Sparse and group regression models in portfolio optimization

Project Supervisor: Professor Mahesan Niranjan
Second Examiner: Professor Vladimiro Sassone

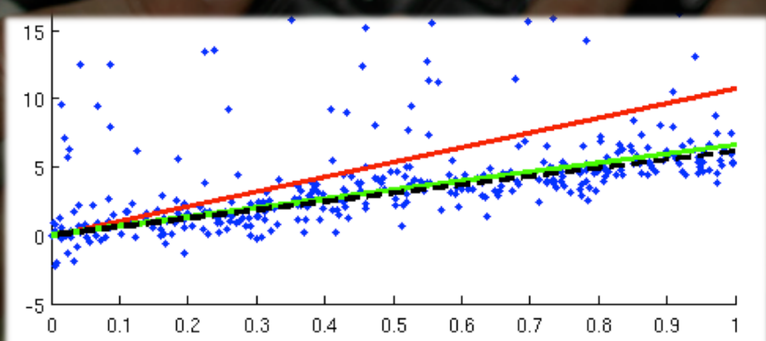
Abstract

Portfolio Optimization

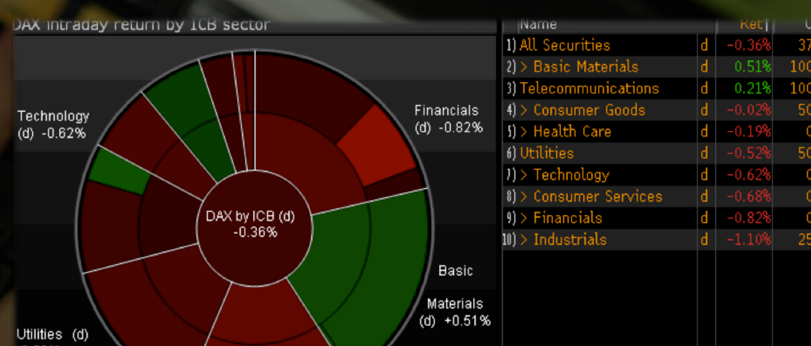
Choose Portfolio (Market Index)

Portfolio Selection (Optimization - Index Tracking)

Numerous Approaches (Revenue, Tracking Error, Cost, etc)



Stock Groups/Classifications (Industry, Sector, Rating, Type, etc)



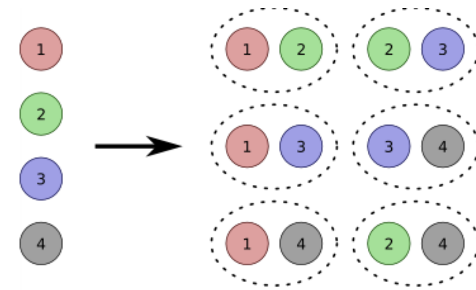
Initial Goals

(Semester 1) Portfolio Selection - Forward Search

Model Selection through a Greedy Forward Search approach.

Simultaneous pursuit of out-of-sample performance and sparsity in index tracking portfolios

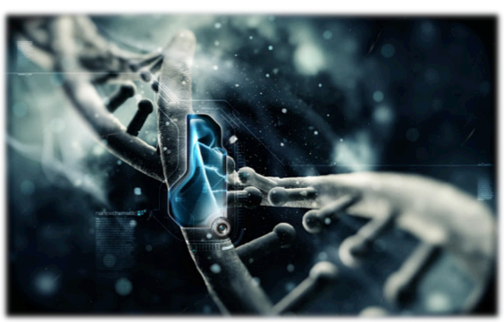
Akiko Takeda · Mahesan Niranjan · Jun-ya Gotoh · Yoshinobu Kawahara



Ridge, Squares, CVaR and Abs

(Semester 2) Portfolio Selection – Sparse Regression Models

Group Level Regression model approach.



