MA1-I SORULAR 11.11.2014

1) $\lim\limits_{x\to a} f(x) = l$ var ise tektir Yani,

$$\lim_{x\to a} f(x) \begin{matrix} \nearrow l_1 \\ \searrow l_2 \end{matrix} \Rightarrow l_1 = l_2 \text{ dir.}$$

$\lim\limits_{x\to a} f(x) = l_1 \overset{t}{\Leftrightarrow} \forall \varepsilon > 0$, $\exists \delta_1(\varepsilon) > 0$ vardır ki
$0 < |x-a| < \delta_1$ iken $|f(x) - l_1| < \varepsilon/2$

$\lim\limits_{x\to a} f(x) = l_2 \overset{t}{\Leftrightarrow} \forall \varepsilon > 0$, $\exists \delta_2(\varepsilon) > 0$ vardır ki
$0 < |x-a| < \delta_2$ iken $|f(x) - l_2| < \varepsilon/2$

$\forall \varepsilon > 0$ için $\exists \delta = \min\{\delta_1, \delta_2\}$ $|x-a| < \delta$ iken

$$0 < |l_1 - l_2| = |l_1 - l_2 + f(x) - f(x)| \leq |f(x) - l_1| + |f(x) - l_2| < \varepsilon/2 + \varepsilon/2 = \varepsilon$$

$\Rightarrow |l_1 - l_2| < \varepsilon \quad (\forall \varepsilon > 0)$

$\Rightarrow l_1 = l_2$

*

* $\forall \varepsilon > 0$ için
$|x-y| < \varepsilon \Rightarrow x = y$

2) $\left.\begin{matrix} \lim\limits_{x\to a} f(x) = l_1 \\ \lim\limits_{x\to a} g(x) = l_2 \end{matrix}\right\} \Rightarrow \lim\limits_{x\to a} f(x) + g(x) \overset{?}{=} l_1 + l_2$

$\lim\limits_{x\to a} f(x) = l_1 \overset{t}{\Leftrightarrow} \forall \varepsilon > 0$, $\exists \delta_1 > 0$ vardır ki $|x-a| < \delta_1 \Downarrow |f(x) - l_1| < \varepsilon/2$

$\lim\limits_{x\to a} g(x) = l_2 \overset{t}{\Leftrightarrow} \forall \varepsilon > 0$, $\exists \delta_2 > 0$ vardır ki $|x-a| < \delta_2 \Downarrow |g(x) - l_2| < \varepsilon/2$

(1)

$\varepsilon > 0$ verilsin. Bir $\delta(\varepsilon) > 0$ varmıdır ki $|x-a| < \delta$

$\Downarrow$

$|f(x)+g(x)-(l_1+l_2)| < \varepsilon$

olsun.

$\delta = \min\{\delta_1, \delta_2\}$ olsun. $|x-a| < \delta$ iken

$\Rightarrow |f(x)+g(x)-(l_1+l_2)| \leqslant |f(x)-l_1| + |g(x)-l_2|$

$< \varepsilon/2 + \varepsilon/2 = \varepsilon$ olur.

Ödev $\left.\begin{array}{l}\lim\limits_{x \to a} f(x) = l_1 \\ \lim\limits_{x \to a} g(x) = l_2\end{array}\right\} \Rightarrow \lim\limits_{x \to a} f(x) \cdot g(x) = l_1 \cdot l_2$ dir.

Gösteriniz

3) $\lim\limits_{x \to 3} 10^{-\frac{1}{(x-3)^2}} = 0$ olduğunu gösteriniz.

$\forall \varepsilon > 0$ için $\exists \delta(\varepsilon) > 0$ var mıdır ki $|x-3| < \delta$

$\Downarrow$

$|10^{-\frac{1}{(x-3)^2}} - 0| < \varepsilon$ olsun.

$|x-3| < \delta \Rightarrow -\delta < x-3 < \delta$ (özel olarak $\delta = 1$ alırsak $x-3 < 1$ olur,

$(x-3)^2 = (x-3)(x-3) < \delta \cdot 1 = \delta$

$-\frac{1}{(x-3)^2} < -\frac{1}{\delta} \Rightarrow 10^{-\frac{1}{(x-3)^2}} < 10^{-\frac{1}{\delta}} = \varepsilon$

$-\frac{1}{\delta} = \log \varepsilon \Rightarrow \delta = -\frac{1}{\log \varepsilon} \Rightarrow \delta(\varepsilon) = \min\{1, -\frac{1}{\log \varepsilon}\}$

4) $\lim_{x \to 4} \frac{2-x}{(x-4)^2} = -\infty$ old. gösteriniz.

