Preliminary Spectral Analysis of Spectroscopic Binary HIP-61732

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1 Introduction

As the two stars of a binary system orbit around their common center of mass, the velocity component along the line of sight with respect to an observer changes periodically. This periodic change in velocities causes a periodic change in the wavelengths of the spectra of the two stars. By observing this change in wavelengths over a span of time, it is possible to make a radial velocity versus time curve. Analysis of the radial velocity data reveals many parameters describing the system. If the spectroscopic binary turns to be of eclipsing type as well, then using the analysis of the light curves once can determine much more information about the given binary system. Therefore, analysis of binaries plays an important role in astrophysical research.

We use the system HIP61732 (RA:189.80°, Dec:+16.51°) for analysis. The observations were performed at the T193 telescope of the Haute-Provence Observatory, with the SOPHIE spectrograph. The data was used for analysis in Halbwachs et al. (2020) and we use their results to compare our results.

2 Theory

The radial velocity V_r as obtained by Tatum(2020) is a function of the true anomaly θ alongwith other constant parameters. The relation is given by:

$$V_r = K[\cos(\theta + \omega) + e \cos\omega] + V_\gamma \tag{1}$$

where, K is the semi-amplitude, ω is the argument of periastron passage, e is the orbital eccentricity and V_γ is the systemic velocity. The semi amplitude K is given as:

$$K = \frac{2\pi a \sin i}{P\sqrt{1 - e^2}} \tag{2}$$

The observational data obtained is the velocity at certain time instants. We have the radial velocities of the binary components as a function of the true anomaly. Therefore, we need a connect between the true anomaly θ and time t. To accomplish this we use the properties of an ellipse which relate several ellipse parameters.

For any ellipse, the mean anomaly M and eccentric anomaly E are related through the eccentricity e in the form:

$$M = E - e \sin E \tag{3}$$

The mean anomaly is in turn related to the time instant t as:

$$M = \frac{2\pi}{P}(t - T) \tag{4}$$

where P is the time period of orbit and T is the time of periastron passage. Combining equations (3) and (4) we get:

$$\frac{2\pi}{P}(t - T) = E - e \sin E \tag{5}$$

The final relation is that of true anomaly θ and eccentric anomaly E, given as:

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+e}{1-e}} \tan\left(\frac{E}{2}\right) \tag{6}$$

Starting from t, to get $V_r(t)$ for a given set of parameters, one needs to follow these steps:

1. Calculate M for a t using equation (4)
2. Use equation (3) to get the E corresponding to the M obtained above. This equation needs to be solved numerically as a closed form of the inverse is not possible to obtain. Methods such as Newton-Raphson can be used to solve for E.
3. Obtain the respective θ from equation (6) and use it to get V_r from equation (1)

The above equations are the modelling equations using which we try to fit the data. We feed the radial velocities measured at some instants and then try to obtain the orbital parameters. Once these parameters are obtained, we can go on to determine the minimum masses and projected semi major axes.

From equation (2), we get projected semi major axis of each component as:

$$a_{1,2}\sin i = \frac{K_{1,2}P\sqrt{1-e^2}}{2\pi} \tag{7}$$

According to the Kepler's third law we have:

$$\begin{aligned} P^2 &= \frac{4\pi^2(a_1+a_2)^3}{G(m_1+m_2)} \\ \Rightarrow m_1+m_2 &= \frac{4\pi^2(a_1+a_2)^3}{GP^2} \end{aligned} \tag{8}$$

We know that the masses are inversely proportional to the semi-major axes, thus we get:

$$\begin{aligned} m_1 + \frac{m_1K_1}{K_2} &= \frac{4\pi^2 \times \frac{P^3(1-e^2)^{\frac{3}{2}}}{8\pi^3\sin^3 i}(K_1+K_2)^3}{GP^2} \\ \Rightarrow \frac{m_1(K_1+K_2)}{K_2} &= \frac{P(1-e^2)^{\frac{3}{2}}(K_1+K_2)^3}{2\pi G\sin^3 i} \end{aligned}$$

And thus we have the minimum masses:

$$m_{1,2}\sin^3 i = \frac{P(1-e^2)^{\frac{3}{2}}K_{2,1}(K_1+K_2)^3}{2\pi G} \tag{9}$$

Since, radial velocity curves provide no information regarding the inclination, we obtain just the lower limit of the masses, that is, $m\sin^3 i$ and the "projected" semi major axes $a\sin i$.

3 Algorithm

We employ Nonlinear Least Squares methodology to fit the radial velocity data and obtain the best fit parameters. Let the modelling equation be defined as $v_{cal}(t)$ defined by the parameters $(P, T, e, K, V_\gamma, \omega)$ and v_i be the data point obtained via observations at time t_i. The task is to minimise the sum of square of differences to obtain the best fit parameters. The sum S is defined as:

$$S = \sum_{i=1}[v_i - v_{cal}(t_i)]^2 \tag{10}$$

The minima occurs when,

$$\frac{\partial S}{\partial P} = \frac{\partial S}{\partial T} = \frac{\partial S}{\partial e} = \frac{\partial S}{\partial K} = \frac{\partial S}{\partial V_\gamma} = \frac{\partial S}{\partial \omega} = 0 \tag{11}$$

Since these partial derivatives are functions of both independent variable and parameters, a closed form solution does not exist. Instead, initial values must be chosen for the parameters. Then, the parameters are refined by iterating, that is: the values are obtained by successive approximation.

