3.2 TIME VALUE OF MONEY

If an investor invests a sum of Rs. 100 in a fixed deposit for five years with an interest rate of 15% compounded annually, the accumulated amount at the end of every year will be as shown in Table 3.1.

Table 3.1 Compound Amounts

(amount of deposit = Rs. 100.00)

Year end	*Interest* (Rs.)	*Compound amount* (Rs.)
0		100.00
1	15.00	115.00
2	17.25	132.25
3	19.84	152.09
4	22.81	174.90
5	26.24	201.14

SIMPLE INTEREST

- $I = P\ i\ N$
- I=INTEREST EARNED.
- P=PRINCIPAL
- i= RATE OF INTEREST.
- N= NO OF INTEREST PERIOD(USUALLY YEARS)
- F=FUTURE SUM OF MONEY
- $F = P + I$, $P + PiN = P(1 + iN)$

SIMPLE INTEREST

- Find the future sum of money to be paid to a lendor for a loan amount of Rs 1000 for two months at a rate of 10%.
- $N=2/12$ year
- $F=1000(1+0.1\times 2/12)$
- Rs 1016.67
- AT WHAT RATE OF SIMPLE INTEREST A SUM OF MONEY WILL DOUBLE IN 8 YEARS.
- A SUM WAS PUT AT CERTAIN RATE FOR 2 YEARS. HAD IT BEEN PUT AT 3% HIGHER RATE, IT WOULD HAVE FETCHED Rs 300 MORE.. FIND THE SUM.

- SUM=X
- 2X=X(1+iN/100), 2=1+iN/100, 1=iN/100, i=100/8 YEARS(12.5)

- X(R+3)2/100 –XR2/100 =300
- 2XR+6X-2XR=300x100
- X=300x100/6 = 5000

The formula to find the future worth in the third column is

$$F = P \times (1 + i)^n$$

where

P = principal amount invested at time 0,

F = future amount,

i = interest rate compounded annually,

n = period of deposit.

The maturity value at the end of the fifth year is Rs. 201.14. This means that the amount Rs. 201.14 at the end of the fifth year is equivalent to Rs. 100.00 at time 0 (i.e. at present). This is diagrammatically shown in Fig. 3.1. This explanation assumes that the inflation is at zero percentage.

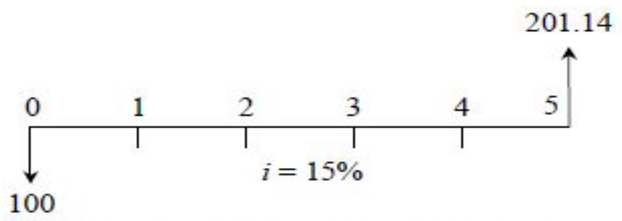

Fig. 3.1 Time value of money.

End of year (n)	*Present worth*	*Compound amount after n year(s)*
0		100
1	86.96	100
2	75.61	100
3	65.75	100
4	57.18	100
5	49.72	100
6	43.29	100
7	37.59	100
8	32.69	100
9	28.43	100
10	24.72	100

The formula to find the present worth in the second column is

$$P = \frac{F}{(1+i)^n}$$

The notations which are used in various interest formulae are as follows:

P = principal amount

n = No. of interest periods

i = interest rate (It may be compounded monthly, quarterly, semiannually or annually)

F = future amount at the end of year n

A = equal amount deposited at the end of every interest period

G = uniform amount which will be added/subtracted period after period to/from the amount of deposit A1 at the end of period 1

3.3.1 Single-Payment Compound Amount

Here, the objective is to find the single future sum (F) of the initial payment (P) made at time 0 after n periods at an interest rate i compounded every period. The cash flow diagram of this situation is shown in Fig. 3.2.

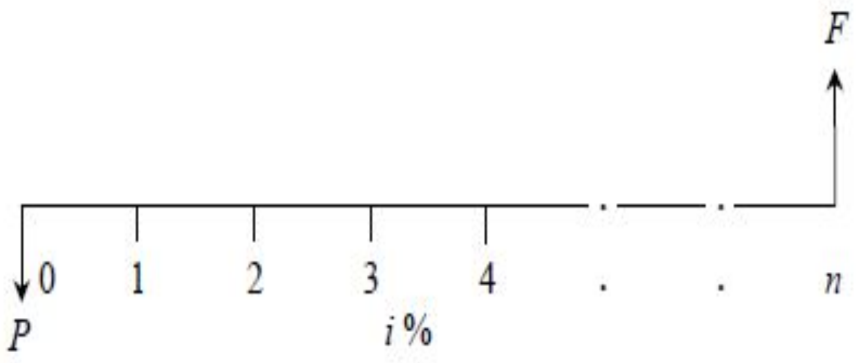

Fig. 3.2 Cash flow diagram of single-payment compound amount.

