Student Information

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Answer 1

a) Since server A and B are independent:

$$f(t_A, t_B) = f(t_A) * f(t_B),\ F(t_A, t_B) = F(t_A) * F(t_B)$$

As it is uniformly distributed, density is constant the $f_x(x) = \dfrac{1}{100}$

$$f(t_A, t_B) = \frac{1}{100} * \frac{1}{100} = \frac{1}{10000} \text{ for } 0 \leq t_A, t_B \leq 100$$

Joint cdf is similar:

$$F(t_A) = \int_0^{t_A} f(t_A)dx = \int_0^{t_A} \frac{dx}{100} = \frac{t_A}{100},$$
$$F(t_B) = \int_0^{t_B} f(t_B)dx = \int_0^{t_B} \frac{dx}{100} = \frac{t_B}{100}$$

$$F(t_A, t_B) = F(t_A) * F(t_B) = \frac{t_A * t_B}{10000} \text{ for } 0 \leq t_A, t_B \leq 100$$

b)

A should response in first 30 second and B in between $40 - 60$ seconds:

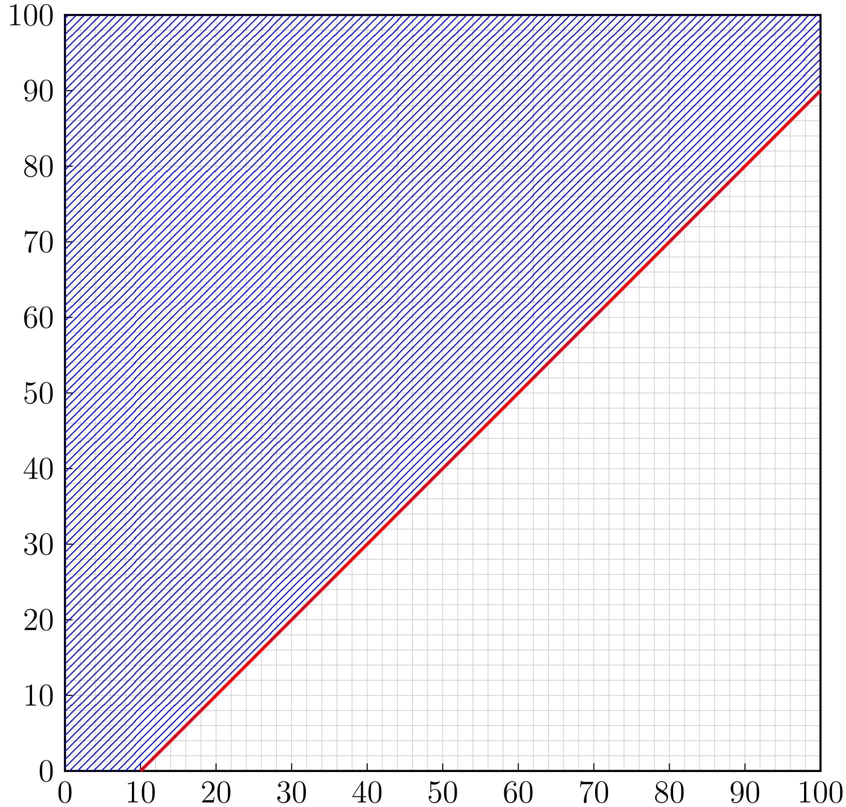
$$F(t_A, t_B) = P(x \leq 30 \cap 40 \leq y \leq 60) = P(x \leq 30) * P(40 \leq y \leq 60)$$

$$P(x \leq 30) = \int_0^{30} \frac{dt}{100} = 0.3,$$
$$P(40 \leq y \leq 60) = \int_{40}^{60} \frac{dt}{100} = 0.2$$

$$P(x \leq 30 \cap 40 \leq y \leq 60) = 0.3 * 0.2 = 0.06 = 6\%$$

c)

Using the given hint, it can be considered an $[0, 100] \times [0, 100]$ square. x-axis is the t_A and y-axis is the t_B.

