INTEGRALI NOTEVOLI

$\int f(x)^{\alpha} f'(x)dx = \frac{f(x)^{\alpha+1}}{\alpha+1} + c$

$\int \frac{f'(x)}{f(x)}dx = \log|f(x)| + c \quad \int \frac{\sin(2x)}{\cos(2x)}dx = \log(\cos(x))$

$\int e^{f(x)} f'(x)dx = e^{f(x)} + c$

$\int \sin f(x) \cdot f'(x)dx = -\cos(f(x)) + c$

$\int \cos f(x) \cdot f'(x)dx = \sin(f(x)) + c$

$\int \sinh f(x) \cdot f'(x)dx = \cosh(f(x)) + c$

$\int \cosh f(x) \cdot f'(x)dx = \sinh(f(x)) + c$

$\int \frac{f'(x)}{1+(f(x))^2}dx = \arctan f(x) + c$

$\int \frac{f'(x)}{\sqrt{1-(f(x))^2}}dx = \arcsin f(x) + c$

$\int \frac{f'(x)}{\sqrt{(f(x))^2+1}}dx = \operatorname{settsinh} f(x) + c$

$\int \frac{f'(x)}{\sqrt{(f(x))^2-1}}dx = \operatorname{settcosh} f(x) + c$

$\int \sqrt{1-f(x)^2} \cdot f'(x)dx = \frac{1}{2}[f(x)\sqrt{1-(f(x))^2} + \arcsin f(x)] + c$

$\int \sqrt{f(x)^2+1} \cdot f'(x)dx = \frac{1}{2}[f(x)\sqrt{(f(x))^2+1} + \operatorname{settsinh} f(x)] + c$

$\int \sqrt{f(x)^2-1} \cdot f'(x)dx = \frac{1}{2}[f(x)\sqrt{(f(x))^2-1} + \operatorname{settcosh} f(x)] + c$

$I_n = \int \frac{1}{(1+x^2)^n}dx = \frac{1}{2(n-1)} \frac{x}{(x^2+1)^{n-1}} + \frac{2n-3}{2(n-1)}$

$\int \tan(x) = \int \frac{\sin(2x)}{\cos(2x)}dx = \log(\cos(x))$

POTENZE:

$a^m \cdot a^n = a^{m+n}$

$(a^m)^n = a^{m \cdot n}$

$a^m \cdot b^m = (ab)^m$

$a^n / a^m = a^{n-m}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

INTEGRALI IMMEDIATI

$x^{\alpha} \rightarrow \frac{x^{\alpha+1}}{(\alpha+1)}$

$\frac{1}{x} \rightarrow \log|x|$

$e^x \rightarrow e^x$

$a^x \rightarrow \frac{a^x}{\log a}$

$\sin x \rightarrow -\cos x$

$\cos x \rightarrow \sin x$

$\tan x \rightarrow -\log|\cos x|$

$\cot x \rightarrow -\log|\sin x|$

$Sh\ x \rightarrow Ch\ x$

$Ch\ x \rightarrow Sh\ x$

$\frac{1}{(\cos x)^2} \rightarrow \tan x$

$\frac{1}{(\sin x)^2} \rightarrow -\cot x$

$\frac{1}{(1+x^2)} \rightarrow \arctan x$

$\frac{1}{\sqrt{1-x^2}} \rightarrow \arcsin x$

$\cos^2 x \rightarrow \frac{1}{2}(x + \sin x \cos x)$

LIMITI NOTEVOLI

$\lim_{x\to\pm\infty} \left(1+\frac{a}{x}\right)^x = e^a$

$\lim_{x\to+\infty} \left(1+\frac{\alpha}{x}\right)^{\beta x} = e^{\alpha\beta}$

$\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$

$\lim_{x\to 0} \frac{\log(1+x)}{x} = 1$

$\lim_{x\to 0} \frac{\log_a(1+x)}{x} = \frac{1}{\log a}$

$\lim_{x\to 0} \frac{e^x-1}{x} = 1$

$\lim_{x\to 0} \frac{a^x-1}{x} = \log a$

$\lim_{x\to 0} \frac{(1+x)^{\alpha}-1}{x} = \alpha$

$\lim_{x\to\infty} \frac{x^{\alpha}}{e^x} = 0$

$\lim_{x\to\infty} \frac{(\log(x))^{\alpha}}{x} = 0$

$\lim_{x\to+\infty} \log_a x = +\infty$

$\lim_{x\to x_0} \log_a x = \log_a x_0$ dove $x_0 > 0$

$\lim_{x\to 0^+} \log_a x = -\infty$

$\lim_{x\to\infty} x^{\frac{1}{x}} = 1$

$\lim_{x\to 0} \sin(x) = 0$

$\lim_{x\to 0} \cos(x) = 1$

$\lim_{x\to 0} \frac{\sin(x)}{x} = 1$

$\lim_{x\to 0} \frac{1-\cos(x)}{x^2} = \frac{1}{2}$

$\lim_{x\to 0} \frac{\tan(x)}{x} = 1$

$\lim_{x\to 0} \frac{\arcsin(x)}{x} = 1$

$\lim_{x\to 0} \frac{\sinh(x)}{x} = 1$ ...

$\lim_{x\to 0} \frac{\tanh(x)}{x} = 1$

$\lim_{x\to 0} \frac{\arctan(x)}{x} = 1$

$\lim_{x\to\infty} \frac{\log_a x}{x^b} = \lim_{x\to\infty} \frac{x^b}{a^x} = \lim_{x\to\infty} \frac{a^x}{x^x} = 0$ con $b > 0$ ed $a > 1$

DERIVATE NOTEVOLI

$(x^n)' = nx^{n-1}$, con $n \in \mathbb{N}$ e $x \in \mathbb{R}$.

$(x^{\alpha})' = \alpha x^{\alpha-1}$, con $\alpha \in \mathbb{R}$ e $x > 0$.

$(a^x)' = a^x \log a$, con $a > 0$, $a \neq 1$.

$(e^x)' = e^x$, per ogni $x \in \mathbb{R}$.

$(\log_a x)' = \frac{1}{x \log a}$, con $a > 0$, $a \neq 1$ e per ogni $x > 0$.

$(\log x)' = \frac{1}{x}$, per ogni $x > 0$.

$(\sin x)' = \cos x$, per ogni $x \in \mathbb{R}$.

$(\cos x)' = -\sin x$, per ogni $x \in \mathbb{R}$.

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x} = 1 + \operatorname{tg}^2 x$, per ogni $x \in \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi\}$, $k \in \mathbb{Z}$.

$(\operatorname{cotg} x)' = -\frac{1}{\sin^2 x} = -1 - \operatorname{cotg}^2 x$, per ogni $x \in \mathbb{R} \setminus \{k\pi\}$, $k \in \mathbb{Z}$.

