



# Short-term Synaptic Plasticity

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A few examples of STP embedded network dynamics



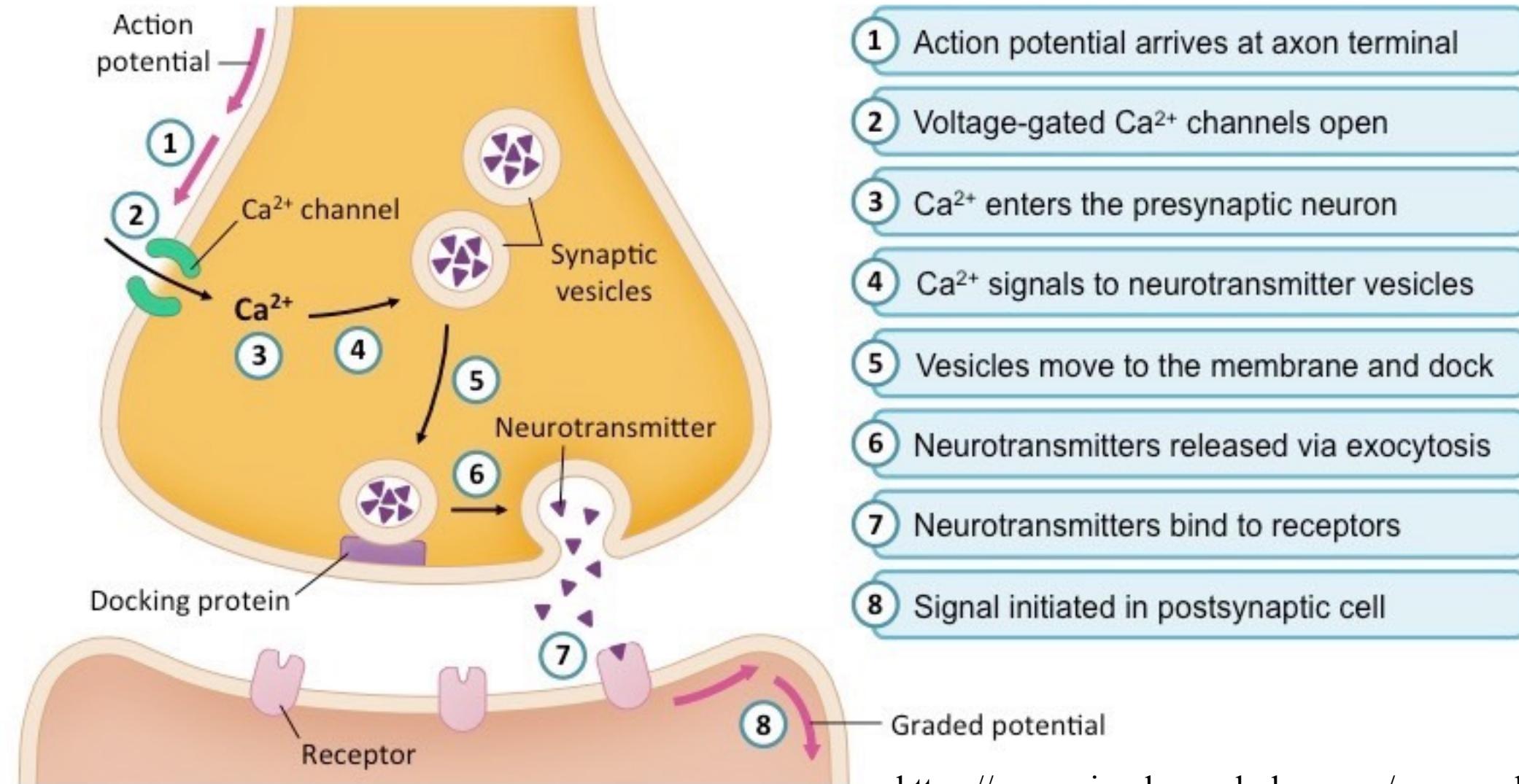
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# 01

## Synaptic transmission and plasticity

# Process of Chemical synaptic transmission



# EPSP and EPSC

EPSP: Excitatory Post Synaptic Potential

EPSC: Excitatory Post Synaptic Current

Post synaptic current:

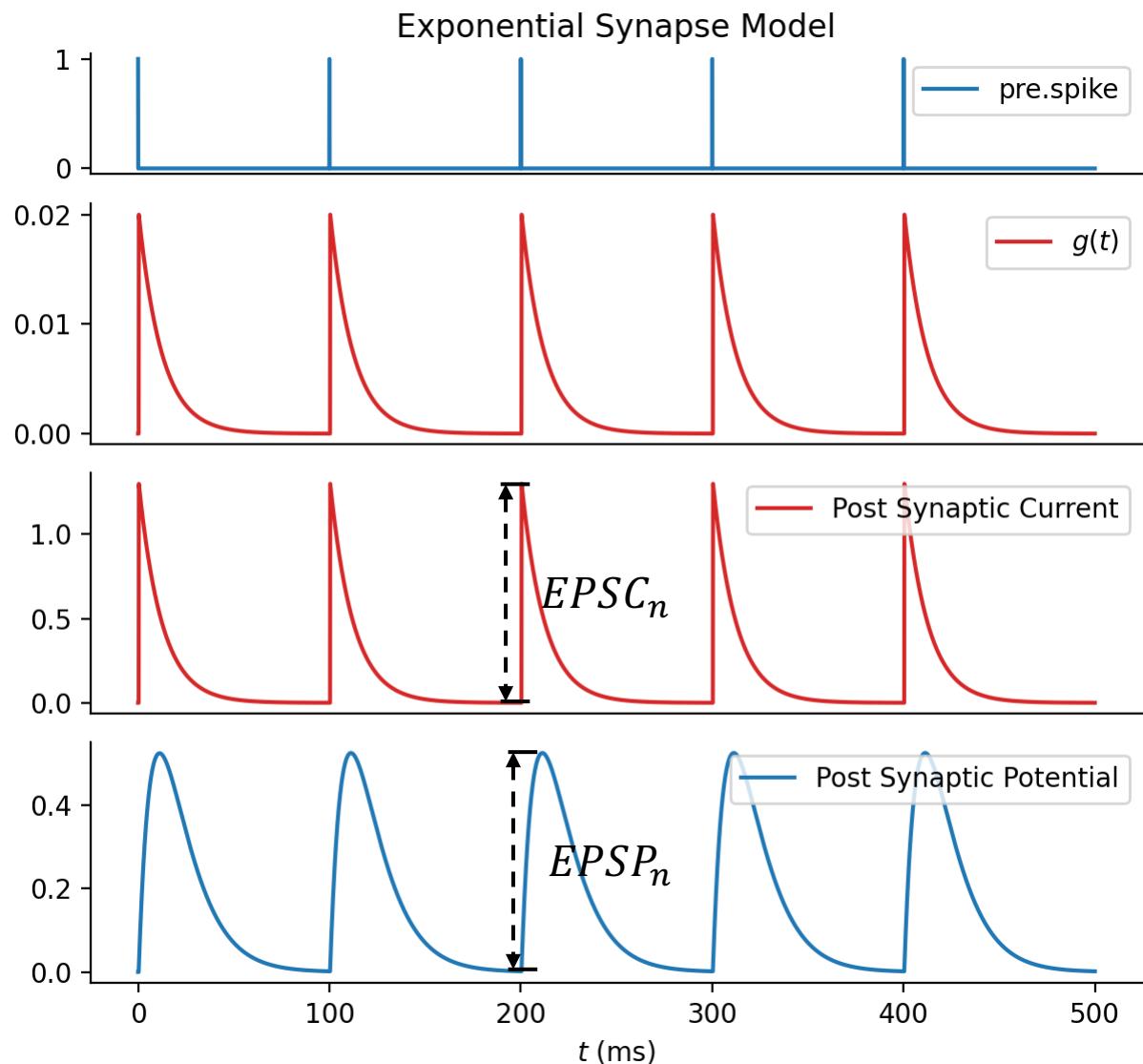
$$I(t) = g(t)[V_{\text{post}}(t) - E_{\text{reversal}}]$$

Dynamics of post-synaptic conductance  
(exponential model):

$$\frac{dg(t)}{dt} = -\frac{g(t)}{\tau_s} + A\delta(t - t_{sp})$$

The synaptic strength is characterized as EPSC,  
which refers to the post synaptic current  
increment at each spike,

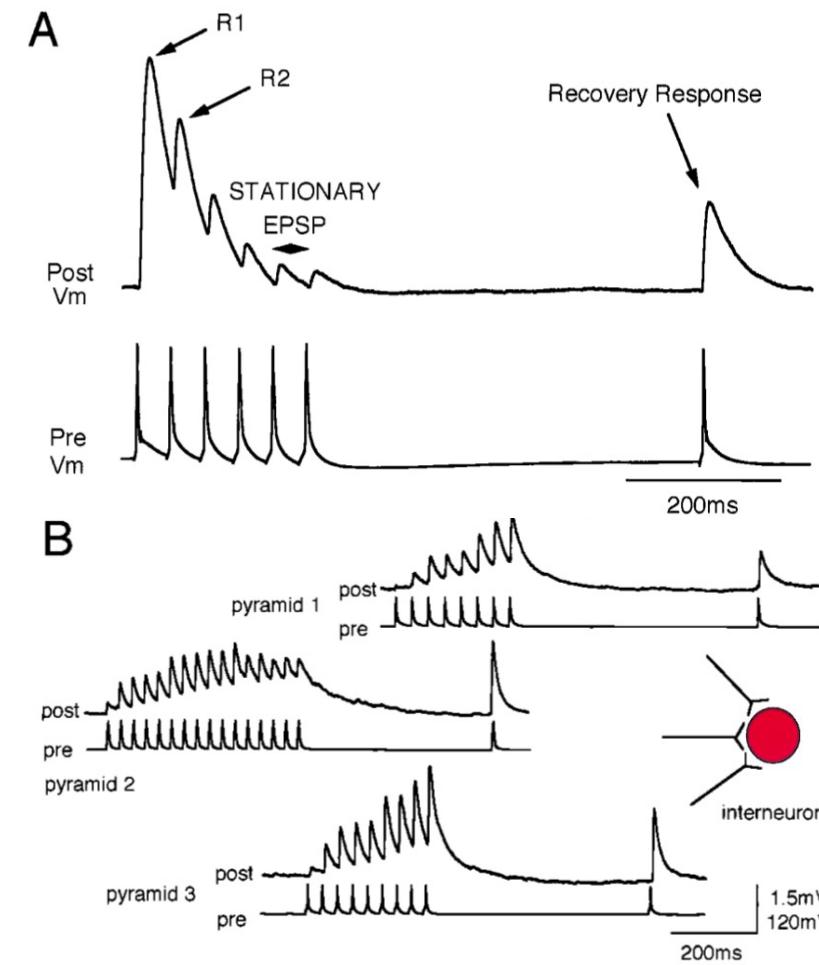
$$EPSC_n = A(V_{\text{rest}} - E_{\text{reversal}})$$



# Synaptic plasticity



Short-term plasticity  
(timescale: 1ms-1s)



Long-term plasticity  
(timescale: > 1mins)

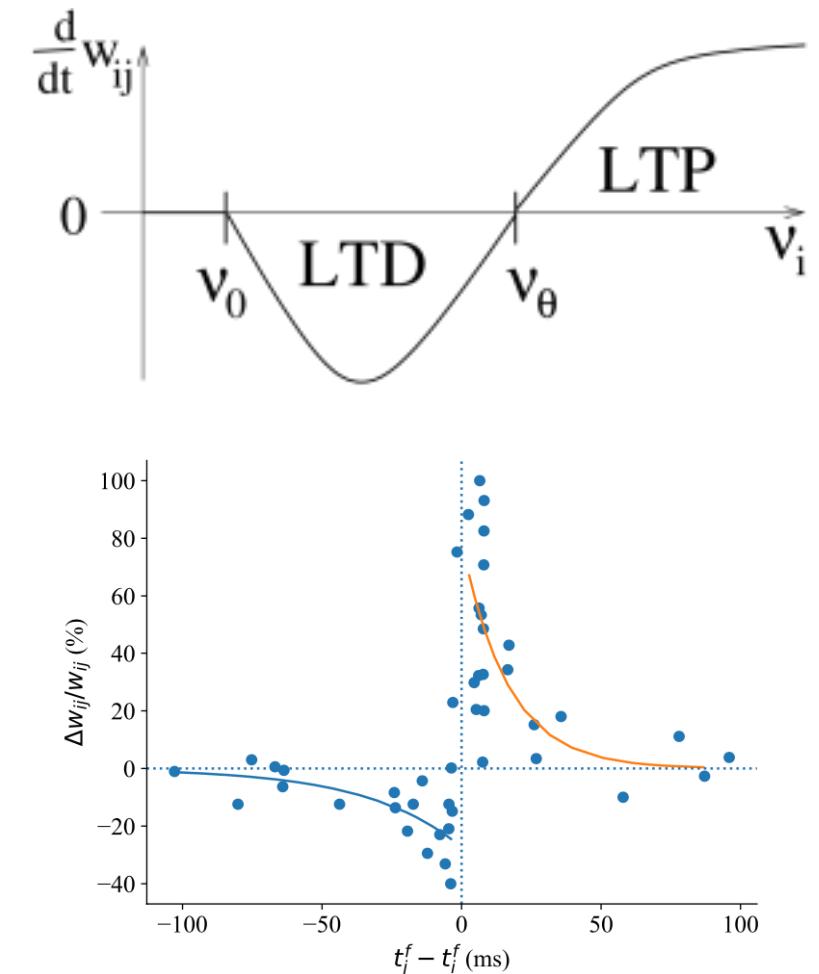


图 5.3: 脉冲时序依赖可塑性<sup>[34]</sup>



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# 02

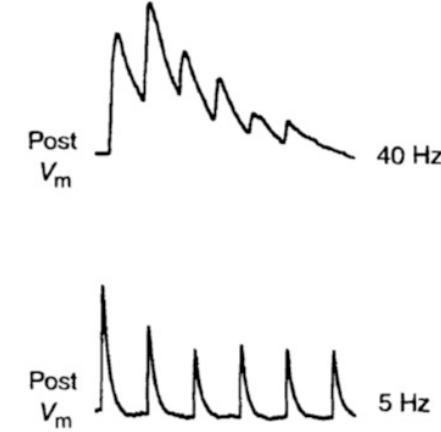
## Phenomenological model of Short-term Plasticity

