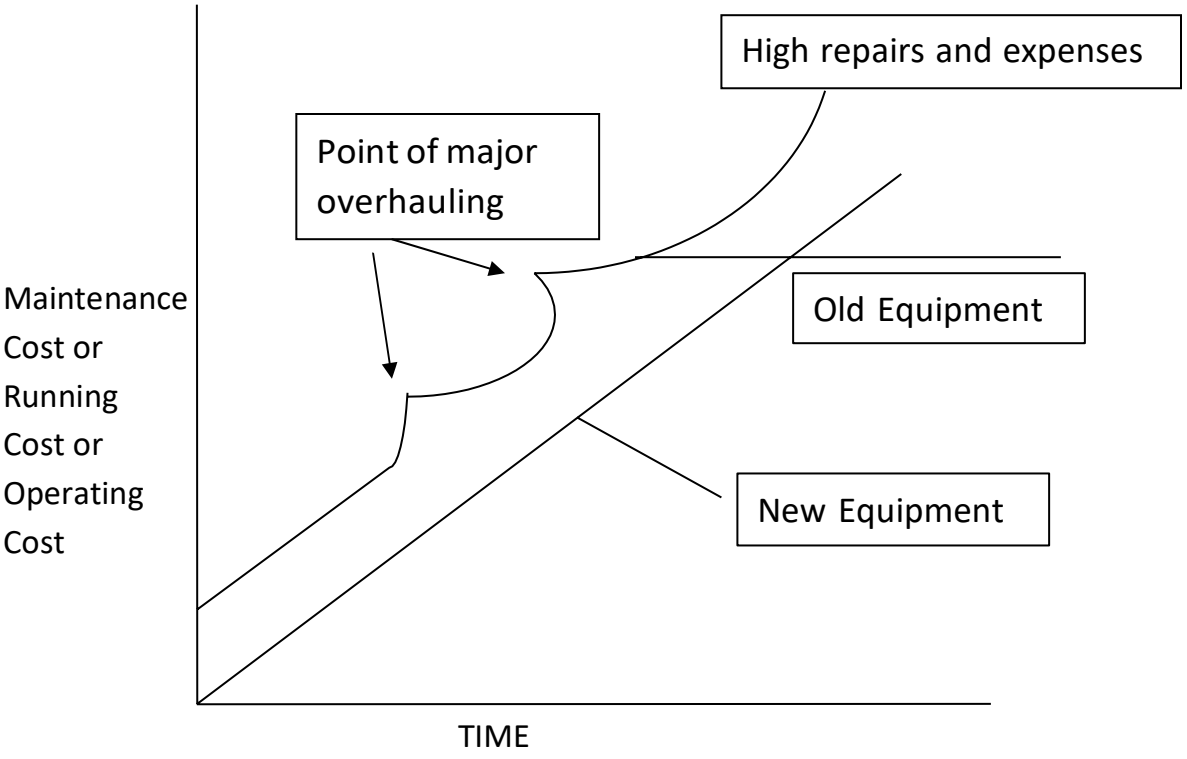
# OPERATIONS RESEARCH

## Replacement Analysis




Reasons for replacement

1. Technological development
2. Poor performance over years
3. Unable to meet the required demands

Capital equipment that deteriorates with time:

It is concerned with the equipment an machinery that deteriorates with time. Many people feel that equipment should not be replaced until it is physically worn off. But, it is not correct, preferable equipment must be constantly renewed and updated otherwise it will be in the risk of failure or it may become obsolete.

Reasons for replacement

1. Deterioration
2. Obsolescence
3. Technological development
4. Inadequacy

Deterioration is the decline in the performance of the equipment as compared to the new equipment. It may occur due to wear and tear. Due to this

a) Increase in the maintenance cost.
b) Reduces the product quality
c) Decreases the rate of production
d) Increases the labor cost
e) Reduces the efficiency of the equipment

Models:

Model1:
"Replacement of items whose maintenance Cost increases with time and the value of the money remains constant during the period"

Model 2: "replacement of items whose maintenance cost increases with time and value of money also changes with time".

Model 3: "Group Replacement policy"

Model1: Notation and symbols
C- Purchase cost of the machinery or equipment

S- Salvage value or resale value or scrap value of the machinery or equipment

$T_c$ total cost increased on the item or equipment during the period y

$T_c = C + m(Y) – S$

Where M(Y) is the cumulative maintenance cost in that period.

G(Y) ⟶ Average cost incurred on the equipment or item during the period.

$G(Y) = T_c / y$

Problem 1

The cost of the machine is Rs 6100/- and its scrap value is Rs 100 at the end of every year. The M.C. found from experience are as follows:

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| M.C | 100 | 250 | 400 | 600 | 900 | 1200 | 1600 | 2000 |

When should the machine be replaced?

Solution:

Given: C=6100

S=100

| Year (Y) | Maintenance | Cumulative Maintenance cost m(y) | Total Cost C-S + m(y) | Average Cost g(y) |
|---|---|---|---|---|
| 1 | 100 | 100 | 6100 | 6100 |
| 2 | 250 | 350 | 6350 | 3175 |
| 3 | 400 | 750 | 6750 | 2250 |
| 4 | 600 | 1350 | 7350 | 1837.5 |
| 5 | 900 | 2250 | 8250 | 1650 |
| 6 | 1200 | 3450 | 9450 | 1575 |
| 7 | 1600 | 5050 | 11050 | 1578.5 |
| 8 | 2000 | 7050 | 13050 | 1631.25 |

It’s clear from the above analysis that the machine has to replaced at the end of $6^{th}$ year or at the beginning of $7^{th}$ year because the maintenance cost of $7^{th}$ year is more than the average cost of the machine i.e 1578.5 > 1575.5

Problem 2

A fleet owner finds from his past experience records that the cost of the machine is Rs 6000/- and the running cost are given below. At what age the replacement is due;-

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Maintenance Cost | 1000 | 1200 | 1400 | 1800 | 2300 | 2800 | 3400 | 4000 |
| Resale Value | 3000 | 1500 | 750 | 375 | 200 | 200 | 200 | 200 |

