Continuous Attractor Neural Network

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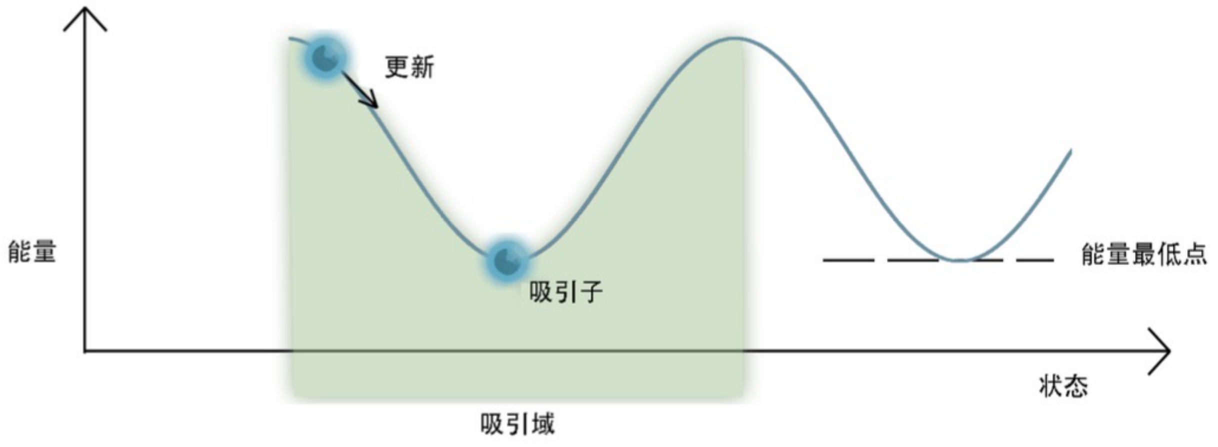
01

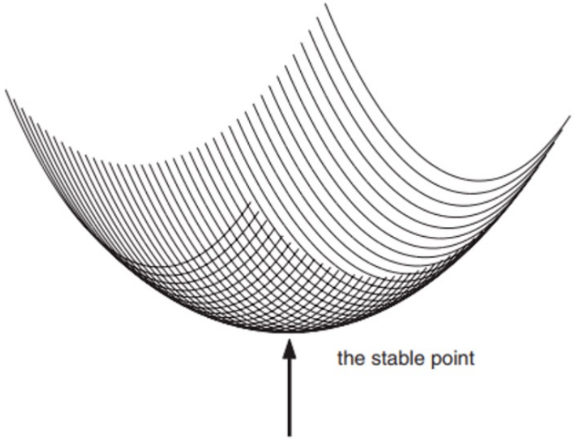
Attractor Neural Network

The concept of attractor dynamics

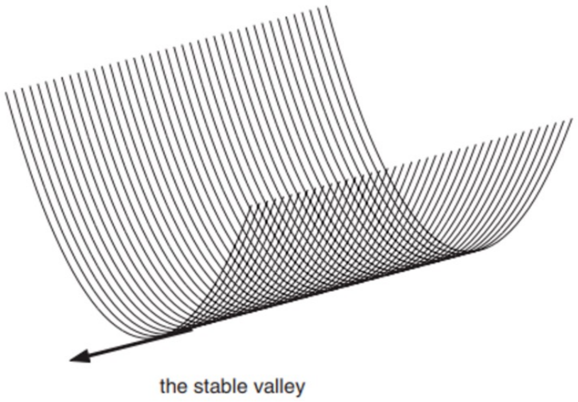
- Key words: Dynamical system, energy function, stable state, basin of attraction
- Different types of attractors:
- Point attractors, Line attractors, Ring attractors, Plane attractors, Cyclic attractors, Chaotic attractors



Discrete (point) attractor



Line attractor



Discrete attractor Network Model: Hopfield Model

$S_i = \pm 1$: the neuronal state

w_{ij} : the neuronal connection

The network dynamics:

$$S_i = \mathrm{sign}\left(\sum_j w_{ij} S_j - \theta\right), \quad \mathrm{sign}(x) = 1, \text{for } x > 0; -1, \text{ otherwise}$$

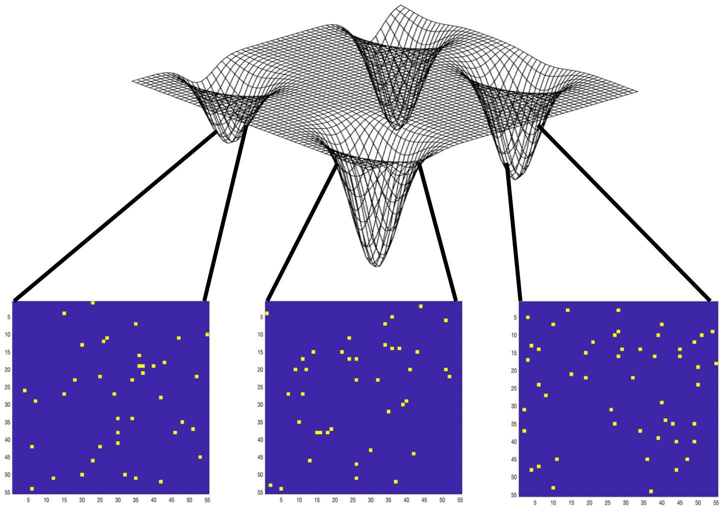
Updating rule: synchronous or asycchronous

Consider the network stores p pattern, ξ_i^{μ}, for μ=1,...p; $i = 1, \ldots N$

Setting $w_{ij} = \dfrac{1}{N} \sum_{\mu=1}^{p} \xi_i^{\mu} \xi_j^{\mu}$

Discrete concepts/items

Discrete attractors



Energy space of Hopfield network

Energy function: $E = -\frac{1}{2}\sum_{i,j} w_{ij}S_iS_j + \theta\sum_{i} S_i$

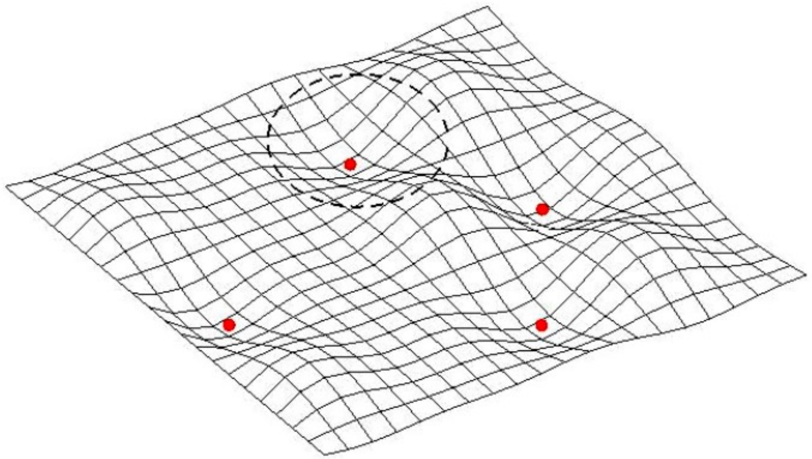
Consider S_i is updated, $S_i(t+1) = sign[\sum_{j} w_{ij}S_j(t) - \theta]$

$$\Delta E = E(t+1) - E(t)$$

$$= -[S_i(t+1) - S_i(t)]\sum_{j} w_{ij}S_j(t) + \theta\,[S_i(t+1) - S_i(t)]$$

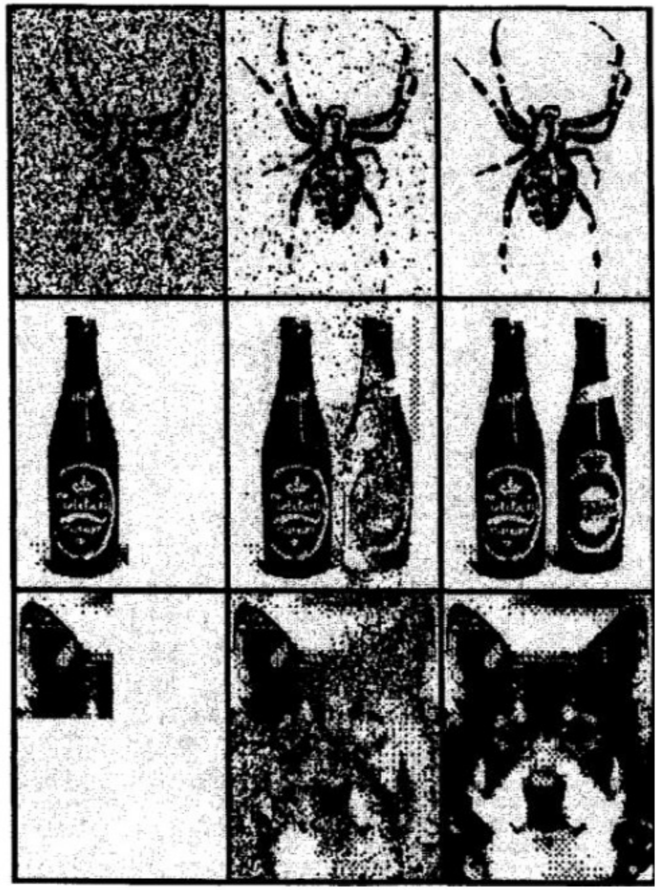
$$= -[S_i(t+1) - S_i(t)][\sum_{j} w_{ij}S_j(t) - \theta]$$

