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MODULE 4

C-4 PRESENT WORTH METHOD OF COMPARISON

4.1 INTRODUCTION

In this method of comparison, the cash flows of each alternative will be reduced to time zero by assuming an interest rate *i*. Then, depending on the type of decision, the best alternative will be selected by comparing the present worth amounts of the alternatives. The sign of various amounts at different points in time in a cash flow diagram is to be decided based on the type of the decision problem. In a cost dominated cash flow diagram, the costs (outflows) will be assigned with positive sign and the profit, revenue, salvages value (all inflows), etc. will be assigned with negative sign. In a revenue/profit-dominated cash flow diagram, the profit, revenue, salvage value (all inflows to an organization) will be assigned with positive sign. The costs (outflows) will be assigned with negative sign. In case the decision is to select the alternative with the minimum cost, then the alternative with the least present worth amount will be selected. On the other hand, if the decision is to select the alternative with the maximum profit, then the alternative with the maximum present worth will be selected.

4.2 REVENUE-DOMINATED CASH FLOW DIAGRAM

A generalized revenue-dominated cash flow diagram to demonstrate the present worth method of comparison is presented in Fig. 4.1.

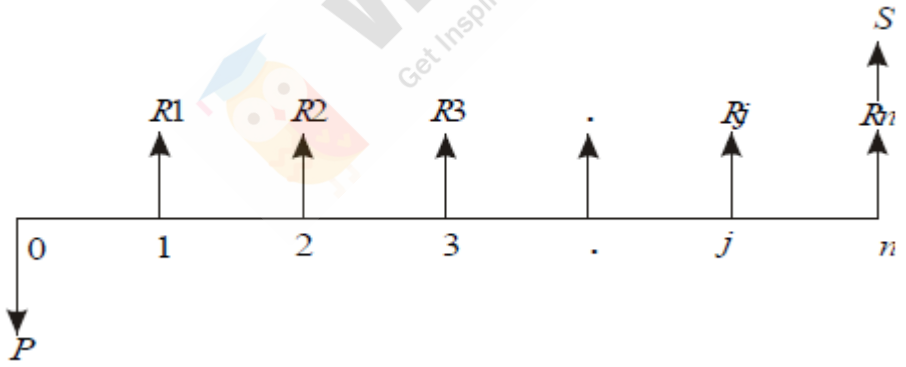


Fig. 4.1 Revenue-dominated cash flow diagram.

In Fig. 4.1, *P* represents an initial investment and *Rj* the net revenue at the end of the *j*th year. The interest rate is *i*, compounded annually. *S* is the salvage value at the end of the *n*th year.

To find the present worth of the above cash flow diagram for a given interest rate, the formula is

$$PW(i) = -P + R1[1/(1+i)1] + R2[1/(1+i)2] + \ldots + Rj[1/(1+i)\,j] + Rn[1/(1+i)n] + S[1/(1+i)n]$$

In this formula, expenditure is assigned a negative sign and revenues are assigned a positive sign. If we have some more alternatives which are to be compared with this alternative, then the corresponding present worth amounts are to be computed and compared. Finally, the

alternative with the maximum present worth amount should be selected as the best alternative.

4.3 COST-DOMINATED CASH FLOW DIAGRAM

A generalized cost-dominated cash flow diagram to demonstrate the present worth method of comparison is presented in Fig. 4.2.

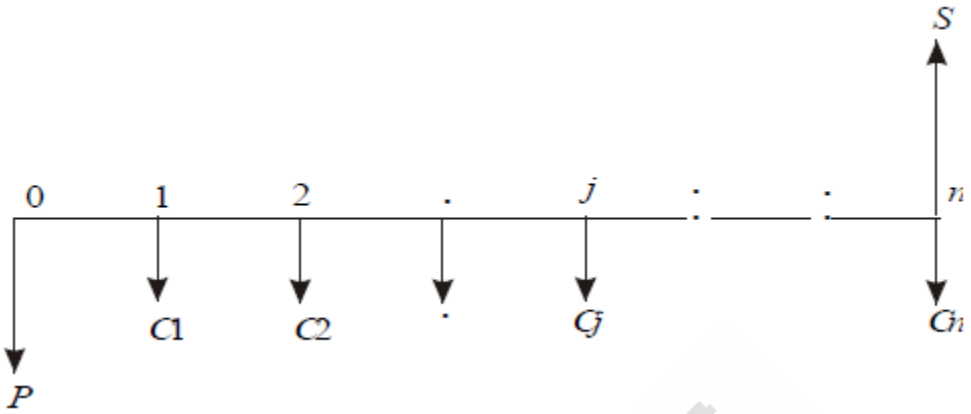


Fig. 4.2 Cost-dominated cash flow diagram.

In Fig. 4.2, *P* represents an initial investment, *Cj* the net cost of operation and maintenance at the end of the *j*th year, and *S* the salvage value at the end of the *n*th year.

To compute the present worth amount of the above cash flow diagram for a given interest rate *i*, we have the formula

$$PW(i) = P + C1[1/(1 + i)1] + C2[1/(1 + i)2] + \ldots + Cj[1/(1 + i)\,j] + Cn[1/(1 + i)n] - S[1/(1 + i)n]$$

In the above formula, the expenditure is assigned a positive sign and the revenue a negative sign. If we have some more alternatives which are to be compared with this alternative, then the corresponding present worth amounts are to be computed and compared. Finally, the alternative with the minimum present worth amount should be selected as the best alternative.

EXAMPLE 4.1 Alpha Industry is planning to expand its production operation. It has identified three different technologies for meeting the goal. The initial outlay and annual revenues with respect to each of the technologies are summarized in Table 4.1. Suggest the best technology which is to be implemented based on the present worth method of comparison assuming 20% interest rate, compounded annually.

Table 4.1

	Initial outlay (Rs.)	*Annual revenue* (Rs.)	*Life* (years)
Technology 1	12,00,000	4,00,000	10
Technology 2	20,00,000	6,00,000	10
Technology 3	18,00,000	5,00,000	10

Solution In all the technologies, the initial outlay is assigned a negative sign and the annual revenues are assigned a positive sign.

TECHNOLOGY 1

Initial outlay, P = Rs. 12,00,000

Annual revenue, A = Rs. 4,00,000

Interest rate, i = 20%, compounded annually

Life of this technology, n = 10 years

The cash flow diagram of this technology is as shown in Fig. 4.3.

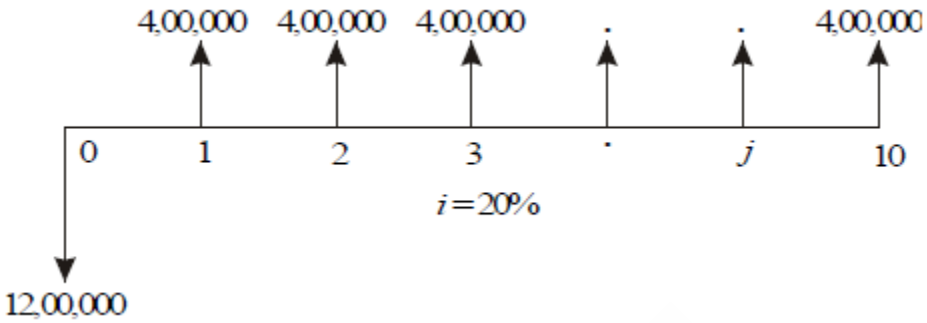


Fig. 4.3 Cash flow diagram for technology 1.

The present worth expression for this technology is

$PW(20\%)1 = -12,00,000 + 4,00,000 \times (P/A, 20\%, 10)$

$= -12,00,000 + 4,00,000 \times (4.1925)$

$= -12,00,000 + 16,77,000$

= Rs. 4,77,000

TECHNOLOGY 2

Initial outlay, P = Rs. 20,00,000

Annual revenue, A = Rs. 6,00,000

Interest rate, i = 20%, compounded annually

Life of this technology, n = 10 years

The cash flow diagram of this technology is shown in Fig. 4.4.

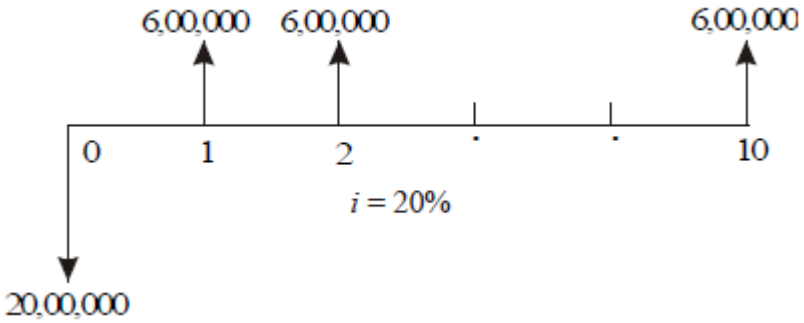


Fig. 4.4 Cash flow diagram for technology 2.

The present worth expression for this technology is

$PW(20\%)2 = -20,00,000 + 6,00,000 \times (P/A, 20\%, 10)$

$= -20,00,000 + 6,00,000 \times (4.1925)$

$= -20,00,000 + 25,15,500$

= Rs. 5,15,500

TECHNOLOGY 3

Initial outlay, P = Rs. 18,00,000

Annual revenue, A = Rs. 5,00,000

Interest rate, i = 20%, compounded annually

Life of this technology, n = 10 years

The cash flow diagram of this technology is shown in Fig. 4.5.

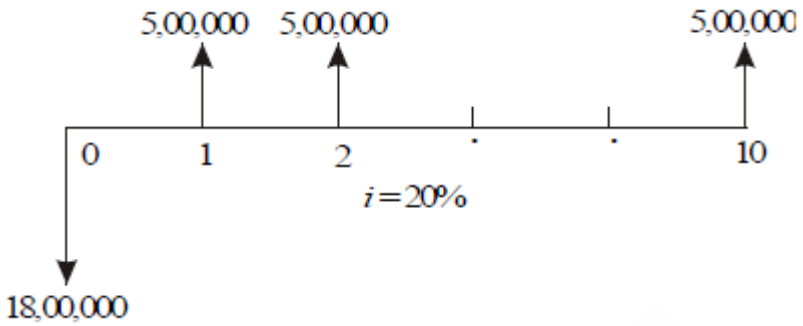


Fig. 4.5 Cash flow diagram for technology 3.

The present worth expression for this technology is

$PW(20\%)3 = -18,00,000 + 5,00,000 \times (P/A, 20\%, 10)$

$= -18,00,000 + 5,00,000 \times (4.1925)$

$= -18,00,000 + 20,96,250$

= Rs. 2,96,250

From the above calculations, it is clear that the present worth of technology 2 is the highest among all the technologies. Therefore, technology 2 is suggested for implementation to expand the production.

EXAMPLE 4.2 An engineer has two bids for an elevator to be installed in a new building. The details of the bids for the elevators are as follows:

Bid	*Engineer's estimates*		
	Initial cost (Rs.)	*Service life* (years)	*Annual operations & maintenance cost* (Rs.)
Alpha Elevator Inc.	4,50,000	15	27,000
Beta Elevator Inc.	5,40,000	15	28,500

Determine which bid should be accepted, based on the present worth method of comparison assuming 15% interest rate, compounded annually.

Solution

Bid 1: Alpha Elevator Inc.

Initial cost, P = Rs. 4,50,000

Annual operation and maintenance cost, A = Rs. 27,000

Life = 15 years

Interest rate, i = 15%, compounded annually.
The cash flow diagram of bid 1 is shown in Fig. 4.6.

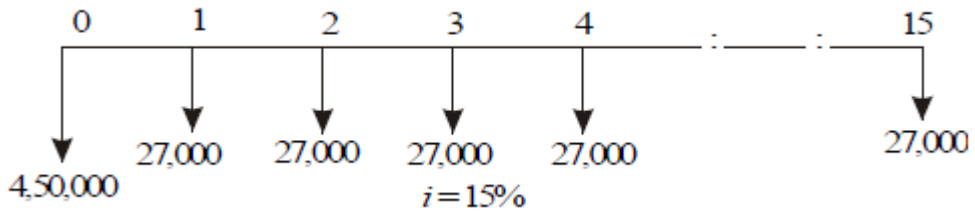


Fig. 4.6 Cash flow diagram for bid 1.

The present worth of the above cash flow diagram is computed as follows:
$PW(15\%) = 4,50,000 + 27,000(P/A, 15\%, 15)$
$= 4,50,000 + 27,000 \times 5.8474$
$= 4,50,000 + 1,57,879.80$
= Rs. 6,07,879.80

Bid 2: Beta Elevator Inc.
Initial cost, P = Rs. 5,40,000
Annual operation and maintenance cost, A = Rs. 28,500
Life = 15 years
Interest rate, i = 15%, compounded annually.

The cash flow diagram of bid 2 is shown in Fig. 4.7.

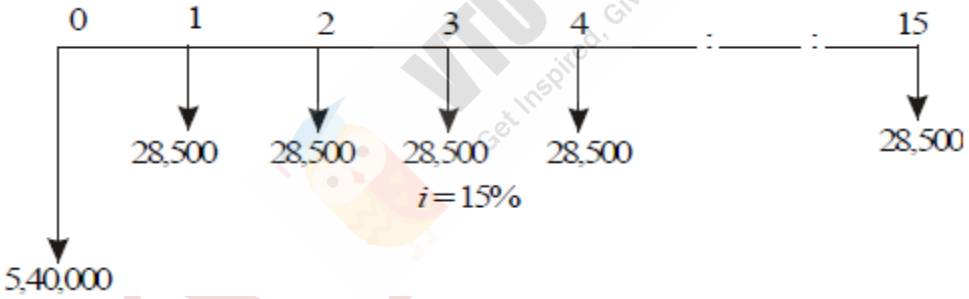


Fig. 4.7 Cash flow diagram for bid 2.

The present worth of the above cash flow diagram is computed as follows:
$PW(15\%) = 5,40,000 + 28,500(P/A, 15\%, 15)$
$= 5,40,000 + 28,500 \times 5.8474$
$= 5,40,000 + 1,66,650.90$
= Rs. 7,06,650.90

The total present worth cost of bid 1 is less than that of bid 2. Hence, bid 1 is to be selected for implementation. That is, the elevator from Alpha Elevator Inc. is to be purchased and installed in the new building.

EXAMPLE 4.3 Investment proposals A and B have the net cash flows as follows

Proposal	*End of years*				
	0	1	2	3	4
A (Rs.)	–10,000	3,000	3,000	7,000	6,000
B (Rs.)	–10,000	6,000	6,000	3,000	3,000

Compare the present worth of A with that of B at i = 18%. Which proposal should be selected?

Solution

Present worth of A at *i* = 18%. The cash flow diagram of proposal A is shown in Fig. 4.8.

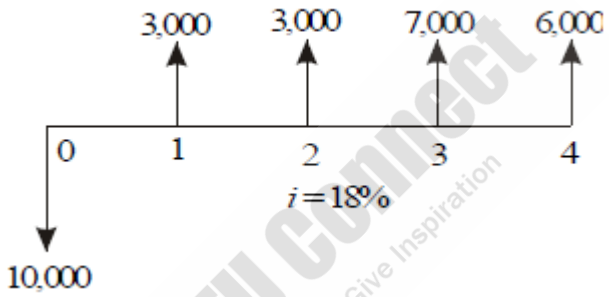


Fig. 4.8 Cash flow diagram for proposal A.

The present worth of the above cash flow diagram is computed as

PWA(18%) = –10,000 + 3,000(P/F, 18%, 1) + 3,000(P/F, 18%, 2) + 7,000(P/F, 18%, 3) + 6,000(P/F, 18%, 4)

= –10,000 + 3,000 (0.8475) + 3,000(0.7182) + 7,000(0.6086) + 6,000(0.5158)

= Rs. 2,052.10

Present worth of B at *i* = 18%. The cash flow diagram of the proposal B is shown in Fig. 4.9.

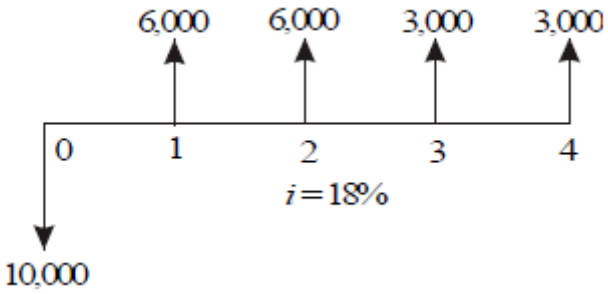


Fig. 4.9 Cash flow diagram for proposal B.

The present worth of the above cash flow diagram is calculated as

PWB(18%) = –10,000 + 6,000(P/F, 18%, 1) + 6,000(P/F, 18%, 2) + 3,000(P/F, 18%, 3) + 3,000(P/F, 18%, 4)
= –10,000 + 6,000(0.8475) + 6,000(0.7182) + 3,000(0.6086) + 3,000(0.5158)
= Rs. 2,767.40

At i = 18%, the present worth of proposal B is higher than that of proposal A. Therefore, select proposal B.

EXAMPLE 4.4 A granite company is planning to buy a fully automated granite cutting machine. If it is purchased under down payment, the cost of the machine is Rs. 16,00,000. If it is purchased under installment basis, the company has to pay 25% of the cost at the time of purchase and the remaining amount in 10 annual equal installments of Rs. 2,00,000 each. Suggest the best alternative for the company using the present worth basis at i = 18%, compounded annually.

Solution There are two alternatives available for the company:
1. Down payment of Rs. 16,00,000
2. Down payment of Rs. 4,00,000 and 10 annual equal installments of Rs. 2,00,000 each

Present worth calculation of the second alternative. The cash flow diagram of the second alternative is shown in Fig. 4.10.

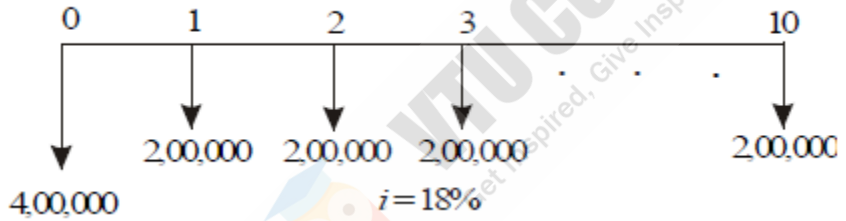


Fig. 4.10 Cash flow diagram for the second alternative.

The present worth of the above cash flow diagram is computed as
PW(18%) = 4,00,000 + 2,00,000(P/A, 18%, 10)
= 4,00,000 + 2,00,000 × 4.4941
= Rs. 12,98,820

The present worth of this option is Rs. 12,98,820, which is less than the first option of complete down payment of Rs. 16,00,000. Hence, the company should select the second alternative to buy the fully automated granite cutting machine.

EXAMPLE 4.5 A finance company advertises two investment plans. In plan 1, the company pays Rs. 12,000 after 15 years for every Rs. 1,000 invested now. In plan 2, for every Rs. 1,000 invested, the company pays Rs. 4,000 at the end of the 10th year and Rs. 4,000 at the end of 15th year. Select the best investment plan from the investor's point of view at i = 12%, compounded annually.

Solution *Plan 1.* The cash flow diagram for plan 1 is illustrated in Fig. 4.11.

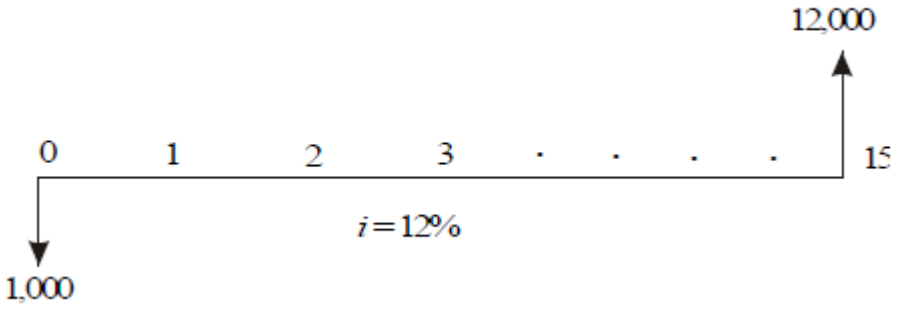


Fig. 4.11 Cash flow diagram for plan 1.

The present worth of the above cash flow diagram is calculated as

$PW(12\%) = -1,000 + 12,000(P/F, 12\%, 15)$

$= -1,000 + 12,000(0.1827)$

= Rs. 1,192.40

Plan 2. The cash flow diagram for plan 2 is shown in Fig. 4.12.

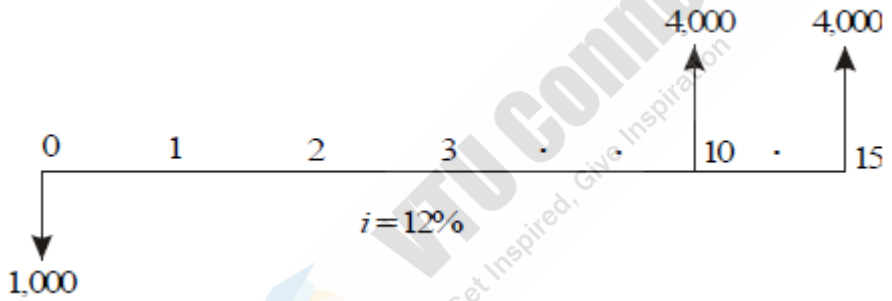


Fig. 4.12 Cash flow diagram for plan 2.

The present worth of the above cash flow diagram is computed as

$PW(12\%) = -1,000 + 4,000(P/F, 12\%, 10) + 4,000(P/F, 12\%, 15)$

$= -1,000 + 4,000(0.3220) + 4,000(0.1827)$

= Rs. 1,018.80

The present worth of plan 1 is more than that of plan 2. Therefore, plan 1 is the best plan from the investor's point of view.

EXAMPLE 4.6 Novel Investment Ltd. accepts Rs. 10,000 at the end of every year for 20 years and pays the investor Rs. 8,00,000 at the end of the 20th year. Innovative Investment Ltd. accepts Rs. 10,000 at the end of every year for 20 years and pays the investor Rs. 15,00,000 at the end of the 25th year. Which is the best investment alternative? Use present worth base with i = 12%.

Solution *Novel Investment Ltd's plan.* The cash flow diagram of Novel Investment Ltd's plan is shown in Fig. 4.13.

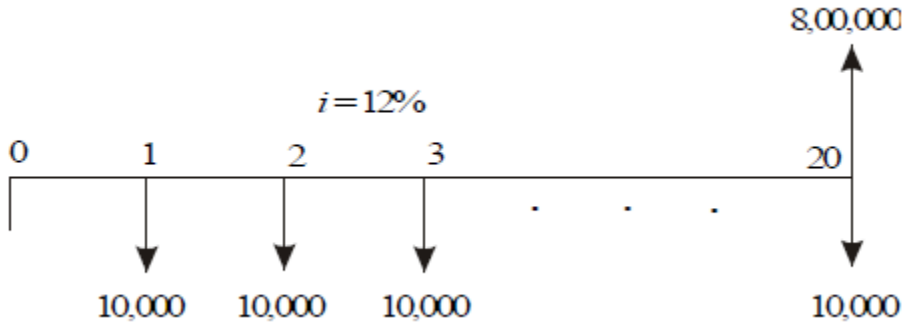


Fig. 4.13 Cash flow diagram for Novel Investment Ltd.

The present worth of the above cash flow diagram is computed as
PW(12%) = –10,000(*P*/*A*, 12%, 20) + 8,00,000(*P*/*F*, 12%, 20)
= –10,000(7.4694) + 8,00,000(0.1037)
= Rs. 8,266
Innovative Investment Ltd's plan. The cash flow diagram of the Innovative Investment Ltd's plan is illustrated in Fig. 4.14

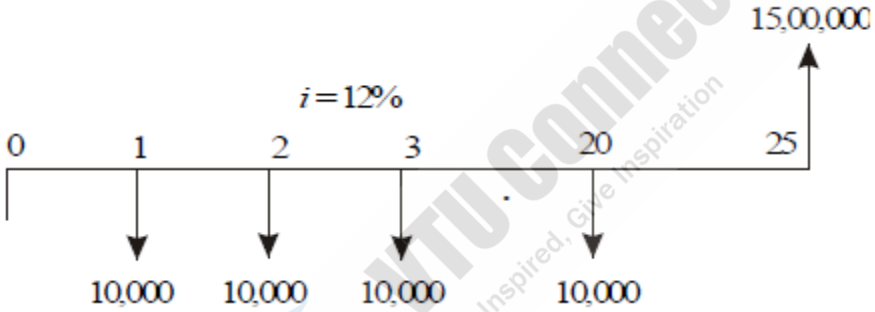


Fig. 4.14 Cash flow diagram for Innovative Investment Ltd.

The present worth of the above cash flow diagram is calculated as
PW(12%) = –10,000(*P*/*A*, 12%, 20) + 15,00,000(*P*/*F*, 12%, 25)
= –10,000(7.4694) + 15,00,000(0.0588)
= Rs. 13,506

The present worth of Innovative Investment Ltd's plan is more than that of Novel Investment Ltd's plan. Therefore, Innovative Investment Ltd's plan is the best from investor's point of view.