Solutions:

| Year (Y) | Maintenance | Cumulative Maintenance cost m(y) | C-S | Total Cost C-S + m(y) | Average Cost g(y) |
|---|---|---|---|---|---|
| 1 | 1000 | 1000 | 3000 | 4000 | 4000 |
| 2 | 1200 | 2200 | 4500 | 6700 | 3350 |
| 3 | 1400 | 3600 | 5250 | 8850 | 2950 |
| 4 | 1800 | 5400 | 5625 | 11025 | 2756.25 |
| 5 | 2300 | 7700 | 5800 | 13500 | 2700 |
| 6 | 2800 | 10500 | 5800 | 16300 | 2716.67 |
| 7 | 3400 | 13900 | 5800 | 19700 | 2814.28 |
| 8 | 4000 | 17900 | 5800 | 23700 | 2962.5 |

The machine should be replaced at the end of $5^{th}$ year of at the beginning of $6^{th}$ year because the maintenance cost of the 6 year is more than the average cost of the machine i.e 2716 > 2700

Problem 3:

A Fleet owner finds from this past experience that the cost/year of running the truck whose purchase price rises to Rs 60000/- are given below Solutions; given C= 60000/-

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Maintenance | 10000 | 12000 | 14000 | 18000 | 23000 | 28000 | 34000 | 40000 |
| Depreciation | 30000 | 45000 | 52500 | 56250 | 58000 | 58000 | 58000 | 58000 |

At what age the truck is to be replaced?

Solution:

| Year (Y) | Maintenance | Cumulative Maintenance cost m(y) | Depreciation C-S | Total Cost C-S + m(y) | Average Cost g(y) |
|---|---|---|---|---|---|
| 1 | 10000 | 10000 | 30000 | 40000 | 40000 |
| 2 | 12000 | 22000 | 45000 | 67000 | 33500 |
| 3 | 14000 | 36000 | 52500 | 88500 | 29500 |
| 4 | 18000 | 54000 | 56250 | 110250 | 27562.5 |
| 5 | 23000 | 77000 | 58000 | 135000 | 27000 |
| 6 | 28000 | 108000 | 58000 | 163000 | 27166.66 |
| 7 | 34000 | 139000 | 58000 | 197000 | 28142.85 |
| 8 | 40000 | 179000 | 58000 | 237000 | 29625 |

The machine should be replaced at the end of $5^{th}$ year or at the beginning of $6^{th}$ year because the maintenance cost of the $6^{th}$ year is more than the average cost of the machine. i.e 27166.66 > 27000.

Problem 4:

An auto rickshaw driver finds from his previous records that the cost/year of running an autorickshaw whose purchase the cost/year of running an autorickshaw whose purchase price is Rs. 7000/- is as follows:-

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Maintenance cost | 1100 | 1300 | 1500 | 1900 | 2400 | 2900 | 3500 | 4100 |
| Resale value | 3100 | 1600 | 850 | 475 | 300 | 300 | 300 | 300 |

Solution:

| Year (Y) | Maintenance | Cumulative Maintenance cost m(y) | C-S | Total Cost C-S + m(y) | Average Cost g(y) |
|---|---|---|---|---|---|
| 1 | 1100 | 1100 | 3900 | 5000 | 5000 |
| 2 | 1300 | 2400 | 5400 | 7800 | 3900 |
| 3 | 1500 | 3900 | 6150 | 10050 | 3350 |
| 4 | 1900 | 5800 | 6525 | 12325 | 3081.25 |
| 5 | 2400 | 8200 | 6700 | 14900 | 2980 |
| 6 | 2900 | 11100 | 6700 | 17800 | 2966.6 |
| 7 | 3500 | 14600 | 6700 | 21300 | 3042.85 |
| 8 | 4100 | 18700 | 6700 | 25400 | 3175 |

The machine should be replaced at the end of $6^{th}$ year or at the beginning of $7^{th}$ year because the maintenance cost of the $7^{th}$ year is more than the average cost of the machine i.e 3042.85 > 2966.6

Problem 5:

Machine A costs Rs 9000/- annual operating cost is Rs 200/- for the first year and then increases by Rs 2000/- every year. Determine the best age at which the machine should be replaced and what would be the average cost of owning is operating cost of the machine?

Machine B costs Rs 10000/- annual operating cost is Rs 400/- for the first year and then increased by Rs. 800/- every year you now have a machine A which is one year old. Should you replace it with B? If so when? Assume machines have no resale value and that the future costs are not discounted.

Solution:-

Machine A:- cost price C = 9000/- ; S = 0

| Year (Y) | Maintenance Cost | Cumulative Maintenance cost m(y) | Total cost ($T_c$ ) C-S + m(y) | Average Cost g(y) |
|---|---|---|---|---|
| 1 | 200 | 200 | 9200 | 9200 |
| 2 | 2200 | 2400 | 11400 | 5700 |
| 3 | 4200 | 6600 | 15600 | 5200 |
| 4 | 6200 | 12800 | 21800 | 5450 |
| 5 | 8200 | 21000 | 30000 | 6000 |

Machine B:- cost price C = 10,000/- ; S = 0

| Year (Y) | Maintenance Cost | Cumulative Maintenance cost m(y) | Total cost ($T_c$ ) C-S + m(y) | Average Cost g(y) |
|---|---|---|---|---|
| 1 | 400 | 400 | 10400 | 10400 |
| 2 | 1200 | 1600 | 11600 | 5800 |
| 3 | 2000 | 3600 | 13600 | 4533.33 |
| 4 | 2800 | 6400 | 16400 | 4100 |
| 5 | 3600 | 10000 | 20000 | 4000 |
| 6 | 4400 | 14400 | 24400 | 4066.67 |

Machine A should be replaced at the end of the 3rd year and the average cost is 5200/- . machine B should be replaced at the end of 5th year and the average cost is Rs 4000/-.

The I year maintenance cost of machine A which is a year old is Rs2200/- < which is less than the average cost of machine B i.e. 4000.