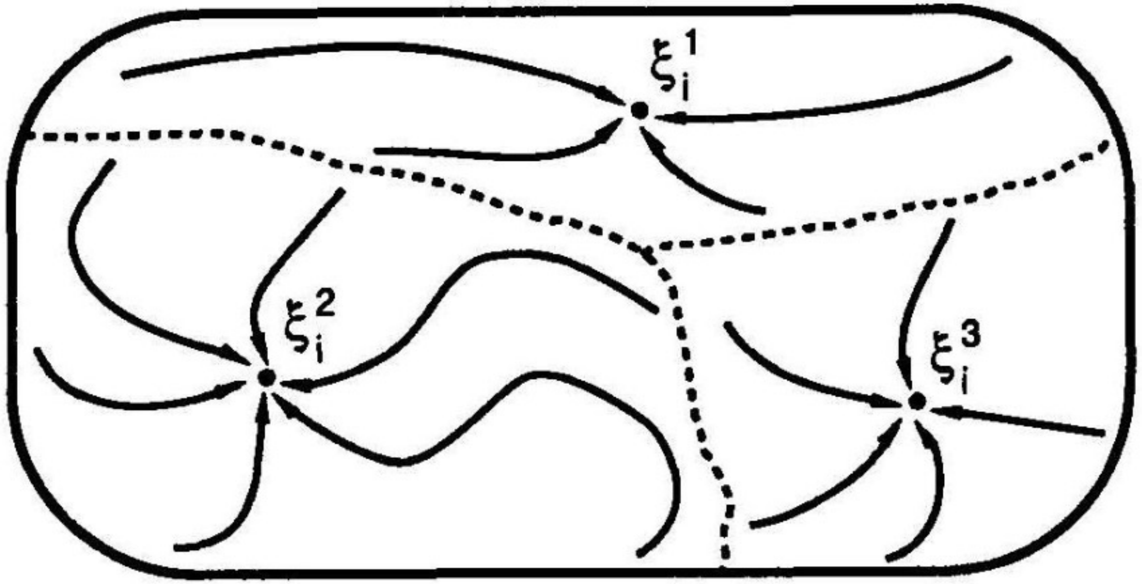
$$\leq 0$$



Auto-associative memory in Hopfield Network

A partial/noisy input can retrieve the related memory pattern

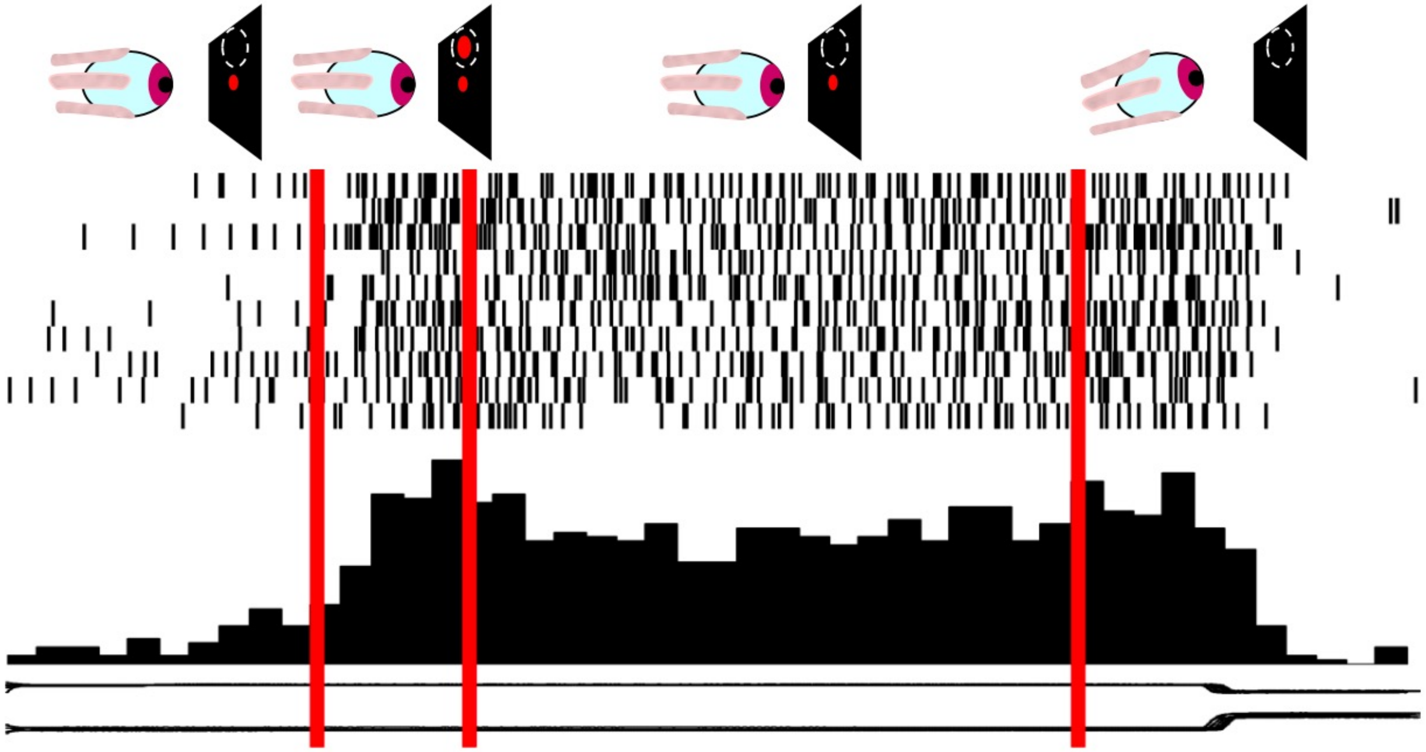




Persistent activity in working memory

After the removal of external input, the neurons in the network encoding the stimulus continue to fire persistently.

Persistent activity recorded during visual working memory task

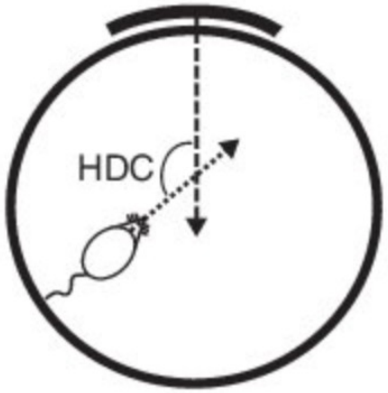


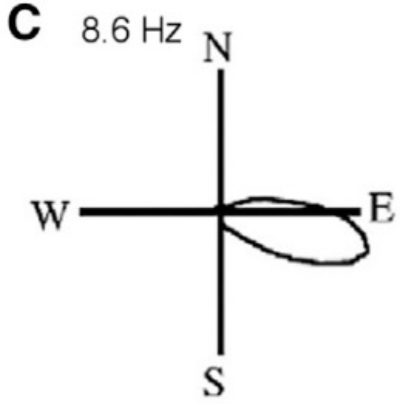
02

Continuous Attractor Neural Network

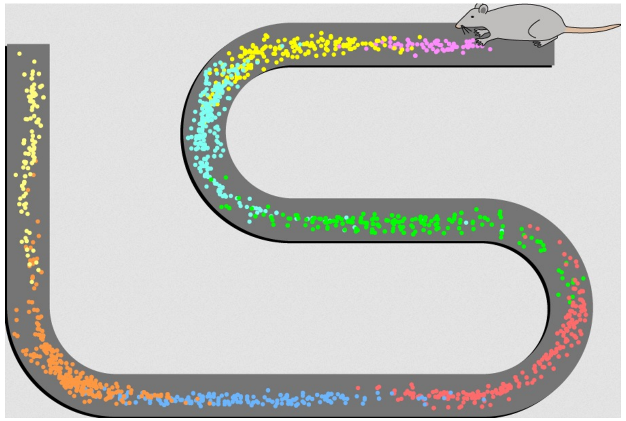
Neural coding of low-dimensional continuous feature

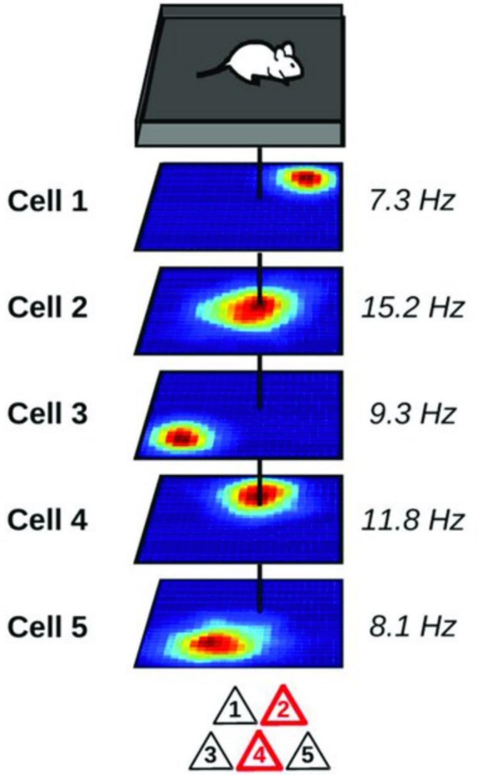
Head-direction cell



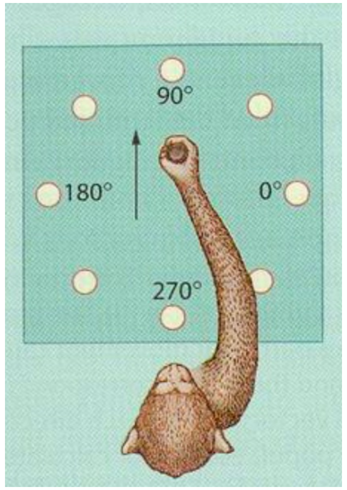


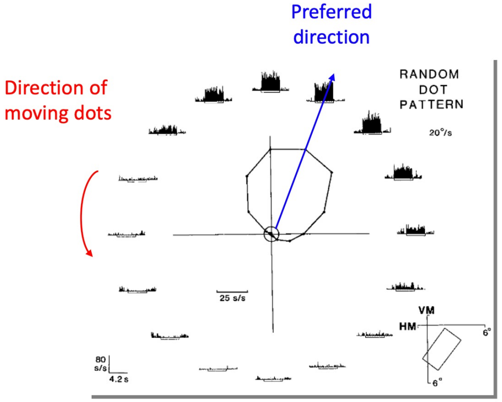
Place cell





Motion-direction cell





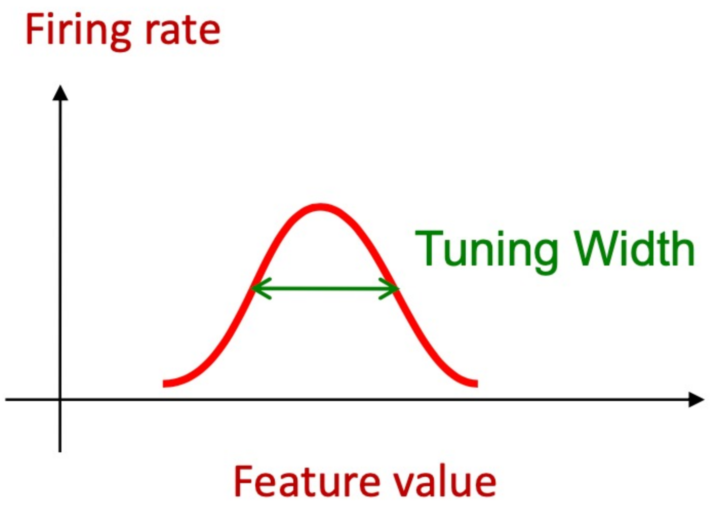
Activities of macaque Middle Temporal (MT) neurons (TD Albright 1984)

Latuske et al., 2018, Frontiers in behavioral neuroscience
Georgepolous et al, 1987, Science

Neural code of Continuous Attractor neural network

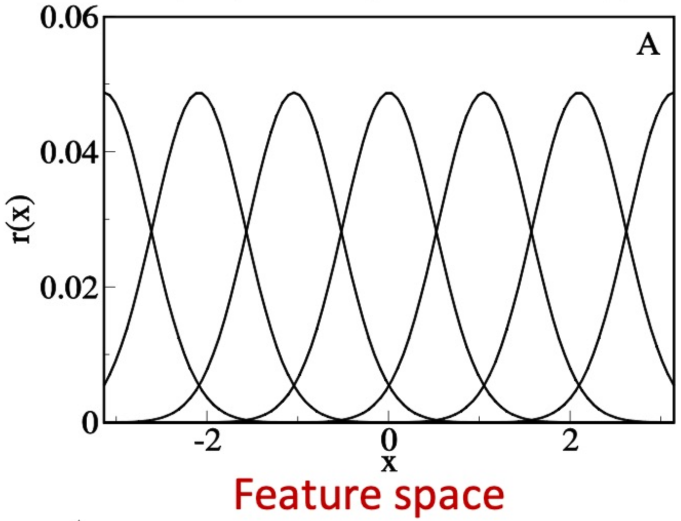
Individual neurons:

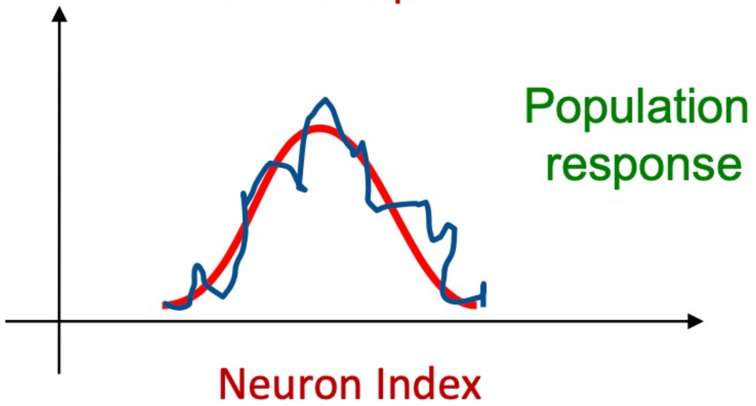
- Preferred feature value
- Bell-shape tuning function