# Short-term depression observed between pyramidal cells

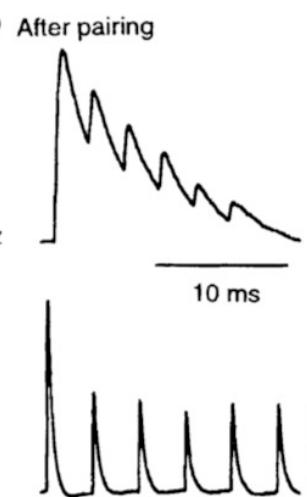
Pyramidal cell – Pyramidal cells



a Before pairing



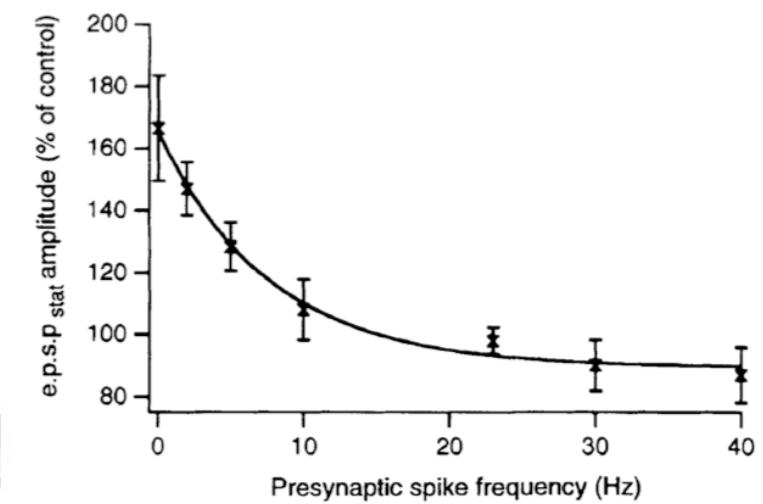
b After pairing



## Redistribution of synaptic efficacy between neocortical pyramidal neurons

Henry Markram & Misha Tsodyks

Department of Neurobiology, The Weizmann Institute for Science,  
Rehovot 76100, Israel



Markram et al., 1996, Nature

# Modeling neuro-transmitter consumption

Dynamics of three-factor STD:

$$\frac{dx(t)}{dt} = \frac{z(t)}{\tau_{rec}} - U_{SE}x(t)\delta(t - t_{sp}),$$

$$\frac{dy(t)}{dt} = -\frac{y(t)}{\tau_{in}} + U_{SE}x(t)\delta(t - t_{sp}),$$

$$x(t) + y(t) + z(t) = 1,$$

$$\frac{dg(t)}{dt} = -\frac{g(t)}{\tau_s} + g_{max}y(t),$$

$x$ : Fraction of available neurotransmitter

$y$ : Fraction of active neurotransmitter

$z$ : Fraction of inactive neurotransmitter

$U_{se}$ : Release probability of active neurotransmitter

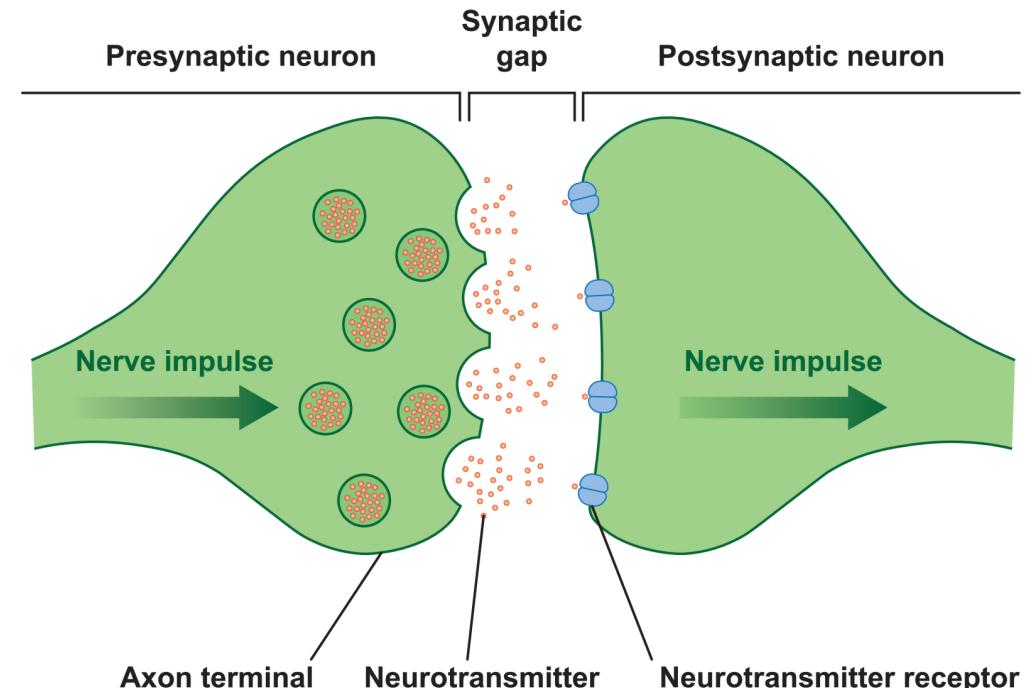
$t_{sp}$ : Pre-synaptic spike time

$g(t)$ : Post-synaptic conductance,

$A$ : total amount of neurotransmitter

$\tau_{in}$  &  $\tau_{rec}$  &  $\tau_s$ : Time constants

## Synaptic Transmission



# Simulate the three-factor STD

Dynamics of three-factor STD:

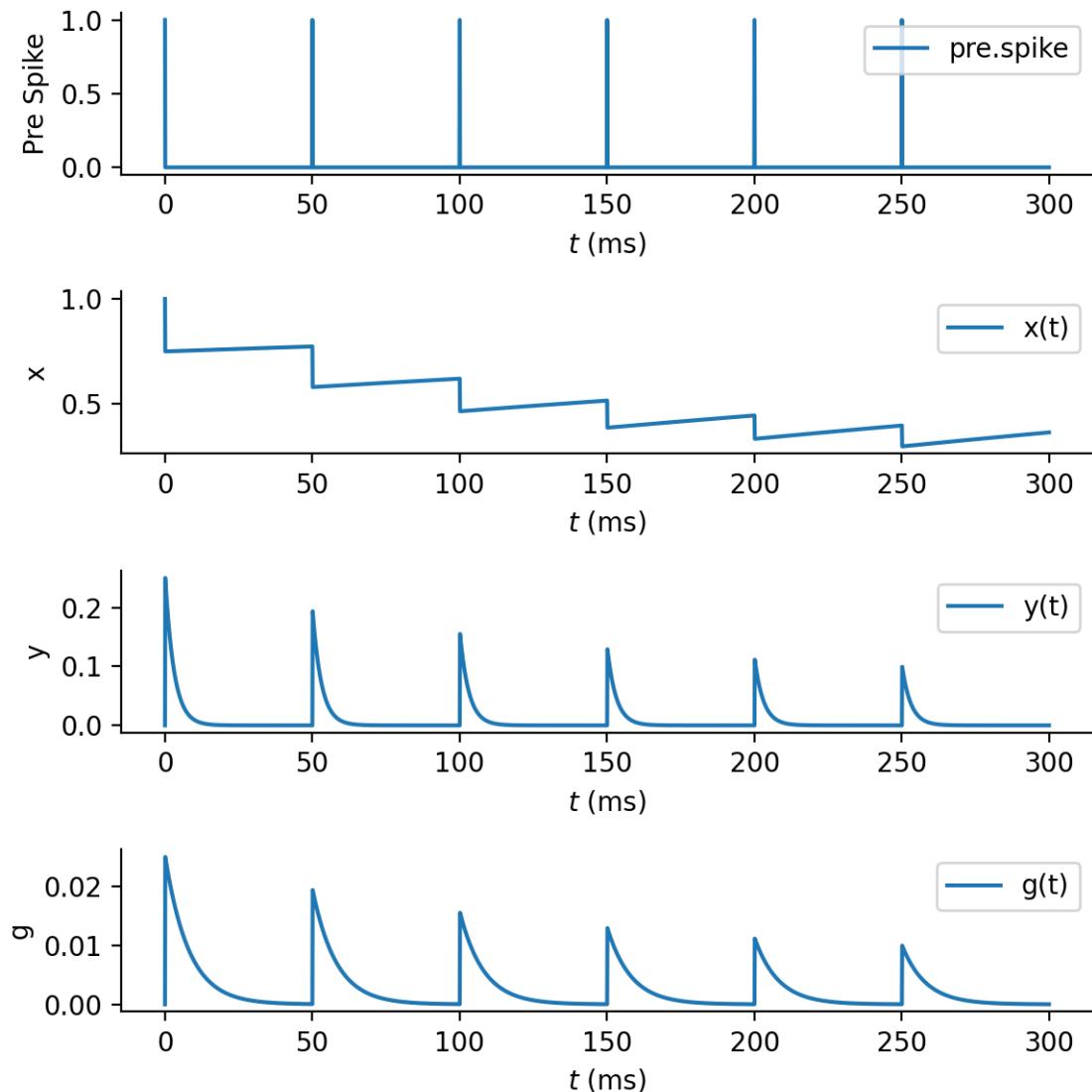
$$\frac{dx(t)}{dt} = \frac{z(t)}{\tau_{rec}} - U_{SE}x(t)\delta(t - t_{sp}),$$

$$\frac{dy(t)}{dt} = -\frac{y(t)}{\tau_{in}} + U_{SE}x(t)\delta(t - t_{sp}),$$

$$x(t) + y(t) + z(t) = 1,$$

$$\frac{dg(t)}{dt} = -\frac{g(t)}{\tau_s} + Ay(t),$$

$$\tau_{rec} = 500ms, \quad \tau_{in} = 3ms, \quad fr = 20hz$$



# Simplify the dynamics of neuro-transmitter consumption

In general, the inactivation time constants is much shorter

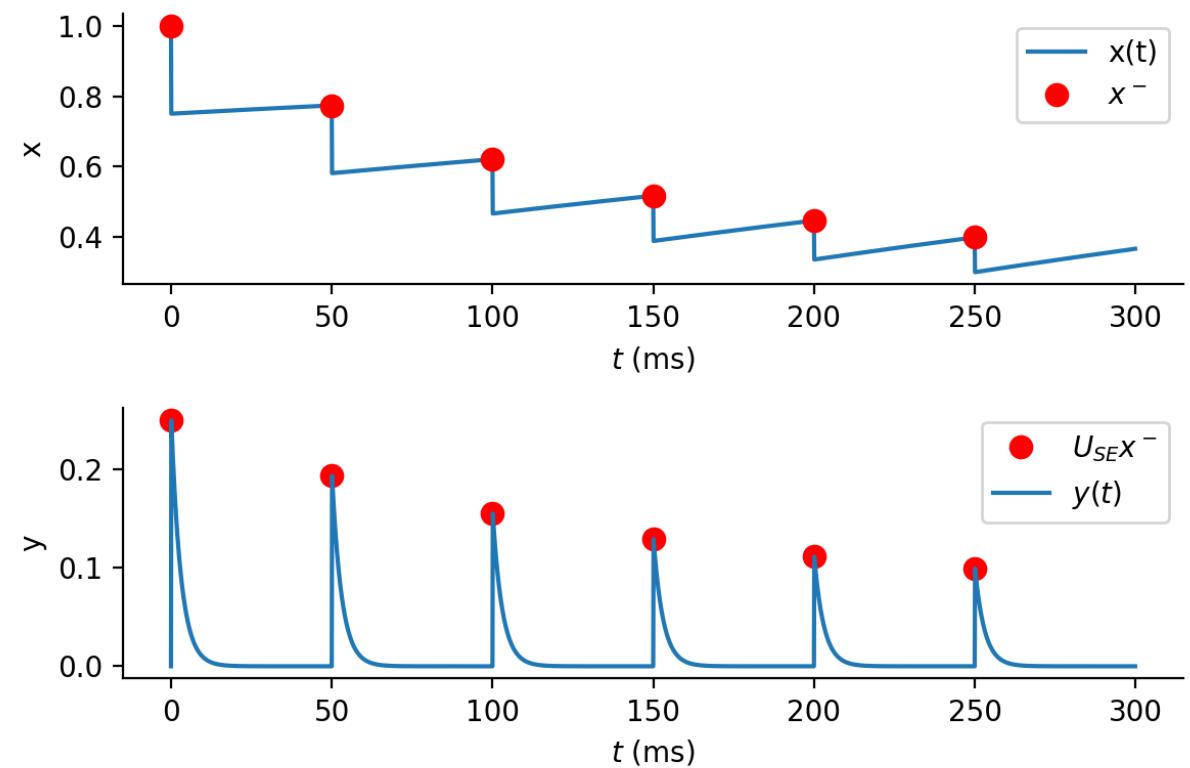
(3ms) than the spike time interval, i.e.,  $\tau_{in} \ll \Delta t$ , so the formulation can be approximately simplified,

