Regulations:

- **Grouping:** You are strongly encouraged to work in pairs.
- **Submission:** You need to submit a pdf file named 'hw3.pdf' to the odtuclass page of the course. You need to use the given template 'hw3.tex' to generate your pdf files. Otherwise you will receive zero.
- **Deadline:** 23:55, April 30, 2024 (Tuesday).
- **Late Submission:** Not allowed.

1. (10 pts) Determine the continuous-time periodic signal $x(t)$ with a period $T = 4$ whose Fourier series coefficients are as follows:

$$a_k = \begin{cases} -1, & \text{k is even,} \\ 1, & \text{k is odd.} \end{cases}$$

2. (15 pts) Consider a continuous-time periodic signal in one full period:

$$x(t) = \begin{cases} 2t, & 0 \le t < 2, \\ 4 - t, & 2 \le t < 4, \end{cases}$$

   where the fundamental period is $T = 4$.

   (a) (10 pts) Find the spectral coefficients of this function.
   (b) (5 pts) Find the spectral coefficients of $\frac{dx}{dt}$ using differentiation property.

3. (15 pts) Consider the following discrete-time periodic signals:

$$x_1[n] = \cos\left(\frac{\pi}{2}n\right),$$
$$x_2[n] = \sin\left(\frac{\pi}{2}n\right),$$
$$x_3[n] = x_1[n]x_2[n].$$

   (a) (9 pts) Find the spectral coefficients of $x_1[n]$, $x_2[n]$ and $x_3[n]$.
   (b) (6 pts) Find the spectral coefficients of $x_3[n]$ using the multiplication property and compare your results with what you found in part (a).

4. (15 pts) Find the discrete-time signal $x[n]$, which has the following spectral coefficients:

$$a_k = \cos\left(k\frac{\pi}{3}\right) + \cos\left(k\frac{\pi}{4}\right).$$

5. (15 pts) Consider the following discrete-time periodic signal:

$$x[n] = \sin\left(\frac{6\pi}{13}n + \frac{\pi}{2}\right).$$

   (a) (5 pts) Find the fundamental period of this signal.
   (b) (10 pts) Find and plot the spectral coefficients of this signal.

6. (15 pts) Consider a causal linear time-invariant system represented by the following frequency response:

$$H(j\omega) = \frac{1}{4j\omega + 3}.$$

   (a) (5 pts) Find the impulse response of the system.
   (b) (10 pts) Find the input $x(t)$, when we observe the following output:

$$y(t) = (e^{-5t} - e^{-10t})u(t).$$

7. (15 pts) Programming.

   In this programming task, write a Python script to analyze a composite continuous-time signal defined by the equation:

   $$x(t) = \cos\left(\frac{\pi t}{3}\right) + 2\cos\left(\pi t + \frac{\pi}{2}\right)$$

   You are required to compute the Fourier series coefficients for x(t) and plot the magnitude and phase of the coefficients.

   Requirements:

   - Use 'numpy' and 'matplotlib' for numerical computations and plotting.
   - Ensure that your code outputs the fundamental period, displays the plot for the Fourier series coefficients, and prints the simplified Fourier series representation.