	**(y)**
i)	$A = \frac{p + q}{p - q}$	**(p)**	j)	$R^2 = k(a^2 - x^2)$	**(x)**
k)	$I = \frac{E - e}{R + r}$	**(r)**	l)	$a^2 = b^2 + c^2 - 2bc\cos\theta$	**(cosθ)**
m)	$p = \frac{W}{g}(v - u)$	**(v)**	n)	$Mg(h + x) = \frac{\sigma^2 V}{2E}$	**(σ)**

Quadratic Equations

A quadratic equation is one in which the highest power of the unknown is 2. As an example $x^2 + 2x - 5 = 0$ is a quadratic equation. In this equation the unknown value x is known as the root of the equation and to solve the equation means to find the value of this root. There are a number of methods of solving these equations and these will be considered below.

Solving quadratic equations by setting equal to zero and factorising

We have already looked at factorisation of quadratic expressions. If it is possible to factorise a quadratic equation this is the quickest and easiest way of solving it.

Worked Example 1:

Solve the equation $x^2 + 2x - 8 = 0$

By factorising the LHS : $(x + 4)(x - 2) = 0$

Either $(x + 4) = 0 \implies x = -4$

or $(x - 2) = 0 \implies x = 2$ ***Note that there are 2 possible solutions to the equation.***

Worked Example 2:

Solve the equation $x^2 - 6x + 9 = 0$

By factorising the LHS : $(x - 3)(x - 3) = 0$

$\therefore \textit{there are two solutions, both } (x - 3) = 0 \implies x = 3$

Worked Example 3:

Solve the equation $x^2 + 12x = 64$

Rearrange the equation so it is set equal to zero: $x^2 + 12x - 64 = 0$

By factorising the LHS : $(x + 16)(x - 4) = 0$

Either $(x + 16) = 0 \implies x = -16$

or $(x - 4) = 0 \implies x = 4$

Worked Example 4:

Solve the equation $4x^2 - 25 = 0$

By factorising the LHS : $(2x + 5)(2x - 5) = 0$

Either $(2x + 5) = 0 \implies x = -\frac{5}{2}$

or $(2x - 5) = 0 \implies x = \frac{5}{2}$

Solving quadratic equations by setting equal to zero and using the quadratic formula

Quadratic equations can be solved using the quadratic formula. In most practical applications of the quadratic equation this will be the case.

If we have the general quadratic equation $ax^2 + bx + c = 0$ then the solutions can be found by solving the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a,b, and c are the coefficients (or numbers) in front of the x^2, x, and constant (number) terms in the quadratic equation. For example, in the equation $2x^2 - 7x + 3 = 0$, $a = 2$, $b = (-7)$, and $c = 3$.

Worked Example 1:

Solve $2x^2 - 7x + 3 = 0$ $\quad a = 2, b = (-7), c = 3$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 2 \times 3}}{2 \times 2}$$

Substitute a,b,c values into quadratic formula

$$= \frac{7 \pm \sqrt{(49 - 24)}}{4}$$

$$= \frac{7 \pm 5}{4}$$

Evaluate the calculation inside the square root sign before taking the square root

$$x = \frac{7 + 5}{4} = 3 \ or \ x = \frac{7 - 5}{4} = \frac{1}{2}$$

Worked Example 2:

Solve $2x^2 - 5x - 3 = 0$ $\quad a = 2, b = (-5), c = (-3)$

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 2 \times (-3)}}{2 \times 2}$ *Substitute a,b,c values into quadratic formula*

$= \frac{5 \pm \sqrt{(25 + 24)}}{4}$

$= \frac{5 \pm 7}{4}$ *Evaluate the calculation inside the square root sign before taking the square root*

$x = \frac{5+7}{4} = 3 \; or \; x = \frac{5-7}{4} = -\frac{1}{2}$

Partially worked example 3:

Solve $2.6x^2 - 12x = 5$

$2.6x^2 - 12x - 5 = 0$ $a = 2.6, b = (-12), c = (-5)$

$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \times 2.6 \times (-5)}}{2 \times 2.6}$ *Substitute a,b,c values into quadratic formula*

$= \frac{\quad \pm \sqrt{(\qquad\qquad)}}{\qquad}$

$= \frac{\quad \pm \quad}{\qquad}$ *Evaluate the calculation inside the square root sign before taking the square root*

$x = 5 \qquad or \; x = \; -0.384$

Partially worked example 4:

Solve $\frac{3x}{2x+1} = \frac{4}{x-1}$

$3x(x-1) = 4(2x+1)$ *Cross multiply denominators to create an equation on one line (no fractions)*

$3x^2 - 3x = 8x + 4$ *Expand*

$3x^2 - 11x - 4 = 0$ *Set equation* $= 0$
$a = 3, b = (-11), c = (-4)$

$$x = \frac{\quad \pm\sqrt{(\qquad\qquad\qquad)}}{\qquad\qquad\qquad\qquad}$$

Substitute a,b,c values into quadratic formula

Evaluate the calculation inside the square root sign before taking the square root

$$x = 4 \; or \; x = -\frac{1}{3}$$

Algebra Exercise 2.7

Solve the following equations using either factorisation or the quadratic formula.

1) $x^2 + 14x + 49 = 0$

2) $x^2 - 8x - 9 = 0$

3) $2x^2 - 8x + 0 = 0$

4) $2x^2 - 11x + 14 = 0$

5) $24x^2 - x - 3 = 0$

6) $2.5x^2 - 7.3x - 1.6 = 0$

7) $7x^2 + 2x - 1 = 0$

8) $\frac{3}{2x-1} - \frac{x}{x+10} = 0$

9) $4x^2 - 23x + 15 = 0$

10) $\frac{3}{x+4} - \frac{2}{x+3} = \frac{1}{x+2}$

11) $\frac{4}{x-10} = \frac{2}{3} - \frac{1}{3+x}$

12) $3.4x^2 + 11.2x + 1.5 = 0$

13) $1 + \frac{2}{x} = \frac{3}{x^2}$

14) $3(x+1) - \frac{19}{x+1} = 14$

Using Logs to Solve Equations with unknown exponents/powers

Recall that inverse operations are used to "back out" or "undo" a variable or number in a formula/equation. So in an equation such as $3^x = \mu$, x is made the subject by removing it from the exponent. This can be achieved using logarithms and log laws.