$$\frac{dy(t)}{dt} = -\frac{y(t)}{\tau_{in}} + U_{SE}x(t)\delta(t - t_{sp})$$

→ {

$$y(t) = U_{SE}x^-\delta_1(t - t_{sp}),$$

$$x^- = \lim_{t \rightarrow t_{sp}^-} x(t)$$



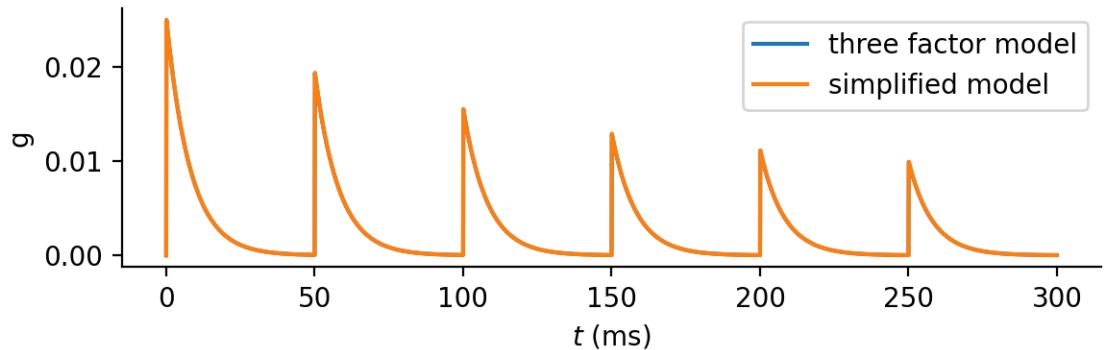
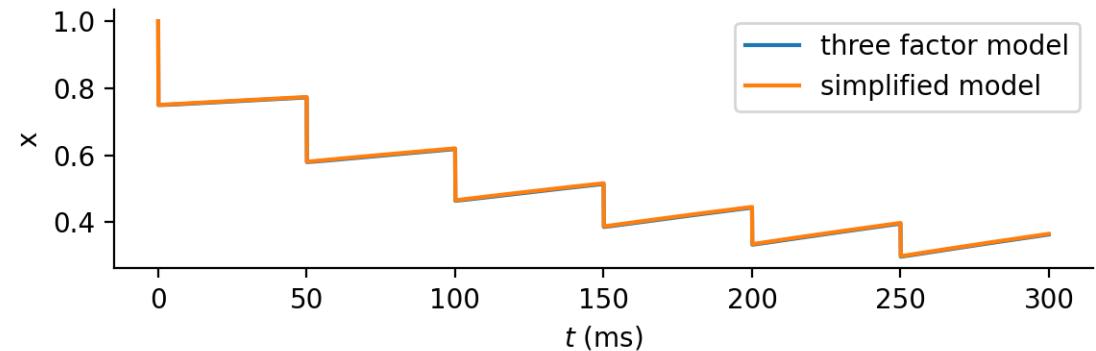
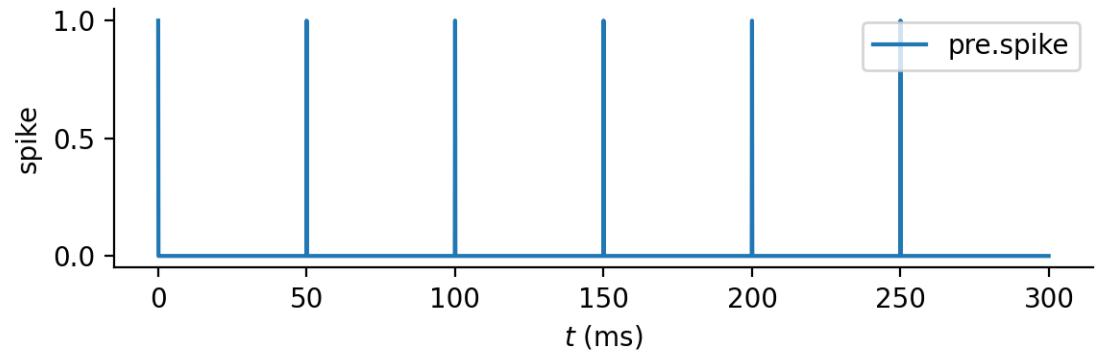
# Simulate STD using simplified dynamics

Simplified model:

$$\frac{dx(t)}{dt} = \frac{1 - x(t)}{\tau_{rec}} - U_{SE}x^{-}\delta(t - t_{sp}),$$

$$\frac{dg(t)}{dt} = -\frac{g(t)}{\tau_s} + AU_{SE}x^{-}\delta(t - t_{sp}),$$

$$EPSC = AU_{SE}x^{-},$$



# Infer model parameters from experimental data

Short term depression model:

$$\frac{dx(t)}{dt} = \frac{1 - x(t)}{\tau_{rec}} - U_{SE}x^{-}\delta(t - t_{sp}),$$

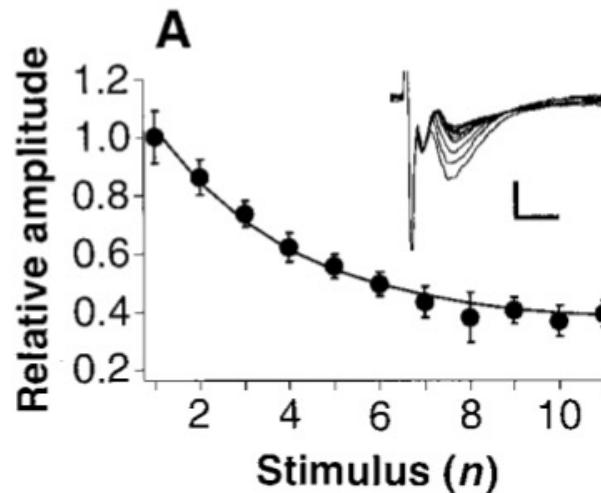
$$EPSC = AU_{SE}x^{-},$$

Iterative expression for EPSCs:

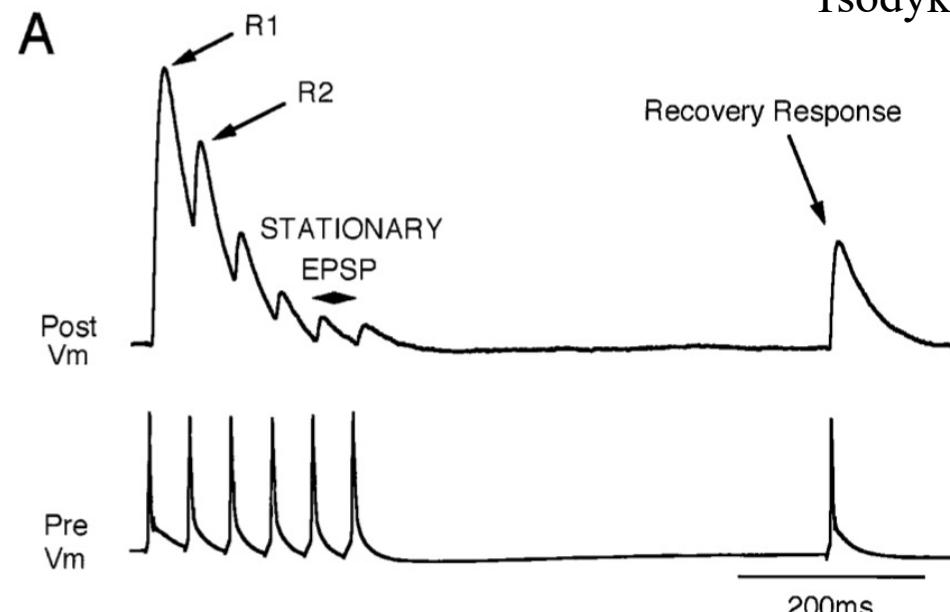
$$x_1^- = 1, \quad EPSC_1 = AU_{SE},$$

$$x_{n+1}^- = 1 - x_n^-(1 - U_{SE})e^{-\frac{\Delta t}{\tau_{rec}}}$$

$$EPSC_{n+1} = AU_{SE} - EPSC_n(1 - U_{SE})e^{-\frac{\Delta t}{\tau_{rec}}}$$

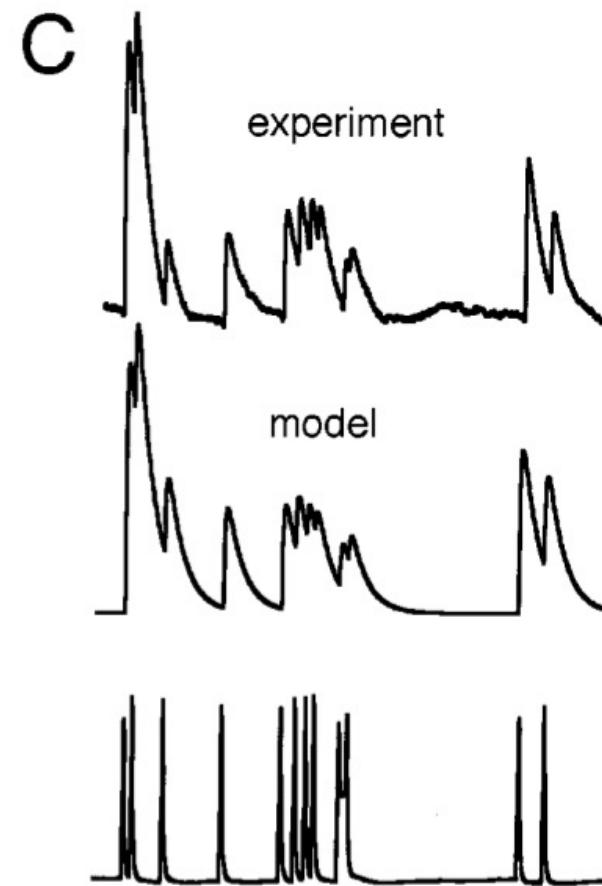
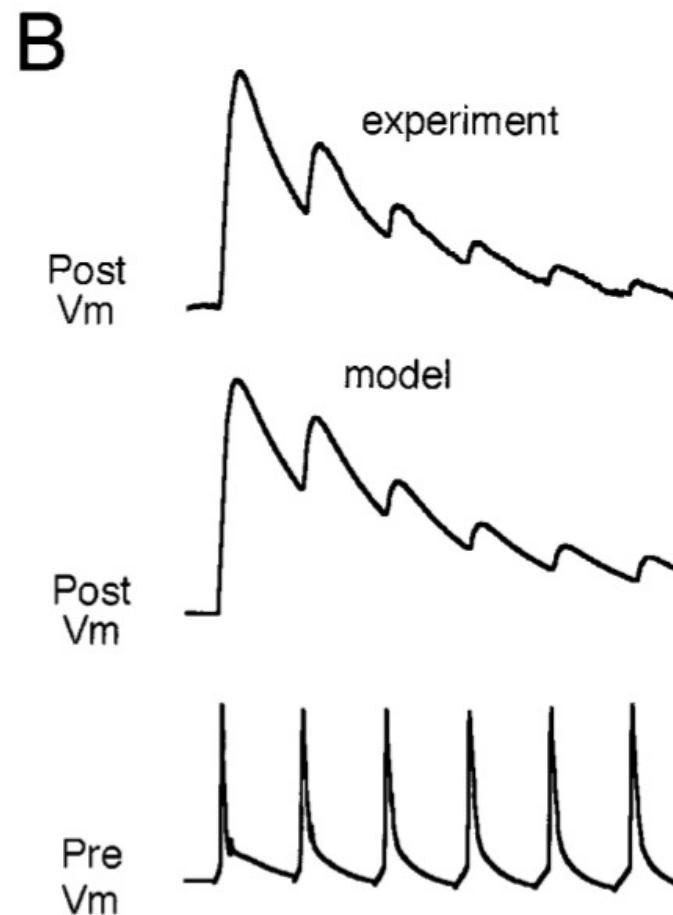