$\forall M > 0$ için $\exists \delta(M) > 0$ var mıdır ki $|x-4| < \delta$

$\Downarrow$

$\frac{2-x}{(x-4)^2} < -M$ ols

$\frac{2-x}{(x-4)^2} = \frac{2-x}{(x-4)(x-4)} < \frac{\delta - 2}{(-1).(-1)} = \frac{\delta - 2}{\delta}$

$-\delta < x-4 < \delta$

$-\delta + 4 < x < \delta$

$\delta - 4 > -x > -\delta$

$\delta - 2 > 2 - x > -\delta$

$\delta = 1$ alalım

$-1 < x - 4 <$

$\Rightarrow 1 - \frac{2}{\delta} = -M$

$\Rightarrow 1 + M = \frac{2}{\delta} \Rightarrow \delta(M) = \frac{2}{1+M}$

$\Rightarrow \delta(M) = \min\{ \frac{2}{1+M}, 1 \}$

5) $\lim_{x \to 3} \frac{x}{x^2-5} = \frac{3}{4}$ old göst.

?

$\forall \varepsilon > 0$, $\exists \delta(\varepsilon) > 0$ $|x-3| < \delta \Rightarrow |f(x) - \frac{3}{4}| < \varepsilon$

$\left| \frac{x}{x^2-5} - \frac{3}{4} \right| = \left| \frac{4x - 3x^2 + 15}{4(x^2-5)} \right| = \frac{|x-3| \, |3x+5|}{4 \, |x^2-5|} < \frac{\delta}{4} \frac{|3x\cdot}{|x^2}$

$|x-3| < \delta \leq 1$

$\Downarrow$

$|x-3| < 1$

$2 < x < 4$

$> 6 < 3x < 12$

$\boxed{11 < 3x+5 < 17}$

$2 < x < 4$

$-1 < x^2 - 5 < 11$

$\Downarrow$

$0 \leq |x^2-5| < 11$

$|x-3| < \delta \leq 1/2$ old. kabul edelim.

o halde $|x-3| < 1/2 \Rightarrow -\frac{1}{2} < x-3 < \frac{1}{2} \Rightarrow \frac{15}{2} < 3x < \frac{21}{2}$

$\frac{25}{2} < 3x+5 < \frac{31}{2}$

④

$$\frac{25}{4} < x^2 < \frac{48}{4} \Rightarrow \frac{5}{4} < x^2-5 < \frac{29}{4}$$

$$\Rightarrow \frac{4}{29} < \frac{1}{x^2-5} < \frac{4}{5}$$

$$\left|f(x)-\frac{3}{4}\right| < \frac{\delta}{4}\cdot\frac{|3x+5|}{|x^2-5|} < \frac{\delta}{\cancel{4}}\cdot\frac{31}{2}\cdot\frac{\cancel{4}}{5} = \varepsilon$$

$$\Rightarrow \delta = \frac{10\varepsilon}{31} \Rightarrow \delta(\varepsilon) = \min\left\{\frac{1}{2}, \frac{10\varepsilon}{31}\right\}$$

6) a) $\lim\limits_{x\to-\infty} \dfrac{\sqrt{x^4+1}-\sqrt{x^4-1}}{\sqrt{x^2+1}-\sqrt{x^2-1}} = ?$ ödev

b) $\lim\limits_{x\to\infty} \sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x} = ?$ $(\infty-\infty)$

b) $\lim\limits_{x\to\infty} \dfrac{\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)\left(\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}\right)}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}$

$$= \lim_{x\to\infty} \frac{\cancel{x}+\sqrt{x+\sqrt{x}}-\cancel{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{\sqrt{x\left(1+\frac{1}{\sqrt{x}}\right)}}{\sqrt{x+\sqrt{x}\cdot\sqrt{1+\frac{1}{\sqrt{x}}}}+\sqrt{x}}$$

$$= \lim_{x\to\infty} \frac{\cancel{\sqrt{x}}\sqrt{1+\frac{1}{\sqrt{x}}}}{\cancel{\sqrt{x}}\left(\sqrt{1+\frac{1}{\sqrt{x}}\sqrt{1+\frac{1}{\sqrt{x}}}}+1\right)} = \frac{1}{1+0\cdot\sqrt{1}+1} = \frac{1}{2}$$

