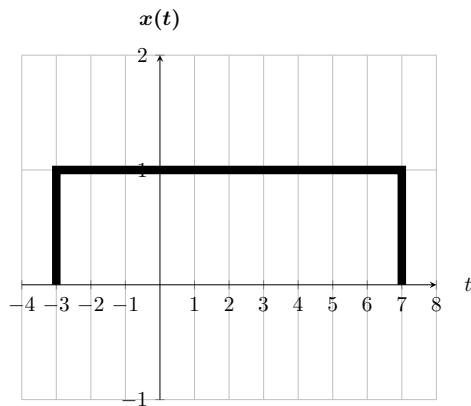




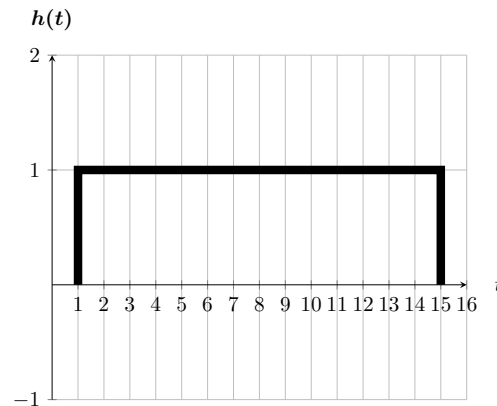
Regulations:

- **Grouping:** You are strongly encouraged to work in pairs.
- **Drawing Plots:** Clearly label the coordinate axes and make sure that your plots are not open to different interpretations.
- **Submission:** You need to submit a pdf file named 'hw2.pdf' to the odtuclass page of the course. You need to use the given template 'hw2.tex' to generate your pdf files. Otherwise you will receive zero.
- **Deadline:** 23:55, April 02, 2024 (Tuesday).
- **Late Submission:** Not allowed.

1. (10 pts) Find the output, $y(t)$, of the system for the following input and impulse response plots:



(a) t vs. $x(t)$.



(b) t vs. $h(t)$.

Figure 1: $x(t)$ and $h(t)$.

2. (15 pts) Find and plot the outputs of the discrete time LTI systems given below for the input $x[n] = \delta[n] + 2\delta[n - 2] - 3\delta[n - 4]$ and the impulse response $h[n] = 2\delta[n + 2] + \delta[n - 2]$:
- (5 pts) $y_1[n] = x[n] * h[n]$
 - (5 pts) $y_2[n] = x[n + 2] * h[n]$
 - (5 pts) $y_3[n] = x[n + 2] * h[n - 2]$
3. (20 pts) Consider a discrete time casual LTI system, represented by the following difference equation:

$$y[n] = \frac{1}{5}x[n - 1] + x[n].$$

- (4 pts) Find the impulse response, $h[n]$, of the system.
 - (4 pts) Find the output, $y[n]$, for the input $x[n] = \delta[n - 2]$.
 - (4 pts) Is this system BIBO stable? Explain.
 - (4 pts) Does this system have memory? Explain.
 - (4 pts) Is this system invertible? If yes, find its inverse.
4. (15 pts) The transfer function of a continuous time LTI system is given as follows:

$$H(\lambda) = \frac{2\lambda}{\lambda^2 - 2\lambda + 1},$$

where the system is initially at rest.

- (5 pts) Find the differential equation which represents this system.
- (5 pts) Find the output this system for $x(t) = 0$.
- (5 pts) Find the output this system for $x(t) = (2t + 1)u(t)$.

5. (15 pts) A discrete time LTI system, which is initially at rest, is represented by the following difference equation:

$$y[n] = \frac{1}{5}y[n-1] + 2x[n-2].$$

- (a) (5 pts) Find the impulse response of this system.
 - (b) (5 pts) Find the transfer function of this system.
 - (c) (5 pts) Find a block diagram representation of this system using the adders and unit delay operators.
6. (10 pts) A continuous time casual LTI system is represented by the following differential equation:

$$y(t) = -\frac{1}{2}y'(t) + 4x(t).$$

- (a) (5 pts) Find a block diagram representation of this system using integrators and adders.
 - (b) (5 pts) Find a block diagram representation of this system using differentiators and adders.
7. (15 pts) Programming.

Write a Python program that calculates and plots the output of a causal Linear Time-Invariant (LTI) system defined by a given difference equation when $x[n] = \delta[n-1]$:

$$y[n] = \frac{1}{4}y[n-1] + x[n],$$

without using built-in convolution functions (e.g., `numpy.convolve()`). Plot the first 5 samples of $y[n]$ using **matplotlib.pyplot**. You should write your code in **Python 3**. You are not allowed to use any library other than **matplotlib.pyplot**.