Name, SURNAME and ID ⇒ KEY

Middle East Technical University
Department of Computer Engineering

CENG 331

Section 2
Fall '2021-2022

Final

- **Duration: 40+20+20** minutes.
- **Exam:**
  - Write your name, surname, and id on ALL THE PAGES!
  - The exam will be held in three sessions: 40 + 40 + 20 minutes. This title page will only appear once in Part I and hence read it carefully since it also applies other parts.
  - An Appendix will be handed to you as reference material. The appendix will not be graded, but you cannot take it out of the exam, and should return it at the end.
  - If you have any questions, write your question on the back of the Appendix, fold it, and pass it to us. We will reply in written form.
  - Make sure you cover both your nose and mouth with your mask. If you fail to do so, we have to remove you from the exam for the safety of other students.
  - This is a **closed book**, **closed notes** exam.
  - Write your answers only in the indicated spaces. Notes/writing on other parts of the page may be neglected during grading.
  - No attempts of cheating will be tolerated. In case such attempts are observed, the students who took part in the act will be prosecuted. The legal code states that students who are found guilty of cheating shall be expelled from the university for **a minimum of one semester**!

111

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```
int main(){
  unsigned result=INTEGER_MAX-UNSIGNED_MAX;
  short unsigned shortresult=INTEGER_MAX+SHORTINT_MIN;
  printf("Hexadecimal: result:%x shortresult=%x",result,shortresult);
```

30 → mixed → un

100 . . . 0

integer_max = 0111...1 = $2^{w-1}-1$

unsigned_max = 11... 1 = $2^{w}-1$

(4 pts) Consider a computer system where int and unsigned numbers are represented with 30 bits and short and short unsigned numbers are represented with 15 bits.

Assuming that the maximum and minimum of the int/unsigned/short/short unsigned values are represented with capitalized constants. For instance, INTEGER_MAX represents the maximum value that an int variable can get.

What will the following C code print?

$2^{w-1} \cancel{-1} - 2^{w} \cancel{+1}$

$-2^{w-1} \times 2$

$2^{w-1}(1-2)$

$\boxed{= -2^{w-1}}$

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```
int main(){
  unsigned result=INTEGER_MAX-UNSIGNED_MAX;
  short unsigned shortresult=INTEGER_MAX+SHORTINT_MIN;
  printf("Hexadecimal: result:%x shortresult=%x",result,shortresult);
```

1000 . . . 0

15

1 (

(a) (4 pts) Consider a computer system where int and unsigned numbers are represented with 30 bits and short and short unsigned numbers are represented with 15 bits.

Assuming that the maximum and minimum of the int/unsigned/short/short unsigned values are represented with capitalized constants. For instance, INTEGER_MAX represents the maximum value that an int variable can get.

What will the following C code print?

2000 0000	3FFF

0111 ... 1

1111100 ... 0

```
int main(){
  unsigned result=INTEGER_MAX-UNSIGNED_MAX;
  short unsigned shortresult=INTEGER_MAX+SHORTINT_MIN;
  printf("Hexadecimal: result:%x shortresult=%x",result,shortresult);
}
```

INTEGER_MAX = 01 1111 1111 1111 1111 1111 1111 1111

UNSIGNED_MAX = 11 1111 1111 1111 1111 1111 1111 1111

~UN_MAX+1 = -UNSIGNED_MAX = 00 0000 0000 0000 0000 0000 0000 0001

result = 10 0000 0000 0000 0000 0000 0000 0000

= 0x2 000 0000

SHORTINT_MIN = 100 0000 0000 0000

INTEGER_MAX = 01 1111 1111 1111 1111 1111 1111 1111

= 011 1111 1111 1111 = 0x3FFF

(b) (4 pts) Write the C expression to generate the bit pattern $0^{w-k}1^k$ where a^k represents k repetitions of symbol a. Assume a w-bit data type. Your code may contain references to parameter k, representing the values of k, but not a parameter representing w.

$$\boxed{(1<<k)-1}$$

$1<<k = 0\cdots 01\underbrace{00\cdots 0}_{k}$

$-1 = 1\cdots 11\,11\cdots 1$

$(1<<k)-1 = 0\cdots 00\,11\cdots 1$

$= 0^{w-k}\,1^{k}$

Name, SURNAME and ID ⇒ [ ]

$0|11110|11111111$ (5, 8) bias $= 2^4 - 1 = 15$

Bias $= 2^4 - 1 = 15$

2 **(12 pts) Floats.** Consider a computer system where float numbers are represented with 14 bits. The float representation consisted of a single sign bit, followed by 5 bits representing the *exp*, with the remaining bits reserved for *frac*.

exp $= 30$

$E = \text{exp} - \text{bias}$ 15

$0|11111|1111$

(a) What is the largest positive real number that can be represented (do not consider infinity)? Write the binary representation in the first box and the result in full precision in decimal in the second box.