7) $\lim\limits_{x\to 0} \dfrac{1-2\cos x+\cos 2x}{x^2} = ?$ $\left(\frac{0}{0}\right)$

$$\lim_{x\to 0}\frac{\cancel{1}-2\cos x+2\cos^2 x-\cancel{1}}{x^2} \left(\tfrac{0}{0}\right) = \lim_{x\to 0}\frac{-2\left(1-2\sin^2\frac{x}{2}\right)+2(1-s}{x^2}$$

$$=\lim_{x\to 0}\frac{-\cancel{2}+4\sin^2\frac{x}{2}+\cancel{2}-2\sin^2 x}{x^2} = 4\lim_{x\to 0}\frac{\sin\frac{x}{2}\cdot\sin\frac{x}{2}}{\frac{x}{2}\cdot\frac{x}{2}\cdot 4}$$

$$-2\lim_{x\to 0}\frac{\sin x\ \sin x}{x\cdot x} = 1\cdot$$

$$= -$$

<u>Ödev</u> 1) f, g, h aynı D tanımlı bölgesine sahip olan fonk.
her $x\in D$ için $f(x)\leq h(x)\leq g(x)$ ve
$\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} g(x) = \ell \Rightarrow \lim\limits_{x\to a} h(x) = \ell$ dir.

2) $\lim\limits_{x\to 0}\dfrac{\sin x}{x} = 1$ dir. Göst.

8) $\lim\limits_{x\to 0}\dfrac{\cos^3 3x-\cos^3 2x}{x^2} = ?$

$$\lim_{x\to 0}\frac{(\cos 3x-\cos 2x)(\cos^2 3x+\cos 3x\ \cos 2x+\cos^2 2x)}{x^2}$$

$$=3\cdot\lim_{x\to 0}\frac{\cos 3x-\cos 2x}{x^2} = 3\cdot\lim_{x\to 0}\frac{-2\cdot\sin\frac{5x}{2}\cdot\sin\frac{x}{2}}{x^2}$$

$$=(-6)\cdot\underbrace{\lim_{x\to 0}\frac{\sin\frac{5x}{2}}{\frac{5x}{2}}}_{1}\cdot\underbrace{\lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}}_{1}\cdot\frac{5}{2}\cdot\frac{1}{2} = -\frac{15}{2}$$

9) $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\cos x - \sin x}{(4x-\pi)\tan x} = ?$

$x - \frac{\pi}{4} = y$ olsun.

$\lim\limits_{y \to 0} \dfrac{\cos\left(y+\frac{\pi}{4}\right) - \sin\left(y+\frac{\pi}{4}\right)}{4y \cdot \tan\left(y+\frac{\pi}{4}\right)}$

$= \lim\limits_{y \to 0} \dfrac{\cos y \cdot \cos\frac{\pi}{4} - \sin y \sin\frac{\pi}{4} - \sin y \cos\frac{\pi}{4} - \cos y \sin\frac{\pi}{4}}{4y \tan\left(y+\frac{\pi}{4}\right)}$

$= \lim\limits_{y \to 0} \dfrac{-\sqrt{2}\sin y}{4y \tan\left(y+\frac{\pi}{4}\right)} = \lim\limits_{y \to 0} \dfrac{\sin y}{y} \cdot \lim\limits_{y \to 0} \dfrac{-\sqrt{2}}{\tan\left(y+\frac{\pi}{4}\right)}$

$= 1\left(-\sqrt{2}/4\right) = -\sqrt{2}/4$

ödev $\lim\limits_{x \to 1/2} (2x^2 - 3x + 1)\tan \pi x = ?$

10) $f : \mathbb{R} \setminus \{3\} \to \mathbb{R}$, $f(x) = \dfrac{|x^2-9|}{x-3}$ fonk. $x = 3$ noktasındaki limitini bulunuz.

$\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} \dfrac{|x^2-9|}{x-3} = \lim\limits_{x \to 3^+} \dfrac{x^2-9}{x-3} = \lim\limits_{x \to 3^+} (x+3) = 6$

$\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^-} \dfrac{|x^2-9|}{x-3} = \lim\limits_{x \to 3^-} \dfrac{-x^2+9}{x-3} = -\lim\limits_{x \to 3^-} x+3 = -6$

$\Rightarrow x = 3$ nokt. limit yoktur.

1) $\forall n \in \mathbb{N}$ için

$$\lim_{x \to n^-} [\![x]\!] = ? \qquad \lim_{x \to n^+} [\![x]\!] = ?$$

$h > 0$ o.ü. $x = n - h$ olsun.

$$\Rightarrow \lim_{x \to n^-} [\![x]\!] = \lim_{h \to 0^+} [\![n - h]\!] = \lim_{h \to 0^+} [\![n - 1 + 1 - h]\!]$$

$$= n - 1 + \lim_{h \to 0^+} [\![1 - h]\!] = n - 1 + 0 = n - 1$$

$h > 0$ o.ü. $x = n + h$ olsun.