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$, per ogni $x \in (-1, 1)$.

$(\arccos x)' = -\frac{1}{1-x^2}$, per ogni $x \in (-1, 1)$.

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$, per ogni $x \in \mathbb{R}$.

$(\sinh x)' = \cosh x$, per ogni $x \in \mathbb{R}$.

$(\cosh x)' = \sinh x$, per ogni $x \in \mathbb{R}$.

$(\operatorname{tgh} x)' = \frac{1}{\cosh^2 x} = 1 - \operatorname{tgh}^2 x$, per ogni $x \in \mathbb{R}$.

$(\operatorname{settsinh} x)' = \frac{1}{\sqrt{1+x^2}}$, per ogni $x \in \mathbb{R}$.

$(\operatorname{settcosh} x)' = \frac{1}{\sqrt{x^2-1}}$, per ogni $x > 1$.

$(\operatorname{setttgh} x)' = \frac{1}{1-x^2}$, per ogni $x \in (-1, 1)$.

$(|x|)' = \frac{|x|}{x}$

$(arctg(\frac{1}{x}))' = -\frac{1}{1+x^2}$

$(\frac{1}{2} \cdot log(\frac{1+x}{1-x}))' = \frac{1}{1-x^2}$

REGOLE DI DERIVAZIONE:

$(f \pm g)'(x) = f'(x) \pm g'(x)$

$(f \cdot g)'(x) = f'(x)g(x) + f(x)g'(x)$

$(f/g)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$ con $g(x) \neq 0$

$f(g(x))' = f'(g(x))\, g'(x)$

$(f^{-1})'(y) = 1/f'(f^{-1}(y))$

$(a \cdot f)' = a \cdot f'$

DERIVATE DI FUNZIONI COMPOSTE

$D[f(x)]^{\alpha} = \alpha[f(x)]^{\alpha-1} \cdot f'(x)$

$D \log(f(x)) = \frac{f'(x)}{f(x)}$

$D\, a^{f(x)} = f'(x) \cdot a^{f(x)} \cdot \ln(a)$

$D\, e^{f(x)} = f'(x) \cdot e^{f(x)}$

$D \sin(f(x)) = f'(x) \cdot \cos(f(x))$

$D \cos(f(x)) = f'(x) \cdot -\sin(f(x))$

$D \tan(f(x)) = \frac{f'(x)}{\cos^2(f(x))}$

$D \cot(f(x)) = -\frac{f'(x)}{\sin^2(f(x))}$

$D \log_a(f(x)) = \frac{f'(x)}{f(x)\log(a)}$

$D \arctan(f(x)) = \frac{f'(x)}{1+f^2(x)}$

$D \arcsin(f(x)) = \frac{f'(x)}{\sqrt{1-f^2(x)}}$

$D \arccos(f(x)) = -\frac{f'(x)}{\sqrt{1-f^2(x)}}$

$D \cosh(f(x)) = f'(x) \cdot \sinh(f(x))$

$D \sinh(f(x)) = f'(x) \cdot \cosh(f(x))$

$D \tanh(f(x)) = \frac{f'(x)}{\cosh^2(f(x))}$

$D \operatorname{settsinh}(f(x)) = \frac{f'(x)}{\sqrt{1+f^2(x)}}$

$D \operatorname{settcosh}(f(x)) = \frac{f'(x)}{\sqrt{f^2(x)-1}}$

$D[f(x)]^{g(x)} =$

$= [f(x)]^{g(x)} \cdot \{g'(x)\log(f(x)) \cdot \frac{f'(x)}{f(x)}$

Integrali per parti

$\int_a^b f(x)'g(x)dx$

$= [f(x)g(x)]_a^b - \int_a^b f(x)g(x)'dx$

Integrali per sostituzione

$\int_{\phi(a)}^{\phi(b)} f(x)dx = \int_{\phi(a)}^{\phi(b)} f(\phi(t))\phi'(t)dt$

Sostituzioni utili

$\int f(e^x)dx$	$e^x = t$
$\int \frac{1}{x} f(\log x)dx$	$\log x = t$
$\int f(\sin x, \cos x)dx$	$\tan\frac{x}{2} = t$
$\int f(\tan x)dx$	$\tan x = t$
$\int \sqrt{1-x^2}dx, \int \frac{1}{\sqrt{1-x^2}}dx$	$x = \sin t$
$\int \sqrt{1+x^2}dx, \int \frac{1}{\sqrt{1+x^2}}dx$	$x = \sinh t$
$\int \sqrt{x^2-1}dx, \int \frac{1}{\sqrt{x^2-1}}dx$	$x = \cosh t$

$|x| \geq 0$

$|x| = 0 \iff x = 0, \ |-x| = |x|$

$|x \cdot y| = |x| \cdot |y|, \ \left|\frac{x}{y}\right| = \frac{|x|}{|y|}$

$\log_a 1 = 0, \ a^{\log_a(b)} = b$

$\log(b \cdot c) = \log(b) + \log(c)$

$\log\left(\frac{b}{c}\right) = \log(b) - \log(c)$

$\log\left(b^c\right) = c \cdot \log(b)$

Formule parametriche

Posto $t = \tan\frac{x}{2}$ si ha :

$\sin x = \frac{2t}{1+t^2}$

$\cos x = \frac{1-t^2}{1+t^2}$

$\tan x = \frac{2t}{1-t^2}$

Cambio base

$\log_a(b) = \frac{\ln(a)}{\ln(b)} \rightarrow a^x = b^{\log_b(a)}$

$\rightarrow \log_a x = \log_a b \cdot \log_b x$

Relazione asintoto

Se $\lim_{x\to x_0} f(x) = 0$

$\sin(f(x)) \approx f(x)$

$e^{f(x)} - 1 \approx f(x)$

$1 - cos(f(x)) \approx \frac{1}{2}(f(x))^2$

$\arctan f(x) \approx f(x)$

$\log(1+f(x)) \approx f(x)$

$a^{f(x)} - 1 \approx \log(a)f(x)$

$\tan(f(x)) \approx f(x)$

$\sinh(f(x)) \approx f(x)$

$\tanh(f(x)) \approx f(x)$

$(1+f(x))^c - 1 \approx c \cdot f(x)$

$\arcsin(f(x)) \approx f(x)$

$\cosh(f(x)) - 1 \approx \frac{1}{2}(f(x))^2$

Prodotti Notevoli

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

$(a+b)^2 = a^2 + b^2 + 2ab$

$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$(a^3 \pm b^3) = (a \pm b)(a^2 \mp ab + b^2)$

$(a+b+c)^3 = a^3 + b^3 + c^3 + 3a^2b + 3a^2c + 3b^2a + 3b^2c + 3c^2a + 3c^2b + 6abc$