The formula to obtain the single-payment compound amount is

$$F = P(1 + i)^n = P(F/P, i, n)$$

where

$(F/P, i, n)$ is called as single-payment compound amount factor.

EXAMPLE 3.1 A person deposits a sum of Rs. 20,000 at the interest rate of 18% compounded annually for 10 years. Find the maturity value after 10 years.

EXAMPLE 3.1 A person deposits a sum of Rs. 20,000 at the interest rate of 18% compounded annually for 10 years. Find the maturity value after 10 years.

Solution

$$P = \text{Rs. } 20{,}000$$
$$i = 18\% \text{ compounded annually}$$
$$n = 10 \text{ years}$$
$$F = P(1 + i)^n = P(F/P, i, n)$$
$$= 20{,}000\ (F/P,\ 18\%,\ 10)$$
$$= 20{,}000 \times 5.234 = \text{Rs. } 1{,}04{,}680$$

The maturity value of Rs. 20,000 invested now at 18% compounded yearly is equal to Rs. 1,04,680 after 10 years.

3.3.2 Single-Payment Present Worth Amount

Here, the objective is to find the present worth amount (P) of a single future sum (F) which will be received after n periods at an interest rate of i compounded at the end of every interest period.

The corresponding cash flow diagram is shown in Fig. 3.3.

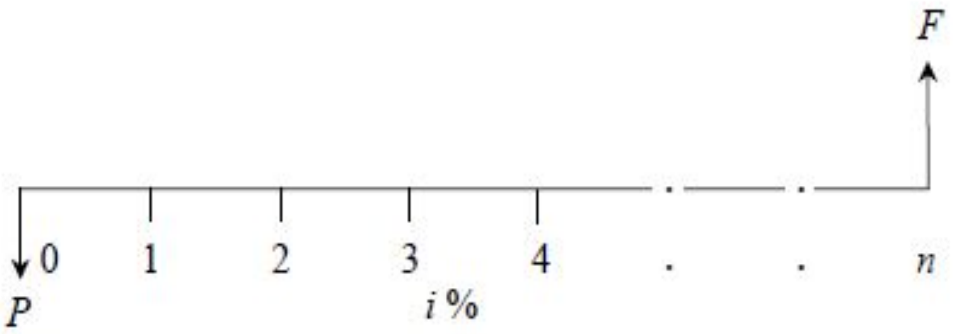

Fig. 3.3 Cash flow diagram of single-payment present worth amount.

The formula to obtain the present worth is

$$P = \frac{F}{(1+i)^n} = F(P/F, i, n)$$

where

$(P/F, i, n)$ is termed as *single-payment present worth factor.*

EXAMPLE 3.2 A person wishes to have a future sum of Rs. 1,00,000 for his son's education after 10 years from now. What is the single-payment that he should deposit now so that he gets the desired amount after 10 years? The bank gives 15% interest rate compounded annually.

Solution

$$F = \text{Rs. } 1{,}00{,}000$$

$$i = 15\%, \text{ compounded annually}$$

$$n = 10 \text{ years}$$

$$P = F/(1 + i)^n = F(P/F, i, n)$$

$$= 1{,}00{,}000\ (P/F, 15\%, 10)$$

$$= 1{,}00{,}000 \times 0.2472$$

$$= \text{Rs. } 24{,}720$$

The person has to invest Rs. 24,720 now so that he will get a sum of Rs. 1,00,000 after 10 years at 15% interest rate compounded annually.

3.3.8 Effective Interest Rate

Let i be the nominal interest rate compounded annually. But, in practice, the compounding may occur less than a year. For example, compounding may be monthly, quarterly, or semi-annually. Compounding monthly means that the interest is computed at the end of every month. There are 12 interest periods in a year if the interest is compounded monthly. Under such situations, the formula to compute the effective interest rate, which is compounded annually, is

$$\text{Effective interest rate, } R = (1 + i/C)^C - 1$$

where,

i = the nominal interest rate

C = the number of interest periods in a year.

EXAMPLE 3.9 A person invests a sum of Rs. 5,000 in a bank at a nominal interest rate of 12% for 10 years. The compounding is quarterly. Find the maturity amount of the deposit after 10 years.

