CENG 384 - Signals and Systems for Computer Engineers
Spring 2024
Homework 1

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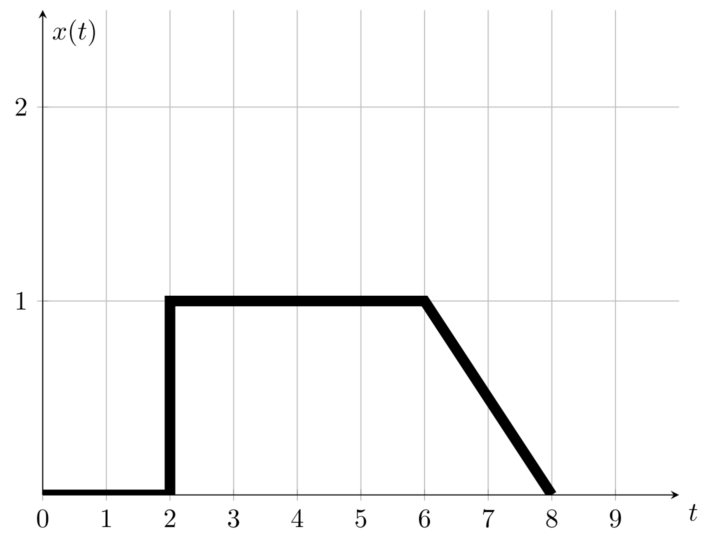
1. (a) $z = \dfrac{\sqrt{2}+\sqrt{2}j}{2+2\sqrt{3}j} = \dfrac{\sqrt{2}+\sqrt{2}j}{2+2\sqrt{3}j} \times \dfrac{2-2\sqrt{3}j}{2-2\sqrt{3}j} = \dfrac{2\sqrt{2}+2\sqrt{2}j-2\sqrt{6}j-2\sqrt{6}j^2}{16} = \dfrac{\sqrt{2}+\sqrt{6}}{8} + \dfrac{(\sqrt{2}-\sqrt{6})j}{8}$

   So the real part is $Re\{z\} = \dfrac{\sqrt{2}+\sqrt{6}}{8}$ and the imaginery part is $Im\{z\} = \dfrac{\sqrt{2}-\sqrt{6}}{8}$

   (b) Magnitude is defined as $\sqrt{a^2+b^2}$ and phase is defined as $\arctan(\dfrac{b}{a})$ for a complex number $z' = a + bj$.

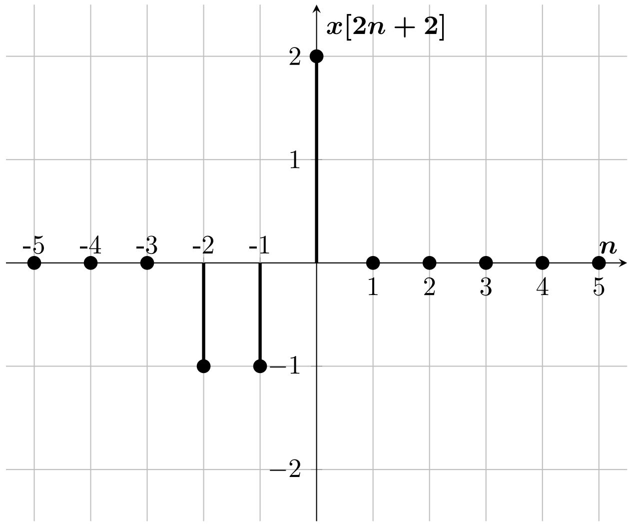
   So Magnitude for the above complex number(z) is $|z| = \sqrt{\left(\dfrac{\sqrt{2}+\sqrt{6}}{8}\right)^2 + \left(\dfrac{\sqrt{2}-\sqrt{6}}{8}\right)^2} = 0.5$