2^{15}

$1 + \frac{2^8 - 1}{2^8}$

0 11110 1111 1111	65408

$(-1)^0 \cdot (2 - 2^{-8}) \cdot 2^{15} = 2^{16} - 2^7$

$\frac{2^8 + 2^8 - 1}{2^8}$

$\frac{2^9 - 1}{2^8}$

(b) What is the smallest positive real number that can be represented above zero? Write the binary representation in the first box and the result as a fraction such as 1/512 in the second box.

$0|00000|00000001$

$1 - \text{bias} = -14$

$2^{-14} \times \frac{1}{2^8}$

0 00000 0000 0001	1/4194304

$(-1)^0 \quad 2^{-8} \cdot 2^{-14} = 2^{-22}$

(c) What is the smallest interval between any two consecutive numbers represented in float representation? Write the result as a fraction such as 1/512.

1/4194304

2^{-22}

(d) What is the largest interval between any two consecutive numbers represented in float representation? Write the result as a decimal number.

128

2^7

Next largest: 0 11110 1111 1110

$(-1)^0 (2 - 2^{-7}) \cdot 2^{15} = 2^{16} - 2^8$

Difference: $(2^{16} - 2^7) - (2^{16} - 2^8)$

$= 2^{16} - 2^7 - 2^{16} + 2^8$

$= 2^7$

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3 **(7 pts) Assembly.** Circle the C-code that the following assembly code is generated from. ***Wrong answers: -2 pts.***

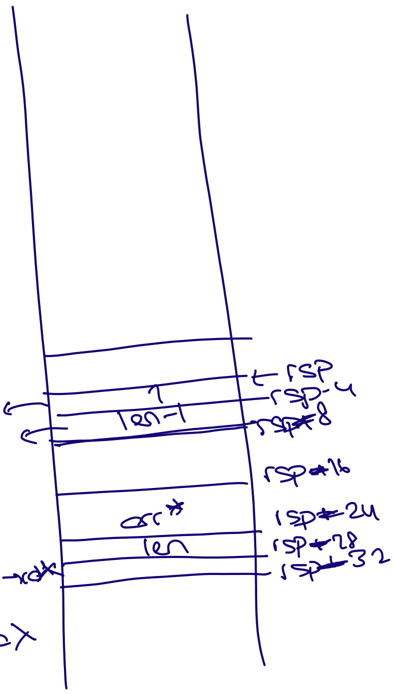
```
func:pushq %rbp
     movq %rsp, %rbp
     movq %rdi, -24(%rbp)
     movl %esi, -28(%rbp)
     movl -28(%rbp), %eax
     subl $1, %eax
     movl %eax, -8(%rbp)
     movl $1, -4(%rbp)
     jmp .L2
.L3: movl -8(%rbp), %eax
     cltq    ;sign extends eax into rax
     leaq 0(,%rax,4), %rdx
     movq -24(%rbp), %rax
     addq %rdx, %rax
     movl (%rax), %eax
     cmpl %eax, -4(%rbp)
     cmovge -4(%rbp), %eax
     movl %eax, -4(%rbp)
     subl $1, -8(%rbp)
.L2: cmpl $0, -8(%rbp)
     jns .L3
     movl -4(%rbp), %eax
     popq %rbp
     ret
```

rdi → arr*
rsi → len

rax / len
len-1

res
i

(len-1) << 2 → rdx
arr* → rax



>0

rax / ui   rax / arr*

res > arr[i] → res = a[i]
res

(a)
```
int func(int* arr, int len){
  int i, res;
  for(i = len-1, res = 1; i >= 0; i--) res += arr[i];
  return res;}
```

(b)
```
int func(int* arr, int len){
  int i, res;
  for(i = len-1, res = 0; i >= 0; i--) res = arr[i] > res ? arr[i] : res;
  return res;}
```

(c)
```
int func(int* arr, int len){
  int i, res;
  for(i = len-1, res = 1; i >= 0; i--) res *= arr[i];
  return res;}
```

(d)
```
int func(int* arr, int len){
  int i, res;
  for(i = len-1, res = 1; i >= 0; i--) res = arr[i] > res ? arr[i] : res;
  return res;}
```

