Middle East Technical University — Department of Computer Engineering

CENG 280

Formal Languages and Abstract Machines

Spring 2022-2023

Homework 6

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Answer for Q1

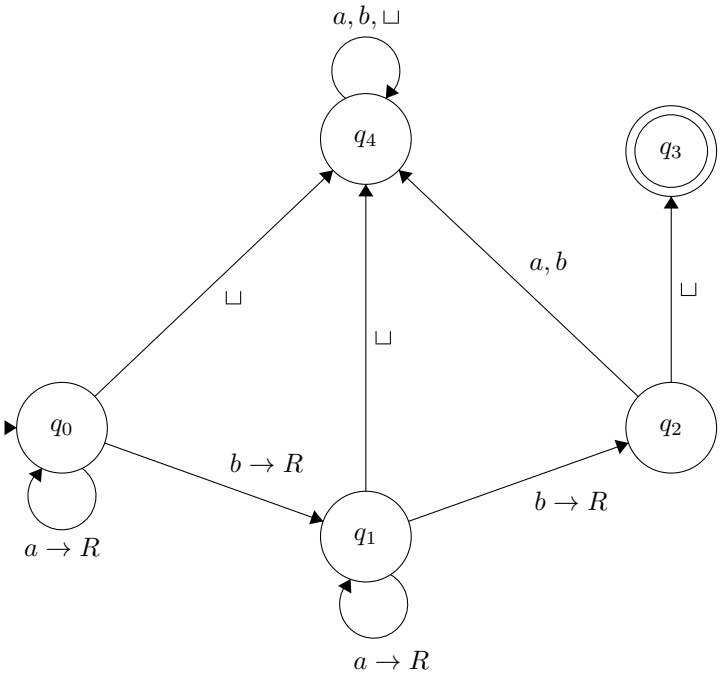
(i) 1954
(ii) Enigma
(iii) Turing test
(iv) The Chemical Basis of Morphogenesis
(v) The Imitation Game

Answer for Q2

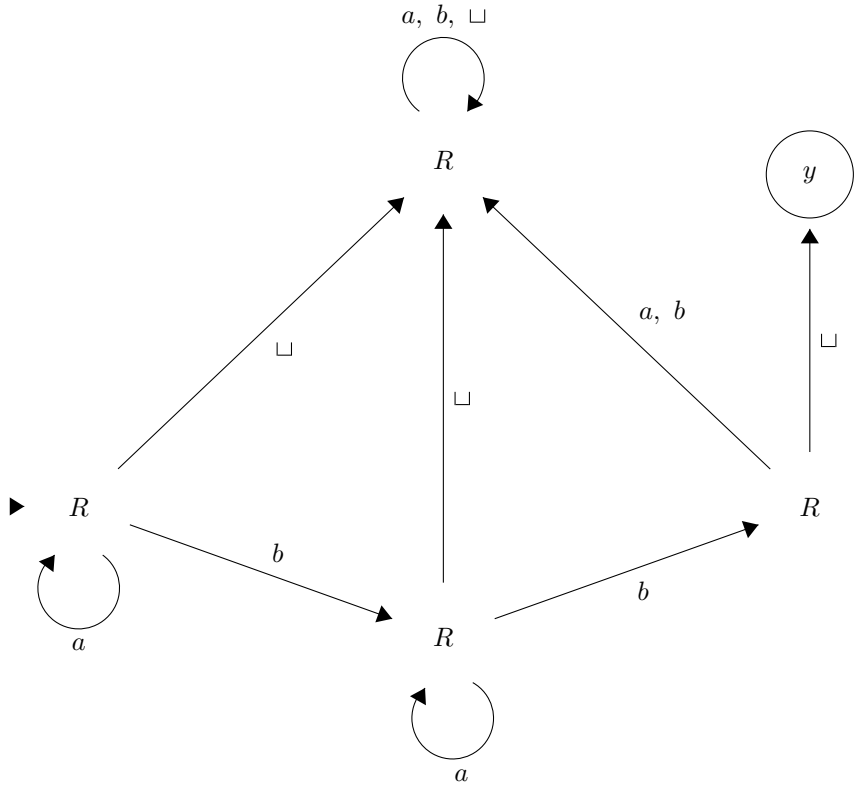
a) $M = (K, \Sigma, \delta, s, H)$
$K = \{q_0, q_1, q_2, q_3, q_4\}$
$\Sigma = \{a, b, \sqcup\}$, $s = q_0$, $H = \{q_3\}$

$q,$	σ	$\delta(q, \sigma)$
q_0	a	$(q_0, \rightarrow)$
q_0	b	$(q_1, \rightarrow)$
q_0	$\sqcup$	$(q_4, \sqcup)$
q_1	a	$(q_1, \rightarrow)$
q_1	b	$(q_2, \rightarrow)$
q_1	$\sqcup$	$(q_4, \sqcup)$
q_2	a	(q_4, a)
q_2	b	(q_4, b)
q_2	$\sqcup$	$(q_4, \sqcup)$
q_4	a	$(q_4, \rightarrow)$
q_4	b	$(q_4, \rightarrow)$
q_4	$\sqcup$	$(q_4, \rightarrow)$

Diagram:



b)



Answer for Q3

We have some assumptions before creating the final Turing machine.

1. Our first tape in the form:

⊔	a	,	b	⊔	⊔	⊔

   Where a and b are binary numbers.

2. 2^{nd} and 3^{rd} tapes are blank initially.

