9

DEPRECIATION

9.1 INTRODUCTION

Any equipment which is purchased today will not work for ever. This may be due to wear and tear of the equipment or obsolescence of technology. Hence, it is to be replaced at the proper time for continuance of any business. The replacement of the equipment at the end of its life involves money. This must be internally generated from the earnings of the equipment. The recovery of money from the earnings of an equipment for its replacement purpose is called *depreciation fund* since we make an assumption that the value of the equipment decreases with the passage of time. Thus, the word "depreciation" means *decrease* in value of any physical asset with the passage of time.

9.2 METHODS OF DEPRECIATION

There are several methods of accounting depreciation fund. These are as follows:

1. Straight line method of depreciation
2. Declining balance method of depreciation
3. Sum of the years—digits method of depreciation
4. Sinking-fund method of depreciation
5. Service output method of depreciation

These are now discussed in detail.

9.2.1 Straight Line Method of Depreciation

In this method of depreciation, a fixed sum is charged as the depreciation amount throughout the lifetime of an asset such that the accumulated sum at the end of the life of the asset is exactly equal to the purchase value of the asset. Here, we make an important assumption that inflation is absent.

Let

P = first cost of the asset,

F = salvage value of the asset,

n = life of the asset,
B_t = book value of the asset at the end of the period t,
D_t = depreciation amount for the period t.

The formulae for depreciation and book value are as follows:

$$D_t = (P - F)/n$$
$$B_t = B_{t-1} - D_t = P - t \times [(P - F)/n]$$

EXAMPLE 9.1 A company has purchased an equipment whose first cost is Rs. 1,00,000 with an estimated life of eight years. The estimated salvage value of the equipment at the end of its lifetime is Rs. 20,000. Determine the depreciation charge and book value at the end of various years using the straight line method of depreciation.

Solution

$$\begin{aligned} P &= \text{Rs. } 1,00,000 \\ F &= \text{Rs. } 20,000 \\ n &= 8 \text{ years} \\ D_t &= (P - F)/n \\ &= (1,00,000 - 20,000)/8 \\ &= \text{Rs. } 10,000 \end{aligned}$$

In this method of depreciation, the value of D_t is the same for all the years. The calculations pertaining to B_t for different values of t are summarized in Table 9.1.

Table 9.1 D_t and B_t Values under Straight line Method of Depreciation

End of year (t)	*Depreciation* (D_t)	*Book value* ($B_t = B_{t-1} - D_t$)
0		1,00,000
1	10,000	90,000
2	10,000	80,000
3	10,000	70,000
4	10,000	60,000
5	10,000	50,000
6	10,000	40,000
7	10,000	30,000
8	10,000	20,000

If we are interested in computing D_t and B_t for a specific period (t), the formulae can be used. In this approach, it should be noted that the depreciation is the same for all the periods.

EXAMPLE 9.2 Consider Example 9.1 and compute the depreciation and the book value for period 5.

$$P = \text{Rs. } 1{,}00{,}000$$
$$F = \text{Rs. } 20{,}000$$
$$n = 8 \text{ years}$$
$$D_5 = (P - F)/n$$
$$= (1{,}00{,}000 - 20{,}000)/8$$
$$= \text{Rs. } 10{,}000 \text{ (This is independent of the time period.)}$$
$$B_t = P - t \times (P - F)/n$$
$$B_5 = 1{,}00{,}000 - 5 \times (1{,}00{,}000 - 20{,}000)/8$$
$$= \text{Rs. } 50{,}000$$

9.2.2 Declining Balance Method of Depreciation

In this method of depreciation, a constant percentage of the book value of the previous period of the asset will be charged as the depreciation amount for the current period. This approach is a more realistic approach, since the depreciation charge decreases with the life of the asset which matches with the earning potential of the asset. The book value at the end of the life of the asset may not be exactly equal to the salvage value of the asset. This is a major limitation of this approach.

Let

P = first cost of the asset,

F = salvage value of the asset,

n = life of the asset,

B_t = book value of the asset at the end of the period t,

K = a fixed percentage, and

D_t = depreciation amount at the end of the period t.

The formulae for depreciation and book value are as follows:

$$D_t = K \times B_{t-1}$$
$$B_t = B_{t-1} - D_t = B_{t-1} - K \times B_{t-1}$$
$$= (1 - K) \times B_{t-1}$$

The formulae for depreciation and book value in terms of P are as follows:

$$D_t = K(1 - K)^{t-1} \times P$$
$$B_t = (1 - K)^t \times P$$

While availing income-tax exception for the depreciation amount paid in each year, the rate K is limited to at the most $2/n$. If this rate is used, then the corresponding approach is called the *double declining balance method of depreciation.*

EXAMPLE 9.3 Consider Example 9.1 and demonstrate the calculations of the declining balance method of depreciation by assuming 0.2 for K.

Solution

P = Rs. 1,00,000

F = Rs. 20,000

n = 8 years

K = 0.2

The calculations pertaining to D_t and B_t for different values of t are summarized in Table 9.2 using the following formulae:

$$D_t = K \times B_{t-1}$$
$$B_t = B_{t-1} - D_t$$

Table 9.2 D_t and B_t according to Declining Balance Method of Depreciation

End of year (n)	*Depreciation* (D_t)	*Book value* (B_t)
0		1,00,000.00
1	20,000.00	80,000.00
2	16,000.00	64,000.00
3	12,800.00	51,200.00
4	10,240.00	40,960.00
5	8,192.00	32,768.00
6	6,553.60	26,214.40
7	5,242.88	20,971.52
8	4,194.30	16,777.22

If we are interested in computing D_t and B_t for a specific period t, the respective formulae can be used.

EXAMPLE 9.4 Consider Example 9.1 and calculate the depreciation and the book value for period 5 using the declining balance method of depreciation by assuming 0.2 for K.