The II year maintenance cost of machine A ( 1 year old) is 4200 > avg cost of machine B (i.e. 4000/-)

The III year maintenance cost of machine A is 6200/- which is greater than the average cost of machine B. i.e. 4000/-.

It is clear from above analysis that machine A that is one year old can be used one more year from now and then replaced with machine B.

Case 2 or Model II

"Replacement of items whose maintenance cost increases with time and the value of money also changes with time"

The maintenance cost varies with time and we want to find out the optimum time period at which the items will be replaced value of money decreases with a constant rate which is known as depreciation ratio or discounted factor which is given by $V= 1/ (1+i)^n$ for the value of 1 rupee where i $\longrightarrow$ rate of interest , n $\longrightarrow$ no. of years

Problem 6:-

A company buys a machine for Rs 6000/-. The maintenance cost are expected to be Rs 300/- in each year for the first 2 years and go up annually as follows 700, 1000, 1500, 2000, and 2500. Assume the money is worth of 20% per year. When the machine should be replaced.

Solution:-

C = 6000/-

I = 20%

Assumption:- in this to solve the problem we assume that the maintenance cost is spent on the machine at the beginning of each year as 1.

| Year | Maintenance cost | Present value of 1 Re | Present value of Maintenance cost | Cumulative Present value Maintenance cost m (y) | Total cost $T_c$ = C-S + m(y) | Cumulative Present value of 1 Re | Weighted Average cost |
|---|---|---|---|---|---|---|---|
| 1 | 300 | 1 | 300 | 300 | 6300 | 1 | 6300 |
| 2 | 300 | 0.833 | 249.9 | 549.9 | 6549.9 | 1.833 | 3573.32 |
| 3 | 700 | 0.694 | 486.1 | 1036 | 7036 | 2.527 | 2784.32 |
| 4 | 1000 | 0.578 | 578 | 1614 | 7614 | 3.105 | 2452.17 |
| 5 | 1500 | 0.482 | 723 | 2337 | 8337 | 3.587 | 2324.22 |
| 6 | 2000 | 0.401 | 802 | 3139 | 9139 | 3.988 | 2291.62 |
| 7 | 2500 | 0.334 | 835 | 3974 | 9974 | 4.322 | 2307.72 |

Specimen calculations for 3rd year

by $V= 1/ (1+i)^n$

$=1/(1.2)^2$

$=0.694$

$=0.694*700 = 486.1$

The machine should be replaced at the end of 6th year or at the beginning of 7th year because the maintenance cost in the 7th year is more than the average cost of the machine 2500 > 2291.62

Problem 7:

A machine cost Rs 10000/- the operating cost is Rs 500/- for the first 5 years and then increased by Rs 100/- every year subsequently from the 6th year onwards. Assuming the money is worth of 10% per year. Find the optimal length of time to hold the machine before replacement.

Solution:-

C=10,000, i = 10%

S = 0

Assumption:

To solve the problem, we assume that maintenance cost spent on the equipment is at the beginning of each year.

Specimen Calculations for 3rd year

$V= 1/(1+i)^n = 1/(1.1)^3 = 0.826$

$= 0.826 \times 500$

$= 413$

| Year | Maintenance cost | Present value of 1 Re | Present value of Maintenance cost | Cumulative Present value Maintenance cost m (y) | Total cost $T_c$ =C-S + m(y) | Cumulative Present value of 1 Re | Weighted Average cost |
|---|---|---|---|---|---|---|---|
| 1 | 500 | 1 | 500 | 500 | 10500 | 1 | 10500 |
| 2 | 500 | 0.909 | 454.5 | 954.5 | 10954.5 | 1.909 | 5738.34 |
| 3 | 500 | 0.826 | 413 | 1367.5 | 11367.5 | 2.735 | 4156.31 |
| 4 | 500 | 0.751 | 375.5 | 1743 | 11743 | 3.486 | 3368.62 |
| 5 | 500 | 0.683 | 341.5 | 2084.5 | 12084.5 | 4.169 | 2898.66 |
| 6 | 600 | 0.621 | 372.6 | 2457.1 | 12457.1 | 4.79 | 2600.65 |
| 7 | 700 | 0.564 | 394.8 | 2851.9 | 12851.9 | 5.354 | 2400.43 |
| 8 | 800 | 0.513 | 410.4 | 3262.3 | 13262.3 | 5.867 | 2260.49 |
| 9 | 900 | 0.466 | 419.4 | 3681.7 | 13681.7 | 6.333 | 2160.38 |
| 10 | 1000 | 0.424 | 424 | 4105.7 | 14105.7 | 6.757 | 2087.57 |
| 11 | 1100 | 0.385 | 423.5 | 4529.2 | 14529.2 | 7.142 | 2034.33 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 12 | 1200 | 0.350 | 420 | 4949.2 | 14949.2 | 7.492 | 1995.36 |
| 13 | 1300 | 0.318 | 413.4 | 5362.6 | 15362.6 | 7.81 | 1967.04 |
| 14 | 1400 | 0.289 | 404.6 | 5767.2 | 15767.2 | 8.099 | 1946.81 |
| 15 | 1500 | 0.263 | 394.5 | 6161.7 | 16161.7 | 8.362 | 1932.76 |
| 16 | 1600 | 0.239 | 382.4 | 6544.1 | 16544.1 | 8.601 | 1923.51 |
| 17 | 1700 | 0.217 | 368.9 | 6913 | 16913 | 8.818 | 1918.01 |
| 18 | 1800 | 0.198 | 356.4 | 7269.4 | 17269.4 | 9.016 | 1915.42 |
| 19 | 1900 | 0.180 | 342 | 7611.4 | 17611.4 | 9.196 | 1915.11 |
| 20 | 2000 | 0.163 | 326 | 7937.4 | 17937.4 | 9.359 | 1916.59 |

Conclusion:

The machine should be replaced at the end of 19$^{th}$ year or at the beginning of 20$^{th}$ year because the maintenance cost in the 20$^{th}$ year is more than the average cost of 19$^{th}$ year i.e. 1916.59 > 1915.11

Problem 8:

As Rs 6000/- for the first 4 years and increasing by Rs 2000/- per year in the 5$^{th}$ and subsequent years. If money is worth of 10% per year, when should the truck be replaced? Assume the truck has no resale value.