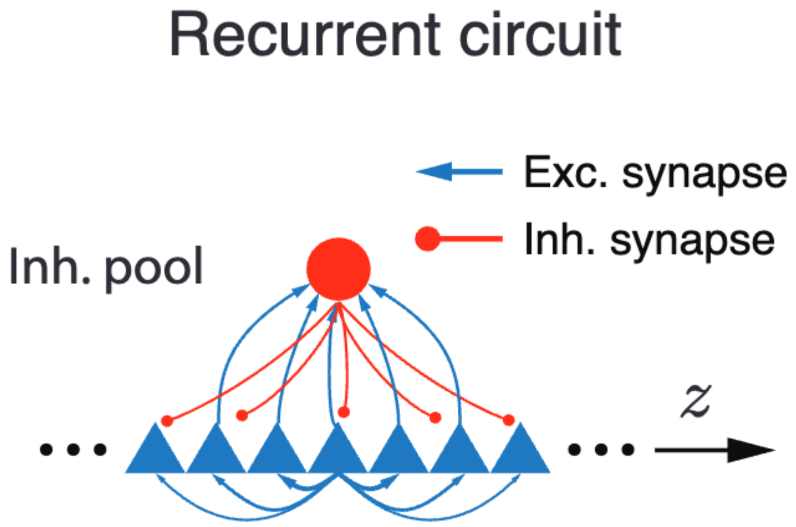
A neural population:

- Overlapped tuning functions covering the whole space
- Largely independent responses





CANN: A rate-based recurrent circuit model



$$\tau \frac{\partial U(x,t)}{\partial t} = -U(\mathrm{x},\mathrm{t}) + \rho \int J(x,x') r(x',t) dx' + I^{ext} \quad (1)$$

$$r(x,t) = \frac{U^2(x,t)}{1 + k\rho \int U^2(x,t) dx} \quad (2)$$

$$J(x,x') = \frac{J_0}{\sqrt{2\pi}a} \exp\left[-\frac{(x-x')^2}{2a^2}\right] \quad (3)$$

Key Structure:

- Bell-shaped recurrent connection strength
- Translation-invariant connection pattern
- Global divisive normalization

Key Mathematic Properties:

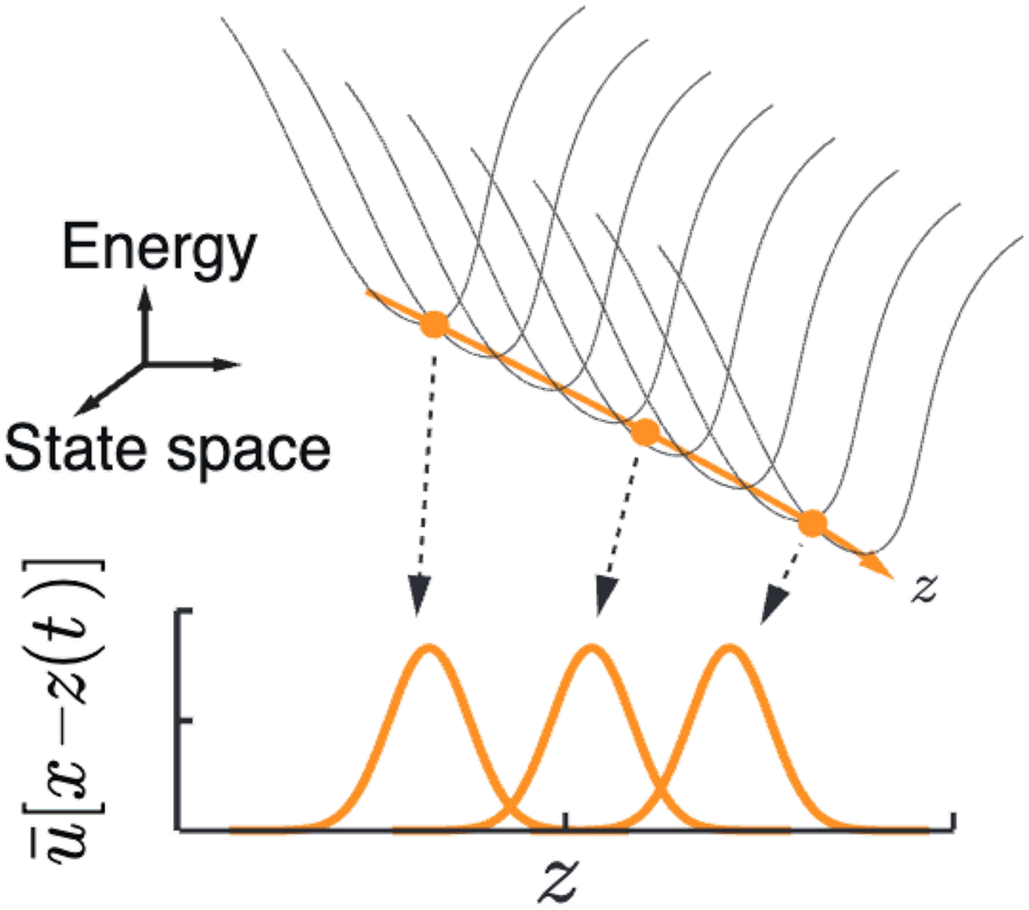
- Recurrent positive-feedback generates attractor, retaining input information
- Divisive normalization avoids exploration
- Translation-invariance ensures many attractors

References: 1. Amari, 1977, 2. Ben-Yishai et al., 1995, 3. Zhang, 1996, 4. Seung, 1996, 5. Deneve et al, 1999, 6. Wu et al, 2002, 2005, 2008, 2010, 2012

A Continuous family of attractor states

$$\overline{U}(x\,|\,z) = \frac{A\rho J}{\sqrt{2}}\exp\left[-\frac{(x-z)^2}{4a^2}\right]$$

$$\bar{r}(x\,|\,z) = A\exp\left[-\frac{(x-z)^2}{4a^2}\right]$$



Stability analysis derive continuous attractor dynamics

Consider small fluctuations around a stationary state at z:

$$\delta U(x|z) = U(x|z) - \bar{U}(x|z)$$

$$\tau \frac{\partial \delta U(x|z)}{\partial t} = -\delta U(x|z) + \rho \int dx' J(x,x') \delta r(x'|z)$$

$$= -\delta U(x|z) + \int dx' F(x,x') \delta U(x')$$

Where

$$F(x,x') = \int dx'' \rho J(x,x'') \frac{\partial \bar{r}(x''|z)}{\partial \bar{U}(x'|z)}$$

$$F(x,x'|z) = \frac{AJ^2\rho^2}{B\sqrt{\pi}a} e^{-(x-x')^2/2a^2} - \frac{kA^3\rho^5 J^4}{\sqrt{3}B^2} e^{-(x-z)^2/4a^2} e^{-(x'-z)^2/4a^2}$$

$$\tau \frac{\partial \boldsymbol{\delta}\mathbf{U}}{\partial t} = -(\mathbf{I} - \mathbf{F})\boldsymbol{\delta}\mathbf{U}, \qquad \boldsymbol{\delta}\mathbf{U} = \{\delta U(x|z)\}, \text{ for all } x$$

Projecting $\boldsymbol{\delta}\mathbf{U}$ on the ith right eigenvector of $\mathbf{F}$

$$(\boldsymbol{\delta}\mathbf{U})_i(t) = (\boldsymbol{\delta}\mathbf{U})_i(0) e^{-(1-\lambda_i)t/\tau}$$

Two cases:

1. **If $\lambda_i < 1$, the projection decays exponentially;**
2. **If $\lambda_i = 1$, the projection is sustained.**

Fung et al. Neural computation, 2010

Spectra of the kernel F

- $\lambda_0 = 1 - 2k\rho A\sqrt{2\pi}a < 1, \quad \mathbf{u}_0(x\,|\,z) = \bar{\mathbf{U}}(x\,|\,z);$
- $\lambda_1 = 1, \qquad \mathbf{u}_1(x\,|\,z) = \dfrac{d\bar{\mathbf{U}}(x\,|\,z)}{dz},$ the tangent of the valley
- $\lambda_n = \dfrac{1}{2^{n-2}}, \qquad \mathbf{u}_n(z) =$ Combination of $\mathbf{v}_n(z)$

$\mathbf{v}_n(z) \sim e^{-(c-z)^2/4a^2}\left(\dfrac{d}{dc}\right)^n e^{-(c-z)^2/2a^2},$ the wave functions of quntumn harmonic osscilator

