Heat-Conduction Tests

CSIRO

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Contents

1 Simple heat conduction in 1D

Porous-flow heat conduction is governed by the equation

$$0 = \frac{\partial}{\partial t}\mathcal{E} + \nabla \cdot \mathbf{F}^{\mathrm{T}} \ , \tag{1.1}$$

Here $\mathcal{E}$ is the energy per unit volume in the rock-fluid system, and $\mathbf{F}^{\mathrm{T}}$ is the heat flux. In the porous-flow module

$$\mathcal{E} = (1-\phi)\rho_R C_R T + \phi \sum_\beta S_\beta \rho_\beta \mathcal{E}_\beta \ , \tag{1.2}$$

when there is no adsorbed species. Here T is the temperature, ϕ is the rock porosity, and S_β is the saturation of phase β. The remainder of the notation is described in the next paragraph. When studying problems involving heat conduction (with no fluid convection)

$$\mathbf{F}^{\mathrm{T}} = -\lambda \nabla T \ , \tag{1.3}$$

where λ is the tensorial thermal conductivity of the rock-fluid system.

The tests described in this section use the following simple forms for each term

- ρ_R, which is the rock-grain density (that is, the density of rock with zero porespace), measured in kg.m^{-3}, is assumed constant in the current implementation of the PorousFlow module.
- C_R, which is the rock-grain specific heat capacity, measured in J.kg^{-1}.K^{-1}, is assumed constant in the current implementation of the PorousFlow module.
- ρ_β, which is the density of fluid phase β, is assumed in this chapter to be a function of the fluid pressure only (this is so Equation (1.1) may be easily solved — more general forms are allowed in the PorousFlow module).
- $\mathcal{E}_\beta$, which is the specific internal energy of the fluid phase β, and is measured in J.kg^{-1}, is assumed in this chapter to be

  $$\mathcal{E}_\beta = C_v^\beta T \ , \tag{1.4}$$

  where C_v^β is the fluid's specific heat capacity at constant volume. This specific heat capacity is assumed constant (so that Equation (1.1) may be easily solved — more general forms are allowed in the PorousFlow module).
- λ is assumed to vary between λ^{dry} and λ^{wet}, depending on the aqueous saturation:

  $$\lambda_{ij} = \lambda_{ij}^{\mathrm{dry}} + S^n \left(\lambda_{ij}^{\mathrm{wet}} - \lambda_{ij}^{\mathrm{dry}}\right) , \tag{1.5}$$

where S is the aqueous saturation, and n is a positive user-defined exponent. More general forms may be easily accommodated in the PorousFlow module, but to date none have been coded.

Under these conditions, Equation (1.1) becomes

$$\dot{T} = \nabla_i \alpha_{ij} \nabla_j T \ . \tag{1.6}$$

The tensor α is

$$\alpha_{ij} = \frac{\lambda_{ij}}{(1-\phi)\rho_R C_R + \rho \sum_\beta S_\beta \rho_\beta C_v^\beta} \ . \tag{1.7}$$

For constant saturation and porepressure, α_{ij} is also constant.

Consider the one-dimensional case where the spatial dimension is the semi-infinite line $x \geq 0$. Suppose that initially the temperature is constant, so that

$$T(x, t=0) = T_0 \quad \text{for} \quad x \geq 0 \ . \tag{1.8}$$

Then apply a fixed-pressure Dirichlet boundary condition at $x = 0$ so that

$$T(x=0, t>0) = T_\infty \tag{1.9}$$

The solution of the above differential equation is well known to be

$$T(x,t) = T_\infty + (T_0 - T_\infty)\,\mathrm{Erf}\left(\frac{x}{\sqrt{4\alpha t}}\right) \ , \tag{1.10}$$

where Erf is the error function.

This is verified by using the following tests on a line of 10 elements.

