Binomial Loss

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June 2019

1 Binomial Loss with count upper bound

This loss function is different than logistic regression based on two ways. Different count for each data sample. Response variable is proportion.
Important Formulas:-

$$\begin{aligned} P(y_i = 1) &= \frac{\exp^{x\beta}}{1+\exp^{x\beta}} + \epsilon_i \\ \log\frac{P(y_i=1)}{1-P(y_i=1)} &= x\hat{\beta} \\ \eta &= x\hat{\beta} \end{aligned} \tag{1}$$

$$lik = \prod_{i=1}^{n} \binom{n_i}{n_i y_i} p(\theta;x)^{\{n_i y_i\}} (1-p(\theta;x))^{\{n_i - n_i y_i\}} \tag{2}$$

$$-log - lik = -\sum_{i=1}^{n} n_i y_i \log(p(\theta;x)) + (n_i - n_i y_i) log(1-p(\theta;x)) \tag{3}$$

$$L = -log - lik = -\sum_{i=1}^{n} n_i y_i \log(p(\theta;x)) + (-n_i + n_i y_i) log(1-p(\theta;x)) \tag{4}$$

$$L_i = -log - lik_i = -n_i y_i \log(p(\theta;x)) + (-n_i + n_i y_i) log(1-p(\theta;x)) \tag{5}$$

$$L_i = -log - lik_i = -n_i y_i \frac{\log(p(\theta;x_i))}{log(1-p(\theta;x_i))} - n_i log(1-p(\theta;x_i)) \tag{6}$$

$$L_i = -log - lik_i = -n_i y_i \eta_i - n_i log(1 - \frac{\exp^{\eta_i}}{1+\exp^{\eta_i}}) \tag{7}$$

$$L_i = -log - lik_i = -n_i y_i \eta_i - n_i log(\frac{1}{1+\exp^{\eta_i}}) \tag{8}$$

$$L_i = -log - lik_i = -n_i y_i \eta_i + n_i log(1+\exp^{\eta_i}) \tag{9}$$

2 Negative Gradient

$$
\begin{aligned}
-\frac{\partial L_i}{\partial \eta_i} &= \frac{\partial n_i y_i \eta_i - n_i log(1+\exp^{\eta_i})}{\partial \eta_i} \\
&= n_i y_i - n_i \frac{\exp^{\eta_i}}{1+\exp^{\eta_i}} \\
&= n_i y_i - n_i \frac{\exp^{\eta_i}}{1+\exp^{\eta_i}} \\
&= n_i y_i - n_i p_i
\end{aligned}
\tag{10}
$$

3 Hessian

$$
\begin{aligned}
\frac{\partial^2 L_i}{\partial \eta_i^2} &= \frac{\partial - n_i y_i + n_i \frac{\exp^{\eta_i}}{1+\exp^{\eta_i}}}{\partial \eta_i} \\
= n_i \frac{(1+\exp^{\eta_i})\exp^{\eta_i} - (\exp^{\eta_i})\exp^{\eta_i}}{(1+\exp^{\eta_i})^2} \\
&= n_i \frac{\exp^{\eta_i}}{(1+\exp^{\eta_i})^2}
\end{aligned}
\tag{11}
$$