CENG 384 - Signals and Systems for Computer Engineers
20232
Written Assignment 1 Solutions

April 3, 2024

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1. (a) (5 pts)

$$
\begin{aligned}
z &= \frac{\sqrt{2}+\sqrt{2}j}{2+2\sqrt{3}j} \\
&= \frac{\sqrt{2}+\sqrt{2}j}{2+2\sqrt{3}j}\,\frac{2-2\sqrt{3}j}{2-2\sqrt{3}j} \\
&= \frac{2\sqrt{2}+2\sqrt{6}+(2\sqrt{2}-2\sqrt{6})j}{16} \\
&= \frac{\sqrt{2}+\sqrt{6}}{8} + \frac{\sqrt{2}-\sqrt{6}}{8}j
\end{aligned}
$$

$$\Re(z) = \frac{\sqrt{2}+\sqrt{6}}{8}, \quad \Im(z) = \frac{\sqrt{2}-\sqrt{6}}{8}$$

   (b) (5 pts)

$$z_1 = \sqrt{2}+\sqrt{2}j = 2e^{j\frac{\pi}{4}}$$

$$z_2 = 2+2\sqrt{3}j = 4e^{j\frac{\pi}{3}}$$

$$z = \frac{z_1}{z_2} = \frac{1}{2}e^{-j\frac{\pi}{12}}$$

$$|z| = \frac{1}{2}, \quad \angle z = -\frac{\pi}{12}\ radians = -15^\circ$$

2. (8 pts) Expand by 2 and shift to the right by 4.

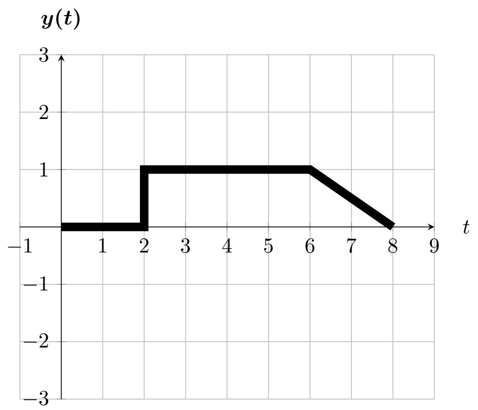

Figure 1: t vs. $y(t)$.

3. (a) (5 pts) $x[n] = \delta[n+3] - \delta[n+2] - \delta[n+1] - \delta[n] + \delta[n-1] + 2\delta[n-2] + \delta[n-3]$

   (b) (7 pts) For $x[1-n]$, reflect $x[n]$ about y-axis and shift to the right by 1. For $x[2n+2]$, we first shrink $x[n]$ by 2 and then shift to the left by 1 and take the integer n values. At the end we sum $x[1-n]$ and $x[2n+2]$.

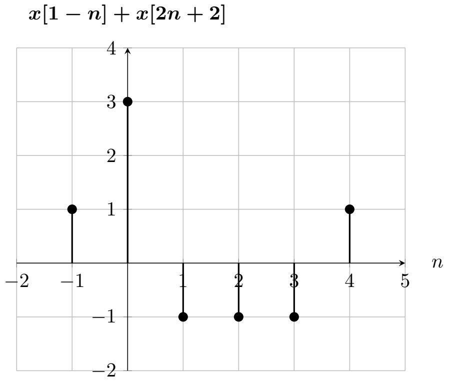

Figure 2: n vs. $x[1-n] + x[2n+2]$.

(c) (5 pts) $x[1-n] + x[2n+2] = \delta[n+1] + 3\delta[n] - \delta[n-1] - \delta[n-2] - \delta[n-3] + \delta[n-4]$

4. (a) (5 pts) $x_1[n] = \cos\left(\frac{5\pi}{2}n\right)$

$x_1[n] \implies N_0 = \frac{2\pi}{\omega_0}m = \frac{4\cancel{5}}{\cancel{5}}$

So the fundamental period is $N_0 = 4$.

(b) (5 pts) $x_2[n] = \sin(5n) \implies N_0 = \frac{2\pi}{\omega_0}m = \frac{2\pi}{5}m$

We don't have an integer m value that makes N_0 integer. So $x[n]$ is not periodic.

(c) (5 pts) $x(t) = 5\sin(4t + \frac{\pi}{3}) = A\sin(\omega_0 t + \phi) \implies T_0 = \frac{2\pi}{|\omega_0|} \implies T_0 = \frac{2\pi}{4} = \frac{\pi}{2}$.