$$\lim_{x \to n^+} [\![x]\!] = \lim_{h \to 0^+} [\![n + h]\!] = \lim_{h \to 0^+} n + [\![h]\!] = n + 0 = n$$

2) a) $\displaystyle\lim_{x \to 0} \frac{1}{1 + 2^{1/x}} = ?$ b) $f(x) = \begin{cases} \dfrac{\sin x}{x}, & x < 0 \\ \dfrac{1 - \cos x}{x}, & x > 0 \end{cases}$

a) $\displaystyle\lim_{x \to 0^+} 2^{1/x} = \infty \Rightarrow \lim_{x \to 0^+} \frac{1}{1 + 2^{1/x}} = 0$

$\displaystyle\lim_{x \to 0^-} 2^{1/x} = 0 \Rightarrow \lim_{x \to 0^-} \frac{1}{1 + 2^{1/x}} = 1$

$\neq$

b) $x = 0$ da limiti var mı?

b) $\lim\limits_{x\to 0^-} \dfrac{\sin x}{x} = 1$

$$\lim_{x\to 0^+} \frac{1-\cos x}{x} = \lim_{x\to 0^+} \frac{1-\left(1-2\sin^2\frac{x}{2}\right)}{x} = \lim_{x\to 0^+} \frac{2\sin^2\frac{x}{2}}{x}$$

$$= \lim_{x\to 0^+} \underbrace{\sin\frac{x}{2}}_{=0} \cdot \underbrace{\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)}_{=1} = 0 \cdot 1 = 0$$

$\Rightarrow x = 0$ da limit yoktur

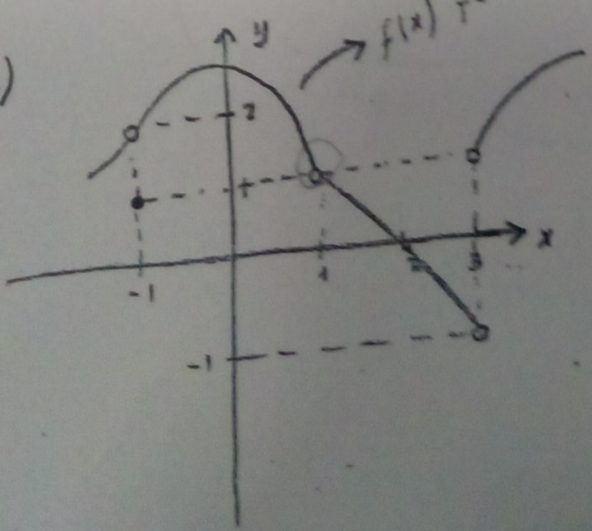
3) a) $\lim\limits_{x\to 0} \dfrac{1-\cos x}{\sin x} = ?$ b) $\lim\limits_{x\to 2} \dfrac{\sin(x^2-4)}{x-2} = ?$

a) $$\lim_{x\to 0} \frac{1-\cos x}{\sin x} = \lim_{x\to 0} \frac{1-\left(1-2\sin^2\frac{x}{2}\right)}{2\sin\frac{x}{2}\cdot\cos\frac{x}{2}} = \lim_{x\to 0} \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = 0$$

b) $$\lim_{x\to 2} \frac{\sin(x^2-4)}{x-2} = \lim_{x\to 2} \frac{\sin(x^2-4)}{x^2-4} \cdot \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2)}{\cancel{x-2}}$$

$$= 4$$

4)



$\lim\limits_{x\to -1} f(x) = 2$

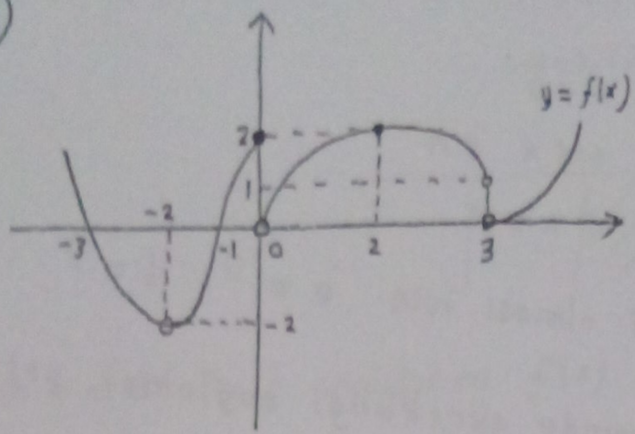
$\lim\limits_{x\to 1} f(x) = 1$

$\lim\limits_{x\to 2} f(x) = 0$

$\lim\limits_{x\to 3} f(x)$ yoktur. Çünkü,

$\lim\limits_{x\to 3^+} f(x) = 1$, $\lim\limits_{x\to 3^-} f(x) = -1$

5)



$f(x)$ fonk. $x=-2$, $x=0$, $x=2$ ve $x=3$ noktalarındaki süreklilik durumunu inceleyiniz

+ $x=-2$ noktasında fonk tanımlı olmadığından fonk $x=-2$ nok. sürekli değildir.