Recall the Law $log_b x^n = nlog_b x$ means that by taking logs, exponent n can be removed as an exponent and multiplied.

Therefore, x can be made the subject of a formula by taking logs of both sides of an equation (either log or ln logarithms).

Example: $3^x = \mu$

$\log(3^x) = \log(\mu)$

$xlog(3) = \log(\mu)$

$x = \dfrac{\log(\mu)}{log(3)}$

Example: Make x the subject of $2^{2x-1} = 25$

$\log 2^{(2x-1)}) = \log 25$

$(2x-1)log(2) = log(25)$

$(2x-1) = \dfrac{log(25)}{log(2)}$

$2x = 1 + \left(\dfrac{log(25)}{log(2)}\right) \qquad \{or\ 2x = 1 + 4.6438\}$

$x = \dfrac{1 + \left(\frac{log(25)}{log(2)}\right)}{2} = \dfrac{5.6439}{2}\ or \quad x = \dfrac{1}{2} + \dfrac{\log(25)}{2\log(2)}$

$x == 2.82$

Similarly, exponential equations using e^x can be rearranged by taking natural logs (ln) of both sides since $ln(e^x) = x$

Example: Solve for k:

$0.5 = e^{9.45k}$

$ln(0.5) = ln(e^{9.45k})$

$ln(0.5) = 9.45k$

$\dfrac{ln(0.5)}{9.45} = k$

Further Worked examples - Transposing Formulae when the subject is an exponent/power	
Transpose $5^{3x} = u$ making x the subject. $log(5^{3x}) = log(u)$ $3xlog5 = log(u)$ $3x = \frac{log(u)}{log5}$ $x = \frac{log(u)}{3log5}$ or $\frac{log(u)}{2.097}$	*Aim to bring the x term down from being an exponent* *Take logs of both sides (either log or ln)* *Use log law $log_b x^n = nlog_b x$*
$Transpose\ pV^{\gamma} = k$ making γ the subject $ln(pV^{\gamma}) = ln(k)$ $ln(p) + \gamma lnV = \ln(k)$ $\gamma lnV = \ln(k) - \ln(p)$ $\gamma = \frac{\ln(k) - \ln(p)}{lnV}$	*Take logs of both sides (either log or ln)* *Use log law $log_b x^n = nlog_b x$ and* *log law $logx + logy = log(xy)$* *rearranging*

Algebra Exercise 2.87

Solve for x in the following (answer to 3 d.p.):

1. $8^x = 76$
2. $14^{2x} = 28$
3. $2^{x+6} = 2^{3x}$
4. $3^{x-1} = 5^x$
5. $e^{(1-3x)} = 10$
6. Transpose the formula to make T the subject of $\theta = \theta_0 e^{kT}$
7. The decay of voltage, v volts across a capacitor at time t seconds is given by $v = 250e^{-t/3}$. Find the time when the voltage is 150V.

Linear Simultaneous Equations with 2 unknowns

We previously looked at linear equations and how to solve and transpose them. In all the cases we required only one equation to find the value of a single unknown. If, however, an equation contains more than one unknown it is not possible to solve it using only one equation. If two unknowns are present then it is necessary to have two equations to get a solution. These types of equations, where they must be solved together to find the values of the unknowns, are known as simultaneous equations.

There are two methods of solving simultaneous equations and these are shown below. Another method of solving simultaneous equations using graphical methods will be considered later in the course.

Example: Solve the following equations for x and y

$$3x + 4y = 23$$
$$2x - y = 8$$

By substitution

Re-arrange the 2nd equation making y *the subject*

$y = 2x - 8$

Substitute y *back into first equation. i.e. replace* y *in first equation with* $(2x - 8)$

$3x + 4(2x - 8) = 23$

Expand and solve for x

$3x + 8x - 32 = 23$

$11x = 55$

$\frac{x}{11} = \frac{55}{11}$

$x = 5$

By elimination

Multiply the 2nd equation by 4 so when added to 1st equation, the y terms will be eliminated.

$4(2x - y) = 4 \times 8$

$3x + 4y = 23$

$\underline{8x - 4y = 32}$

$11x + 0 = 55$

$\frac{x}{11} = \frac{55}{11}$

$x = 5$

Algebra Exercise 2.9

Solve the following simultaneous equations using either substitution or elimination methods.

1) $x + 3y = -5$
   $-x + 4y = 12$

2) $x + 2y = -1$
   $4x - 3y = 18$

3) $3x - 2y = 12$
   $x + 3y = -7$

4) $3x + 2y = 5$
   $6y = 2 + 4x$

5) $2x + 3y = 8$
   $-5x - 4y = 1$

6) $2.5x + 0.75 - 3y = 0$
   $1.6x = 1.08 - 1.2y$

7) $\frac{1}{x+y} = \frac{4}{27}$
   $\frac{1}{2x-y} = \frac{4}{33}$

Linear Simultaneous Equations with 3 Unknowns

Three unknowns can be solved provided there are three simultaneous equations to work with. There are several ways to solve three simultaneous equations. One method involves solving by eliminating one variable at a time. For example:

Solve for $x, y\ and\ z$
$3x + 2y - z = 4 \quad (1)$
$2x + y + z = 7 \quad (2)$
$x - y + z = 2 \quad (3)$

<u>Step 1</u>
Visually, we can see if the first 2 equations are added, z will be eliminated

$3x + 2y - z = 4 \quad (1) + (2)$
$2x + y + z = 7$
____
$5x + 3y = 11 \quad (4)$

<u>Step2</u>
Visually, we could add equation (1) and (3) to also eliminate z

$3x + 2y - z = 4 \quad (1) + (3)$
$x - y + z = 2$
____
$4x + y = 6 \quad (5)$

<u>Step 3</u>
Manipulate equations (4) and (5) to eliminate one of the remaining variables. E.g. multiply equation (5) by 3 gives equation (6). Then subtract (4) from (6)..