Pathways-Driven Sparse Regression Identifies Pathways and Genes Associated with High-Density Lipoprotein Cholesterol in Two Asian Cohorts

Matt Silver[1,2*], Peng Chen[3], Ruoying Li[4], Ching-Yu Cheng[3,5,6], Tien-Yin Wong[5,6], E-Shyong Tai[3,4], Yik-Ying Teo[3,7,8,9,10], Giovanni Montana[1¤]

Lasso, Group Lasso & Sparse Group Lasso

Intuition

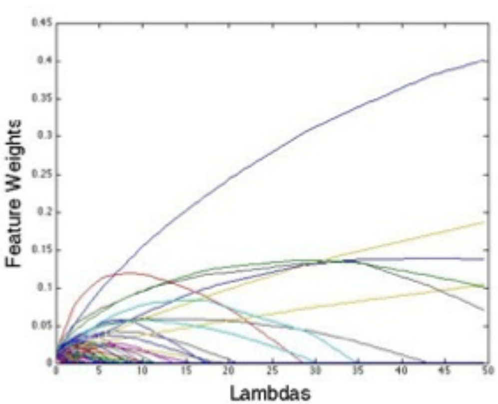
$$R = \begin{bmatrix} R_{1,1} & \dots & R_{1,n} \\ \dots & \dots & \dots \\ R_{T,1} & \dots & R_{T,n} \end{bmatrix} = Returns$$

$$\pi = \begin{bmatrix} \pi_1 \\ \dots \\ \pi_n \end{bmatrix} = Weights$$

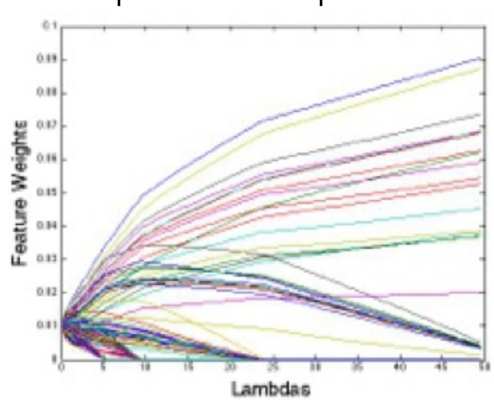
$$R_t^{\mathrm{T}}\pi = \quad I = \begin{bmatrix} I_1 \\ \dots \\ I_T \end{bmatrix} = Index$$

$$\min_{\pi} \; \|I - R^{\mathrm{T}}\pi\|_2^2 + \lambda \sum_{g=1}^{m} \sqrt{p_g}\,\|\bar{\pi}_g\|_2 + \lambda\,|\pi|$$

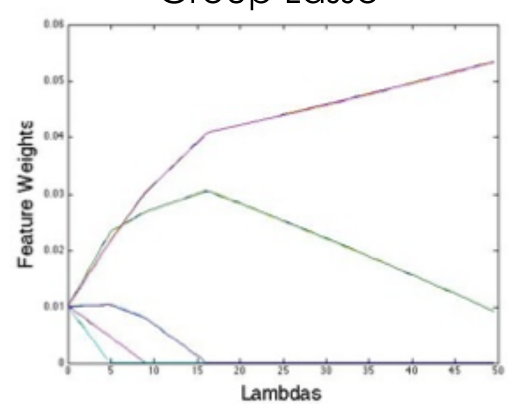
Lasso



Sparse Group Lasso



Group Lasso



Research Objectives

Design, implement and test regression models
Implement feature level and group level regression models (Ridge, Squares, SGL, etc.)

Evaluate performance: Feature vs. group level approach
Present visual and intuitive results in order to provide concrete conclusions

Encourage Research in group sparsity models
Multiple Sparse Group Regression concept proposed to encourage further research

Build an intuitive, reusable programming library
Provide the code used for the implementation in a presentable and structured manner