EXAMPLE 4.7 A small business with an initial outlay of Rs. 12,000 yields Rs. 10,000 during the first year of its operation and the yield increases by Rs. 1,000 from its second year of operation up to its 10th year of operation. At the end of the life of the business, the salvage value is zero. Find the present worth of the business by assuming an interest rate of 18%, compounded annually.

Solution
Initial investment, *P* = Rs. 12,000
Income during the first year, *A* = Rs. 10,000

Annual increase in income, G = Rs. 1,000
n = 10 years
i = 18%, compounded annually
The cash flow diagram for the small business is depicted in Fig. 4.15.

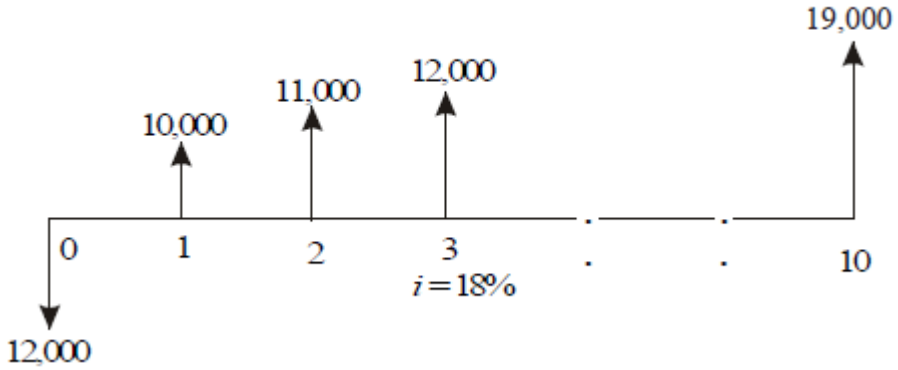


Fig. 4.15 Cash flow diagram for the small business.

The equation for the present worth is
$PW(18\%) = -12{,}000 + (10{,}000 + 1{,}000 \times (A/G, 18\%, 10)) \times (P/A, 18\%, 10)$
$= -12{,}000 + (10{,}000 + 1{,}000 \times 3.1936) \times\_ 4.4941$
$= -12{,}000 + 59{,}293.36$
= Rs. 47,293.36
The present worth of the small business is Rs. 47,293.36.

C-5 FUTURE WORTH METHOD

5.1 INTRODUCTION

In the future worth method of comparison of alternatives, the future worth of various alternatives will be computed. Then, the alternative with the maximum future worth of net revenue or with the minimum future worth of net cost will be selected as the best alternative for implementation.

5.2 REVENUE-DOMINATED CASH FLOW DIAGRAM

A generalized revenue-dominated cash flow diagram to demonstrate the future worth method of comparison is presented in Fig. 5.1.

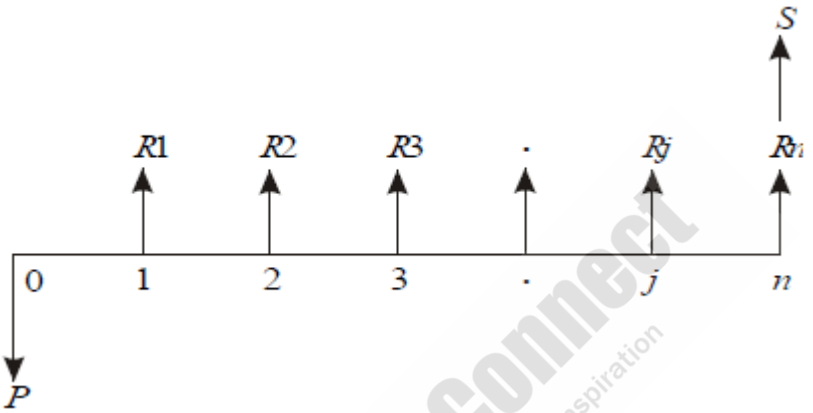


Fig. 5.1 Revenue-dominated cash flow diagram.

In Fig. 5.1, *P* represents an initial investment, *Rj* the net-revenue at the end of the *j*th year, and *S* the salvage value at the end of the *n*th year. The formula for the future worth of the above cash flow diagram for a given interest rate, *i* is

$FW(i) = -P(1 + i)n + R1(1 + i)n-1 + R2(1 + i)n-2 + ...+ Rj(1 + i)n-j + ... + Rn + S$

In the above formula, the expenditure is assigned with negative sign and the revenues are assigned with positive sign. If we have some more alternatives which are to be compared with this alternative, then the corresponding future worth amounts are to be computed and compared. Finally, the alternative with the maximum future worth amount should be selected as the best alternative.

5.3 COST-DOMINATED CASH FLOW DIAGRAM

A generalized cost-dominated cash flow diagram to demonstrate the future worth method of comparison is given in Fig. 5.2.

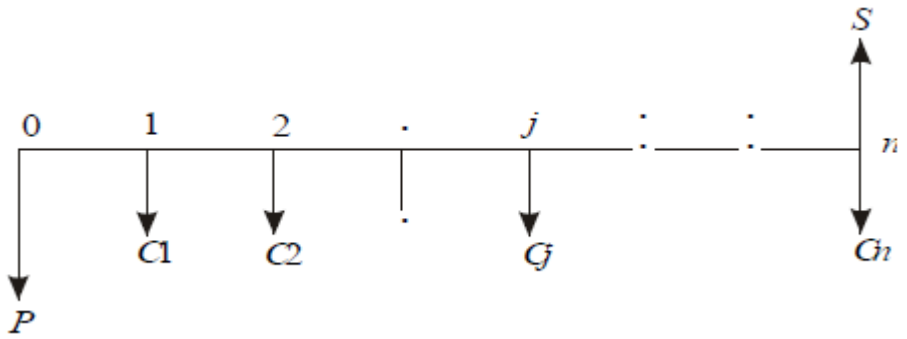


Fig. 5.2 Cost-dominated cash flow diagram.

In Fig. 5.2, *P* represents an initial investment, *Cj* the net cost of operation and maintenance at the end of the *j*th year, and *S* the salvage value at the end of the *n*th year. The formula for the future worth of the above cash flow diagram for a given interest rate, *i* is

$FW(i) = P(1 + i)n + C1(1 + i)n–1 + C2(1 + i)n–2 + ... + Cj(1 + i)n–j + ... + Cn – S$

In this formula, the expenditures are assigned with positive sign and revenues with negative sign. If we have some more alternatives which are to be compared with this alternative, then the corresponding future worth amounts are to be computed and compared. Finally, the alternative with the minimum future worth amount should be selected as the best alternative.

5.4 EXAMPLES

In this section, several examples highlighting the applications of the future worth method of comparison are presented.

EXAMPLE 5.1 Consider the following two mutually exclusive alternatives:

	End of year				
Alternative	0	1	2	3	4
A (Rs.)	−50,00,000	20,00,000	20,00,000	20,00,000	20,00,000
B (Rs.)	−45,00,000	18,00,000	18,00,000	18,00,000	18,00,000

At *i* = 18%, select the best alternative based on future worth method of comparison.

Solution Alternative A

Initial investment, *P* = Rs. 50,00,000

Annual equivalent revenue, *A* = Rs. 20,00,000

Interest rate, *i* = 18%, compounded annually

Life of alternative A = 4 years

The cash flow diagram of alternative A is shown in Fig. 5.3.

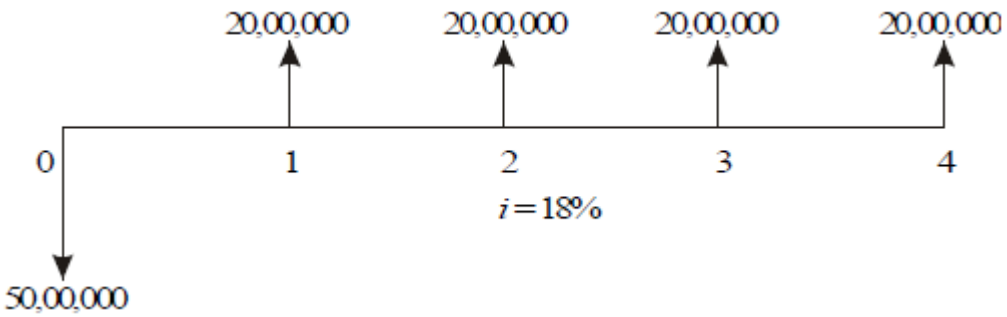


Fig. 5.3 Cash flow diagram for alternative A.

The future worth amount of alternative B is computed as

FWA(18%) = –50,00,000(F/P, 18%, 4) + 20,00,000(F/A, 18%, 4)

= –50,00,000(1.939) + 20,00,000(5.215)

= Rs. 7,35,000

Alternative B

Initial investment, P = Rs. 45,00,000

Annual equivalent revenue, A = Rs. 18,00,000

Interest rate, i = 18%, compounded annually

Life of alternative B = 4 years

The cash flow diagram of alternative B is illustrated in Fig. 5.4.

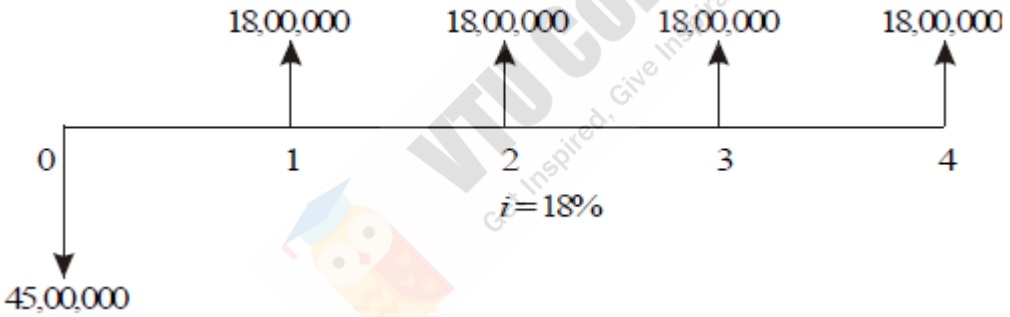


Fig. 5.4 Cash flow diagram for alternative B.

The future worth amount of alternative B is computed as

FWB(18%) = – 45,00,000(F/P, 18%, 4) + 18,00,000 (F/A, 18%, 4)

= –45,00,000(1.939) + 18,00,000(5.215)

= Rs. 6,61,500

The future worth of alternative A is greater than that of alternative B. Thus, alternative A should be selected.

EXAMPLE 5.2 A man owns a corner plot. He must decide which of the several alternatives to select in trying to obtain a desirable return on his investment. After much study and calculation, he decides that the two best alternatives are as given in the following table:

	Build gas station	*Build soft ice-cream stand*
First cost (Rs.)	20,00,000	36,00,000
Annual property taxes (Rs.)	80,000	1,50,000
Annual income (Rs.)	8,00,000	9,80,000
Life of building (years)	20	20
Salvage value (Rs.)	0	0

Evaluate the alternatives based on the future worth method at i = 12%.

***Alternative 1*—Build gas station**

First cost = Rs. 20,00,000

Net annual income = Annual income – Annual property tax

= Rs. 8,00,000 – Rs. 80,000

= Rs. 7,20,000

Life = 20 years

Interest rate = 12%, compounded annually

The cash flow diagram for this alternative is depicted in Fig. 5.5.

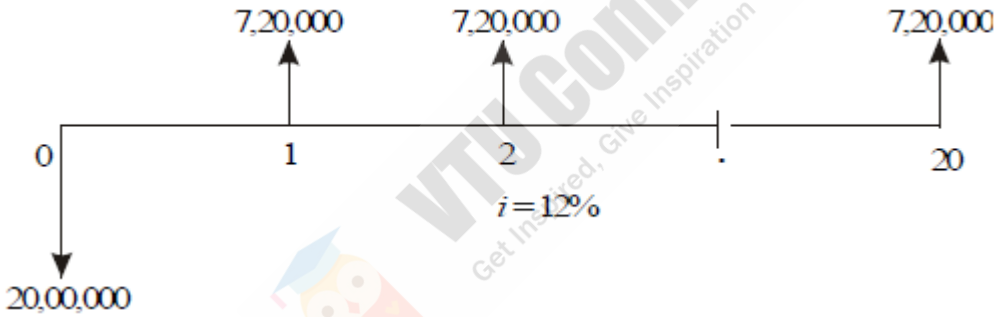


Fig. 5.5 Cash flow diagram for alternative 1.

The future worth of alternative 1 is computed as

$FW1(12\%)$ = –20,00,000 (F/P, 12%, 20) + 7,20,000 (F/A, 12%, 20)

= –20,00,000(9.646) + 7,20,000 (72.052)

= Rs. 3,25,85,440

***Alternative 2*—Build soft ice-cream stand**

First cost = Rs. 36,00,000

Net annual income = Annual income – Annual property tax

= Rs. 9,80,000 – Rs. 1,50,000

= Rs. 8,30,000

Life = 20 years

Interest rate = 12%, compounded annually

The cash flow diagram for this alternative is shown in Fig. 5.6.

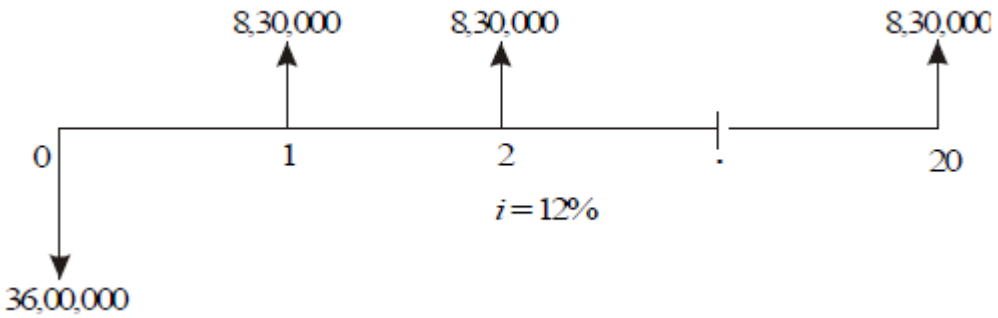


Fig. 5.6 Cash flow diagram for alternative 2.

The future worth of alternative 2 is calculated as

$FW2(12\%) = -36,00,000(F/P, 12\%, 20) + 8,30,000\ (F/A, 12\%, 20)$

$= -36,00,000\ (9.646) + 8,30,000\ (72.052)$

= Rs. 2,50,77,560

The future worth of alternative 1 is greater than that of alternative 2. Thus, building the gas station is the best alternative.

EXAMPLE 5.3 The cash flow diagram of two mutually exclusive alternatives is given in Figs. 5.7 and 5.8.

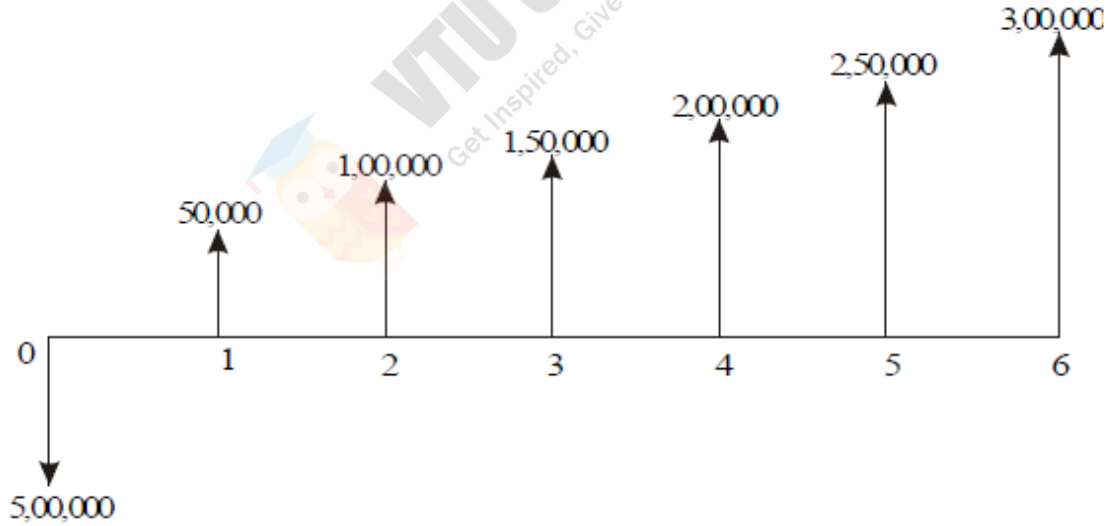


Fig. 5.7 Cash flow diagram for alternative 1.

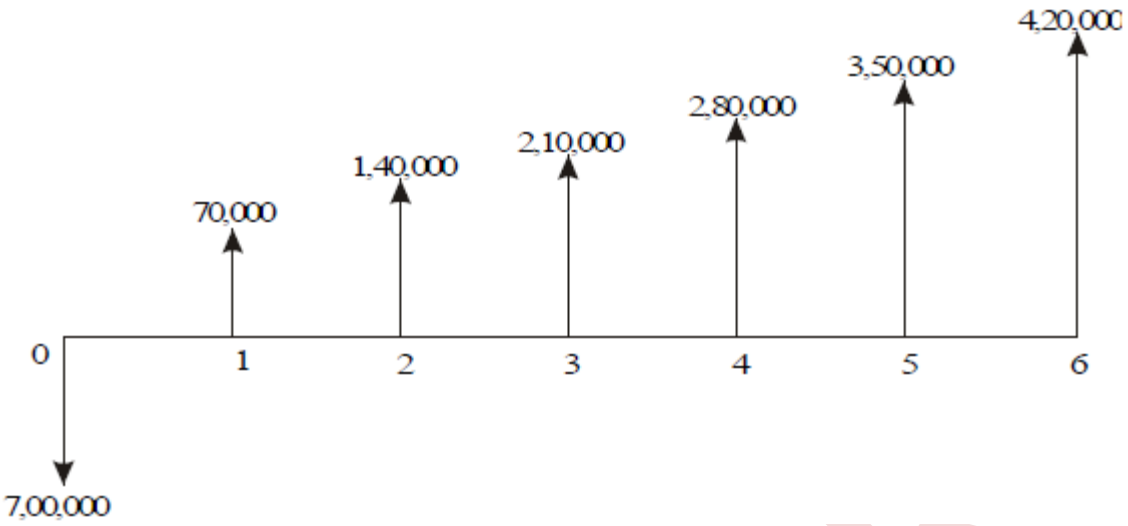


Fig. 5.8 Cash flow diagram for alternative 2.

(a) Select the best alternative based on future worth method at i = 8%.
(b) Rework part (a) with i = 9% and 20%
(*a*) *Evaluation at* i = 8%

***Alternative 1*—This comes under equal payment gradient series.**
P = Rs. 5,00,000
$A1$ = Rs. 50,000
G = Rs. 50,000
i = 8%
n = 6 years
The formula for the future worth of alternative 1 is
$FW1(8\%) = -P(F/P, 8\%, 6) + [A1 + G(A/G, 8\%, 6)] \times (F/A, 8\%, 6)$
= – 5,00,000(1.587) + [50,000 + 50,000(2.2764)] × 7.336
= – 79,35,000 + 1,63,820 ×7.336
= –79,35,000 + 12,01,784
= Rs. 4,08,283.52

***Alternative 2*—This comes under equal payment gradient series.**
P = Rs. 7,00,000
$A1$ = Rs. 70,000
G = Rs. 70,000
i = 8%
n = 6 years
The formula for the future worth of alternative 2 is
$FW2(8\%) = -P(F/P, 8\%, 6) + [A1 + G(A/G, 8\%, 6)] \times (F/A, 8\%, 6)$
$FW2(8\%) = -7,00,000 \times 1.587 + [70,000 + 70,000 \times 2.2764] \times 7.336$
= –11,10,900 + 16,82,497
= Rs. 5,71,596.93

The future worth of alternative 2 is more than that of alternative 1. Therefore, alternative 2 must be selected.

(b) (i) Evaluation at** i = 9%: **Alternative 1

P = Rs. 5,00,000
$A1$ = Rs. 50,000
G = Rs. 50,000
n = 6 years
The formula for the future worth of alternative 1 is as follows:
$FW1(9\%) = -P(F/P, 9\%, 6) + [A1 + G(A/G, 9\%, 6)] \times (F/A, 9\%, 6)$
= – 5,00,000 (1.677) + [50,000 + 50,000 (2.2498)] ×7.523
= – 8,38,500 + 12,22,412.27
= Rs. 3,83,912.27

Alternative 2

P = Rs. 7,00,000
$A1$ = Rs. 70,000
G = Rs. 70,000
n = 6 years
The formula for the future worth of the alternative 2 is
$FW2(9\%) = -P(F/P, 9\%, 6) + [A1 + G(A/G, 9\%, 6)] \_ (F/A, 9\%, 6)$
= –7,00,000 _ 1.677 + [70,000 + 70,000 _ 2.2498] _ 7.523
= –11,73,900 + 17,11,377.18
= Rs. 5,37,477.18
The future worth of alternative 2 is more than that of alternative 1. Therefore, alternative 2 must be selected.

(ii) Evaluation at** i = 20%: **Alternative 1

P = Rs. 5,00,000
$A1$ = Rs. 50,000
G = Rs. 50,000
n = 6 years
The formula for the future worth of alternative 1 is
$FW1(20\%) = -P(F/P, 20\%, 6) + [A1 + G(A/G, 20\%, 6)] \times (F/A, 20\%, 6)$
= –5,00,000(2.986) + [50,000 + 50,000 (1.9788)] _ 9.93
= –14,93,000 + 14,78,974.20
= Rs. –14,025.80
The negative sign of the future worth amount indicates that alternative 1 incurs loss.

Alternative 2

P = Rs. 7,00,000
$A1$ = Rs. 70,000
G = Rs. 70,000
n = 6 years

The formula for the future worth of alternative 2 is
$FW2(20\%) = -P(F/P, 20\%, 6) + [A1 + G(A/G, 20\%, 6)] \times (F/A, 20\%, 6)$
$= -7,00,000 \_ 2.986 + [70,000 + 70,000 \times 1.9788] \times 9.93$
$= -20,90,200 + 20,70,563.88$
= Rs. –19,636.12

The negative sign of the above future worth amount indicates that alternative 2 incurs loss. Thus, none of the two alternatives should be selected.

EXAMPLE 5.4 M/S Krishna Castings Ltd. is planning to replace its annealing furnace. It has received tenders from three different original manufacturers of annealing furnace. The details are as follows.

	Manufacturer		
	1	2	3
Initial cost (Rs.)	80,00,000	70,00,000	90,00,000
Life (years)	12	12	12
Annual operation and maintenance cost (Rs.)	8,00,000	9,00,000	8,50,000
Salvage value after 12 years	5,00,000	4,00,000	7,00,000

Which is the best alternative based on future worth method at i = 20%?

Solution Alternative 1—Manufacturer 1
First cost, P = Rs. 80,00,000
Life, n = 12 years
Annual operating and maintenance cost, A = Rs. 8,00,000
Salvage value at the end of furnace life = Rs. 5,00,000
The cash flow diagram for this alternative is shown in Fig. 5.9.

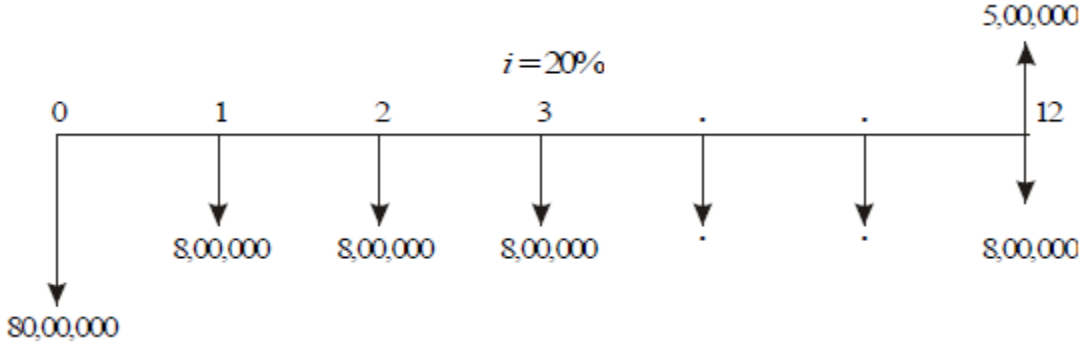


Fig. 5.9 Cash flow diagram for manufacturer 1.

