Q1.

Answer: TSP is NP-Complete if

1. TSP is in NP and
2. TSP is NP-Hard

1. TSP is in NP if we can verify that the potential solution in polynomial time.

    Given a Graph G(V,E), target sum K and Potential solution S. can we verify the solution in polynomial time.

    Verification steps:

    - Check if every vertex in G is visited exactly once by S. and only the first vertex is visited twice to make it a cycle.
    - Calculate the distance of S and compare it with the target sum K.
    - Total time = O(n) for visiting the n vertices in G + O(n^2) for computing the distance matrix.
    - If the distance matrix is already computed O(n) for matrix lookup.

    Thus it takes polynomial time to verify the solution of TSP. Hence, it is in NP

2. TSP is NP-Hard if every problem in NP is polynomial reducible to TSP

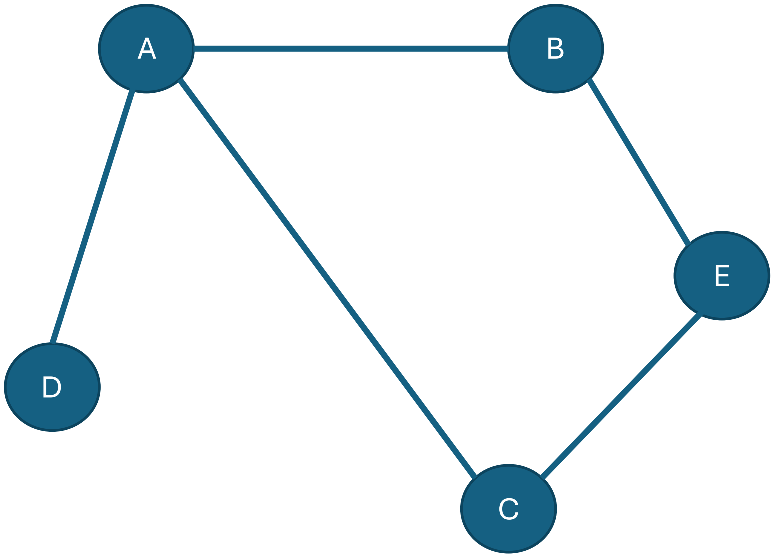
    Proof via Hamiltonian cycle:

    We show that Hamiltonian cycle is reducible to TSP

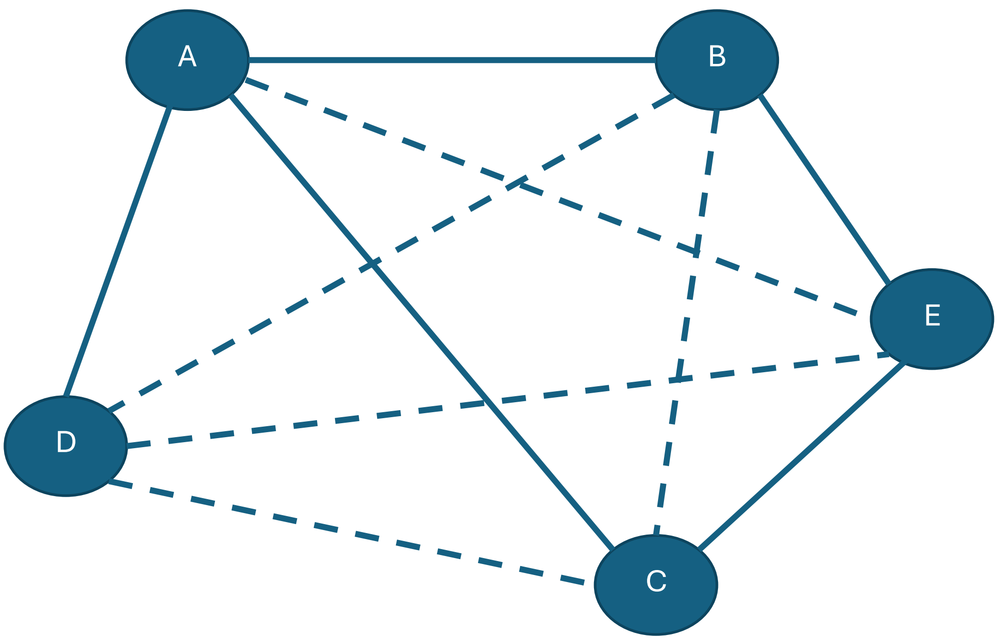
    Given G(V,E) with n vertices, the hamiltonian cycle problem asks if there is a cycle that visits each vertex exactly once except the starting vertex.

    Lets take an instance of hamiltonian cycle.

    G=(V={A,B,C,D,E}, E={AB,BC,EC,AC,AD})



We can see G(V,E) is a subgraph of Kn; n = 5. Add missing edges to make it complete.



Add a function to calculate distance of the edges

F(d) = $\int_{0\ if\ e \notin E}^{1\ if\ e \in E}$

Let target Distance = 0

Now this is a TSP problem. This reduction is done in polynomial time and since hamiltonian cycle is NP-Complete, we can conclude TSP is a NP hard problem.

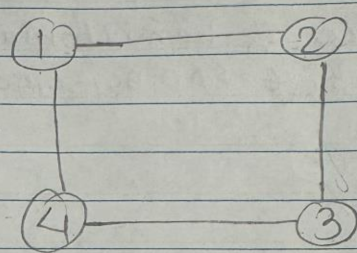
Finally TSP is NP-Complete.

Q2.

a. False. For B to be in NPH, it must be true that every NP problem is polynomial reducible to B.
b. False.
c. True.
d. False. We can only conclude B is NP-Hard. B may not be in NP.

Q3.

③. $G = \{V = (1,2,3,4),\ E = ((1,2), (2,3), (3,4), (1,4))$



Smallest vertex cover is $\{1,3\}$ or $\{2,4\}$
with size $s = 2$

let's run vertex Cover Approx Algorithm:

$E \leftarrow \{(\cancel{1,2}), (\cancel{2,3}), (\cancel{3,4}), (\cancel{1,4})\}$

$C \leftarrow$ new set $\{1, 2, 3, 4\}$

①. take edge $(1,2)$
Add 1 and 2 to C.
delete all edges from E which are incident to vertex 1 or 2 i.e; $(1,2), (2,3), (1,4)$

②. take edg $(3,4)$
Add 3 and 4 to C
delete all edges from E which are incident to vertex 3 or 4 i.e; $(3,4)$

③. No edges left.
$C = \{1,2,3,4\}$ is the vertex cover with size $= 4$
$= 2 * s$ #

Q4.

(4).

The vertex cover decision problembelongs if we can verify its solution in polynomial time.

Input: Graph $G(V, E)$, vertex cover V_c where $V_c \in V$ and required size k.

steps (I) check the elements in V_c to be unique andless than or equal to k.

(II). loop through all edges of Graph and make sure at least one vertex of the edge is in the Vertex Cover V_c.

(I) takes $O(n)$; n is number of vertex in V_c

(II) takes $O(m^2)$; m is number of vertex in Graph.

This takes polynomial Time. Hence the vertex cover decision problem belongs to NP.