+ $f(0)=2$

$\lim\limits_{x\to 0^+} f(x)=0$, $\lim\limits_{x\to 0^-} f(x)=2$ old. $\lim\limits_{x\to 0} f(x)$ yoktur

$\Rightarrow$ f fonk. $x=0$ noktasında süreksizdir. Fakat ,

$\lim\limits_{x\to 0^-} f(x)=f(0)=2$ old. f fonk. $x=0$ noktasında soldan süreklidir.

+ $f(2)=2$

$\lim\limits_{x\to 2^-} f(x)=2$, $\lim\limits_{x\to 2^+} f(x)=2$

$\Rightarrow$ f fonk $x=2$ noktasında süreklidir.

+ $x=3$ nok. fonk. tanımlı olmadığından sürekli değildir.

6)
$$f(x)=\begin{cases} 2\cos x & , \ x<0 \\ a\cos x+b & , \ 0\leqslant x\leqslant \pi \\ -\sin x & , \ x>\pi \end{cases}$$

sürekli olması için $a=?$ $b=?$

fonk her yerde sürekli olması için $x=0$ ve $x=\pi$ noktalarında sürekliliği sağlamak yeterlidir

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 2\cos x = 2 \ , \ \lim_{x\to 0^+} a\cos x + b = a+b$$

$$\Rightarrow \boxed{a+b=2}$$

$$\lim_{x\to \pi^-} f(x) = \lim_{x\to \pi^-} a\cos x + b = -a+b \ , \ \lim_{x\to \pi^+} -\sin x = 0$$

$$\Rightarrow \boxed{-a+b=0}$$

$$\Rightarrow a=b=1$$

$$\lim_{x\to 0} f(x) = 2 = f(0) \ , \ \lim_{x\to \pi} f(x) = 0 = f(\pi)$$

7) $m\in\mathbb{R}$ o.ü. $f(x)=\dfrac{3x^2-4x-1}{x^2-(m+1)x+1}$ fonk. her $m\in\mathbb{R}$ için sürekli ise m nin alabileceği değerler kümesi ?

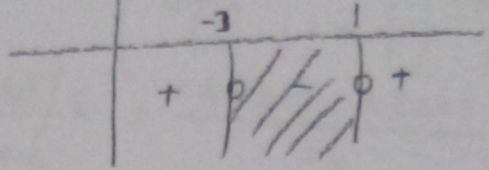
$3x^2-4x-1$ fonk. her $x\in\mathbb{R}$ için süreklidir.

$x^2-(m+1)x+1$ fonksiyonun da her $x\in\mathbb{R}$ için süreklidi

Fakat $f(x)$ in her $x\in\mathbb{R}$ için sürekli olabilmesi için $x^2-(m+1)x+1\neq 0$ olmalıdır. Yani $\Delta<0$ olmalıdır.

$(m+1)^2 - 4.1.1 < 0 \Rightarrow m^2 + 2m - 3 < 0$

$(m+3)(m-1) < 0$



$-3 < m < 1$

$\Rightarrow \forall m \in (-3,1)$ aralığında $f(x)$ fonk $(\forall x \in \mathbb{R}$ için$)$ süreklidir

8) a) $\lim\limits_{x \to -\infty} \left(\sqrt{x^2+1} - \sqrt{x^2+2x-3}\right) = ?$ b) $\lim\limits_{x \to 0} \dfrac{\cos^3 3x - \cos^3 2x}{x^2} = ?$

a) $\lim\limits_{x \to -\infty} \left(\sqrt{x^2+1} - \sqrt{x^2+2x-3}\right) = \infty - \infty$

$$\lim_{x \to -\infty} \frac{\left(\sqrt{x^2+1} - \sqrt{x^2+2x-3}\right)\left(\sqrt{x^2+1} + \sqrt{x^2+2x-3}\right)}{\sqrt{x^2+1} + \sqrt{x^2+2x-3}}$$

$$= \lim_{x \to -\infty} \frac{-2x+4}{\sqrt{x^2\left(1+\frac{1}{x^2}\right)} + \sqrt{x^2\left(1+\frac{2}{x}-\frac{3}{x^2}\right)}}$$

$$= \lim_{x \to -\infty} \frac{-2x+4}{-x\sqrt{1+\frac{1}{x^2}} - x\sqrt{1+\frac{2}{x}-\frac{3}{x^2}}}$$

$$= \lim_{x \to -\infty} \frac{x\left(-2+\frac{4}{x}\right)}{\cancel{x}\left(-\sqrt{1+\frac{1}{x^2}} - \sqrt{1+\frac{2}{x}-\frac{3}{x^2}}\right)} = \frac{-2}{-\sqrt{1}-\sqrt{1}} = 1$$

b) $\lim\limits_{x \to 0} \dfrac{\cos^3 3x - \cos^3 2x}{x^2} = \lim\limits_{x \to 0} \dfrac{(\cos 3x - \cos 2x)(\cos^2 3x + \cos 3x \cos 2x + \cos^2 2x)}{x^2}$