The future worth amount of alternative 1 is computed as
$FW1(20\%) = 80,00,000\ (F/P, 20\%, 12) + 8,00,000\ (F/A, 20\%, 12) - 5,00,000$
$= 80,00,000(8.916) + 8,00,000\ (39.581) - 5,00,000$
= Rs. 10,24,92,800

Alternative 2— Manufacturer 2

First cost, P = Rs. 70,00,000
Life, n = 12 years
Annual operating and maintenance cost, A = Rs. 9,00,000
Salvage value at the end of furnace life = Rs. 4,00,000
The cash flow diagram for this alternative is given in Fig. 5.10

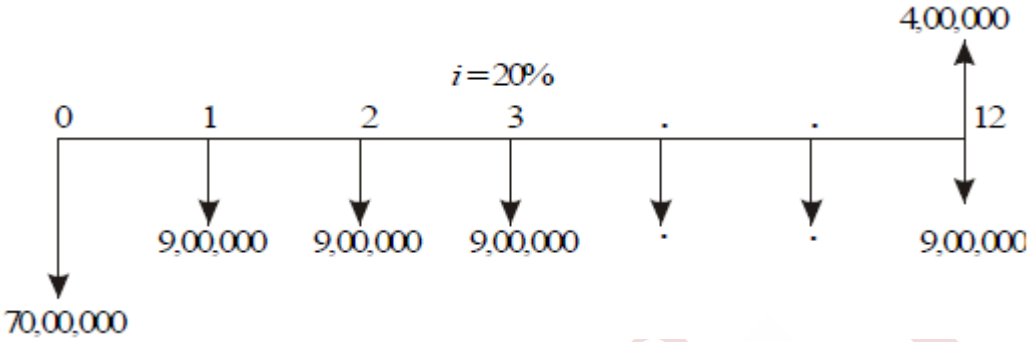


Fig. 5.10 Cash flow diagram for manufacturer 2.

The future worth amount of alternative 2 is computed as
$FW2(20\%) = 70,00,000(F/P, 20\%, 12) + 9,00,000(F/A, 20\%, 12) – 4,00,000$
$= 70,00,000(8.916) + 9,00,000\ (39.581) – 4,00,000$
= Rs. 9,76,34,900

Alternative 3—Manufacturer 3

First cost, P = Rs. 90,00,000
Life, n = 12 years
Annual operating and maintenance cost, A = Rs. 8,50,000
Salvage value at the end of furnace life = Rs. 7,00,000
The cash flow diagram for this alternative is illustrated in Fig. 5.11.

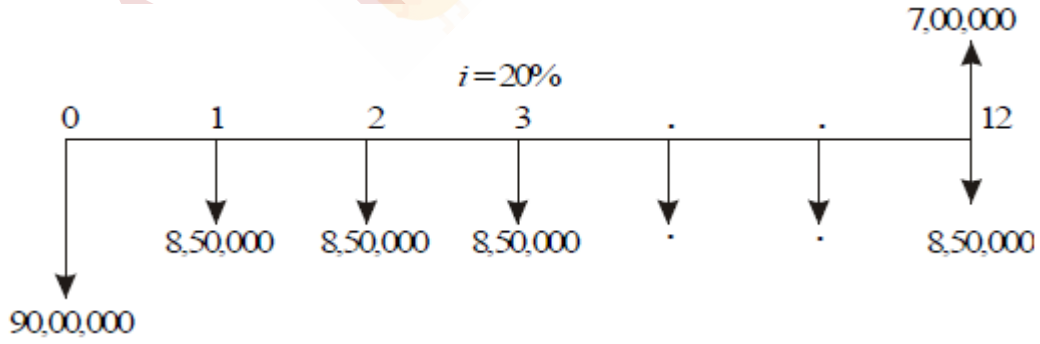


Fig. 5.11 Cash flow diagram for manufacturer 3.

The future worth amount of alternative 3 is calculated as
$FW3(20\%) = 90,00,000(F/P, 20\%, 12) + 8,50,000(F/A, 20\%, 12) – 7,00,000$
$= 90,00,000(8.916) + 8,50,000\ (39.581) – 7,00,000$
= Rs. 11,31,87,850

The future worth cost of alternative 2 is less than that of the other two alternatives. Therefore, M/s. Krishna castings should buy the annealing furnace from manufacturer 2.

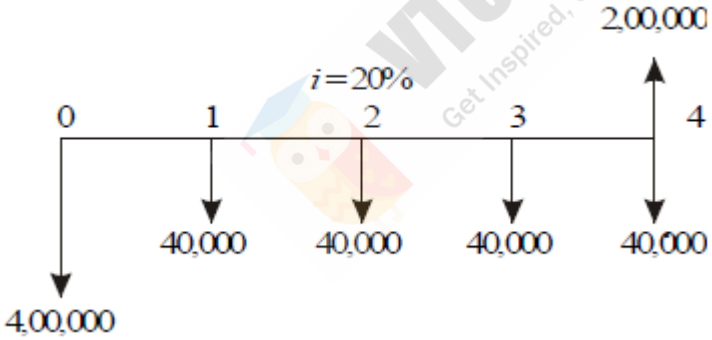
EXAMPLE 5.5 A company must decide whether to buy machine *A* or machine *B*:

	Machine A	*Machine B*
Initial cost	Rs. 4,00,000	Rs. 8,00,000
Useful life, in years	4	4
Salvage value at the end of machine life	Rs. 2,00,000	Rs. 5,50,000
Annual maintenance cost	Rs. 40,000	0

At 12% interest rate, which machine should be selected? (Use future worth method of comparison).

Solution Machine A

Initial cost of the machine, P = Rs. 4,00,000
Life, n = 4 years
Salvage value at the end of machine life, S = Rs. 2,00,000
Annual maintenance cost, A = Rs. 40,000
Interest rate, i = 12%, compounded annually.
The cash flow diagram of machine *A* is given in Fig. 5.12.



The future worth function of Fig. 5.12 is
$FWA(12\%) = 4,00,000 \_ (F/P, 12\%, 4) + 40,000 \times (F/A, 12\%, 4) – 2,00,000$
$= 4,00,000 \times (1.574) + 40,000 \times (4.779) – 2,00,000$
= Rs. 6,20,760

Machine B

Initial cost of the machine, P = Rs. 8,00,000
Life, n = 4 years
Salvage value at the end of machine life, S = Rs. 5,50,000
Annual maintenance cost, A = zero.
Interest rate, i = 12%, compounded annually.
The cash flow diagram of the machine *B* is illustrated in Fig. 5.13.

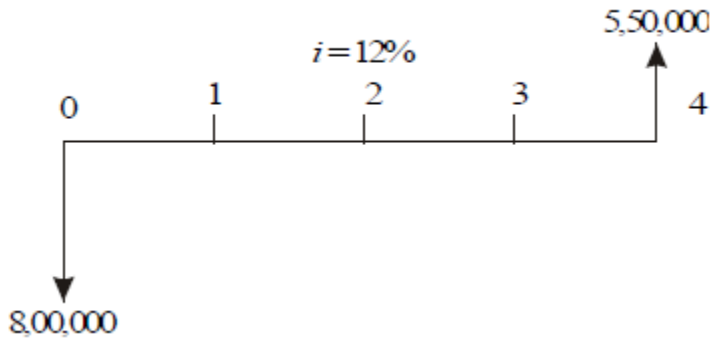


Fig. 5.13 Cash flow diagram for machine *B*.

The future worth function of Fig 5.13 is

FWB(12%) = 8,00,000 × (*F*/*P*, 12%, 4) – 5,50,000

= 8,00,000 × (1.574) – 5,50,000

= Rs. 7,09,200

The future worth cost of machine *A* is less than that of machine *B*. Therefore, machine *A* should be selected.

C-6 ANNUAL EQUIVALENT METHOD

6.1 INTRODUCTION

In the annual equivalent method of comparison, first the annual equivalent cost or the revenue of each alternative will be computed. Then the alternative with the maximum annual equivalent revenue in the case of revenue-based comparison or with the minimum annual equivalent cost in the case of costbased comparison will be selected as the best alternative.

6.2 REVENUE-DOMINATED CASH FLOW DIAGRAM

A generalized revenue-dominated cash flow diagram to demonstrate the annual equivalent method of comparison is presented in Fig. 6.1.

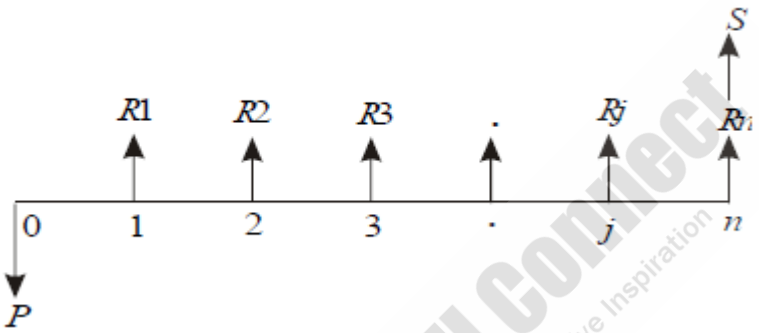


Fig. 6.1 Revenue-dominated cash flow diagram.

In Fig. 6.1, *P* represents an initial investment, *Rj* the net revenue at the end of the *j*th year, and *S* the salvage value at the end of the *n*th year. The first step is to find the net present worth of the cash flow diagram using the following expression for a given interest rate, *i*:

$PW(i) = -P + R1/(1 + i)1 + R2/(1 + i)2 + ... + Rj/(1 + i)\,j + ... + Rn/(1 + i)n + S/(1 + i)n$

In the above formula, the expenditure is assigned with a negative sign and the revenues are assigned with a positive sign. In the second step, the annual equivalent revenue is computed using the following formula:

$$A = PW(i)\,\frac{i(1+i)^n}{(1+i)^n - 1}$$

$$= PW(i)\,(A/P, i, n)$$

where $(A/P, i, n)$ is called *equal payment series capital recovery factor*. If we have some more alternatives which are to be compared with this alternative, then the corresponding annual equivalent revenues are to be computed and compared. Finally, the alternative with the maximum annual equivalent revenue should be selected as the best alternative.

6.3 COST-DOMINATED CASH FLOW DIAGRAM

A generalized cost-dominated cash flow diagram to demonstrate the annual equivalent method of comparison is illustrated in Fig. 6.2.

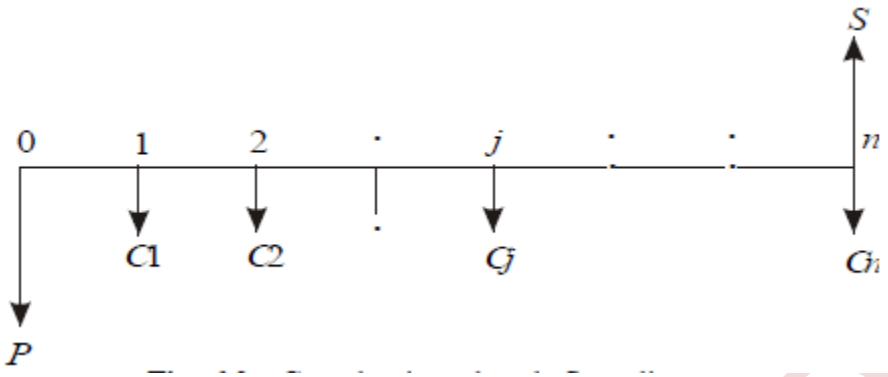


Fig. 6.2 Cost-dominated cash flow diagram.

In Fig. 6.2, *P* represents an initial investment, *Cj* the net cost of operation and maintenance at the end of the *j*th year, and *S* the salvage value at the end of the *n*th year. The first step is to find the net present worth of the cash flow diagram using the following relation for a given interest rate, *i*.

$PW(i) = P + C1/(1 + i)1 + C2/(1 + i)2 + ... + Cj/(1 + i)\,j + ... + Cn/(1 + i)n – S/(1 + i)n$

In the above formula, each expenditure is assigned with positive sign and the salvage value with negative sign. Then, in the second step, the annual equivalent cost is computed using the following equation:

$$A = PW(i)\,\frac{i(1+i)^n}{(1+i)^n - 1}$$

$$= PW(i)\,(A/P, i, n)$$

where $(A/P, i, n)$ is called as equal-payment series capital recovery factor. As in the previous case, if we have some more alternatives which are to be compared with this alternative, then the corresponding annual equivalent costs are to be computed and compared. Finally, the alternative with the minimum annual equivalent cost should be selected as the best alternative. If we have some non-standard cash flow diagram, then we will have to follow the general procedure for converting each and every transaction to time zero and then convert the net present worth into an annual equivalent cost/ revenue depending on the type of the cash flow diagram. Such procedure is to be applied to all the alternatives and finally, the best alternative is to be selected.

6.4 ALTERNATE APPROACH

Instead of first finding the present worth and then figuring out the annual equivalent cost/revenue, an alternate method which is as explained below can be used. In each of the cases presented in Sections 6.2 and 6.3, in the first step, one can find the future worth of the cash flow diagram of each of the alternatives. Then, in the second step, the annual equivalent cost/revenue can be obtained by using the equation:

$$A = F\frac{i}{(1+i)^n - 1}$$
$$= F(A/F, i, n)$$

where $(A/F, i, n)$ is called *equal-payment series sinking fund factor.*

6.5 EXAMPLES

In this section, the application of the annual equivalent method is demonstrated with several numerical examples.

EXAMPLE 6.1 A company provides a car to its chief executive. The owner of the company is concerned about the increasing cost of petrol. The cost per litre of petrol for the first year of operation is Rs. 21. He feels that the cost of petrol will be increasing by Re.1 every year. His experience with his company car indicates that it averages 9 km per litre of petrol. The executive expects to drive an average of 20,000 km each year for the next four years. What is the annual equivalent cost of fuel over this period of time?. If he is offered similar service with the same quality on rental basis at Rs. 60,000 per year, should the owner continue to provide company car for his executive or alternatively provide a rental car to his executive? Assume i = 18%. If the rental car is preferred, then the company car will find some other use within the company.

Solution

Average number of km run/year = 20,000 km
Number of km/litre of petrol = 9 km
Therefore,
Petrol consumption/year = 20,000/9 = 2222.2 litre
Cost/litre of petrol for the 1st year = Rs. 21
Cost/litre of petrol for the 2nd year = Rs. 21.00 + Re. 1.00 = Rs. 22.00
Cost/litre of petrol for the 3rd year = Rs. 22.00 + Re. 1.00 = Rs. 23.00
Cost/litre of petrol for the 4th year = Rs. 23.00 + Re. 1.00 = Rs. 24.00
Fuel expenditure for 1st year = 2222.2 ×21 = Rs. 46,666.20
Fuel expenditure for 2nd year = 2222.2 × 22 = Rs. 48,888.40
Fuel expenditure for 3rd year = 2222.2 ×23 = Rs. 51,110.60
Fuel expenditure for 4th year = 2222.2×24 = Rs. 53,332.80
The annual equal increment of the above expenditures is Rs. 2,222.20 (G).
The cash flow diagram for this situation is depicted in Fig. 6.3.

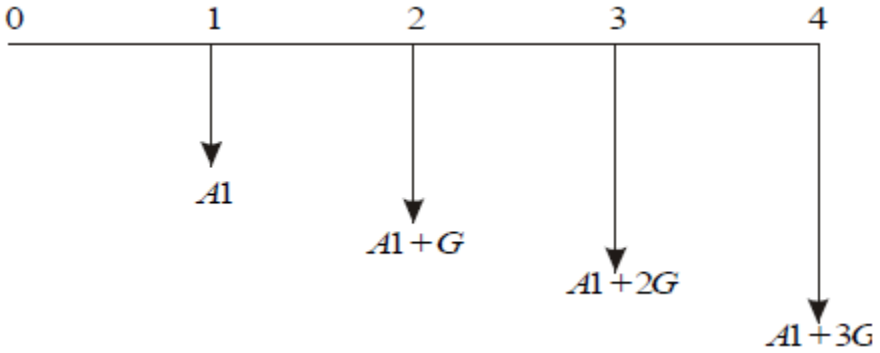


Fig. 6.3 Uniform gradient series cash flow diagram.

In Fig. 4.3, $A1$ = Rs. 46,666.20 and G = Rs. 2,222.20
$A = A1 + G(A/G, 18\%, 4)$
= 46,666.20 + 2222.2(1.2947)
= Rs. 49,543.28
The proposal of using the company car by spending for petrol by the company will cost an annual equivalent amount of Rs. 49,543.28 for four years. This amount is less than the annual rental value of Rs. 60,000. Therefore, the company should continue to provide its own car to its executive.

EXAMPLE 6.2 A company is planning to purchase an advanced machine centre. Three original manufacturers have responded to its tender whose particulars are tabulated as follows:

Manufacturer	*Down payment* (Rs.)	*Yearly equal installment* (Rs.)	*No. of installments*
1	5,00,000	2,00,000	15
2	4,00,000	3,00,000	15
3	6,00,000	1,50,000	15

Determine the best alternative based on the annual equivalent method by assuming i = 20%, compounded annually.

Solution Alternative 1
Down payment, P = Rs. 5,00,000
Yearly equal installment, A = Rs. 2,00,000
n = 15 years
i = 20%, compounded annually
The cash flow diagram for manufacturer 1 is shown in Fig. 6.4.

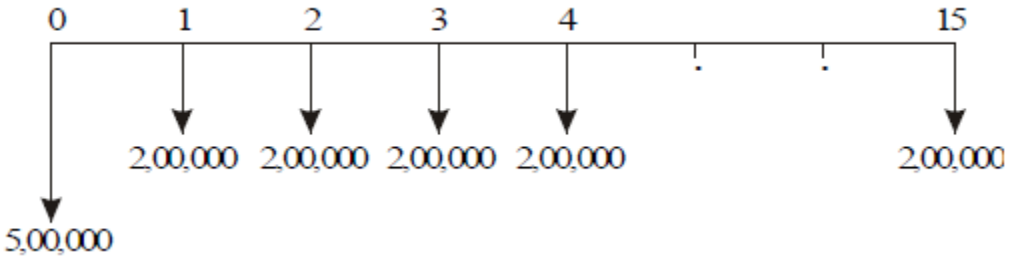


Fig. 6.4 Cash flow diagram for manufacturer 1.
The annual equivalent cost expression of the above cash flow diagram is
$AE1(20\%) = 5,00,000(A/P, 20\%, 15) + 2,00,000$
= 5,00,000(0.2139) + 2,00,000
= 3,06,950

Alternative 2
Down payment, P = Rs. 4,00,000

Yearly equal installment, A = Rs. 3,00,000
n = 15 years
i = 20%, compounded annually
The cash flow diagram for the manufacturer 2 is shown in Fig. 6.5.

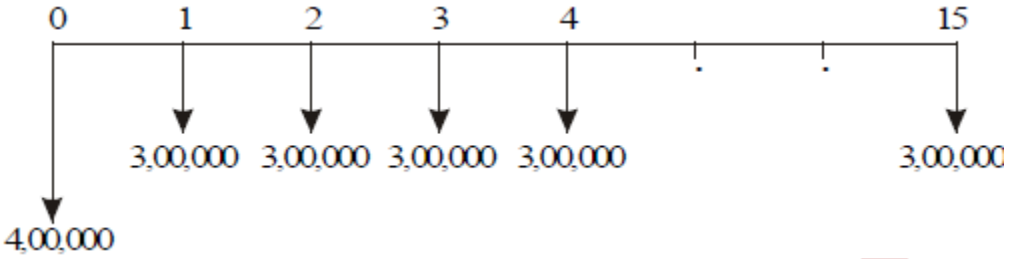


Fig. 6.5 Cash flow diagram for manufacturer 2.

The annual equivalent cost expression of the above cash flow diagram is
$AE2(20\%) = 4,00,000(A/P, 20\%, 15) + 3,00,000$
$= 4,00,000(0.2139) + 3,00,000$
= Rs. 3,85,560.

Alternative 3
Down payment, P = Rs. 6,00,000
Yearly equal installment, A = Rs. 1,50,000
n = 15 years
i = 20%, compounded annually
The cash flow diagram for manufacturer 3 is shown in Fig. 6.6.

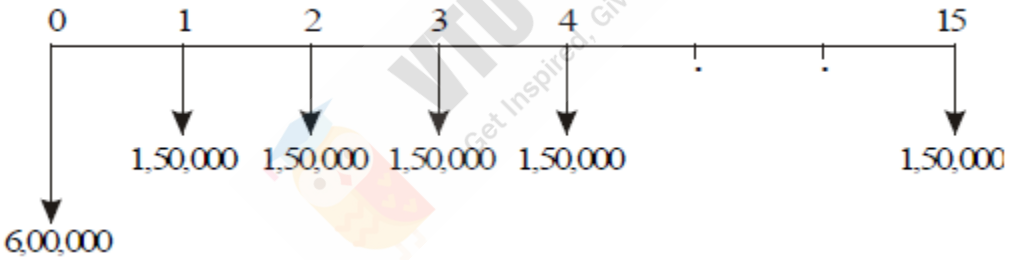


Fig. 6.6 Cash flow diagram for manufacturer 3.

The annual equivalent cost expression of the above cash flow diagram is
$AE3(20\%) = 6,00,000(A/P, 20\%, 15) + 1,50,000$
$= 6,00,000(0.2139) + 1,50,000$
= Rs. 2,78,340.

The annual equivalent cost of manufacturer 3 is less than that of manufacturer 1 and manufacturer 2. Therefore, the company should buy the advanced machine centre from manufacturer 3.

EXAMPLE 6.3 A company invests in one of the two mutually exclusive alternatives. The life of both alternatives is estimated to be 5 years with the following investments, annual returns and salvage values.

	Alternative	
	A	B
Investment (Rs.)	– 1,50,000	– 1,75,000
Annual equal return (Rs.)	+ 60,000	+ 70,000
Salvage value (Rs.)	+ 15,000	+ 35,000

Determine the best alternative based on the annual equivalent method by assuming i = 25%
.

Solution Alternative A

Initial investment, P = Rs. 1,50,000
Annual equal return, A = Rs. 60,000
Salvage value at the end of machine life, S = Rs. 15,000
Life = 5 years
Interest rate, i = 25%, compounded annually
The cash flow diagram for alternative A is shown in Fig. 6.7.

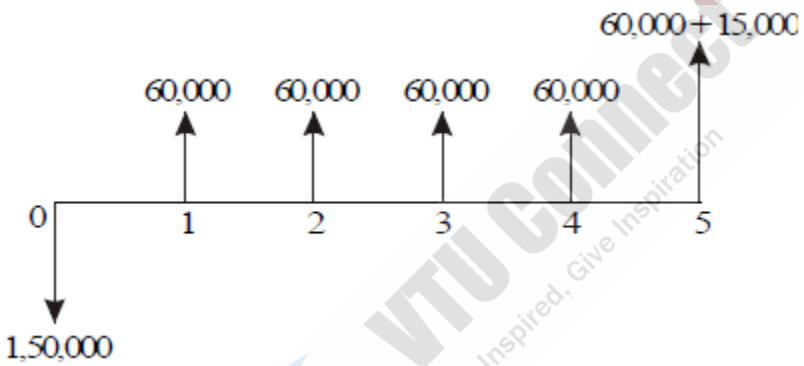


Fig. 6.7 Cash flow diagram for alternative A.

The annual equivalent revenue expression of the above cash flow diagram is as follows:
AEA(25%) = –1,50,000(A/P, 25%, 5) + 60,000 + 15,000(A/F, 25%, 5)
= –1,50,000(0.3718) + 60,000 + 15,000(0.1218)
= Rs. 6,057

Alternative B

Initial investment, P = Rs. 1,75,000
Annual equal return, A = Rs. 70,000
Salvage value at the end of machine life, S = Rs. 35,000
Life = 5 years
Interest rate, i = 25%, compounded annually
The cash flow diagram for alternative B is shown in Fig. 6.8.

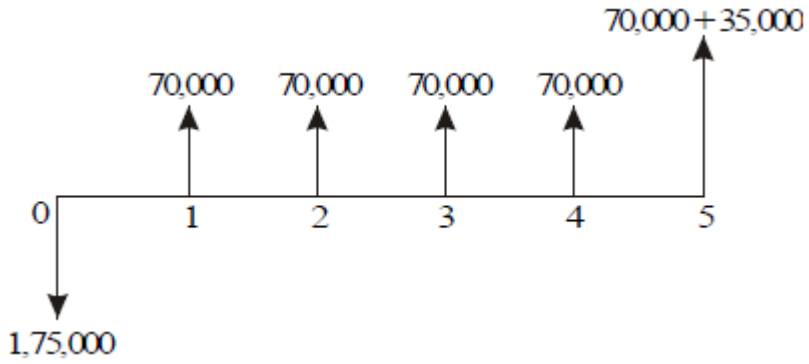


Fig. 6.8 Cash flow diagram for alternative B.

The annual equivalent revenue expression of the above cash flow diagram is
$AE\text{B}(25\%) = -1,75,000(A/P, 25\%, 5) + 70,000 + 35,000(A/F, 25\%, 5)$
$= -1,75,000(0.3718) + 70,000 + 35,000(0.1218)$
= Rs. 9,198

The annual equivalent net return of alternative B is more than that of alternative A. Thus, the company should select alternative B.

EXAMPLE 6.4 A certain individual firm desires an economic analysis to determine which of the two machines is attractive in a given interval of time. The minimum attractive rate of return for the firm is 15%. The following data are to be used in the analysis:

	Machine X	*Machine Y*
First cost	Rs. 1,50,000	Rs. 2,40,000
Estimated life	12 years	12 years
Salvage value	Rs. 0	Rs. 6,000
Annual maintenance cost	Rs. 0	Rs. 4,500

Which machine would you choose? Base your answer on annual equivalent cost.

