

Temporal locality \rightarrow item itself

Spatial locality \rightarrow nearby addresses

Direct Mapped Cache Simulation ($E=1$)

$M = 16$ byte addresses $2^4 \Rightarrow 4$ bit

$B = 2$ bytes / block $\Rightarrow 2^1 = 1 = |B|$

$S = 4$ sets $= 2^2 \Rightarrow |S| = 2$

$E = 1$ Blocks / set

		Tag		SI	BO			
PA								
		✓	Tag	Block				
00	1	1	M[8]	M[9]	1	0001	hit	
01	0				7	0111	miss	
10	0				8	1000	miss	
11	1	0	M[6]	M[7]	0	0000	miss	

$$\text{Cache size} = C = S \times E \times B$$

For E -way set associative case, we need to compare all tags on that set with our tag

\rightarrow no match: One line is selected for eviction and replacement

2-way set Associative Cache Simulation ($E=2$)

$M = 16$ byte addresses $= 2^4 \Rightarrow |PA| = 4$

$B = 2$ bytes / block $= 2^1 \Rightarrow |B| = 1$

$S = 2$ sets $= 2^1 \Rightarrow |S| = 1$

$|Tag| = 2$



$E = 2$ Blocks / set

0	0000	miss
1	0001	hit
7	0111	miss
8	1000	miss
0	0000	hit

0	✓	Tag	Block		✓	Tag	Block	
	1	00	M[0]	M[1]	1	10	M[8]	M[9]
1	✓	Tag	Block		✓	Tag	Block	
	1	01	M[6]	M[7]				

Write Operations

What to do on a write-hit? → The line you want to write is on the cache.

Write-through (write immediately to the memory)

Write both in cache and to memory



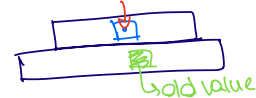
Write-back (defer write to mem until replacement of line)

Need a dirty bit (line different from mem or not)



D=0 → no need to update the mem
D=1 →

What to do on a write-miss? → You want to write into a mem address which is not in the cache.



Write-allocate

Load into cache, update line in cache

No-write-allocate

Writes immediately to the memory

↳ When we have to replace the block, update data back to mem if the write bit is set.

↳ otherwise, don't.

Typical

Write-through + no-write-allocate

Write-back + write allocate

Örnek: L1 i-cache, d-cache

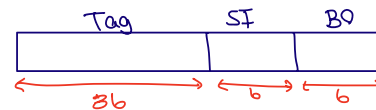
32 KB, 8 way, Mem Address = 48 bit

$$C = 32 \text{ KB} = 2^5 \times 2^{10} = 2^{15}$$

$$E = 8 = 2^3 \rightarrow 8 \text{ lines in each set}$$

$$S = 2^6 \quad |S| = 6$$

$$C = 2^{15} = S \times E \times B \Rightarrow \frac{2^{15}}{2^3 \times 2^6} = 2^6 = B \quad |B| = 6$$



TYPES OF CACHE MISSES

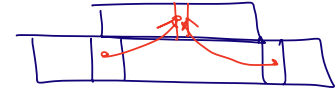
Cold (compulsory) miss

The cache is empty

Capacity miss

The set of active cache blocks (working set) is larger than the cache.

Conflict miss → Most likely to happen in direct-mapped access



Multiple data objects all map to the same level k block

CACHE PERFORMANCE METRICS

Miss rate

Fraction of memory references not found in cache (misses/accesses)

$$= 1 - \text{hit rate}$$

Hit time

Time to deliver a line in the cache to the processor

↳ Includes time to determine whether the line is in the cache

Miss Penalty

Additional time required because of a miss



Hit rate vs. Miss rate

Assume $\begin{cases} \text{cache hit time} \rightarrow 1 \text{ cycle} \\ \text{miss penalty} \rightarrow 100 \text{ cycles} \end{cases}$

$$0.97 \text{ hits} : 1 \text{ cycle} + 0.03 * 100 \text{ cycles} = 4 \text{ cycles}$$

$$0.99 \text{ hits} : 1 \text{ cycle} + 0.01 * 100 \text{ cycles} = 2 \text{ cycles}$$

Hitrate 0.97 is twice

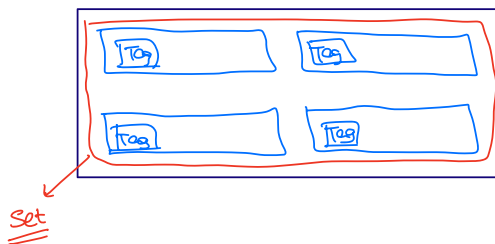
as good as 0.99

↳ use miss rate

$$\text{Given } C = S \times E \times B$$

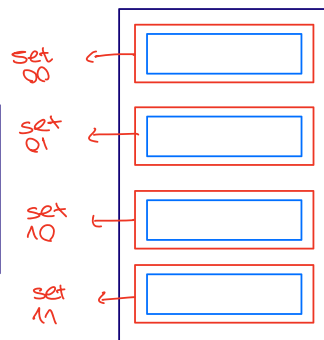
Max number of Tag comparisons needed

Fully Associative ($S=1$)