$4x + y = 6 \quad (5) \times 3$
____
$12x + 3y = 18 \quad (6)$
$5x + 3y = 11 \quad (4)$
____
$7x = 7 \quad (6) - (4)$
$\therefore x = 1$

<u>Step 4</u>
Substitute $x = 1$ into equation (4) to solve for y

$5(1) + 3y = 11$
$3y = 11 - 5$
$3y = 6$
$\therefore y = 2$

<u>Step 5</u>
Substitute $x = 1$and $y = 2$ into any of the original equations to solve for z.

Subst. into equation (3) $x - y + z = 2$
$1 - 2 + z = 2$
$z = 2 - 1 + 2$
$\therefore z = 3$

Therefore $x = 1, y = 2\ and\ z = 3$

Algebra Exercise 2.10

Solve the following simultaneous equations:

1. $2x + y - z = 0$
   $3x + 2y + z = 4$
   $5x + 3y + 2z = 8$

2. $3x + 2y + z = 14$
$7x + 3y + z = 22.5$
$4x - 4y - z = -8.5$

3. Kirchhoff's laws are used to determine the current equations in an electrical network and result in the following:
$i_1 + 8i_2 + 3i_3 = -31$
$3i_1 - 2i_2 + i_3 = -5$
$2i_1 - 3i_2 + 2i_3 = 6$
Determine the values of $i_1, i_2, and\ i_3$.

4. The forces in three members of a framework are $F_1, F_2, and\ F_{3.}$ They are related by the following set of simultaneous equations.
$1.4F_1 + 2.8F_2 + 2.8F_3 = 5.6$
$4.2F_1 - 1.4F_2 + 5.6F_3 = 35.0$
$4.2F_1 + 2.8F_2 - 1.4F_3 = -5.6$
Find the values of $F_1, F_2, and\ F_{3.}$

Converting Word Sentences into Mathematical Equations

As we know, Algebra uses mathematics like a language. In fact it is easy to convert word sentences into mathematics using some techniques of algebra.

Once the problem is converted into equations it is easy to then solve these equations.

Given below are some simple examples of how word phrases and sentences are converted to expressions and equations.

- the sum of a and b — *a+b*
- the product of 5 and a — *5a*
- the difference between x and y — x-y
- 14 decreased by 9 — *14 – 9*
- 8 more than x — *x + 8*
- the product of a number and 6 equals 18 — *6x = 18*
- 3 times a number equals the number increased by 12 — *3x = x + 12*
- 4 times the sum of a number and 5 is equal to the product of the number and 12 — *4(x + 5) = 12x*

Word Problems Leading to Linear Equations

We will now look at solving word problems. This section looks at word problems that result in linear equations. In later sections we will solve word problems leading to simultaneous and quadratic equations.

The following steps are helpful for all word problems.

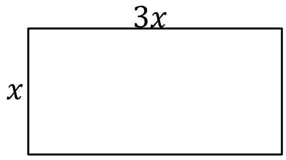
1 Read through the problem carefully. Restate in your own words if necessary. Underline the actual question (often found in the last sentence).
2 Draw a diagram if necessary.
3 Make an assumption statement choosing the variable. In most cases the quantity asked for in the problem is best chosen as the variable. e.g. “let the length = x”
4 It may help to restate the problem by replacing words with the variable chosen and by numbers.
5 Using the data given in the problem form an equation/s. (see ‘ Converting word sentences…’ above)
6 Solve the equation as shown before under ‘solving linear equations’.
7 Cross check that your answer is correct
8 State in English the answer to the original question.

Worked Example 1:

A rectangle is 3 times as long as it is wide. Its perimeter is 32cm. Find its width.

Answer:

Draw a sketch. Let the width (shorter side) be x cm.



Perimeter = $2 \times 3x + 2 \times x = 32$

$$6x + 2x = 32$$
$$8x = 32$$
$$x = \frac{32}{8}$$
$$x = 4$$

Answer: The width of the rectangle is 4cm

Cross Check Answer:

If $x = 4, \quad 3x = 12$

So perimeter $= (2\text{x}12) + (2\text{x}4) = 24 + 8 = 32$. OK

Worked Example 2:

Matthew paid $2 for one bread roll, an apple and a can of juice for lunch.
He paid 20 cents more for the apple than the bread roll and 15 cents more for the bread roll than for the juice. How much did the roll cost?

Bread roll + apple + juice = 200 cents	*Create an equation to represent the problem. NOTE: Do not mix units so work in either cents or dollars (cents in this case)*
$x + (x + 20) + (x - 15) = 200$	*Let the bread roll cost x cents. So the apple cost $x + 20$ cents and the juice cost $x - 15$ cents.*
$3x + 5 = 200$	*Collect like terms*
$3x = 195$	*Gather x terms on one side or equation and numbers on the other*
$\frac{3x}{3} = \frac{195}{3}$	*Divide both side by 3 to cancel the 3 from the $3x$ on the LHS*
$x = 65\ cents\ for\ the\ bread\ roll$	

Worked Example 3:

Two boy's ages add up to 24 years and one boy is six years older than the other. Find their ages.

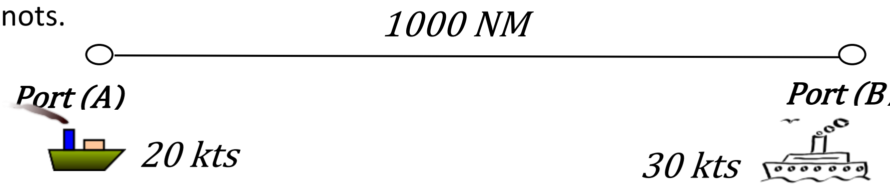
Age of younger boy + age of older boy=24	*Create an equation to represent the problem.*
$x + (x + 6) = 24$	*Let the age of the younger boy be* x *years. So the age of the older boy is* $x + 6$ *years.*
$2x = 24 - 6$	*Collect like terms and gather* x *terms on one side or equation and numbers on the other.*
$\frac{2x}{2} = \frac{18}{2}$	*Divide both side by 2 to cancel the 2 from the* $2x$ *on the LHS*

$x = 9$, *the age of the younger boy and*
$\therefore$ age of older boy is $9 + 6 = 15$ years

How old will they be when their ages add up to 34 years? Answer (14, 20).