$a^{n+1} - b^{n+1} = (a-b)(a^n + a^{n-1} \cdot b + \ldots + a b^{n-1} + b^n)$

$a^b = e^{b \cdot \log a}$

$a^b = c^{b \cdot \log_c a}$

$\cos^2\alpha = \frac{1+\cos(2\alpha)}{2}$

$\sin^2\alpha = \frac{1-\cos(2\alpha)}{2}$

$\tan^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{1-\cos(2\alpha)}{1+\cos(2\alpha)}$ per $\alpha \neq \frac{\pi}{2} + k\pi$ con $k \in \mathbb{Z}$

$cos^2(\theta) + sen^2(\theta) = 1 \implies cos(\theta) = \sqrt{1+sen^2(\theta)} \wedge sen(\theta) = \sqrt{1+cos^2(\theta)}$

α(radianti)	α (gradi)	senα	cosα	tangα	cosecα	secα	cotanα
0	0°	0	1	0	N.E.	1	N.E.
π/6	30°	1/2	√3/2	√3/3	2	2/√3	3/√3
π/4	45°	√2/2	√2/2	1	2/√2	2/√2	1
π/3	60°	√3/2	1/2	√3	2/√3	2	1/√3
π/2	90°	1	0	N.E.	1	N.E.	0
2/3π	120°	√3/2	-1/2	-√3	2/√3	-2	-1/√3
3/4π	135°	√2/2	-√2/2	-1	2/√2	-2/√2	-1
5/6π	150°	1/2	-√3/2	-√3/3	2	-2/√3	-3/√3
π	180°	0	-1	0	N.E.	-1	N.E.
7/6π	210°	-1/2	-√3/2	√3/3	-2	-2/√3	3/√3
5/4π	225°	-√2/2	-√2/2	1	-2/√2	-2/√2	1
4/3π	240°	-√3/2	-1/2	√3	-2/√3	-2	1/√3
3/2π	270°	-1	0	N.E.	-1	N.E.	0
5/3π	300°	-√3/2	1/2	-√3	-2/√3	2	-1/√3
7/4π	315°	-√2/2	√2/2	-1	-2/√2	2/√2	-1
11/6π	330°	-1/2	√3/2	-√3/3	-2	2/√3	-3/√3
2π	360°	0	1	0	N.E.	1	N.E.

$\sin(2\alpha) = 2\sin\alpha\cos\alpha$

$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$

$\tan(2\alpha) = \frac{2\tan\alpha}{1-\tan^2\alpha}$ per $\alpha \neq \frac{\pi}{2} + k\pi$ e $\alpha \neq \pm\frac{\pi}{4} + k\pi$ con $k \in \mathbb{Z}$

$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$

$\sin\alpha\sin\beta = -\frac{1}{2}[\cos(\alpha+\beta) - \cos(\alpha-\beta)]$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$ per $\alpha, \beta, \alpha+\beta \neq \frac{\pi}{2} + k\pi$ con $k \in \mathbb{Z}$

$\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}$ per $\alpha, \beta, \alpha+\beta \neq k\pi$ con $k \in \mathbb{Z}$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$ per $\alpha, \beta, \alpha-\beta \neq \frac{\pi}{2} + k\pi$ con $k \in \mathbb{Z}$

$\cot(\alpha-\beta) = \frac{\cot\alpha\cot\beta + 1}{\cot\beta - \cot\alpha}$ per $\alpha, \beta, \alpha-\beta \neq k\pi$ con $k \in \mathbb{Z}$

Seconda relazione fondamentale

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$, per $\alpha \neq \frac{\pi}{2} + k\pi$ con $k \in \mathbb{Z}$

$\cos^2\alpha = \frac{1}{1+\tan^2\alpha}$, per $\alpha \neq \frac{\pi}{2} + k\pi$ con $k \in \mathbb{Z}$

$\cos\alpha = \pm\frac{1}{\sqrt{1+\tan^2\alpha}}$

Terza relazione fondamentale

$\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$, per $\alpha \neq k\pi$ con $k \in \mathbb{Z}$

Quarta relazione fondamentale

$\sec\alpha = \frac{1}{\cos\alpha}$, per $\alpha \neq \frac{\pi}{2} + k\pi$ con $k \in \mathbb{Z}$

Quinta relazione fondamentale

$\csc\alpha = \frac{1}{\sin\alpha}$, per $\alpha \neq k\pi$ con $k \in \mathbb{Z}$

VETTORI

$$V_1 \cdot V_2 = |V_1| \cdot |V_2| \cdot cos(\theta)$$

$$cos(\alpha) = \frac{\Delta P_x}{|P|}$$

$$\begin{cases} v_{0x} = v_0 \cos(\alpha) \\ v_{0y} = v_0 \sin(\alpha) \end{cases}$$

$$v = \sqrt{v_{0x}^2 + v_{0y}^2}$$

$$\tan(\alpha) = \frac{v_{0y}}{v_{0x}}$$

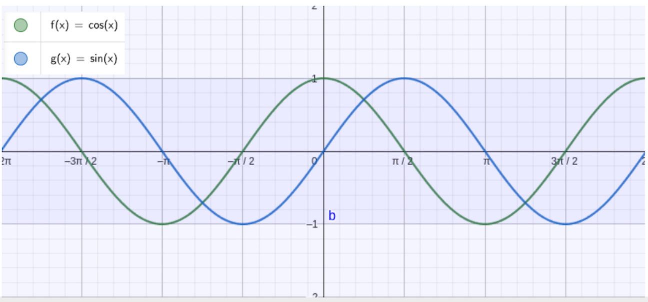
$$v_x = v_{0x}$$

$$\begin{cases} x = x_0 + v_{0x} t \\ y = -\frac{1}{2} g t^2 + v_{0y} t + y_0 \end{cases}$$

$$\begin{cases} t = \dfrac{x - x_0}{v_{0x}} \\ y = -\dfrac{g}{2 v_0^2 \cos^2(\alpha)} x^2 + \left(\dfrac{g x_0}{v_0^2 \cos^2(\alpha)} + \tan(\alpha) \right) x - \dfrac{g x_0^2}{2 v_0^2 \cos^2(\alpha)} - \tan(\alpha) x_0 + y_0 \end{cases}$$