Note the decay time constant is : $\dfrac{\tau}{1-\lambda_n}$

Fung et al. Neural computation, 2010

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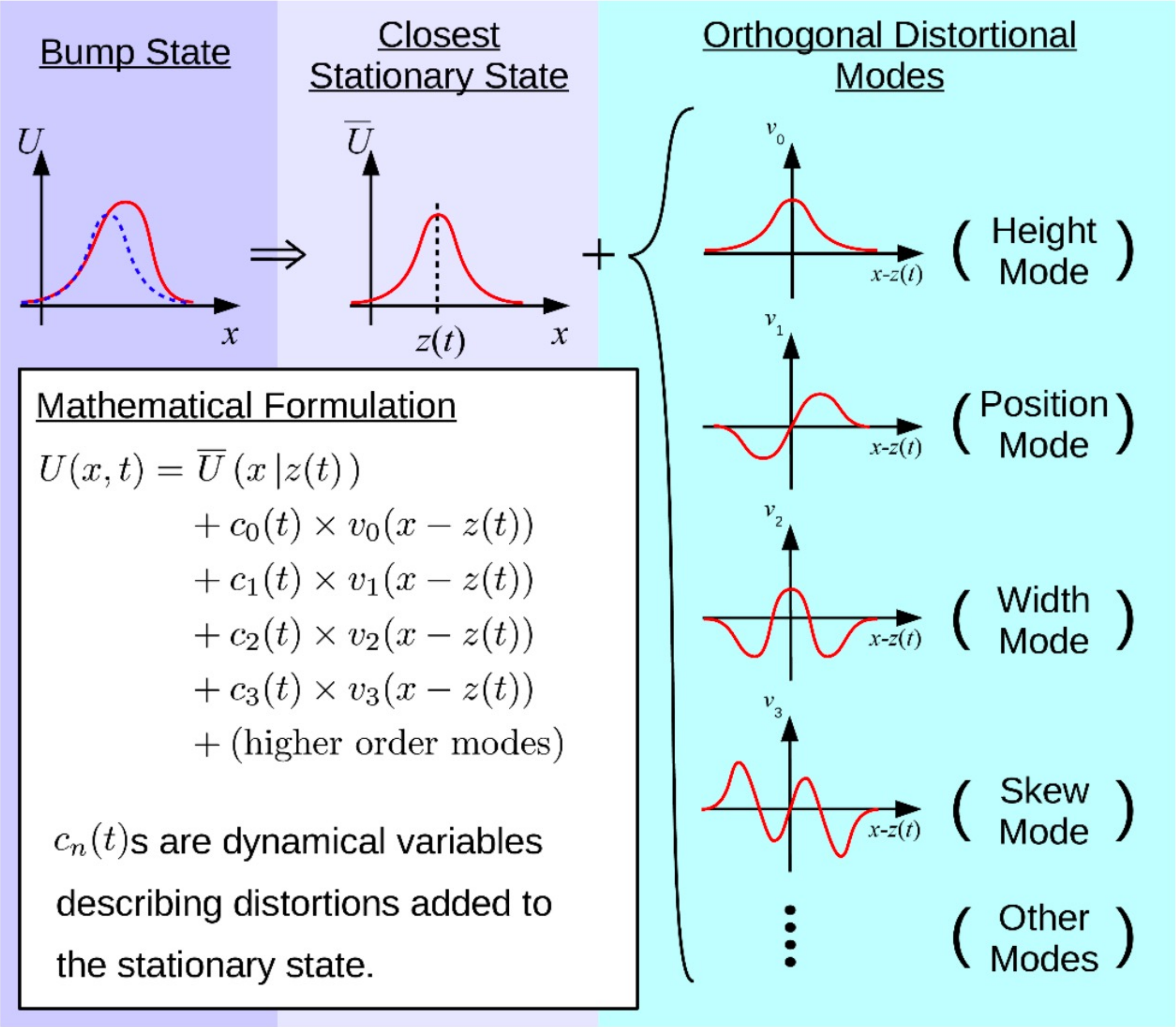
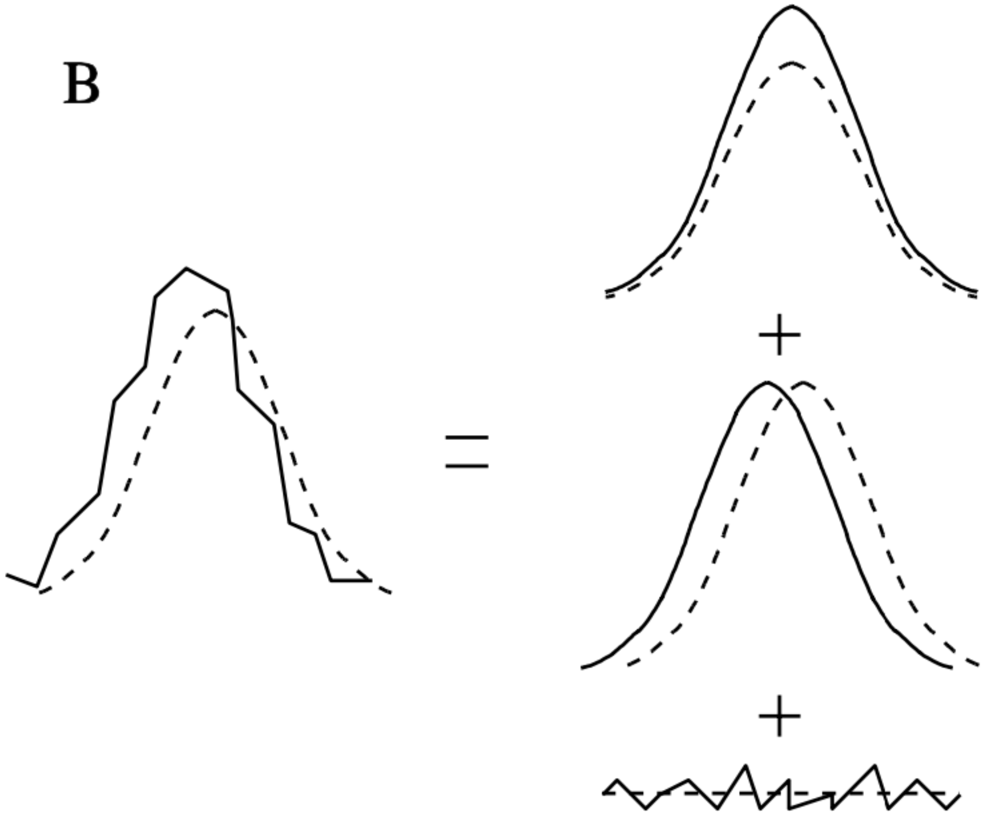
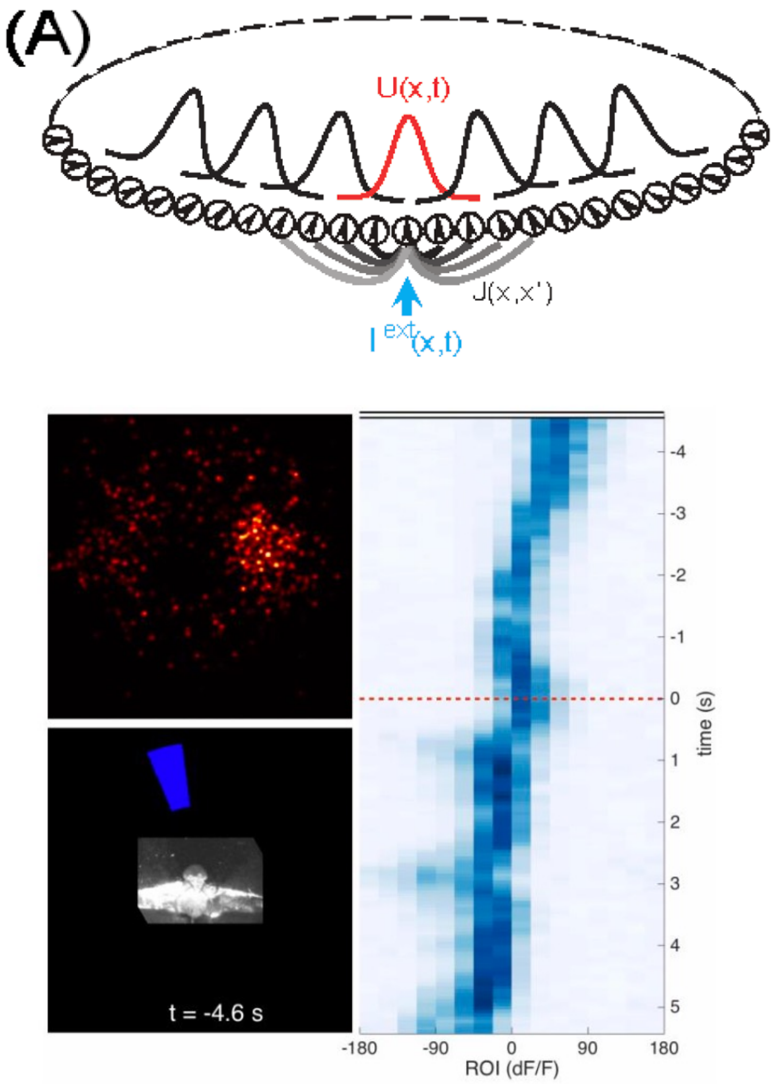
Only bump position shift survives

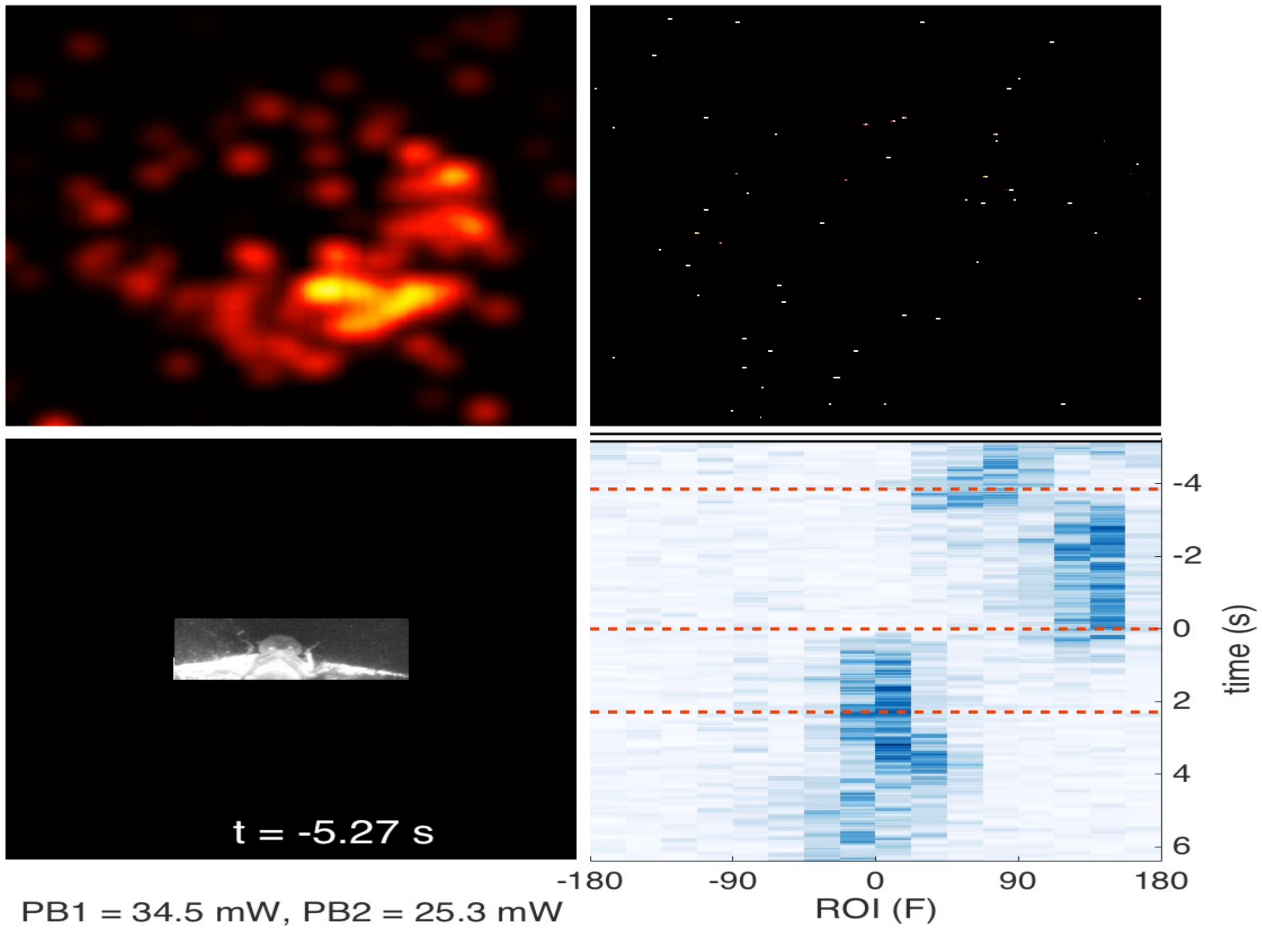




Wu et al., 2010, f1000research

1D CANN for head-direction cell in fruit fly





Sung Soo Kim et al. 2017

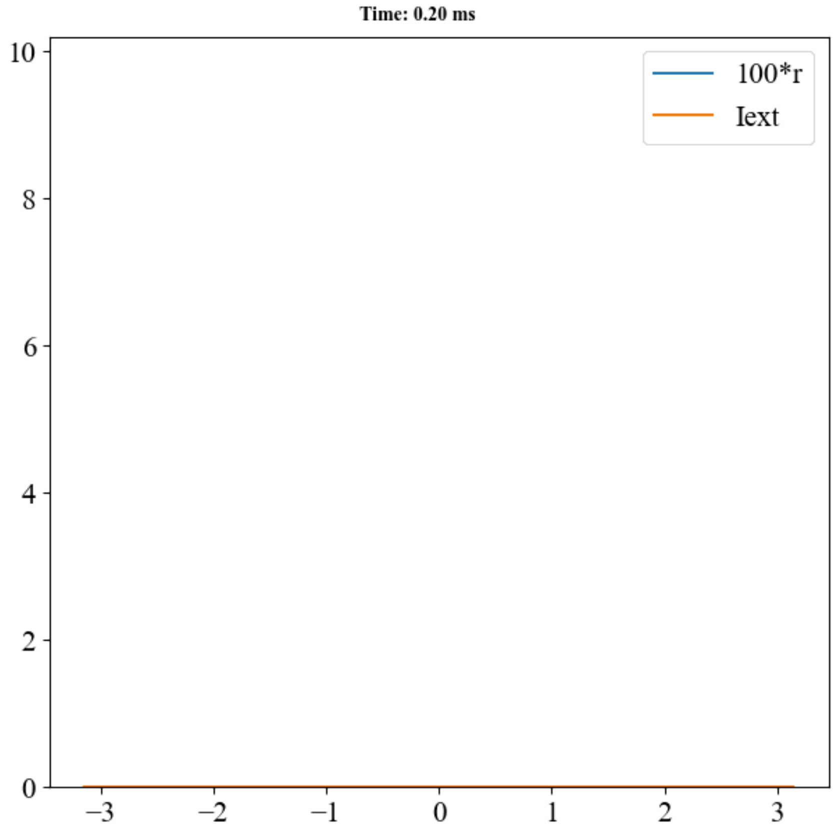
03

Computation with CANN

Persistent activity for working memory

When the global inhibition is not too strong, the network spontaneously hold bump activity:

$$k < \frac{\rho J_0^2}{8\sqrt{2\pi} a}$$



$$\tilde{U}(x|z) = U_0 \exp\left[-\frac{(x-z)^2}{4a^2}\right],$$

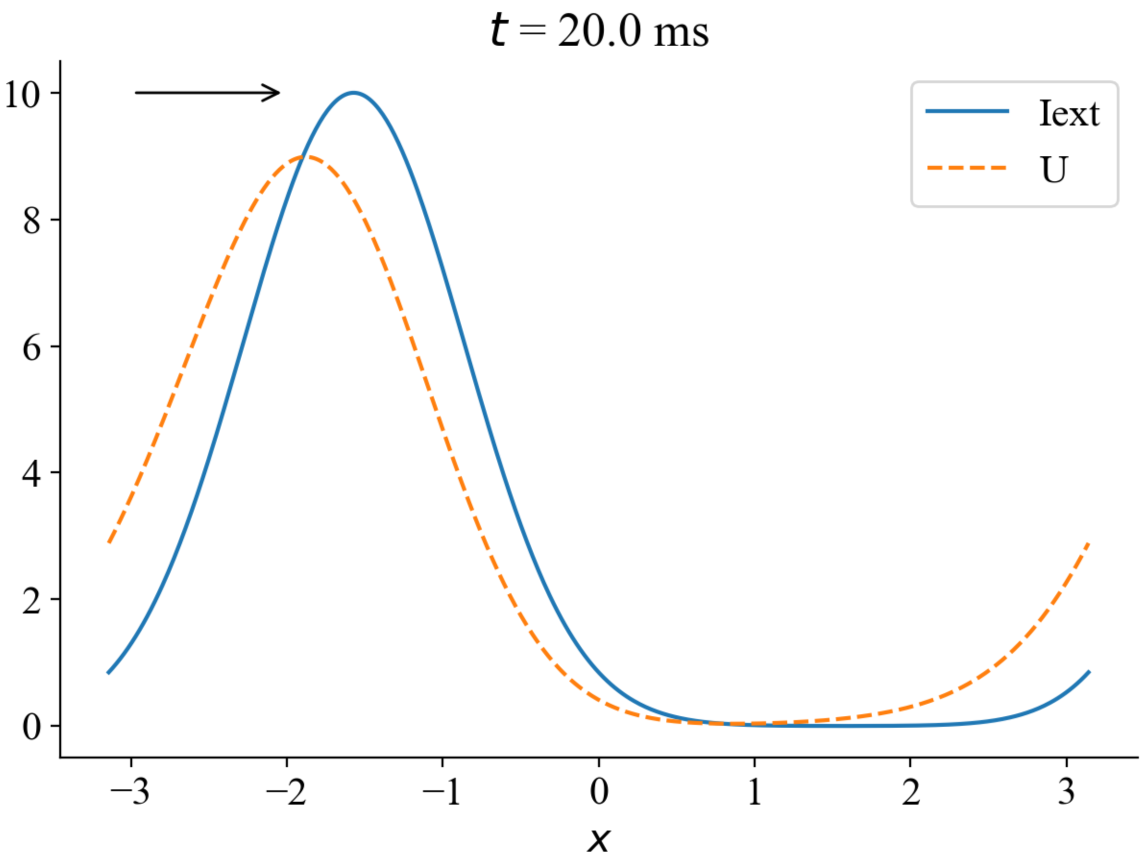
$$\tilde{r}(x|z) = r_0 \exp\left[-\frac{(x-z)^2}{2a^2}\right],$$

$$U_0 = [1 + (1 - k/k_c)^{1/2}] A/(4\sqrt{\pi} a k)$$

$$r_0 = [1 + (1 - k/k_c)^{1/2}]/(2\sqrt{2\pi} a k \rho).$$

Wu et al. Neural computation, 2008

Smooth tracking by CANN



Project the network dynamics on $\mathbf{v}_1(t)$

$$\tau \frac{\partial \mathbf{U} * \mathbf{v}_1}{\partial t} = -\mathbf{U} * \mathbf{v}_1 + (\mathbf{J} * \mathbf{r}) * \mathbf{v}_1 + \mathbf{I}^{ext} * \mathbf{v}_1$$

Consider

$$I^{ext}(t) = \alpha \bar{U}(x \mid z_0) + \sigma \xi_c(t)$$

$$\mathbf{U} * \mathbf{v}_1 \equiv \int_x dx U(x \mid z) v_1(x \mid z)$$

$$\tau \frac{dz}{dt} = -\alpha (z - z_0) e^{-(z-z_0)^2/8a^2} + \beta \xi(t)$$

$1st$ term: the force of the signal that pulls the bump back to the stimulus position

$2nd$ term : random shift

Fung et al. Neural computation, 2010

Smooth tracking by CANN

Define $s = z_0 - z$

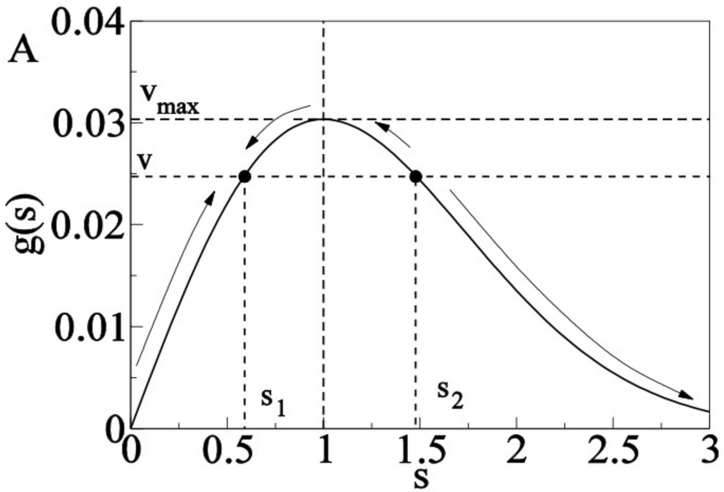
$$\frac{ds}{dt} = \frac{dz_0}{dt} - \frac{dz}{dt}$$

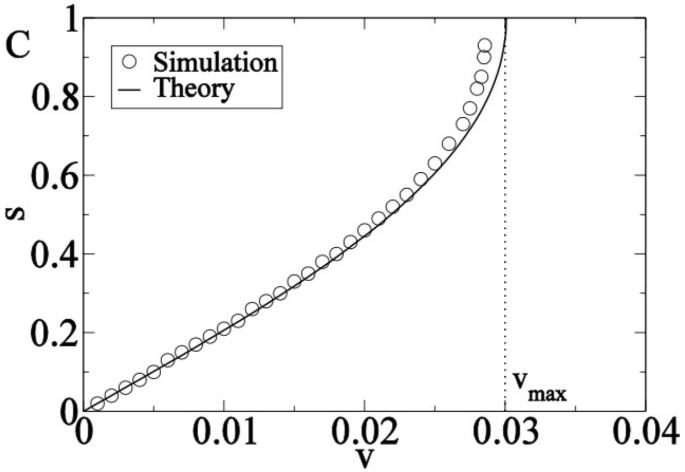
$$= v - \frac{\alpha s}{\tau} e^{-s^2/8a^2}$$

$$= v - g(s)$$

Condition for successful tracking :

$$v = g(s)$$

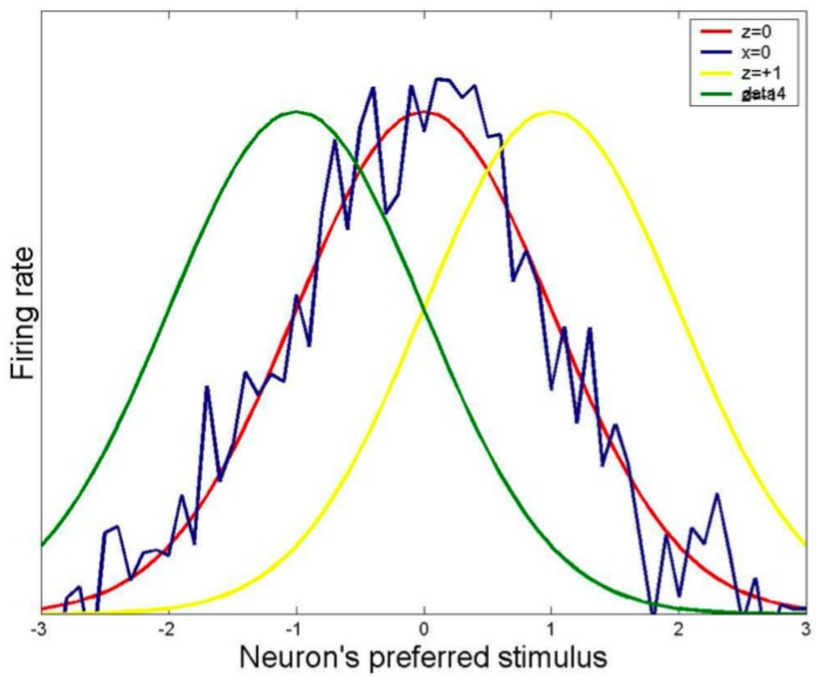




Fung et al. Neural computation, 2010

Population decoding via template matching

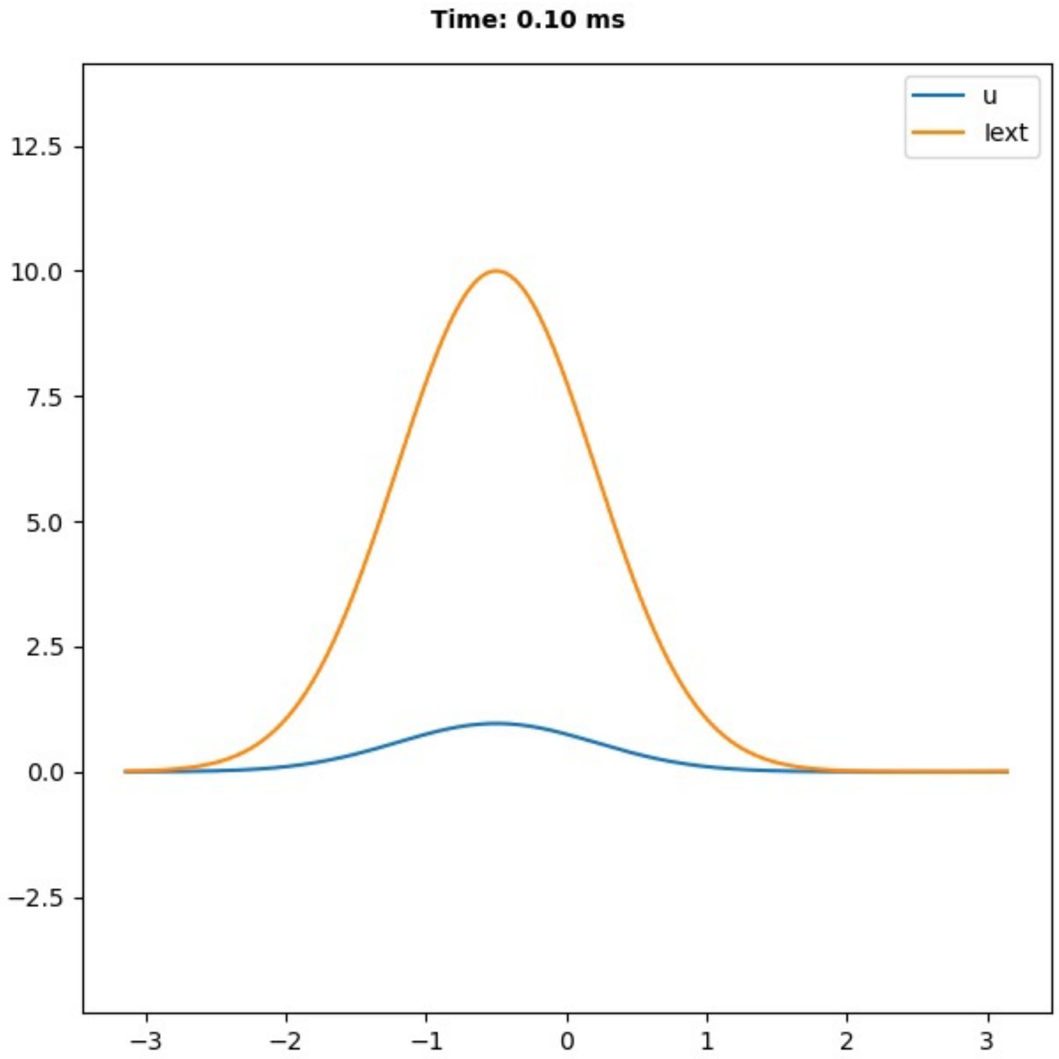
$$\hat{x} = \max_z \sum_i r_i f_i(z)$$



- The noisy bump is the population activity when the stimulus x=*0*.
- Among three positions, the red one ($z \approx 0$) has the maximum overlap with the observed data.