Worked Example 4:

Two ships depart ports A and B travelling towards each other. Port A and port B are 1000NM apart. Ship (A) departs from port A travelling at an average speed of 20 knots. Ship (B) departs from port (B) travelling towards port (A) at an average speed of 30 knots.



Calculate each of the following:

a) The time taken to reach the point where the ships pass each other.

Distance A has travelled at any time is : $d = 20t$

$Distance\ B\ has\ travelled\ at\ any\ time\ is\ d = 30t$

Ships pass when combined distance covered is 1000

$20t + 30t = 1000$

$50t = 1000$

$\frac{50t}{50} = \frac{1000}{50}$

$t = 20\ hours$

b) The distance travelled by the ship A, when it passes ship B.

Distance A has travelled after 20 hours:

$d = 20t = 20 \times 20 = 400\ NM$

c) The distance travelled by the ship B, when it passes ship A.

Distance B has travelled after 20 hours:

$d = 30t = 30 \times 20 = 600\ NM$

d) The time until the ships are first 150NM apart.

Ships are first 150 NM apart when combined distance is 1000-150=850 NM

$20t + 30t = 850$

$50t = 850$

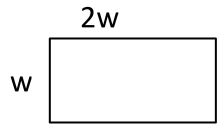
$\frac{50t}{50} = \frac{850}{50}$

$t = 17\ hours$

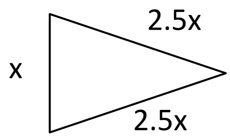
Algebra Exercise 2.11

1. a) Ship X is 9 years older than ship Y. The sum of their ages is 33. Find the age of each ship.
   b) Ship X is 9 years older than ship Y. The product of their ages is 252. Find the age of each ship.

2. A rectangle's width, w, is half its length. Its perimeter is 15mm. Find its dimensions.



3. The two equal sides of an isosceles triangle are 2.5 times the length of the other side. Give the dimensions of the triangle if the perimeter is 24mm



4. Two ships leave different ports A and B 100 miles apart at 0800 hours, each heading towards the opposite port (on reciprocal courses). One ship A steams at 20 knots and ship B at 15 knots.
   a) Find the time when they are first 45 miles apart,
   b) Find the time when they pass each other and their distance from port A.
   c) Find the time when they will be 30 miles apart after passing?

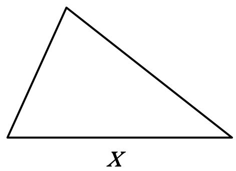
5. Two ships steam towards each other from two positions 60 nautical miles apart, one at 12 knots and the other at 9 knots. Find when they are first 18 nautical miles apart.

6. A ballast tank containing 150 tonnes is emptied by one pump alone in 20 minutes and by another pump alone in 35 minutes at full speed. If the two pumps are working together each at half speed, how long will it take to empty the tank?

7. At noon ship A leaves port steaming at 8 knots. Two hours later ship B leaves the same port, on the same course, and steaming at 12 knots. How far from port, and what time will ship A be overtaken by ship B? Also, find at what time B will be 8 nautical miles astern of A.

8. At midnight Ship A leaves port steaming at 15 knots. 4 hours later ship B leaves the same port, on the same course, and steaming at 20 knots. How far from port will ship A be overtaken by ship B?

9. The perimeter of a triangle is 28.5cm, and its base is 3cm longer than one of its sides and 4.5cm longer than the other, Find the length of all three sides. (hint: let x be the base length)



10. A and B are two ports 466 miles apart. Ship X leaves A for B steaming at 8 knots. Two hours later ship Y leaves A steaming at 20 knots and on arriving at B waits for 20 hours before steaming back along the same track. How far will they be from B when they meet?

11. Four equally spaced buoys A, B, C and D are in line and 5 miles apart from A to D. A vessel steamed from A to B at full speed and from B to C at 3/4 speed and from C to D at 2/3 of full speed. If the average speed from A to D was 9 knots, find the full speed?

Word Problems Leading to Quadratic Equations

These are approached in exactly the same manner as word problems leading to linear (simple) equations except that here the unknown quantity *x* appears squared with or without another term with plain *x.*

Helpful steps for solving problems leading to quadratic equations:

1. state clearly which number your variable (e.g. *x)* represents
2. Draw a diagram perhaps.
3. convert the problem into an equation. You need to carefully go over all given information and convert it into an algebraic form.
4. If the problem is converted into the form of a quadratic equation, it can be solved by using the quadratic formula i.e.

quadratic equation $ax^2 + bx + c = 0$ is solved by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

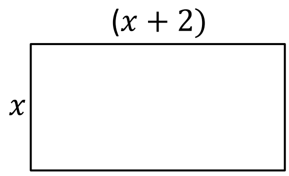
You have seen that the solution to a quadratic equation has two values. Often only one of the solutions will be relevant to the problem.

Worked Example 1:

One side of a rectangular tarpaulin is 2 m. longer than the other, and the area is 20 sq. m. Find the length of its sides.

Let length of one side be x metres and the length of other side is (x + 2) metres.

Then



$$x(x + 2) = 20$$

$$x^2 + 2x = 20$$

$$x^2 + 2x - 20 = 0$$

Using the quadratic formula with a=1, b=2 and c=(-20) gives the solutions

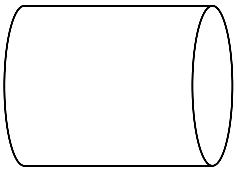
$x = 3.58 \; or \; x = -5.58$

Therefore the sides of the tarpaulin are 3.58 m and 5.58 m.

Note: The negative value of x is rejected in practical problems.