Solution

P = Rs. 1,00,000

F = Rs. 20,000

n = 8 years

K = 0.2

$$D_t = K(1 - K)^{t-1} \times P$$
$$D_5 = 0.2(1 - 0.2)^4 \times 1,00,000$$
$$= \text{Rs. } 8,192$$

$$B_t = (1 - K)^t \times P$$
$$B_5 = (1 - 0.2)^5 \times 1,00,000$$
$$= \text{Rs. } 32,768$$

9.2.3 Sum-of-the-Years-Digits Method of Depreciation

In this method of depreciation also, it is assumed that the book value of the asset decreases at a decreasing rate. If the asset has a life of eight years, first the sum of the years is computed as

$$\text{Sum of the years} = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$$
$$= 36 = n(n + 1)/2$$

The rate of depreciation charge for the first year is assumed as the highest and then it decreases. The rates of depreciation for the years 1–8, respectively are as follows: 8/36, 7/36, 6/36, 5/36, 4/36, 3/36, 2/36, and 1/36.

For any year, the depreciation is calculated by multiplying the corresponding rate of depreciation with $(P - F)$.

$$D_t = \text{Rate} \times (P - F)$$
$$B_t = B_{t-1} - D_t$$

The formulae for D_t and B_t for a specific year t are as follows:

$$D_t = \frac{n - t + 1}{n(n + 1)/2}\,(P - F)$$

$$B_t = (P - F)\,\frac{(n - t)}{n}\,\frac{(n - t + 1)}{(n + 1)} + F$$

EXAMPLE 9.5 Consider Example 9.1 and demonstrate the calculations of the sum-of-the-years-digits method of depreciation.

Solution

P = Rs. 1,00,000

F = Rs. 20,000

n = 8 years

$$\text{Sum} = n(n + 1)/2 = 8 \times 9/2 = 36$$

The rates for years 1–8, are respectively 8/36, 7/36, 6/36, 5/36, 4/36, 3/36, 2/36 and 1/36.

The calculations of D_t and B_t for different values of t are summarized in Table 9.3 using the following formulae:

$$D_t = \text{Rate} \times (P - F)$$
$$B_t = B_{t-1} - D_t$$

Table 9.3 D_t and B_t under Sum-of-the-years-digits Method of Depreciation

End of year (n)	*Depreciation* (D_t)	*Book value* (B_t)
0		1,00,000.00
1	17,777.77	82,222.23
2	15,555.55	66,666.68
3	13,333.33	53,333.35
4	11,111.11	42,222.24
5	8,888.88	33,333.36
6	6,666.66	26,666.70
7	4,444.44	22,222.26
8	2,222.22	20,000.04

If we are interested in calculating D_t and B_t for a specific t, then the usage of the formulae would be better.

EXAMPLE 9.6 Consider Example 9.1 and find the depreciation and book value for the 5th year using the sum-of-the-years-digits method of depreciation.

Solution

P = Rs. 1,00,000

F = Rs. 20,000

n = 8 years

$$D_t = \frac{n - t + 1}{n(n + 1)/2}\,(P - F)$$

$$D_5 = \frac{8 - 5 + 1}{8(8 + 1)/2}\,(1,00,000 - 20,000)$$

$$= \text{Rs. } 8,888.88$$

$$B_t = (P - F)\,\frac{n - t}{n}\;\frac{n - t + 1}{n + 1} + F$$

$$B_5 = (1,00,000 - 20,000)\,\frac{8 - 5}{8}\;\frac{8 - 5 + 1}{8 + 1} + 20,000$$

$$= 80,000 \times (3/8) \times (4/9) + 20,000$$

$$= \text{Rs. } 33,333.33$$

9.2.4 Sinking Fund Method of Depreciation

In this method of depreciation, the book value decreases at increasing rates with respect to the life of the asset. Let

P = first cost of the asset,

F = salvage value of the asset,

n = life of the asset,

i = rate of return compounded annually,

A = the annual equivalent amount,

B_t = the book value of the asset at the end of the period t, and

D_t = the depreciation amount at the end of the period t.

The loss in value of the asset $(P - F)$ is made available an the form of cumulative depreciation amount at the end of the life of the asset by setting up an equal depreciation amount (A) at the end of each period during the lifetime of the asset.

$$A = (P - F) \times [A/F, i, n]$$

The fixed sum depreciated at the end of every time period earns an interest at the rate of i% compounded annually, and hence the actual depreciation amount will be in the increasing manner with respect to the time period. A generalized formula for D_t is

$$D_t = (P - F) \times (A/F, i, n) \times (F/P, i, t - 1)$$

The formula to calculate the book value at the end of period t is

$$B_t = P - (P - F)\ (A/F, i, n)\ (F/A, i, t)$$

The above two formulae are very useful if we have to calculate D_t and B_t for any specific period. If we calculate D_t and B_t for all the periods, then the tabular approach would be better.

EXAMPLE 9.7 Consider Example 9.1 and give the calculations regarding the sinking fund method of depreciation with an interest rate of 12%, compounded annually.

Solution

P = Rs. 1,00,000

F = Rs. 20,000

n = 8 years

i = 12%

$$\begin{aligned} A &= (P - F) \times [A/F, 12\%, 8] \\ &= (1{,}00{,}000 - 20{,}000) \times 0.0813 \\ &= \text{Rs. } 6{,}504 \end{aligned}$$

In this method of depreciation, a fixed amount of Rs. 6,504 will be depreciated at the end of every year from the earning of the asset. The depreciated amount will earn interest for the remaining period of life of the asset at an interest rate of 12%, compounded annually. For example, the calculations of net depreciation for some periods are as follows:

Depreciation at the end of year 1 (D_1) = Rs. 6,504.

Depreciation at the end of year 2 (D_2) = 6,504 + 6,504 × 0.12

= Rs. 7,284.48

Depreciation at the end of the year 3 (D_3)

$$= 6{,}504 + (6{,}504 + 7{,}284.48) \times 0.12$$
$$= \text{Rs. } 8{,}158.62$$

Depreciation at the end of year 4 (D_4)

$$= 6{,}504 + (6{,}504 + 7{,}284.48 + 8{,}158.62) \times 0.12$$
$$= \text{Rs. } 9{,}137.65$$

These calculations along with book values are summarized in Table 9.4.