Funzione	Notazione	Dominio	Codominio	Radici	Andamento	Funzione inversa
arcoseno	arcsen, arcsin, asin, sen⁻¹[1]	$[-1,1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	0	↗	seno
arcocoseno	arccos, acos, cos⁻¹	$[-1,1]$	$[0,\pi]$	1	↘	coseno
arcotangente	arctan, arctg, atan, tan⁻¹	$\mathbb{R}$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	0	↗	tangente
arcocotangente	arccot, arccotg, arcctg, acot, cot⁻¹	$\mathbb{R}$	$(0,\pi)$	$+\infty$	↘	cotangente
arcosecante	arcsec, asec, sec⁻¹	$(-\infty,-1] \cup [1,+\infty)$	$[0,\pi]$	1	*crescente, con una discontinuità in* $[-1,1]$	secante
arcocosecante	arccsc, arccosec, acsc, csc⁻¹	$(-\infty,-1] \cup [1,+\infty)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$\pm\infty$	*decrescente, con una discontinuità in* $[-1,1]$	cosecante

Funzione	Notazione	Dominio	Immagine	Radici	Periodo	Funzione inversa
seno	sen, sin	$\mathbb{R}$	$[-1,1]$	$\mathbb{Z}\pi$	2π	arcoseno
coseno	cos	$\mathbb{R}$	$[-1,1]$	$\frac{\pi}{2} + \mathbb{Z}\pi$	2π	arcocoseno
tangente	tan, tg	$\mathbb{R} \setminus \left(\frac{\pi}{2} + \mathbb{Z}\pi\right)$	$\mathbb{R}$	$\mathbb{Z}\pi$	π	arcotangente
cotangente	cot, cotg, ctg	$\mathbb{R} \setminus \mathbb{Z}\pi$	$\mathbb{R}$	$\frac{\pi}{2} + \mathbb{Z}\pi$	π	arcocotangente
secante	sec	$\mathbb{R} \setminus \left(\frac{\pi}{2} + \mathbb{Z}\pi\right)$	$(-\infty,-1] \cup [1,+\infty)$	*nessuna*	2π	arcosecante
cosecante	csc, cosec	$\mathbb{R} \setminus \mathbb{Z}\pi$	$(-\infty,-1] \cup [1,+\infty)$	*nessuna*	2π	arcocosecante



$$\cos(\pi - \alpha) = -\cos\alpha \quad \cos(-\alpha) = \cos\alpha$$

$$\sin(\pi - \alpha) = \sin\alpha \quad \sin(-\alpha) = -\sin\alpha$$

$$\tan(\pi - \alpha) = -\tan\alpha \quad \tan(-\alpha) = -\tan\alpha$$

$$\cos(\pi + \alpha) = -\cos\alpha$$

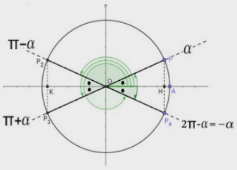
$$\sin(\pi + \alpha) = -\sin\alpha$$

$$\tan(\pi + \alpha) = \tan\alpha$$

$$\cos(2\pi - \alpha) = \cos\alpha$$

$$\sin(2\pi - \alpha) = -\sin\alpha$$

$$\tan(2\pi - \alpha) = -\tan\alpha$$



Formule degli angoli complementari (la loro somma è un angolo retto)

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$$

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$$

$$\tan\left(\frac{\pi}{2} - \alpha\right) = \cot\alpha$$

Formule degli angoli che differiscono di un angolo retto

$$\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin\alpha$$

$$\sin\left(\frac{\pi}{2} + \alpha\right) = \cos\alpha$$

$$\tan\left(\frac{\pi}{2} + \alpha\right) = -\cot\alpha$$

$$\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1 + \cos\alpha}{2}} \quad \text{per } \alpha \neq \pi + 2k\pi$$

$$\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1 - \cos\alpha}{2}}$$

$$\tan\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}}$$

Formule dell'angolo aggiunto [modifica | modifica wikitesto]

- $a \sin x + b \cos x = \sqrt{a^2 + b^2} \sin(x + \phi)$,

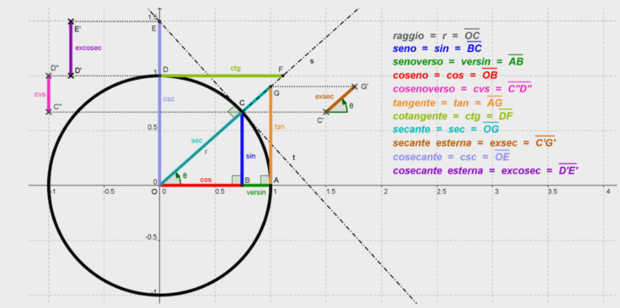
dove l'angolo ϕ è un qualunque angolo che soddisfa

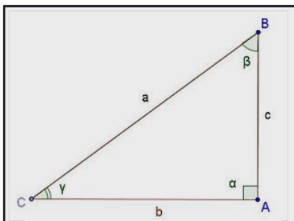
$$\begin{cases} \cos\phi = \dfrac{a}{\sqrt{a^2+b^2}} \\ \sin\phi = \dfrac{b}{\sqrt{a^2+b^2}}. \end{cases}$$

Se si sceglie l'angolo ϕ nell'intervallo $(-\pi, \pi]$, si può esplicitare nel seguente modo:

$$\phi = \begin{cases} \arctan\left(\frac{b}{a}\right), & \text{se } a > 0, \\ \arctan\left(\frac{b}{a}\right) + \pi, & \text{se } a < 0 \text{ e } b \geq 0, \\ \arctan\left(\frac{b}{a}\right) - \pi, & \text{se } a < 0 \text{ e } b < 0, \\ \frac{\pi}{2}, & \text{se } a = 0 \text{ e } b > 0, \\ -\frac{\pi}{2}, & \text{se } a = 0 \text{ e } b < 0. \end{cases}$$

L'angolo ϕ non è definito se $a = b = 0$, in tal caso l'uguaglianza iniziale si riduce all'identità $0 = 0$.





$$a = \frac{c}{\sin\gamma} \Rightarrow c = a \cdot \sin\gamma$$

$$a = \frac{b}{\cos\gamma} \Rightarrow b = a \cdot \cos\gamma$$

$$\frac{c}{b} = \frac{\sin\gamma}{\cos\gamma} \Rightarrow c = b \cdot \tan\gamma$$

$$\frac{b}{c} = \frac{\cos\gamma}{\sin\gamma} \Rightarrow b = c \cdot \cot\gamma$$

$$a = \frac{b}{\sin\beta} \Rightarrow b = a \cdot \sin\beta$$

$$a = \frac{c}{\cos\beta} \Rightarrow c = a \cdot \cos\beta$$

$$\frac{b}{c} = \frac{\sin\beta}{\cos\beta} \Rightarrow b = c \cdot \tan\beta$$

$$\frac{c}{b} = \frac{\cos\beta}{\sin\beta} \Rightarrow c = b \cdot \cot\beta$$

INTEGRALI NOTEVOLI

$\int f(x)^\alpha f'(x)dx = \frac{f(x)^{\alpha+1}}{\alpha+1}+c$

$\int \frac{f'(x)}{f(x)}dx = \log|f(x)|+c \quad \int \frac{\sin(2x)}{\cos(2x)}dx = \log(\cos(x))$