METHOD 1

No. of interest periods per year = 4

No. of interest periods in 10 years = $10 \times 4 = 40$

Revised No. of periods (No. of quarters), $N = 40$

Interest rate per quarter, $r = 12\%/4$

$= 3\%$, compounded quarterly.

$$F = P(1 + r)^N = 5{,}000(1 + 0.03)^{40}$$

$$= \text{Rs. } 16{,}310.19$$

METHOD 2

No. of interest periods per year, $C = 4$

Effective interest rate, $R = (1 + i/C)^C - 1$

$= (1 + 12\%/4)^4 - 1$

$= 12.55\%$, compounded annually.

$$F = P(1 + R)^n = 5{,}000(1 + 0.1255)^{10}$$

$$= \text{Rs. } 16{,}308.91$$

3.3.3 Equal-Payment Series Compound Amount

In this type of investment mode, the objective is to find the future worth of n equal payments which are made at the end of every interest period till the end of the nth interest period at an interest rate of i compounded at the end of each interest period. The corresponding cash flow diagram is shown in Fig. 3.4.

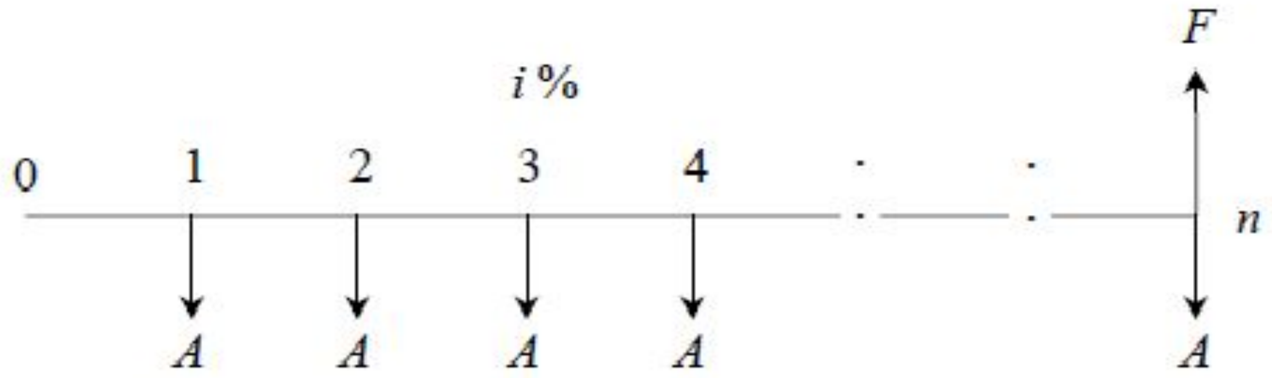

Fig. 3.4 Cash flow diagram of equal-payment series compound amount.

In Fig. 3.4,

A = equal amount deposited at the end of each interest period

n = No. of interest periods

i = rate of interest

F = single future amount

$$F = A\,\frac{(1+i)^n - 1}{i} = A(F/A, i, n)$$

where

$(F/A, i, n)$ is termed as *equal-payment series compound amount factor.*

EXAMPLE 3.3 A person who is now 35 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for the next 25 years starting from the end of the next year. The bank gives 20% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.

The corresponding cash flow diagram is shown in Fig. 3.5.