Table 9.4 D_t and B_t according to Sinking Fund Method of Depreciation

End of year t	*Fixed depreciation* (Rs.)	*Net depreciation* D_t (Rs.)	*Book value* B_t (Rs.)
0	6,504	–	1,00,000.00
1	6,504	6,504.00	93,496.00
2	6,504	7,284.48	86,211.52
3	6,504	8,158.62	78,052.90
4	6,504	9,137.65	68,915.25
5	6,504	10,234.17	58,681.08
6	6,504	11,462.27	47,218.81
7	6,504	12,837.74	34,381.07
8	6,504	14,378.27	20,002.80

$B_t = B_{t-1} - D_t$

EXAMPLE 9.8 Consider Example 9.1 and compute D_5 and B_7 using the sinking fund method of depreciation with an interest rate of 12%, compounded annually.

Solution

P = Rs. 1,00,000

F = Rs. 20,000

n = 8 years

i = 12%

$$D_t = (P - F)\ (A/F, i, n)\ (F/P, i, t - 1)$$
$$D_5 = (P - F)\ (A/F, 12\%, 8)\ (F/P, 12\%, 4)$$
$$= (1{,}00{,}000 - 20{,}000) \times 0.0813 \times 1.574$$
$$= \text{Rs. } 10{,}237.30$$

This is almost the same as the corresponding value given in the table. The minor difference is due to truncation error.

$$B_t = P - (P - F)\ (A/F, i, n)\ (F/A, i, t)$$
$$B_7 = P - (P - F)\ (A/F, 12\%, 8)\ (F/A, 12\%, 7)$$
$$= 1{,}00{,}000 - (1{,}00{,}000 - 20{,}000) \times 0.0813 \times 10.089$$
$$= 34{,}381.10$$

9.2.5 Service Output Method of Depreciation

In some situations, it may not be realistic to compute depreciation based on time period. In such cases, the depreciation is computed based on service rendered by an asset. Let

P = first cost of the asset

F = salvage value of the asset

X = maximum capacity of service of the asset during its lifetime

x = quantity of service rendered in a period.

Then, the depreciation is defined per unit of service rendered:

Depreciation/unit of service = $(P - F)/X$

Depreciation for x units of service in a period = $\dfrac{P-F}{X}(x)$

EXAMPLE 9.9 The first coat of a road laying machine is Rs. 80,00,000. Its salvage value after five years is Rs. 50,000. The length of road that can be laid by the machine during its lifetime is 75,000 km. In its third year of operation, the length of road laid is 2,000 km. Find the depreciation of the equipment for that year.

Solution

P = Rs. 80,00,000

F = Rs. 50,000

X = 75,000 km

x = 2,000 km

Depreciation for x units of service in a period = $\dfrac{P-F}{X}x$

$$\text{Depreciation for year 3} = \frac{(80{,}00{,}000 - 50{,}000)}{75{,}000} \times 2{,}000$$

$$= \text{Rs. } 2{,}12{,}000$$

QUESTIONS

1. Define the following:
   (a) Depreciation
   (b) Book value
2. Distinguish between declining balance method of depreciation and double declining balance method of depreciation.
3. The Alpha Drug Company has just purchased a capsulating machine for Rs. 20,00,000. The plant engineer estimates that the machine has a useful

life of five years and a salvage value of Rs. 25,000 at the end of its useful life. Compute the depreciation schedule for the machine by each of the following depreciation methods:

(a) Straight line method of depreciation

(b) Sum-of-the-years digits method of depreciation

(c) Double declining balance method of depreciation

4. A company has recently purchased an overhead travelling crane for Rs. 25,00,000. Its expected life is seven years and the salvage value at the end of the life of the overhead travelling crane is Rs. 1,00,000. Using the straight line method of depreciation, find the depreciation and the book value at the end of third and fourth year after the crane is purchased.

5. An automobile company has purchased a wheel alignment device for Rs. 10,00,000. The device can be used for 15 years. The salvage value at the end of the life of the device is 10% of the purchase value. Find the following using the double declining balance method of depreciation:

(a) Depreciation at the end of the seventh year

(b) Depreciation at the end of the twelfth year

(c) Book value at the end of the eighth year

6. A company has purchased a bus for its officers for Rs. 10,00,000. The expected life of the bus is eight years. The salvage value of the bus at the end of its life is Rs. 1,50,000. Find the following using the sinking fund method of depreciation:

(a) Depreciation at the end of the third and fifth year

(b) Book value at the end of the second year and sixth year

7. Consider Problem 4 and find the following using the sum-of-the-years-digits method of depreciation:

(a) Depreciation at the end of the fourth year

(b) Depreciation at the end of the seventh year

(c) Book value at the end of the fifth year

(d) Book value at the end of the eighth year

8. A company has purchased a Xerox machine for Rs. 2,00,000. The salvage value of the machine at the end of its useful life would be insignificant. The maximum number of copies that can be taken during its lifetime is 1,00,00,000. During the fourth year of its operation, the number of copies taken is 9,00,000. Find the depreciation for the fourth year of operation of the Xerox machine using the service output method of depreciation.

9. A heavy construction firm has been awarded a contract to build a large concrete dam. It is expected that a total of eight years will be required to complete the work. The firm will buy Rs.1,80,00,000 worth of special equipment for the job. During the preparation of the job cost estimate, the following utilization schedule was computed for the special equipment:

Year	1	2	3	4	5	6	7	8
Hours/yr	6,000	4,000	4,000	1,600	800	800	2,200	2,200

At the end of the job, it is estimated that the equipment can be sold at auction for Rs. 18,00,000.

Prepare the depreciation schedule for all the years of operation of the equipment using the service output method of depreciation.

(b) What other considerations could affect the decision to launch the pilot project? Should the money already invested in research and development be a consideration? Why?
(c) Compare the range-of-estimates method of evaluation with sensitivity analysis.

11.16 Three revenue-producing projects are being considered. Estimates of future returns are uncertain because the projects involve new products. The initial investment is expected to provide adequate production capability for any reasonable project life, and the lives of all projects are considered to be equal. Regardless of useful life, there will be no salvage value.

Project *I*	Has a first cost of \$200,000 and uniform annual net revenue of \$65,000
Project *O*	Has a low initial cost of \$100,000 and will return \$50,000 the first year, but net revenue will decline each year by an amount $0.5G$, where G is a uniform gradient
Project *U*	Has a high initial cost of \$250,000 and returns \$50,000 the first year, with the expectation that new revenue will increase by an annual uniform amount G

Compare the three projects by constructing sensitivity graphs according to the following assumptions, and discuss the results.
(a) Let the project life be 6 years and $G = \$7500$ to test project preference for sensitivity to minimum rates of return up to 25 percent.
(b) Let the project life be 6 years and MARR = 12 percent to test project preference for sensitivity to values of G from 0 to \$15,000.
(c) Let MARR = 12 percent and $G = \$7500$ to test project preference for sensitivity to project life.

