MIDDLE EAST TECHNICAL UNIVERSITY
DEPARTMENT OF COMPUTER ENGINEERING

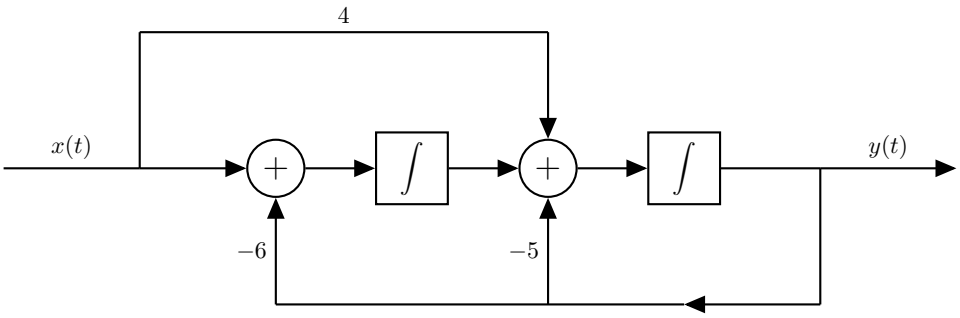
CENG 384

Spring 2024
Homework 4

Regulations:

- **Grouping:** You are strongly encouraged to work in pairs.
- **Submission:** You need to submit a pdf file named 'hw4.pdf' to the odtuclass page of the course. You need to use the given template 'hw4.tex' to generate your pdf files. Otherwise you will receive zero.
- **Deadline:** 23:55, May 19, 2024 (Sunday).
- **Late Submission:** Not allowed.

1. (20 pts) Consider an LTI system given by the following block diagram:



   (a) (5 pts) Find the differential equation which represents this system.
   (b) (5 pts) Find the frequency response of this system.
   (c) (5 pts) Find the impulse response of this system from its frequency response.
   (d) (5 pts) Find the output $y(t)$ for the input $x(t) = \frac{1}{4}e^{-t/4}u(t)$ using the frequency response.

2. (20 pts) Consider the following LTI system defined by the frequency response below:

$$H(j\omega) = \frac{j\omega + 4}{-\omega^2 + 5j\omega + 6}$$

   (a) (5 pts) Find the differential equation which represents this system.
   (b) (5 pts) Find the impulse response of this system.
   (c) (5 pts) Find $Y(j\omega)$ when the input is $x(t) = e^{-4t}u(t) - te^{-4t}u(t)$.
   (d) (5 pts) Find the output $y(t)$ using the result you found in part c.

3. (20 pts) Consider an LTI system represented by the following impulse response and frequency response pair:

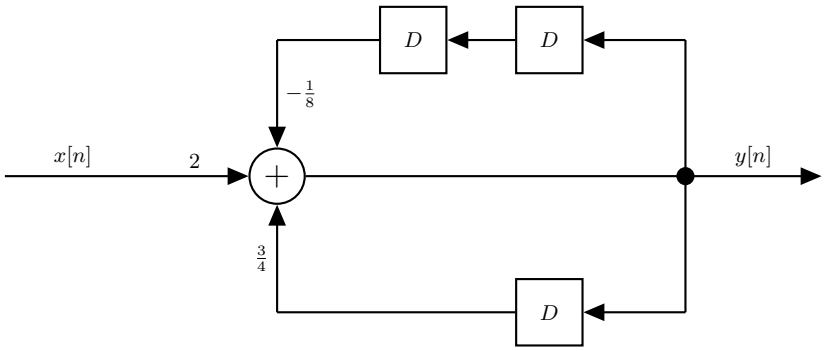
$$h[n] \longleftrightarrow H(e^{j\omega}),$$

   with the following input-output pair:

$$x[n] = \left(\frac{2}{3}\right)^n u[n],$$
$$y[n] = n\left(\frac{2}{3}\right)^{n+1} u[n].$$

   (a) (5 pts) Find the frequency response of this system.
   (b) (5 pts) Find the impulse response of this system from its frequency response.
   (c) (5 pts) Find the difference equation which represents this system.
   (d) (5 pts) Find a block diagram representation of this system using unit delay operators and adders.

4. (20 pts) Consider an LTI system given by the following block diagram:



where D is the unit-delay operator.

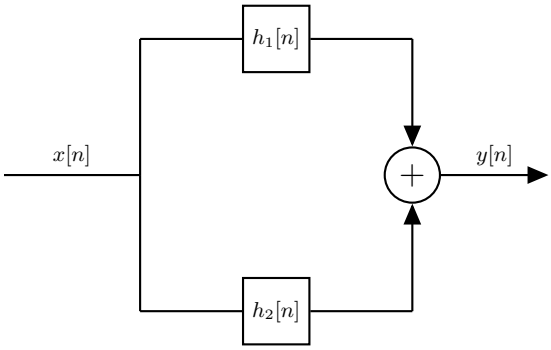
(a) (5 pts) Find the difference equation which represents this system.

(b) (5 pts) Find the frequency response of this system.

(c) (5 pts) Find the impulse response of this system from its frequency response.

(d) (5 pts) Find the output $y[n]$ for the input $x[n] = \left(\frac{1}{4}\right)^n u[n]$ using the frequency response.

5. (10 pts) Suppose that two discrete-time LTI systems with the impulse responses $h_1[n]$ and $h_2[n]$ are connected in parallel.



We have the following information about this combined system:

i. The frequency response of the combined system is $H(e^{j\omega}) = \frac{5e^{-j\omega}-12}{e^{-2j\omega}-7e^{-j\omega}+12}$.

ii. The impulse response of the first system is $h_1[n] = \left(\frac{1}{3}\right)^n u[n]$.

Find $h_2[n]$, the impulse response of the second system.

6. (10 pts) Programming

Write a Python script that computes and plots the DTFT (Discrete-Time Fourier Transform) of the given signal:

$$x[n] = \left(\frac{1}{2}\right)^{|n|}.$$

You can utilize the `numpy` library for efficient numerical calculations and `matplotlib` to generate a clear and informative plot of the DTFT. You should clearly define the parameters involved, including the signal $x[n]$ and you can define a maximum and minimum limit for n. Generate a plot that displays the Fourier Transform $X(e^{j\omega})$ on the y-axis and frequency ω on the x-axis. You can consider a suitable frequency range for visualization.