(e)
```
int func(int* arr, int len){
  int i, res;
  for(i = 0, res = 1; i < len; i++) res = arr[i] > res ? arr[i] : res;
  return res;}
```

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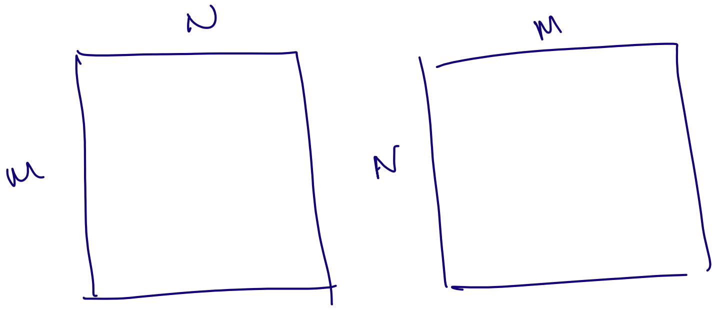
4 **(8 pts) Assembly.** Consider the following source code, where `M` and `N` are constants declared with `#define`:

```
long P[M][N];
long Q[N][M];
long sum_element(long i, long j) {
    return P[i][j] + Q[j][i];
}
```



In compiling this program, gcc generates the following assembly code:

```
sum_element:
 leaq 0(,%rdi,8), %rdx
 subq %rdi, %rdx
 addq %rsi, %rdx
 leaq (%rsi,%rsi,4), %rax
 addq %rax, %rdi
 movq Q(,%rdi,8), %rax
 addq P(,%rdx,8), %rax
 ret
```

8 rdi i

$\frac{rdx}{8i-i+j}$ $\frac{rax}{5j}$

$\frac{rdi}{5j+i}$

$\frac{rax}{\underbrace{8(5j+i)+Q}_{Q} + \underbrace{8(7i+j)+P}_{P}}$

Use your reverse engineering skills to determine the values of `M` and `N` based on this assembly code.

M=5 N=7

$\boxed{M=5\ ,\ N=7}$

Reference to P is at byte offset $8(7i+j)$

Reference to Q is at byte offset $8(5j+i)$

⇒ P has 7 columns

Q has 5 columns

**** End of Part I ****

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$\boxed{5}$ **(10 pts) Cache and Virtual Memory.** Consider a computer system with the following specifications:

- Virtual memory: 2^{38} bytes
- Physical memory: 2^{32} bytes
- Page size: 2048 bytes $= 2^{11}$
- Data TLB: Direct-mapped cache with 64 entries

VM

$|VA| = 38$

$|PA| = 32$

$|PO| = 11$

VA: VPN | VPO — 27, 11

VPN: TLB T | TLB I — 21, 6

$E=1$ $\quad 2^6 \Rightarrow S1=6$

How many TLB misses will occur during the execution of the following program due to accesses to A? Assume that no other data accesses, other than accesses to A take place during the execution of the program. Assume that &A=0x0, and TLB is cold initially, i.e. no prior accesses were made to A before. Show your work below in order to get partial grades.

$\boxed{2^{21}}$

```
#define N 1<<30
int sum=0, A[N];
for (i=0; i<N;i++)
    sum+=A[i];
```

VA: VPN (27) | VPO (11)

VPN: 21 | 6

TLB Tag $\quad$ TLB Index

Access A[0], A[1] . . . A[i]

0x0 $\quad$ $0x0 + 1 * \underbrace{4}_{\text{sizeof(int)} = 4}$ $\quad$ $0x0 + i * 4$

One page holds $2^{11}/2^2 = 2^9$ elements of A.

TLB miss happens whenever we switch from one page to another.

A has a total of 2^{30} elements

$\Rightarrow$ # of TLB misses $= 2^{30}/2^9 = 2^{21}$

The size of Physical Memory is irrelevant

The size of TLB is also irrelevant, since we iterate only once.

Name, SURNAME and ID $\Rightarrow$

$\boxed{6}$ **(15 pts) Virtual Memory 1.** Consider a computer system with the following specifications:

- Virtual memory: 2^{38} bytes
- Physical memory: 2^{18} bytes
- Page size: 2^{10} bytes

VA

← 28 →		← 10 →
VPN1	VPN2	VPO

PA

← 8 →	← 10 →
PPN	PPO

We want to have a virtual memory system with a 2-level page system, such that the page tables at the second (i.e. last) level fits in a single page. Assume that the Page Table Entry (PTE) is only big enough to store the Valid bit and the Physical Page Number (PPN), and the size of the PTE can only be an power of 2 bytes (i.e. size of PTE can be 2, 4, 8 but not 3, or 6 bytes).