11.17 An electronics circuit-board manufacturer is considering the purchase of a new robot for component insertion. The robot will cost \$50,000 and is expected to be technologically obsolete in 6 years, at which time the manufacturer plans to donate it to the local university for \$5000. It is estimated to save \$24,000 per year in labor costs with annual operating costs being \$7500. The robot is a 5-year MACRS recovery asset. The manufacturer has an effective income tax rate of 42 percent and an after-tax MARR of 12 percent. Investigate the proposal's sensitivity to (a) annual savings, (b) operating costs, and (c) salvage value on an after-tax basis. Ignore inflation.

11.18 A project will have a life of 10 years and is to be evaluated at a discount rate of 9 percent. It has an initial cost of \$1 million, and expected annual costs are \$100,000. However, it is possible that these annual costs could consistently increase by \$10,000 each year or could decrease by \$5000 per year. The most likely annual benefits are \$350,000, but there is a chance that benefits will decrease by \$25,000 per year.
(a) What is the worst possible B/C ratio that could occur with the given scenario?
(b) What maximum first cost could be incurred for an acceptable project in which annual cash flows follow the expected (most likely) estimates?

CHAPTER 12

BREAK-EVEN ANALYSIS

No matter of fact can be mathematically demonstrated, though it may be proved in such a manner as to leave no doubt on the mind.

Richard Whately, *Logic IV*, 1826

In sensitivity analysis, if a decision is reversed, either from acceptance to rejection or between competing alternatives, then as a certain parameter is varied over a range of possible values, the decision is sensitive to that parameter; otherwise it is insensitive. Break-even analysis expresses a similar concept. Here the value of the parameter at decision reversal is determined. Break-even analysis is a limited form of sensitivity analysis. We are interested in determining a set of values for which an investment alternative is justified economically.

Many economic comparisons are a form of break-even analysis. The lease-or-buy question from Chap. 10 could be rephrased to ask about what level of service or time leasing becomes more expensive than buying. The point where the two alternatives are equal is the *break-even point*. Most sensitivity studies involve an indifference level for a given cash flow element at which two alternatives are equivalent—the break-even point for the given element. The choice between the two then rests on a judgment about which side of the break-even point the element will likely register.

In this chapter, break-even analysis is directed to the point at which operations merely break even, neither making nor losing money; changes in operations are evaluated according to their effect on this point. Break-even analysis, known also as *cost-volume-profit analysis*, is widely used for financial studies because it is simple and extracts useful insights from a modest amount of data. The studies necessarily include an examination of production costs and operating policies.

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Break-even analyses focus on cost-volume-profit relations that hold only over a short run. Over the long run, the relationships are altered by internal factors (new products, production facilities, etc.) and external impacts (competition, state of the general economy, etc.). Many of the internal activities that affect long-run changes are initiated from analyses of current cost-volume-profit conditions. Thus, a break-even analysis is like a medical checkup: The physical examination reveals the current state of health and provides clues about what should be done to become or stay healthy.

12.1 BASIC CONCEPTS

In break-even analysis, costs and revenues are expressed as a function of production rates. The cost-revenue-profit relations are exposed by breaking down a unit of output into its component dollar values. The rectangular block in Fig. 12.1*a* represents a unit of output. This output can be a product, such as an automobile, or it can be a service, such as collecting garbage from a subscriber. The unit is divided into three segments that classify the producer's interests. The overall height or price for which it can be sold is a function of the consumer's regard for the item.

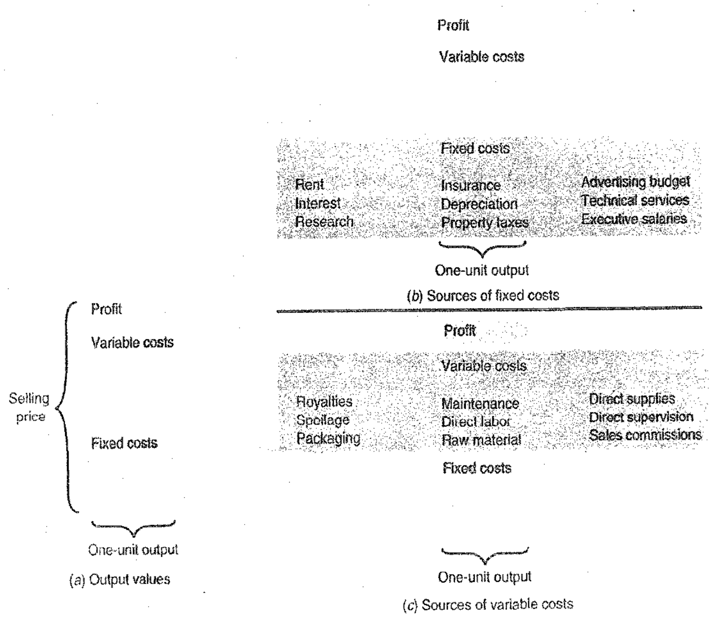

FIGURE 12.1
Unit costs.

Costs can be classified into two major categories: fixed and variable. Costs that remain relatively constant regardless of the level of activity are known as *fixed*, or *indirect, costs*. This description implies that the fixed level is maintained whether output is zero or at 100 percent capacity. In some cases this assumption is not valid; fixed costs may tend to increase as output increases, and they can vary with time. However, the change is usually not significant for short-run studies.

Some of an organization's expenditures that can be considered as fixed are shown in Fig. 12.1*b*. These costs may be thought of as "preparation" expenses. They arise from measures taken to provide the means to produce a product or service. Before painters can paint a house, they must have paintbrushes. Whether they paint one house or a dozen with the brushes, the expense has already been incurred and shows as a fixed cost. The painter's insurance and advertisements for work would also be indirect costs.

Costs that are generally proportional to output are called *variable*, or *direct, costs*. Such costs are relatively easy to determine because they are directly associated with a specific product or service. When there is no output, variable costs are zero. The input material and the time required to make a unit give rise to variable costs. For example, the specific type and quantity of paint used by house painters is a variable cost. The more houses they paint, the more paint they use; the quantity used is a function of their output. In a similar manner, the time they spend painting is a direct cost. Some sources for variable costs are shown in Fig. 12.1*c*.

