SUPPLEMENTAL MATERIAL FOR SILK ET AL.: *FEMALE-MALE RELATIONSHIPS INFLUENCE THE FORM OF FEMALE-FEMALE RELATIONSHIPS IN OLIVE BABOONS*

Varying Effects Structure for Non-directional Dyadic Models

Aim. Consider a population where there are N individuals, and a possible $M = N!/(2!(N-2)!)$ unique dyads. Our outcome variable is at the level of the dyad ij, containing two members i and j, and it has no directionality (i.e $i \rightarrow j = j \rightarrow i$). As individuals i and j occur in other dyads, the average outcome for dyad ij may be decomposed into an average deviation for each individual across all dyads as well as an average deviation for the specific dyad ij. We model therefore varying intercepts for both individuals i and j and the dyad ij.

The Model. Let y_{ijt} be the observed outcome for dyad ij at time t. The linear predictor for this outcome is defined by:

$$\Phi_{ijt} = \alpha + I_i + I_j + D_{ij} \tag{1}$$

where:

- α is the mean y across dyads and individuals
- I_i is a varying effect (or offset from α) for individual i across all dyads
- I_j is a varying effect for individual j across all dyads
- D_{ij} is the dyad level varying effect indexed by ij

This implies N parameters at the individual level and M dyad parameters.

Using a link function, Equation 1 can be applied to distributional assumptions of data commonly employed in a generalized linear mixed effects model framework (i.e. Gamma, Binomial, Poisson). In this manuscript, our outcome variable (DSI) was continuous and lower bound by 0 (where individuals were not observed to interact). Our data were well described by a mixture model of a Bernoulli distribution and a Gamma distribution, also known as a zero-augmented Gamma model. The zero-augmented Gamma model takes the following form:

$$y_{ijt} \sim \text{Bernoulli}(1 - p_{ijt})\text{Gamma}(\lambda, k) \tag{2}$$

$$\text{logit}(p_{ijt}) = \alpha_z + I_{zi} + I_{zj} + D_{zij} + \beta_z x_{ijt} \tag{3}$$

$$\log(\lambda_{ijt}) = \alpha_g + I_{gi} + I_{gj} + D_{gij} + \beta_g x_{ijt} \tag{4}$$

It is conceptually, and mathematically, similar to the more familiar zero-inflated Poisson model. The Bernoulli component (Eq. 3) estimates p, the probability of observing a DSI of 0. The Gamma component estimates (Eq. 4) the mean DSI score, λ, and a shape parameter, k, given a DSI > 0 This is similar to Eq. 1 except that our outcome is estimated by a joint likelihood, by multiplying two different probability distributions together (Eq. 2). Each component of the model estimated a unique parameter for the Bernoulli (indexed by z) and Gamma (indexed by g). Covariates for each component z and g are introduced via ordinary fixed effects, the β terms above. These may be dyad-specific covariates, as the varying intercepts on dyad do not preclude use of dyad-specific predictors. This is an important advantage of varying effects over traditional fixed effects.

The covariance structure for all individual level parameters is:

$$\begin{pmatrix} Iz \\ Ig \end{pmatrix} \sim \text{Normal}\left(0, \begin{bmatrix} \sigma_z^2 & \sigma_z\sigma_g\rho \\ \sigma_z\sigma_g\rho & \sigma_g^2 \end{bmatrix}\right) \tag{5}$$

This tells the model to estimate a correlation between varying effects parameters between the Bernoulli and Poisson components. A similar multivariate normal structure for dyadic parameters is calculated as well. All together, this implies $2N$ parameters for individuals, $2M$ parameters for dyads, 4 scale parameters for the variances of these effects, and 2 correlation parameters to achieve partial pooling across outcome components z and g.

Hyperparameters for scale and correlation parameters were weakly informative, designed to introduce conservatism into estimates. We used half-Cauchy(0,2) priors for each scale parameter and a flat "Onion method" correlation matrix prior for each correlation parameter. See McElreath 2016, Chapter 13, for discussion and examples. Example map2stan model code used in this analysis can be found at `https://github.com/bjbarrett/PapioDSI`.