(a) What is the size of the PTE in bytes? $\boxed{2}$

$|VPN2| = 9$

(b) How many PTEs does a second-level page table contain? $\boxed{2^{10}/2 = 2^{9}}$ 512

$|VPN1| = 28 - |VPN2| = 19$

(c) How many PTEs does the first-level page table contain? $\boxed{2^{19}}$

(d) How many bytes is the first level page table? $\boxed{2^{19} * 2 = 2^{20}}$

PTE

← 1 →	← 8 →
V	PPN

$\Rightarrow |PTE| = \underline{2 \text{ bytes}}$

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7 **(15 pts) Virtual Memory 2.** The following problem concerns the way virtual addresses are translated into physical addresses.

VA: VPN (8) | VPO (9)

PA: PPN (6) | PPO (9)

- The memory is byte addressable.
- Memory accesses are to **1-byte words** (not 4-byte words).
- Virtual addresses are 17 bits wide. $|VA|=17$
- Physical addresses are 15 bits wide. $|PA|=15$
- The page size is 512 bytes. $= 2^9$ $|VPO|=|PPO|=9$
- The TLB is 4-way set associative with 16 total entries. $|\text{TLB Index}|=2$
- The cache is physically addressed.
- The cache is 2-way set associative, with a 4 byte line size and 16 total lines. $|\text{Cache Index}|=3$

SI=2

TLBT (6) | TLBI (2)

$|\text{Cache Offset}|=2$

In the following tables, **all numbers are given in hexadecimal**. The contents of the TLB, the page table for the first 32 pages, and the cache are as follows:

PPN: Tag (1) | SI (3) | BO (2)

TLB			
Index	Tag	PPN	Valid
0	09	4	1
	12	2	1
	10	0	1
	08	5	1
1	05	7	1
	13	1	0
	10	3	0
	18	3	0
2	04	1	0
	0C	1	0
	12	0	0
	$\boxed{03}$	1	0
3	06	7	0
	03	1	0
	07	5	0
	02	2	0

TLB Miss

Page Table					
VPN	PPN	Valid	VPN	PPN	Valid
00	6	1	10	0	1
01	5	0	11	5	0
02	3	1	12	2	1
03	4	1	13	4	0
04	2	0	14	6	0
05	7	0	15	2	0
06	1	0	16	4	0
07	3	0	17	6	0
08	5	1	18	1	1
09	4	0	19	2	0
0A	3	0	1A	5	0
0B	2	0	1B	7	0
0C	5	0	1C	6	0
0D	6	0	1D	2	0
0E	7	1	1E	3	0
0F	0	0	1F	1	0

2-way Set Associative Cache												
Index	Tag	Valid	Byte 0	Byte 1	Byte 2	Byte 3	Tag	Valid	Byte 0	Byte 1	Byte 2	Byte 3
0	19	1	99	11	23	11	00	0	99	11	23	11
1	15	0	4F	22	EC	11	00	1	55	59	0B	41
2	1B	1	00	02	04	08	0B	1	01	03	05	07
3	06	0	84	06	B2	9C	FD	1	84	06	B2	9C
4	07	0	43	6D	8F	09	05	0	43	6D	8F	09
5	0D	1	36	32	00	78	1E	1	A1	B2	C4	DE
6	11	0	A2	37	68	31	00	1	BB	77	33	00
7	16	1	11	C2	11	33	1E	1	00	C0	0F	00

- (1.5 pts) The box below shows the format of a virtual address. Indicate (by labeling the diagram) the fields (if they exist) that would be used to determine the following: (If a field doesn't exist, don't draw it on the diagram.)

VPO	The virtual page offset
VPN	The virtual page number
TLBI	The TLB index
TLBT	The TLB tag

VPN								VPO								
16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
TLBT						TLBI										

- (1.5 pts) The box below shows the format of a physical address. Indicate (by labeling the diagram) the fields that would be used to determine the following:

PPO	The physical page offset
PPN	The physical page number
CO	The block offset within the cache line
CI	The cache index
CT	The cache tag

PPN						PPO								
14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
CT										CI			CO	

0001 1101 1101 1110

For the given virtual address, indicate the TLB entry accessed, the physical address, and the cache byte value returned **in hex**. Indicate whether the TLB misses, whether a page fault occurs, and whether a cache miss occurs.