Solution Machine X
First cost, P = Rs. 1,50,000
Life, n = 12 years
Estimated salvage value at the end of machine life, S = Rs. 0.
Annual maintenance cost, A = Rs. 0.
Interest rate, i = 15%, compounded annually.
The cash flow diagram of machine X is illustrated in Fig. 6.9.

Fig. 6.9 Cash flow diagram for machine X.

The annual equivalent cost expression of the above cash flow diagram is

$AEX(15\%) = 1,50,000(A/P, 15\%, 12)$

$= 1,50,000(0.1845)$

= Rs. 27,675

Machine Y

First cost, P = Rs. 2,40,000

Life, n = 12 years

Estimated salvage value at the end of machine life, S = Rs. 60,000

Annual maintenance cost, A = Rs. 4,500

Interest rate, i = 15%, compounded annually.

The cash flow diagram of machine Y is depicted in Fig. 6.10.

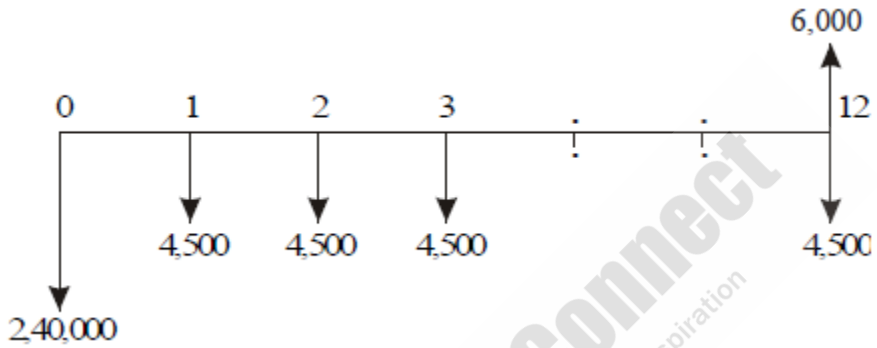


Fig. 6.10 Cash flow diagram for machine Y.

The annual equivalent cost expression of the above cash flow diagram is

$AEY(15\%) = 2,40,000(A/P, 15\%, 12) + 4,500 – 6,000(A/F, 15\%, 12)$

$= 2,40,000(0.1845) + 4,500 – 6,000(0.0345)$

= Rs. 48,573

The annual equivalent cost of machine X is less than that of machine Y. So, machine X is the more cost effective machine.

EXAMPLE 6.5 Two possible routes for laying a power line are under study. Data on the routes are as follows:

		Around the lake	*Under the lake*
Length		15 km	5 km
First cost	(Rs.)	1,50,000/km	7,50,000/km
Useful life	(years)	15	15
Maintenance cost	(Rs.)	6,000/km/yr	12,000/km/yr
Salvage value	(Rs.)	90,000/km	1,50,000/km
Yearly power loss	(Rs.)	15,000/km	15,000/km

If 15% interest is used, should the power line be routed around the lake or under the lake?

Solution Alternative 1— Around the lake

First cost = 1,50,000 _ 15 = Rs. 22,50,000

Maintenance cost/yr = 6,000 _ 15 = Rs. 90,000

Power loss/yr = 15,000 _ 15 = Rs. 2,25,000

Maintenance cost and power loss/yr = Rs. 90,000 + Rs. 2,25,000

= Rs. 3,15,000

Salvage value = 90,000 _ 15 = Rs. 13,50,000

The cash flow diagram for this alternative is shown in Fig. 6.11.

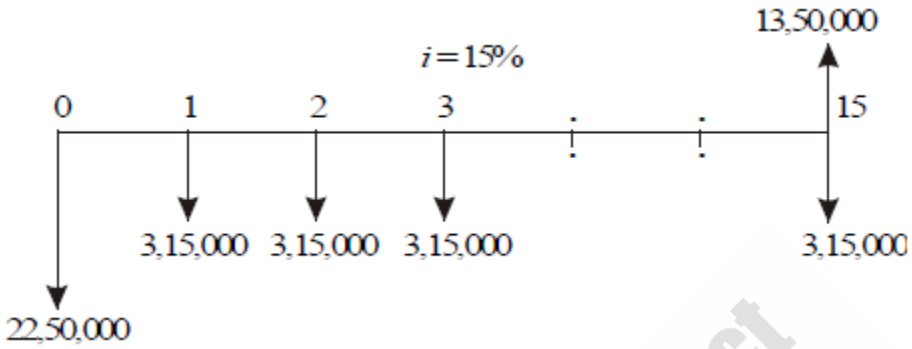


Fig. 6.11 Cash flow diagram for alternative 1.

The annual equivalent cost expression of the above cash flow diagram is

$AE1(15\%) = 22,50,000(A/P, 15\%, 15) + 3,15,000 – 13,50,000(A/F, 15\%, 15)$

= 22,50,000(0.1710) + 3,15,000 – 13,50,000(0.0210)

= Rs. 6,71,400

Alternative 2—Under the lake

First cost = 7,50,000×5 = Rs. 37,50,000

Maintenance cost/yr = 12,000 × 5 = Rs. 60,000

Power loss/yr = 15,000 ×5 = Rs. 75,000

Maintenance cost and power loss/yr = Rs. 60,000 + Rs. 75,000

= Rs. 1,35,000

Salvage value = 1,50,000 ×5 = Rs. 7,50,000

The cash flow diagram for this alternative is shown in Fig. 6.12.

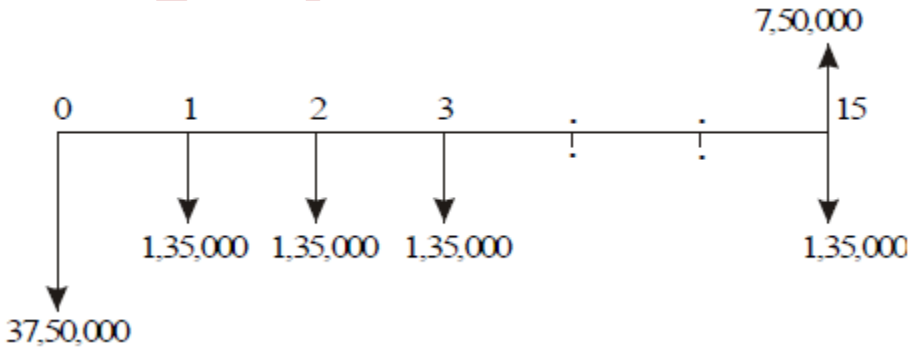


Fig. 6.12 Cash flow diagram for alternative 2.

The annual equivalent cost expression of the above cash flow diagram is

$AE2(15\%) = 37,50,000(A/P, 15\%, 15) + 1,35,000 – 7,50,000(A/F, 15\%, 15)$

= 37,50,000(0.1710) + 1,35,000 – 7,50,000(0.0210)
= Rs. 7,60,500

The annual equivalent cost of alternative 1 is less than that of alternative 2. Therefore, select the route around the lake for laying the power line.

EXAMPLE 6.6 A suburban taxi company is analyzing the proposal of buying cars with diesel engines instead of petrol engines. The cars average 60,000 km a year with a useful life of three years for the petrol taxi and four years for the diesel taxi. Other comparative details are as follows:

	Diesel	*Petrol*
Vehicle cost (Rs.)	3,90,000	3,60,000
Fuel cost per litre (Rs.)	8	20
Mileage in km/litre	30	20
Annual repairs (Rs.)	9,000	6,000
Annual insurance premium (Rs.)	15,000	15,000
Resale value at the end of vehicle life (Rs.)	60,000	90,000

Determine the more economical choice if interest rate is 20%, compounded annually.

Solution Alternative 1— Purchase of diesel taxi
Vehicle cost = Rs. 3,90,000
Life = 4 years
Number of litres/year 60,000/30 = 2,000 litres
Fuel cost/yr = 2,000 _ 8 = Rs. 16,000
Fuel cost, annual repairs and insurance premium/yr
= Rs. 16,000 + Rs. 9,000 + Rs. 15,000 = Rs. 40,000
Salvage value at the end of vehicle life = Rs. 60,000
The cash flow diagram for alternative 1 is shown in Fig. 6.13.

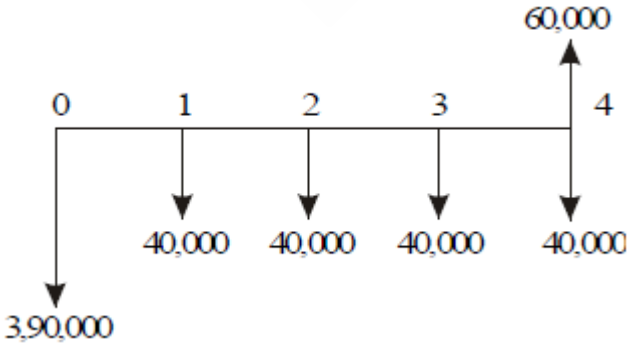


Fig. 6.13 Cash flow diagram for alternative 1.

The annual equivalent cost expression of the above cash flow diagram is
AE(20%) = 3,90,000(*A*/*P*, 20%, 4) + 40,000 – 60,000(*A*/*F*, 20%, 4)
= 3,90,000(0.3863) + 40,000 – 60,000(0.1863)
= Rs. 1,79,479

Alternative 2— Purchase of petrol taxi

Vehicle cost = Rs. 3,60,000
Life = 3 years
Number of litres/year 60,000/20 = 3,000 litres
Fuel cost/yr = 3,000 ×20 = Rs. 60,000
Fuel cost, annual repairs and insurance premium/yr
= Rs. 60,000 + Rs. 6,000 + Rs. 15,000 = Rs. 81,000
Salvage value at the end of vehicle life = Rs. 90,000
The cash flow diagram for alternative 2 is shown in Fig. 6.14.

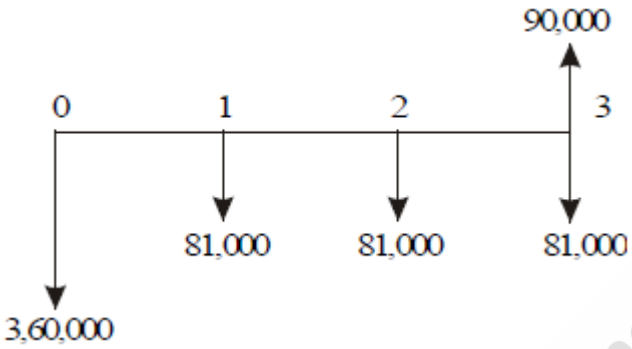


Fig. 6.14 Cash flow diagram for alternative 2.

The annual equivalent cost expression of the above cash flow diagram is
$AE(20\%) = 3{,}60{,}000(A/P, 20\%, 3) + 81{,}000 - 90{,}000(A/F, 20\%, 3)$
$= 3{,}60{,}000(0.4747) + 81{,}000 - 90{,}000(0.2747)$
= Rs. 2,27,169

The annual equivalent cost of purchase and operation of the cars with diesel engine is less than that of the cars with petrol engine. Therefore, the taxi company should buy cars with diesel engine. (*Note:* Comparison is done on common multiple lives of 12 years.)

EXAMPLE 6.7 Ramu, a salesman, needs a new car for use in his business. He expects that he will be promoted to a supervisory job at the end of third year and so his concern now is to have a car for the three years he expects to be "on the road". The company will reimburse their salesman each month the fuel cost and maintenance cost. Ramu has decided to drive a low-priced automobile. He finds, however, that there are two different ways of obtaining the automobile. In either case, the fuel cost and maintenance cost are borne by the company.
(a) Purchase for cash at Rs. 3,90,000.
(b) Lease a car. The monthly charge is Rs. 10,500 on a 36-month lease
payable at the end of each month. At the end of the three-year period, the car is returned to the leasing company. Ramu believes that he should use a 12% interest rate compounded monthly in determining which alternative to select. If the car could be sold for Rs. 1,20,000 at the end of the third year, which option should he use to obtain it?

Alternative 1—Purchase car for cash

Purchase price of the car = Rs. 3,90,000

Life = 3 years = 36 months
Salvage value after 3 years = Rs. 1,20,000
Interest rate = 12% (nominal rate, compounded annually)
= 1% compounded monthly
The cash flow diagram for alternative 1 is shown in Fig. 6.15.

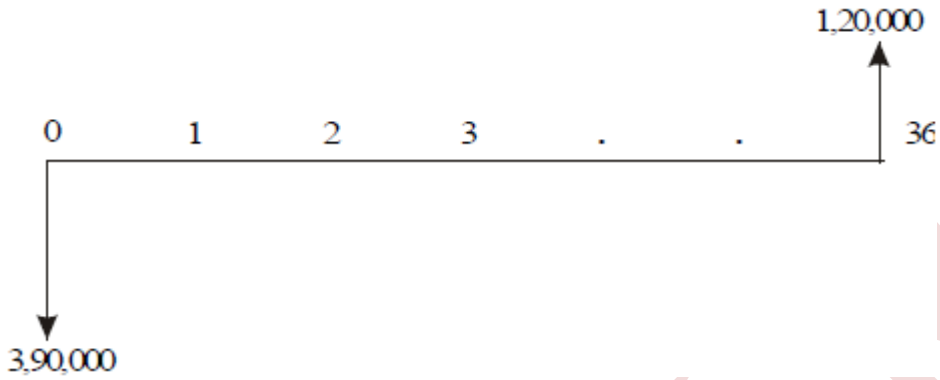


Fig. 6.15 Cash flow diagram for alternative 1.

The monthly equivalent cost expression [*ME*(1%)] of the above cash flow diagram is
ME(1%) = 3,90,000(*A*/*P*, 1%, 36) – 1,20,000(*A*/*F*, 1%, 36)
= 3,90,000(0.0332) – 1,20,000(0.0232)
= Rs. 10,164

Alternative 2—Use of car under lease
Monthly lease amount for 36 months = Rs. 10,500
The cash flow diagram for alternative 2 is illustrated in Fig. 6.16.

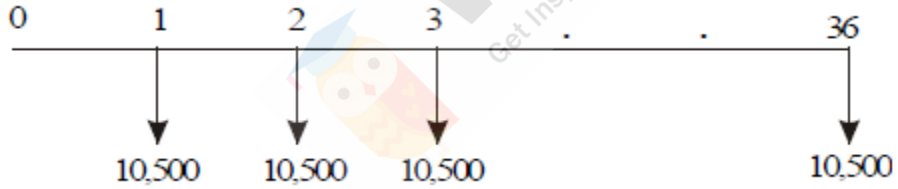


Fig. 6.16 Cash flow diagram for alternative 2.

Monthly equivalent cost = Rs.10,500.

The monthly equivalent cost of alternative 1 is less than that of alternative 2. Hence, the salesman should purchase the car for cash.

EXAMPLE 6.8 A company must decide whether to buy machine A or machine B

	Machine A	*Machine B*
Initial cost (Rs.)	3,00,000	6,00,000
Useful life (years)	4	4
Salvage value at the end of machine life (Rs.)	2,00,000	3,00,000
Annual maintenance (Rs.)	30,000	0

At 15% interest rate, which machine should be purchased?

Solution Machine A

Initial cost = Rs. 3,00,000
Useful life (years) = 4
Salvage value at the end of machine life = Rs. 2,00,000
Annual maintenance = Rs. 30,000
Interest rate = 15%, compounded annually
The cash flow diagram of machine A is depicted in Fig. 6.17.

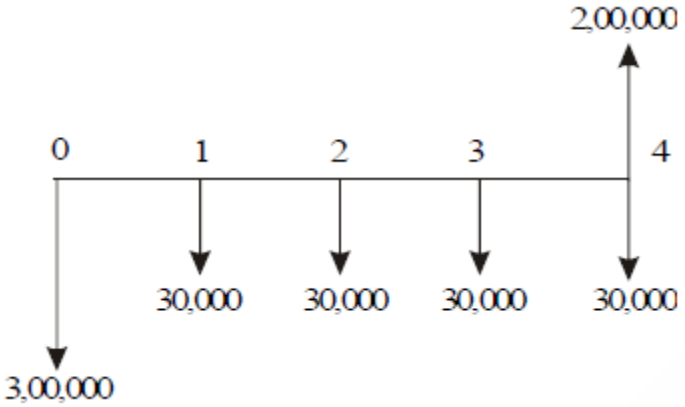


Fig. 6.17 Cash flow diagram for machine A.

The annual equivalent cost expression of the above cash flow diagram is
$AE(15\%) = 3,00,000(A/P, 15\%, 4) + 30,000 – 2,00,000(A/F, 15\%, 4)$
$= 3,00,000(0.3503) + 30,000 – 2,00,000(0.2003)$
= Rs. 95,030

Machine B

Initial cost = Rs. 6,00,000
Useful life (years) = 4
Salvage value at the end of machine life = Rs. 3,00,000
Annual maintenance = Rs. 0.
Interest rate = 15%, compounded annually
The cash flow diagram of machine B is illustrated in Fig. 6.18.

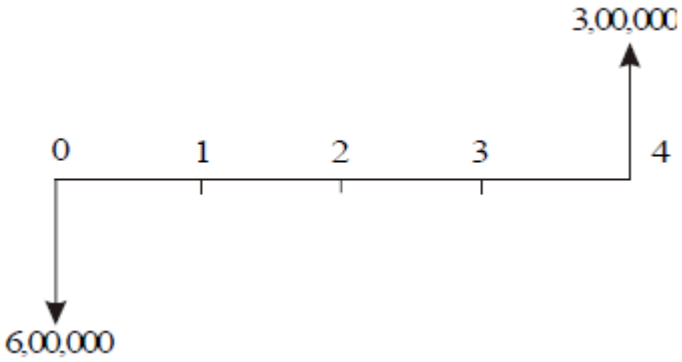


Fig. 6.18 Cash flow diagram for machine B.

The annual equivalent cost expression of the above cash flow diagram is
$AE(15\%) = 6,00,000(A/P, 15\%, 4) – 3,00,000(A/F, 15\%, 4)$

= 6,00,000(0.3503) – 3,00,000(0.2003)
= Rs. 1,50,090

Since the annual equivalent cost of machine A is less than that of machine B, it is advisable to buy machine A.

EXAMPLE 6.9 Jothi Lakshimi has arranged to buy some home recording equipment. She estimates that it will have a five year useful life and no salvage value at the end of equipment life. The dealer, who is a friend has offered Jothi Lakshimi two alternative ways to pay for the equipment.
(a) Pay Rs. 60,000 immediately and Rs. 15,000 at the end of one year.
(b) Pay nothing until the end of fourth year when a single payment of Rs. 90,000 must be made. If Jothi Lakshimi believes 12% is a suitable interest rate, which alternative is the best for her?

Solution Alternative 1
Down payment = Rs. 60,000
Payment after one year = Rs. 15,000
The cash flow diagram for alternative 1 is shown in Fig. 6.19.

Fig. 6.19 Cash flow diagram for alternative 1.

The present worth equation of the above cash flow diagram is
$PW(12\%) = 60,000 + 15,000(P/F, 12\%, 1)$
= 60,000 + 15,000(0.8929)
= 73,393.50
The above present worth is represented in Fig. 6.20.

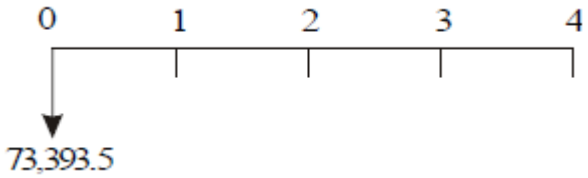


Fig. 6.20 Resultant cash flow diagram.

The annual equivalent expression of the above cash flow diagram is
$AE(12\%) = 73,393.5(A/P, 12\%, 4)$
= 73,393.5(0.3292)
= Rs. 24,161.14

Alternative 2
Payment after four years = Rs. 90,000
The cash flow diagram for alternative 2 is shown in Fig. 6.21.

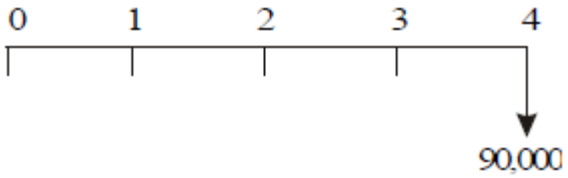


Fig. 6.21 Cash flow diagram of alternative 2.

The annual equivalent cost expression of the above cash flow diagram is

$AE(12\%) = 90{,}000(A/F, 12\%, 4)$

$= 90{,}000(0.2092)$

= Rs. 18,828

The annual equivalent cost of alternative 2 is less than that of alternative 1. Hence, Jothi Lakshimi should select alternative 2 for purchasing the home equipment.

C-7 RATE OF RETURN METHOD

7.1 INTRODUCTION

The rate of return of a cash flow pattern is the interest rate at which the present worth of that cash flow pattern reduces to zero. In this method of comparison, the rate of return for each alternative is computed. Then the alternative which has the highest rate of return is selected as the best alternative. In this type of analysis, the expenditures are always assigned with a negative sign and the revenues/inflows are assigned with a positive sign. A generalized cash flow diagram to demonstrate the rate of return method of comparison is presented in Fig. 7.1.

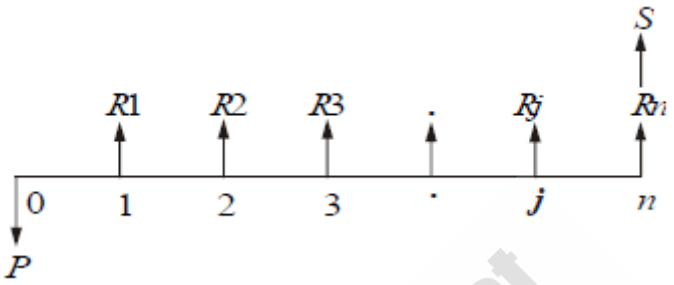


Fig. 7.1 Generalized cash flow diagram.

In the above cash flow diagram, P represents an initial investment, Rj the net revenue at the end of the jth year, and S the salvage value at the end of the nth year. The first step is to find the net present worth of the cash flow diagram using the following expression at a given interest rate, i.

$PW(i) = -P + R1/(1 + i)1 + R2/(1 + i)2 + ... + Rj/(1 + i)\,j + ... + Rn/(1 + i)n + S/(1 + i)n$

Now, the above function is to be evaluated for different values of i until the present worth function reduces to zero, as shown in Fig. 7.2.

In the figure, the present worth goes on decreasing when the interest rate is increased. The value of i at which the present worth curve cuts the X-axis is the rate of return of the given proposal/project. It will be very difficult to find the exact value of i at which the present worth function reduces to zero.

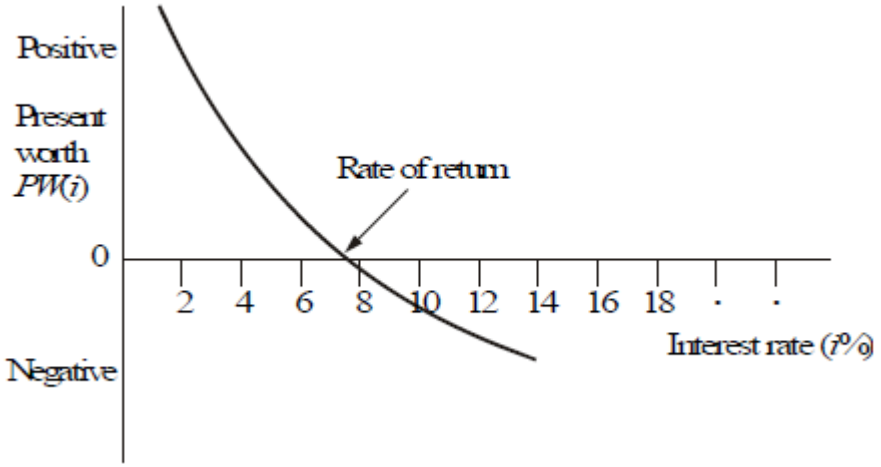


Fig. 7.2 Present worth function graph.

So, one has to start with an intuitive value of *i* and check whether the present worth function is positive. If so, increase the value of *i* until *PW*(*i*) becomes negative. Then, the rate of return is determined by interpolation method in the range of values of *i* for which the sign of the present worth function changes from positive to negative.

7.2 EXAMPLES

In this section, the concept of rate of return calculation is demonstrated with suitable examples.

EXAMPLE 7.1 A person is planning a new business. The initial outlay and cash flow pattern for the new business are as listed below. The expected life of the business is five years. Find the rate of return for the new business.

Period	0	1	2	3	4	5
Cash flow (Rs.)	–1,00,000	30,000	30,000	30,000	30,000	30,000

Solution

Initial investment = Rs. 1,00,000
Annual equal revenue = Rs. 30,000
Life = 5 years
The cash flow diagram for this situation is illustrated in Fig. 7.3.