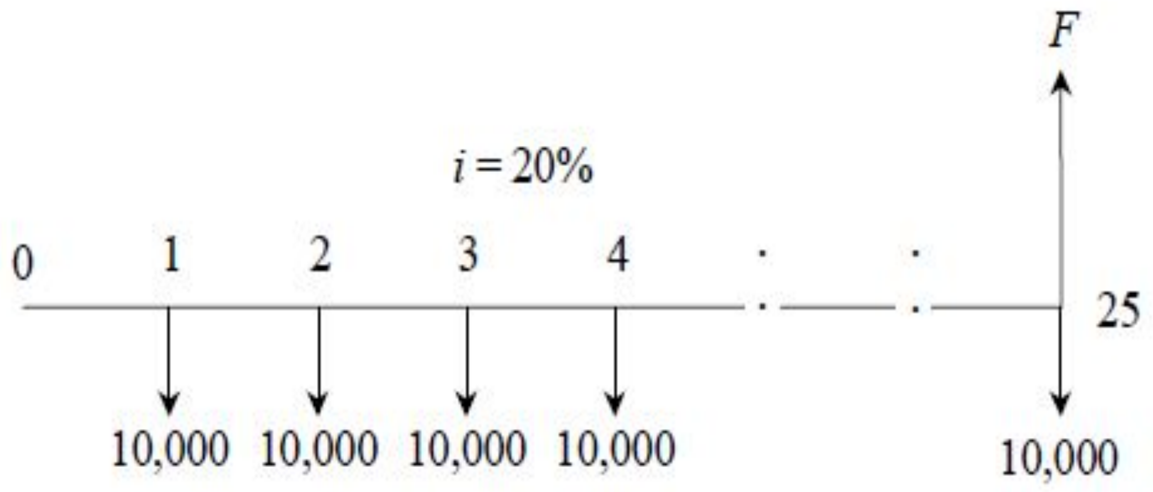

Fig. 3.5 Cash flow diagram of equal-payment series compound amount.

$$F = A\frac{(1+i)^n - 1}{i}$$

$$= A(F/A, i, n)$$

$$= 10{,}000(F/A, 20\%, 25)$$

$$= 10{,}000 \times 471.981$$

$$= \text{Rs. } 47{,}19{,}810$$

3.3.4 Equal-Payment Series Sinking Fund

In this type of investment mode, the objective is to find the equivalent amount (A) that should be deposited at the end of every interest period for n interest periods to realize a future sum (F) at the end of the nth interest period at an interest rate of i.

The corresponding cash flow diagram is shown in Fig. 3.6.

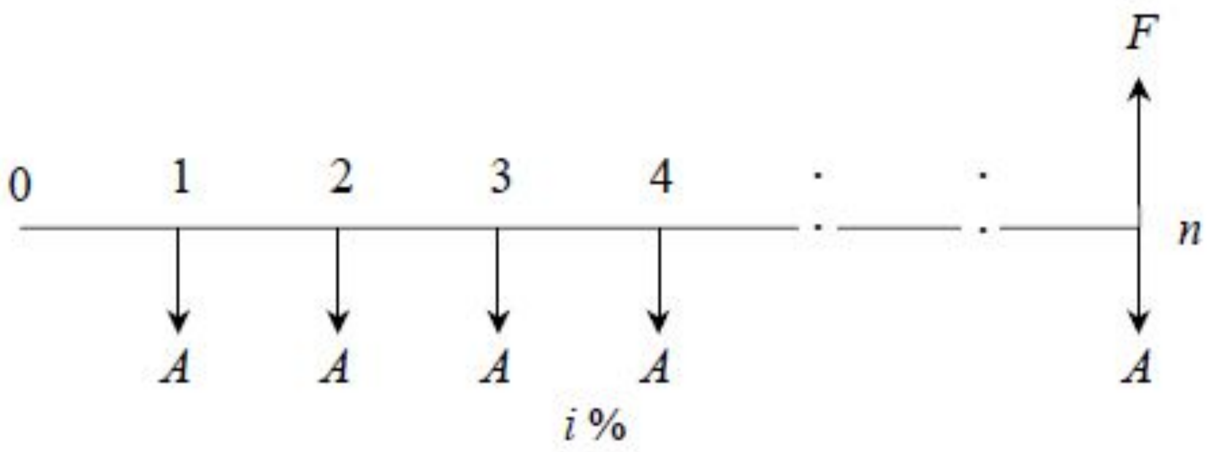

Fig. 3.6 Cash flow diagram of equal-payment series sinking fund.

The formula to get F is

$$A = F \frac{i}{(1+i)^n - 1} = F(A/F, i, n)$$

where

$(A/F, i, n)$ is called as *equal-payment series sinking fund factor.*

EXAMPLE 3.4 A company has to replace a present facility after 15 years at an outlay of Rs. 5,00,000. It plans to deposit an equal amount at the end of every year for the next 15 years at an interest rate of 18% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 15 years.

The corresponding cash flow diagram is shown in Fig. 3.7.