Motivation

Opportunity to contribute to current research

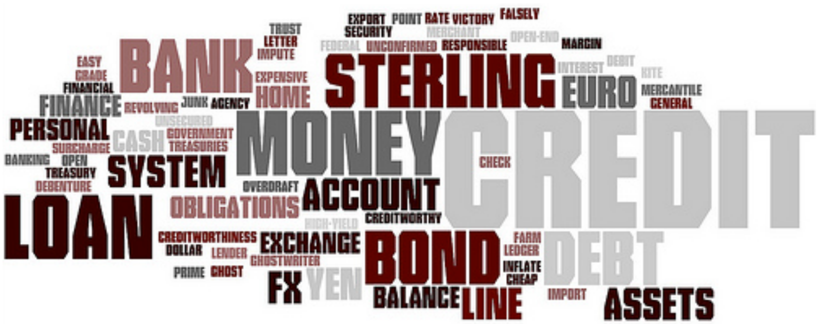
Successful applications in genetics

Novel approach to portfolio optimization

Extremely relevant to previous and future projects

Main Challenges

Financial Concepts



Machine Learning Understanding

Equation 8 - α-Sparse Group Lasso

$$\min_{\pi\in\mathbb{R}^n} \frac{1}{2n}\left\| I - \sum_{g=1}^{m} R_g^{T}\bar{\pi}_g \right\|_2^2 + (1-\alpha)\lambda \sum_{g=1}^{m} \sqrt{p_g}\left\|\bar{\pi}_g\right\|_2 + \alpha\lambda\|\pi\|_1$$

Equation 1 - Tracking Error

$$\frac{1}{T}\sqrt[1/\varphi]{\left[\sum_{t=1}^{T} \left|I_t - R_t^{T}\pi\right|\right]^{\varphi}}$$

Equation 2 - CVaR

$$\min_{\pi,\alpha,z} \alpha + \frac{1}{(1-\beta)T}\sum_{t=1}^{T} z_t$$

Equation 7 - Group Lasso

$$\min_{\pi\in\mathbb{R}^n} \frac{1}{2}\left\| I_t - \sum_{g=1}^{m} R_{g,t}^{T}\bar{\pi}_g \right\|_2^2 + \lambda\sum_{g=1}^{m} \sqrt{p_g}\|\bar{\pi}\|_2$$

Equation 3 - Abs

$$\min_{\pi} \|I - R^{T}\pi\|_1$$

Equation 5 - Ridge

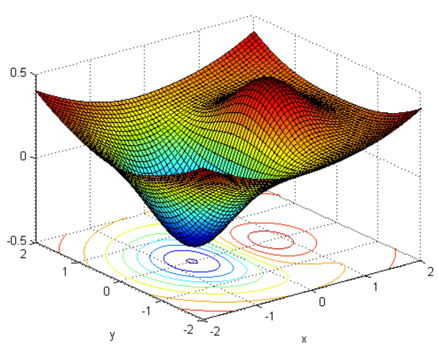
$$\min_{\pi} f^{\tau}(\pi) = \sum_{t=1}^{T} \left|I_t - R_t^{T}\pi\right| + \lambda\|\pi\|_2^2$$

Equation 6 - Lasso

$$\min_{\pi} \sum_{i=1}^{T} \left|I_t - R_t^{T}\pi\right| + \lambda\,|\pi|$$

Programming

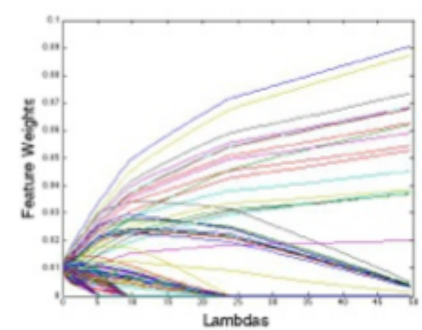
Convex Programming Libraries



Matrix Notation Algorithms

$$\begin{array}{cc} \text{loop} & \text{branch} \rightarrow \\ \downarrow & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Implementation of ZCSGL