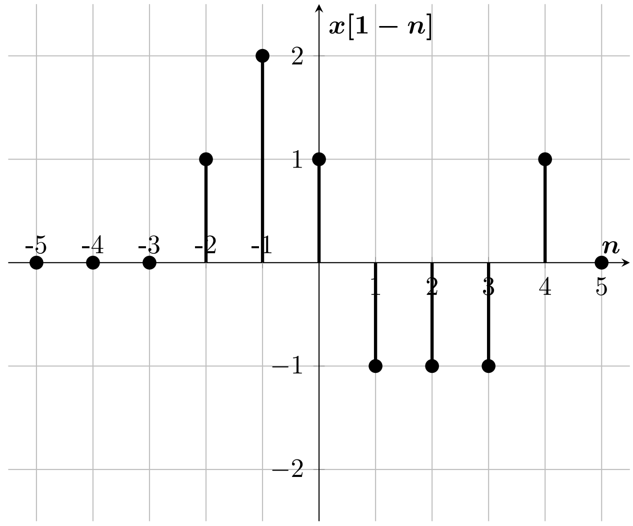
   And phase is $\arctan\left(\dfrac{\frac{\sqrt{2}-\sqrt{6}}{8}}{\frac{\sqrt{2}+\sqrt{6}}{8}}\right) \approx -0.2618$ in radians, which equals to $-15°$

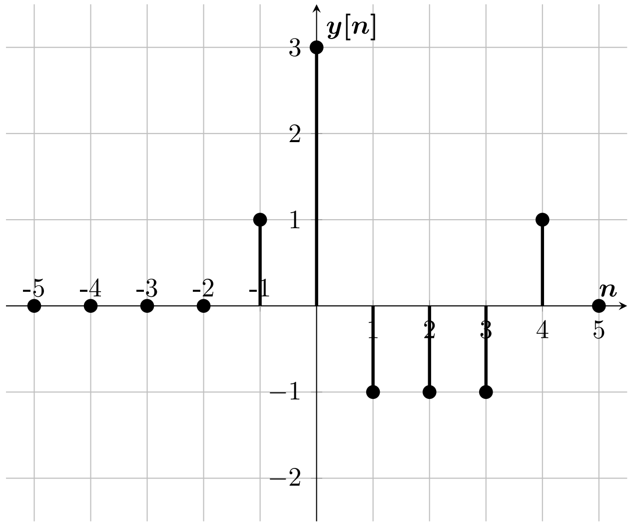
2. 

3. (a) $x[n] = \delta[n-1] + 2\delta[n-2] + \delta[n-3] - \delta[n] - \delta[n+3] - \delta[n+2] - \delta[n+1]$

   (b)







(c)

$$y[n] = \delta[n+1] + 3\delta[n] - \delta[n-1] - \delta[n-2] - \delta[n-3] + \delta[n-4]$$

4. (a) $x_1[n] = \cos(\frac{5\pi}{2}n)$ should be equal to $\cos(\frac{5\pi}{2}(n+N))$ for some integer N. Then $\dfrac{5\pi}{2} * N = 2\pi k$ for some integer k.

$$\frac{N}{k} = \frac{4}{5}$$

N should be a multiple of 4 and k should be a multiple of 5. So, the smallest N that satisfies the equation is 4. Then the function is periodic and its fundamental period is 4.

(b) $x_2[n] = \sin(5n)$ should be equal to $\sin(5(n+N))$ for some integer N. Then $5 * N = 2\pi k$ for some integer k.

$$\frac{N}{k} = \frac{2\pi}{5}$$

N should be a multiple of 2π and k should be a multiple of 5. So, the smallest N that satisfies the equation is 2π. But this is not an integer. So, we can say that the signal is aperiodic.

(c) $x_3(n) = 5\sin(4t + \frac{\pi}{3})$ should be equal to $5\sin(4(t+T) + \frac{\pi}{3})$ for some real number T. Then $4 * T = 2\pi k$ for some integer $k > 0$.

$$\frac{T}{k} = \frac{\pi}{2}$$

So, the smallest k that satisfies the equation is $k = 1 > 0$. Then the smallest T that satisfies k=1 is T=$\dfrac{\pi}{2}$. Since there exists such T and k, we conclude that this function is periodic and its fundamental period is $\dfrac{\pi}{2}$.

5. The Dirac Delta Funcation has two main properties. These are:

   (a) $\delta(t) = \begin{cases} \infty & : t = 0 \\ 0 & : \text{otherwise} \end{cases}$

   (b) $\int_{-\infty}^{\infty} \delta(t)dt = 1$

   The equation can be shown as follows:

$$|a| \times \delta(at) = \delta(t)$$

   If we integrate the both sides of the equation, we get:

$$\int_{-\infty}^{\infty} |a| \times \delta(at)dt = \int_{-\infty}^{\infty} \delta(t)dt$$

   (a) If we make the substitution $u = at$, we get (assuming a is a **positive** real number):

$$\int_{-\infty}^{\infty} a \times \delta(u)\frac{du}{a} = \int_{-\infty}^{\infty} \delta(t)dt$$

   Using the second property of the Dirac Delta Function, we get:

$$\int_{-\infty}^{\infty} a \times \delta(u)\frac{du}{a} = 1$$

   Finally, we get:

$$\int_{-\infty}^{\infty} \delta(u)du = 1$$

   This is the second property of the Dirac Delta Function. So, the equation is **true**.