Worked example 2

The surface area of a closed cylindrical tank is $48m^2$. Calculate the radius of the tank if the height of the tank is 4m. (formula for surface area of closed cylinder is $2\pi rh + 2\pi r^2$)



$h = 4\ m$

Answer

1. *Draw a diagram*
2. *Since the surface area has been given ($48m^2$) write down the formula for this*
   $48 = 2\pi rh + 2\pi r^2$ *(NOTE: remember to consider whether all parts of the formula are required .e.g. if the cylinder was open-ended you may only need one (or zero) πr^2term in the formula)*
3. *Substitute other known values (i.e. $h = 4$) into the formula.*
   $48 = 8\pi r + 2\pi r^2$
4. *Recognising this is a quadratic equation, rearrange and solve for r.*
   $0 = 2\pi r^2 + 8\pi r - 48 \quad [a = 2\pi, b = 8\pi, c = (-48)]$

$$r = \frac{-8\pi \pm \sqrt{(8\pi)^2 - 4 \times 2\pi \times (-48)}}{4\pi}$$

$$r = \frac{-8\pi \pm \sqrt{1838.03}}{4\pi}$$

$$r = \frac{-25.13+42.87}{12.57} \text{ or } r = \frac{-25.13-42.87}{12.57}$$

Gives solutions 1.412 and -5.411. Reject negative solution so answer is r=1.412m

Partially worked example 3

One side of a rectangular steel plate is 500mm shorter than the other, and the area is 800000 mm^2. Find the length of its sides.

x

$x - 500$

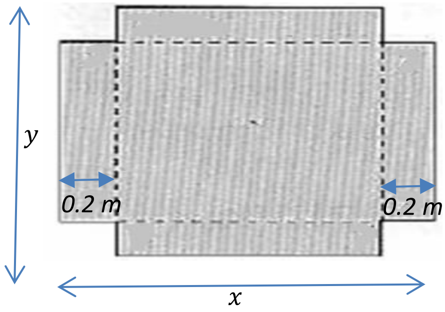
Answer

1. *Draw a diagram*
2. *Since the area has been given ($800000mm^2$) write down the formula for this*
   $800000 = x(x - 500)$
   $0 = x^2 - 500x - 800000$ *(expanding and setting=0)*
3. *Solve using quadratic formula…*
   Gives solutions 1178.7 and -678.7. Reject negative solution so answer is x =1178.7mm^2, so lengths of the sides are 1178.7mm and 678.7mm

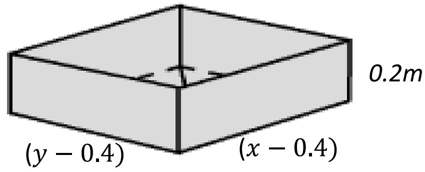
Algebra Exercise 2.12

1. The hypotenuse of a right–angled triangle is 10 m. Find the other two sides if their sum is 12 m.

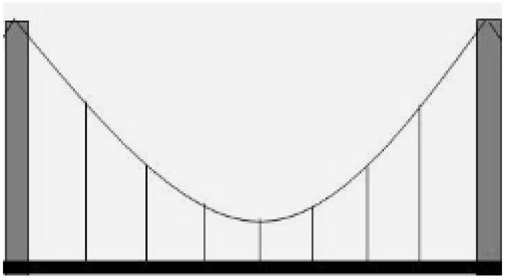
2. A steel tray is constructed from a rectangular sheet of steel. 0.2 m is cut from each corner of the sheet of steel and the sheet is then folded to form a tray as shown in the diagram. The volume of this tray is 0.048 m^3. Calculate the dimensions of the original sheet of steel given that the length, x is twice the breadth, y.



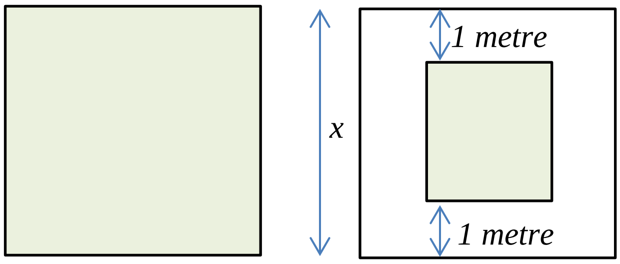
Diagrams not drawn to scale



3. When a cable L metres long is suspended between two points D metres apart, the sag is s metres. Calculate the distance between the supports if the cable length is 25 metres and the sag is 3 metres, given the equation of the curve of the cable (catenary) is $L = \frac{80s^2}{D} - D$ *(hint: substitute the given values for L and s, multiply both sides of the equation by D to eliminate the fraction before solving)*.



4. A concrete path was constructed by taking 1 metre from the each side of a square lawn. The area of the concrete path is 16 m^2. Find original area of the lawn before the path was constructed.



4. A 100 mm long steel rod is of square section and its surface area is equal to the surface area of a sphere 7.05mm radius. Find the dimensions of the rod.

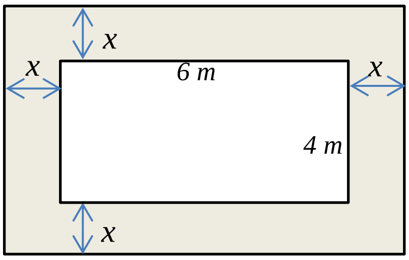
6. The total surface area of a circular cylinder of height 30 mm is equal to the surface area of a sphere of radius $\sqrt{140}\ mm$. What is the radius of the cylinder?

7. A raft of depth 4m is fitted with a steel band 36 m in length around the waterline. If the volume of the raft is 320 m^3, find its length and breadth.

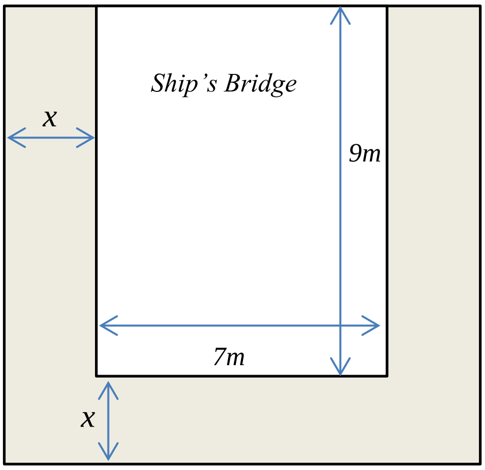
8. A hatchway has an area of 328 m^2 and its perimeter is 73 m. Find its length and breadth.