$\int e^{f(x)} f'(x)dx = e^{f(x)}+c$

$\int \sin f(x)\cdot f'(x)dx = -\cos(f(x))+c$

$\int \cos f(x)\cdot f'(x)dx = \sin(f(x))+c$

$\int \sinh f(x)\cdot f'(x)dx = \cosh(f(x))+c$

$\int \cosh f(x)\cdot f'(x)dx = \sinh(f(x))+c$

$\int \frac{f'(x)}{1+(f(x))^2}dx = \arctan f(x)+c$

$\int \frac{f'(x)}{\sqrt{1-(f(x))^2}}dx = \arcsin f(x)+c$

$\int \frac{f'(x)}{\sqrt{(f(x))^2+1}}dx = \text{settsinh} f(x)+c$

$\int \frac{f'(x)}{\sqrt{(f(x))^2-1}}dx = \text{settcosh} f(x)+c$

$\int \sqrt{1-f(x)^2}\cdot f'(x)dx = \frac{1}{2}[f(x)\sqrt{1-(f(x))^2} + \arcsin f(x)]+c$

$\int \sqrt{f(x)^2+1}\cdot f'(x)dx = \frac{1}{2}[f(x)\sqrt{(f(x))^2+1} + \text{settsinh} f(x)]+c$

$\int \sqrt{f(x)^2-1}\cdot f'(x)dx = \frac{1}{2}[f(x)\sqrt{(f(x))^2-1} + \text{settcosh} f(x)]+c$

$I_n = \int \frac{1}{(1+x^2)^n}dx = \frac{1}{2(n-1)}\frac{x}{(x^2+1)^{n-1}} + \frac{2n-3}{2(n-1)}$

$\int \tan(x) = \int \frac{\sin(2x)}{\cos(2x)}dx = \log(\cos(x))$

POTENZE:

$a^m \cdot a^n = a^{m+n}$

$(a^m)^n = a^{m\cdot n}$

$a^m \cdot b^m = (ab)^m$

$a^m / a^n = a^{m-n}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

INTEGRALI IMMEDIATI

$x^\alpha \to \frac{x^{\alpha+1}}{(\alpha+1)}$

$\frac{1}{x} \to \log|x|$

$e^x \to e^x$

$a^x \to \frac{a^x}{\log a}$

$\sin x \to -\cos x$

$\cos x \to \sin x$

$\tan x \to -\log|\cos x|$

$\cot x \to -\log|\sin x|$

$Sh\ x \to Ch\ x$

$Ch\ x \to Sh\ x$

$\frac{1}{(\cos x)^2} \to \tan x$

$\frac{1}{(\sin x)^2} \to -\cot x$

$\frac{1}{(1+x^2)} \to \arctan x$

$\frac{1}{\sqrt{1-x^2}} \to \arcsin x$

$\cos^2 x \to \frac{1}{2}(x+\sin x\cos x)$

LIMITI NOTEVOLI

$\lim_{x\to\pm\infty}\left(1+\frac{a}{x}\right)^x = e^a$

$\lim_{x\to+\infty}\left(1+\frac{\alpha}{x}\right)^{\beta x} = e^{\alpha\beta}$

$\lim_{x\to0}(1+x)^{\frac{1}{x}} = e$

$\lim_{x\to0}\frac{\log(1+x)}{x} = 1$

$\lim_{x\to0}\frac{\log_a(1+x)}{x} = \frac{1}{\log a}$

$\lim_{x\to0}\frac{e^x-1}{x} = 1$

$\lim_{x\to0}\frac{a^x-1}{x} = \log a$

$\lim_{x\to0}\frac{(1+x)^\alpha-1}{x} = \alpha$

$\lim_{x\to+\infty}\frac{x^\alpha}{e^x} = 0$

$\lim_{x\to+\infty}\frac{(\log(x))^\alpha}{x} = 0$

$\lim_{x\to+\infty}\log_a x = +\infty$

$\lim_{x\to x_0}\log_a x = \log_a x_0$ dove $x_0 > 0$

$\lim_{x\to0^+}\log_a x = -\infty$

$\lim_{x\to\infty}x^{\frac{1}{x}} = 1$

$\lim_{x\to0}\sin(x) = 0$

$\lim_{x\to0}\cos(x) = 1$

$\lim_{x\to0}\frac{\sin(x)}{x} = 1$

$\lim_{x\to0}\frac{1-\cos(x)}{x^2} = \frac{1}{2}$

$\lim_{x\to0}\frac{\tan(x)}{x} = 1$

$\lim_{x\to0}\frac{\arcsin(x)}{x} = 1$

$\lim_{x\to0}\frac{\sinh(x)}{x} = 1$ ...

$\lim_{x\to0}\frac{\tanh(x)}{x} = 1$

$\lim_{x\to0}\frac{\arctan(x)}{x} = 1$

$\lim_{x\to\infty}\frac{\log_a x}{x^b} = \lim_{x\to\infty}\frac{x^b}{a^x} = \lim_{x\to\infty}\frac{a^x}{x^x} = 0$

con $b>0$ ed $a>1$

DERIVATE NOTEVOLI

$(x^n)' = nx^{n-1}$, con $n\in\mathbb{N}$ e $x\in\mathbb{R}$.

$(x^\alpha)' = \alpha x^{\alpha-1}$, con $\alpha\in\mathbb{R}$ e $x>0$.

$(a^x)' = a^x\log a$, con $a>0$, $a\neq1$.

$(e^x)' = e^x$, per ogni $x\in\mathbb{R}$.

$(\log_a x)' = \frac{1}{x\log a}$, con $a>0$, $a\neq1$ e per ogni $x>0$.

$(\log x)' = \frac{1}{x}$, per ogni $x>0$.

$(\sin x)' = \cos x$, per ogni $x\in\mathbb{R}$.

$(\cos x)' = -\sin x$, per ogni $x\in\mathbb{R}$.

$(\text{tg}\,x)' = \frac{1}{\cos^2 x} = 1+\text{tg}^2 x$, per ogni $x\in\mathbb{R}\setminus\{\frac{\pi}{2}+k\pi\}$, $k\in\mathbb{Z}$.

$(\text{cotg}\,x)' = -\frac{1}{\sin^2 x} = -1-\text{cotg}^2 x$, per ogni $x\in\mathbb{R}\setminus\{k\pi\}$, $k\in\mathbb{Z}$.

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$, per ogni $x\in(-1,1)$.

$(\arccos x)' = -\frac{1}{1-x^2}$, per ogni $x\in(-1,1)$.

$(\text{arctg}\,x)' = \frac{1}{1+x^2}$, per ogni $x\in\mathbb{R}$.