(Ans: end of 10$^{th}$ year Rs 18000.40/-)

Problem 9:

A manufacturer is offered two machines A & B. A is priced at Rs 5000/- and running cost are estimated at Rs. 800/- in each of the 1$^{st}$ 5 years, then increasing by Rs 200/- per year in the 6$^{th}$ and subsequent years. The machine B which has the same capacity as A, costs. Rs 2500/- but will have a running cost of Rs 1200/- per year for the 1$^{st}$ 6 years and then increasing by Rs 200/- per year thereafter. If money is worth of 10% per year which machine should be purchased? Assume the scrap value is zero at the end of year. ( Ans : M/c A :- 9$^{th}$ year end : Rs 1750

M/c B:- 8$^{th}$ Year end : Rs 1680.39)

Problem 10:

A lorry is priced at Rs250, 000. The operating cost is estimated at the rate of Rs 25000 for each year for the first 5 years and increasing by Rs 5000/- per year in the 6$^{th}$ and subsequent years. Decide when the lorry should be replaced. Assume the scrap value is negligible at the end. The worth of money is 25% per year.

(Ans : End of 16$^{th}$ year : Rs.82664.6)

Problem 11:

Assume that the present value of Rs 1/- to be spent in a year time is Rs 0.9 and purchase cost of the machine is Rs 3000/-. The running costs are given below.

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Running cost | 500 | 600 | 800 | 1000 | 1300 | 1600 | 2000 |

When should the machine be replaced.

( Ans : End of $6^{th}$ year : Rs 1539.85)

Solution: Assumption maintenance cost spent on the machinery is at the beginning of each year

$V= 1/ (1+i)^n$

$0.9 = 1/ (1+i)^1 = 11\%$

| Year | Maintenance cost | Present value of 1 Re | Present value of Maintenance cost | Cumulative Present value Maintenance cost m (y) | Total cost $T_c$ =C-S + m(y) | Cumulative Present value of 1 Re | Weighted Average cost |
|---|---|---|---|---|---|---|---|
| 1 | 500 | 1 | 500 | 500 | 3500 | 1 | 3500 |
| 2 | 600 | 0.9 | 540 | 1040 | 4040 | 1.9 | 2126.31 |
| 3 | 800 | 0.811 | 648.8 | 1688.8 | 4688.8 | 2.711 | 1729.54 |
| 4 | 1000 | 0.731 | 731 | 2419.8 | 5419.8 | 3.442 | 1574.6 |
| 5 | 1300 | 0.658 | 855.4 | 3275.2 | 6275.2 | 4.1 | 1530.53 |
| 6 | 1600 | 0.593 | 948.8 | 4224 | 7224 | 4.693 | 1539.31 |
| 7 | 2000 | 0.534 | 1068 | 5292 | 8292 | 5.227 | 1586.37 |

Specimen Calculations for $3^{rd}$ year

$V= 1/ (1+i)^n$

$= 1/ (1.11)^2 = 0.811$

$= 0.811 \times 800$

$= 648.8$

Conclusion: machinery should be replaced at the end of $5^{th}$ year and at the beginning of $6^{th}$ year because the maintenance cost in the $6^{th}$ year is more than the average cost of machine in $5^{th}$ year i.e. 1539.37 > 1530.53

Problem 12

If you wish to have a return of 10% on your investment which of the following plans would you prefer

| Particulars | Plan A | Plan B |
|---|---|---|
| First cost in Rs or fixed cost or capital or initial investment | 2,00,000 | 2,50,000 |
| Scrap value after 15 years in Rs. | 1,50,000 | 1,80,000 |
| Excess of annual revenue over annual disbursement or cash inflow | 25,000 | 30,000 |

Solution:

Using the present value analysis

Consider the plan A initial investment = 200000/-

Present value of Initial investment =200000/-

Scrap value after 15 years =150000/-

Present value of scrap value is = $1/ (1.1)^{15}$ X 150000 = 35,908.8

Average annual cash inflow =26000/-

Present value of cash flow

P.V = $1/ (1.1)^{1}$ X 25000+$1/ (1.1)^{2}$ X 25000 + $1/ (1.1)^{3}$ X25000 +--------+$1/ (1.1)^{15}$ X25000

Present worth factor = $(1+i)^{n} - 1/ i (1+i)^{n}$ = 7.606 when n=15, i=0.1

The present value of cash flow = 25000X 7.606 =190150/-

Net present value = P V of inflows - PV of outflows

= [(PV of salvage) + (PV of inflow)] – initial investment

=35908 + 190150 -200000

= 26058/-

Consider Plan B:

Consider the plan A initial investment = 250000/-

Present value of Initial investment =250000/-

Scrap value after 15 years =160000/-

Present value of scrap value is = $1/ (1.1)^{15}$ X180000 = 43,090.56

Average annual cash inflow =30000/-

Present value of cash flow

P.v = $1/ (1.1)^{1}$ X30000+$1/ (1.1)^{2}$ X30000 + $1/ (1.1)^{3}$ X30000 +--------+$1/ (1.1)^{15}$ X30000

Present worth factor = $(1+i)^{n} - 1/ i (1+i)^{n}$ = 7.606 when n=15, i=0.1

The present value of cash flow = 30000X 7.606 =228180/-

Net present value = P V of inflows - PV of outflows

= [(PV of salvage) + (PV of inflow)] – initial investment

=43090 + 228180 -250000

= 21270/-

Since the net present value is more for plan A, so Select Plan A

Problem 13

Using the present value analysis which crane is to be selected. Assume the interest as 10%

| Particulars | Crane A | Crane B |
|---|---|---|
| Capital | 25000 | 18000 |
| Maintenance cost per year | 1000 | 700 |
| Scrap value at the end of 10 years | 25000 | 30000 |

Solution : Using the present value analysis:-

Consider Plan A:

Capital = 25000/-

Present value of Initial investment =25000/-

Scrap value after 10 years =8000/-

Present value of scrap value is = $1/ (1.1)^{10}$ X8000 = 3084.35

Average annual cash outflow (machine and labour cost)