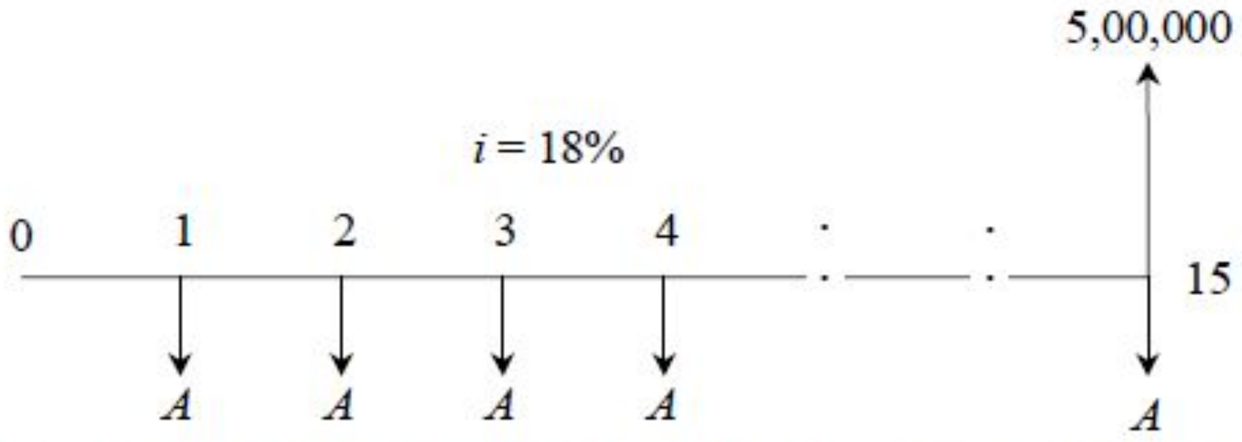

Fig. 3.7 Cash flow diagram of equal-payment series sinking fund.

$$A = F \frac{i}{(1+i)^n - 1} = F(A/F, i, n)$$

$$= 5{,}00{,}000(A/F, 18\%, 15)$$

$$= 5{,}00{,}000 \times 0.0164$$

$$= \text{Rs. } 8{,}200$$

The annual equal amount which must be deposited for 15 years is Rs. 8,200.

3.3.5 Equal-Payment Series Present Worth Amount

The objective of this mode of investment is to find the present worth of an equal

The formula to compute P is

$$P = A\,\frac{(1+i)^n - 1}{i(1+i)^n} = A(P/A,\ i,\ n)$$

where

$(P/A,\ i,\ n)$ is called *equal-payment series present worth factor.*

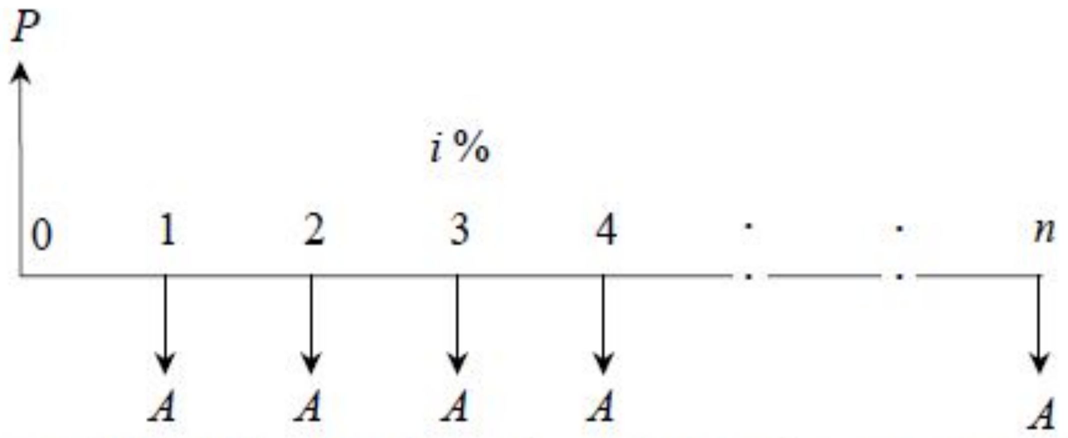

Fig. 3.8 Cash flow diagram of equal-payment series present worth amount.

EXAMPLE 3.5 A company wants to set up a reserve which will help the company to have an annual equivalent amount of Rs. 10,00,000 for the next 20 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made now as the reserve amount.