9. A rectangle has a perimeter of 14 cm. If the length is decreased by 1 cm. and the breadth increased by 1 cm, the area of the new rectangle is 12 cm^2. Find the original dimensions.

10. A rectangular swimming pool measures 6 m by 4 m. It is surrounded by a path of constant width, x. The area of the pathway (shaded) is 11m^2. Calculate the width of the pathway.



11. A U-shaped deck surrounds a ship's bridge on 3 sides. The ship's bridge is rectangular, measuring 7 m by 9 m as shown in the diagram. The surrounding deck area (shaded area in the diagram) is equal to the area of the bridge. The deck width is x metres wide. Form an equation to represent this situation and calculate the width of the deck.

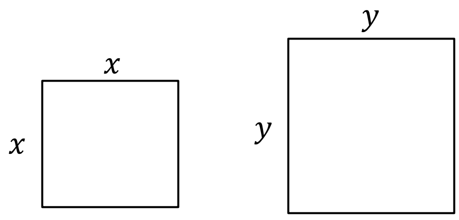


Practical Problems Involving Simultaneous Equations

Worked Example 1:

The difference between the perimeters of two squares is 12mm and the difference in their areas is 45 mm^2. Determine the dimensions of the two squares.

Answer:



Drawing a sketch may be helpful. Let the sides of the smaller square be x and let the sides of the larger square be y

Form equations for perimeter and area since these values have been given in the question.

Difference is perimeters = 12 : $4y - 4x = 12$

Difference is areas = 45 : $y^2 - x^2 = 45$

Rearrange first equation and substitute into second equation

$substitute\ y = x + 3\ into\ y^2 - x^2 = 45$

$(x+3)^2 - x^2 = 45$

Rearrange and solve for x

$x^2 + 6x + 9 - x^2 = 45$

$6x = 45 - 9$

$6x = 36$

$\frac{6x}{6} = \frac{36}{6}$

$x = 6\ mm$

Substitute answer for x into one of the original equations to find y

$y = x + 3$

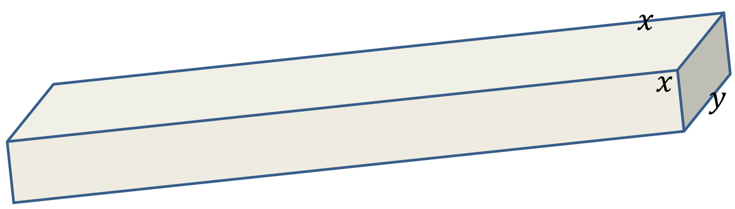
$y = 6 + 3$

$y = 9\ mm$

Worked Example 2:

A strip of sheet metal of width 1.5 m is required to be bent at the sides to form a rectangular channel. Determine the depth of the channel given that the cross-sectional area will be 0.25 m^2

Answer:



Drawing a sketch may be helpful. Let the depth (vertical sides) of the channel x and let the base of the channel be y

Form equations for width of the sheet metal and cross-section area since these values have been given in the question.

Width of sheet metal $= 1.5$: $\quad 2x + y = 1.5$

Cross-section area $= 0.25$: $\quad xy = 0.25$

Rearrange first equation and substitute into second equation

$substitute\ y = 1.5 - 2x\ into\ xy = 0.25$

$x(1.5 - 2x) = 0.25$

Rearrange and solve for x noting this is a quadratic equation so there will be two solutions

$a = 2, b = (-1.5), c = 0.25$

$1.5x - 2x^2 = 0.25$

$0 = 2x^2 - 1.5x + 0.25$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1.5) \pm \sqrt{(-1.5)^2 - 4 \times 2 \times 0.25}}{2 \times 2}$$

$$x = \frac{1.5 \pm \sqrt{0.25}}{4}$$

$$x = \frac{1.5 + 0.5}{4}\ or\ x = \frac{1.5 - 0.5}{4}$$

$x = 0.5\ \ or\ x = 0.25$

There are two possible solutions. Either the depth of the channel is 0.5 m or 0.25 m

Worked Example 3:

A vessel travels down a stream with the current (c), for a distance of 20 nautical miles in 2.5 hours. She then returns upstream for the same distance in 5 hours. Assuming the current (c) is constant, find the speed (v), of the vessel in still water and speed (c) of the current.

Answer:

Write expressions for the total speed downstream (vessel traveling with the current) and the total speed upstream (vessel traveling against the current).

$speed\ downstream = (v + c)$

$speed\ upstream = (v - c)$

Form equations for the distance down-stream

$d_{down} = vt \quad d_{down} = v$

$20 = (v + c) \times 2.5$

$20 = 2.5v + 2.5c$ *(1)*

and the distance upstream

$d_{up} = vt$

$20 = (v - c) \times 5$

$20 = 5v - 5c$ *(2)*

Solve equations (1) and (2) for v and c
Rearranging (2):

$$5c = 5v - 20$$
$$\frac{5c}{5} = \frac{5v}{5} - \frac{20}{5}$$
$$c = v - 4 \qquad (3)$$

Substitute (3) into (1) and solve for v

$$20 = 2.5v + 2.5(v - 4)$$
$$20 = 2.5v + 2.5v - 10$$
$$20 + 10 = 5v$$
$$\frac{30}{5} = \frac{5v}{5}$$
$$v = 6 \textit{ knots}$$

Substitute v=6 into (3) to solve for c

$$c = 6 - 4$$
$$c = 2 \textit{ knots}$$

Algebra Exercise 2.13

1. One ship is 25 years older than another and the sum of their ages is 49 years. Find the age of each.

2. Ship A leaves port at 1000 hrs at 9 knots and ship B leaves at 1130 hrs at 11 knots. How far will A have sailed and what time is it when A is overtaken by B? when

3. If 6 large and 4 small copper ingots weigh 2040 kg and 4 large and 10 small copper ingots weigh 2680 kg, find the weight of 10 large and 8 small ingots.