Abbott et al., 1997, Nature  
Tsodyks et al., 1997, PNAS



# Prediction for complex post-synaptic patterns

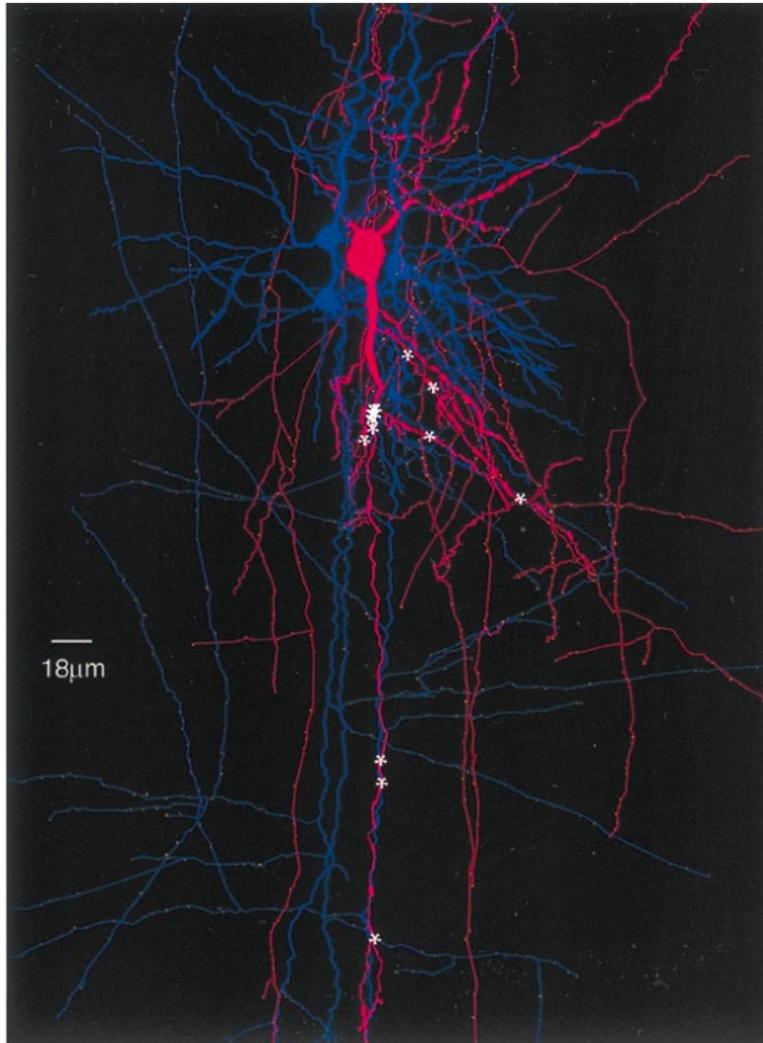
Simulate with complex pre-synaptic spike trains and compare with *vitro* experimental results (patch-clamp)



Tsodyks et al., 1997, PNAs

# Short-term facilitation observed between pyramidal cells and interneurons

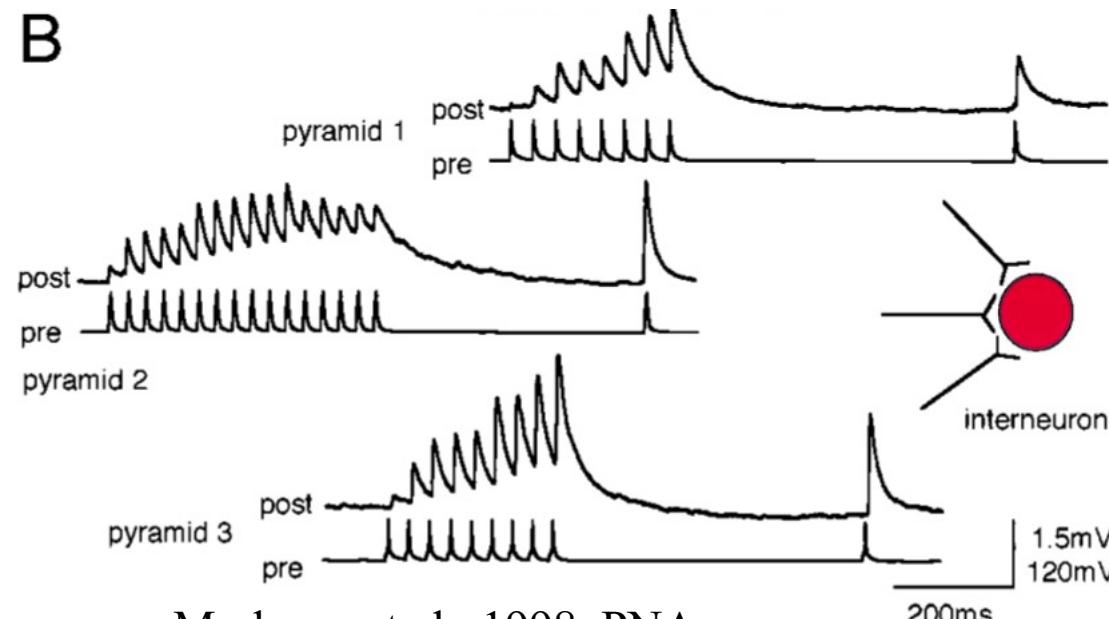
pyramidal cell – interneurons



*Proc. Natl. Acad. Sci. USA*  
Vol. 95, pp. 5323–5328, April 1998  
Neurobiology

**Differential signaling via the same axon of neocortical pyramidal neurons**

HENRY MARKRAM\*, YUN WANG, AND MISHA TSODYKS



Markram et al., 1998, PNAs

# Modeling neuro-transmitter release probability

The release probability can also be modelled as a dynamical variable  $u(t)$ ,

$$\frac{du(t)}{dt} = \frac{-u(t)}{\tau_f} + U_{SE}(1 - u^-)\delta(t - t_{sp}),$$

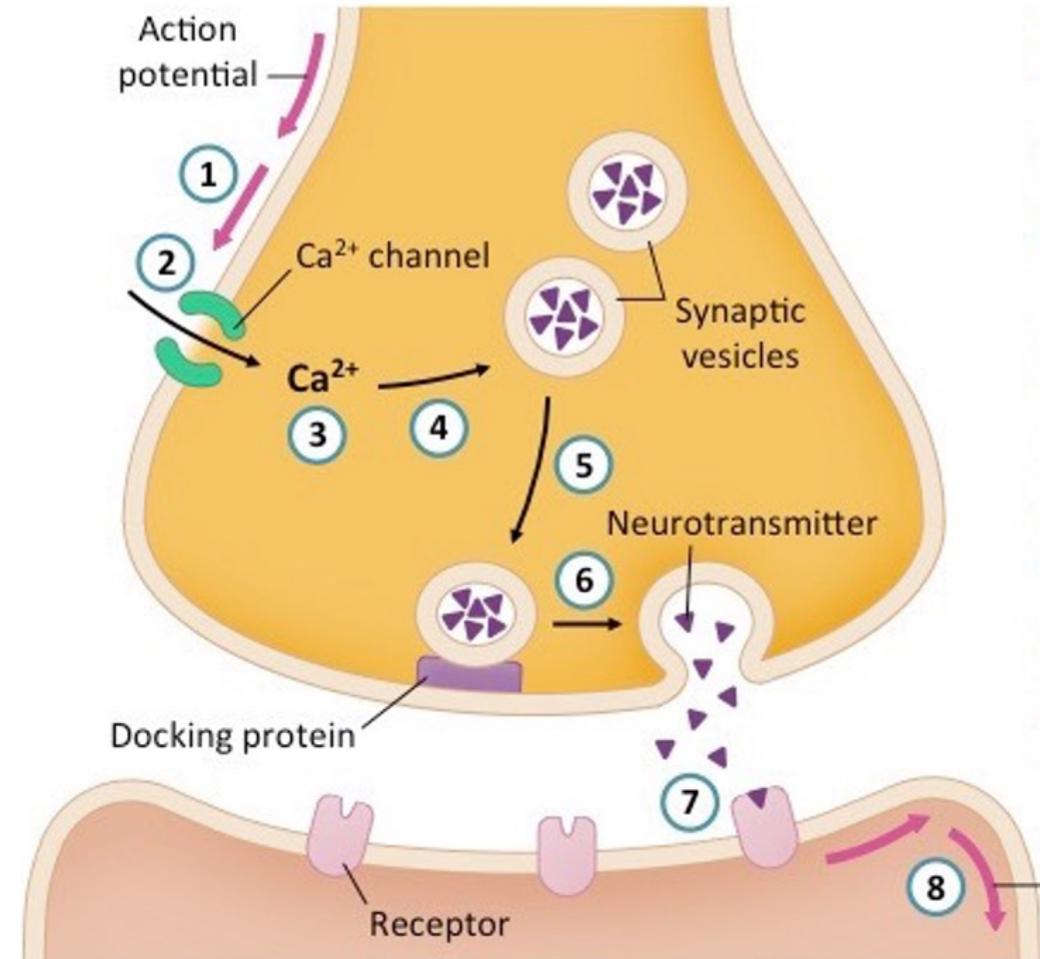
$$\frac{dx(t)}{dt} = \frac{1 - x(t)}{\tau_d} - u(t)x^-\delta(t - t_{sp} + \delta t),$$

$$\frac{dg(t)}{dt} = -\frac{g(t)}{\tau_s} + Au(t)x^-\delta(t - t_{sp} + \delta t),$$

$$EPSC = Au(t)x^-,$$

$U_{SE}$  might reflect the concentration of  $Ca^{2+}$

Tsodyks et al., 1998, Neural computation



# Modeling neuro-transmitter release probability

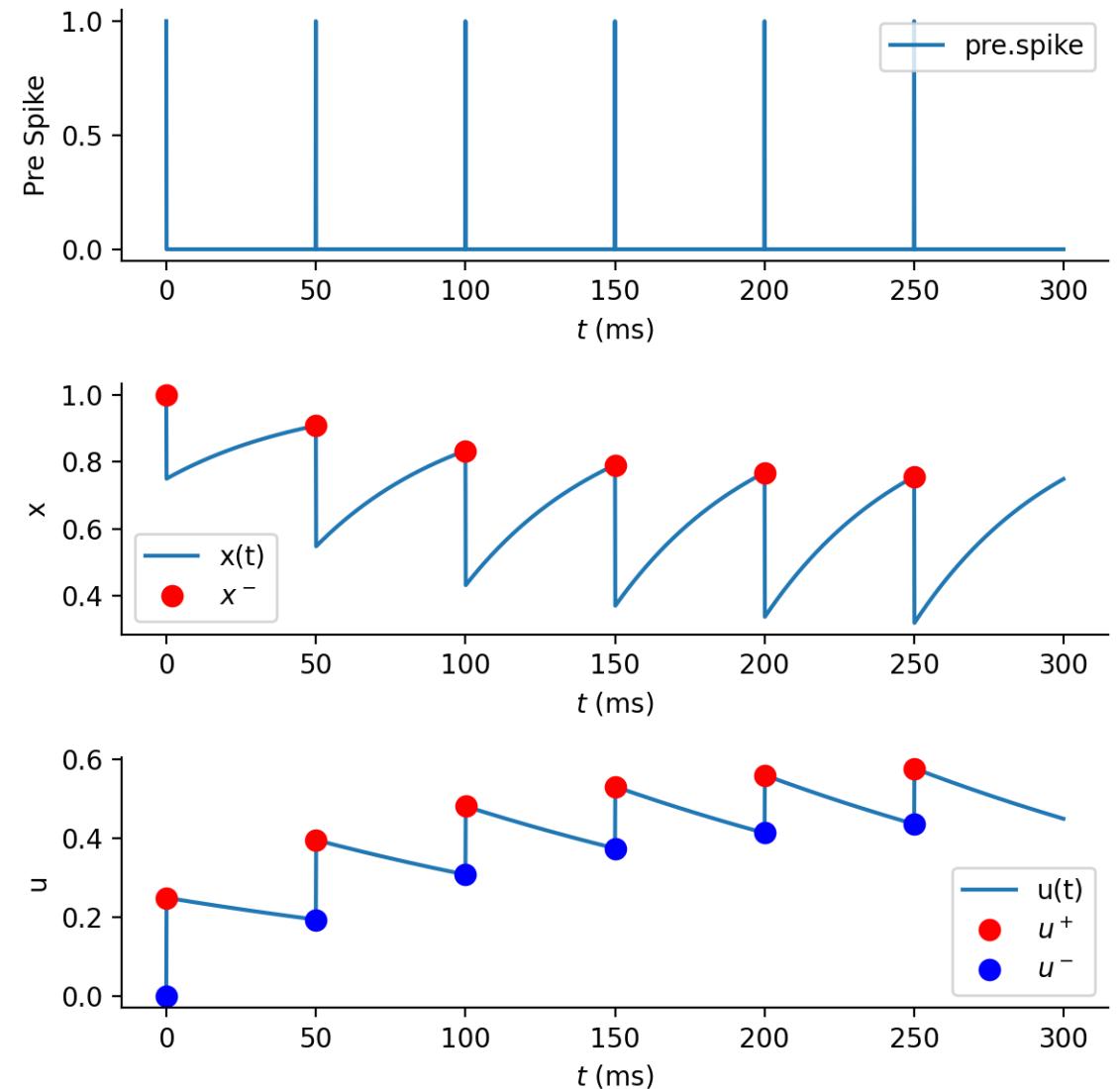
The release probability can also be modelled as a dynamical variable  $u(t)$ ,