Revenue results from sales of output. Profits represent the difference between revenue and total costs. The dimension of quantity must be included to examine the competitive aspects of profit. A single unit of output is relatively immune to competition. In isolated instances, a fair-sized output distributed in a local area to satisfy a particular need is also shielded from competition. However, as output quantity expands, competition is an increasingly apparent factor. Profit is the cause and effect of competitiveness.

A profit (or loss) figure attracts a great amount of attention. It is a handy yardstick of success. Like a thermometer, it measures only the level achieved; it does not control the source it measures. Unlike a thermometer, however, its continued low readings may convince the financial temperature takers to eliminate the source.

There are basically three ways to increase profit: increase the selling price, increase the value to increase sales, and decrease the selling price to increase sales. The profit expansion descriptions are oriented to consumers' interests. The issues become more complicated when we look at them from the producer's viewpoint. Figure 12.2 shows some of the consequences of selling-price manipulations.

The original price-cost-quantity conditions are shown in Fig. 12.2*a*. The *total revenue* is the product of n units sold at selling price P. The *total cost* is the sum of variable and fixed costs incurred in producing n units.

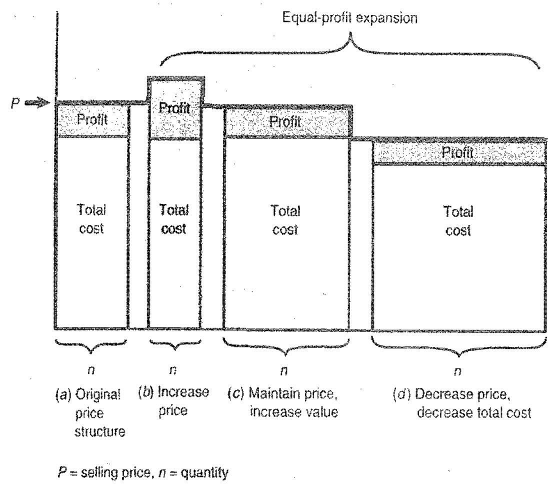

FIGURE 12.2 Methods for expanding profit.

Profit is the difference between revenue and total cost (when revenue exceeds costs).

Figure 12.2*b* through *d* shows increased profit. Conditions of free enterprise are assumed—similar products or services are available from a number of vendors. The shaded profit areas of Fig. 12.2*b*, *c*, and *d* are equal to or larger than the profit in *a*. The dangers and limitations of profit expansion methods are as follows:

Increased selling price. Competing products or services set an upper limit to price increases. This limit is ultimately controlled by the consumers. Their willingness to pay is a function of the value they expect to receive and their loyalty to a product. Prices higher than those of competing products of equivalent value will reduce the number of units sold. The shrinking share of the market eventually causes a decline in total profit.

Unchanged selling price. One way to increase profit without changing the selling price is to sell more units by increasing the value. The greater value perceived by the consumer can result from better quality, more quantity, or more effective advertising. All these measures increase the total cost of the producer. Higher total cost leads to a lower margin of profit per unit sold. If the market is unstable, a very low profit margin can seriously limit recuperative powers during market fluctuations.

A straightforward means to increase profit while holding prices constant is to reduce total cost. Such a task is the continuous aim of engineers and managers. The problem is that it becomes increasingly difficult to make more and more savings in an established operation. At first it is easy. When a product or service is new, it meets a high current demand that compensates for operational inefficiencies. As competition forces the price down, the "fat" is removed from operations. Further effort to reduce costs meets diminishing returns. It is like trying to make a horse run faster. A small whip may help at first, but using ever-larger whips fails to force proportional returns in ever-greater speeds.

Reduced selling price. New areas of cost reduction are exposed by changing the level of operations, or capacity. A greater output often allows new methods to be incorporated. Some of the savings resulting from the new methods are passed on to consumers in the form of a lower selling price. In theory, the decreased price should lead to the sale of more units, which in turn satisfies the conditions for incorporating the new methods.

Limitations are inherent throughout the cost reduction–lower price–increased sales cycle. Cost reductions are limited by minimum levels of quality, maximum levels of expenditure for new equipment, and basic labor or material costs that resist lowering. Reduced prices may be an insufficient incentive to attract enough new sales. However, with reasonable care the cycle rewards the producer and leads to a better standard of living for the consumer.

12.2 LINEAR BREAK-EVEN ANALYSIS

In linear break-even analysis, revenue and variable costs are directly proportional to output. There are three primary conditions for linear break-even analysis:

1. Income is only from operations under consideration.
2. Fixed costs, per-unit variable costs, and per-unit sales prices remain constant over time and over output.
3. All units produced are sold.

12.2.1 Break-Even Charts

The break-even chart presents two curves: a total-cost curve and a curve showing income from sales. The name *break-even chart* is derived from the concept it depicts—the volume or level at which the revenue and the total cost of operations exactly break even. At this point, one additional unit made and sold would produce a profit. Until the break-even point is attained, the producer operates at a loss for the period.

Properties of a typical break-even chart are displayed in Fig. 12.3. The vertical scale shows the revenue and costs in monetary units. The horizontal scale indicates the volume of activity n during the period pictured. The

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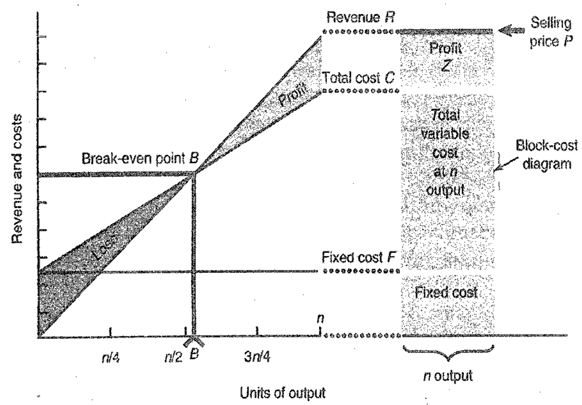

FIGURE 12.3 Standard format for break-even charts.

units of volume can be in sales dollars, number of units produced and sold, or output quantity expressed as a percentage of total capacity.