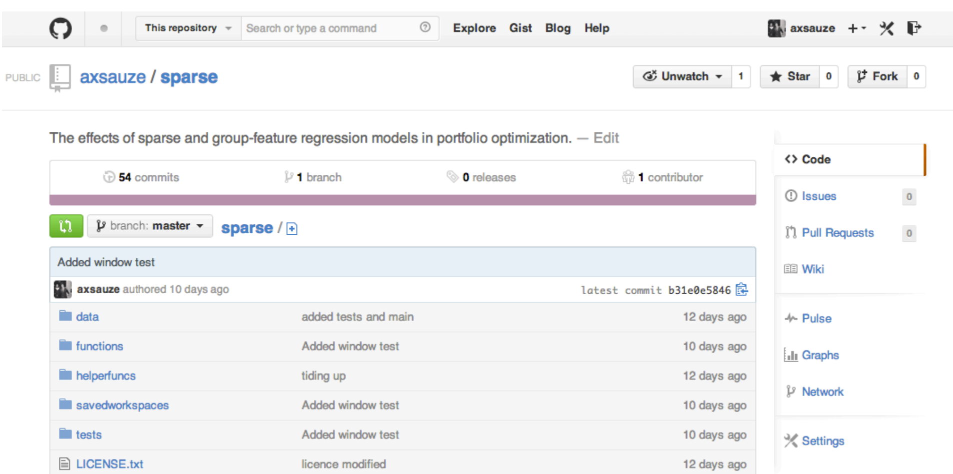


Achievements

Zero-Constrained Sparse Group Lasso

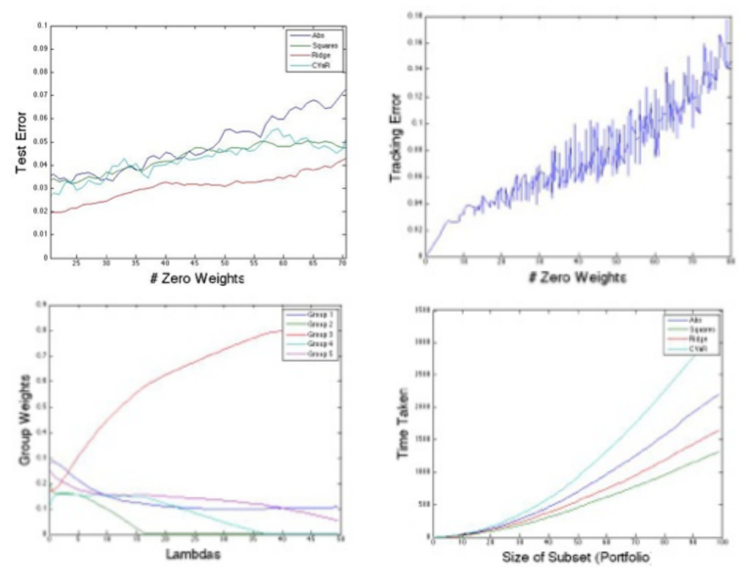
$$\min_{\boldsymbol{\pi}\in\mathbb{R}^n} \left\|\boldsymbol{I}-\boldsymbol{R}^{\mathrm{T}} sum(\widehat{\widetilde{\boldsymbol{\pi}}},2)\right\| + \lambda_1\ sum\left(\sqrt{p^{\mathrm{T}}} * \sqrt{sum(\widehat{\widetilde{\boldsymbol{\pi}}}.\,\hat{}\,2,2)}\right) + \lambda_2 \left\|sum(\widehat{\widetilde{\boldsymbol{\pi}}},2)\right\|_1$$

$$\text{s.t.}\quad \boldsymbol{\pi}_i > \boldsymbol{0} \quad \wedge \quad \widehat{\widetilde{\boldsymbol{\pi}}}[\sim\boldsymbol{\Psi}_g, g] = 0$$



