

Binomial Loss

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1 Binomial Loss with count upper bound

This loss function is different than logistic regression based on two ways. Different count for each data sample. Response variable is proportion. Important Formulas:-

$$\begin{aligned} P(y = 1) &= \frac{\exp^{x\beta}}{1 + \exp^{x\beta}} + \epsilon_i \\ \log \frac{P(y = 1)}{1 - P(y = 1)} &= x\hat{\beta} \\ \eta &= x\hat{\beta} \end{aligned} \tag{1}$$

$$lik = \prod_{i=1}^n \binom{n_i}{y_i} p(\theta; x)^{y_i} (1 - p(\theta; x))^{n_i - y_i} \tag{2}$$

$$-log - lik = - \sum_{i=1}^n \{y_i \log(p(\theta; x)) + (n_i - y_i) \log(1 - p(\theta; x))\} \tag{3}$$

$$L_i = -log - lik_i = -y_i \log(p(\theta; x)) + (-n_i + y_i) \log(1 - p(\theta; x)) \tag{4}$$

$$L_i = -log - lik_i = -y_i \frac{\log(p(\theta; x_i))}{\log(1 - p(\theta; x_i))} - n_i \log(1 - p(\theta; x_i)) \tag{5}$$

$$L_i = -log - lik_i = -y_i \eta_i - n_i \log(1 - \frac{\exp^{\eta_i}}{1 + \exp^{\eta_i}}) \tag{6}$$

$$L_i = -log - lik_i = -y_i \eta_i - n_i \log(\frac{1}{1 + \exp^{\eta_i}}) \tag{7}$$

$$L_i = -log - lik_i = -y_i \eta_i + n_i \log(1 + \exp^{\eta_i}) \tag{8}$$

2 Negative Gradient

$$\begin{aligned} -\frac{\partial L_i}{\partial \eta_i} &= \frac{\partial y_i \eta_i - n_i \log(1 + \exp^{\eta_i})}{\partial \eta_i} \\ &= y_i - n_i \frac{\exp^{\eta_i}}{1 + \exp^{\eta_i}} \\ &= y_i - n_i \frac{\exp^{\eta_i}}{1 + \exp^{\eta_i}} \\ &= y_i - n_i p_i \end{aligned} \tag{9}$$

3 Hessian

$$\begin{aligned} \frac{\partial^2 L_i}{\partial \eta_i^2} &= \frac{\partial -y_i + n_i \frac{\exp^{\eta_i}}{1 + \exp^{\eta_i}}}{\partial \eta_i} \\ &= n_i \frac{(1 + \exp^{\eta_i}) \exp^{\eta_i} - (\exp^{\eta_i}) \exp^{\eta_i}}{(1 + \exp^{\eta_i})^2} \\ &= n_i \frac{\exp^{\eta_i}}{(1 + \exp^{\eta_i})^2} \end{aligned} \tag{10}$$