The horizontal line above the x axis shows the fixed costs F, which are constant throughout the range of volume. The sloping line originating at the intersection of the fixed-cost line and the vertical axis represents variable costs V plus the fixed costs. For linear relations, variable costs are directly proportional to volume; each additional unit produced adds an identical increment of cost. The sum of variable and fixed costs is the total cost C. The sloping line from the origin of the graph is the revenue line. Revenue R is also assumed to be directly proportional to the number of units produced and sold at price P.

The break-even point B occurs at the intersection of the total-cost and revenue lines. It thus specifies the dollar volume of sales and the unit volume of output at which an operation neither makes nor loses money. The vertical distance between the revenue line and the total-cost line indicates a profit Z to the right of B and a loss to the left.

A block-cost diagram used in the discussion of profit expansion is shown alongside the break-even chart in Fig. 12.3. For a specific output level n, the costs, revenue, and profit are the same in both formats. The break-even chart further indicates the profit or loss expectation at levels of output other than the specific quantity n. This feature helps explain such statements as "A very low profit margin can seriously limit recuperative powers during market fluctuations." *A very low profit margin* means that the output is barely on the profit side of the breakeven point. An unstable market could easily cause sales to fall below point B and show a loss for the period.

A break-even analysis often tends to oversimplify the decision environment. This is an attribute for presentation purposes and for gross evaluations. It can also be a shortcoming for problems in which detailed measures are needed. A decision to lower the break-even point for an operation can result from a study of total revenue and costs, but the study alone seldom reveals the in-plant operations that engineers and managers must conduct to implement the decision. The inability to identify tactical procedures is not a defect of a break-even analysis; it merely indicates that decision makers should be aware of the limitations of the approach in order to apply it appropriately. The validity of a break-even chart is directly proportional to the accuracy of the data incorporated in the chart. When several products are lumped together and represented by one line on a chart, there is a distinct possibility that poor performance by one product may go undetected. A firm should have a good cost accounting system, but data from past performances are not always indicative of future performance. However, examining graphs of previous break-even conditions calls attention to developing trends in revenues and costs.

Break-even relationships suggest where engineering efforts can be of most use to an organization. Field or factory-floor engineers can observe the present state of financial affairs and use such observations to guide their cost control activities. As an engineer's managerial responsibilities increase, the interplay of price, cost, and quantity becomes a greater concern. Then the combined effect of the system's operations and the underlying economic principles steer strategic decisions.

12.2.2 Algebraic Relationships

The graph format is convenient for clarifying or presenting economic relationships. It is possible to obtain quantities for particular conditions by scaling values from the chart. However, the same conditions can be easily quantified by formulas. Calculations generally provide greater accuracy. Using the symbols already defined, we have

$$\frac{\text{Revenue}}{\text{Period}} = R = nP$$

$$\frac{\text{Total cost}}{\text{Period}} = C = nV + F$$

$$\frac{\text{Gross profit}}{\text{Period}} = Z = R - C = n(P - V) - F$$

$$\frac{\text{Net profit}}{\text{Period}} = Z' = Z(1 - t)$$

where n can also be a fraction of total capacity when P and V represent total dollar volume at 100 percent capacity, and t is the tax rate.

At the break-even point, profit equals zero. To determine the output to simply break even, we have, at B,

$$Z = 0 = R - C = n(P - V) - F$$

and letting $n = B$, we have

$$B = \frac{F}{P - V}$$

The term $P - V$ is called the *contribution*. It indicates the portion of the selling price that contributes to paying off the fixed cost. At $n = B$ the sum of contributions from B units equals the total fixed cost. The contribution of each unit sold beyond $n = B$ is an increment of profit.

EXAMPLE 12.1 **Break-Even Analysis by the Numbers**

An airline is evaluating its feeder routes. These routes connect smaller cities to major terminals. The routes are seldom very profitable themselves, but they feed passengers into the major flights which yield better returns. One feeder route has a maximum capacity of 1000 passengers per month. The contribution from the fare of each passenger is 75 percent of the \$120 ticket price. Fixed costs per month are \$63,000. Determine the break-even point and net profit when the effective income tax rate is 40 percent.

Solution

To find the average percentage of seats that must be sold on each flight to break even, the cost and revenue data could be converted to the graphical break-even format shown in Fig. 12.4. The same information displayed in the break-even chart is supplied by the following calculations:

Total maximum revenue per month is

$$nP = 1000 \text{ passengers} \times \$120/\text{passenger} = \$120{,}000$$

$$\text{Total contribution} = 0.75 \times \$120{,}000 = \$90{,}000$$

$$B(\% \text{ of capacity}) = \frac{F \times 100\%}{\text{contribution}} = \frac{\$63{,}000}{\$90{,}000}(100\%) = 70\%$$

or

$$B(\text{passengers}) = \frac{F}{P - V} = \frac{\$63{,}000/\text{month}}{0.75 \times \$120/\text{passenger}} = 700 \text{ passengers per month}$$

With a tax rate t of 40 percent, the net profit at full capacity is

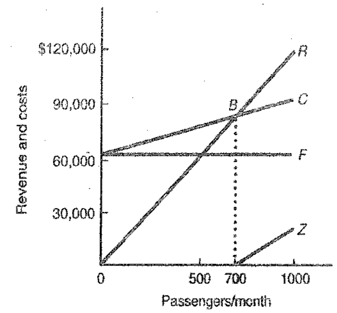

FIGURE 12.4 Break-even chart for an airline operation.

$$\begin{aligned}
\text{Net profit} &= Z(1 - t) \\
&= (R - C)(1 - t) \\
&= [n(P - V) - F](1 - t) \\
&= [1000 \text{ passengers}(0.75 \times \$120/\text{passenger}) - \$63{,}000](0.60) \\
&= \$27{,}000 \times 0.60 = \$16{,}200
\end{aligned}$$

A gross-profit line is shown in the lower right corner of the chart, starting at $B = 700$. Net profit is a fraction of Z, which depends on the tax rate for the total earnings of the organization.

12.2.3 Breakeven Point Alternatives

Any change in costs or selling price affects the break-even point. We observed the gross effects of profit expansion as a function of selling price. Now we can consider the interaction of revenue, variable costs, and fixed costs in terms of output.