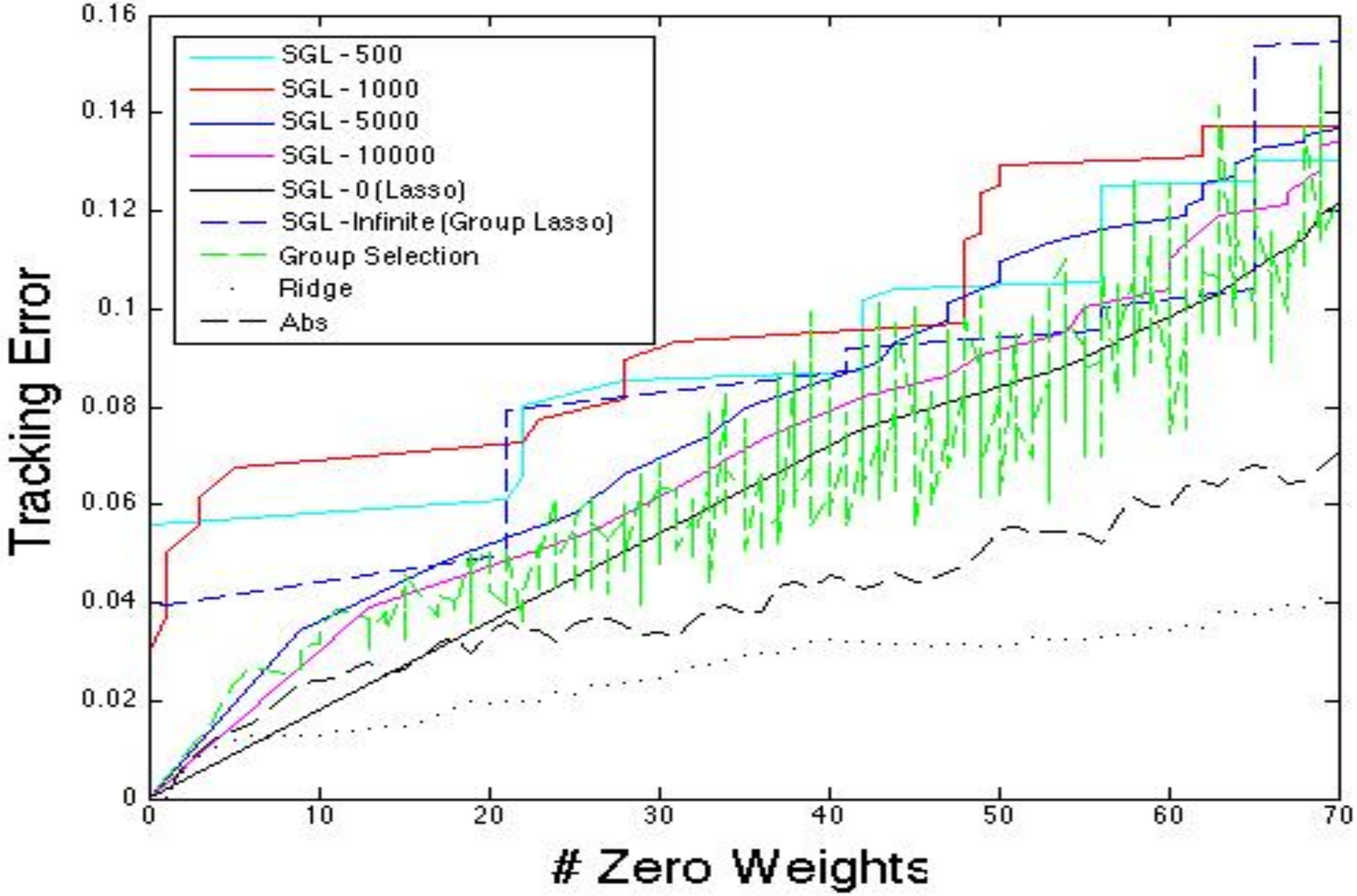
Algorithmic Implementations

- Time Series Modeling (p.6)
- Lasso, Ridge, Squares, Abs
- Spectral Clustering Algorithm
- Zero-Constrained-GL (p. 16)
- Zero-Constrained-SGL (p. 16)