Present value of cash flow

P.V = $1/ (1.1)^{1}$ X1000+$1/ (1.1)^{2}$ X1000 + $1/ (1.1)^{3}$ X1000 +--------+$1/ (1.1)^{10}$ X1000

Present worth factor = $(1+i)^{n} - 1/ i (1+i)^{n}$ = 6.145 when n=10, i=0.1

The present value of cash flow = 1000X 6.145 = 6145 /-

Net present value = P V of inflows - PV of outflows

= PV of salvage - (PV of inflow+ initial investment)

=3084.35 – (25000 + 6145)

= 28060.65/-

Consider Plan B:

Capital = 18000/-

Present value of Initial investment =18000/-

Scrap value at the end of 10 years = 3000/-

Present value of scrap value is = $1/ (1.1)^{10}$ X 3000 = 1156.63/-

Average annual cash outflow (machine and labour cost)

Present value of cash flow

P.V= $1/ (1.1)^{1}$ X 700+$1/ (1.1)^{2}$ X 700 + $1/ (1.1)^{3}$ X 700 +--------+$1/ (1.1)^{10}$ X 700

Present worth factor = $(1+i)^{n} - 1/ i (1+i)^{n}$ = 6.145 when n=10, i=0.1

The present value of cash flow = 700X 6.145 = 4301.5 /-

Net present value = P V of inflows - PV of outflows

= PV of salvage - (PV of inflow+ initial investment)

=1156.63 – (18000 + 4301.5)

= 21144.87/-

Since, the net present value is more for machine B, so select machine B

## Group Replacement Policy:-

Replacement of items that fail completely

We always come across situation where the probability of failure in any s/m increases with time. The nature of the s/m may be such that if any item fails then it may result in complete breakdown of the s/m. this breakdown implies loss of production, work-in-progress.

Individual replacement policy:-

Whenever an item fails it should be replaced immediately.

Group replacement policy:- 2 steps

In the first step, it consists of individual replacement at the time of failure of any unit in the s/m.

In the second step, there is a group replacement of some live units at some suitable time.

In the group replacement we decide that all items in a s/m should be replaced after a certain interval of time irrespective of the fact that the items have failed or not with a provision that if any item fails before this time it should be replaced immediately. It requires

a) Rate of individual replacement during the period
b) Total cost incurred for individual and group replacement during the chosen period. The period for which the total cost incurred is minimum will be the optimum period for replacement.

Problem 14

The following mortality rate have been observed from a certain type of light bulbs

| Week | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| % of bulbs failing by the end of week. | 10 | 25 | 50 | 80 | 100 |
| Individual failure at the end of week. | 0.1 | 0.15 | 0.25 | 0.3 | 0.2 |

There are 1000 bulbs in use, the cost is Rs 1/- is represented an individual bulb which has burnt out if all the bulbs are replaced simultaneously. It would cost 25 paise per bulb. It is proposed to replace all bulbs at fixed intervals determine optimum replacement policy for the bulbs.

At what group replacement prize for bulb would a policy of strictly individual replacement becomes preferable to the adopted policy.

Solution:-

Assumption: in this to solve the problems we assume that the bulbs that fail during the week are replaced at the end of the week.

Let Ni denote the number of replacements made at the end of the ith week. Let $N_0$ is initially the no. of items in the s/m.

If it is not given assume it as 100 or 1000

$N_1$ = expected no. of bulbs being replaced at the end of I week

$N_1 = N_0 \ XP_1$

$= 1000X0.1$

$N_1 = 100$

$N_2 = N_0 \ XP_2 + N_1 \ XP_1$

$= 1000X0.15 + 100x0.1$

$= 281$

$N_3 = N_0 XP_3 + N_1 XP_2 + N_2 X P_1$

$=1000X0.25 + 100X0.15 + 160X0.1$

$= 281$

Expected life of the bulbs $= \sum_{I=1}^{n} X_i P_i$

Where $X_i$ no of weeks &

$P_i$ is the corresponding probability

$= (1X0.1) + (2X0.15) + 3X0.25) + (4X0.3) + (5X0.2)$

$= 3.35$ weeks.

Average no. of failures per week = (initial no. of items in the s\m) / (expected life)

= No. of failure / expected life

$= 1000 /3.35$

$= 298.6$

$\approx 299$

Cost of individual replacement per bulb =1 Rs

Total cost of individual replacement per week =299 x1 Rs

=299

Consider the group replacement

| End of the week | Total cost of group replacement | Average cost per week |
|---|---|---|
| 1 | (1000X0.25) + (100X1) = 350/- | 350/1 =350/- |
| 2 | 1000X0.25) + (100X1) + (160X1) = 510/- | 510/2 = 255/- |
| 3 | 1000X0.25) + (100X1) + (160X1) +(281X1)= 791/- | 791/3 = 263.66/- |

Conclusion: with reference to above data, it is preferred the group replacement once in two weeks and Avg cost per week is 255/-

Where in the case of individual replacement

Average cost is 299/-

So prefer group replacement

b) $(1000 \times X) + (100x 1) +(160x1)/2 > 299$

$x > 0.338$

i.e. 34 paise

Problem 15:-

A automatic safety electric switches attached to a press has the following probability.

| No of Years | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Probability of failure | 0.05 | 0.1 | 0.15 | 0.2 | 0.35 | 0.1 | 0.05 |

If the average cost to replace the single switch is Rs 15/- but, this comes down to Rs. 3/- when the replacement is carried out on the group basis. Find the optimum replacement plan.

Solution:

| Week | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Probability of failure | 0.05 | 0.1 | 0.15 | 0.2 | 0.35 | 0.1 | 0.05 |

Assumption: - here to solve the problem assume that the no. of switches that fail during the year are replaced just before the end of the year.

Let $N_i$ Denote the no of replacement mode at the end of the ith year

$N_1$ - expected no of switched being replaced at the end of the 1st year

Let $N_0$ is initially the no. of items in the system.