Wu et al. Neural computation, 2008

Population decoding via template matching



$$\tau_s \frac{\partial U(x,t)}{\partial t} = -U(x,t) + \rho \int dx' J(x,x') r(x',t) + \varepsilon I^{ext}(x,t)$$

For small inputs, the final position

$$\hat{z} = \max_z \int dx \bar{U}(x \mid z) I^{ext}(x)$$

Wu et al. Neural computation, 2008

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04

Computation and Dynamics of Adaptive CANN

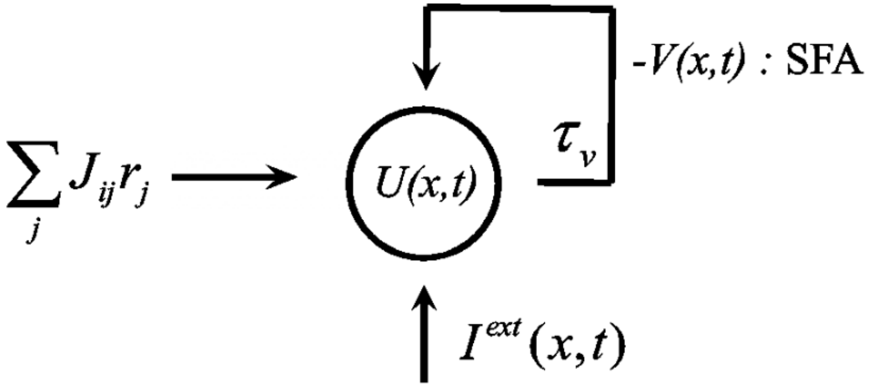
Adaptive Continuous Attractor neural network

$$\tau \frac{dU(x,t)}{dt} = -U(x,t) + \rho \int dx' J(x-x') r(x',t) - V(x,t) + I^{ext}(x,t)$$

$$\tau_v \frac{dV(x,t)}{dt} = -V(x,t) + mU(x,t)$$

V(x,t) represents the SFA effect,

$$V(x,t) = \frac{m}{\tau_v} \int_{-\infty}^{t} e^{-\frac{t-t'}{\tau_v}} U(x,t') dt'$$



Spike frequency Adaptation (SFA):

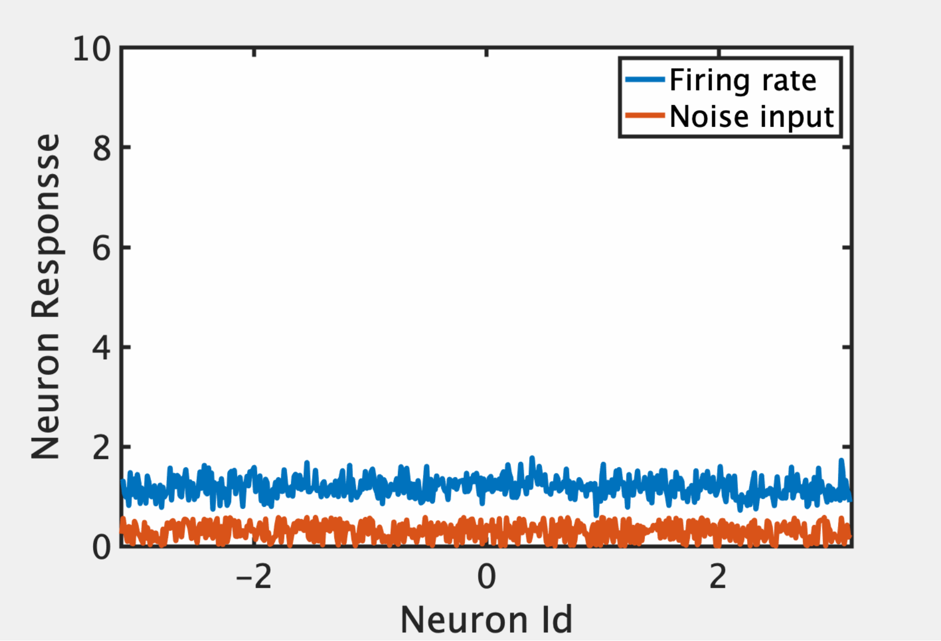
- Neuronal response attenuates after experiencing prolonged firing.
- Slow negative feedback modulation to neuronal response.

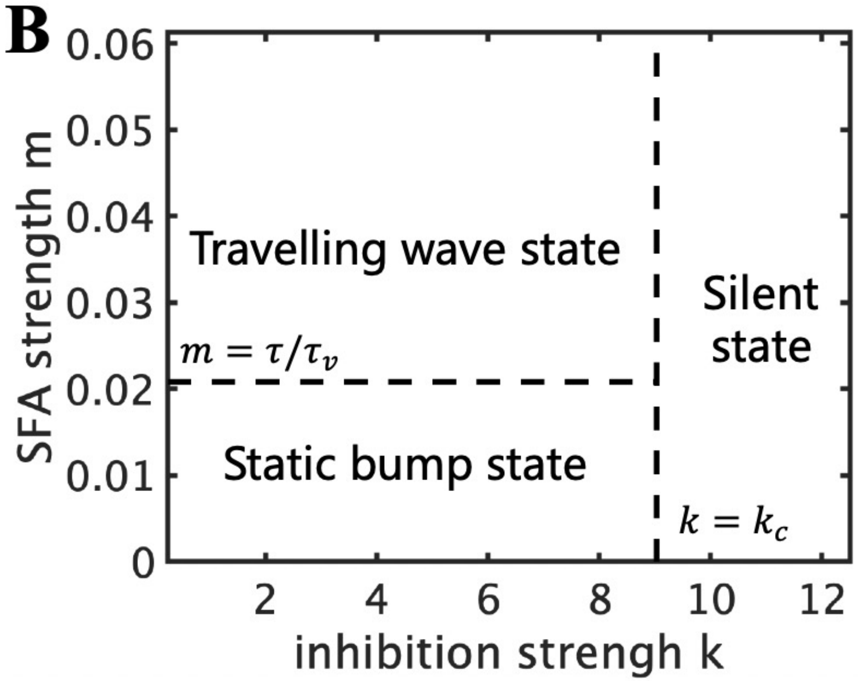
Mi et al., NIPS 2014

Intrinsic mobility of A-CANN

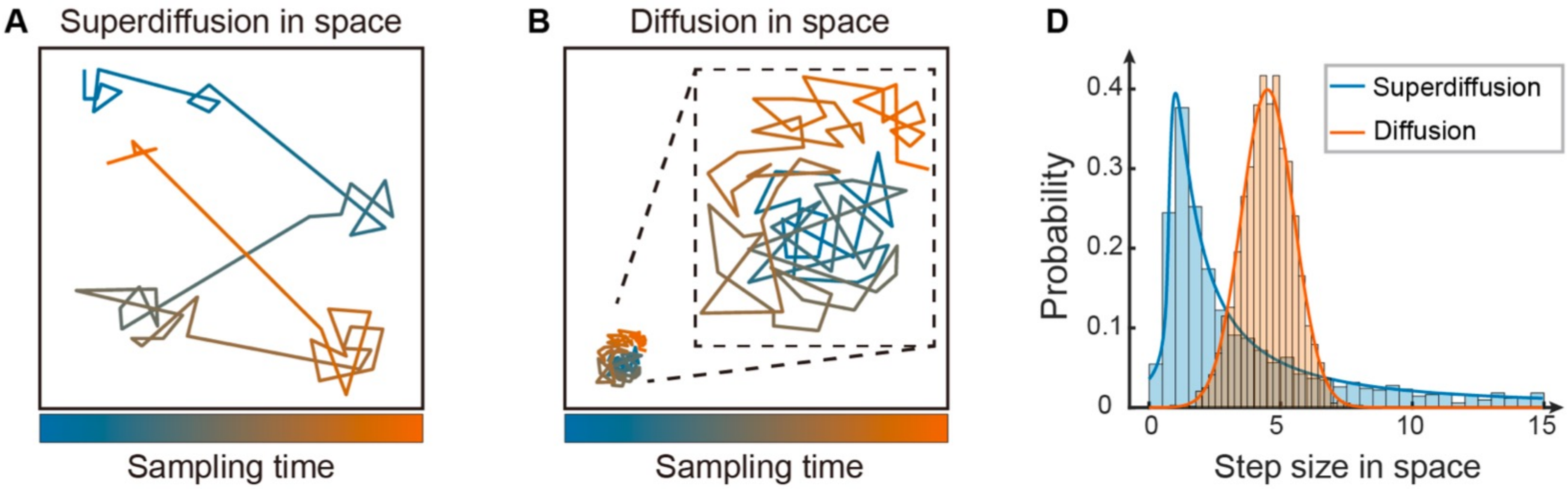
Traveling Wave: a moving bump in the network without relying on external drive.
The mechanism: SFA suppresses localized neural activity and triggers the bump to move.

$$m > \frac{\tau}{\tau_v}, \text{ Travelling wave}$$





Levy flights vs. Brownian motion



$$p(x) \sim x^{-u}$$

or

$$p(x) \sim x^{-1-\alpha}$$

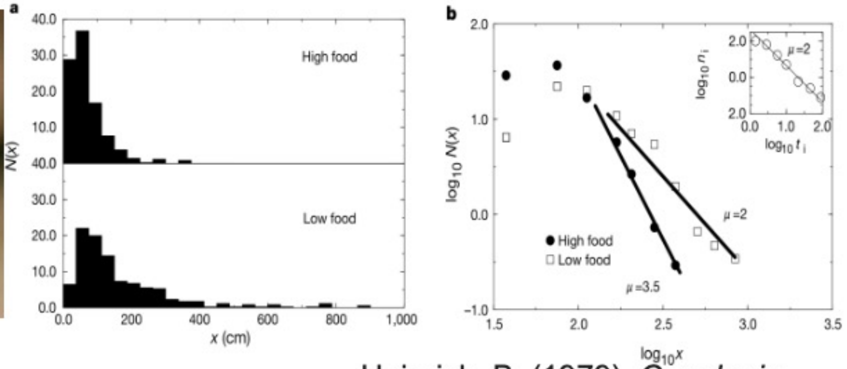
$0 < \alpha < 2$

Superdiffusion / Lévy motion

$\alpha \geq 2$

Diffusion / Brownian motion

Lévy flights in ecology and human cognitive behaviors



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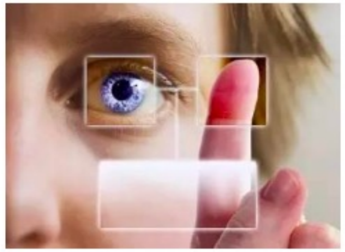

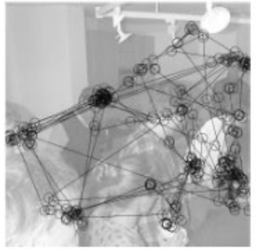

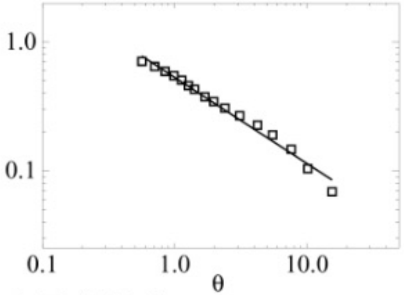

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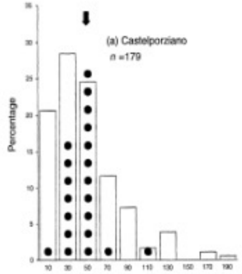
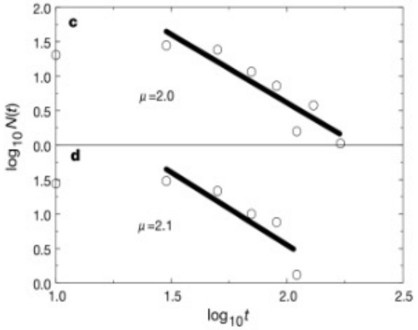
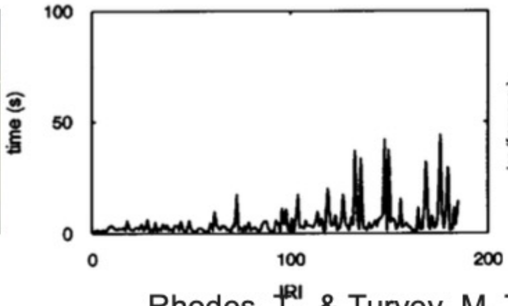
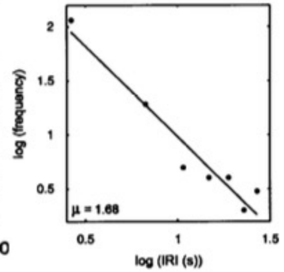