Final Results

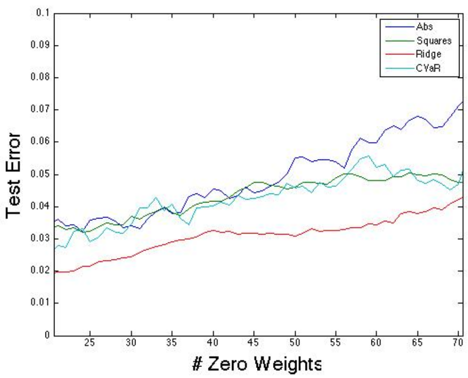
(Tracking Error – All Data p.18-22)



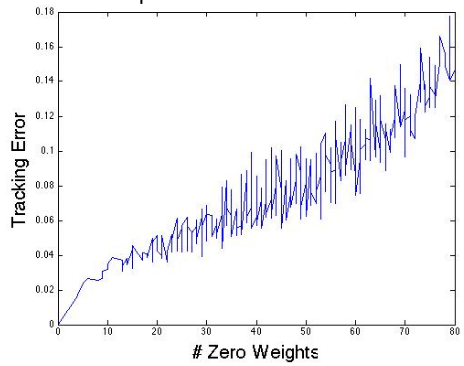
Final Results

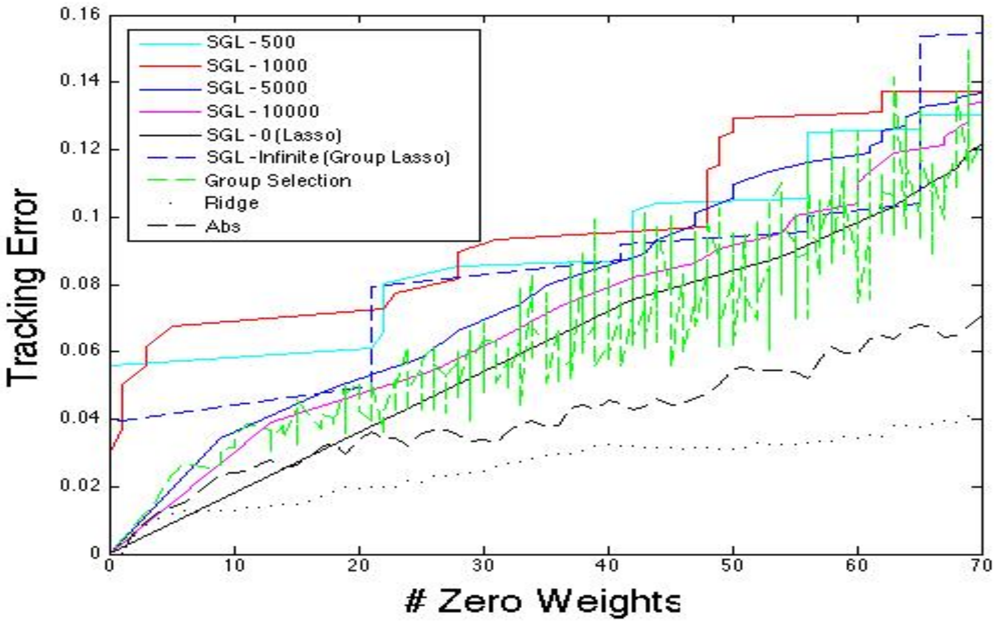
(Tracking Error – All Data p.18-22)