$= 3 \cdot \lim\limits_{x \to 0} \dfrac{\cos 3x - \cos 2x}{x^2} = 3 \cdot \lim\limits_{x \to 0} \dfrac{-2 \sin \frac{5x}{2} \cdot \sin \frac{x}{2}}{x^2}$

$= (-6) \cdot \underbrace{\lim\limits_{x \to 0} \dfrac{\sin \frac{5x}{2}}{\frac{5x}{2}}}_{=1} \cdot \underbrace{\lim\limits_{x \to 0} \dfrac{\sin \frac{x}{2}}{\frac{x}{2}}}_{=1} \cdot \dfrac{5}{2} \cdot \dfrac{1}{2} = -\dfrac{15}{2}$

g) $\lim\limits_{x \to \frac{\pi}{8}} \dfrac{8x - \pi}{\sin 2x - \cos 2x} = ?$

$\left(\frac{0}{0}\right)$

$\sin 2x - \cos 2x = \cos\left(\frac{\pi}{2} - 2x\right) - \cos 2x$

$= -2 \cdot \sin \frac{\pi}{4} \cdot \sin\left(\frac{\pi}{4} - 2x\right)$

$\Rightarrow \lim\limits_{x \to \frac{\pi}{8}} \dfrac{8x - \pi}{-2 \sin \frac{\pi}{4} \, \sin\left(\frac{\pi}{4} - 2x\right)}$

$4 \cdot \left(2x - \frac{\pi}{4}\right)$ ''

$= \lim\limits_{x \to \frac{\pi}{8}} \dfrac{1}{-2 \frac{\sqrt{2}}{2}} \cdot \lim\limits_{x \to \frac{\pi}{8}} \dfrac{8x - \pi}{\sin\left(\frac{\pi}{4} - 2x\right)}$

$= -\dfrac{\sqrt{2}}{2} \cdot (-4) = 2\sqrt{2}$

10) $\lim\limits_{x \to e} \dfrac{e - x}{1 - \ln x} = ?$ $\left(\dfrac{0}{0}\right)$ $\left(\lim\limits_{x \to \infty} \left(1 + \dfrac{1}{x}\right) = \lim\limits_{x \to 0} (x+1)^{1/x} = e \right)$

$$= \lim_{x \to e} \frac{x - e}{\ln x - \ln e} = \lim_{x \to e} \frac{x - e}{\ln\left(\frac{x}{e}\right)}$$

$\dfrac{x}{e} = t + 1 \Rightarrow x = et + e$ $\qquad x \to e \Rightarrow t \to 0$

$$= \lim_{t \to 0} \frac{et + e - e}{\ln(t+1)} = \lim_{t \to 0} \frac{e}{\ln(t+1)^{1/t}} = e \cdot \frac{1}{\ln e} = e$$

1) $\lim\limits_{x \to 0} \dfrac{x - \tan x}{x - \sin x} = ?$

$$\lim_{x \to 0} \frac{x - \tan x}{x - \sin x} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x \to 0} \frac{1 - (1 + \tan^2 x)}{1 - \cos x} = \lim_{x \to 0} \frac{-\tan^2 x}{1 - \cos x}$$

$$= \lim_{x \to 0} \frac{-2\tan x\,(1 + \tan^2 x)}{\sin x} = \lim_{x \to 0} \frac{-2(1 + \tan^2 x)}{\cos x} = -2$$

2) a) $\lim\limits_{x \to \infty} \dfrac{e^{1/x} - 1}{\frac{1}{x}} = ?$ b) $\lim\limits_{x \to \infty} \dfrac{\ln x}{\ln(\ln x)} = ?$

$$a) \lim_{x \to \infty} \frac{e^{1/x} - 1}{\frac{1}{x}} \overset{\left(\frac{0}{0}\right)}{=} \lim_{t \to 0} \frac{e^t - 1}{t} = \lim_{t \to 0} \frac{e^t}{1} = 1$$