$$\frac{du(t)}{dt} = \frac{-u(t)}{\tau_f} + U_{SE}(1 - u^-)\delta(t - t_{sp}),$$

$$\frac{dx(t)}{dt} = \frac{1 - x(t)}{\tau_d} - u^+ x^- \delta(t - t_{sp}),$$

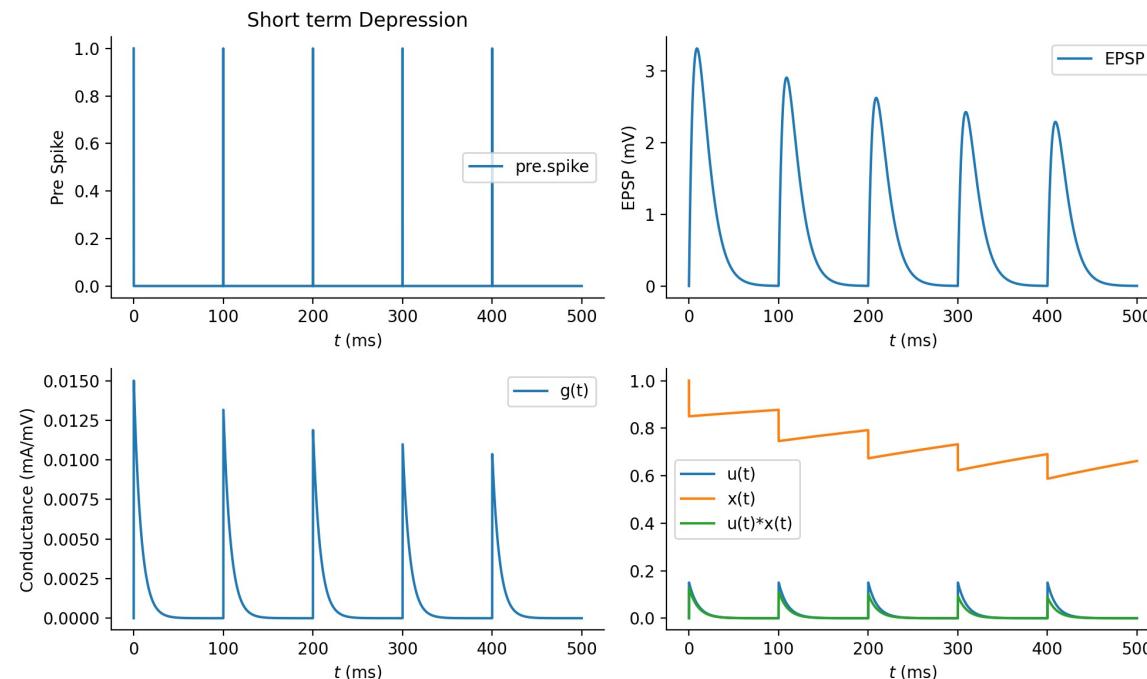
$$\frac{dg(t)}{dt} = -\frac{g(t)}{\tau_s} + A u^+ x^- \delta(t - t_{sp}),$$

$$EPSC = A u^+ x^-, \quad u^+ = \lim_{t-t_{sp} \rightarrow 0^+} u(t),$$

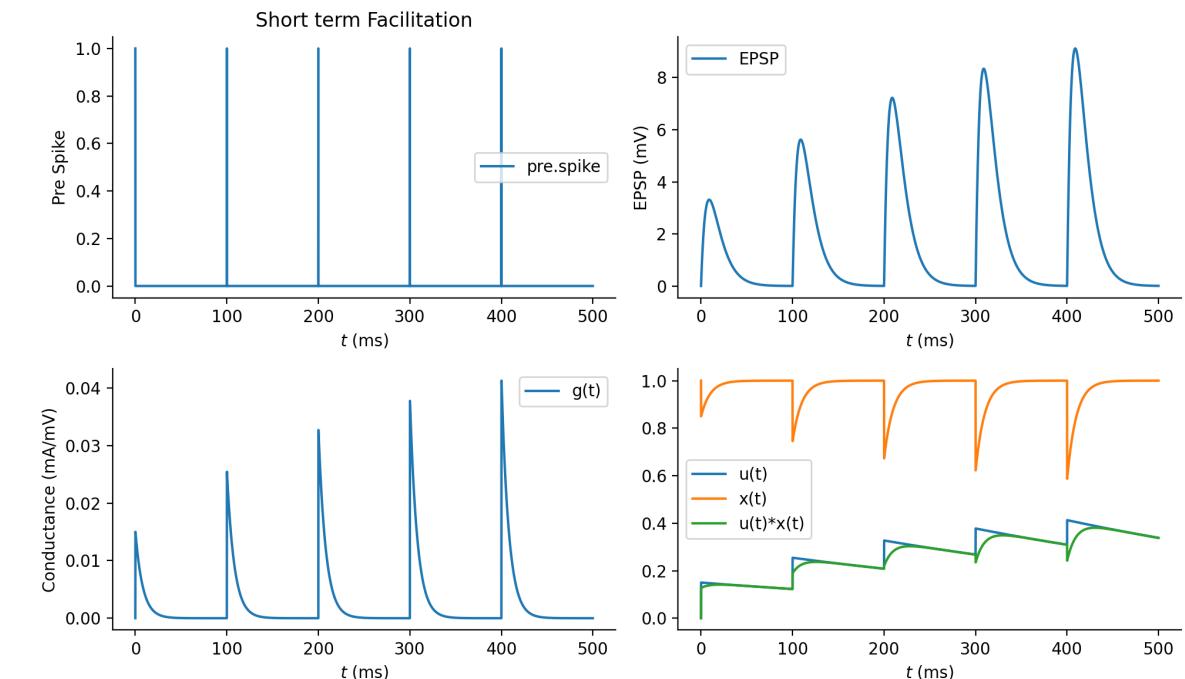


# STD and STF under different parameter regime

Short-term depression:  $\tau_f = 10ms, \tau_d = 500ms$



Short-term facilitation:  $\tau_f = 500ms, \tau_d = 10ms$



# Derivation of iterative expressions for EPSCs

Iterative expression for  $x_n, u_n, EPSC_n$ :

$$\begin{aligned} u_1^+ &= U_{SE}, \quad x_1^- = 1, \\ x_{n+1}^- &= 1 - x_n^- (1 - u_n^+) e^{-\frac{\Delta t}{\tau_{rec}}}, \\ u_{n+1}^+ &= u_n^+ e^{-\frac{\Delta t}{\tau_f}} + U_{SE} \left( 1 - u_n^+ e^{-\frac{\Delta t}{\tau_f}} \right), \\ EPSC_{n+1} &= A u_n^+ x_n^-, \end{aligned}$$

Steady state of  $x_n, u_n, EPSC_n$ :

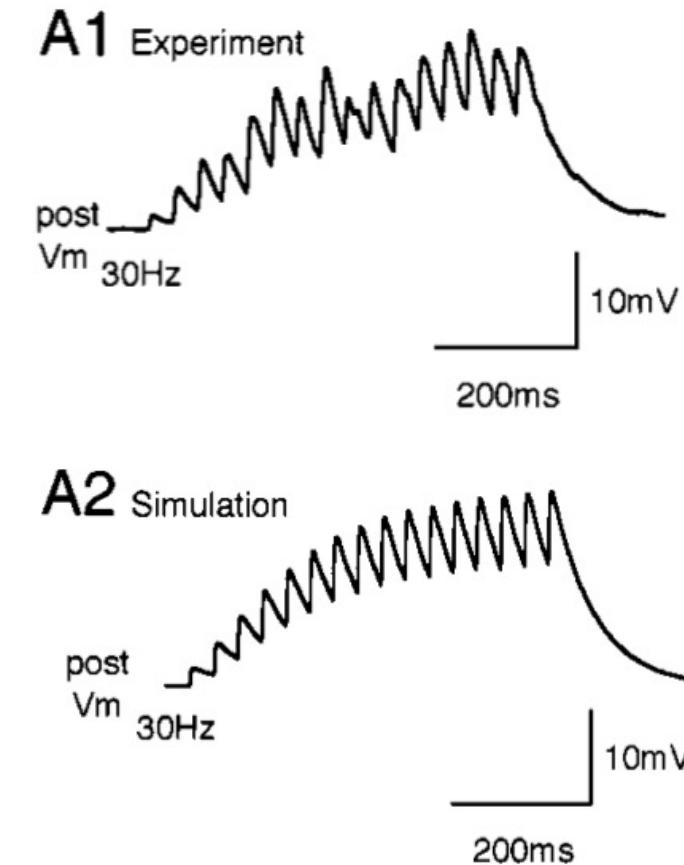
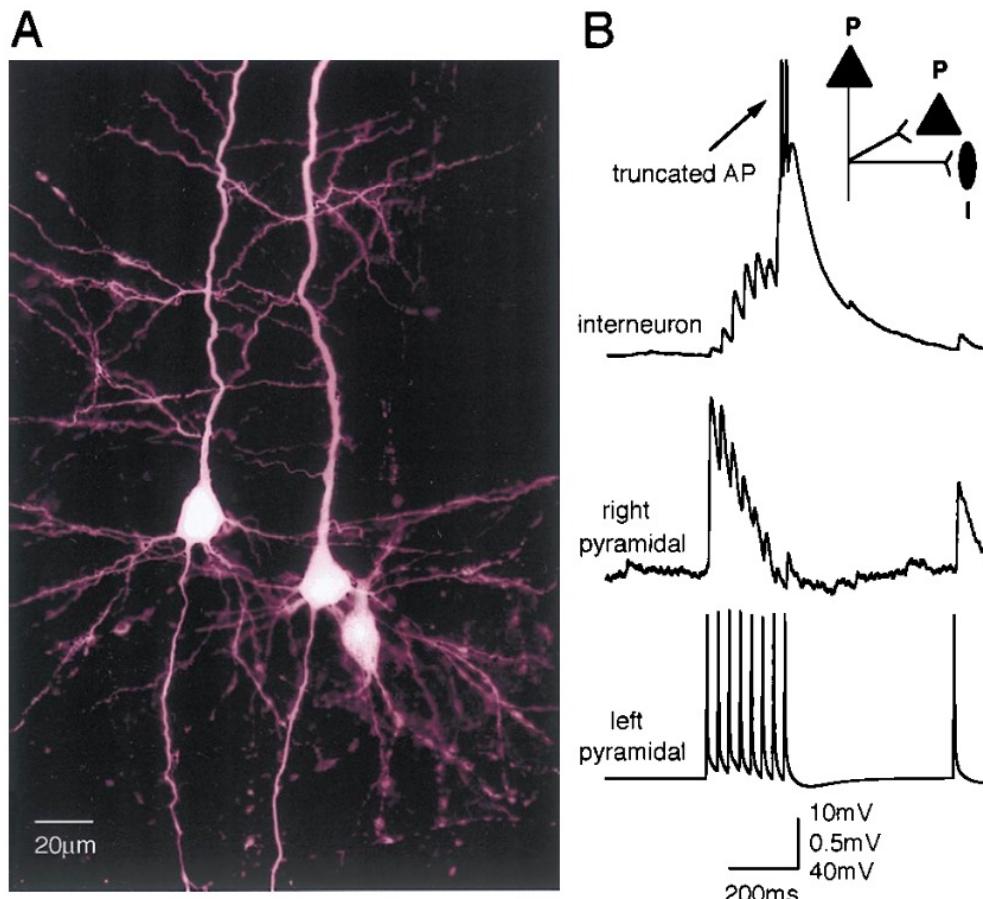
$$\begin{aligned} u_{st}^+ &= \frac{U_{SE}}{1 - (1 - U_{SE}) e^{-\frac{\Delta t}{\tau_f}}} \geq U_{SE} = u_1^+, \\ x_{st}^- &= \frac{1}{1 + (1 - u_{st}^+) e^{-\frac{\Delta t}{\tau_{rec}}}} \leq 1 = x_1^-, \\ EPSC_{st} &= A u_{st}^+ x_{st}^- \end{aligned}$$

# Prediction for complex post-synaptic patterns

Infer model parameters by fitting experiments:

$$EPSC_{n+1} = Au_n x_n$$

Simulate with complex pre-synaptic spike trains and compare with *vitro* experimental results (patch-clamp)