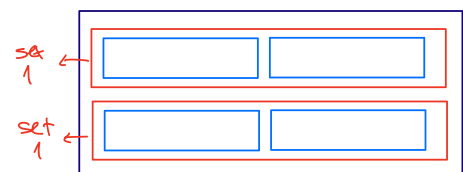
Address Tag BO

1-tag comparison needed
Direct-mapped ($E=1$)



Tag SI BO
↔ 2

($E=2$)
2-way Set Associative



Tag SI BO
↔ 1

THE MEMORY MOUNTAIN

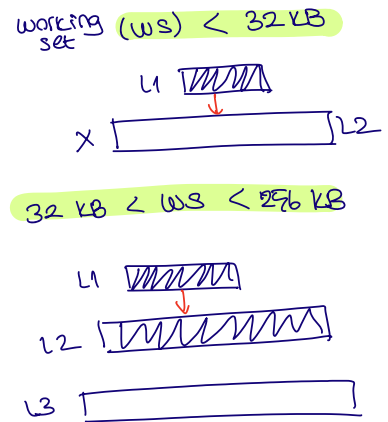
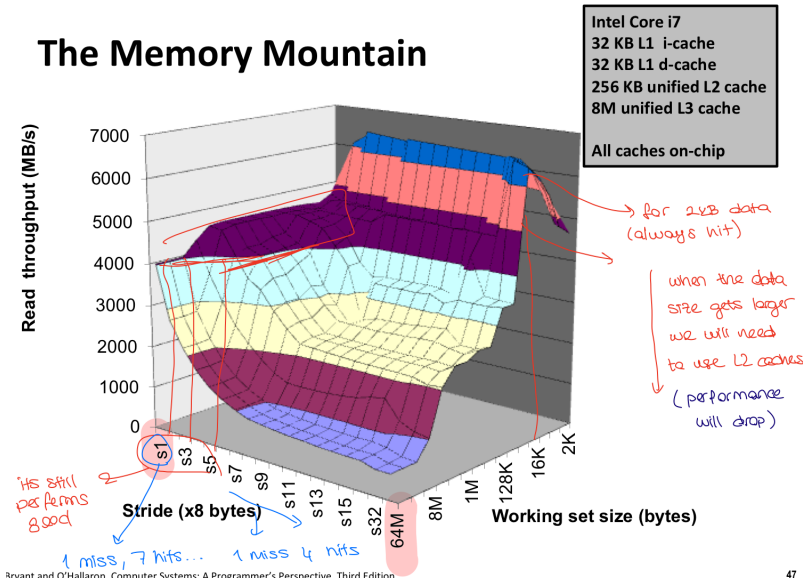
Read throughput

Number of bytes read from mem per second (MB/s)

Memory Mountain

Measured read throughput as a function of spatial and temporal locality

The Memory Mountain

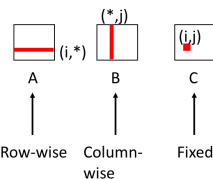


Matrix Multiplication (ijk)

2 loads, 0 stores
misses/iter = 1.25

```
/* ijk */
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Inner loop:



Misses per inner loop iteration:

A	B	C
0.25	1.0	0.0

→ I'm accessing this 1 time (negligible)

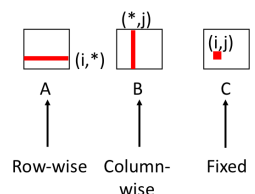
|a_{ij}| = 8 bytes
Block size = 32 bytes

2 loads, 1 store
misses/iter = 0.5

Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Inner loop:



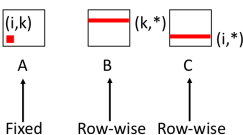
Misses per inner loop iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (ikj)

```
/* ikj */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

Inner loop:



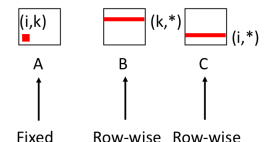
Access only 1 time

Misses per inner loop iteration:

A	B	C
0.0	0.25	0.25

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

Inner loop:



Misses per inner loop iteration:

A	B	C
0.0	0.25	0.25

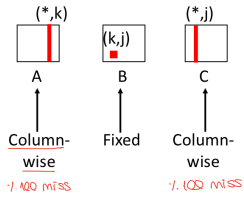
Matrix Multiplication (jki)

```

/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}

```

Inner loop:



Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0

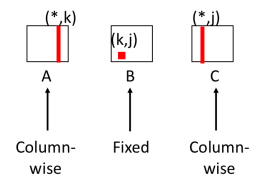
Matrix Multiplication (kji)

```

/* kji */
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}

```

Inner loop:



Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0