A lower break-even point is a highly desirable objective. It means that the organization can meet fixed costs at a lower level of output or utilization. A sales level well above the break-even output is a sign of healthiness. Three methods of lowering the break-even point are shown in Fig. 12.5. The original operating conditions are shown as light lines and are based on the following data:

$$V = \$7 \text{ per unit}$$

$$P = \$12 \text{ per unit}$$

$$R(\text{at } n = 100 \text{ units}) = \$1200$$

$$C(\text{at } n = 100 \text{ units}) = \$1100$$

$$F = \$400$$

$$B = 80 \text{ units}$$

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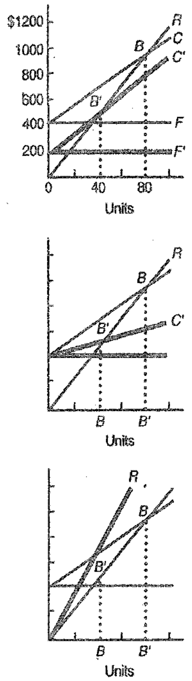

Fixed cost reduction: By reducing the fixed costs by half, B is halved. Therefore the new fixed cost F' is

$$F' = \frac{\$400}{2} = \$200$$

and the new breakeven output B' becomes

$$B' = \frac{F'}{P - V} = \$200$$

and

$$B' = \frac{\$200}{(\$12 - \$7)/\text{unit}} = 40 \text{ units}$$

Variable cost reduction: Knowing that B should equal 40 units, we can solve for the associated V' by

$$B' = \frac{F}{P - V'}$$

or

$$V' = P - \frac{F}{B'} = \$12/\text{unit} - \frac{\$400}{40 \text{ units}}$$

$$= \$2/\text{unit}$$

Selling price increase: Raising the selling price P' increases the slope of the revenue line and augments the contribution. The contribution required for B to equal 40 units is

$$\text{Contribution} = \frac{F}{B'} = \frac{\$400}{40 \text{ units}}$$

$$= \$10/\text{unit}$$

which leads to

$$P' = V + \text{contribution}$$
$$= \$7/\text{unit} + \$10/\text{unit} = \$17/\text{unit}$$

FIGURE 12.5
Break-even alternatives.

The bold lines depict the measures necessary to reduce the breakeven point by one-half, from 80 to 40 units.

EXAMPLE 12.2

Making More Money by Raising Costs and Decreasing the Price

A firm produces package waste-disposal units that sell for $35,000 each. Variable costs are $20,000 per unit, and fixed costs are $600,000. The plant can produce a maximum of 80 units per year. It is currently operating at 60 percent capacity. The firm is contemplating the effects of reducing the selling price by $2000 per unit, adding a feature to each unit that will increase the variable costs by $1000, and allocating an extra $120,000 per year for advertising. These actions are designed to sell enough additional units to raise plant utilization to 90 percent. Evaluate the alternatives.

Solution

Under current conditions

$$B = \frac{F}{P - V} = \frac{\$600{,}000}{(\$35{,}000 - \$20{,}000)/\text{unit}} = 40 \text{ units}$$

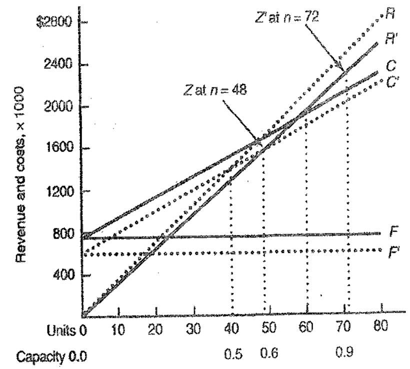

FIGURE 12.6
Original and anticipated conditions for profit improvement from increased plant utilization.

Since the company now sells 0.60 × 80 = 48 units per year, the gross annual profit is

$$Z = \text{units sold beyond } B \times \text{contribution per unit}$$
$$= (48 - 40) \text{ units } (\$15{,}000/\text{unit}) = \$120{,}000$$

Both the current conditions (dotted lines) and the anticipated conditions (solid lines) are displayed in Fig. 12.6. As graphed, B increases as a result of the added expenditures, to

$$B' = \frac{\$600{,}000 + \$120{,}000}{(\$33{,}000 - \$21{,}000)/\text{unit}} = \frac{\$720{,}000}{\$12{,}000/\text{unit}} = 60 \text{ units}$$

and the gross profit expected at 90 percent capacity also increases to

$$Z = [0.90(80 \text{ units}) - 60 \text{ units}][(\$33{,}000 - \$21{,}000)/\text{unit}]$$
$$= 12 \text{ units} \times \$12{,}000/\text{unit} = \$144{,}000$$

The firm could also follow a course of action in which only one or two of the alternatives are pursued. If the advertising budget is eliminated but price and modifications are retained, the same profit ($144,000) could be obtained at an output of

$$n = \frac{Z + F}{P - V} = \frac{\$144{,}000 + \$600{,}000}{(\$33{,}000 - \$21{,}000)/\text{unit}}$$
$$= \frac{\$744{,}000}{\$12{,}000/\text{unit}} = 62 \text{ units}$$

and

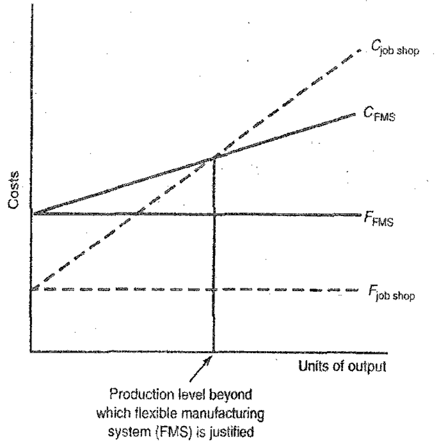

FIGURE 12.10 Break-even chart for comparing manufacturing alternatives.

production costs. The total cost at low production volumes is high for the flexible system due to underutilization of equipment. The break-even point represents the production volume required to justify the investment in the flexible manufacturing system.

12.3 NONLINEAR BREAK-EVEN ANALYSIS

Cost and revenue functions do not always follow convenient linear patterns. Often realistic cost relationships develop a nonlinear pattern, as typified by Table 12.1. The first four columns in the table relate output n to total fixed cost F, total variable cost, denoted by TV, and total cost C. The right side of the table shows average and marginal costs derived from the figures tabulated on the left.