1. A transient analysis with no fluids. The parameters chosen are $\lambda_{ij} = \mathrm{diag}(2.2)$, $\phi = 0.9$, $\rho_R = 0.5$ and $C_R = 2.2$ is chosen, so that $\alpha_{ij} = 1/(0.9 \times 0.5)$.

2. A transient analysis with 2 fluid phases. The parameters chosen are $\lambda^{\mathrm{dry}} = 0.3$, $\lambda^{\mathrm{wet}} = 1.7$ and $S = 0.5$, so that $\lambda_{ij} = \mathrm{diag}(1)$. $\rho_{\mathrm{gas}} = 0.4$, $\rho_{\mathrm{water}} = 0.3$, $\rho_R = 0.25$, $C_v^{\mathrm{gas}} = 1$, $C_v^{\mathrm{water}} = 2$, $C_R = 1.0$, and $\phi = 0.8$. With these parameters, $\alpha_{ij} = 1/(0.9 \times 0.5)$.

An example verification is shown in Figure 1.1. These tests are part of the automatic test suite.

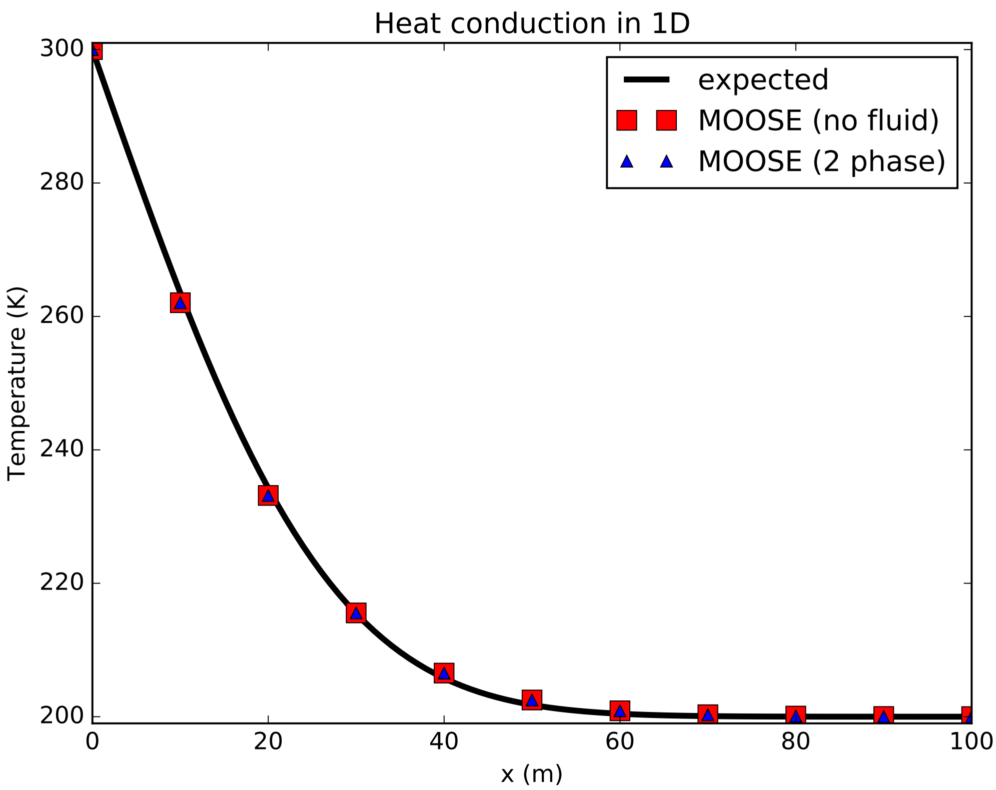

Figure 1.1: Comparison between the MOOSE result (in dots), and the exact analytic expression given by Eqn (1.10). This test had 10 elements in the x direction, with $0 \leq x \leq 100$ m, and ran for a total of 10^2 seconds with 10 timesteps.