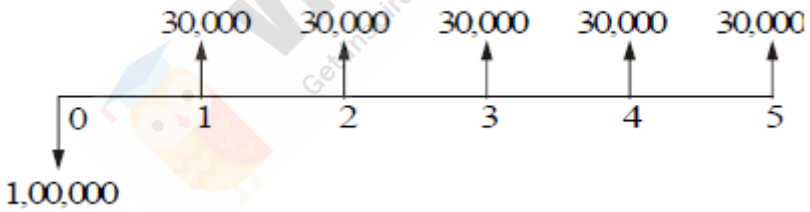


Fig. 7.3 Cash flow diagram.

The present worth function for the business is
PW(*i*) = –1,00,000 + 30,000(*P*/*A*, *i*, 5)
When *i* = 10%,
PW(10%) = –1,00,000 + 30,000(*P*/*A*, 10%, 5)
= –1,00,000 + 30,000(3.7908) = Rs. 13,724.
When *i* = 15%,
PW(15%) = –1,00,000 + 30,000(*P*/*A*, 15%, 5)
= –1,00,000 + 30,000(3.3522) = Rs. 566.
When *i* = 18%,
PW(18%) = –1,00,000 + 30,000(*P*/*A*, 18%, 5)
= –1,00,000 + 30,000(3.1272) = Rs. – 6,184

$$i = 15\% + \frac{566 - 0}{566 - (-6184)} \times (3\%)$$

$$= 15\% + 0.252\%$$

$$= 15.252\%$$

Therefore, the rate of return for the new business is 15.252%.

EXAMPLE 7.2 A company is trying to diversify its business in a new product line. The life of the project is 10 years with no salvage value at the end of its life. The initial outlay of the project is Rs. 20,00,000. The annual net profit is Rs. 3,50,000. Find the rate of return for the new business.

Solution

Life of the product line (n) = 10 years
Initial outlay = Rs. 20,00,000
Annual net profit = Rs. 3,50,000
Scrap value after 10 years = 0
The cash flow diagram for this situation is shown in Fig. 7.4.

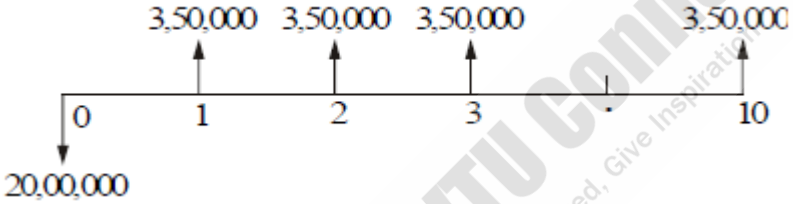


Fig. 7.4 Cash flow diagram.

The formula for the net present worth function of the situation is
$PW(i) = -20,00,000 + 3,50,000(P/A, i, 10)$
When $i = 10\%$,
$PW(10\%) = -20,00,000 + 3,50,000(P/A, 10\%, 10)$
$= -20,00,000 + 3,50,000(6.1446)$
= Rs. 1,50,610.
When $i = 12\%$,
$PW(12\%) = -20,00,000 + 3,50,000(P/A, 12\%, 10)$
$= -20,00,000 + 3,50,000(5.6502)$
= Rs. –22,430.

$$i = 10\% + \frac{1,50,610 - 0}{1,50,610 - (-22,430)} \times (2\%)$$

= 11.74 %
Therefore, the rate of return of the new product line is 11.74%

EXAMPLE 7.3 A firm has identified three mutually exclusive investment proposals whose details are given below. The life of all the three alternatives is estimated to be five years with negligible salvage value. The minimum attractive rate of return for the firm is 12%.

	Alternative		
	*A*1	*A*2	*A*3
Investment	Rs. 1,50,000	Rs. 2,10,000	Rs. 2,55,000
Annual net income	Rs. 45,570	Rs. 58,260	Rs. 69,000

Find the best alternative based on the rate of return method of comparison.

Solution Calculation of rate of return for alternative *A*1
Initial outlay = Rs. 1,50,000
Annual profit = Rs. 45,570
Life = 5 years
The cash flow diagram for alternative *A*1 is shown in Fig. 7.5.

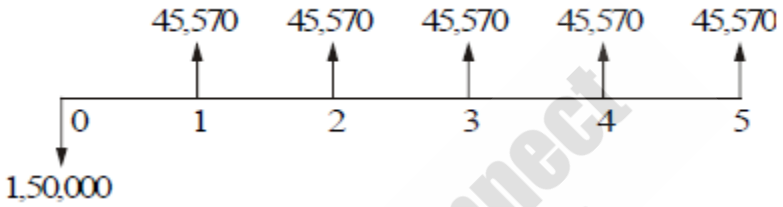


Fig. 7.5 Cash flow diagram for alternative *A*1.

The formula for the net present worth of alternative *A*1 is given as
$PW(i) = -1,50,000 + 45,570(P/A, i, 5)$
When $i = 10\%$,
$PW(10\%) = -1,50,000 + 45,570(P/A, 10\%, 5)$
$= -1,50,000 + 45,570(3.7908)$
= Rs. 22,746.76
When $i = 12\%$,
$PW(12\%) = -1,50,000 + 45,570(P/A, 12\%, 5)$
$= -1,50,000 + 45,570(3.6048)$
= Rs. 14,270.74
When $i = 15\%$,
$PW(15\%) = -1,50,000 + 45,570(P/A, 15\%, 5)$
$= -1,50,000 + 45,570(3.3522)$
= Rs. 2,759.75
When $i = 18\%$,
$PW(18\%) = -1,50,000 + 45,570(P/A, 18\%, 5)$
$= -1,50,000 + 45,570(3.1272)$
= Rs. –7,493.50
Therefore, the rate of return of the alternative *A*1 is

$$i = 15\% + \frac{2,759.75 - 0}{2,759.75 - (-7,493.50)} \times (3\%)$$

= 15% + 0.81%

= 15.81%

Calculation of rate of return for alternative *A*2
Initial outlay = Rs. 2,10,000
Annual profit = Rs. 58,260
Life of alternative *A*2 = 5 years
The cash flow diagram for alternative *A*2 is shown in Fig. 7.6.

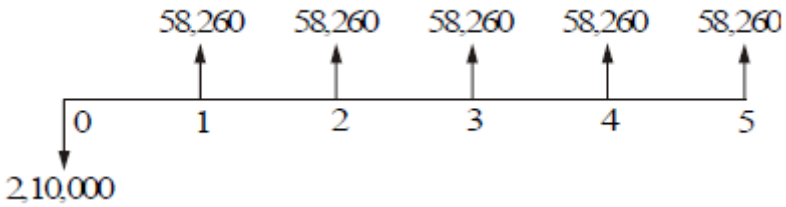


Fig. 7.6 Cash flow diagram for alternative *A*2.

The formula for the net present worth of this alternative is
$PW(i) = -2,10,000 + 58,260\ (P/A, i, 5)$
When $i = 12\%$,
$PW(12\%) = -2,10,000 + 58,260(P/A, 12\%, 5)$
$= -2,10,000 + 58,260(3.6048)$
= Rs. 15.65
When $i = 13\%$,
$PW(13\%) = -2,10,000 + 58,260(P/A, 13\%, 5)$
$= -2,10,000 + 58,260\ (3.5172)$
= Rs. –5,087.93
Therefore, the rate of return of alternative *A*2 is

$$i = 12\% + \frac{15.65 - 0}{15.65 - (-5,087.93)} \times (1\%)$$

= 12 % + 0%
= 12%

Calculation of rate of return for alternative *A*3
Initial outlay = Rs. 2,55,000
Annual profit = Rs. 69,000
Life of alternative *A*3 = 5 years
The cash flow diagram for alternative *A*3 is depicted in Fig. 7.7.

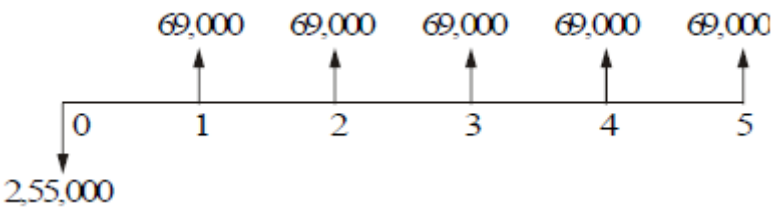


Fig. 7.7 Cash flow diagram for alternative *A*3.

The formula for the net present worth of this alternative *A*3 is
$PW(i) = -2,55,000 + 69,000(P/A, i, 5)$

When i = 11%,
$PW(11\%) = -2{,}55{,}000 + 69{,}000(P/A, 11\%, 5)$
$= -2{,}55{,}000 + 69{,}000\ (3.6959) = \text{Rs. } 17.1$
When i = 12%,
$PW(12\%) = -2{,}55{,}000 + 69{,}000(P/A, 12\%, 5)$
$= -2{,}55{,}000 + 69{,}000\ (3.6048) = \text{Rs. } -6{,}268.80$
Therefore, the rate of return for alternative $A3$ is

$$i = 11\% + \frac{17.1 - 0}{17.1 - (-6{,}268.80)} \times 1\%$$

= 11%
The rates of return for the three alternatives are now tabulated.

Alternative	*A1*	*A2*	*A3*
Rate of return	15.81%	12%	11%

From the above data, it is clear that the rate of return for alternative $A3$ is less than the minimum attractive rate of return of 12%. So, it should not be considered for comparison. The remaining two alternatives are qualified for consideration. Among the alternatives $A1$ and $A2$, the rate of return of alternative $A1$ is greater than that of alternative $A2$. Hence, alternative $A1$ should be selected.

EXAMPLE 7.4 For the cash flow diagram shown in Fig. 7.8, compute the rate of return. The amounts are in rupees.

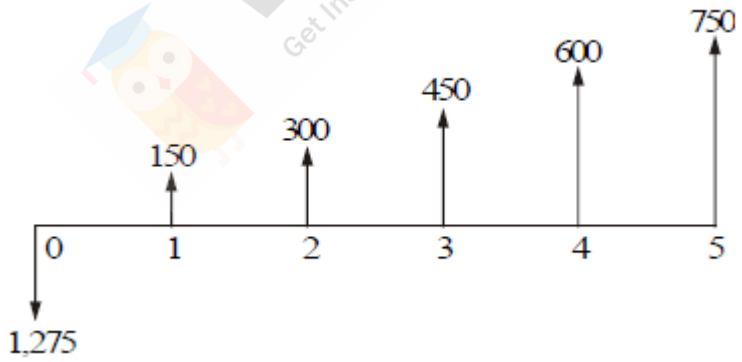


Fig. 7.8 Cash flow diagram.

Solution For the positive cash flows of the problem,
$A1$ = Rs. 150, G = Rs. 150
The annual equivalent of the positive cash flows of the uniform gradient series is given by
$A = A1 + G(A/G, i, n)$
$= 150 + 150(A/G, i, 5)$
The formula for the present worth of the whole diagram
$= -1{,}275 + [150 + 150(A/G, i, 5)] \times (P/A, i, 5)$

$PW(10\%) = -1{,}275 + [150 + 150(A/G, 10\%, 5)] \times (P/A, 10\%, 5)$
$= -1{,}275 + [150 + 150(1.8101)] \times (3.7908)$
= Rs. 322.88
$PW(12\%) = -1{,}275 + [150 + 150(A/G, 12\%, 5)] \times (P/A, 12\%, 5)$
$= -1{,}275 + [150 + 150(1.7746)] \times (3.6048)$
= Rs. 225.28
$PW(15\%) = -1{,}275 + [150 + 150(A/G, 15\%, 5)] \times (P/A, 15\%, 5)$
$= -1{,}275 + [150 + 150(1.7228)] \_ (3.3522)$
= Rs. 94.11
$PW(18\%) = -1{,}275 + [150 + 150(A/G, 18\%, 5)] \times (P/A, 18\%, 5)$
$= -1{,}275 + [150 + 150(1.6728)] \times (3.1272)$
= Rs. –21.24
Therefore, the rate of return for the cash flow diagram is

$$i = 15\% + \frac{94.11 - 0}{94.11 - (-21.24)} \times 3\%$$

$$= 15\% + 2.45\% = 17.45\%$$

EXAMPLE 7.5 A company is planning to expand its present business activity. It has two alternatives for the expansion programme and the corresponding cash flows are tabulated below. Each alternative has a life of five years and a negligible salvage value. The minimum attractive rate of return for the company is 12%. Suggest the best alternative to the company.

	Initial investment (Rs.)	*Yearly revenue* (Rs.)
Alternative 1	5,00,000	1,70,000
Alternative 2	8,00,000	2,70,000

Solution Alternative 1
Initial outlay = Rs. 5,00,000
Annual revenue = Rs. 1,70,000
Life of alternative 1 = 5 years
The cash flow diagram for alternative 1 is illustrated in Fig. 7.9.

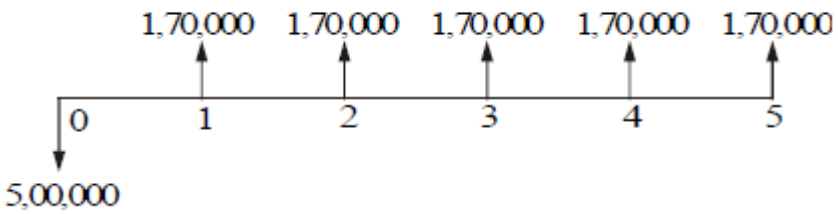


Fig. 7.9 Cash flow diagram for alternative 1.

The formulae for the net present worth of alternative 1 are as follows:
$PW1(i) = -5{,}00{,}000 + 1{,}70{,}000(P/A, i, 5)$
$PW1(15\%) = -5{,}00{,}000 + 1{,}70{,}000(P/A, 15\%, 5)$

= –5,00,000 + 1,70,000(3.3522)
= Rs. 69,874
*PW*1(17%) = –5,00,000 + 1,70,000(*P*/*A*, 17%, 5)
= –5,00,000 + 1,70,000(3.1993)
= Rs. 43,881
*PW*1(20%) = –5,00,000 + 1,70,000(*P*/*A*, 20%, 5)
= –5,00,000 + 1,70,000(2.9906)
= Rs. 8,402
*PW*1(22%) = –5,00,000 + 1,70,000(*P*/*A*, 22%, 5)
= –5,00,000 + 1,70,000(2.8636)
= Rs. –13,188
Therefore, the rate of return of alternative 1 is

$$i = 20\% + \frac{8,402 - 0}{8,402 - (-13,188)} \times 2\%$$

$$= 20.78\%$$

Alternative 2
Initial outlay = Rs. 8,00,000
Annual revenue = Rs. 2,70,000
Life = 5 years
The cash flow diagram for alternative 2 is depicted in Fig. 7.10.

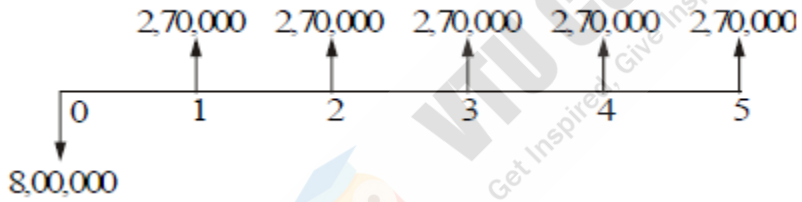


Fig. 7.10 Cash flow diagram for alternative 2.

The formula for the net present worth of alternative 2 is:
*PW*2(*i*) = – 8,00,000 + 2,70,000(*P*/*A*, *i*, 5)
*PW*2(20%) = – 8,00,000 + 2,70,000(*P*/*A*, 20%, 5)
= – 8,00,000 + 2,70,000(2.9906)
= Rs. 7,462
*PW*2(22%) = –8,00,000 + 2,70,000 (*P*/*A*, 22%, 5)
= – 8,00,000 + 2,70,000 (2.8636)
= Rs. –26,828
Thus, the rate of return of alternative 2 is

$$i = 20\% + \frac{7,462 - 0}{7,462 - (-26,828)} \times 2\%$$

$$= 20.435\%$$

Since the rate of return of alternative 1 is greater than that of the alternative 2, select alternative 1.

Module-4

PRESENT WORTH, FUTURE WORTH, ANNUAL EQUIVALENT AND RATE OF RETURNS METHODS

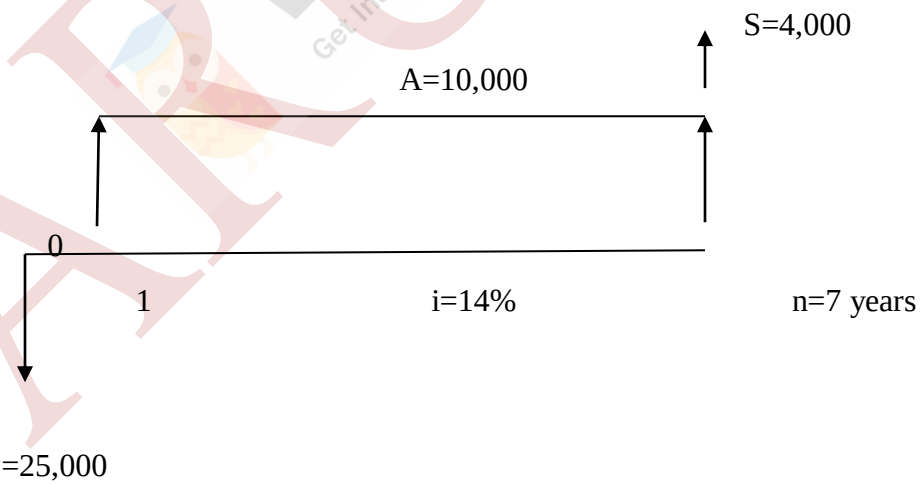
1. An entrepreneur owning a small scale industry wants to buy milling machine. He has 3 machines in view, from different manufacturers. The initial investments, annual revenue, salvage values and the lives of three machines are given in the table. i=14%

	initial investments (Rs.)	annual revenue (Rs)	salvage values	Life (Years)
Machine 1	25,000	10,000	4,000	7
Machine 2	45,000	15,000	6,500	7
Machine 3	70,000	20,000	9,000	7

Solution:

Machine 1

CFD



PW_1=-p+A(P/A,i,n)+S(P/F,I,n)

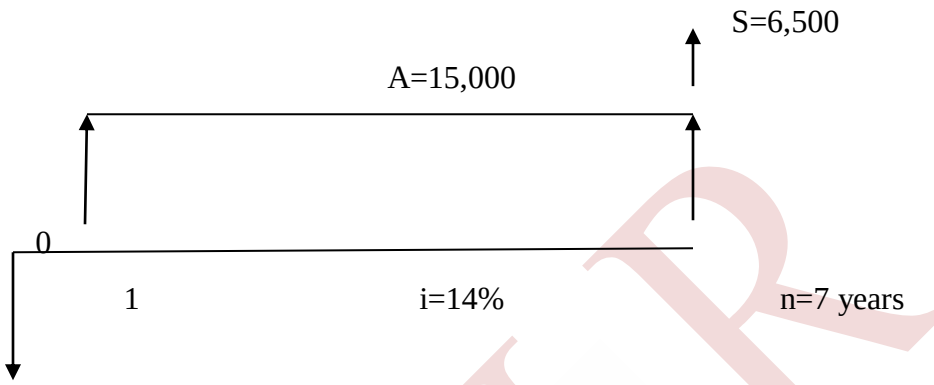
=-25,000+10,000×(P/A, 14%,7)+ 4,000×(P/F, 14%,7)

=-25,000+10,000×(4.2883)+ 4,000×(03996)

=Rs 19481.4

Machine 2

CFD



P=45,000

PW_1=-p+A(P/A,i,n)+S(P/F,I,n)

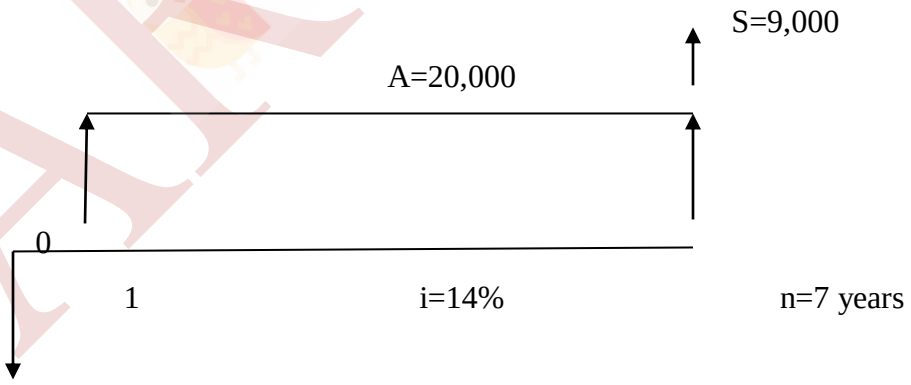
=-45,000+15,000×(P/A, 14%,7)+ 6,500×(P/F, 14%,7)

=-45,000+15,000×(4.2883)+ 6,500×(03996)

=Rs 21921.9

Machine 3

CFD



P=70,000

PW_1=-p+A(P/A,i,n)+S(P/F,I,n)

=-70,000+20,000×(P/A, 14%,7)+ 9,000×(P/F, 14%,7)

=-70,000+20,000×(4.2883)+ 9,000×(03996)

=Rs 19362.4

Suggestion: Revenue dominated system ,so select highest,machine 2 is suggested

2. A Businessman has two investment proposals P and Q in front of him to help him expand his operations. The net cash of the proposals are as follows:

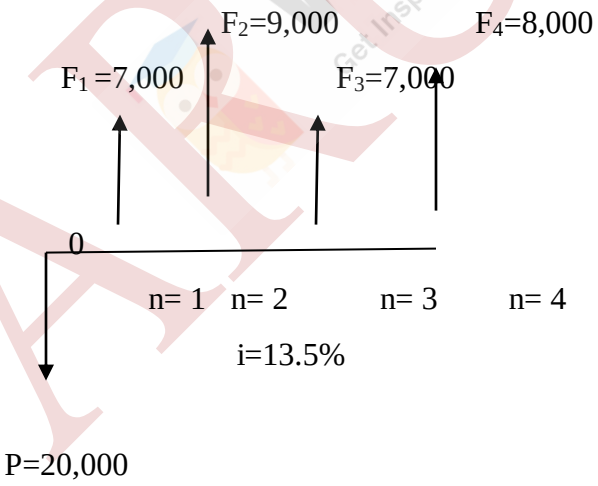
Proposal	End of years				
	0	1	2	3	4
P(Rs)	-20,000	7,000	9,000	7,000	8,000
Q(Rs)	-20,000	10,000	6,000	7,000	6,000

Compare the present worth of proposal P and Q at i=13.5% which proposal should be selected.

Solution:

Proposal P

CFD



$PW(P)=-p+F_1(P/F,i,n)+ F_2(P/F,i,n)+ F_3(P/F,i,n)+ F_4(P/F,i,n)$

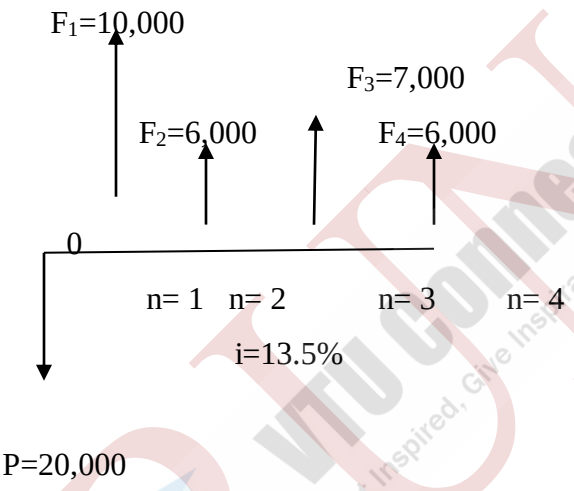
=- 20,000+7,000(P/F, 13.5%,1)+ 9,000(P/F, 13.5%,2)+7,000(P/F, 13.5%,3)+8,000(P/F, 13.5%,4)

=- 20,000+7,000(0.8811)+ 9,000(0.7763)+7,000(0.6839)+8,000(0.6026)

=Rs 2762.5 (-)

Proposal Q

CFD



PW(P)=-p+F_1(P/F,i,n)+ F_2(P/F,i,n)+ F_3(P/F,i,n)+ F_4(P/F,i,n)

=- 20,000+10,000(P/F, 13.5%,1)+ 6,000(P/F, 13.5%,2)+7,000(P/F, 13.5%,3)+6,000(P/F, 13.5%,4)

=- 20,000+10,000(0.8811)+ 6,000(0.7763)+7,000(0.6839)+6,000(0.6026)

=Rs 1871.7 (-)

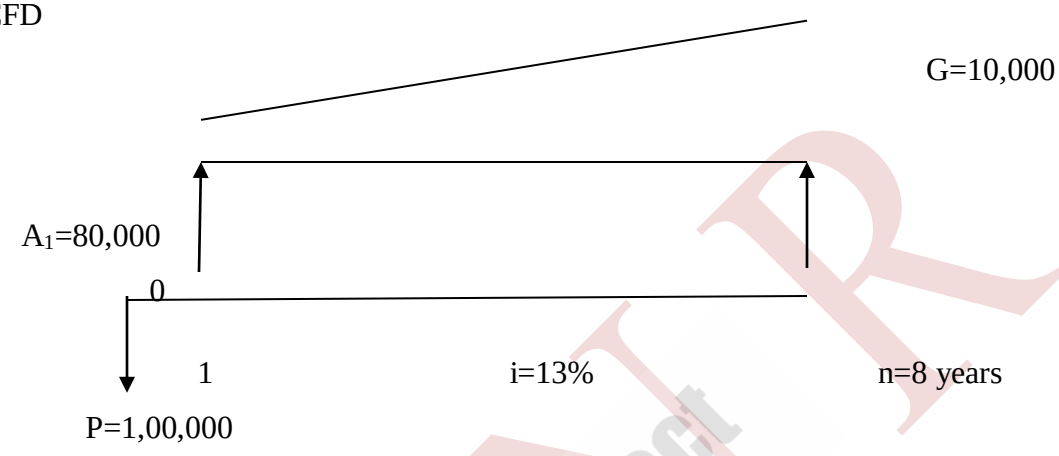
Suggestion: Cost dominated system ,so select smallest negative value, Proposal Q is suggested

3. A Xerox machine in a new locality with an initial outlay of Rs. 1,00,000 yields Rs.80,000 during first year of its operations and the yield increases by Rs.10,000 from its second year up to the 8th year of operation. At the end of the life of business, the machine

becomes scrap and has zero salvage value. Find the present worth of the business assuming a rate of interest of 13%, compounded annually.