If it is not given assume it as 100

$N_1 = N_0 \ XP_1$

$= 100X0.05$

$N_1 = 5$

$N_2 = N_0 \ XP_2 + N_1 \ XP_1$

$= 100 X 0.1 + 5x0.05$

$= 10.25$

$N_3 = N_0 \ XP_3 + N_1 \ XP_2 + N_2 X P_1$

$=100X0.15 + 5X0.1 + 10.25X0.05$

$= 16.0125$

$N_4 = N_0 \ XP_4 + N_1 \ XP_3 + N_2 \ X \ P_2 + N_3 \ XP_1$

=100x0.2+5x0.15+10.25x0.1+16.0125x0.05

=22.5

Expected life of the switches

=1x0.05+2x0.1+3x0.15+4x0.2+5x0.35+6x0.1+7x0.05

=4.2 years

Avg. no failures per years = No. of items in s/m initially/expected life

= 100/4.2

=24

Avg cost of individual replacement / switch = Rs 15/-.

Total cost of individual replacement per year

=24x15 =360/-

Consider Group Replacement

| End of the year | Total cost of group replacement | Average cost per Year |
|---|---|---|
| 1 | 100x3+5x15 =375 | 375/1=375 |
| 2 | 100x3+5X15+10.25x15=528.75 | 528.75/2=264 |
| 3 | 100x3+5X15+10.25x15+16x15=768.75 | 768.75/3=256.3 |
| 4 | 100x3+5X15+10.25x15+16x15+22.5x15=1106.25 | 1106.25/4=276.5 |

Prefer the group replacement once in three years and the avg cost /year =Rs256.3/-. Whereas, in case of individual replacement. The avg cost per year is Rs 360/-. So prefer group replacement.

Problem 16:

The following mortality list rates have been observed for a certain type of tube lights

| Week | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Probability of failure to date | 0.04 | 0.18 | 0.29 | 0.46 | 0.68 | 0.92 | 1 |

The cost of replacing a tube light individually is Rs.2.35. the cost of group replacement is Rs. 0.65/tube. Determine the optimum replacement policy.

Solution:

Let $N_0$ =100

Here to solve the problem, assume that the no. of tube light that fail during the week are replaced at the end of the week.

$N_i$ - expected no of switched being replaced at the end of the 1st week

$N_1$ - expected no of switched being replaced at the end of the 1st week

$N_1 = N_0 \ XP_1$

$= 100X0.04$

$N_1 = 4$

$N_2 = N_0 \ XP_2 + N_1 \ XP_1$

$= 100 X 0.14 + 4x0.04$

$= 14.16$

$N_3 = N_0 \ XP_3 + N_1 \ XP_2 + N_2 X P_1$

$=100X0.11 + 4X0.14 + 14.16X0.04$

$= 12.1264$

$N_4 = N_0 \ XP_4 + N_1 \ XP_3 + N_2 X P_2 + N_3 \ XP_1$

$=100x0.17+4x0.11+14.16x0.14+12.1264x0.04$

$=19.9$

Expected life of the tube light

=1x0.04+2x0.14+3x0.11+4x0.17+5x0.22+6x0.24+7x0.08

=4.43 years

Avg. no failures per week = No. of items in s/m initially/expected life

= 100/4.43

$=22.57 \approx 23$

Avg cost of individual replacement / tube = Rs 2.35/-.

Total cost of individual replacement per week

=23x2.35 =54.05/-

Consider Group Replacement

| End of the week | Total cost of group replacement | Average cost per week |
|---|---|---|
| 1 | 100x0.65+4x2.35 =74.4 | 74.4/1=74.4 |
| 2 | 100x0.65+4x2.35+14.16x2.35=107.67 | 107.67/2=53.83 |
| 3 | 100x0.65+4X2.35+14.16x2.35+12.1264x2.35=136.17 | 136.17/3=45.39 |
| 4 | 100x0.65+4X2.35+14.16x2.35+12.1264x2.35+19.9x2.35= 182.93 | 182.93/4=45.73 |

Conclusion: prefer group replacement once in three week where the avg cost/year =Rs45.39/-. In case of individual replacement, the avg cost / week =54.05/-

So prefer group replacement

Problem 17:

A computer contains 10000 resistors. When a resistor fails it is replaced. The cost of replacing a resistor individually is Rs 1/-. If all the resistors are at the same time the cost / resistor is reduced by 65 paise the probability of serving at the end of the month is given below. What is the optimum replacement plan?

| Month | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Probability of survival | 1 | 0.97 | 0.9 | 0.7 | 0.3 | 0.15 | 0 |

Solutions:

| Month | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Probability of failure cumulative | 0 | 0.03 | 0.1 | 0.3 | 0.7 | 0.85 | 1 |
| Probability of failure in each month | 0 | 0.03 | 0.07 | 0.2 | 0.4 | 0.15 | 0.15 |

Given $N_0$ =10000

Assume that the no of resistors are that fail during the month are replaced at the end of the month.

Let $N_i$ denote the no of replacements at the end of ith month.

$N_1$ = expected no of resistors being replaced at the end of 1st month.

$N_1 = N_0 \ XP_1$

= 10000X0.03

$N_1 = 300$

$N_2 = N_0 \ XP_2 + N_1 \ XP_1$

= 10000 X 0.07 + 300x0.03

= 709

$N_3 = N_0 \ XP_3 + N_1 \ XP_2 + N_2 \ X \ P_1$

=10000X0.2 + 300X0.07 + 709X0.03

= 2042.27

$N_4 = N_0 \ XP_4 + N_1 \ XP_3 + N_2 \ X \ P_2 + N_3 \ XP_1$

=10000x0.4+300x0.2+709x0.07+2042.27x0.03

=4170.89

Expected life of the tubes

=1x0.03+2x0.07+3x0.2+4x0.4+5x0.15+6x0.15

=4.02 years

Avg. no failures per month = No. of items in s/m initially/expected life

= 10000/4.02

=2487.56 ≈2488

Avg cost of individual replacement resistor = Rs 1/-.