$(\sinh x)' = \cosh x$, per ogni $x\in\mathbb{R}$.

$(\cosh x)' = \sinh x$, per ogni $x\in\mathbb{R}$.

$(\text{tgh}\,x)' = \frac{1}{\cosh^2 x} = 1-\text{tgh}^2 x$, per ogni $x\in\mathbb{R}$.

$(\text{settsinh}\,x)' = \frac{1}{\sqrt{1+x^2}}$, per ogni $x\in\mathbb{R}$.

$(\text{settcosh}\,x)' = \frac{1}{\sqrt{x^2-1}}$, per ogni $x>1$.

$(\text{setttgh}\,x)' = \frac{1}{1-x^2}$, per ogni $x\in(-1,1)$.

$(|x|)' = \frac{|x|}{x}$

$(arctg(\frac{1}{x}))' = -\frac{1}{1+x^2}$

$(\frac{1}{2}\cdot log(\frac{1+x}{1-x}))' = \frac{1}{1-x^2}$

REGOLE DI DERIVAZIONE:

$(f\pm g)'(x) = f'(x)\pm g'(x)$

$(f\cdot g)'(x) = f'(x)\,g(x) + f(x)\,g'(x)$

$(f/g)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$ con $g(x)\neq0$

$f(g(x))' = f'(g(x))\,g'(x)$

$(f^{-1})'(y) = 1/f'(f^{-1}(y))$

$(a\cdot f)' = a\cdot f'$

DERIVATE DI FUNZIONI COMPOSTE

$D[f(x)]^\alpha = \alpha[f(x)]^{\alpha-1}\cdot f'(x)$

$D\log(f(x)) = \frac{f'(x)}{f(x)}$

$D\,a^{f(x)} = f'(x)\cdot a^{f(x)}\cdot\ln(a)$

$D\,e^{f(x)} = f'(x)\cdot e^{f(x)}$

$D\sin(f(x)) = f'(x)\cdot\cos(f(x))$

$D\cos(f(x)) = f'(x)\cdot-\sin(f(x))$

$D\tan(f(x)) = \frac{f'(x)}{\cos^2(f(x))}$

$D\cot(f(x)) = -\frac{f'(x)}{\sin^2(f(x))}$

$D\log_a(f(x)) = \frac{f'(x)}{f(x)\log(a)}$

$D\arctan(f(x)) = \frac{f'(x)}{1+f^2(x)}$

$D\arcsin(f(x)) = \frac{f'(x)}{\sqrt{1-f^2(x)}}$

$D\arccos(f(x)) = -\frac{f'(x)}{\sqrt{1-f^2(x)}}$

$D\cosh(f(x)) = f'(x)\cdot\sinh(f(x))$

$D\sinh(f(x)) = f'(x)\cdot\cosh(f(x))$

$D\tanh(f(x)) = \frac{f'(x)}{\cosh^2(f(x))}$

$D\,\text{settsinh}(f(x)) = \frac{f'(x)}{\sqrt{1+f^2(x)}}$

$D\,\text{settcosh}(f(x)) = \frac{f'(x)}{\sqrt{f^2(x)-1}}$

$D[f(x)]^{g(x)} =$

$= [f(x)]^{g(x)}\cdot\{g'(x)\log(f(x))\cdot\frac{f'(x)}{f(x)}$

Sostituzioni utili

$\int f(e^x)dx$	$e^x = t$
$\int \frac{1}{x}f(\log x)dx$	$\log x = t$
$\int f(\sin x,\cos x)dx$	$\tan\frac{x}{2} = t$
$\int f(\tan x)dx$	$\tan x = t$
$\int\sqrt{1-x^2}dx, \int\frac{1}{\sqrt{1-x^2}}dx$	$x = \sin t$
$\int\sqrt{1+x^2}dx, \int\frac{1}{\sqrt{1+x^2}}dx$	$x = \sinh t$
$\int\sqrt{x^2-1}dx, \int\frac{1}{\sqrt{x^2-1}}dx$	$x = \cosh t$

Integrali per parti

$\int_a^b f(x)'g(x)dx$

$= [f(x)g(x)]_a^b - \int_a^b f(x)g(x)'dx$

Integrali per sostituzione

$\int_{\phi(a)}^{\phi(b)} f(x)dx = \int_{\phi(a)}^{\phi(b)} f(\phi(t))\phi'(t)dt$

$|x|\geq0$

$|x| = 0 \iff x = 0,\ |-x| = |x|$

$|x\cdot y| = |x|\cdot|y|\ ,\ \left|\frac{x}{y}\right| = \frac{|x|}{|y|}$

$\log_a 1 = 0,\ a^{\log_a(b)} = b$

$\log(b\cdot c) = \log(b)+\log(c)$

$\log\left(\frac{b}{c}\right) = \log(b)-\log(c)$

$\log(b^c) = c\cdot\log(b)$

Formule parametriche

Posto $t = \tan\frac{\pi}{2}$ si ha :

$\sin x = \frac{2t}{1+t^2}$

$\cos x = \frac{1-t^2}{1+t^2}$

$\tan x = \frac{2t}{1-t^2}$

Cambio base

$\log_a(b) = \frac{\ln(a)}{\ln(b)} \to a^x = b^{\log_b(a)}$

$\to \log_a x = \log_a b\cdot\log_b x$

Relazione asintoto

Se $\lim_{x\to x_0} f(x) = 0$

$\sin(f(x)) \approx f(x)$

$e^{f(x)}-1 \approx f(x)$

$1-cos(f(x)) \approx \frac{1}{2}(f(x))^2$

$\arctan f(x) \approx f(x)$

$\log(1+f(x)) \approx f(x)$

$a^{f(x)}-1 \approx \log(a)f(x)$

$\tan(f(x)) \approx f(x)$

$\sinh(f(x)) \approx f(x)$

$\tanh(f(x)) \approx f(x)$

$(1+f(x))^c-1 \approx c\cdot f(x)$

$\arcsin(f(x)) \approx f(x)$

$\cosh(f(x))-1 \approx \frac{1}{2}(f(x))^2$

Prodotti Notevoli

$(a+b+c)^2 = a^2+b^2+c^2+2ab+2ac+2bc$

$(a+b)^2 = a^2+b^2+2ab$

$(a+b)^3 = a^3+b^3+3a^2b+3ab^2$

$(a^3\pm b^3) = (a\pm b)(a^2\mp ab+b^2)$

$(a+b+c)^3 = a^3+b^3+c^3+3a^2b+3a^2c+3b^2a+3b^2c+3c^2a+3c^2b+6abc$

$a^{n+1}-b^{n+1} = (a-b)(a^n+a^{n-1}b+\ldots+ab^{n-1}+b^n)$

$a^b = e^{b\cdot\log a}$

$a^b = c^{b\cdot\log_c a}$

$\cos^2\alpha = \frac{1+\cos(2\alpha)}{2}$

$\sin^2\alpha = \frac{1-\cos(2\alpha)}{2}$

$\tan^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{1-\cos(2\alpha)}{1+\cos(2\alpha)}$ per $\alpha\neq\frac{\pi}{2}+k\pi$ con $k\in\mathbb{Z}$