4. A right angled plane triangle has a perimeter of 40cm. One of the smaller sides is 8cm. Find the length of the other two.

5. The linear law of a lifting machine is given by F = a + bm where m is the mass lifted, F = effort applied and a and b are constants. In a certain lifting machine it was found that when m = 30 kg, F = 35N and when m = 70kg then F = 55N. Find the constants, a and b, express the law of this machine and find the effort required to lift a mass of 60kg.

6. Velocity v is given by the formula v = u + at. If v is 20 when t =2 and v is 40 when t = 7 find the values of u and a.

7. The molar heat capacity of a solid compound is given by the equation c = a+ bt, where a and b are constants. When c = 52, T = 100 and when c = 172 T = 400. Write two simultaneous equations using this data and solve to determine the values of a and b.

8. 3500 cartons of apples are to be loaded into a ship. The ship has 18 compartments in total. There are two sizes of compartment. The large compartments (L) hold 250 cartons, and the remaining small compartments (S) can hold 150 cartons. All compartments are full. How many cartons will be placed in the Small compartments and how many in the Large compartments?

ANSWERS

Algebra Exercise 2.1

1.

a) $7x^2$

b) $y^2 - 2xy$

c) $-4a + 7$

d) $4a^2b - 10ab^2$ *or* $2ab\,(2a - 5b)$

e) $4a - 2a^2$ *or* $2a\,(2 - a)$

f) $7x^2y - 7xy^2$ or $7xy\,(x - y)$

g) $-11x - 3y$ *or* $-(11x + 3y)$

h) xy

2.

a) $-20a^2$

b) $-27x^3$

c) $-12x^4y^2$

d) $9x^2$

e) $-\frac{x}{6y}$

f) $\frac{-a^2}{2b}$

g) $-9a^2$

h) $\frac{-20xy^2 \times 8x^3}{-4x^2} = 40x^2y^2$

3.

a) $4x^3$

b) $\frac{1}{8x^6}$

c) $27x^3$

d) $\left(\frac{3}{x^3}\right)^{-2} = \left(\frac{x^3}{3}\right)^2 = \frac{x^3}{3} \times \frac{x^3}{3} = \frac{x^6}{9}$

e) $(16x^2)^{\frac{3}{4}} = 16^{\frac{3}{4}}(x^2)^{\frac{3}{4}} = 8x^{\frac{3}{2}}$ *or* $8\sqrt{x^3}$

f) $\frac{1}{2xy^2}$

g) $\frac{6}{m^3}$

h) $(2xy^2)^{-3} = \frac{1}{(2xy^2)^3} = \frac{1}{(2xy^2)\times(2xy^2)\times(2xy^2)} = \frac{1}{8x^3y^6}$

i) $\frac{(3xy)^2}{3x^{-1}} = \frac{(3xy)\times(3xy)\times x}{3} = \frac{9x^3y^2}{3} = 3x^3y^2$

j) $\frac{2y^{\frac{3}{2}}}{(4y)^{\frac{1}{2}}} = \frac{2y^{\frac{3}{2}}}{4^{\frac{1}{2}}y^{\frac{1}{2}}} = \frac{2y^{\left(\frac{3}{2}-\frac{1}{2}\right)}}{2} = y^1 = y$

(k) $\frac{(2x)^{-3}\times 4x^2y}{y} = \frac{4x^2y}{(2x)^3y}$

$$= \frac{4x^2y}{(2x)\times(2x)\times(2x)y} = \frac{4}{8x} = \frac{1}{2x}$$

Algebra Exercise 2.2

1.

a) $2x$

b) $x^2 - 2x - 1$

c) $3a^2 + 5ab + 2b^2$

d) $x - 7y + 7$

e) $a^2 + 23b - 10b^2$

f) $-5z^2 + 7xz$

2.

a) $x^2 + 3x - 18$

b) $2x^2 + 5xy + 2y^2$

c) $8x^2 - 18x - 35$

d) $4x^2 + 4xy + y^2$

e) $9a^2 - 24ab + 16b^2$

f) $4x^2 - 9y^2$

g) $-x^2 - 3x + 10$

Algebra Exercise 2.3

1.

a) $7a(1 + 4c)$

b) $4xy(x - 2y)$

c) $5y(xy - 4z)$

d) $\pi R(Rh + 2)$

e) $\sin\beta(\sin\beta - 3\cos\beta)$

f) $(x + 5)(x - 4)$

g) $2(a+2b)(a-2b)$

h) $(x+4)(x+2)$

i) $(a-1)(a-18)$

j) $(x+5)(x-4)$

k) $8(a^2 - 25b^2)$ or $8(a + 5b)(a - 5b)$

l) $(x + 1)(x - 3)$

m) $(x + 15)(x - 4)$

n) $(x + 12)(x + 3)$

Algebra Exercise 2.4

1.

a) $\frac{26}{15}$

b) $\frac{a}{5}$

c) $\frac{3c}{4}$

d) $\frac{11}{6a}$

e) $\frac{11}{2a}$

f) $\frac{5b-3a}{2ab}$

g) $\frac{R_2 + R_1}{R_1 R_2}$

h) $\frac{-2x}{(x+1)(x-1)}$

i) $-\frac{36}{x-6}$

Algebra Exercise 2.5

Ex 2.5 Q1.

a) $\frac{3}{2}$

b) $\frac{5}{x^2}$

c) $\frac{4 - x^2}{x^2}$

d) $\frac{60p^5 - pq^2 - 360}{30p^2q}$

e) *1*

f) $x - y$

g)	$\frac{a-2b}{2b}$	h)	$x+y$	i)	$\frac{1}{a-b}$
j)	$\frac{1}{a-b}$	k)	$\frac{-2}{(x+2)(x+4)}$ or $\frac{-2}{x^2+6x+8}$		

Ex2.5 Q2.

