Accelerated Failure Time

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May 2019

1 Error - Normal Distribution

1.1 Uncensored Data

1.1.1 Loss Function

Loss $function_i$ = -log-lik_i = -$log(f_{\hat{Y_i}}(\hat{y_i}))$
where,

$$\begin{aligned}
z_i &= \frac{lny_i - ln\hat{y}_i}{\sigma} \sim f \\
f(z) &= \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}} \\
f_{\hat{Y}}(\hat{y}) &= f_Z(z)\frac{\partial z}{\partial y} \\
f_{\hat{Y}}(\hat{y}) &= \frac{exp(\frac{(ln(z)-ln(\hat{y}))^2}{2\sigma^2}}{\hat{y}\sigma\sqrt{2\pi}} \\
Lossfunction_i &= -log(\frac{exp(\frac{(ln(z_i)-ln(\hat{y}_i))^2}{2\sigma^2}}{\hat{y}_i\sigma\sqrt{2\pi}})
\end{aligned} \tag{1}$$

1.1.2 Negative Gradient

$-\frac{\partial Lossfunction_i}{\partial \hat{y}_i} = \frac{\partial log(f_{\hat{Y}_i}(\hat{y}_i))}{\partial \hat{y}_i} = \frac{1}{f_{\hat{Y}_i}(\hat{y}_i)} * \frac{\partial f_{\hat{Y}_i}(\hat{y}_i)}{\partial \hat{y}_i}$

$$\begin{aligned}
\frac{\partial f_{\hat{Y}}(\hat{y})}{\partial \hat{y}} &= \frac{\partial}{\partial \hat{y}}[f_Z(z)\frac{\partial z}{\partial \hat{y}}] \\
&= \frac{\partial f_Z(z)}{\partial \hat{y}}\frac{\partial z}{\partial \hat{y}} + f_Z(z)\frac{\partial^2 z}{\partial \hat{y}^2} \\
&= \frac{\partial f_Z(z)}{\partial z}\frac{\partial z^2}{\partial \hat{y}^2} + f_Z(z)\frac{\partial^2 z}{\partial \hat{y}^2} \\
&= \frac{-zf_Z(z)}{\sigma^2\hat{y}^2} + \frac{f_Z(z)}{\sigma\hat{y}^2}
\end{aligned} \tag{2}$$

$$\frac{\partial z}{\partial \hat{y}} = \frac{-1}{\sigma \hat{y}}$$
$$\frac{\partial^2 z}{\partial \hat{y}^2} = \frac{1}{\sigma \hat{y}^2} \tag{3}$$
$$\frac{\partial f_Z(z)}{\partial z} = -z f_Z(z)$$

$$\begin{aligned} -\frac{\partial Lossfunction_i}{\partial \hat{y}_i} &= \frac{1}{f_{\hat{Y}_i}(\hat{y}_i)} * \frac{\partial f_{\hat{Y}_i}(\hat{y}_i)}{\partial \hat{y}_i} \\ &= \frac{1}{f_{\hat{Y}_i}(\hat{y}_i)} * \{\frac{-z_i f_{Z_i}(z_i)}{\sigma^2 \hat{y_i}^2} + \frac{f_{Z_i}(z_i)}{\sigma \hat{y_i}^2}\} \\ &= \frac{f_{Z_i}(z_i)}{f_{\hat{Y}_i}(\hat{y}_i)} * \{\frac{-z_i}{\sigma^2 \hat{y_i}^2} + \frac{1}{\sigma \hat{y}^2}\} \end{aligned} \tag{4}$$

$$\begin{aligned} f_{Z_i}(z_i) &= \frac{e^{-z_i^2/2}}{\sqrt{2\pi}} = \frac{e^{-\frac{(lny_i - ln\hat{y}_i)^2}{2*\sigma^2}}}{\sqrt{2\pi}} \\ f_{\hat{Y}_i}(\hat{y}_i) &= \frac{e^{-\frac{(lny_i - ln\hat{y}_i)^2}{2*\sigma^2}}}{\sqrt{2\pi} * \sigma * \hat{y}_i} \\ \frac{f_{Z_i}(z_i)}{f_{\hat{Y}_i}(\hat{y}_i)} &= \sigma * \hat{y}_i \end{aligned} \tag{5}$$

Coming back to equation 4

$$\begin{aligned} -\frac{\partial Lossfunction_i}{\partial \hat{y}_i} &= \frac{f_{Z_i}(z_i)}{f_{\hat{Y}_i}(\hat{y}_i)} * \{\frac{-z_i}{\sigma^2 \hat{y_i}^2} + \frac{1}{\sigma \hat{y}^2}\} \\ &= \sigma * \hat{y}_i * \{\frac{-z_i}{\sigma^2 \hat{y_i}^2} + \frac{1}{\sigma \hat{y}^2}\} \\ &= -\frac{lny_i - ln\hat{y}_i}{\hat{y}_i \sigma^2} + \frac{1}{\hat{y}_i} \\ &= \frac{\sigma^2 - ln\frac{y_i}{\hat{y}_i}}{\sigma^2 \hat{y}_i} \end{aligned} \tag{6}$$