Markram et al., 1998, PNAs



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# 03

## Effects on Information Transmission

# Mean-field Analysis of STP model

STP based on spiking time

$$\frac{du(t)}{dt} = \frac{-u(t)}{\tau_f} + U_{SE}(1 - u^-)\delta(t - t_{sp}),$$

$$\frac{dx(t)}{dt} = \frac{1 - x(t)}{\tau_d} - u^+ x^- \delta(t - t_{sp}),$$

$$\frac{dg(t)}{dt} = -\frac{g(t)}{\tau_s} + A u^+ x^- \delta(t - t_{sp}),$$

$$u^+ = \lim_{t-t_{sp} \rightarrow 0^+} u(t),$$

time average

STP based on firing rate

$$\frac{du(t)}{dt} = \frac{-u(t)}{\tau_f} + U_{SE}[1 - u(t)]R(t),$$

$$\frac{dx(t)}{dt} = \frac{1 - x(t)}{\tau_d} - u^+ x R(t),$$

$$g(t) = \tau_s A u^+ x R(t),$$

$$u^+ = u(t) + U_{SE}[1 - u(t)],$$

# Mean-field Analysis of STP model

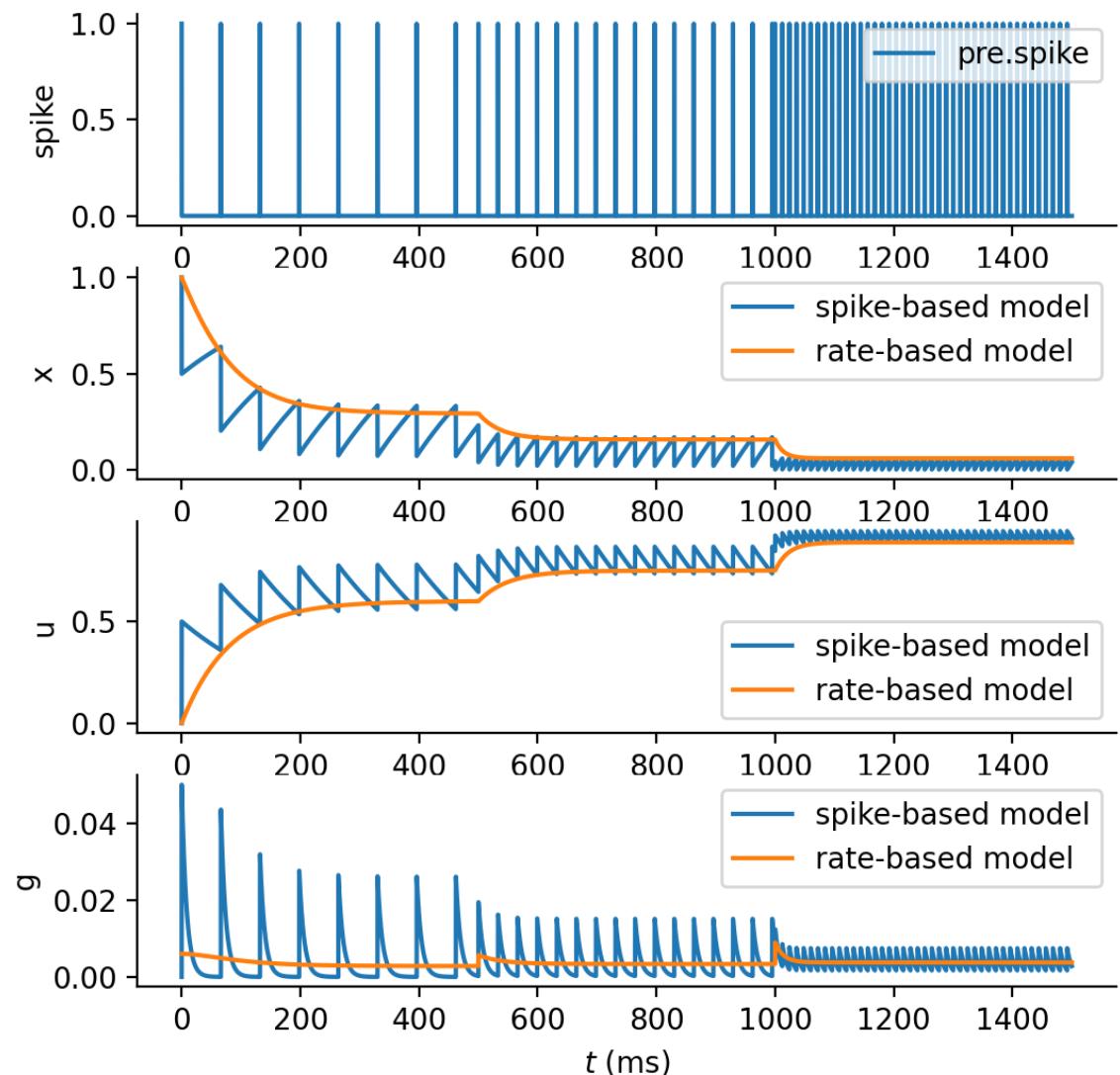
STP based on firing rate

$$\frac{du(t)}{dt} = \frac{-u(t)}{\tau_f} + U_{SE}[1 - u(t)]R(t),$$

$$\frac{dx(t)}{dt} = \frac{1 - x(t)}{\tau_d} - u^+ x R(t),$$

$$g(t) = \tau_s A u^+ x R(t),$$

$$u^+ = u(t) + U_{SE}[1 - u(t)],$$

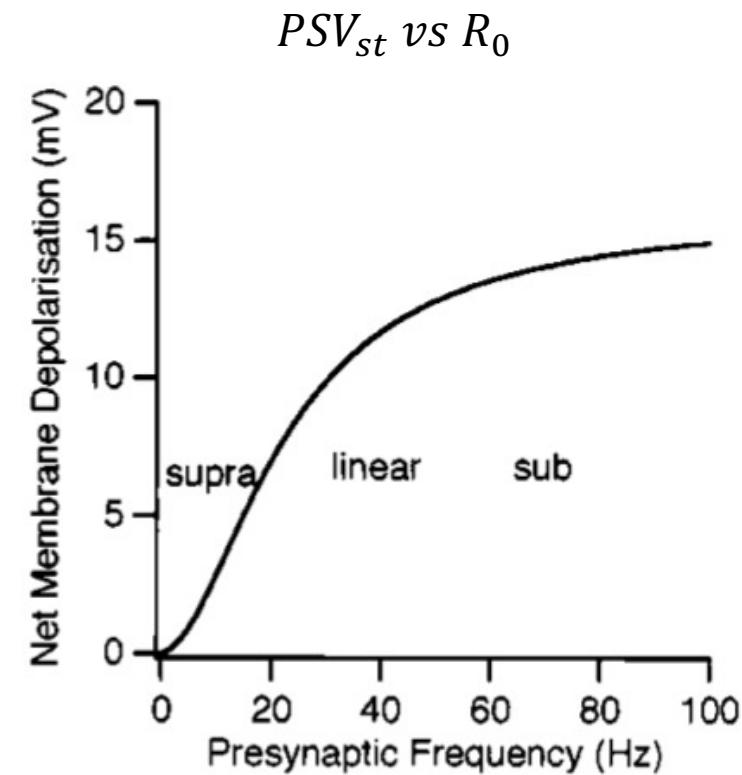
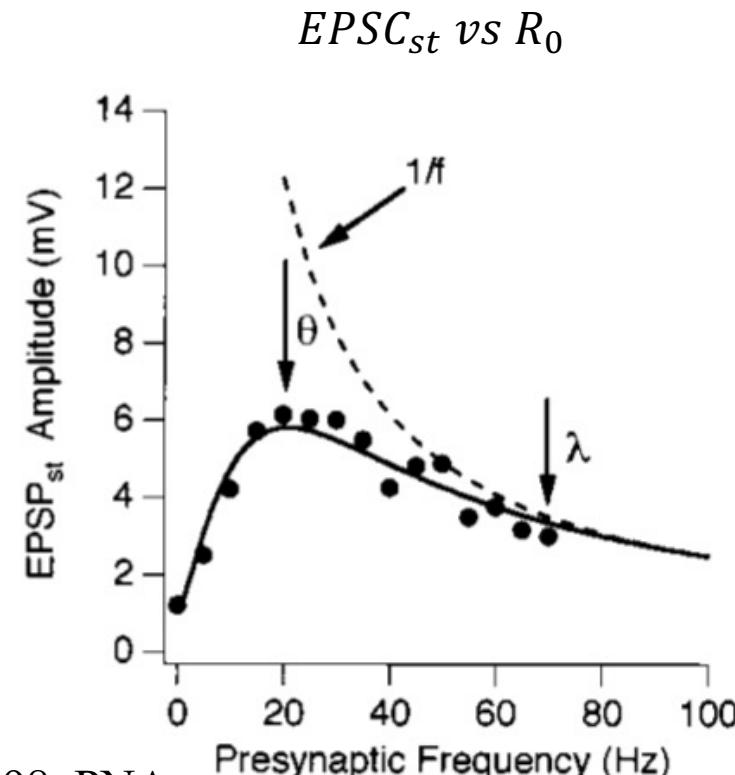


# Theoretical analysis of the rate model

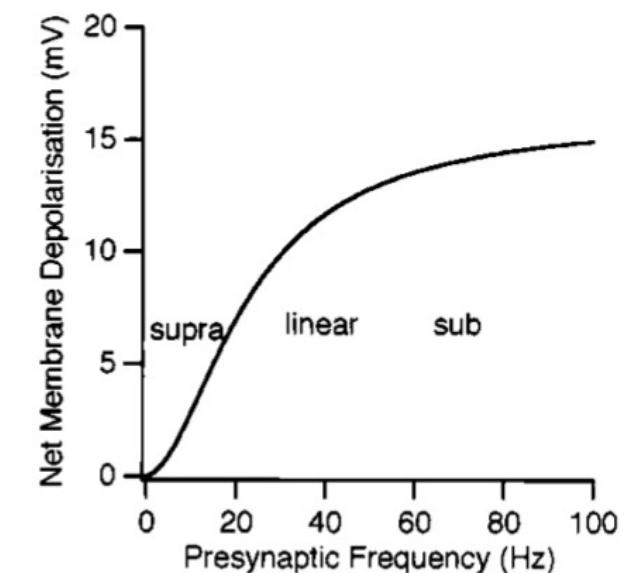
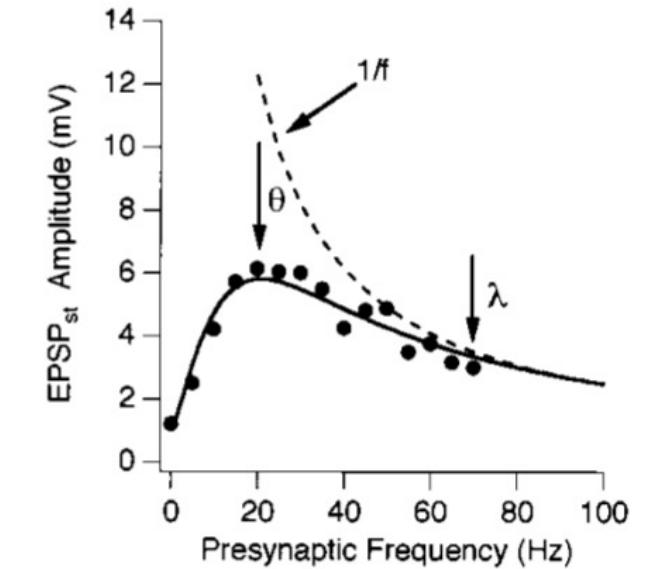
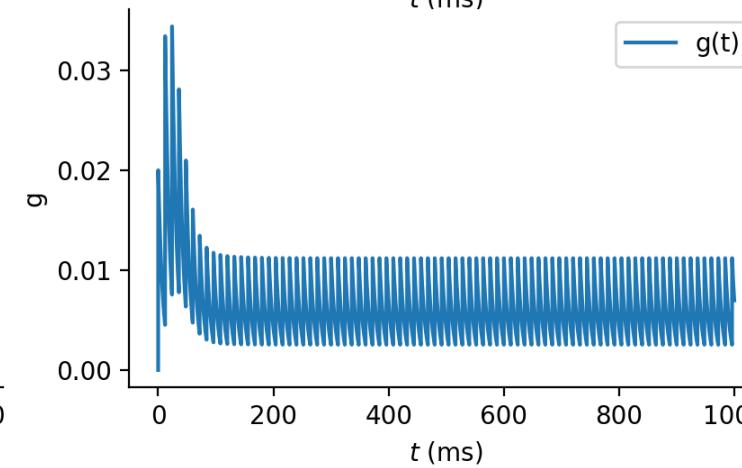
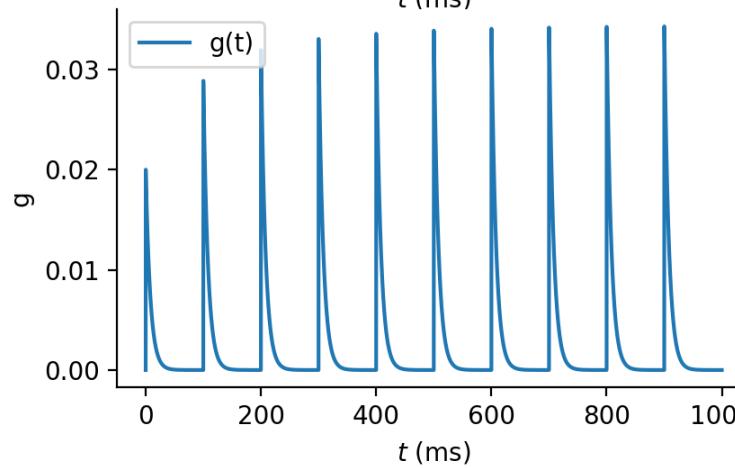
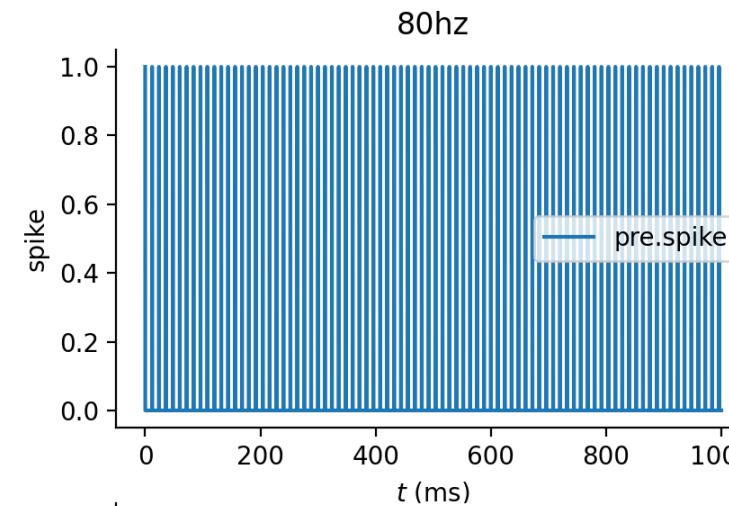
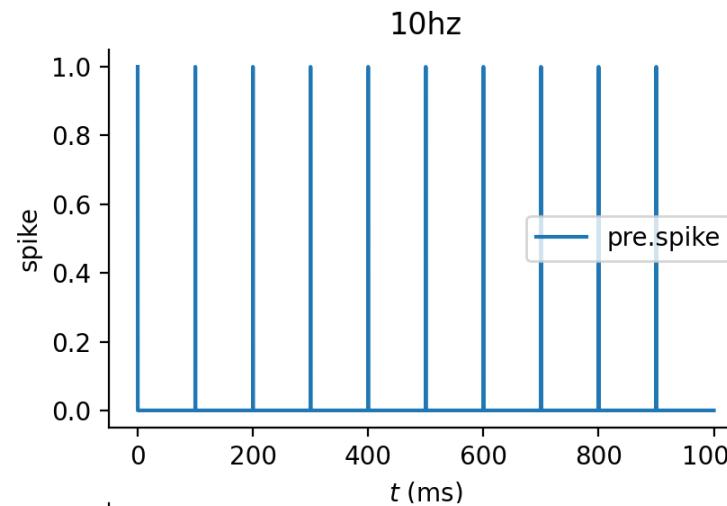
Suppose the pre-synaptic firing rate keeps as constant, we can calculate the stationary response