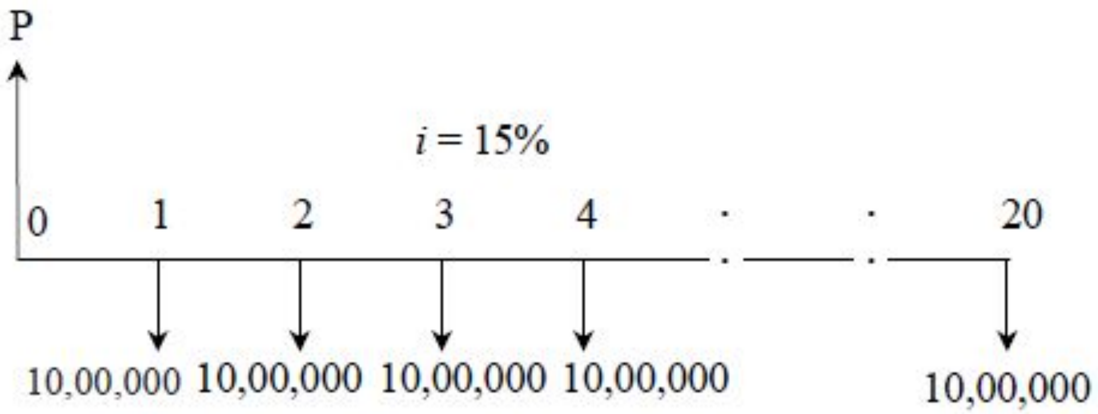

Fig. 3.9 Cash flow diagram of equal-payment series present worth amount.

$$P = A\frac{(1+i)^n - 1}{i(1+i)^n} = A(P/A, i, n)$$

$$= 10{,}00{,}000 \times (P/A, 15\%, 20)$$

$$= 10{,}00{,}000 \times 6.2593$$

$$= \text{Rs. } 62{,}59{,}300$$

The amount of reserve which must be set-up now is equal to Rs. 62,59,300.

3.3.6 Equal-Payment Series Capital Recovery Amount

The objective of this mode of investment is to find the annual equivalent amount (A) which is to be recovered at the end of every interest period for n interest periods for a loan (P) which is sanctioned now at an interest rate of i compounded at the end of every interest period (see Fig. 3.10).

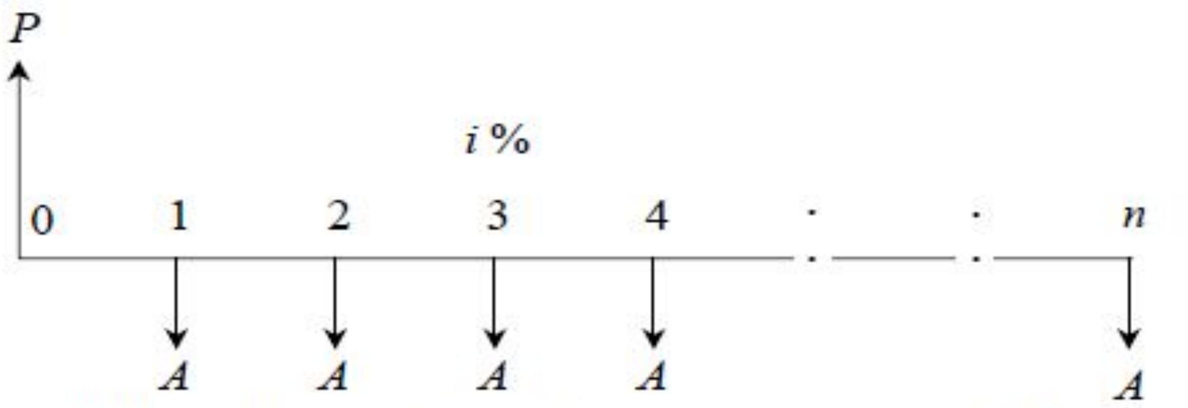

Fig. 3.10 Cash flow diagram of equal-payment series capital recovery amount.

In Fig. 3.10,

P = present worth (loan amount)
A = annual equivalent payment (recovery amount)
i = interest rate
n = No. of interest periods

The formula to compute P is as follows:

The formula to compute P is as follows:

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1} = P(A/P, i, n)$$

where,

$(A/P, i, n)$ is called *equal-payment series capital recovery factor*.

EXAMPLE 3.6 A bank gives a loan to a company to purchase an equipment worth Rs. 10,00,000 at an interest rate of 18% compounded annually. This amount should be repaid in 15 yearly equal installments. Find the installment amount that the company has to pay to the bank.

3.3.7 Uniform Gradient Series Annual Equivalent Amount

The objective of this mode of investment is to find the annual equivalent amount of a series with an amount $A1$ at the end of the first year and with an equal increment (G) at the end of each of the following $n - 1$ years with an interest rate i compounded annually.

The corresponding cash flow diagram is shown in Fig. 3.12.