$cos^2(\theta)+sen^2(\theta) = 1 \implies cos(\theta) = \sqrt{1+sen^2(\theta)} \wedge sen(\theta) = \sqrt{1+cos^2(\theta)}$

α(radianti)	α (gradi)	senα	cosα	tangα	cosecα	secα	cotanα
0	0°	0	1	0	N.E.	1	N.E.
π/6	30°	1/2	√3/2	√3/3	2	2/√3	3/√3
π/4	45°	√2/2	√2/2	1	2/√2	2/√2	1
π/3	60°	√3/2	1/2	√3	2/√3	2	1/√3
π/2	90°	1	0	N.E.	1	N.E.	0
2/3π	120°	√3/2	-1/2	-√3	2/√3	-2	-1/√3
3/4π	135°	√2/2	-√2/2	-1	2/√2	-2/√2	-1
5/6π	150°	1/2	-√3/2	-√3/3	2	-2/√3	-3/√3
π	180°	0	-1	0	N.E.	-1	N.E.
7/6π	210°	-1/2	-√3/2	√3/3	-2	-2/√3	3/√3
5/4π	225°	-√2/2	-√2/2	1	-2/√2	-2/√2	1
4/3π	240°	-√3/2	-1/2	√3	-2/√3	-2	1/√3
3/2π	270°	-1	0	N.E.	-1	N.E.	0
5/3π	300°	-√3/2	1/2	-√3	-2/√3	2	-1/√3
7/4π	315°	-√2/2	√2/2	-1	-2/√2	2/√2	-1
11/6π	330°	-1/2	√3/2	-√3/3	-2	2/√3	-3/√3
2π	360°	0	1	0	N.E.	1	N.E.

$\sin(2\alpha) = 2\sin\alpha\cos\alpha$

$\cos(2\alpha) = \cos^2\alpha-\sin^2\alpha = 1-2\sin^2\alpha = 2\cos^2\alpha-1$

$\tan(2\alpha) = \frac{2\tan\alpha}{1-\tan^2\alpha}$ per $\alpha\neq\frac{\pi}{2}+k\pi$ e $\alpha\neq\pm\frac{\pi}{4}+k\pi$ con $k\in\mathbb{Z}$

$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$

$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$

$\sin\alpha\sin\beta = -\frac{1}{2}[\cos(\alpha+\beta)-\cos(\alpha-\beta)]$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta+\cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta-\sin\alpha\sin\beta$

$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$ per $\alpha,\beta,\alpha+\beta\neq\frac{\pi}{2}+k\pi$ con $k\in\mathbb{Z}$

$\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta-1}{\cot\alpha+\cot\beta}$ per $\alpha,\beta,\alpha+\beta\neq k\pi$ con $k\in\mathbb{Z}$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta-\cos\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta+\sin\alpha\sin\beta$

$\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$ per $\alpha,\beta,\alpha-\beta\neq\frac{\pi}{2}+k\pi$ con $k\in\mathbb{Z}$

$\cot(\alpha-\beta) = \frac{\cot\alpha\cot\beta+1}{\cot\beta-\cot\alpha}$ per $\alpha,\beta,\alpha-\beta\neq k\pi$ con $k\in\mathbb{Z}$

Seconda relazione fondamentale

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$, per $\alpha\neq\frac{\pi}{2}+k\pi$ con $k\in\mathbb{Z}$

$\cos^2\alpha = \frac{1}{1+\tan^2\alpha}$, per $\alpha\neq\frac{\pi}{2}+k\pi$ con $k\in\mathbb{Z}$

$\cos\alpha = \pm\frac{1}{\sqrt{1+\tan^2\alpha}}$

Terza relazione fondamentale

$\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$, per $\alpha\neq k\pi$ con $k\in\mathbb{Z}$

Quarta relazione fondamentale

$\sec\alpha = \frac{1}{\cos\alpha}$, per $\alpha\neq\frac{\pi}{2}+k\pi$ con $k\in\mathbb{Z}$

Quinta relazione fondamentale

$\csc\alpha = \frac{1}{\sin\alpha}$, per $\alpha\neq k\pi$ con $k\in\mathbb{Z}$

VETTORI

$$V_1 \cdot V_2 = |V_1| \cdot |V_2| \cdot cos(\theta)$$

$$cos(\alpha) = \frac{\Delta P_x}{|P|}$$

$$\begin{cases} v_{0x} = v_0 \cos(\alpha) \\ v_{0y} = v_0 \sin(\alpha) \end{cases}$$

$$v = \sqrt{v_{0x}^2 + v_{0y}^2}$$

$$\tan(\alpha) = \frac{v_{0y}}{v_{0x}}$$

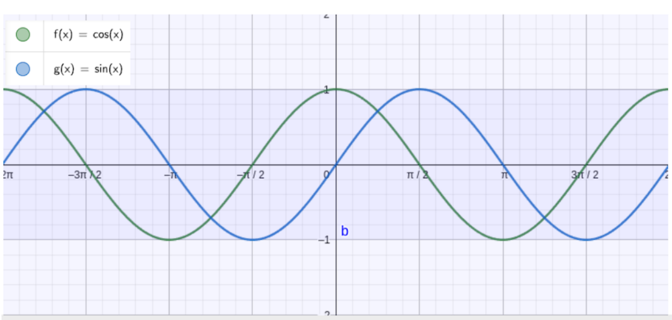
$$v_x = v_{0x}$$

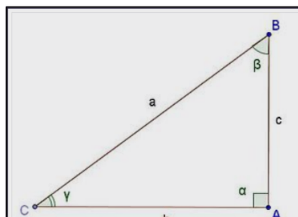
$$\begin{cases} x = x_0 + v_{0x} t \\ y = -\frac{1}{2} g t^2 + v_{0y} t + y_0 \end{cases}$$

$$\begin{cases} t = \dfrac{x - x_0}{v_{0x}} \\ y = -\dfrac{g}{2 v_0^2 \cos^2(\alpha)} x^2 + \left(\dfrac{g x_0}{v_0^2 \cos^2(\alpha)} + \tan(\alpha) \right) x - \dfrac{g x_0^2}{2 v_0^2 \cos^2(\alpha)} - \tan(\alpha) x_0 + y_0 \end{cases}$$