Characteristic patterns of average and marginal costs based on Table 12.1 are pictured in Fig. 12.11.

Average fixed cost. Since fixed costs are independent of output, their per-unit amount declines as output increases. This feature is recognized when business people speak of "higher sales spreading the overhead":

$$\text{Average fixed cost} = \frac{F}{n}$$

Average variable cost. The typical saucer-shaped average-cost curve declines at first, reaches a minimum, and then increases thereafter. It reflects the law of diminishing returns. Initially, combining variable resources TV with fixed resources F produces increasing returns; but a point is reached where more and more variable resources must be applied

TABLE 12.1

Cost in dollars for nonlinear break-even analysis

Total product n	Total fixed cost F	Total variable cost TV	Total cost C	Average fixed cost F/n	Average total variable cost TV/n	Average total cost C/n	Marginal cost $\Delta C/\Delta n$
0	3,000	0	3000	—	—	—	
1	3,000	700	3700	3,000	700	3,700	700
2	3,000	1,300	4300	1,500	650	2,150	600
3	3,000	1,800	4800	1,000	600	1,600	500
4	3,000	2,400	5400	750	600	1,350	600
5	3,000	3,100	6100	600	620	1,220	700
6	3,000	3,900	6,900	500	650	1,150	800
7	3,000	4,900	7,900	429	700	1,129	1,000
8	3,000	6,200	9,200	375	775	1,150	1,300
9	3,000	7,800	10,800	333	867	1,200	1,600

to obtain each additional unit of output. Stated another way, a fixed plant is underemployed when its output is below the minimum average-cost point. As output expands, more complete utilization of the plant's capital equipment will make production more efficient. But continually increasing variable costs will eventually create a condition in which overcrowding and overutilization of equipment impair efficiency:

$$\text{Average total variable cost} = \frac{\text{TV}}{n}$$

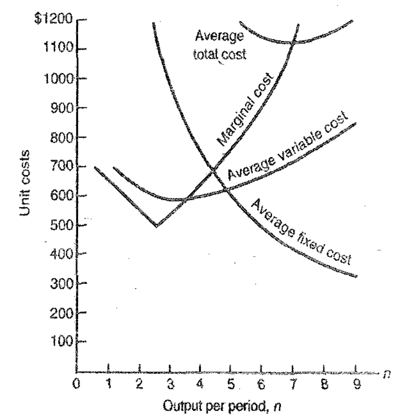

FIGURE 12.11 Relationship of average and marginal costs. Minimum points on the average- and total-cost curves occur where they cross the marginal-cost curve.

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Average total cost. Because the average total cost is simply the sum of average fixed and average variable costs, it shows the combined effects of spreading out fixed charges and diminishing returns from variable resources:

$$\text{Average total cost} = \frac{C}{n}$$

Marginal cost. The key to the cost pattern in Fig. 12.11 is contained in marginal-cost concepts. Marginal cost is calculated from either C or TV as the *extra increment of cost required to produce an additional unit of output*. If the last increment of cost is smaller than the average of all previous costs, it pulls the average down. Thus, *average total cost declines until it equals marginal cost. Equivalently, the rising marginal-cost curve also intersects the average total variable cost curve at its minimum point*:

$$\text{Marginal cost} = \frac{\Delta C}{\Delta n}$$

12.3.1 Marginal Revenue and Profit

Both nonlinear revenue and cost schedules may be expressed as formulas. When such equations are available, their analysis is not much more difficult than that of linear models. Since an assumption of linearity makes all monetary increments constant over an extended range of output, nonlinear models call more attention to marginal relationships.

Marginal revenue is the additional money received from selling one more unit at a specified level of output. For linear revenue functions, the marginal revenue is a constant value P. That is, for each additional unit sold, the total revenue is increased by P dollars. Consequently, a greater output automatically increases the total profit:

$$\text{Marginal revenue} = \frac{\Delta R}{\Delta n}$$

When the linear relationship is replaced by an expression such as

$$\text{Selling price } P = 21{,}000n^{-1/2} \qquad \text{dollars per unit}$$

the price of each unit is not so obvious. Such expressions are examined with differential calculus. For the price function above, the rate of change of revenue with output is

$$\text{Marginal revenue} = \frac{dR}{dn} = \frac{d(nP)}{dn} = \frac{d(21{,}000n^{1/2})}{dn}$$
$$= 10{,}500n^{-1/2}$$

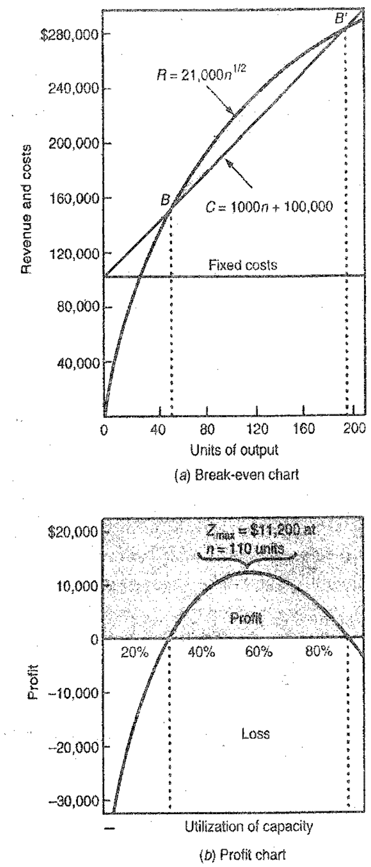

FIGURE 12.12 Nonlinear break-even charts for decreasing marginal revenue and linear costs.

Figure 12.12*a* shows a decelerating revenue rate and linear costs. Decreasing marginal revenue is more realistic because it takes into account the likely condition that could result from a policy of lowering prices in order to achieve a higher plant utilization. The effect is that of increasing revenue, but at a decreasing rate.

The nonlinear revenue curve fixes two break-even points. Between these two points the firm operates at a profit. Outside the break-even points a loss is incurred, as shown in the profit chart, Fig. 12.12*b*.

The graphs are based on the following data:

n = 1 unit produced and sold per period

V = \$1000 per unit