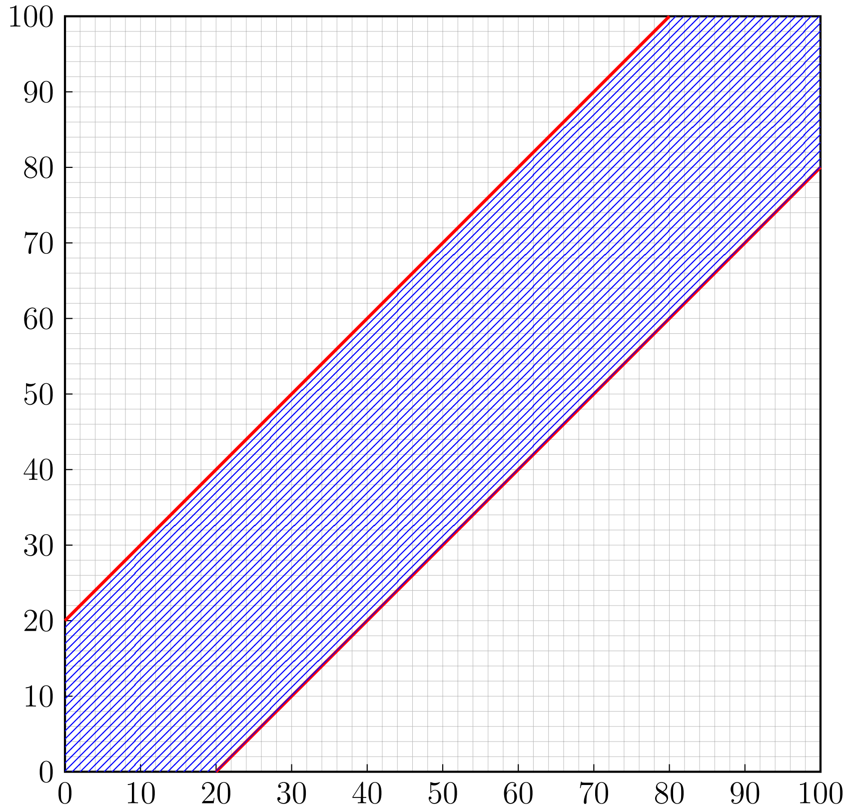


Blue area is the probability we want, $\dfrac{Blue\ Area}{Whole\ Area}$

$$\frac{Blue\ Area}{Whole\ Area} = \frac{100 \times 100 - \dfrac{90 \times 90}{2}}{100 \times 100} = \frac{10000 - 4050}{10000} = 0.595$$

d)

We need to find $|t_A - t_B| \leq 20$. Assume x-axis is t_A and y-axis is t_B.



Blue area is the probability we want, $\frac{Blue\ Area}{Whole\ Area}$

$$\frac{Blue\ Area}{Whole\ Area} = \frac{10000 - \frac{80 \times 80}{2} \times 2}{10000} = 0.36$$

Answer 2

a)

The expected value for a sample of 150 people is:

$$\mu = 150 * 0.6 = 90$$

Standard deviation:

$$\sigma^2 = 150 * 0.6 * (1 - 0.6)$$

$$\sigma = 6$$

With this information, we'll standardize:

$$P\{X > 150 * 0.65\} = P\{X > 97.5\}$$
$$P\{X > 97.5\} = 1 - P\{X < 97.5\}$$
$$1 - P\{X < 97.5\} = 1 - P\{Z < \frac{97.5 - 90}{6} = 1 - \phi(1.25)$$
$$1 - \phi(1.25) = 0.1056$$

Here I used $1 - \mathit{stdnormal\_cdf}(1.25)$ on octave.

b)

The expected value for a sample of 150 people is:

$$\mu = 150 * 0.1 = 15$$

Standard deviation:

$$\sigma^2 = 150 * 0.1 * (1 - 0.1)$$
$$\sigma = 3.674$$

With this information, we'll standardize:

$$P\{X < 150 * 0.15\} = P\{X > 22.5\}$$
$$P\{X < 22.5\} = P\{Z < \frac{22.5 - 15}{3.674}\}$$
$$P\{Z < \frac{22.5 - 15}{3.674}\} = \phi(2.041) = 0.9794$$

Here I used $\mathit{stdnormal\_cdf}(2.041)$ on octave.