Funzione	Notazione	Dominio	Codominio	Radici	Andamento	Funzione inversa
arcoseno	arcsen, arcsin, asin, sen^{-1}[1]	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	0	↗	seno
arcocoseno	arccos, acos, cos^{-1}	$[-1, 1]$	$[0, \pi]$	1	↘	coseno
arcotangente	arctan, arctg, atan, tan^{-1}	$\mathbb{R}$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	0	↗	tangente
arcocotangente	arccot, arccotg, arcctg, acot, cot^{-1}	$\mathbb{R}$	$(0, \pi)$	$+\infty$	↘	cotangente
arcosecante	arcsec, asec, sec^{-1}	$(-\infty, -1] \cup [1, +\infty)$	$[0, \pi]$	1	*crescente, con una discontinuità in* $[-1, 1]$	secante
arcocosecante	arccsc, arccosec, acsc, csc^{-1}	$(-\infty, -1] \cup [1, +\infty)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$\pm\infty$	*decrescente, con una discontinuità in* $[-1, 1]$	cosecante

Funzione	Notazione	Dominio	Immagine	Radici	Periodo	Funzione inversa
seno	sen, sin	$\mathbb{R}$	$[-1, 1]$	$\mathbb{Z}\pi$	2π	arcoseno
coseno	cos	$\mathbb{R}$	$[-1, 1]$	$\frac{\pi}{2} + \mathbb{Z}\pi$	2π	arcocoseno
tangente	tan, tg	$\mathbb{R} \setminus \left(\frac{\pi}{2} + \mathbb{Z}\pi\right)$	$\mathbb{R}$	$\mathbb{Z}\pi$	π	arcotangente
cotangente	cot, cotg, ctg	$\mathbb{R} \setminus \mathbb{Z}\pi$	$\mathbb{R}$	$\frac{\pi}{2} + \mathbb{Z}\pi$	π	arcocotangente
secante	sec	$\mathbb{R} \setminus \left(\frac{\pi}{2} + \mathbb{Z}\pi\right)$	$(-\infty, -1] \cup [1, +\infty)$	*nessuna*	2π	arcosecante
cosecante	csc, cosec	$\mathbb{R} \setminus \mathbb{Z}\pi$	$(-\infty, -1] \cup [1, +\infty)$	*nessuna*	2π	arcocosecante





$$a = \frac{c}{\sin\gamma} \quad \Rightarrow \quad c = a \cdot \sin\gamma$$

$$a = \frac{b}{\cos\gamma} \quad \Rightarrow \quad b = a \cdot \cos\gamma$$

$$\frac{c}{b} = \frac{\sin\gamma}{\cos\gamma} \quad \Rightarrow \quad c = b \cdot \tan\gamma$$

$$\frac{b}{c} = \frac{\cos\gamma}{\sin\gamma} \quad \Rightarrow \quad b = c \cdot \cot\gamma$$

$$a = \frac{b}{\sin\beta} \quad \Rightarrow \quad b = a \cdot \sin\beta$$

$$a = \frac{c}{\cos\beta} \quad \Rightarrow \quad c = a \cdot \cos\beta$$

$$\frac{b}{c} = \frac{\sin\beta}{\cos\beta} \quad \Rightarrow \quad b = c \cdot \tan\beta$$

$$\frac{c}{b} = \frac{\cos\beta}{\sin\beta} \quad \Rightarrow \quad c = b \cdot \cot\beta$$

$$\cos(\pi - \alpha) = -\cos\alpha \quad \cos(-\alpha) = \cos\alpha$$

$$\sin(\pi - \alpha) = \sin\alpha \quad \sin(-\alpha) = -\sin\alpha$$

$$\tan(\pi - \alpha) = -\tan\alpha \quad \tan(-\alpha) = -\tan\alpha$$

$$\cos(\pi + \alpha) = -\cos\alpha$$

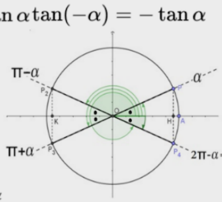
$$\sin(\pi + \alpha) = -\sin\alpha$$

$$\tan(\pi + \alpha) = \tan\alpha$$

$$\cos(2\pi - \alpha) = \cos\alpha$$

$$\sin(2\pi - \alpha) = -\sin\alpha$$

$$\tan(2\pi - \alpha) = -\tan\alpha$$



Formule degli angoli complementari (la loro somma è un angolo retto)

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$$

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$$

$$\tan\left(\frac{\pi}{2} - \alpha\right) = \cot\alpha$$

Formule degli angoli che differiscono di un angolo retto

$$\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin\alpha$$

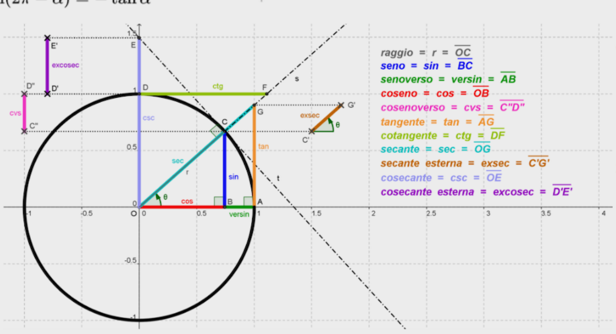
$$\sin\left(\frac{\pi}{2} + \alpha\right) = \cos\alpha$$

$$\tan\left(\frac{\pi}{2} + \alpha\right) = -\cot\alpha$$

$$\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1 + \cos\alpha}{2}} \quad \text{per } \alpha \neq \pi + 2k\pi$$

$$\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1 - \cos\alpha}{2}}$$

$$\tan\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}}$$



Formule dell'angolo aggiunto [modifica | modifica wikitesto]

- $a \sin x + b \cos x = \sqrt{a^2 + b^2} \sin(x + \phi)$,

dove l'angolo ϕ è un qualunque angolo che soddisfa

$$\begin{cases} \cos\phi = \frac{a}{\sqrt{a^2+b^2}} \\ \sin\phi = \frac{b}{\sqrt{a^2+b^2}}. \end{cases}$$

Se si sceglie l'angolo ϕ nell'intervallo $(-\pi, \pi]$, si può esplicitare nel seguente modo:

$$\phi = \begin{cases} \arctan(\frac{b}{a}), & \text{se } a > 0, \\ \arctan(\frac{b}{a}) + \pi, & \text{se } a < 0 \text{ e } b \geq 0, \\ \arctan(\frac{b}{a}) - \pi, & \text{se } a < 0 \text{ e } b < 0, \\ \frac{\pi}{2}, & \text{se } a = 0 \text{ e } b > 0, \\ -\frac{\pi}{2}, & \text{se } a = 0 \text{ e } b < 0. \end{cases}$$

L'angolo ϕ non è definito se $a = b = 0$, in tal caso l'uguaglianza iniziale si riduce all'identità $0 = 0$.