3. $M^a_\times$ and M^a_- work like following:

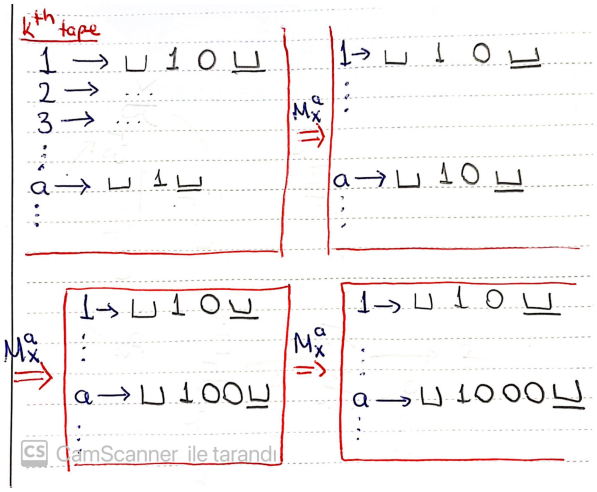


Figure 1: $M^a_\times$ mechanism

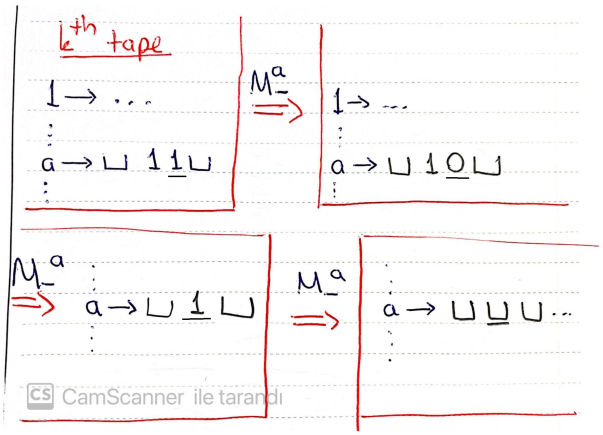


Figure 2: M^a_- mechanism

Basically, $M^a_\times$ takes 1^{st} tape and multiplies it with a^{th} tape and changes a^{th} tape's content with the result of the multiplication, the reading head appears at the first blank space after the string (Figure 1).

The function of M^a_- is to decrease the value in the a^{th} tape by 1 and write the binary result back to the same tape. If there is/are some 0(s) appearing after the first blank space, M^a_- erases it, and the reading head appears at the last non-blank character (Figure 2).

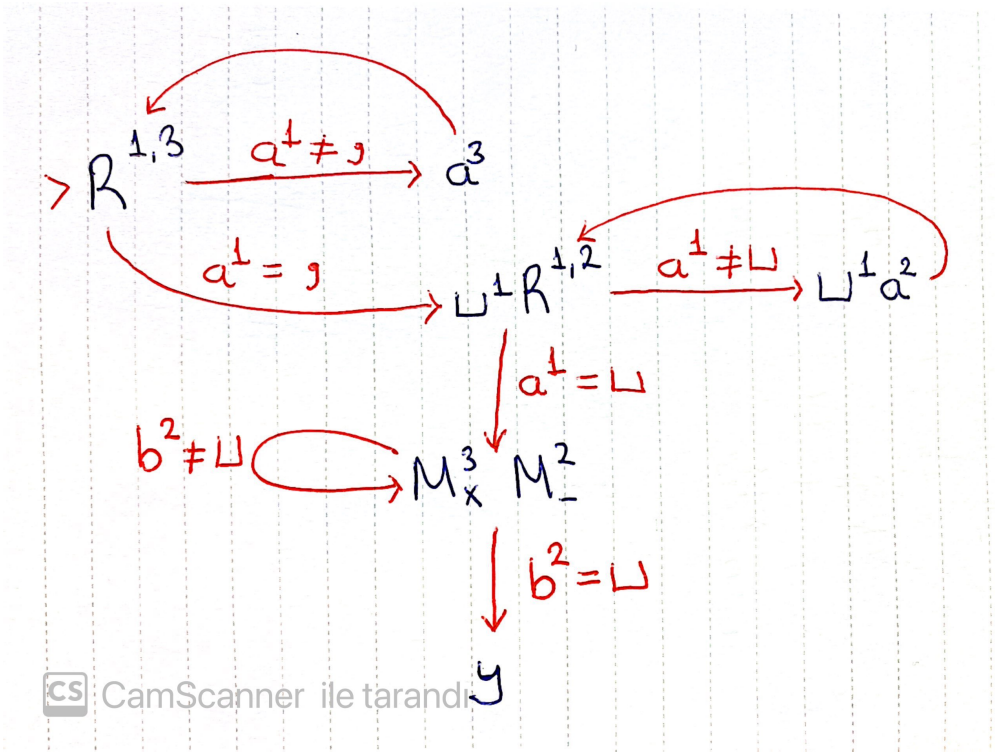


Figure 3: Final Turing Machine

Final Turing Machine:

1. Copy the first element of the input from 1^{st} tape to 3^{rd}. Repeat until the reading head of the first tape reads ,.

2. At this point first part of the input (a) was copied to the third tape. When the head of the first tape reads "," erase it and copy from the first tape to the second tape till it hits the blank space in the first tape.

3. While the head at the second tape is reading a non-blank character, multiply 3^{rd} tape's value with the first tape and write it to the third.

4. When the reading head of the second tape reads a blank space, TM moves the halting state y, which means we multiplied a with a, b times, in the 3^{rd} tape's content, we can read the binary representation of the a^b.