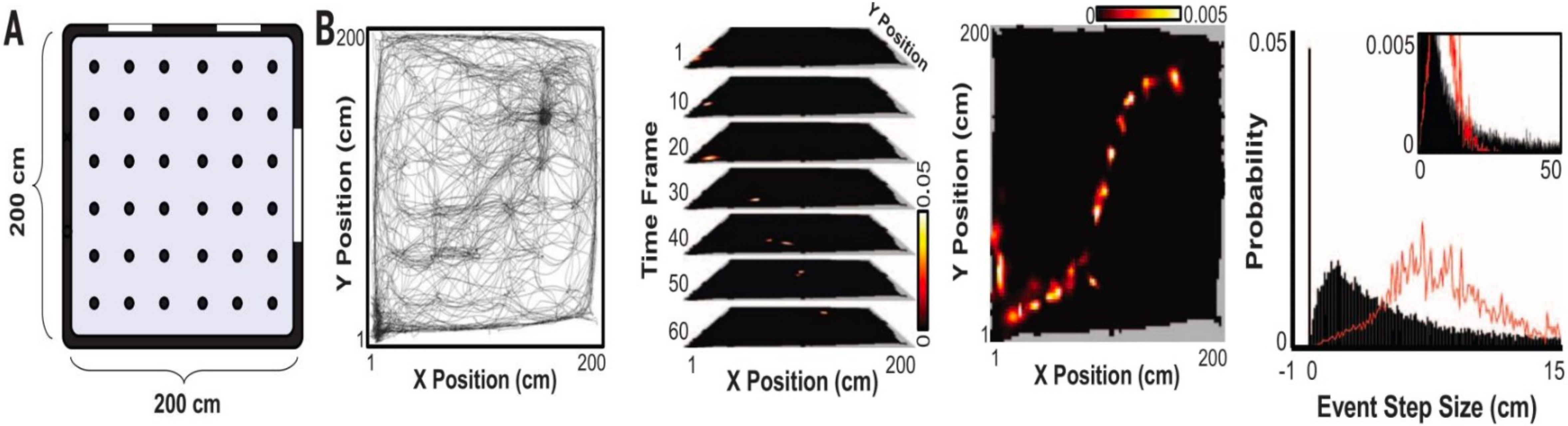
Focardi et al. (1996) *Journal of Animal Ecology*





Rhodes, T., & Turvey, M. T. (2007). *Physica A*

Levy flight in hippocampal re-activation



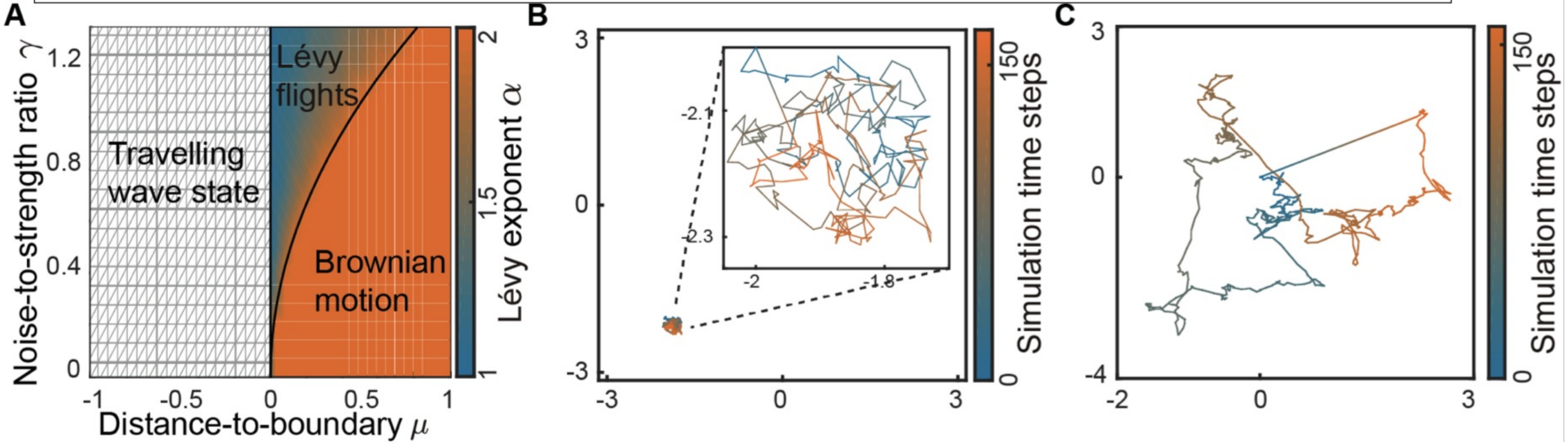
Pfeiffer, B. E., & Foster, D. J. (2015). *Science*.

Noisy adaptation generates Levy flight in CANN

$$\tau \frac{\mathrm{d}\mathbf{z}}{\mathrm{d}t} = m\mathbf{s} + \frac{\sigma_U}{\tilde{A}_u}\sqrt{\frac{2}{\pi}}\xi_{\mathbf{z}}(t),$$

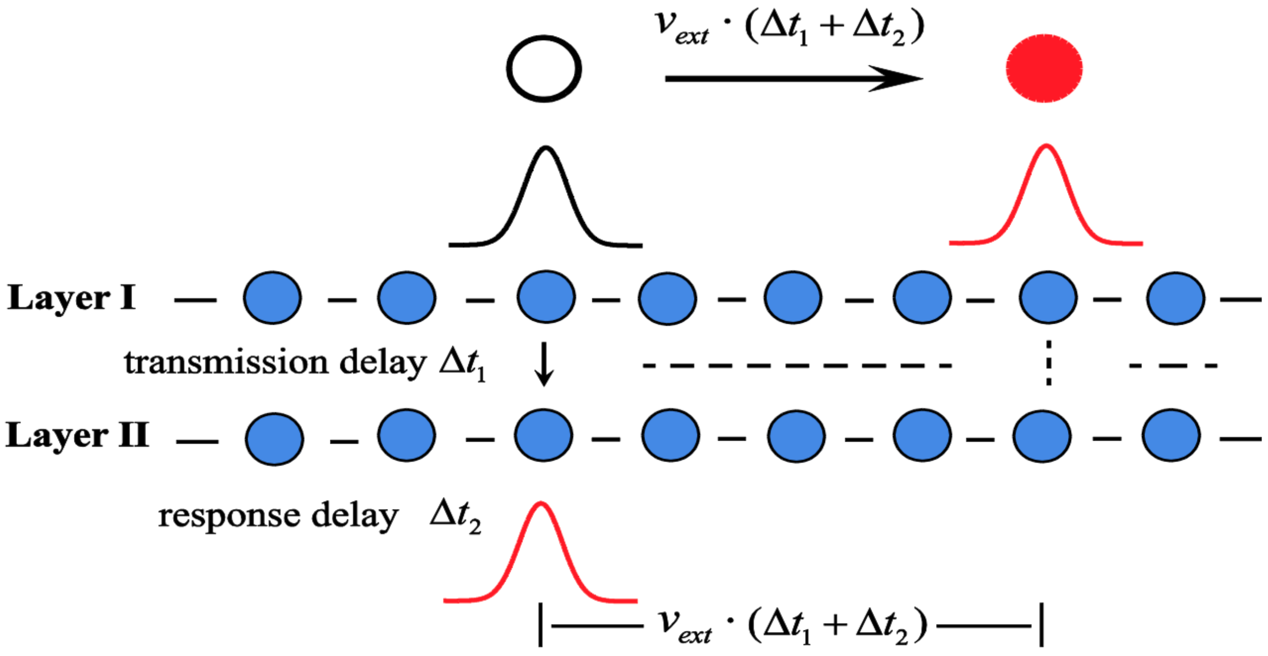
$$\tau_v \frac{\mathrm{d}\mathbf{s}}{\mathrm{d}t} = -\left[1 - \frac{\tau_v}{\tau}m + \frac{\sigma_m}{2\sqrt{\pi}am}\xi_m(t)\right]\mathbf{s} + \sqrt{\frac{2}{\pi}\left(\frac{\tau_v \sigma_U}{\tau \tilde{A}_u}\right)^2 + \frac{1}{2\pi}\left(\frac{\sigma_m}{m}\right)^2}\xi_{\mathbf{s}}(t).$$

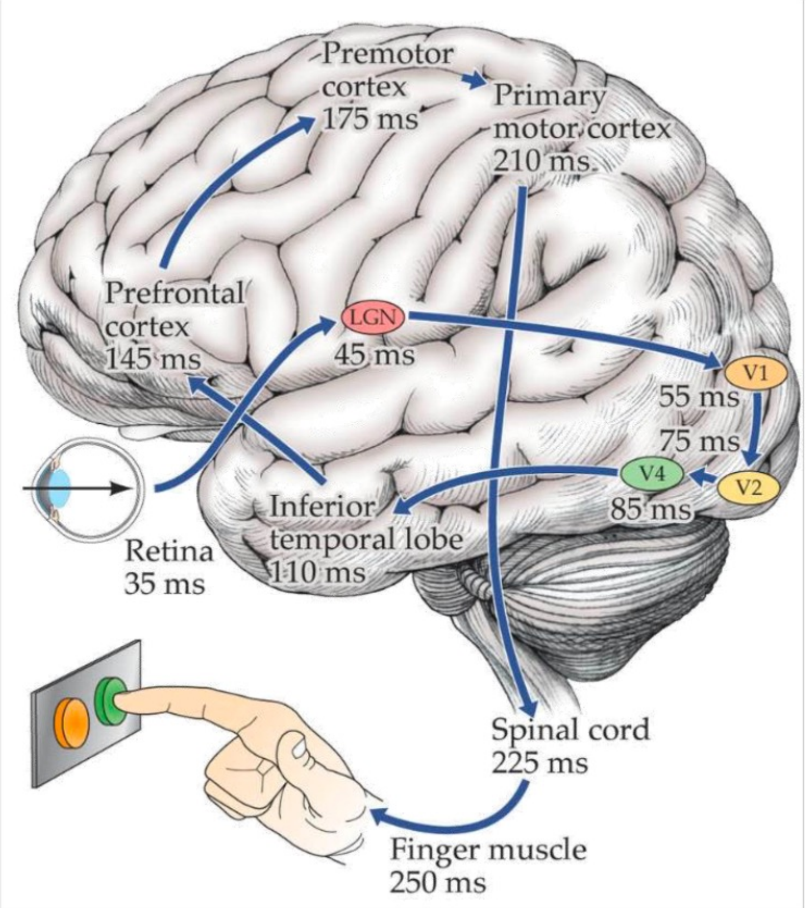
$$\boxed{p(\|\Delta\mathbf{z}\|) \sim \|\Delta\mathbf{z}\|^{-1-(1+2\mu/\gamma^2)}}$$



Dong, X.*, Chu, T.*, Huang, T., Ji, Z., & Wu, S. (2021). Noisy Adaptation Generates Lévy Flights in Attractor Neural Networks. Advances in Neural Information Processing Systems, 34, 16791-16804. *: Equally Contributed