Total cost of individual replacement per month

=2488x1 =2488/-

Consider Group Replacement

| End of the Month | Total cost of group replacement | Average cost per Month |
|---|---|---|
| 1 | 10000x0.35+300x1 =3800/- | 3800/1= 3800 |
| 2 | 10000x0.35+300x1+709x1=4509/- | 4509/2= 2254.5 |
| 3 | 10000x0.35+300X1+709x1+2042.27x1=6551.27 | 6551.27/3= 2183.7 |
| 4 | 10000x0.35+300X1+709x1+2042.27x1+4170.89x1=10722.16 | 10722.16/4= 2680.54 |

Conclusion: prefer group replacement once in three months where the avg cost/month =Rs 2183.7/-. In case of individual replacement, the avg cost / month =2488/-

So prefer group replacement

Problem 18:

Suppose a special purpose type of light never last longer than 2 weeks. There is a chance of 0.3 that a bulb will fail at the end of 1st week cost per for individual replacement is Rs 1.25 is the cost/bulb in the group replacement is Rs 0.50. it is cheapest to replace bulbs.

a) Individually
b) Every week
c) Every 2nd week

Solution:

| Week | 1 | 2 |
|---|---|---|
| Prob. 0f Failure | 0.3 | 0.7 |

Let $N_0$ =100

Assumption:- No of bulbs that fail during the week are replaced at the end of week

Let $N_i$ denote the no of replacements at the end of ith week.

$N_1$ = expected no of resistors being replaced at the end of 1st week

$N_1 = N_0 \ XP_1$

$= 100X0.3$

$N_1 = 300$

$N_2 = N_0 \ XP_2 + N_1 \ XP_1$

$= 100 X 0.7 + 30x0.3$

$= 79$

Expected life of the tubes

=1x0.3+2x0.7

=1.7 weeks

Avg. no failures per week = No. of items in s/m initially/expected life

= 100/1.7

=58.82 ≈59

Avg cost of individual replacement / bulb = Rs 1.25/-.

Total cost of individual replacement per week

=59x1.25 =73.75/-

Consider Group Replacement

| End of the week | Total cost of group replacement | Average cost per week |
|---|---|---|
| 1 | 100x0.5+30x1.25 =87.5/- | 87.5/1=87.5 |
| 2 | 100x0.5+30x1.25+79x1.25=186.25/- | 186.25/2= 93.125 |

Conclusion:

Avg cost if replaced individually is 73.75/-

Whereas if replaced after every week will cost Rs 87.5/- and if replaced after two weeks it will cost Rs 93.125/-

So prefer individual replacement.

Problem 19:

The following mortality rates have been observed for a certain type of light bulbs

| Weeks | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| % of bulbs surviving by the end of week | 100 | 90 | 75 | 50 | 20 | 0 |
| | 1 | 0.9 | 0.75 | 0.5 | 0.2 | 0 |

It costs Rs. 2/- to replace an individual bulb which has burnt out. If all bulbs are replaced simultaneously the cost would e reduced by Rs 1.5/- per bulb. Determine the optimum replacement policy.

Solution:

| Week | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Prob. of failure cumulative | 0 | 0.1 | 0.25 | 0.5 | 0.8 | 1 |
| Prob. of failure in each month | 0 | 0.1 | 0.15 | 0.25 | 0.3 | 0.2 |

Assumption: - to solve the problem we assume that the no of bulbs that fail during the week are replaced at the end of the week.

Let $N_i$ denote the no of replacements made at the end of the ith week.

Let $N_0$ is initially the no. of Items in the system.

Let $N_0$ =100

$N_1 = N_0 \ XP_1$

$= 100X0$

$N_1 = 0$

$N_2 = N_0 \ XP_2 + N_1 \ XP_1$

$= 100 X 0.1 + 0x0$

$= 10$

$N_3 = N_0 \ XP_3 + N_1 \ XP_2 + N_2 X P_1$

$=100X0.15 + 0X0.1 +10X0$

$= 15$

$N_4 = N_0 \ XP_4 + N_1 \ XP_3 + N_2 X P_2 + N_3 \ XP_1$

$=100x0.25+0x0.15+1x0.1+15x0$

$=26$

Expected life of the tubes

=1x0+2x0.1+3x0.15+4x0.25+5x0.3+6x0.2

=4.35 weeks

Avg. no failures per week = No. of items in s/m initially/expected life

= 100/4.35

=22.98 ≈23 bulbs

cost of individual replacement bulb = Rs 2/-.

Total cost of individual replacement per week

=2x23 =46/-

Consider Group Replacement

| End of the week | Total cost of group replacement | Average cost per week |
|---|---|---|
| 1 | 100x0.5+0x2 =50/- | 50/1= 50 |
| 2 | 100x0.5+0x2+10x2=70/- | 70/2= 35 |
| 3 | 100x0.5+0X2+10x2+15x2=100 | 100/3= 33.33 |
| 4 | 100x0.5+0X2+10x2+15x2+26x2=152 | 152/4= 38 |

Conclusion: prefer group replacement once in three months where the avg cost/week =Rs 33.33/-. In case of individual replacement, the avg cost / week =46/-

So prefer group replacement

Problem 20:-

Supplies of bins are located throughout a plant when bin is empty. The foreman calls to the store room and the fill it. There are 10 bins. Located over the plant it has been suggested that it might be cheap to routine.

Individual bins as required it is estimated that the time required to service a single bin costs Rs3/-. But, it would cost only Rs 10/- to service all bins at one trip. The probability of bins being empty is the function of time. It is recognized that an empty bin must be filled whenever this occurs. What is the best policy for supply.

| Time since refill (in shift) | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Prob. of being empty | 0.1 | 0.2 | 0.5 | 1 |

Solution:

Given $N_0$ =10

| Time since refills (shifts) | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Prob. Of failure | 0.1 | 0.1 | 0.3 | 0.5 |

Assumption: no of bins that are full during the shift are serviced at the end of rack shift

Let $N_i$ = no of replacement at the end of $i^{th}$ shift

$N_1 = N_0 \ XP_1$

$= 10x0.1$

$N_1 = 1$

$N_2 = N_0 \ XP_2 + N_1 \ XP_1$

$= 10 X 0.1 + 1x0.1$

$= 1.1$

$N_3 = N_0 \ XP_3 + N_1 \ XP_2 + N_2 X P_1$

=10X0.3 + 1X0.1 +1.1X0.1 = 3.21

$N_4 = N_0 XP_4 + N_1 XP_3 + N_2 X P_2 + N_3 XP_1$

=10x0.5+1x0.3+1.1x0.1+3.21x0.1

=5.731

Expected life of bins

=1x0.1+2x0.1+3x0.3+4x0.5

=3.2

Avg. no failures per shift = No. of items in s/m initially/expected life

= 10/3.2

=3.125 ≈3

Avg cost of individual replacement per bin = Rs 3/-.