$$u_{st} = \frac{U_{SE}R_0\tau_f}{1 + U_{SE}R_0\tau_f}, \quad u_{st}^+ = U_{SE} \frac{1 + R_0\tau_f}{1 + U_{SE}R_0\tau_f}, \quad x_{st} = \frac{1}{1 + u_{st}^+\tau_dR_0},$$

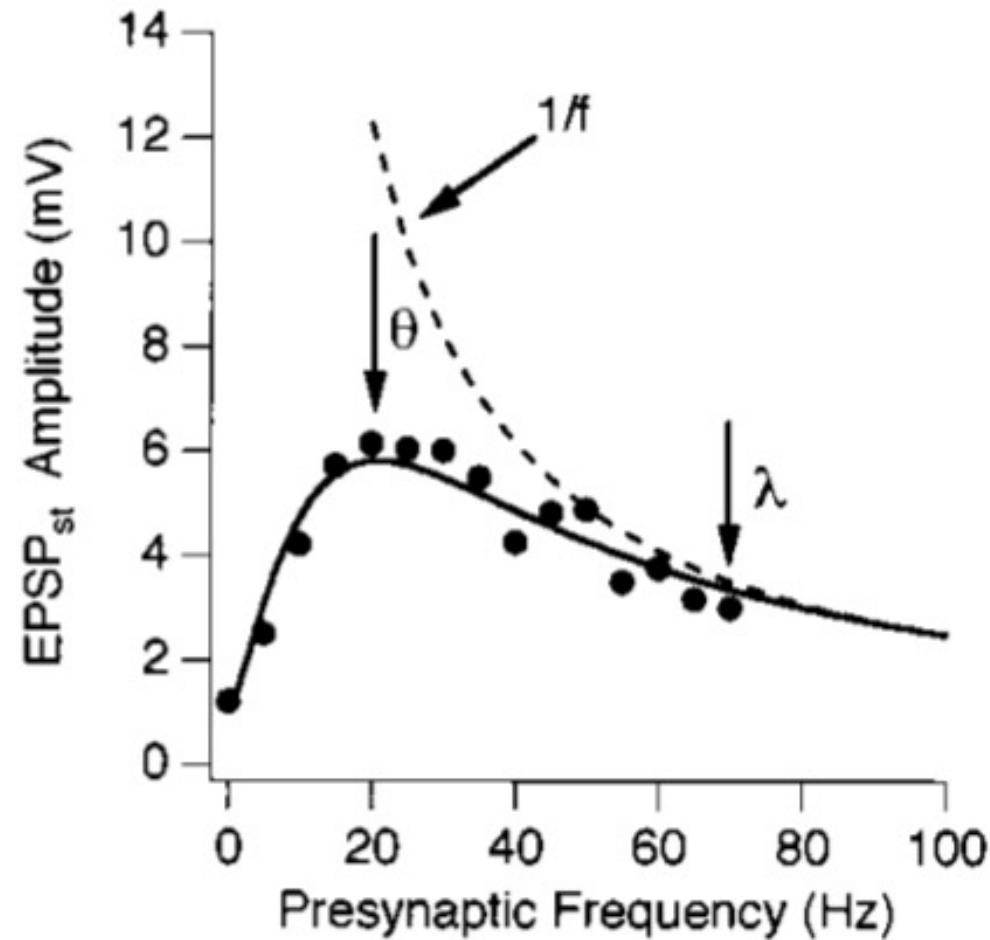
$$EPSC_{st} = Au_{st}^+x_{st} = A \frac{u_{st}^+}{1 + u_{st}^+\tau_dR_0}, \quad PSV_{st} \propto g_{st} = \tau_s A u_{st}^+ x_{st} R_0 = A \frac{u_{st}^+ R_0}{1 + u_{st}^+\tau_dR_0},$$



# Frequency-dependent facilitation/depression



# Frequency-dependent Gain control of spike information



$$u_{st}^+ = U_{SE} \frac{1 + R_0 \tau_f}{1 + U_{SE} R_0 \tau_f},$$

$$\chi_{st} = \frac{1}{1 + u_{st}^+ \tau_d R_0},$$

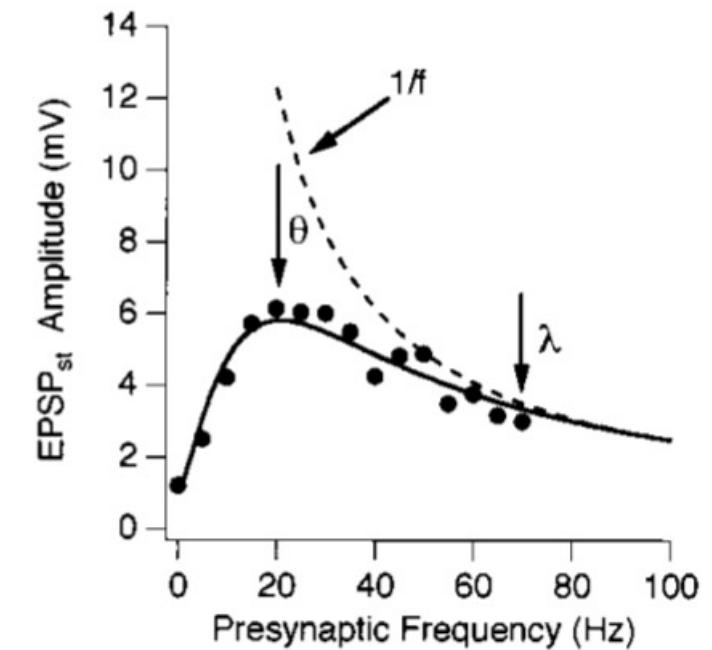
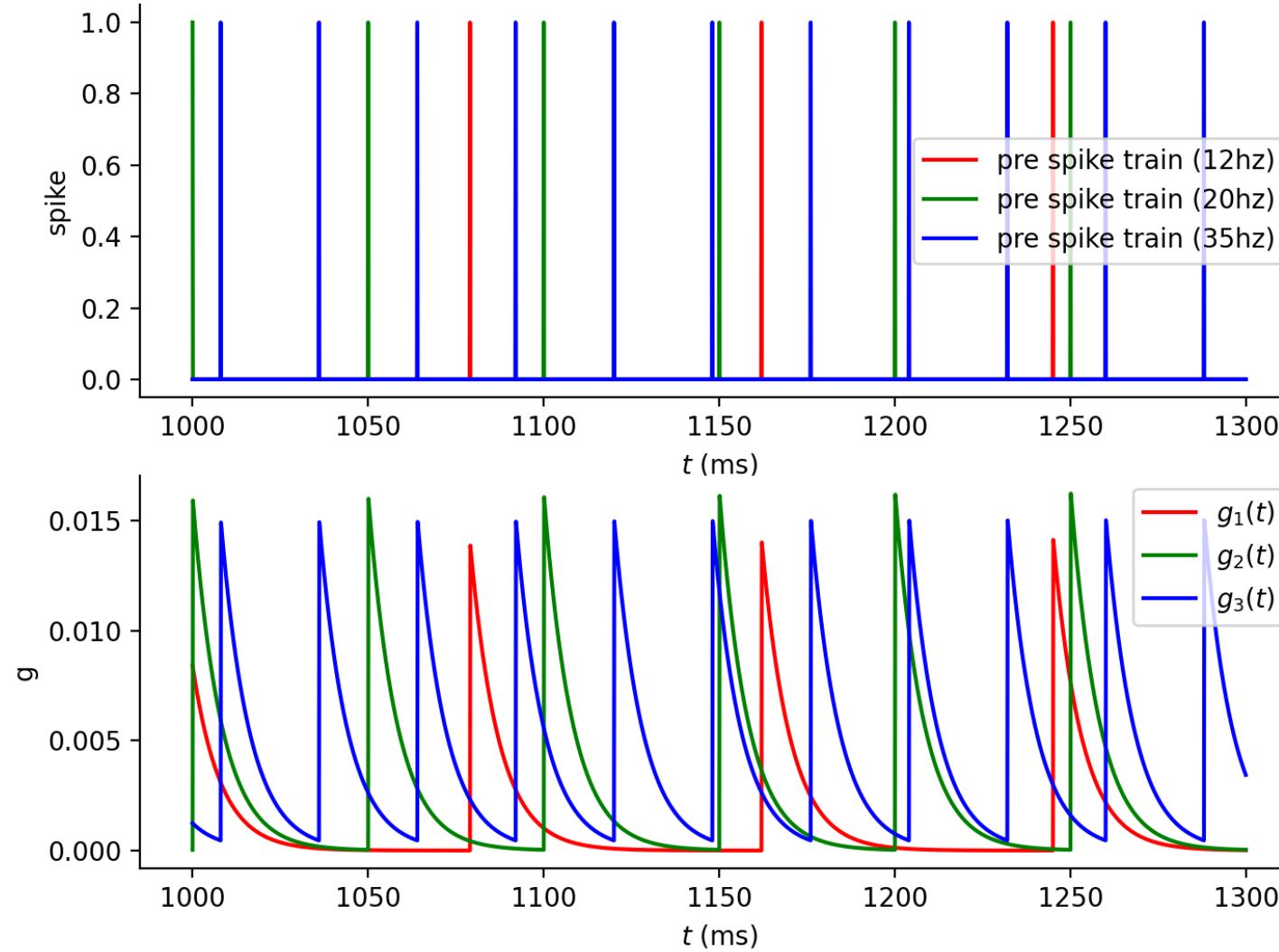
$$EPSC_{st} = A u_{st}^+ \chi_{st} = A \frac{u_{st}^+}{1 + u_{st}^+ \tau_d R_0},$$