   (b) If we make the substitution $u = at$, we get (assuming a is a **negative** real number):

$$\int_{\infty}^{-\infty} |a| \times \delta(u)\frac{du}{a} = \int_{-\infty}^{\infty} \delta(t)dt$$

$$\frac{|a|}{a} \times \int_{\infty}^{-\infty} \delta(at)dt = 1$$

$$\frac{-a}{a} \times \int_{\infty}^{-\infty} \delta(at)dt = 1$$

$$-\int_{\infty}^{-\infty} \delta(at)dt = 1$$

   Finally, we get:

$$\int_{-\infty}^{\infty} \delta(u)du = 1$$

   This is the second property of the Dirac Delta Function. So, the equation is **true**.

6. (a) $y[n] = 4 * x[n-2] + 2 * x[n-3]$

(b) Yes, the system is commutative. Assume that the input signal is $x[n]$. If we apply the S_2 first:

$$y_1[n] = x[n-2]$$

After that, if we apply the S_1:

$$y_2[n] = 4 * x[n-2] + 2 * x[n-3]$$

It is the same result as the result of applying the S_1 first and then applying the S_2. So, the system is commutative.

(c) We know that:

$$y[n] = 4 * x[n-2] + 2 * x[n-3]$$

Assume that $x[n]$ is the linear combination of $x_1[n]$ and $x_2[n]$:

$$x[n] = \alpha * x_1[n] + \beta * x_2[n]$$

Then we can show that:

$$y[n] = 4 * (\alpha * x_1[n-2] + \beta * x_2[n-2]) + 2 * (\alpha * x_1[n-3] + \beta * x_2[n-3])$$

If we distribute the terms, we get:

$$y[n] = \alpha * (4 * x_1[n-2] + 2 * x_1[n-3]) + \beta * (4 * x_2[n-2] + 2 * x_2[n-3])$$

This is the same as:

$$y[n] = \alpha * y_1[n] + \beta * y_2[n]$$

So, the system is linear.

(d) In order to show that the system is time-invariant, we need to show that if the input signal is delayed by k samples, the output signal is also delayed by k samples. Assume that the input signal is $x[n]$. Then the output signal is:

$$y[n] = 4 * x[n-2] + 2 * x[n-3]$$

If we delay the input signal by k samples, we get:

$$x[n-k] \rightarrow y[n-k] = 4 * x[n-k-2] + 2 * x[n-k-3]$$

If we compare the output signal with the original output signal, we can see that the output signal is also delayed by k samples. So, the system is time-invariant.

7. (a)

```python
from sympy import symbols, Function, simplify
import random

n = symbols('n')
x = Function('x')(n)

system = n * x

# For a system to be linear it should satisfy superposition and scaling properties:
# If y1[n] = H(x1[n]) and y2[n] = H(x2[n]), then H(a*x1[n] + b*x2[n]) = a*y1[n] + b*y2[n]

a = random.randint(-100, 100)
b = random.randint(-100, 100)

linear_combination_a = system.subs({x: a*x}) + system.subs({x: b*x})
simplified_a = simplify(linear_combination_a - a*system - b*system)
is_linear_a = simplified_a == 0


if is_linear_a:
    print("The given system is a Linear system")
else:
    print("The given system (a) is a Non-Linear system")
```

(b)

```python
from sympy import symbols, Function, simplify
import random

n = symbols('n')
x = Function('x')(n)

system = x**2

# For a system to be linear it should satisfy superposition and scaling properties:
# If y1[n] = H(x1[n]) and y2[n] = H(x2[n]), then H(a*x1[n] + b*x2[n]) = a*y1[n] + b*y2[n]

a = random.randint(-100, 100)
b = random.randint(-100, 100)

linear_combination_b = system.subs({x: a*x}) + system.subs({x: b*x})
simplified_b = simplify(linear_combination_b - a*system - b*system)
is_linear_b = simplified_b == 0

if is_linear_b:
    print("The given system is a Linear system")
else:
    print("The given system is a Non-Linear system")
```