S=0

CFD



$PW=-P+(A_1+G(A/G,I,n))\times(P/A,I,n)+S(P/F,I,n)$

$=-1,00,000+(80,000+10,000(A/G,13\%,8))\times(P/A, 13\%,8)$

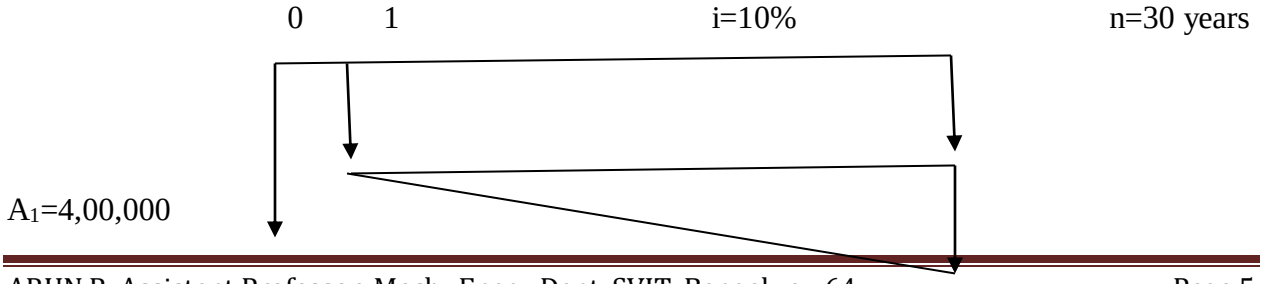
$=-1,00,000+(80,000+10,000(2.8685))\times(4.7988)$

= Rs.`421557.578

4. A small dam and an irrigation system are expected to cost Rs.300,000,00.annual maintenance and operating costs are expected to be Rs.4,00,000 for the first year and will increase at a rate of 10 percent per year. Determine the equivalent present worth of building dam and operating the system with interest of 10 percent over a 30 year life. (VTU July '05)

Solution:

CFD



G=40,000

P=300,000,00

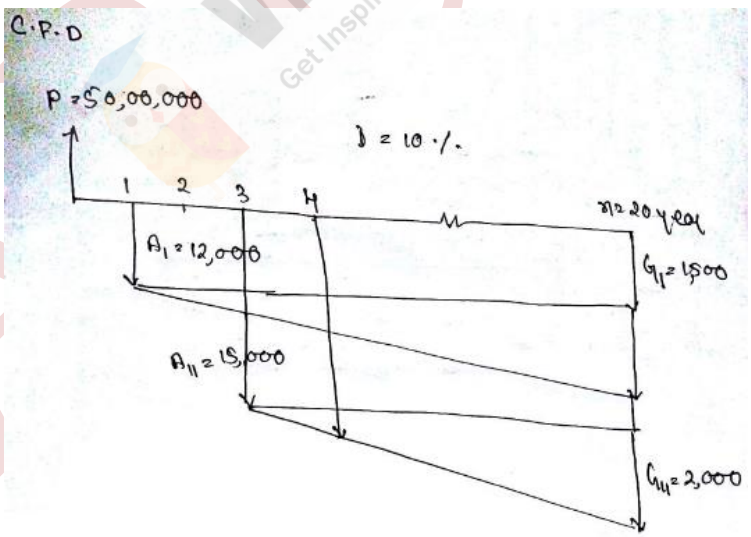
PW= $-P-(A_1+G(A/G,I,n))\times(P/A,I,n)$

=-300,000,00-(4,00,000+40,000 (A/G, 10 %,30)) ×(P/A, 10%,30)

=-300,000,00+(4,00,000+40,000 (8.1762)) ×(9.4269)

=Rs.36853808.76 (-)

5. 5 Million rupees are donated to a college, 20 students are to be awarded scholarships over the next 20 years. The scholarships are each of Rs.12,000. Per year the first year and increase at a rate of Rs.1,500 per year over the following 19 years. Starting with end of the third year, Rs.15,000 are to be spent for maintenance of the college building. The cost rises linearly at the rate of Rs.2,000 per starting with year 4. Assuming 10% interest rate. Determine how much money will be available to construct an auditorium now (VTU july 2006)



$$PW = +P - A\left[P/A, i, n\right]^{20} - A\left[P/A, i, n\right]^{18} \rightarrow F \times \left[P/F, i, n\right]^{3}$$

$$\therefore \quad A = A_1 + G_1\left[A/G, i, n\right]$$

$$= +P - \left[\left[A_1 + G_1\left[A/G, i, n\right]^{20} \times \left[P/A\ i, n\right]^{20}\right] - \left[\left[A_1 + G_1\left[A/G, i, n\right]^{18}\right.\right.$$

$$\times \left[P/A, i, n\right]^{18} \times \left[P/F, i, n\right]^{3}$$

$$= 50,00,000 - \left[\left[12,000 + 1500\left[A/G\ 10\%, 20\right] \times \left[P/A\ 10\%, 20\right]\right]\right.$$

$$- \left[\left[15,000 + 2,000\left[A/G\ 10\%, 18\right] \times \left[P/A\ 10\%, 18\right]\right] \times \left[P/F\ 10\%, 3\right]\right]$$

$$= 50,00,000 - \left[\left[12,000 + 1500\left[6.5081\right]\right] \times \left[8.5136\right]\right]$$

$$- \left[\left[15,000 + 2,000\left[6.0526\right]\right] \times \left[8.2014\right]\right] \times \left[0.7513\right]$$

$$PW = 46,47,712$$

OR

$$PW = P\left[P/P, i, n\right] - A\left[P/A, i, n\right]^{20} - A\left[P/A, i, n\right]^{18}$$

$$= 50,00,000\left[P/P\ 10\%, 20\right] - \left[\left[12,000 + 1500 \times \left[A/G, 10\%, 20\right]\right]\right.$$

$$\times \left[P/A\ 10\%, 20\right]$$

$$- \left[\left[15,000 + 2,000 \times \left[A/G, i, 18\right]\right] \times \left[P/A\ 10\%, 18\right]\right.$$

$= 50,00,000\,[6.7275] - [[12,000 + 1500 \times [6.5081]] \times [57.2750]$

$- [[15,000 + 2,000 \times [6.0526]] \times [45.5992]]$

$33637500 - 1246427.14 - 1235978.436$

$PW = 31155097.42$

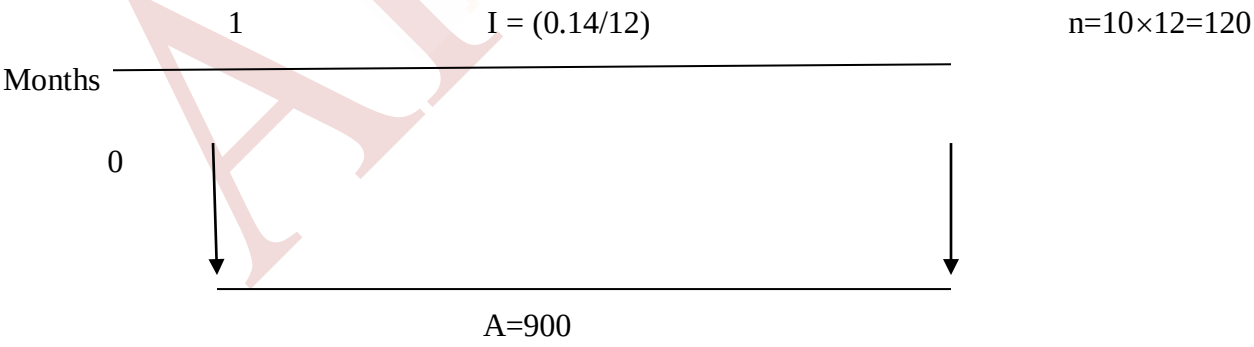
$P = P[P/F, i, n]$

$= 31155097.42\,[P/F\ 10\%\ 20] = 4629647.477$

6. A person wanted to buy a motorbike to his son. Upon down payment the motorbike would cost him Rs.55,000/- while in the instalment scheme, he had to pay an EMI of Rs.900/- for 10 years at the rate of interest of 14%, compounded monthly. Suggest the person which scheme is more economical.

THERE ARE TWO SCHEME FIRST DIRECT DOWN PAYMENTS AND SECOND IS INSTALMENT SCHEME

SECOND IS INSTALMENT SCHEME



PW=-A(P/A,I,n)

$$= -A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$= -900\left[\frac{\left(1+\frac{0.14}{12}\right)^{120} - 1}{\frac{0.14}{12}\left(1+\frac{0.14}{12}\right)^{120}}\right]$$

PW = Rs.57964

Ans. *Since the PW of the down payment is less than PW of instalment, down payment is suggested.*

7. A company needs a mini-bus to bring its employees to work and take them home. It has two alternatives:

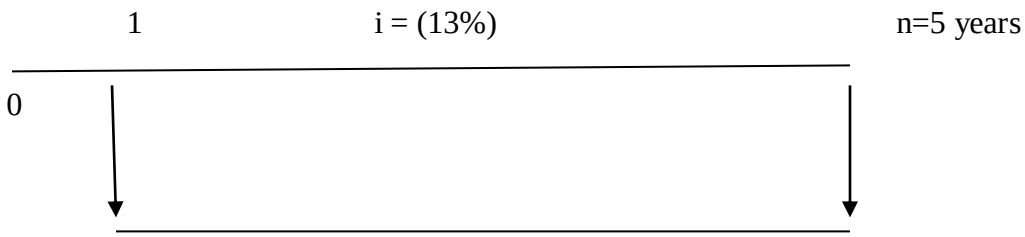
(i)To rent a vehicle for a payment of Rs.3 Lakhs per year for the next 5 years.

(ii) To buy a second hand vehicle for Rs.3Lakhs with an operating and maintenance cost of .Rs.2 Lakhs per year. The salvage value of the vehicle after the five year would be about 1 lakh.

Select the best alternative based on Present Worth method of comparison ,using an interest rate of 13% compounded annually.

Ans:

(i)To rent a vehicle for a payment of Rs.3 Lakhs per year for the next 5 years.

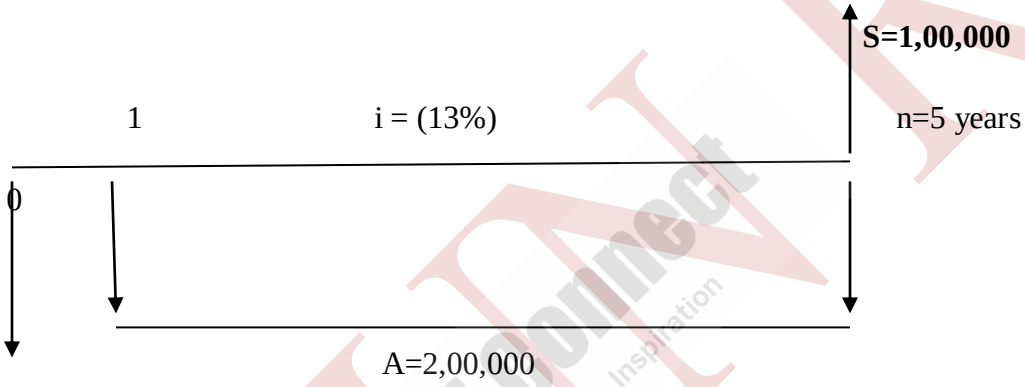


A=3,00,000

PW=-A(P/A,i,n)

=-3,00,000(P/A,i,n)= -3,00,000(P/A,13%,5)= -3,00,000(3.5172)=Rs 10,55,160 (-)

(ii) To buy a second hand vehicle for Rs.3Lakhs with an operating and maintenance cost of .Rs.2 Lakhs per year. The salvage value of the vehicle after the five year would be about 1 lakh.



P=-3,00,000

PW=-P -A(P/A,i,n)+S(P/F,i,n)

=-3,00,000-2,00,000(P/A,13%,5)+ 1,00,000(P/F,13%,5)

=-3,00,000-2,00,000(3.5172)+ 1,00,000(0.5428)=Rs 9,49,160 (-)

ANS:IT IS A COST DOMINATE SYSTEM SO SUGGESTED Rs 9,49,160 (-)

8. (a) An aircraft assembly fixture has a purchase price of Rs.9,00,000 and classed as a 5 year property. Use of the fixture is expected to result in an annual before tax savings of Rs.3,00,000 for a period of 6 years, at the end of which time it will be obsolete and virtually worthless. Applying the appropriate accelerated schedule, determine.

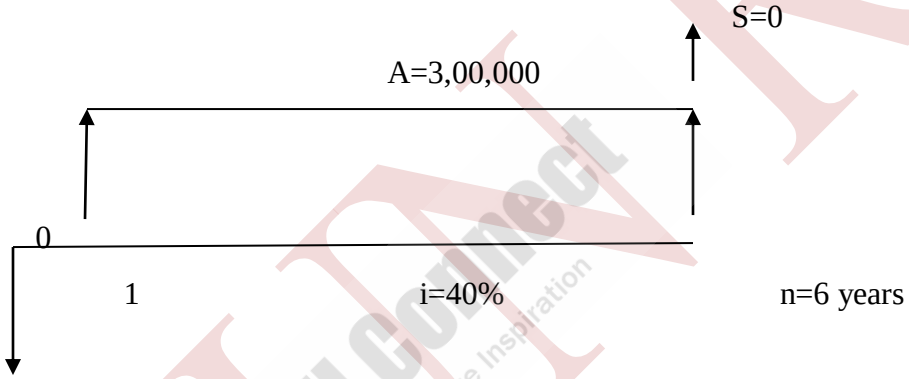
(i) The before tax Present Worth of the investment at an interest rate of 40 percent.

(ii) The after tax Present Worth of the investment with an effective tax rate of 40 percent and an interest rate of 20 percent. (VTU July '05)

ANS:

(i) The before tax Present Worth of the investment at an interest rate of 40 percent.

CFD



P=9,00,000

PW_1=-p+A(P/A,i,n)+S(P/F,I,n)

=-9,00,000+3,00,000×(P/A, 40%,6)+ 0×(P/F, 40%,6)

=-9,00,000+3,00,000×(2.1680)

=Rs 2,49,600 (-)

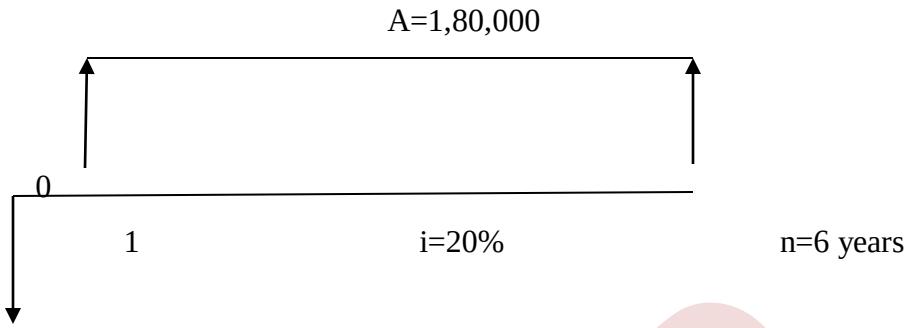
(ii) The after tax Present Worth of the investment with an effective tax rate of 40 percent and an interest rate of 20 percent.

Annual Saving per year (A)=3,00,000-0.40×3,00,000=Rs 1,80,000

OR

(A)= 0.60×3,00,000=Rs 1,80,000

CFD



P=9,00,000

PW_1=-p+A(P/A,i,n)+S(P/F,I,n)

=-9,00,000+1,80,000×(P/A, 20%,6)+ 0×(P/F, 20%,6)

=-9,00,000+1,80,000×(3.3255)=Rs 3,01,410 (-)

4. 1. An entrepreneur owning a small scale industry wants to buy milling machine. He has 3 machines in view, from different manufacturers. The initial investments, annual revenue, salvage values and the lives of three machines are given in the table. i=14%

	initial investments (Rs.)	annual revenue (Rs)	salvage values	Life (Years)
Machine 1	25,000	10,000	4,000	7
Machine 2	45,000	15,000	6,500	7
Machine 3	70,000	20,000	9,000	7

1. Two holiday cottages are under consideration. Compare the Present Worth of the cost of 24 years service, at an interest rate of 5 percent, when neither cottage has a realizable salvage value. (VTU Jan 2006)

	Cottage 1	Cottage 2
First cost	Rs.4,500	Rs.10,000
Estimated life	12 years	24 years
Annual maintenance cost	Rs.1,000	Rs.720

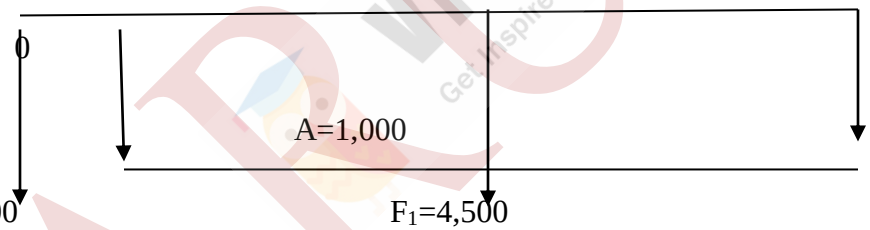
Cottage 1

12 | 12,24
2 | 1,2
1,1
LCM=12×2=24

CFD

24/12=2 (TIMES)

1 i = (5%) n=12 n=24 years



PW=-P -A(P/A,i,n)+F_1(P/F,i,n)

= -4,500 -1,000(P/A,5%,24)+4,500(P/F,5%,12)

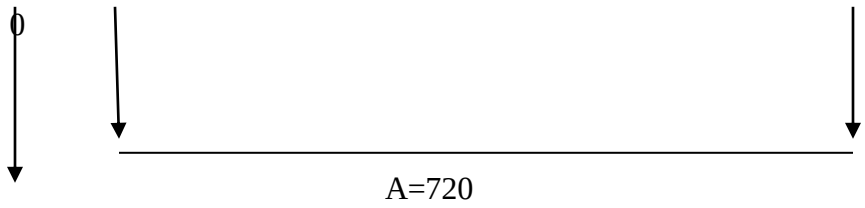
= -4,500 -1,000(13.7986)+4,500(0.5568)=Rs 20,804 (-)

Cottage 2

24/24=1 (TIMES)

CFD

1 i = (5%) n=24 years



P=10,000

PW=-P -A(P/A,i,n)

=-10,000 -720(P/A,5%,24)= -10,000 -720(13.7987)=Rs 19,935 (-)

ANS:IT IS A COST DOMINATE SYSTEM, SO SUGGESTED COTTAGE 2

2. The following alternatives are available to accomplish an objective of 12 year duration.

	Plan A	Plan A	Plan A
Life cycle(yrs)	6	3	4
First cost(Rs.)	2,000	8,000	10,000
Annual cost (Rs.)	3.200	700	500

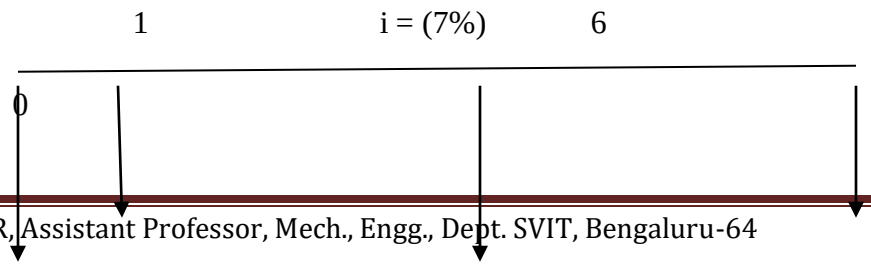
Compare the present worth of the alternatives using an interest rate of 7 percent. (VTU July 2006)

Plan A:

2 6,3,4
3 3 3 2
2 1 1 2
1 1 1

LCM=2×3×2=12 12/6=2 (TIMES)

CFD



A=3,200

P=2,000 F_1=2,000

PW=-P -A(P/A,i,n)-F_1(P/F,i,n)

= -2,000 -3,200 (P/A,7%,12)- 2,000 (P/F,7%,6)

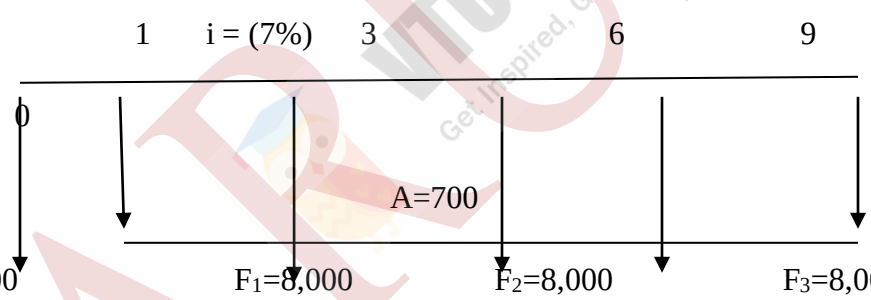
= -2,000 -3,200 (7.9427)- 2,000 (0.6664)=Rs 28,749 (-)

Plan B:

2 6,3,4
3 3 3 2
2 1 1 2
1 1 1

LCM=2×3×2=12 12/3=4 (TIMES)

CFD



PW=-P -A(P/A,i,n)-F_1(P/F,i,n) –F_2(P/F,i,n) –F_3(P/F,i,n)

=-8,0000 -700(P/A,7%,12)-8,000(P/F,7%,3) –8,000(P/F,7%,6) –8,000(P/F,7%,9)

=-8,0000 -700(7.9427)-8,000(0.8163) –8,000(0.6664) –8,000(0.5439)=Rs 29,772 (-)

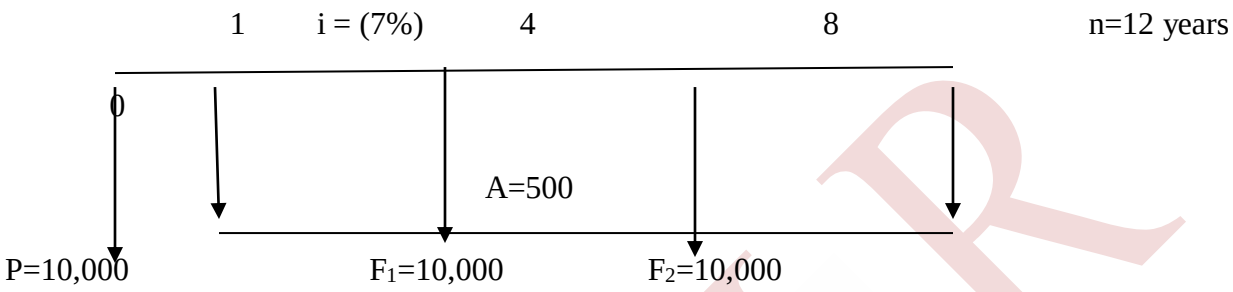
Plan C:

2 6,3,4
3 3 3 2
2 1 1 2

1 1 1

LCM=2×3×2=12 12/4=3 (TIMES)

CFD



PW=-P -A(P/A,i,n)-F_1(P/F,i,n) –F_2(P/F,i,n)

=-10,000-500(P/A,7%,12)-10,000(P/F,7%,4) -10,000(P/F,7%,8)