Answer 3

We're asked to find $P\{170 < x < 180\} = P\{X < 180\} - P\{X < 170\}$ with normal distribution. We can standardize it between $[0, 1]$.

$$P\{X < 180\} = P\{Z < \frac{180 - 175}{7}\} = P\{Z < 0.714\} = \phi(0.714) = 0.7625$$

$$P\{X < 170\} = P\{Z < \frac{170 - 175}{7}\} = P\{Z < -0.714\} = 1 - \phi(0.714) = 0.2375$$

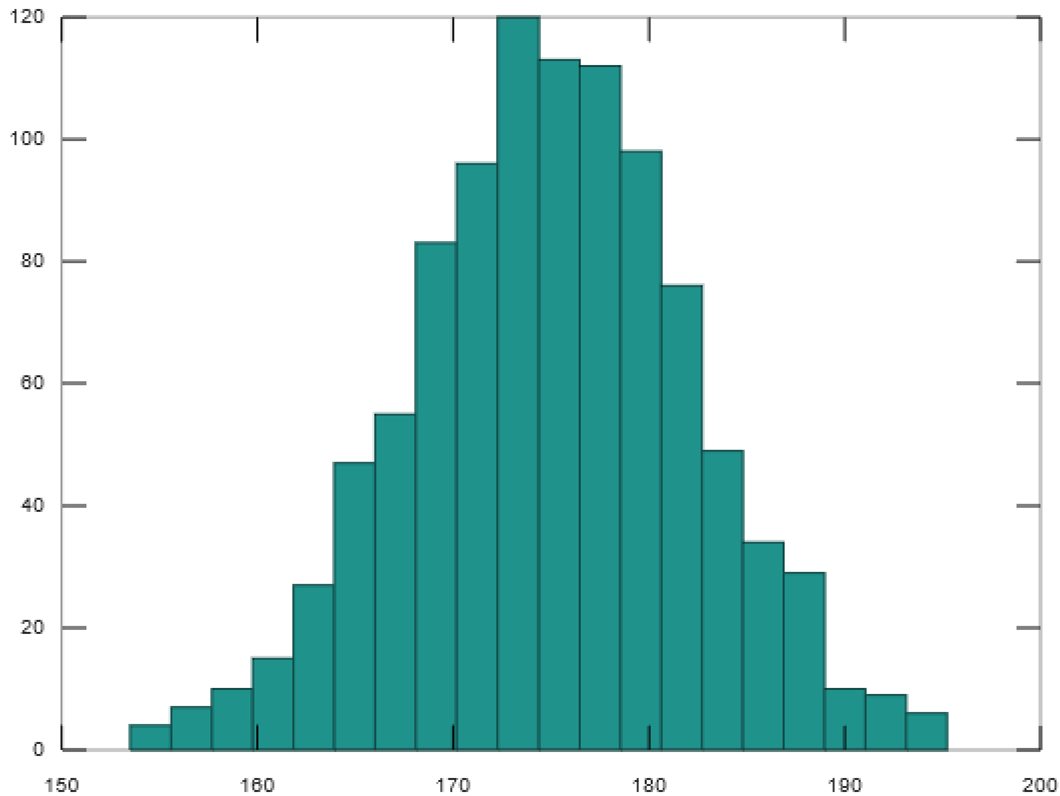
$$P\{X < 180\} - P\{X < 170\} = 0.7625 - 0.2375 = 0.525$$

Here I used *stdnormal_cdf*(0.714) on octave.

Answer 4

a)