If there is a cache miss, enter "-" for "Cache Byte returned". If there is a page fault, enter "-" for "PPN" and leave parts C and D blank.

Virtual address: `1DDE`

VPN VPO

- (2 pts) Virtual address format (one bit per box)

16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
0	0	0	0	1	1	1	0	1	1	1	0	1	1	1	1	0

TLB Tag TLBI

- (4 pts) Address translation

Parameter	Value
VPN	0x 0E
TLB Index	0x 2
TLB Tag	0x 3
TLB Hit? (Y/N)	N
Page Fault? (Y/N)	N
PPN	0x 7

PPN PPO

- (2 pts) Physical address format (one bit per box)

14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
0	0	0	1	1	1	1	1	1	0	1	1	1	1	0

CI Cache/Byte Offset

- (4 pts) Physical memory reference

Parameter	Value
Byte offset	0x 2
Cache Index	0x 7
Cache Tag	0x 7E
Cache Hit? (Y/N)	N
Cache Byte returned	0x —

**** End of Part II ****

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8 **(15 pts) Processor design.** We want to add a "smart call" instruction to the Y86-64 sequential implementation (SEQ). The new instruction

`scall D(rA), rB`

will call the function located at address D+R[rA], and store return address at register rB, instead of pushing it on the stack.

The new instruction, named `scall`, will be a 10-byte instruction as follows:

0	1	2	3	4	5	6	7	8	9
`E0`	`ra:rB`				D				

where `E0` is the 1-byte coding `icode:ifun`, `ra:rB` encode the registers involved and `D` is an immediate, 8 byte value. Fill out the following form to describe what needs to happen during each stage of this new instruction.

(a) (2 pts) Write down the stages of execution.

Stage	Computation
Fetch	icode:ifun ← M_1[PC] rA : rB ← M_1[PC+1] valC ← M_8[PC+2] valP ← PC+10
Decode	valA ← R[rA]
Execute	valE ← valA + valC
Memory	–
Write-back	R[rB] ← valP
PC-update	PC ← valE

(b) (13 pts) Write down the HCL code that needs to be changed using the SEQ HCL code in the appendix as a reference.

```
wordsig ISCALL 'I_ISCALL'
bool instr_valid = icode in { ......., ISCALL };
```

bool need_regids = icode in { ...... , ISCALL }

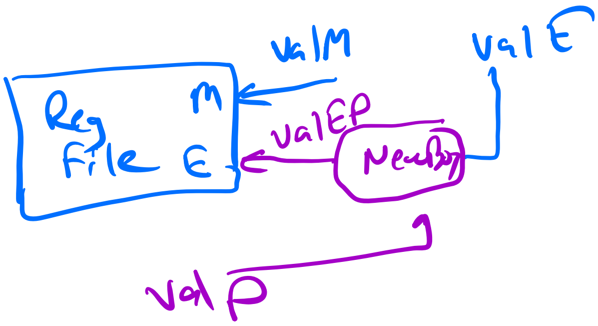
bool need_valC = icode in { .... , ISCALL }

word srcA = icode in { ... , ISCALL } : rA

word dstE = icode in { ... , ISCALL } : rB

Need an extra control box

word valEP = { icode == ISCALL : valP
: valE }



word aluA = [icode in { .. , ISCALL } : valA;
;
]

word aluB = [icode in { ...., ISCALL } : valC;
:
]

word newPC = [icode == ISCALL : valE;
:
]

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9 **(10pts) Processor design.** Consider the final version of the 5-stage PIPE processor developed during the lectures.

(a) (2 pts) How many bubbles are injected in the pipeline during the execution of the following instructions? (2 pts)

- instruction causing load-use hazard: `LU=.` ....
- call instruction: `C= .` ....
- correctly predicted branch instruction: `CP=..` ....
- mispredicted branch instruction: `MB=..` ....
- ret instruction: `R=.` ....

Now consider the execution of s program with the following characteristics:

- The number of instructions: 1400
- The number of all data hazards: 560
- The number of load-use hazards: 140
- The number of call instructions: 70
- The number of branches: 140
- The probability of mispredicting branches: 0.4
- The number of ret instructions: 14

(b) (6 pts) What is the CPI (Cycles per Instruction) for the execution of this program in terms of `LU, C, CP, MB, R`?

(c) (2 pts) What is the CPI (Cycles per Instruction) for the execution of this program as a numerical value?

**** End of Exam ****