5. (10 pts) For $a > 0$:

$$\begin{aligned}
\int_{-\infty}^{+\infty} |a|\,\delta(at)\,dt &= \int_{-\infty}^{+\infty} |a|\,\delta(u)\,\frac{du}{a} \quad \text{(Substitute } u \to a*t, du = a*dt\text{)} \\
&= \frac{|a|}{a}\int_{-\infty}^{+\infty} \delta(u)\,du \\
&= \int_{-\infty}^{+\infty} \delta(u)\,du \\
&= \int_{-\infty}^{+\infty} \delta(t)\,dt
\end{aligned}$$

For $a < 0$: Similar to $a > 0$ case, when we substitute $u \to a*t$, Since u is negative, when $= +\infty$, u becomes $-\infty$ and when $-\infty$, u becomes $+\infty$. This causes the swap on limits of the integral.

$$\begin{aligned}
\int_{-\infty}^{+\infty} |a|\,\delta(at)\,dt &= \int_{+\infty}^{-\infty} |a|\,\delta(u)\,\frac{du}{a} \quad \text{(Substitute } u \to a*t, du = a*dt\text{)} \\
&= \frac{|a|}{a}\int_{+\infty}^{-\infty} \delta(u)\,du \\
&= -\int_{+\infty}^{-\infty} \delta(u)\,du \\
&= \int_{-\infty}^{+\infty} \delta(u)\,du \\
&= \int_{-\infty}^{+\infty} \delta(t)\,dt
\end{aligned}$$

6. (a) (5 pts) $y_1[n] = 4x[n] + 2x[n-1]$

$y_2[n] = y_1[n-2] = y[n] = 4x[n-2] + 2x[n-3]$

(b) (5 pts) $y_1[n] = x[n-2]$

$y_2[n] = 4y_1[n] + 2y_1[n-1] = y[n] = 4x[n-2] + 2x[n-3]$

The system equation we found here is the same as the one found in part a, therefore the series connection of these sub systems is **commutative**.

(c) (5 pts) $x_1[n] \longrightarrow 4x_1[n-2] + 2x_1[n-3] = y_1[n]$

$x_2[n] \longrightarrow 4x_2[n-2] + 2x_2[n-3] = y_2[n]$

Check if superposition property holds: $ax_1[n] + bx_2[n] \xrightarrow{?} ay_1[n] + by_2[n]$

$ax_1[n] + bx_2[n] \longrightarrow 4ax_1[n-2] + 2ax_1[n-3] + 4bx_2[n-2] + 2bx_2[n-3] = ay_1[n] + by_2[n]$

The superposition property holds. So the overall system is **linear**.

(d) (5 pts) $x[n] \longrightarrow y[n]$

$x[n-n_0] \xrightarrow{?} y[n-n_0]$

$x[n-n_0] \longrightarrow 4x[n-n_0-2] + 2x[n-n_0-3] = y[n-n_0]$

We obtain the same amount of shift at the output. Thus, the system is **time invariant**.

7. (a) (10 pts) The given system is a Linear system.

(b) (10 pts) The given system is a Non-Linear system.

```
import sympy as sp

# Define symbols
n = sp.symbols('n')
x = sp.Function('x')(n)  # x as a function of t

# Define the system's behavior as a symbolic expression
def system_output_a(x):
    return n * x
def system_output_b(x):
    return x**2
# Define symbolic expressions for two inputs and their linear combination
x1 = sp.Function('x1')(n)  # First input as a function of t
x2 = sp.Function('x2')(n)  # Second input as a function of t
a, b = sp.symbols('a b')  # Linear combination coefficients

# Calculate the system's output for each input
y1_a = system_output_a(x1)
y2_a = system_output_a(x2)
y1_b = system_output_b(x1)
y2_b = system_output_b(x2)

# Superposition for inputs
x_combined = a * x1 + b * x2

# System's output for the combined input
y_combined_a = system_output_a(x_combined)
y_combined_b = system_output_b(x_combined)

# Calculate the combined output based on superposition principle
y_superposition_a = a * y1_a + b * y2_a
y_superposition_b = b * y1_b + b * y2_b

# Check if the system satisfies the superposition principle
if y_combined_a.simplify() == y_superposition_a.simplify():
    print('The given system is a Linear system')
else:
    print('The given system is a Non-Linear system')

if y_combined_b.simplify() == y_superposition_b.simplify():
    print('The given system is a Linear system')
else:
    print('The given system is a Non-Linear system')
```