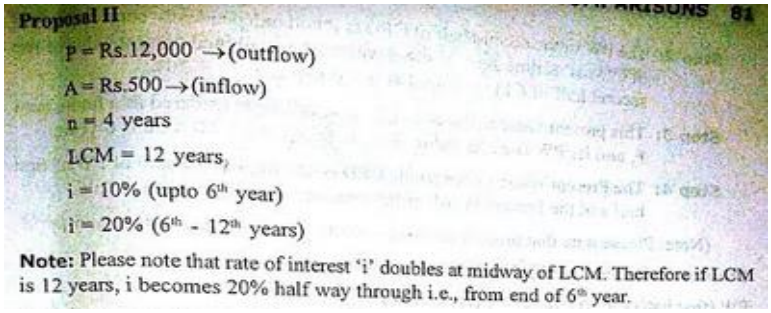
=-10,000-500(7.9427)-10,000(0.7629) -10,000(0.5820)=Rs 27,420 (-)

ANS:IT IS A COST DOMINATE SYSTEM, SO SUGGESTED PLAN C

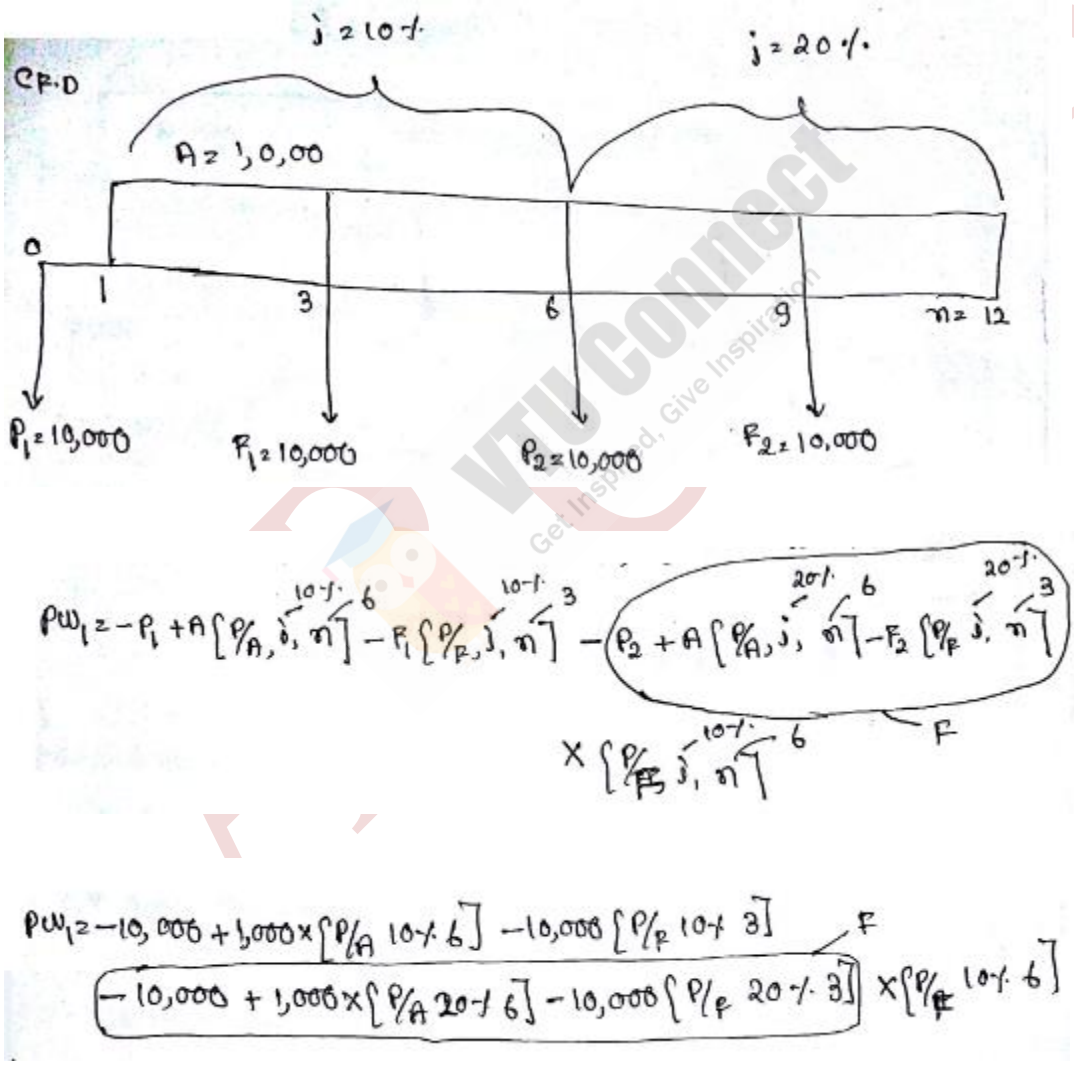
3. Analyse and take a decision on the following proposals based on engineering economy analysis. Proposal one has a life of three years with an investment investment of Rs. 10,000/- and the second proposal has a life of 4 years at an of Rs. 12,000/- Annual gain in operating expenses in both the cases are Rs.1, 000 and Rs. 500 respectively. Assume an interest rate of 10% which correct gets doubled at LCM midway. Comment on the proposal using) method? (V TU Jan 2008)

Proposals I

Investment, P = Rs.10,000 →(outflow)
Annual gains, A = Rs.1,000 →(inflow)
Life, n = 3 years
Least Common multiple, LCM = 12 years
Rate of interest, i = 10% (up to 6th year)
i = 20% (6 – 12 years)



Proposals I



$$PW_1 = -P_1 + A\left[P/A, i, n\right]^{10\%}_{6} - F_1\left[P/F, i, n\right]^{10\%}_{3} - \left(P_2 + A\left[P/A, i, n\right]^{20\%}_{6} - F_2\left[P/F, i, n\right]^{20\%}_{3}\right) \times \left[P/F, i, n\right]^{10\%}_{6}$$

$$PW_1 = -10,000 + 1,000 \times \left[P/A\ 10\%\ 6\right] - 10,000\left[P/F\ 10\%\ 3\right] - \left(-10,000 + 1,000 \times \left[P/A\ 20\%\ 6\right] - 10,000\left[P/F\ 20\%\ 3\right]\right) \times \left[P/F\ 10\%\ 6\right]$$

$PW_{12} = -10,000 + 1,000 \times [4.3553] - 10,000 [0.7513]$

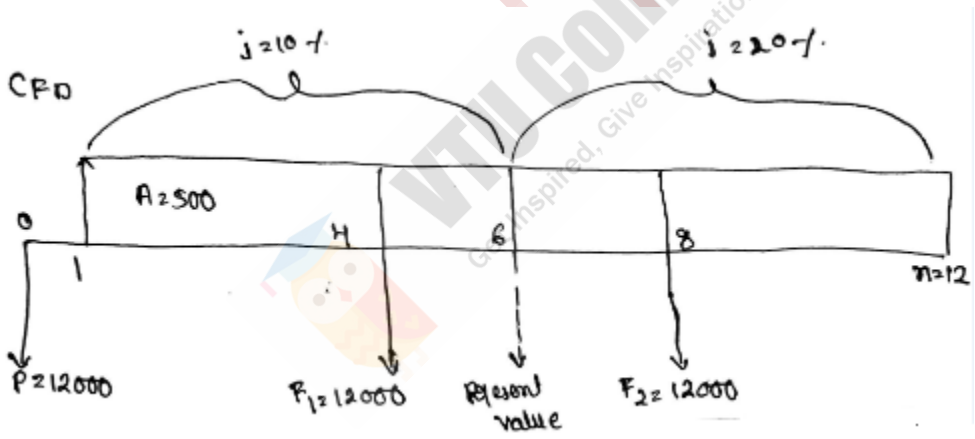
$[-10,000 + 1,000 \times [3.3255] - 10,000 [0.5787]] \times [0.5645]$

$= -18157.7 - 12461.5 \times 0.5645$

$= Rs\ 20192.21\ [-ve]$

Proposals II

CFD



$PW = -P + A[P/A, i, n]_6^{10\%} - F_1[P/F, i, n]_4^{10\%} + \{A[P/A, i, n]_6^{20\%} - F_2[P/F, i, n]_2^{20\%}\} \times [P/F, i, n]_6^{10\%}$

$PW_2 = -12000 + 500[P/A\ 10\%\ 6] - 12{,}000[P/F\ 10\%\ 4]$
$[[+ 500[P/A\ 20\%\ 6] - 12000[P/F\ 20\%\ 2]] \times [P/F\ 10\%\ 6]$

$PW_2 = -12000 + 500 \times [4.3553] - 12{,}000[0.6830]$
$+ [[500[3.3255] - 12{,}000[0.6944]] \times [0.5645]$

$PW_2 = -18018.35 - 6670.05 \times 0.5645$

PW_2 Rs 21,783.59 (-ve)

ANS:IT IS A COST DOMINATE SYSTEM, SO SUGGESTED Proposals I

FUTURE-WORTH METHOD OF COMPARISON

1. Consider the following two mutually exclusive alternatives.

Alterations	0	1	2	3	4
X(Rs.)	— 2,50,000	1,00,000	1,00,000	1,00,000	1,00,000
Y(Rs.)	— 3,00,000	1,40,000	1,10,000	90,000	1,00,000

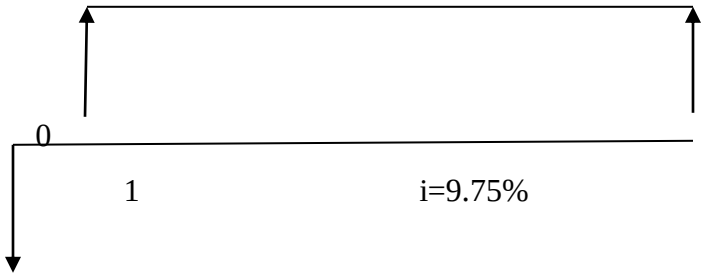
Select the better alternative based on the future-worth method of comparison. If rate of interest is 9.75%, compounded annually.

Note: only formula have to be used.

Alterations: X (Rs.)

CFD

A=1,00,000



P=2,50,000

FW_X=-P(F/P,i,n)+A(F/A,i,n)

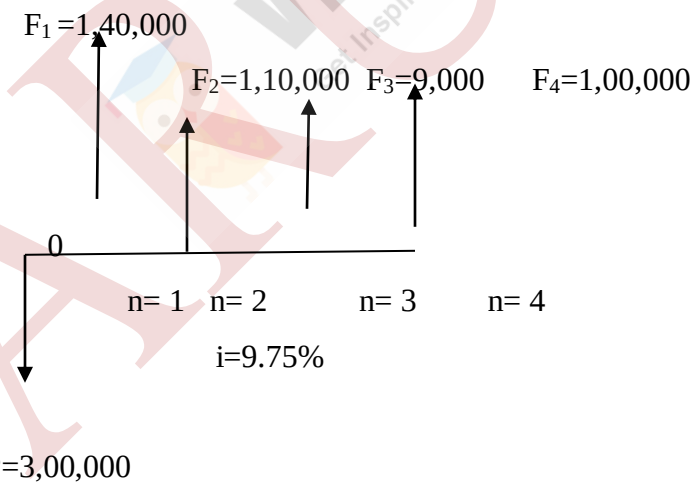
$=-P\ ((1+i)^n)\ + A\times\left(\frac{((1+i)^n-1)}{i}\right)$

$=-2{,}50{,}000\ ((1+0.0975)^4)\ + 1{,}00{,}000\times\left(\frac{((1+0.0975)^4-1)}{0.0975}\right)$

=-362708.82+462395.18= Rs 99686.36

Alterations: Y (Rs.)

CFD



P=3,00,000

$PW(P)=-p+F_1(P/F,i,n)+ F_2(P/F,i,n)+ F_3(P/F,i,n)+ F_4(P/F,i,n)$

$=-P+\left(\frac{F}{(1+i)^n}\right)+\left(\frac{F}{(1+i)^n}\right)+\left(\frac{F}{(1+i)^n}\right)+\left(\frac{F}{(1+i)^n}\right)$

$=-3{,}00{,}000+\left(\frac{1{,}40{,}000}{(1+0.0975)^1}\right)+\left(\frac{1{,}10{,}000}{(1+0.0975)^2}\right)+\left(\frac{90{,}000}{(1+0.0975)^3}\right)+\left(\frac{1{,}00{,}000}{(1+0.0975)^4}\right)$

=-3,00,000+1,27,562.62+9,13,23.72+68,081.46+68,925.81=Rs 55893.61 (P)

FW_Y=P(F/Pi,n)

= 55893.61 $((1 + 0.0975)^4)$ = Rs.81092.42

ANS:IT IS A REVENUE DOMINATE SYSTEM, SO SUGGESTED Proposals X

2. Two motorcycles of brand 'A' and ' B ' are available on the following terms:

(i) Motorcycle 'A' - Make a down payment of Rs.5,000 and then Rs.6,000 at the end of each year for 7 years.

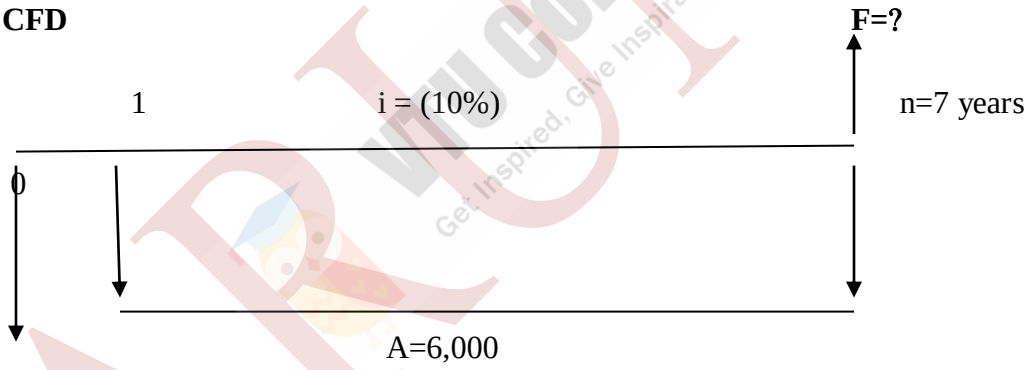
(ii) Motorcycle 'B' - Make a down payment of Rs.15,000 and no payment for the next 3 years. From end of the 4th year annual payments of Rs.12,000 for the next 4 years. (VTU Jan 2006)

ANS:

(i) Motorcycle 'A'

ASSUME=10%

CFD



P=-5,000

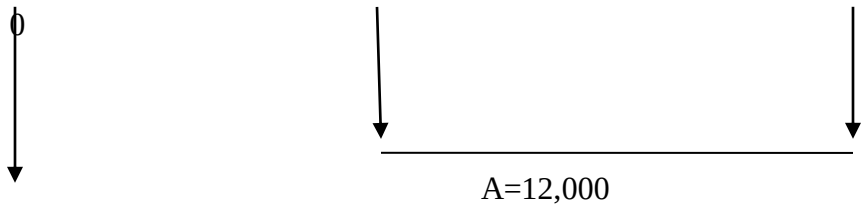
FW=-P(F/P,I,n)+A(F/A,I,n)

=-5,000(F/P,10%,7)+6,000(F/A,10%,7)=-5,000(1.9487)+6,000(9.4872)=Rs.66,666(-)

(ii) Motorcycle 'B'

ASSUME=10%

CFD



P=-15,000

FW=-P(F/P,I,n)+A(F/A,I,n)

=-15,000(F/P,10%,7)+12,000(F/A,10%,4)=-15,000(1.9487)+12,000(4.6410)=Rs.84,922(-)

ANS:IT IS A COST DOMINATE SYSTEM, SO SUGGESTED Motorcycle 'A'

Example: 3. A software engineer wanting to buy a car has two models in his mind, a petrol run Optra Chevrolet or a diesel run Skoda Octavia. The comparative data are as follows.

Particulars	Optra-Chevrolet (Petrol) (Rs.)	Skoda Octavia (Diesel) (Rs.)
1. Vehicle cost	8,50,000	10,00,000
2. Fuel cost per litre	58	36
3. Mileage in km/litre	8 km	11km
4. Distance travelled per year	12000	12000
5. Annual maintenance cost	18000	36000
6. Expected life	12	12
7. Salvage value	400000	400000

If the software engineer considers 10.5% as good value for money, Suggest him the better option. find FW.

ANS:

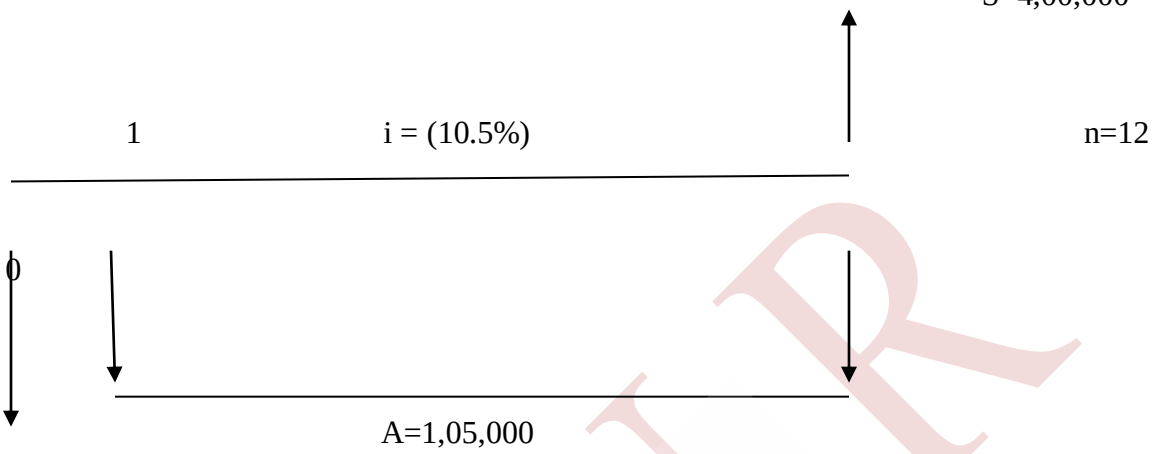
Optra-Chevrolet (Petrol) (Rs.)

A=$\frac{Distance\ travelled}{Mileage}$×Litres of petrol+Maintenance cost/Year

A=$\frac{12000}{8}$×58+18000=Rs.1,05,000

S=4,00,000

CFD



P=-8,50,000

FW=-P(F/P,I,n) -A(F/A,I,n)+S

=-8,50,000(F/P,10.5%,12) -1,05,000(F/A,10.5%,12)+4,00,000

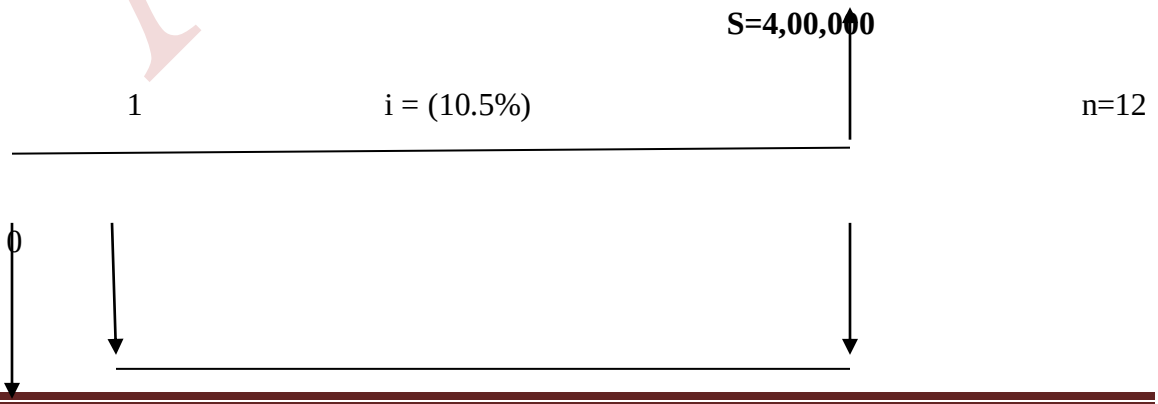
=-8,50,000(3.3140) -1,05,000(22.0377)+4,00,000=Rs 47,30,858 (-)

Skoda Octavia (Diesel) (Rs.)

A=$\frac{Distance\ travelled}{Mileage}$×Litres of petrol+Maintenance cost/Year

A=$\frac{12000}{11}$×36+36000=Rs.75,272

CFD **S=4,00,000**



A=75,272

P=-10,00,000

FW=-P(F/P,I,n) -A(F/A,I,n)+S

=-10,00,000 (F/P,10.5%,12) -75,272 (F/A,10.5%,12)+4,00,000

=-10,00,000 (3.3140) -75,272 (22.0377)+4,00,000=Rs 47,38,521 (-)

ANS:IT IS A COST DOMINATE SYSTEM, SO SUGGESTED Optra-Chevrolet (Petrol)

Conditions for Present Worth Comparison

All present worth comparisons that are made have a common background. In other words, before making present worth comparison, it is necessary to follow some basic assumptions as follows :

1. Cash flows are known : While doing present worth calculation we assume that all future estimates of revenues and costs are known.

2. Inflation is not considered : Present worth calculations do not consider inflation which brings down the value of money with time.

3. Interest rates are known : Interest rates at any given point of time is a complex function of the economy of the country. Present worth calculations are made assuming that interest rates do not change with time.

4. Comparisons are made prior to tax : All revenues and expenditures in any business are subject to various taxes of the state. These taxes vary with magnitude of revenue or expenditure. Present worth comparisons, at an elementary level, are made without taking taxes into consideration.

5. Comparisons do not consider intangible factors : Intangible factors are those which are non quantifiable. In other words, factors like brand image, good will of the company, ecological friendliness, management strength and attitude, quality of workforce etc., are not considered in present worth comparisons, although they all are of considerable value to the company.

6. Comparisons an made assuming constant availability of funds : Availability of capital (especially liquid cash) for any expenditures in future is assumed as and when required to get economical benefit. But this is seldom easy, in reality.

BASIC PRESENT WORTH COMPARISONS

As we have seen in several worked problems, present worth comparisons give the current value of all future revenues and costs, assuming a constant rate of interest i. In other words, the present worth of any business can be estimated when all future investments and returns are well known. When comparing alternatives, present worth calculations done so far have assumed the same life for all alternatives. Present worth comparisons can also be made between alternatives

which have unequal lives, which will be dealt later. When making basic present worth comparison, two general patterns emerge :

1. PRESENT - WORTH EQUIVALENCE

In this, the present worth of a series of future transactions are made at time zero. The purpose is to secure one figure that represents all the future transactions. This figure can then be compared with a corresponding figure that represents transactions from a competing alternative, or it can be compared with the option of doing nothing. A do-nothing option is always an alternative, even if it results only in postponement. As we have seen in the worked examples, the present worth comparisons are made both on revenue - dominated cash flow problems and cost-dominated cash flow problems.

2. NET PRESENT WORTH

In this, there is always an investment at time zero followed by possible receipts and expenditures at future time periods. In other words, there is an initial outlay followed by a series of revenue and disbursements. This is the most frequently encountered pattern, which leads to the fundamental relation.

Net present worth = Pw (revenues) - Pw (costs)

The criteria for choosing between mutually exclusive alternatives is to select the one that maximizes the net present worth or simply the one that yields the larger positive Pw. A negative Pw means that the alternative does not satisfy the rate-of-return requirement.

In addition to mutually exclusive alternatives where we can select only one alternative, we could have independent alternatives where more than one alternative may be selected. The criterion for consideration of independent alternatives is that they should have a Pw which is equal to or greater than zero.

ASSETS WITH UNEQUAL LIVES

All cases discussed in the worked examples section are problems which concern alternatives which have equal lives. In other words, all the present-worth comparisons that were made so far was for co-terminated projects which implies that lives involved have a common end point. Alternatives have been always compared on the basis of equivalent outcomes. However in real time situations, comparisons have to be made among alternatives which may have unequal lives. In such a case two prominent methods, described as under, are used.

1. Common multiple method
2. Study period method

1. COMMON MULTIPLE METHOD

In this method, a common multiple of all the different service lives of various alternatives is chosen. The alternatives are all then compared against this common service life, and the appropriate alternative is chosen, depending on whether it is cost dominated or revenue dominated. In other words, all the alternatives are made to coterminate by selecting an analysis period that spans a common multiple of the lives of the involved assets. For example, if four alternatives have lives of 2, 3, 4 and 6 years, the least common multiple which is 12 years, is chosen. Then the asset with a life of two years would be replaced 6 times during the analysis period. The assets with 2, 4 and 6 years lives would be replaced 4, 3, 2 times, respectively.

The method of least common multiple is used under the assumption that assets will be repeatedly replaced by successors having identical cost characteristics. in other words, it is assumed that neither inflation nor interest rates is going to change, with passage of time.

This method is more applicable to such alternatives whose least common multiple life is not a large number.

2. STUDY PERIOD METHOD

In this method, a common study-period for all the alternatives which is proportionate to the length of the project or the period of time the assets are expected to be in service, is selected. An

appropriate study period reflects the replacement circumstances. Setting up of a study-period could depend upon the length of :

(i) The shortest life of all competing alternatives, or

(ii) The known duration of required services, or

(iii) The time before which a better replacement becomes available.

A study period comparison based on the shortest life of all competing alternatives gives protection against technological obsolescence. A study period comparison usually presumes that all assets will be disposed of at the end of the period. Sometimes it therefore becomes necessary to estimate the income that can be realized from the sale of an asset which can still provide useful service.