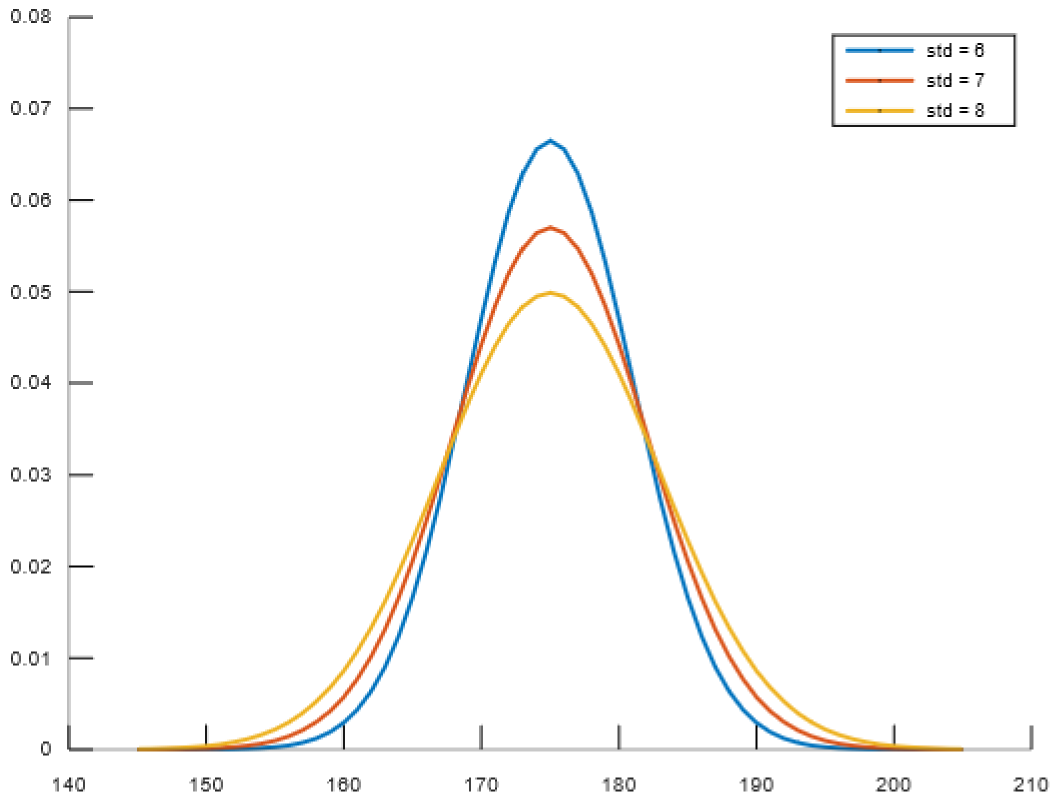
```
mean = 175;
std = 7;
n = 1000;
% r is a vector with random heights with n elements
r = normrnd(mean, std, 1, n);
hist(r,20)
```



This code produces and illustrates a normal distribution for 1000 iterations, providing some information related to the characteristics of data following a normal distribution.

b)

```
mean = 175;
sigmas = [6 7 8];
x = mean - 30 : mean + 30; % range of possible values
figure;
for i = 1:length(sigmas)
    y = normpdf(x, mean, sigmas(i));
    plot(x, y, 'LineWidth', 2);
end
legend('std = 6', 'std = 7', 'std = 8');
```



Increasing sigma makes the distribution wider and flatter, while decreasing sigma makes it narrower and more peaked.

c)

```
mean = 175;
std = 7;
n = 150;

probs = [0.45 0.50 0.55];

% initialize a '0' array/vector 3x1000
count_list = zeros(length(probs), 1000);
```

```
% iterate for all probabilities
for i = 1:length(probs)

    % expected value in our sample space
    % since number of adults cannot be a float
    % we use ceil() function to convert it to an integer
    y = ceil(probs(i) * n);

    % iterate 1000 times
    for j = 1:1000

        % sum all possibilities between 170-180cm
        x = normrnd(mean,std,n,1);
        num_adults_between = sum(x >= 170 & x <= 180);

        %
        count_list(i, j) = num_adults_between >= y;
    end
end

% iterate through count_list find mean of the all rows and print
   out
for i = 1:length(probs)
    fprintf('at least %.0f%%: %.3f\n', probs(i) * 100, sum(
       count_list(i,:)) / 1000);
end
```

```
octave:17> source("hw2_3.m")
at least 45%: 0.971
at least 50%: 0.760
at least 55%: 0.286
octave:18> source("hw2_3.m")
at least 45%: 0.966
at least 50%: 0.749
at least 55%: 0.264
octave:19> source("hw2_3.m")
at least 45%: 0.969
at least 50%: 0.774
at least 55%: 0.280
octave:20> source("hw2_3.m")
at least 45%: 0.970
at least 50%: 0.782
at least 55%: 0.255
octave:21> source("hw2_3.m")
at least 45%: 0.963
at least 50%: 0.758
at least 55%: 0.287
octave:22> source("hw2_3.m")
at least 45%: 0.960
at least 50%: 0.783
at least 55%: 0.267
```

The simulation shows a high probability of a significant proportion of the population having heights between 170-180 cm in a group of 150 adults. However, the estimates may have slight variability due to random sampling.