Peak frequency:

$$\theta \sim \frac{1}{\sqrt{U \tau_f \tau_d}}$$

Markram et al., 1998, PNAs

# Simulation of Frequency-dependent Gain control





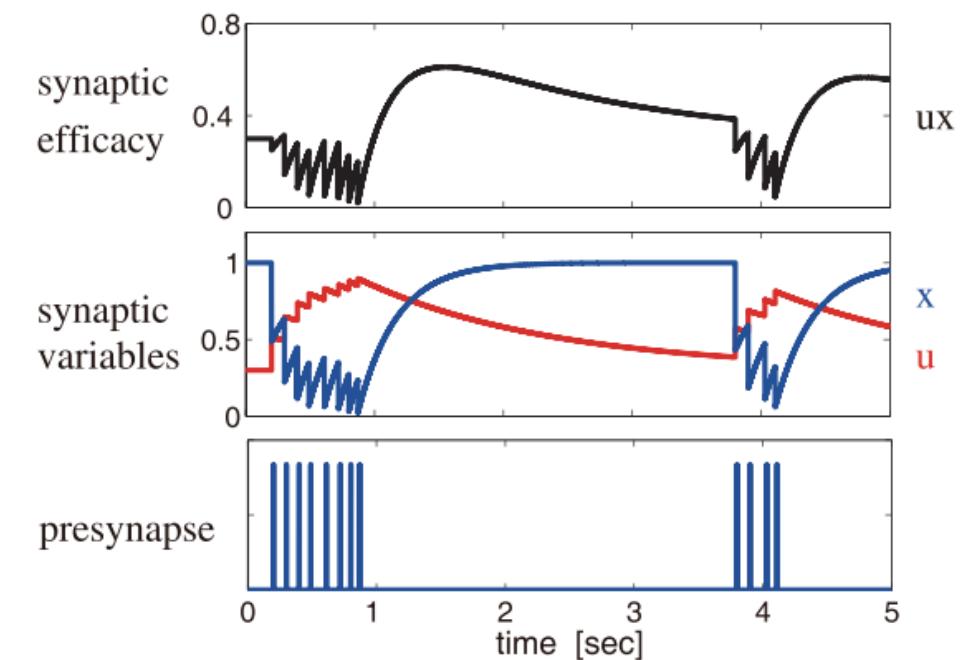
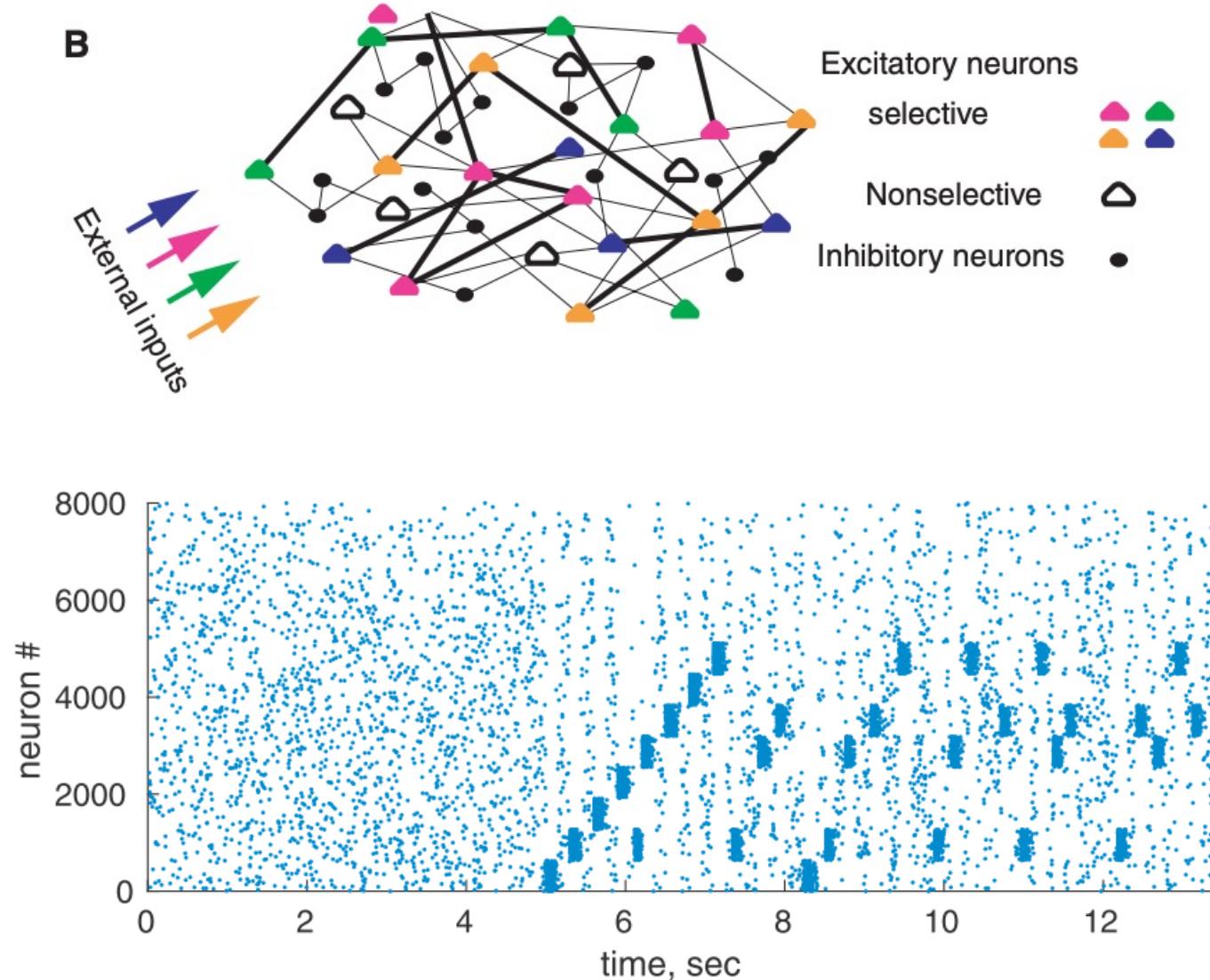
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# 04

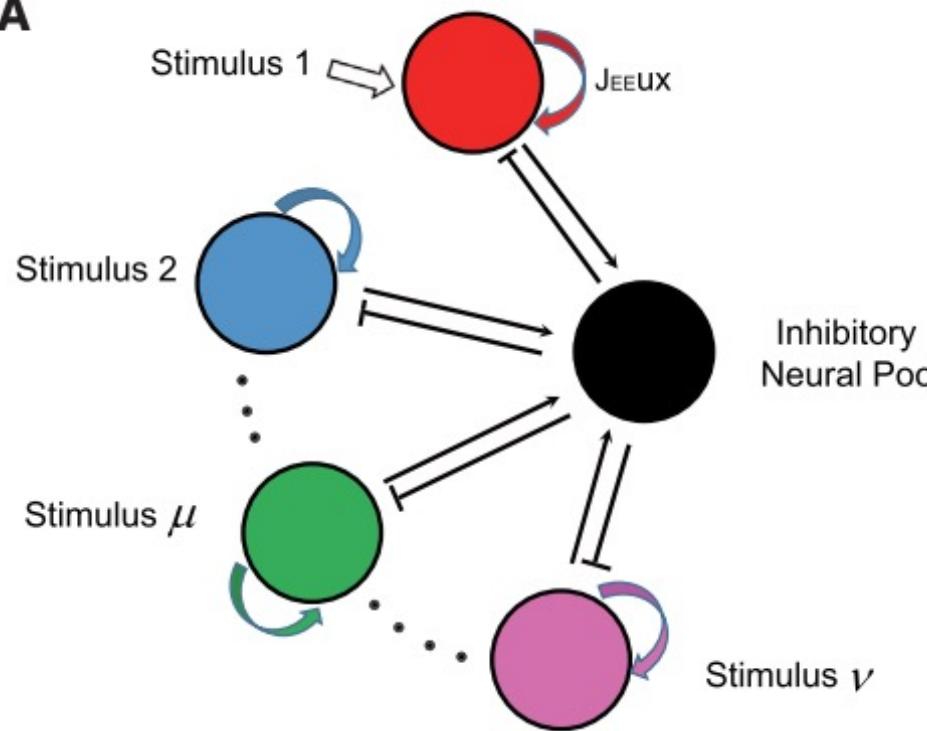
## Effects on Network dynamics

# STP modeling Working memory

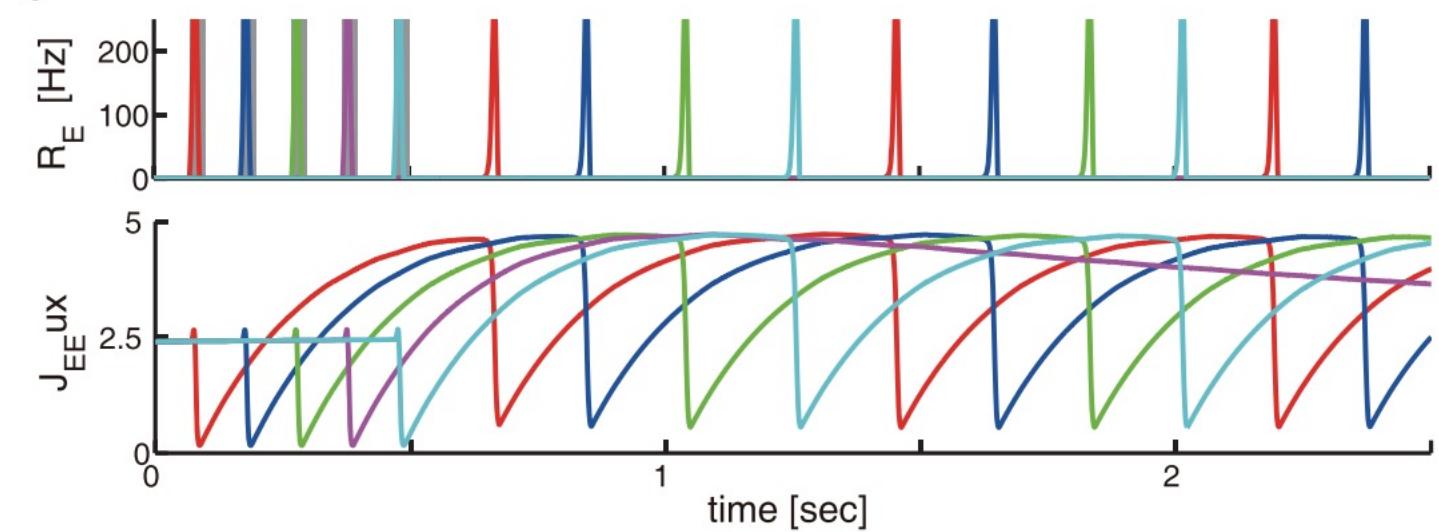
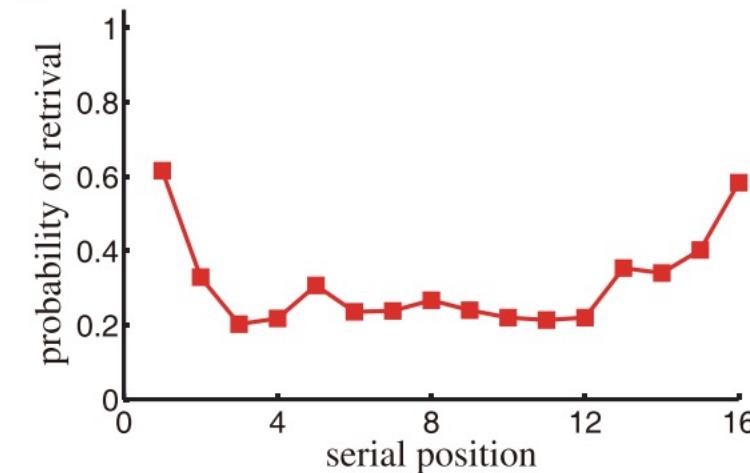
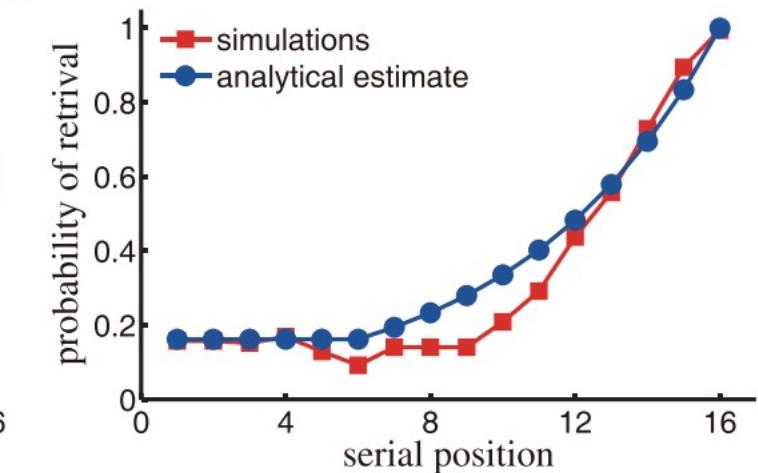


Mongillo et al., 2008, Science

# STP modeling Working memory

**A**


Mi et al., 2017, Neuron

**C**

**D**

**E**


# References

- [1] Markram, H., & Tsodyks, M. (1996). Redistribution of synaptic efficacy between neocortical pyramidal neurons. *Nature*, 382(6594), 807-810.
- [2] Tsodyks, M. V., & Markram, H. (1997). The neural code between neocortical pyramidal neurons depends on neurotransmitter release probability. *Proceedings of the national academy of sciences*, 94(2), 719-723.
- [3] Abbott, L. F., Varela, J. A., Sen, K., & Nelson, S. B. (1997). Synaptic depression and cortical gain control. *Science*, 275(5297), 221-224.
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- [5] Tsodyks, M., Pawelzik, K., & Markram, H. (1998). Neural networks with dynamic synapses. *Neural computation*, 10(4), 821-835.
- [6] Olivia Guy Evans (2023), Synapse: Definition, Parts, Types & Function. SimplyPsychology, <https://www.simplypsychology.org/synapse.html>.
- [7] Mongillo, G., Barak, O., & Tsodyks, M. (2008). Synaptic theory of working memory. *Science*, 319(5869), 1543-1546.
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# Q&A

Thanks for your listening!