$\left(\frac{1}{x} = t \text{ olsun}\right)$

$$b) \lim_{x \to \infty} \frac{\ln x}{\ln(\ln x)} = \lim_{x \to \infty} \frac{\frac{1}{x}}{\frac{\frac{1}{x}}{\ln x}} = \lim_{x \to \infty} \frac{\frac{1}{x}}{x}$$

$x \ln x = \lim\limits_{x \to \infty} \ln x$

$= \infty$

3) $\lim\limits_{x \to 0} \dfrac{3\tan 4x - 12\tan x}{3\sin 4x - 12\sin x} = ?$

$$\lim_{x \to 0} \frac{3\tan 4x - 12\tan x}{3\sin 4x - 12\sin x} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x \to 0} \frac{3(1 + \tan^2 4x).4 - 12(1 + \tan^2 x)}{12\cos 4x - 12\cos x}$$

$$= \lim_{x \to 0} \frac{\tan^2 4x - \tan^2 x}{\cos 4x - \cos x} = \lim_{x \to 0} \frac{(\tan 4x - \tan x)(\tan 4x + \tan x)}{\cos 4x - \cos x}$$

$$= \lim_{x \to 0} \frac{\left(\frac{\sin 4x}{\cos 4x} - \frac{\sin x}{\cos x}\right)\left(\frac{\sin 4x}{\cos 4x} + \frac{\sin x}{\cos x}\right)}{\cos 4x - \cos x}$$

$$= \lim_{x\to 0} \frac{(\sin 4x\cos x - \sin x\cos 4x)(\sin 4x\cos x + \sin x\cos 4x)}{\cos^2 x \cdot \cos^2 4x\ (\cos 4x - \cos x)}$$

$$= \lim_{x\to 0} \frac{\sin 3x \cdot \sin 5x}{\cos^2 x \cdot \cos^2 4x \cdot \left(-2 \cdot \sin\frac{5x}{2} \cdot \sin\frac{3x}{2}\right)}$$

$$= \lim_{x\to 0} \frac{2\cancel{\sin\frac{3x}{2}} \cdot \cos\frac{3x}{2} \cdot 2\cancel{\sin\frac{5x}{2}} \cdot \cos\frac{5x}{2}}{\cos^2 x\ \cos^2 4x \cdot \left(-2\cancel{\sin\frac{5x}{2}} \cdot \cancel{\sin\frac{3x}{2}}\right)} = \frac{4}{-2} = -2$$

4) a) $\lim\limits_{x\to 0^+} x^{\sin x} = ?$ b) $\lim\limits_{x\to \frac{\pi}{2}} (\sin x)^{\tan x} = ?$

a) $y = x^{\sin x} \Rightarrow \ln y = \sin x \ln x$

$$\lim_{x\to 0^+} \sin x \ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{\sin x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-\cos x}{\sin^2 x}} = \lim_{x\to 0^+} \frac{-\sin^2 x}{x\cos x}$$

$$= \lim_{x\to 0^+} \sin x \cdot \frac{\sin x}{x} \cdot \left(-\frac{1}{\cos x}\right) = 0 \cdot 1 \cdot (-1) = 0$$

$$\Rightarrow \lim_{x\to 0^+} \ln y = 0 \Rightarrow \lim_{x\to 0^+} y = e^0 = 1$$

b) $y = (\sin x)^{\tan x} \Rightarrow \ln y = \tan x \ln \sin x$

$$\lim_{x\to \frac{\pi}{2}} \tan x \ln \sin x = \lim_{x\to \frac{\pi}{2}} \frac{\ln \sin x}{\cos x} \sin x = 1 \cdot \lim_{x\to \frac{\pi}{2}} \frac{\ln \sin x}{\cos x}$$

$$\left(\tfrac{0}{0}\right) = \lim_{x\to \frac{\pi}{2}} \frac{\frac{\cos x}{\sin x}}{-\sin x} = \lim_{x\to \frac{\pi}{2}} \frac{-\cos x}{\sin^2 x} = 0$$

$$\Rightarrow \lim_{x\to \frac{\pi}{2}} \ln y = 0 \Rightarrow \lim_{x\to \frac{\pi}{2}} y = e^0 = 1$$

5) $a, b > 0$ için

$$\lim_{x\to 0} (a^x + b^x - 1)^{1/x} = ?$$

$$y = (a^x + b^x - 1)^{1/x} \Rightarrow \ln y = \ln(a^x + b^x - 1)^{1/x} = \frac{1}{x}\ln(a^x + b^x - 1)$$

$$\lim_{x\to 0} \frac{1}{x}\ln(a^x + b^x - 1) \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to 0} \frac{a^x \ln a + b^x \ln b}{a^x + b^x - 1} = \ln a + \ln b = \ln(a.b)$$

$$\Rightarrow \lim_{x\to 0} \ln y = \ln(a.b) \Rightarrow \lim_{x\to 0} y = e^{\ln(a.b)} = a.b$$

6) $y = 4x - x^2$ eğrisinin $(2,5)$ noktasından geçen teğetlerin denklemlerini bulunuz.