Feature Model Selection

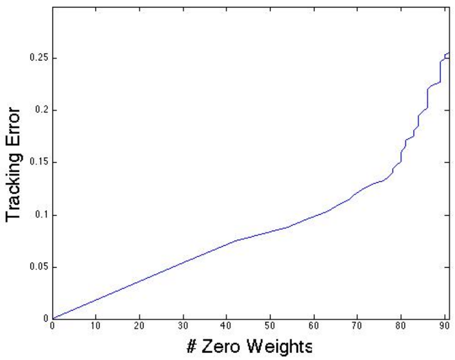


Group Model Selection

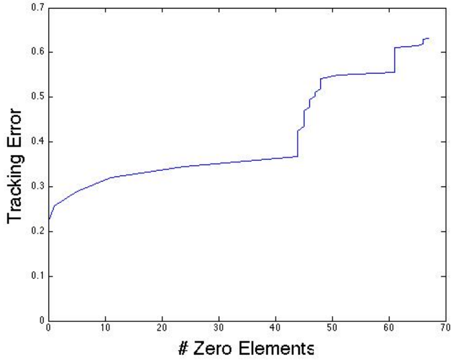




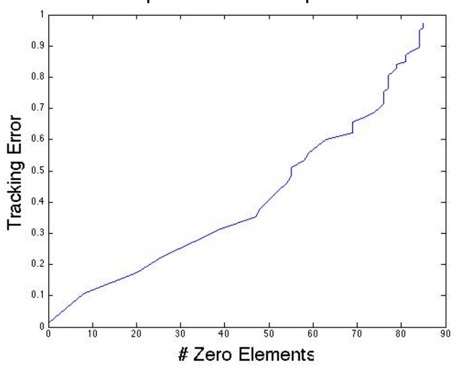
Lasso



Group Lasso



Sparse Group Lasso



Expansion

Observation

Modelling chaotic systems such as financial stock markets requires consideration of complex correlations between groups of stocks.

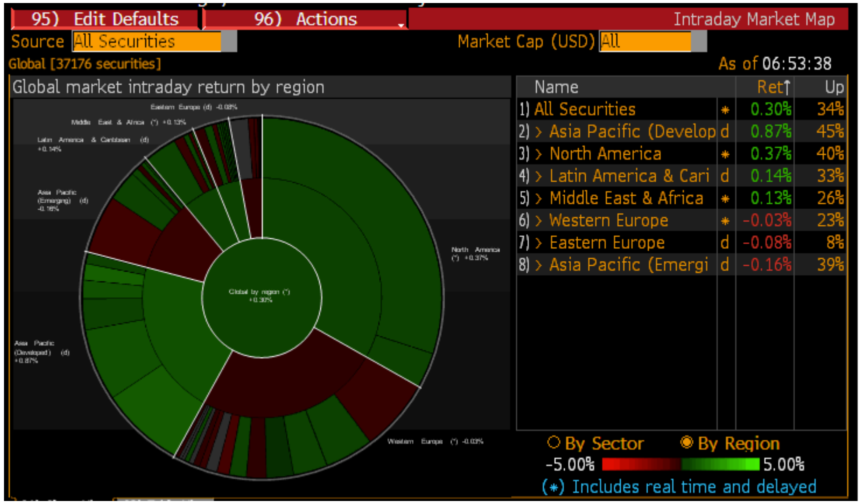
Objective

Enable regression models to induce sparsity based on multiple groupings of the data.

Proposed Solution

Multiple-SGL Concept

$$\min_{\boldsymbol{\pi}\in\mathbb{R}^n} \quad K\|\boldsymbol{I}-\boldsymbol{R}^{\mathrm{T}}\boldsymbol{\pi}\| + \lambda_1\sqrt{\hat{p}} * \sqrt{sum(\boldsymbol{\pi}[\boldsymbol{\psi}]^2, 2)} + \lambda_2\|\boldsymbol{\pi}\|_1$$



Professional Opinion

Denver Trouton
FX Options Applications Lead
Bloomberg LP

"Several Teams working on similar principles"

- Multi-asset class support (equities, FX, commodities, bonds, CDS, IRS)
- Support for portfolios of composite securities (e.g. ETFs, mutual funds, indices)
- Efficient frontier optimization
- Long-only as well as long-short optimization
- Risk (volatility), Value-at-Risk, Conditional Value-at-Risk goal and/or constraints, possibly relative to benchmark
- Weight constraints on groups of securities (e.g. sectors, countries, issuers, ratings, etc)
- Bounding number of trades, buy, sells, positions, longs, shorts (potentially per grouping)

Bloomberg

Bloomberg Portfolio Optimizer

Alper Atamtürk, Sridhar Gollamudi, Kevin He

December 17, 2013

Version 2.10.4

Thank you very much!

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