a) 2

b) πr

c) $x-3$

d) $x+5$

e) $\frac{x+8}{2}$

f) $\frac{x}{x-12}$

g) $x+6$

Algebra Exercise 2.6

1.

a)	*9*	b)	*4*	c)	*8*
d)	*7*	e)	*6*	f)	*20*
g)	*27*	h)	*9*	i)	*– 7*
j)	9	k)	*–21*	l)	*3*
m)	*14*	n)	*3*	o)	*4*
p)	$\frac{1}{12}$	q)	$\frac{10}{3}$	r)	$\frac{2}{9}$
s)	$\frac{5}{4}$	t)	*1*	u)	1
v)	$-\frac{11}{5}$				

Algebra Exercise 2.65

1. $a+b-c=d$
2. $d=c-a-b$
3. $\frac{Q}{6}=R$
4. $3STW=R$
5. $T=\frac{R}{3SW}$
6. $y-mx=c$
7. $\frac{y-c}{m}=x$
8. $y(1-q)=p$
9. $y=\frac{p}{1-q}$
   $y(1-q)=p$
   $y-qy=p$
   $y-p=qy$
   $\frac{y-p}{y}=q \quad or \quad 1-\frac{p}{y}=q$
10. $Q=\frac{2}{3}R+15$
    $Q-15=\frac{2}{3}R$
    $\frac{3(Q-15)}{2}=R \quad or\ equivalent$

Algebra Exercise 2.66

1.

a) $\mathrm{I} = \frac{\mathrm{V}}{\mathrm{R}}$ b) $c = \sqrt{\frac{E}{M}}$

c) $l = g\left(\frac{t}{2\pi}\right)^2 \ or \ l = \frac{gt^2}{4\pi^2}$

d) $t = \frac{L - L_0}{AL_0} \ or \ t = \frac{L}{AL_0} - \frac{1}{A}$

e) $L = \frac{1}{w^2C}$

f) $R_1 = \frac{R\,R_2}{R_2 - R}$

g) $x = \frac{aT}{k} + a \ or \ x = \frac{aT - ka}{k}$

h) $y = \frac{Qx}{P+Q}$

i) $p = \frac{q(1+A)}{A-1} \ or \ p = \frac{q+qA}{A-1}$

j) $x = \sqrt{\frac{ka^2 - R^2}{k}} \ or \ x = \sqrt{a^2 - \frac{R^2}{k}}$

k) $r = \frac{E-e}{I} - R \ or \ r = \frac{E-e-RI}{I}$

l) $\cos\theta = \frac{b^2 + c^2 - a^2}{2bc}$

m) $v = \frac{pg}{w} + u \ or \ v = \frac{pg+uw}{w}$

n) $\sigma = \sqrt{\frac{2EMg\,(h+x)}{V}} \ or \ \sigma = \sqrt{\frac{2EMgh+2EMgx)}{V}}$

Algebra Exercise 2.7

1) *x= –7* 2) *x = –1 or 9*

3) *x = 4or 0* 4) *x = 2 or 3.5*

5) *x = – 1/3 or 3/8* 6) *x= 3.124or –0.2*

7) *x = 0.26 or–0.55* 8) *x = –3or +5*

9) *x = 5or ¾* 10) *x = –5/2*

11) *x = 16 ½ or*–2 12) *x = –0.14or –3.15*

13) *x = 1, –3 or*0 14) *x = 4.77or –2.1*

Algebra Exercise 2.87

1. 2.083
2. 0.631
3. 3
4. -2.151
5. -0.434
6. $\theta = \theta_0 e^{kT} \quad (T\ the\ subject)$
   $\frac{\theta}{\theta_0} = e^{kT}$
   $ln\left(\frac{\theta}{\theta_0}\right) = \ln(e^{kT})$
   $ln\left(\frac{\theta}{\theta_0}\right) = kT$
   $\frac{1}{k} ln\left(\frac{\theta}{\theta_0}\right) = T\ or\ equivalent$
7. $t = 1.5\ seconds$

Algebra Exercise 2.9

1. $x = -8\ and\ y = 1$
2. $x = 3\ and\ y = -2$
3. $x = 2\ and\ y = -3$
4. $x = 1\ and\ y = 1$
5. $x = -5\ and\ y = 6$
6. $x = 0.3, y = 0.5$
7. $x = 5\ and\ y = 1.75$

Algebra Exercise 2.10

$\boldsymbol{x = 2,\ y = -2,\ z = 2}$

$\boldsymbol{x = 1.5, \quad y = 2.5, \quad z = 4.5}$

$\boldsymbol{i_1 = -5, i_2 = -4, i_3 = 2}$

$\boldsymbol{F_1 = 2, F_2 = -3, F_3 = 4}$

Algebra Exercise 2.11

1. a) X =21, Y = 12 b) X =21, Y = 12
2. dimensions 5mm by 2.5mm
3. dimensions 4mm and 10mm
4. a) 0934 b) 1051, 57.1 miles from A c) 1143
5. After 2 hours
6. 25.4 min
7. 48 nm from port, at 1800 hours when B overtakes A;
   B is 8 miles astern at 1600.
8. t=1600, A is $16 \times 15 = 240\ nm$ from port when overtaken by B
9. base=12cm, sides are 9cm and 7.5cm
10. 74 miles
11. 11.5 knots

Algebra Exercise 2.12

1. 9.74 and 2.26
2. sheet of steel dimensions 1 m x 0.8 m
3. 17.1 m
4. original lawn dimensions are 5m by 5m
5. 100mm x 1.6 mm x 1.6 mm
6. 7.5 mm
7. L = 10 m, B = 8 m
8. 20.5m x 16m
9. 4 cm x 3 cm or 5cm x 2 cm
10. 0.5 m
11. 2.15m

Algebra Exercise 2.13

1. 37 and 12 years
2. A is overtaken 8.25 hours after leaving. A has travelled 74.25 NM in that time
3. 3640 kg
4. 15, 17
5. a = 20 b=0.5F = 50 N
6. u = 12, a = 4
7. a = 12, b = 0.4
8. 2000 in large compartments, 1500 in small compartments