Total cost of individual replacement per shift

=3x3 =9/-

Consider Group Replacement:

| End of the shift | Total cost of group replacement | Average cost per shift |
|---|---|---|
| 1 | 10x1+1x3 =13/- | 13/1=13 |
| 2 | 10x1+1x3+1.1x31=16.3/- | 16.3/2=8.15 |
| 3 | 10x1+1X3+1.1x3+3.21x3=25.93 | 25.93/3=8.54 |

Conclusion: prefer group replacement once in two months where the avg cost/shift =Rs 8.15/-. In case of individual replacement, the avg cost / shift =9/-
So prefer group replacement.

Problem 21:-

There is a large no. of light bulbs all of which must be kept in working order. If a bulb fails in service it costs Rs 1/- to replace it. But, if all bulb fails in service in the same operation. It costs only 35 paise a bulb. The proportion of bulb failing in successive time internals is known.

| Weeks | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Proportion of failing | 0.09 | 0.16 | 0.24 | 0.36 | 0.12 | 0.03 |

Problem 22:-

The life of street bulbs is normally distributed about a mean of 6 weeks with a standard deviation of 1 week. The cost of replacing an individual bulb is Rs 6/- per bulb. A decision is made to replace all bulbs simultaneously at fixed intervals and to replace the individual bulb as they fail .

If the cost of group replacement is Rs 2.35/- per bulb, find out the optimum group replacement policy?

Solution:

$\mu=6$

$\sigma=1$ Range -3 to +3

$f(x)^n = 1/\sigma\sqrt{2\pi}\ (e^{-(x-\mu)2/2C})$ else 0

$Z= x - \mu / \sigma$

| Week (x) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Z | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| Prob. (cu) | 0 | 0 | 0 | 0.023 | 0.159 | 0.5 | 0.84 | 0.971 | 1 |
| Prob. In each week | 0 | 0 | 0 | 0.023 | 0.136 | 0.341 | 0.34 | 0.131 | 0.029 |

Assumption:-

In this to solve the problem we assume that the street bulbs that fail during the week are replaced at the end of the week.

Let $N_i$ =denote the no of replacement at the end week

Let $N_0$ =100

$N_1 = N_0 \ XP_1$

$= 100x00$

$N_1 = 0$

$N_2 = N_0 \ XP_2 + N_1 \ XP_1$

$= 100 X 0 +0x0$

$= 0$

$N_3 = N_0 \ XP_3 + N_1 \ XP_2 + N_2 X P_1$

$=0$

$N_4 = N_0 XP_4 + N_1 XP_3 + N_2 X P_2 + N_3 XP_1$

=100x0.023+0+0+0+0

=13.6

$N_5 = N_0 XP_5 + N_1 XP_4 + N_2 X P_3 + N_3 XP_2 + N_4 XP_1$

= 100x0.136+0+0+0+0

= 13.6

$N_5 = N_0 XP_5 + N_1 XP_4 + N_2 X P_3 + N_3 XP_2 + N_4 XP_1 + N_5 XP_0$

=100x0.341+0+0+0+0+0

=34.1

Expected life of bins

=1x0+2x0+3x0+4x0.023+5x0.136+6x0.341+7x0.34+8x0.131+9x0.029

=6.507 weeks

Avg. no failures per week = No. of items in s/m initially/expected life

= 100/6.507

=15.36 ≈ 15

Avg cost of individual replacement per bulb = Rs 6/-.

Total cost of individual replacement per week

=6x15 =Rs90/-

Consider Group Replacement:

| End of the week | Total cost of group replacement | Average cost per week |
|---|---|---|
| 1 | 100x2.35+0=235/- | 235/1=2353 |
| 2 | 100x2.35+0+0=235 | 235/2=117.5 |
| 3 | 100x2.35+0+0+0=235/- | 235/3=78.33 |
| 4 | 100x2.35+0+0+0+2.31x6=248.86 | 248.86/4=62.215 |
| 5 | 100x2.35+0+0+0+2.31x6+13.6x6=330.46 | 330.46/5=66.092 |

Conclusion: with reference to above data, prefer group replacement once in four weeks where the average cost is Rs 62.215/-. In case of individual replacement avg. cost is =Rs90/-

So prefer group replacement.

Problem 23:-

A trucking company has kept records on tyre life. Although failure is a random process it is a function of time. The following data taken from 1000 failure have been summarized.

| Thousands of kms | No. of failures |
|---|---|
| 10 | 50 |
| 20 | 100 |
| 30 | 250 |
| 40 | 400 |
| 50 | 200 |

A scheduled tyre change costs Rs 3,500 including the tyre a failure results in a loss of travel time and required use of emergency tools which takes even longer. Resulting, a net loss of Rs 5000. In addition, this ruins tyre body which has a value of Rs 100 what would the least cost policy for tyre change?

Problem 24:-

The following failure rates have been observed for a certain type of light bulbs

| Weeks | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| % failing by end of week | 10 | 25 | 50 | 30 | 100 |

There are 1000 bulbs in use and its cost Rs2/- to replace an individual bulb which has burnt out. If all bulbs were replaced simultaneously, it cost 50 paise per bulb. It is proposed to replace all bulbs at fixed intervals at what intervals should all the bulbs be replaced?

At what group replacement price/bulb would a policy of strictly individual replacement becomes preferable to the adopted policy.

[end of 2nd week, GR Rs 510/- > Rs 076/ bulb]

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