The above algorithm can be implemented by either `scipy.optimize.curve_fit` function or the `lmfit` package of Python3. We used the latter in analysing the radial velocity data and obtain the best fit parameters. We used the Lomb-Scargle Periodogram (VanderPlas 2018), a classic method for finding periodicity in irregularly-sampled data, to obtain the initial guess for P, which gave a fairly good value, usually within 5% error range on providing a rough range in which P would be lying. The periodogram can be implemented through `astropy.timeseries.LombScargle` class in Python3. Our curve fit code was sensitive to the initial guess value of eccentricity hence we iterated from 0 to 1 at the step of 0.1, providing at each iteration the respective value as the guess for eccentricity and finally obtain the best fit parameters.

4 Analysis Results

On giving the range for period P as 300 to 900 days, the LombScargle Periodogram gave the approximate value of period as 583.828 days. Using that, the following best-fit parameters were obtained:

Parameter		Our Result	Halbwachs et al. (2020)
Period($days$)	P	595.214	595.18 ± 0.20
Systemic Velocity(km/s)	V_γ	-15.774	-15.956 ± 0.032
Eccentricity	e	0.3394	0.3393 ± 0.0019
Argument of Periastron($degs$)	ω	64.584	64.85 ± 0.67
Time of Periastron Passage($days$)	T	118.082	118.62
Semi-Amplitudes(km/s)	K_1	9.207	9.197 ± 0.021
	K_2	13.057	13.068 ± 0.034
Projected axes(Gm)	$a_1 \sin i$	70.895	70.81 ± 0.17
	$a_2 \sin i$	100.540	100.61 ± 0.27
Minimum Masses($M_\odot$)	$m_1 \sin^3 i$	0.3323	0.3326 ± 0.0022
	$m_2 \sin^3 i$	0.2343	0.2341 ± 0.0014

Table 1: Comparison of our results with the values given in Halbwachs et al. (2020)

Note: Due to periodic motion, several values of time of periastron passage(T) will give the same RV values. This happens for values of $T - P$, T, $T + P$, $T + 2P$ and so on. Therefore, we mention here the $T \mod P$ value which tallies and provides confirmation that our result is correct.

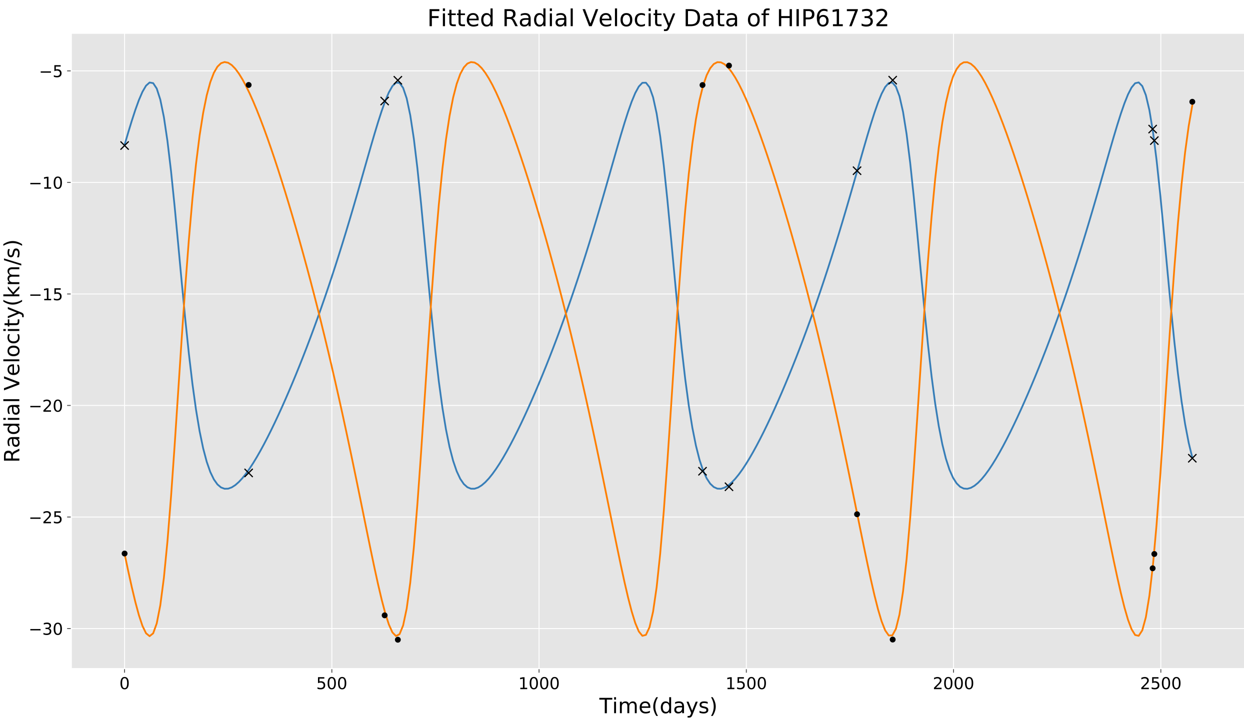

Figure 1: The radial velocity curve plotted using obtained parameters. The crosses and dots represent the observed data of individual component of the system.

5 References

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