Student Information

Full Name : BAŞAR YILMAZ
Id Number : 2644409

Answer 1

a)

Expected value of blue dice is:

$$E(B) = 1 * \frac{1}{6} + 2 * \frac{1}{6} + 3 * \frac{1}{6} + 4 * \frac{1}{6} + 5 * \frac{1}{6} + 6 * \frac{1}{6} = 21/6 = 3.5$$

Expected value of yellow dice is:

$$E(Y) = 1 * \frac{1}{8} + 3 * \frac{3}{8} + 4 * \frac{1}{8} + 8 * \frac{1}{8} = 3$$

Expected value of red dice is:

$$E(R) = 2 * \frac{5}{10} + 3 * \frac{2}{10} + 4 * \frac{2}{10} + 6 * \frac{1}{10} = 3$$

b) In order to answer that question we are supposed to calculate the expected values of rolling three dice and rolling three blue dice individually. Let's say, the expected value of rolling three different dice is:

$$E(B + Y + R) = E(B) + E(Y) + E(R) = 3.5 + 3 + 3 = 9$$

and expected value of rolling three blue dice is:

$$E(3 * B) = 3 * E(B) = 3.5 + 3.5 + 3.5 = 10.5$$

We know the values from part a), so we can compare them. The expected value of rolling three blue dice is greater than rolling three different colors. We should roll three blue dice to make it maximum.

c) If it is guaranteed that the yellow dice's value is 8 **its expected value:** $E(Y) = 8$ **can be inferred. We should change the equation in part b) as**

$$E(B + R + 8) = E(B) + E(R) + 8 = 3.5 + 3 + 8 = 14.5$$

So, in this case, the expected value of rolling three different colors will exceed the $E(3 * B) = 10.5$. **We should roll three different dice at this point.**

d) It's known the value is 3, and we want it to be red. Say the possibility of getting three is $P(E)$. We need to find $P(R|E)$. By Bayes Rule,

$$P(R|E) = \frac{P(E|R) * P(R)}{P(E)}$$

$$P(E|R) = \frac{2}{10}$$

$P(R) = \frac{1}{3}$ **(since each color has equal probability in random choosing)**

$$P(E) = P(B \cap E) + P(Y \cap E) + P(R \cap E)$$

$$= P(3|B) * P(B) + P(3|Y) * P(Y) + P(3|R) * P(R)$$

$$= \frac{1}{6} * \frac{1}{3} + \frac{2}{10} * \frac{1}{3} + \frac{3}{8} * \frac{1}{3} = \frac{89}{360}$$

So $P(R|E) = \frac{\frac{1}{3} * \frac{1}{5}}{\frac{89}{360}} = \frac{24}{89}$

e) The only possibilities to get 5 are rolling (1 blue and 4 yellow) or (2 blue and 3 yellow) or (4 blue and 1 yellow).

Blue	Yellow
1	4
2	3
4	1

$$P(B_1) * P(Y_4) + P(B_2) * P(Y_3) + P(B_4) * P(Y_1) = \frac{1}{6} * \frac{1}{8} + \frac{1}{6} * \frac{3}{8} + \frac{1}{6} * \frac{3}{8} = \frac{7}{48} \approx 0.1458$$

Answer 2

a) We can use Poisson approximation to the binomial distribution, since $n = 80 >= 30$ **and** $p = 0.025 <= 0.05$ **(we will use this as** $\lambda = n * p = 80 * 0.025 = 2$**)**

$P\{x >= 4\} = 1 - F(3)$ **(where** $F(x)$ **is binomial cdf)**

Since conditions are met, we will use Poisson approximation:

$P\{y >= 4\} = 1 - F(3)$ **(in this case** $F(y)$ **is Poisson distribution cdf)**

$1 - F(3) = 1 - 0.8571 = 0.1429$

b) Our answer $P\{$**Buying phone in two days**$\}$ **is equivalent to** $1 - P\{$**No discount on A for 2 days**$\} * P\{$**No discount on B for 2 days**$\}$**.**

For A **we can use the Poisson approximation to the binomial distribution same as part a); yet, the** λ **is no longer the same as the average discount amount for 2 days is not the same with 1 day. So our new** $\lambda = 2 * 2 = 4$**.**

$P\{$**No discount on A for 2 days**$\} = P\{X = 0\} \approx 0.018$**. (where** $P\{X\}$ **is Poisson pdf)**

$P\{$**No discount on B for 2 days**$\} = P\{Y = 0\} = 0.81$**. (where** $P\{Y\}$ **is binomial pdf)**

$1 - (0.018 * 0.81) = 0.98542$

Answer 3

Average total value for the first option: 9.4620

Average total value for the second option: 10.546

Percentage of the cases where the total value of the second option is greater than the first option: 56.30%

Our observation of rolling dice 1000 times is really close to our expected values for two expected values. Repeating the simulation changes the averages; yet, they are always almost the same as the expected values computed in Q1.

Octave Code:

```
% array of face values for each dice
blue = [1 2 3 4 5 6];
red = [2 2 2 2 2 3 3 4 4 6];
yellow = [1 1 1 3 3 3 4 8];

% iteration num
n = 1000;

% a) rolling one dice
total1 = zeros(n,1); % nx1 zero matrix for storing total
for i = 1:n
    % Roll one die of each color
    blue_roll = blue(randi(6));
    yellow_roll = yellow(randi(8));
    red_roll = red(randi(10));

    % Calculate the total value
    total1(i) = red_roll + blue_roll + yellow_roll;
end

% average for all total values
avg1 = mean(total1);

% b) rolling three blue dice
total2 = zeros(n,1);
for i = 1:n
    % Roll three blue dice
    % an array of 1x3
    blue_rolls = blue(randi(6,1,3));
```

```
    % Calculate the total value
    total2(i) = sum(blue_rolls);
end

% average for all total values
avg2 = mean(total2);

% percentage of case2 > case1
percent = sum(total2 > total1)/n * 100;
```