Değme noktası $(a, 4a - a^2)$ olsun.

$y' = 4 - 2x \Rightarrow m = 4 - 2a$ teğetin eğimi

Teğet denklemi. $y - y_0 = m.(x - x_0)$

$$y - 4a + a^2 = (4 - 2a)(x - a)$$

$$\Rightarrow 5 - 4a + a^2 = (4 - 2a)(2 - a) = 8 - 8a + 2a^2$$

$$\Rightarrow a^2 - 4a + 3 = 0 \Rightarrow a = 3,\ a = 1$$

$$(a-3)(a-1) = 0$$

$a = 1$ için teğetin denklemi $y - 3 = 2(x - 1) \Rightarrow y = 2x + 1$

$a = 3$ için teğetin denklemi $y - 3 = -2(x - 3) \Rightarrow y = -2x + !$

7) $f(x) = \begin{cases} 2\sin x & , x < 0 \\ 3x^2+2x & , x \geq 0 \end{cases}$ fonk. (0,0) noktasına teğetinin denklemini bulu..

Teğetin eğimi $f'(0)$ dır.

$$m = f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{f(x)}{x}$$

$$\lim_{x\to 0^+} \frac{f(x)}{x} = \lim_{x\to 0^+} \frac{3x^2+2x}{x} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to 0^+} (3x+2) = 2$$

$$\lim_{x\to 0^-} \frac{f(x)}{x} = \lim_{x\to 0^-} \frac{2\sin x}{x} = 2$$

$$\Rightarrow f'(0) = m = 2$$

Teğetin denklemi $y - y_0 = m.(x-x_0)$

$$\Rightarrow y - 0 = 2(x-0) \Rightarrow y = 2x$$

8) $x^2+y^2=25$ eğrisinin hangi noktasındaki teğetinin eğimi $\frac{3}{4}$ olur?

$y' = \frac{3}{4}$ olmalıdır.

$2x + 2yy' = 0 \Rightarrow x + y.\frac{3}{4} = 0 \Rightarrow y = -\frac{4}{3}x$

$x^2+y^2=25 \Rightarrow x^2 + \frac{16}{9}x^2 = 25 \Rightarrow x^2 = 9$

$\Rightarrow x = \mp 3$

$x=3$ için $y = -\frac{4}{3}x \Rightarrow y = -\frac{4}{3}.3 = -4$ $\boxed{(3,-4)}$

$x=-3$ için $y = -\frac{4}{3}.(-3) = 4$ $\boxed{(-3,4)}$

ENES ARICA

9) $y = x^3 - 5x + 3$ eğrisinin,

a) $y = -2x$ doğrusuna paralel

b) $y = -\frac{x}{7}$ doğrusuna dik

c) ox-ekseni ile 45 derecelik açı yapan teğetlerin denklemini bulunuz

a) $m = -2$ olacağından $y' = -2$

$y' = 3x^2 - 5 = -2 \Rightarrow x = \mp 1$

$x = -1$ için $y = -1 + 5 + 3 = 7$ $(-1, 7)$

$x = 1$ için $y = 1 - 5 + 3 = -1$ $(1, -1)$

Teğetin denklemi $y - y_0 = m(x - x_0)$

$(-1, 7)$ için $y - 7 = -2(x + 1) \Rightarrow y = -2x + 5$

$(1, -1)$ için $y + 1 = -2(x - 1) \Rightarrow y = -2x + 1$

b) $m = 7$ olmalıdır. (Diklikte $m_1 m_2 = -1$)

$(-\frac{1}{7})$ $(?)$

$\Rightarrow y' = 3x^2 - 5 = 7 \Rightarrow x = \mp 2$

$x = -2$ için $y = -8 - 5(-2) + 3 = 5$ $(-2, 5)$

$x = 2$ için $y = 1$ $(2, 1)$

$(-2, 5)$ için $y - 5 = 7(x + 2) \Rightarrow y = 7x + 19$

$(2, 1)$ için $y - 1 = 7(x - 2) \Rightarrow y = 7x - 13$

c) $m = \tan 45 = 1$ olmalıdır. Çözümü (a) ve (b) deki gibidir

Teğet Denk. $\left(\begin{array}{l} y = x + 3 + 4\sqrt{2} \\ y = x + 3 - 4\sqrt{2} \end{array} \right)$

tasına ... mini bulun