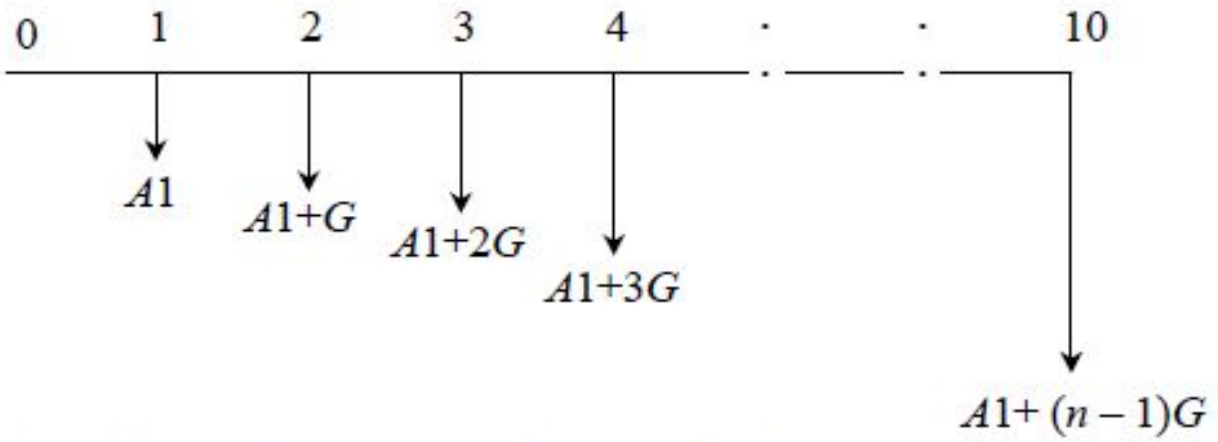

Fig. 3.12 Cash flow diagram of uniform gradient series annual equivalent amount.

The formula to compute A under this situation is

$$A = A1 + G\,\frac{(1+i)^n - in - 1}{i(1+i)^n - i}$$

$$= A1 + G\,(A/G,\ i,\ n)$$

A person is planning for his retired life. He has 10 more years of service. He would like to deposit 20% of his salary, which is Rs. 4,000, at the end of the first year, and thereafter he wishes to deposit the amount with an annual increase of Rs. 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th year of the above series.

The cash flow diagram is shown in Fig. 3.13.

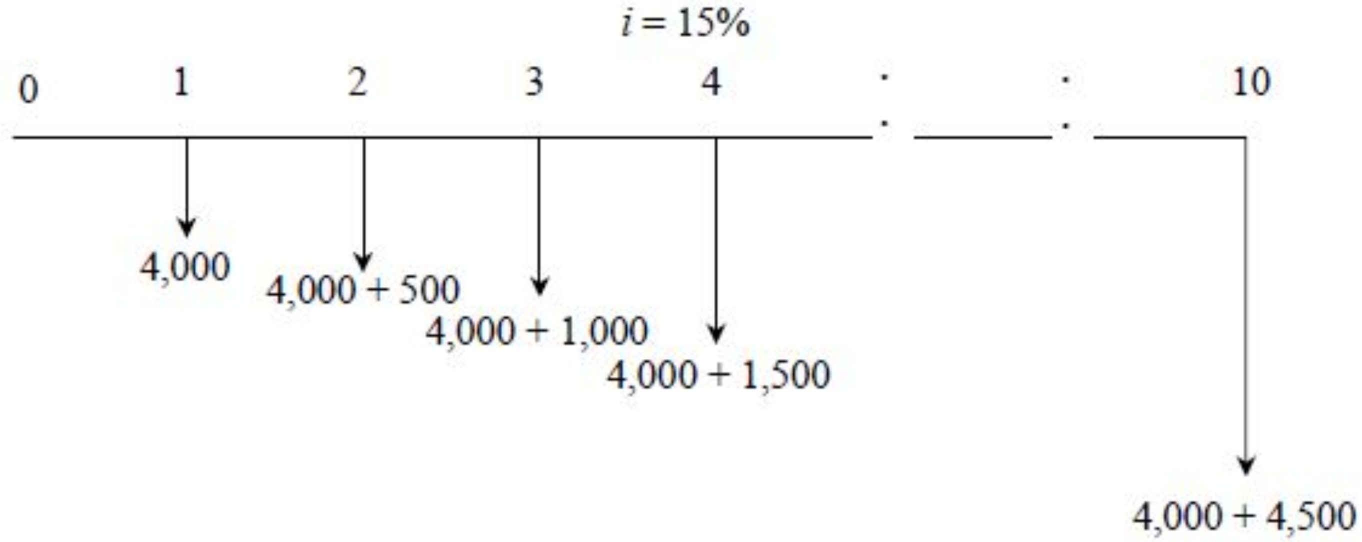

Fig. 3.13 Cash flow diagram of uniform gradient series annual equivalent amount.