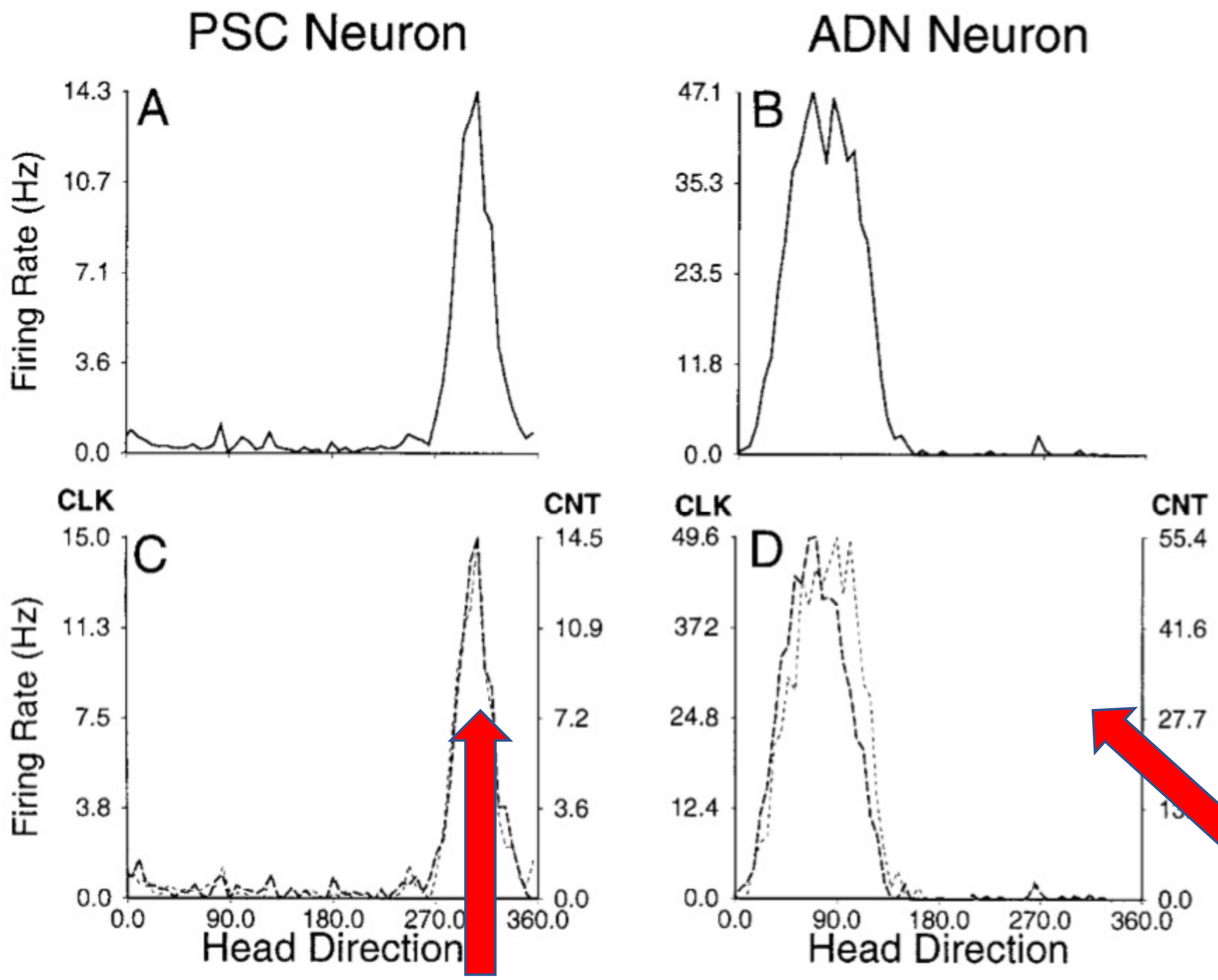
Time Delay in Neural Signal Transmission

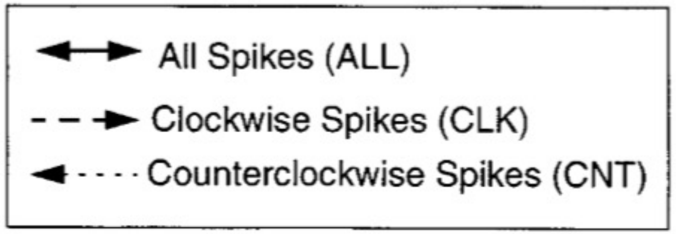




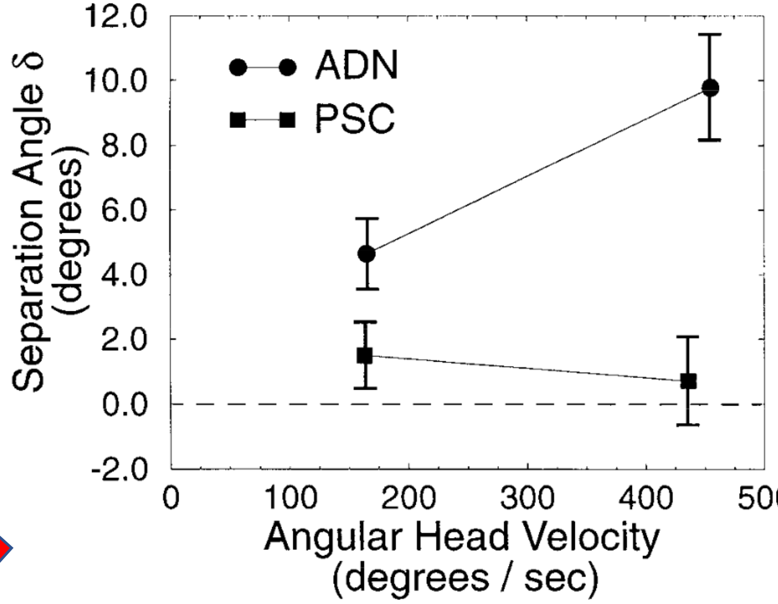
(e.g. Maunsell and Gibson 1992; Raiguel et al. 1989; Nowak et al. 1995; Schmolesky et al. 1998; Thorpe, Fize, & Marlot 1996)

Anticipatory Head Direction Signals in Anterior Thalamus





(Blair et al., 1995, The Journal of Neuroscience,)



The shift of the bump depend on the rotation of the head

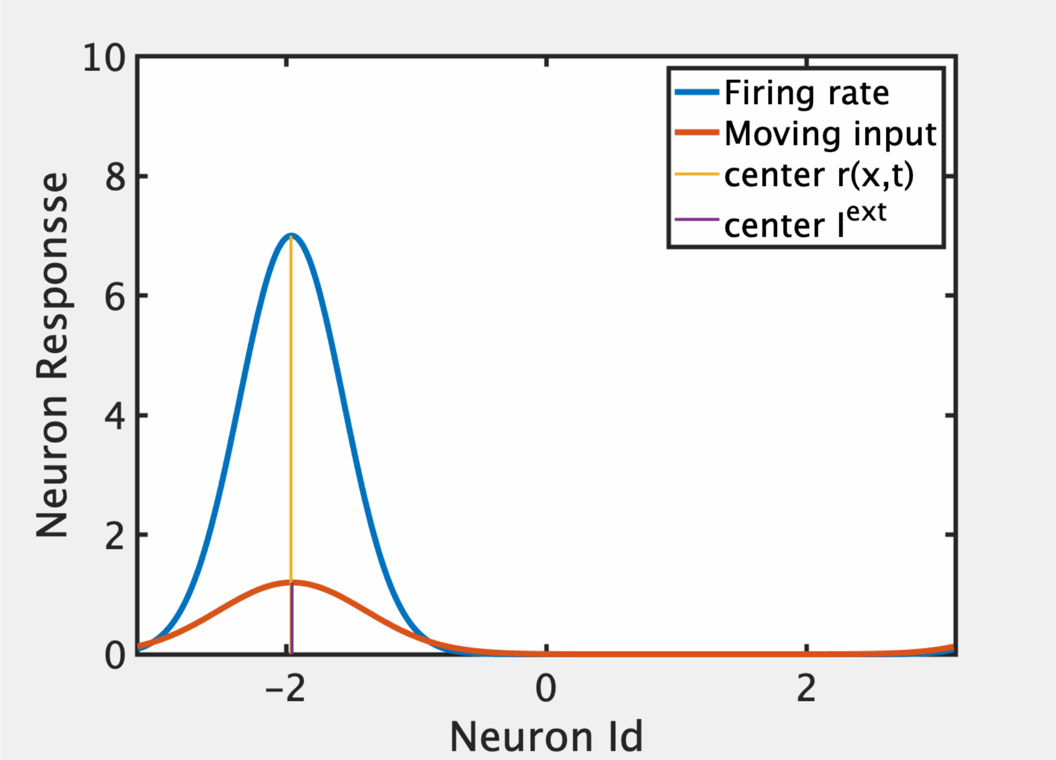
Adaptive CANN accounts for anticipative tracking

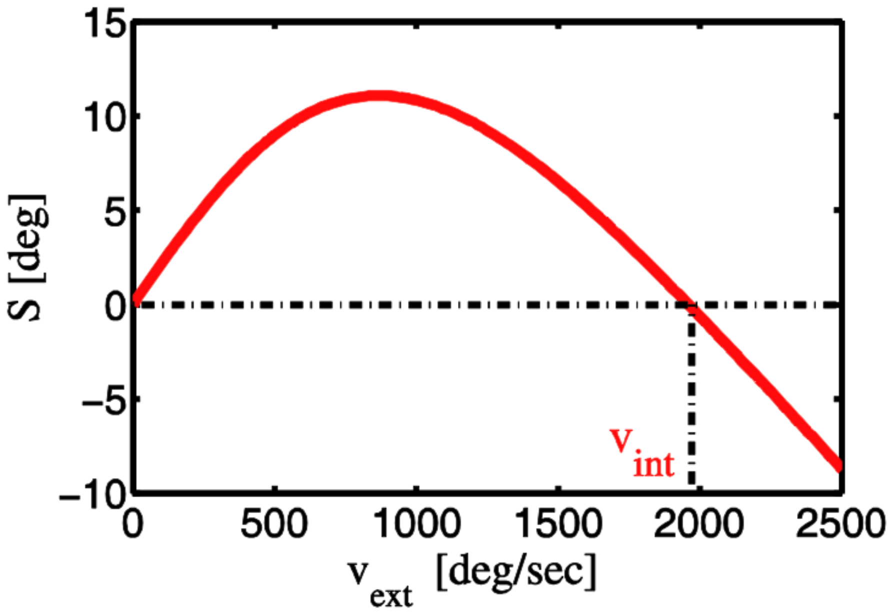
The bump can track the moving input with constant distance s, which satisfies:

$$se^{-\frac{s^2}{8a^2}} = \frac{A_U v_{ext} m\tau_v}{\alpha}\left[e^{-\frac{\tau_v^2 v_{ext}^2}{4a^2}} - \frac{\tau}{m\tau_v}\right] \quad (7),$$

The condition for $s > 0$ is $v_{int} > v_{ext}$.

$v_{int} > v_{ext}, Anticipative\ Tracking$





Mi et al. Nips, 2014

CANN with STP

$$\tau \frac{\mathrm{d}U(x,t)}{\mathrm{d}t} = -U(\mathrm{x},\mathrm{t}) + \rho \int g^{+}(x)h(x',t)J(x,x')r(x',t)dx' + I^{ext}(x,t) \quad (1)$$

$$\frac{dg(x,t)}{dt} = -\frac{g(x,t)}{\tau_f} + G(1-g^{-}(x))r(x',t) \quad (2)$$

$$\frac{dh(x,t)}{dt} = \frac{1-h(x,t)}{\tau_d} - g^{+}(x)h(x,t)r(x',t) \quad (3)$$

$$r(x,t) = \frac{U^2(x,t)}{1+k\rho\int U^2(x,t)dx} \quad (4)$$

Q&A

Thanks for your listening!