ASSETS HAVING INFINITE LIVES

Them are many assets that we come across which may be considered to have infinite lives. For example bridges, dams, railway tunnels etc., can provide extended service and can be assumed to last forever. In such cases of long lived assets, Capitalized cost is calculated which is nothing but the sum of initial investment and the present worth of all operational and maintenance costs. Capitalized cost is calculated in the same way as in a present-worth comparison, where 'n' (service life) equals infinity. This makes the analysis very sensitive to the rate of interest (i)

w.k.t $$(P/A, i, n) = \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

If $$n \to \infty$$

ie., $$\lim_{n\to\infty}(P/A, i, n) = \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

So if 'P' represents the initial investment, we have

Capitalized Cost = P + A (P / A, i, ∞)

$$= P + A \times \frac{1}{i} = P + \frac{A}{i}$$

Where A=uniform annual costs (disbursements)

I= rate of interest (forever)

P= initial out lay (first cost)

If revenues are not considered, then

$$\text{Capitalized Cost} = P + \frac{\text{annual disbursements}}{i}$$

1. RS 10 crores was granted by the management of an engineering, college tor the construction of its new Mechanical Sciences Block. Annual Maintenance for the block is estimated to be Rs. 10 Lakhs. In addition, Rs. 12 lakhs will be needed every 10 years for painting and major repairs.

If the budget granted has to take care of perpetual maintenance, how much of the amount can be used for initial construction costs ? Deposited finch can earn 6% rate of interest, compounded annually. Assume that taxes and inflation do not come into picture.

Solution

Given Capitalized cost = Rs. 10 Crores

Annual Maintenance Costs = Rs. 10 Lakhs Per Year

Painting costs = Rs. 12 Lakhs / 10 years (to be annualized)

Rate of interest, i = 6% (Compounded annually)

Service life, n = co Initial investment or first cost P = ?

Capitalized cost = Initial investment + $\frac{TOTAL\ ANNUAL\ COSTS}{rate\ of\ interest}$

or Initial construction cost = Capitalized cost — TAC /I

We have, Total annual costs = Annual Maintenance cost + Annual repair & Painting cost

But repair and painting costs are give for every 10 years which have to: annualized. For that, equal payment series sinking fund formula, ie., F (A/F, i, n) is used

Therefore, in this problem

$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

Where A = Annual amount = ?

F = Sum at the n^{th} period = 1200000

n = number of years = 10 years

i = Rate of interest (Compounded annually) = 6%

$$A = 1200000\left[\frac{0.06}{(1+0.06)^{10} - 1}\right]$$

A = Rs. 91041

∴ Total annual costs = 1000000 + 91041

TAC = 1091041

∴ Initial cost = capitalized cost – $\frac{TAC}{i}$

$$= 100000000 - \frac{1091041}{0.06}$$

$$= 100000000 - 18184025$$

Initial construction cost = Rs. 8,18,15,974

Funds to be deposited to earn interest for the sake of maintenance = Rs. 1814025.

PAY-BACK COMPARISON

" This investment will pay for itself in 2 years approximately / $21/_2$ years / n years" is a typical comment heard in business circles. This means that any investment made in any business will take a certain duration before it starts realizing profit.

In other words, in any business, the initial income will go towards initial costs of fixed capital and working capital as well as interests calculated on this, before enjoying actual profit. In order to find a rough duration that an investment takes to pay for itself is known as payback period. Payback period is nothing but an estimate of the elapsed time before a proposed investment begins to show profit. A simple formula to find an approximate pay back period is

$$\text{Payback period} = \frac{requierd\ investment}{annual\ receipts - annual\ expenditure}$$

$$\text{Payback period} = \frac{P}{R-C}$$

$$\text{OR Payback period} = \frac{FIRST\ COST}{NET\ ANNUAL\ SAVING}$$

Payback periods could be used to compare alternatives. Such a comparison would be a very simple and direct way of knowing how long will it take before the amount invested is recovered. Payback comparisons usually ignore effects of both depreciation as well as interests.

1. Following are the estimates of three alternative investments made on 3 different machines in an industry. Find out which machine has the fastest payback period.

	Particulars	Machine A	Machine B	Machine C

1	Initial Investment	30,000	38,000	42,000
2	Annual receipts	20,000	23,500	26,000
3	Annual expenditures	5,500	6,500	7,000
4	Economic life	4 years	4 years	4 years

Machine A

Given

Required Investment P = 30000

Annual receipts R = 20000

Annual expenditure C = 5500

Economic life, n = 4 years (redundant information)

$$\therefore \text{Payback period} = \frac{\text{Required investment}}{\text{Annual receipts} - \text{Annual expenditure}} = \frac{P}{R-C}$$

$$= \frac{30000}{20000-5500}$$

$$= 2.06 \text{ years}$$

Machine B

P = Rs.38000

R = Rs.23500

C = Rs.6500

$$\text{Payback period} = \frac{P}{R-C} = \frac{38000}{23500-6500}$$

Payback period = 2.23 years

Machine C

P = Rs.42000

R = Rs.26000

C = Rs.6500

$$\text{Payback period} = \frac{P}{R-C} = \frac{42000}{26000-7000} = 2.21 \text{ years}$$

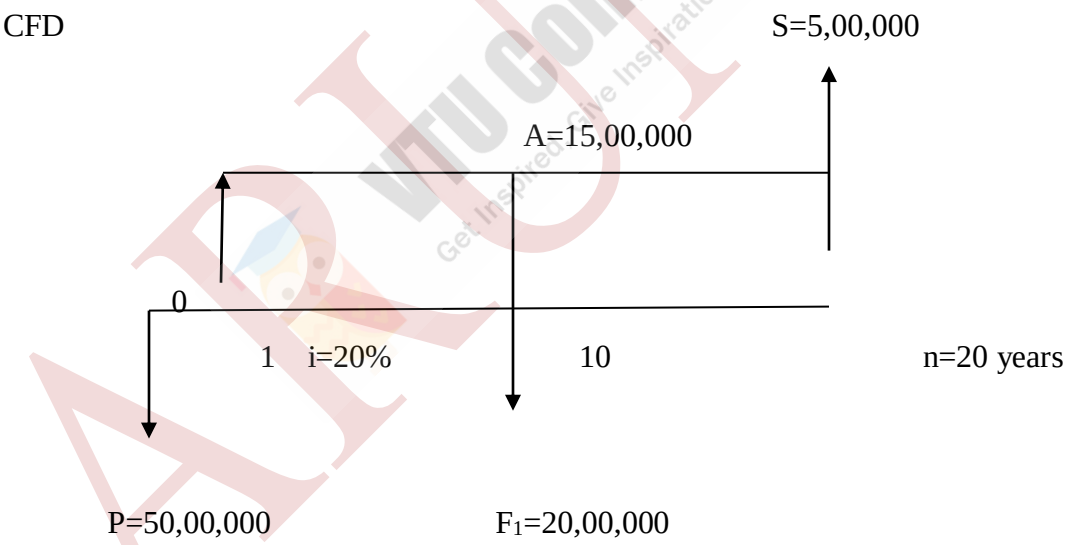
Answer:

Payback period of machine A is the least

Example: The details of the feasibility report of a project are shown below. Check the feasibility of the project based on present worth method if i = 20% Initial outlay = Rs. 50,00,000 /-Life of project = 20 years Annual equivalent revenue = Rs. 15,00,000 /-Modernizing cost at the end of 10th year= Rs. 20,00,000 /-Salvage value at the end of project life= Rs. 5,00,000 /-

Solution :

CFD



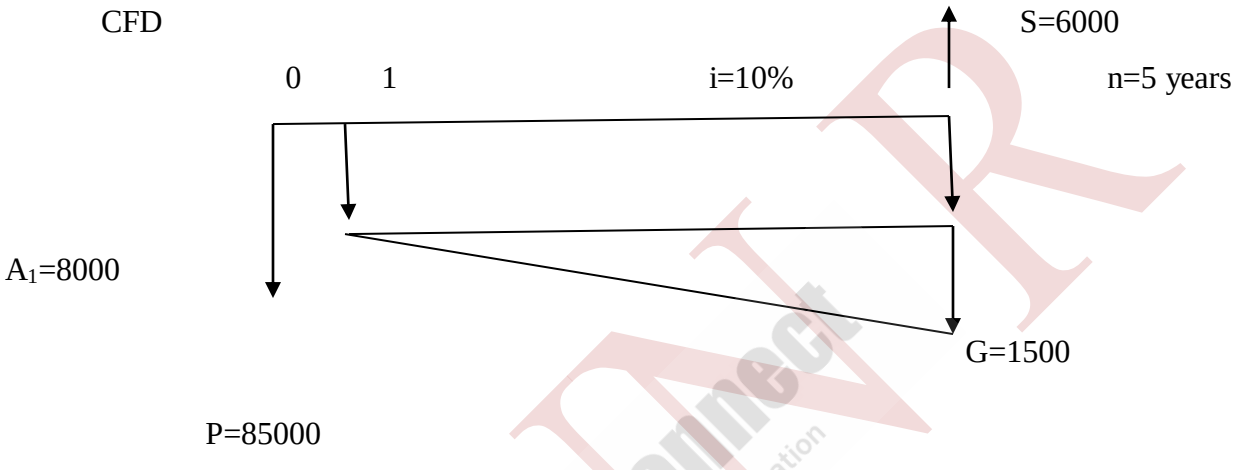
PW=-P+A(P/A,i,n)-F_1(P/F,i,n)+S(P/F,i,n)

=-50,00,000 +15,00,000 (P/A, 20%,20)- 20,00,000 (P/F, 20%,10)+ 5,00,000 (P/F, 20%,20)

=-50,00,000 +15,00,000 (4.86958)- 20,00,000 (0.161506)+ 5,00,000 (0.02608)=Rs 1994398 (-)

Example Megha electronics is considering the purchase of a new programmable circuit tester to improve its product quality. The equipment has a first cost of Rs. 85,000 and the salvage value is

predicted to be Rs. 6000 after a service life of 5 years. Maintenance and operating costs are expected to be Rs. 8000 for the first year of operation and increase by Rs. 1500 per year for each additional year of use. Using an interest rate of 10% determine what annual savings must be obtained through the use of this equipment to make it economically justifiable. (VTU Julay 2005)



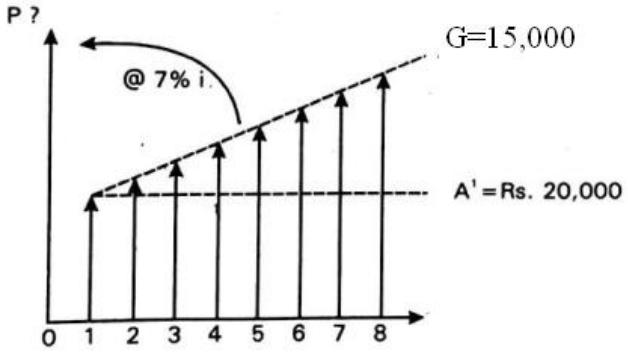
PW= -P-(A_1+G(A/G,I,n))×(P/A,I,n) +S(P/Fi,n)

=-85000-(8000+1500 (A/G, 10 %,5)) ×(P/A, 10%,5)+S(P/F10%,5)

=-85000-(8000+1500 (1.8101)) ×(3.7908)+6000(0.6209)=RS 121893.59(-) (P)

AW=P(A/P,I,n)=- 121893.59 (A/P,10%,5)= -121893.59 (0.2636)=Rs=32131.15 (-)

Example ABC company, which has had a rapid growth in business, finds that it is running out of storage space for raw materials. XYZ company has offered a contract for leasing a nearby storage facility at Rs 20,000 per year with annual increases of Rs 1500 for 8 years. Payments are to be made at the end of each year, starting 1 year from now. The prevailing interest rate is 7 percent. What lump sum paid today would be equivalent to the 8 year lease payment plan?



First step:

Convert the increasing annual payments to a uniform series.

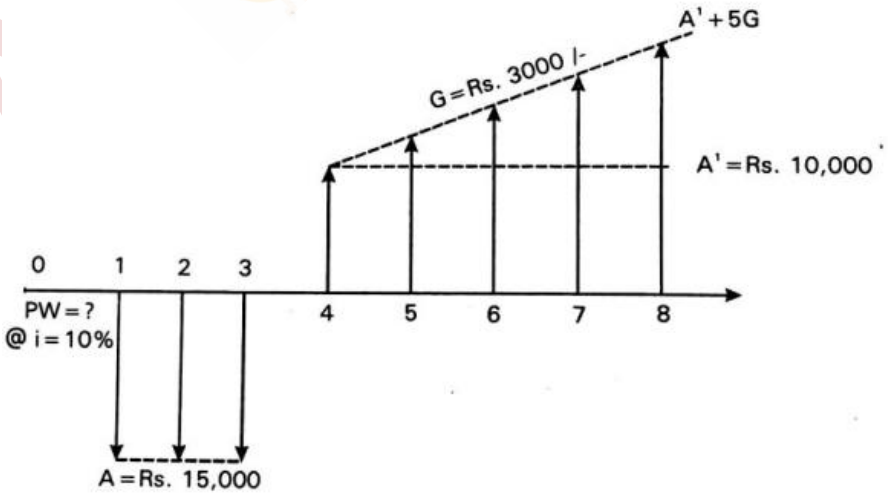
$$A = A' + G\,(A/G, 7, 8)$$
$$= \text{Rs } 20{,}000 + \text{Rs } 1500\,(3.14654)$$
$$= \text{Rs } 24{,}719.81\,/\text{-}$$

Second step:

Translate the annuity into present worth.

$$P = A\,(P/A, 7, 8)$$
$$= \text{Rs. } 24719.81\,(5.9713)$$
$$= \text{Rs. } 1{,}47{,}609.40\,/\text{-}$$

Example An investor can make three end of year payments of Rs 15,000 which are expected to generate receipts of Rs 10,000 at the end of year 4 that will increase annually by Rs 3000 for the following 4 years. If the investor can earn a rate of return of 10% on other 8 year investments, is this alternative attractive?



Present worth,

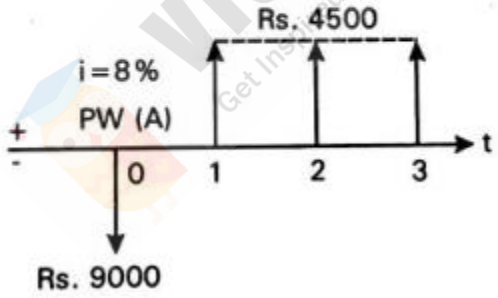
$$PW = -\text{Rs } 15{,}000\,(P/A, 10, 3) + [A^1 + G\,(A/G, 10, 5)][P/A, 10, 5][P/F, 10, 3]$$
$$= -\text{Rs } 15{,}000\,(2.48685) + [\text{Rs } 10{,}000 + \text{Rs } 3000\,(1.81013)]\,(3.79079)\,(0.75131)$$
$$= -\text{Rs } 37{,}302.75 + \text{Rs } 43946.65$$
$$= \text{Rs } 6643.9\ /\text{-}$$

Example Two devices are available to perform a necessary function for 3 years. The initial cost (negative) for each device at time 0 and subsequent annual savings (positive) are shown in the following table. The required interest rate is 8 percent.

	Year			
	0	1	2	3
Device A	9000	4500	4500	4500
Device B	14,500	6000	6000	8000

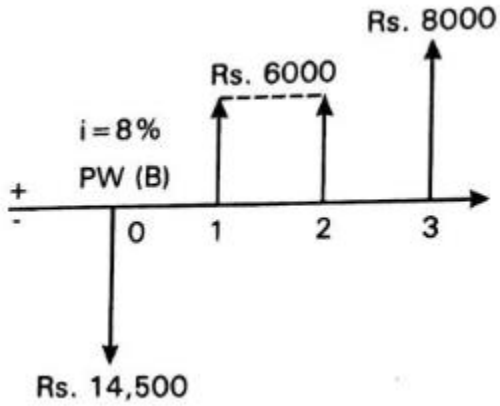
The cash flow diagram is shown below.

Device A



PW of device *A*,

$$= -9000 + 4500\,(P/A, 8, 3)$$
$$= -9000 + 4500\,(2.57710)$$
$$= \text{Rs. } 2596.95\ /\text{-}$$



PW of device B,

$$= -14{,}500 + 6000\,(P/A, 8, 2) + 8000\,(P/F, 8, 3)$$
$$= -14{,}500 + 6000\,(1.78326) + 8000\,(0.79383)$$
$$= \text{Rs. } 2550.20\ /-$$

Two type of trucks are available for transportation use. They are needed for 10 years. The details are,

	Truck A	Truck B
First cost	Rs. 10,00,000	Rs, 15,00,000
Estimated annual maintenance cost	Rs. 20,000	Rs. 15,000
Estimated life	5 years	10 years
Estimated salvage value	Rs. 2,00,000	Rs. 5,00,000

Both the truck deliver same amount of work. Assume interest rate of 7%. Which truck is to be preferred on present worth basis? Use CFD for your analysis. (VTU Model Paper)

Solution :

This is a problem with unequal lives. We can use common multiple method for making a comparison.

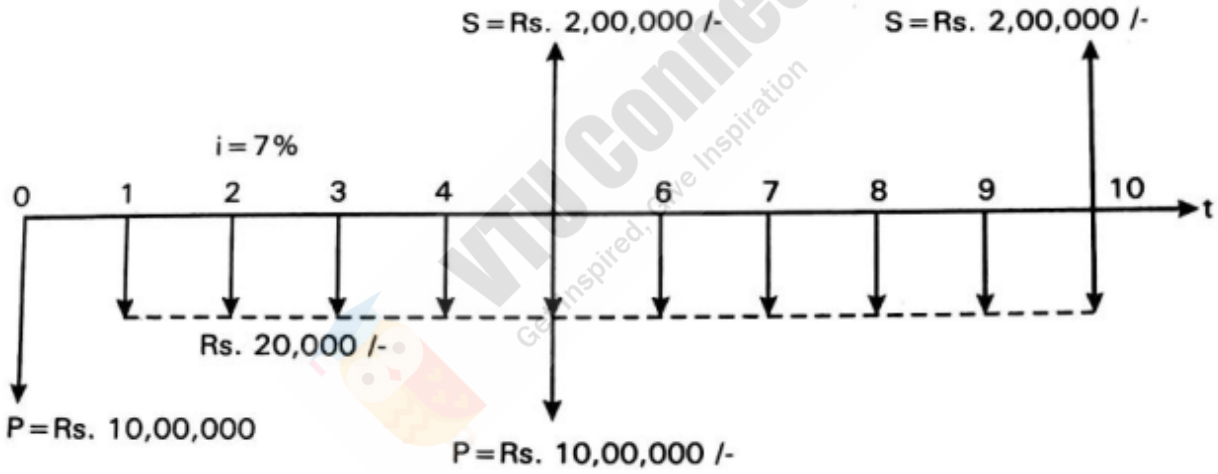
In this case the least common multiple is 10 years. Let us make the comparison based on this value of $LCM = 10$ years.

Truck A

Given data ;

P = Rs. 10,00,000 /-
A = Rs. 20,000 /- (maintenance cost)
N = 5 years
i = 7%
S = Rs. 2,00,000 /-

The CFD diagram for truck A is shown below.



We have present worth of truck A

$$PW_A = \text{Rs. } 10,00,000 + A\,(P/A, i, 10) + 8,00,000\,(P/F, i, 5) - 2,00,000\,(P/F, i, 10)$$

$$= 10,00,000 + 20,000\,(7.02358) + 8,00,000\,(0.71299) - 2,00,000\,(0.50835)$$

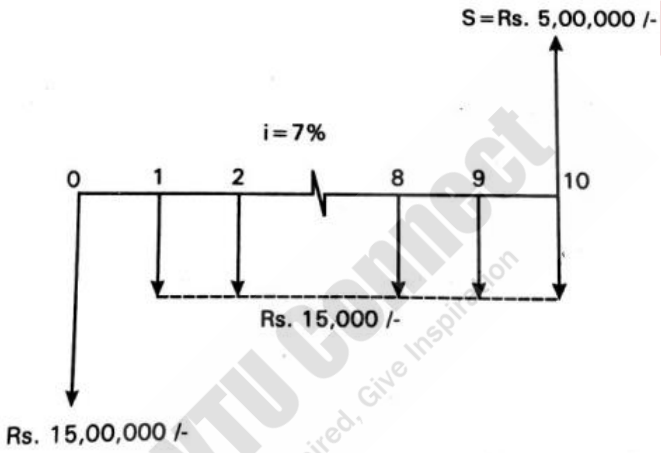
$$= \text{Rs. } 16,09193.85 \text{ /-} \quad (-\text{ve})$$

Truck B

Given data ;

P = Rs. 15,00,000 /-
A = Rs. 15,000 /-
N = 10 years
S = Rs. 5,00,000 /-
i = 7% compounded annually

The CFD diagram for Truck B is shown below.



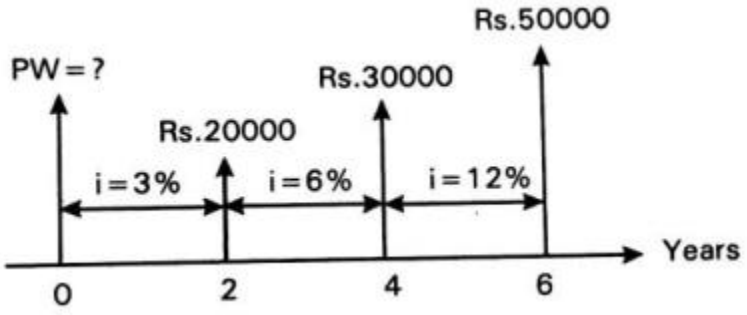
We have present worth of truck B,

PW_B = Rs. 15,00,000 + A (P/A, 7, 10) – 5,00,000 (P/F, 7, 10)
= 15,00,000 + 15,000 (7.02358) – 5,00,000 (0.50835)
= Rs. 13,51,178.7 /- (– ve)

Comparing the present worth of truck A and truck B, **truck B is chosen.**

Example An entrepreneur intending to start a new business knows that the first few years are the most difficult. To lessen the chance of failure, a loan plan for start-up capital is proposed in which interest paid during the first two years will be at 3% at 6% for the next 2 years of the 6 years loan. How large a loan can be justified for proposed repayments at the end of years 2,4 and 6 respectively Rs 20,000, Rs 30,000, and Rs 50,000?

Solution The cash flow diagram for the given problem is shown below.



PW of Rs 20,000 payment,

= Rs 20,000 (*P/F*, 3, 2)
= Rs 20,000 (0.94260)
= **Rs 18,852 /-**

PW of Rs 30,000 payment,

= Rs 30,000 (*P/F*, 6, 2) (*P/F*, 3, 2)
= Rs 30,000 (0.89000) (0.94260)
= **Rs 25,167.42 /-**

PW of Rs 50,000 payment,

= Rs 50,000 (*P/F*, 12, 2) (*P/F*, 6, 2) (*P/F*, 3, 2)
= Rs 50,000 (0.79719) (0.89000) (0.94260)
= **Rs 33,438.69 /-**

PW (total loan),

= Rs 18,852 + Rs 25,167.42 + Rs 33,438.69
= **Rs 77,458.11 /-**

The supervisor of a small machine shop has received three suggestions for reducing production costs. Suggestion *A* is to buy new jigs and fixtures; *B* is to rebuild on existing machine to improve its performance and C is to purchase a new machine to replace some manual labor. Estimates have been made for the three alternative investments.

	Alternatives		
	A	B	C
First cost, Rs	18000	23500	42000
Economic life, year	4	4	8
Net annual savings, Rs	6450	8400	11000
Payback period years	2.8	2.8	3.8

The supervisor selects alternative *B* explaining that because of the limited capital for investments, shorter payback periods are preferable. With alternatives *A* and *B* having the same payback period, *B* is favoured because the annual savings are greater than for *A*. What are the fallacies in this reasoning?

Solution

The reasoning is not considering the fact that both *A* and *B* would have to reinvest after four years whereas option *C* need not. Also a 0% rate of interest is assumed.

Assuming 8% interest rate and constant value of rupee, we can calculate the present worth for the alternatives as follows.

$(PW)_A$ = – Rs 18000 – Rs 18000 (*P/F*, 8, 4) + Rs 6450 (*P/A*, 8, 8)
= – Rs 18000 – Rs 18000 (0.73503) + Rs 6450 (5.74664)
= **Rs 5835.3 /-**

$(PW)_B$ = – Rs 23500 – Rs 23500 (*P/F*, 8, 4) + Rs 8400 (*P/A*, 8, 8)
= – Rs 23500 – Rs 23500 (0.73503) + Rs 8400 (5.74664)
= **Rs 7498.6 /-**

$(PW)_C$ = – Rs 42000 + Rs 11000 (*P/A*, 8, 8)
= – Rs 42000 + Rs 11000 (5.74664)
= **Rs 21213.0 /-**

Thus alternative *C* is preferred because the investment cost involved is only once.