$$
\begin{aligned}
A &= A1 + G\,\frac{(1+i)^n - in - 1}{i(1+i)^n - i} \\
&= A1 + G(A/G,\, i,\, n) \\
&= 4{,}000 + 500(A/G,\, 15\%,\, 10) \\
&= 4{,}000 + 500 \times 3.3832 \\
&= \text{Rs. } 5{,}691.60
\end{aligned}
$$

This is equivalent to paying an equivalent amount of Rs. 5,691.60 at the end of every year for the next 10 years. The future worth sum of this revised series at the end of the 10th year is obtained as follows:

$$
\begin{aligned}
F &= A(F/A, i, n) \\
&= A(F/A, 15\%, 10) \\
&= 5{,}691.60(20.304) \\
&= \text{Rs. } 1{,}15{,}562.25
\end{aligned}
$$

At the end of the 10th year, the compound amount of all his payments will be Rs. 1,15,562.25.

EXAMPLE 3.8 A person is planning for his retired life. He has 10 more years of service. He would like to deposit Rs. 8,500 at the end of the first year and thereafter he wishes to deposit the amount with an annual decrease of Rs. 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th year of the above series.

The cash flow diagram is shown in Fig. 3.14.

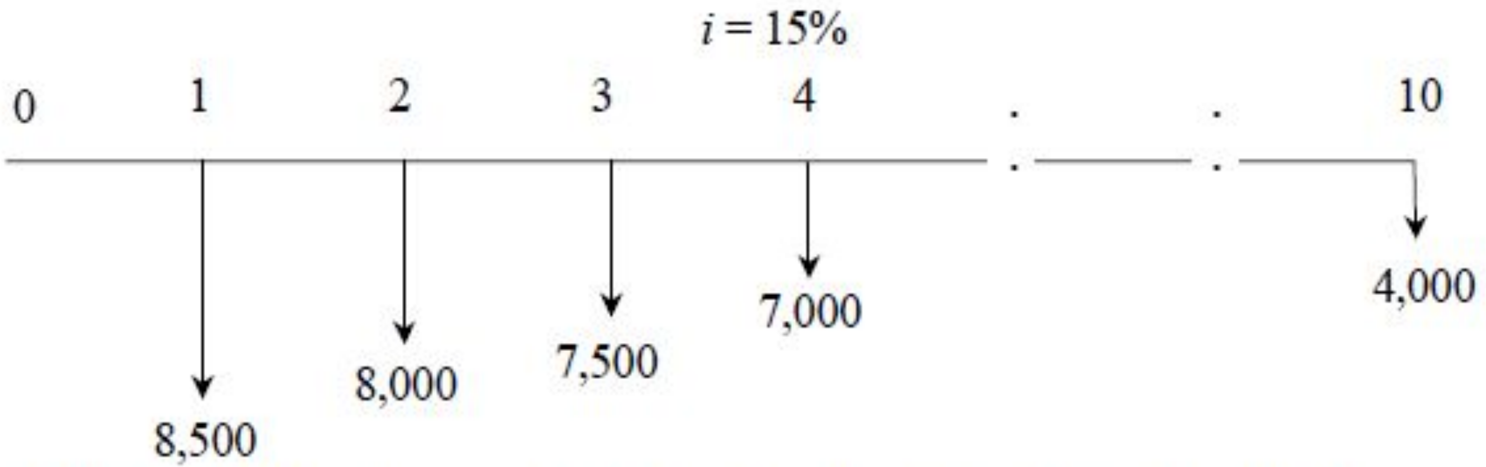

Fig. 3.14 Cash flow diagram of uniform gradient series annual equivalent amount.

$$A = A1 - G\,\frac{(1+i)^n - in - 1}{i(1+i)^n - i}$$

$$= A1 - G\,(A/G, i, n)$$

$$= 8{,}500 - 500(A/G, 15\%, 10)$$

$$= 8{,}500 - 500 \times 3.3832$$

$$= \text{Rs. } 6{,}808.40$$

This is equivalent to paying an equivalent amount of Rs. 6,808.40 at the end of every year for the next 10 years.

The future worth sum of this revised series at the end of the 10th year is obtained as follows:

$$
\begin{aligned}
F &= A(F/A, i, n) \\
&= A(F/A, 15\%, 10) \\
&= 6{,}808.40(20.304) \\
&= \text{Rs. } 1{,}38{,}237.75
\end{aligned}
$$

At the end of the 10th year, the compound amount of all his payments is Rs. 1,38,237.75.

Time taken for doubling the money in compound interest.

- 4 %…………………….18YEARS
- 6%...................12 YEARS
- 8%....................9 YEARS