Buckley-Leverett Tests

CSIRO

April 16, 2016

Contents

Chapter 1

Single Phase

MOOSE is compared with a simple single-phase one-dimensional Buckley-Leverett problem[1]. The single-phase fluid moves in a region $0 \leq x \leq 15\,\mathrm{m}$. A fully-saturated front initially sits at position $x = 5$, while the region $x > 5$ is initially unsaturated. With zero suction function $P_c = 0$, there is no diffusion of the sharp front, and it progresses towards the right (increasing x). This is a difficult problem to simulate numerically as maintaining the sharp front is hard. The front's speed is independent of the relative permeability, since the fluid is flowing from a fully-saturated region (where $\kappa_{\mathrm{rel}} = 1$). This problem is therefore a good test of the upwinding.

In the simulation below, the pressure at the left boundary is kept fixed at $P_0 = 0.98\,\mathrm{MPa}$, while the right boundary is kept fixed at $P_{15} = -20000\,\mathrm{Pa}$, so the difference is 1 MPa. The medium's permeability is set to $\kappa = 10^{-10}\,\mathrm{m}^2$ and its porosity is $\phi = 0.15$. It is not possible to use a zero suction function in the MOOSE implementation, but using the van Genuchten parameters $\alpha = 10^{-3}\,\mathrm{Pa}^{-1}$ and $m = 0.8$ approximates it. The fluid viscosity is $\mu = 10^{-3}\,\mathrm{Pa.s}$.

The initial condition is

$$P(t=0) = \left\{ \begin{array}{ll} P_0 - (P_0 - P_{15})x/5 & \text{for} \quad x < 5 \\ P_{15} & \text{for} \quad x \geq 5 \end{array} \right. , \tag{1.1}$$

which is shown in Figure 1.1. With the suction function defined above this gives

$$S(t=0) = \left\{ \begin{array}{ll} 1 & \text{for} \quad x \leq 4.9 \\ 0.061 & \text{for} \quad x \geq 5 \end{array} \right. \tag{1.2}$$

Good approximations for the pressure $P(x,t)$ and the front position $f(t)$

[1] SE Buckley and MC Leverett (1942) "Mechanism of fluid displacements in sands". Transactions of the AIME **146** 107–116

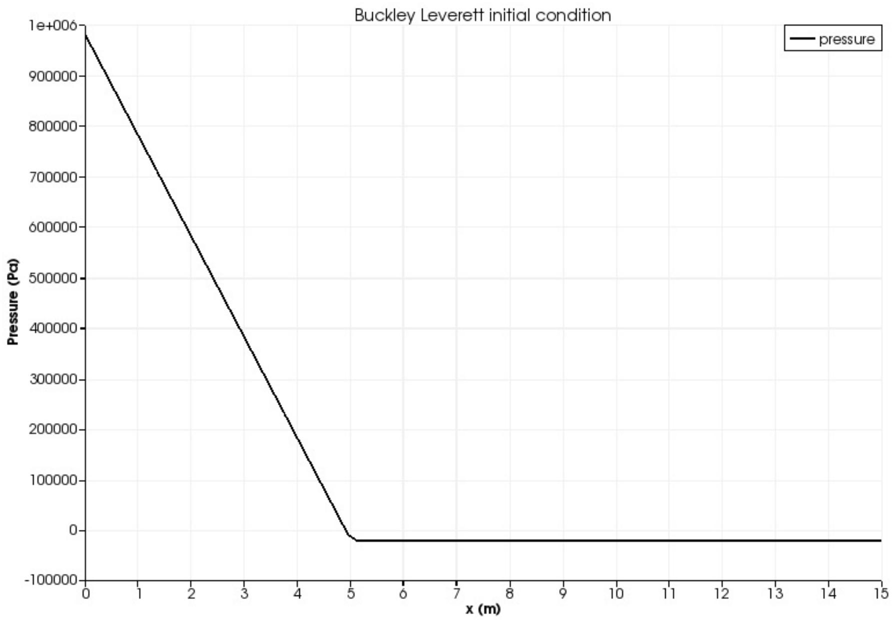

Figure 1.1: Initial setup of the Buckley-Leverett problem where porepressure is a piecewise linear function. The region $x \leq 4.9$ is fully saturated, while the region $x > 5$ has saturation 0.061. During simulation the value $P(x = 0) = 0.98 \times 10^6$ MPa is held fixed.

may be determined from

$$\begin{aligned} \frac{\mathrm{d}f}{\mathrm{d}t} &= -\frac{\kappa}{\phi\mu}\left.\frac{\partial P}{\partial x}\right|_{x=f} , \\ P(x,t) &= \begin{cases} P_0 - (P_0 - P_{15})x/f & \text{for } x \leq f \\ P_{15} & \text{for } x > f \end{cases} , \end{aligned} \tag{1.3}$$

which has solution

$$f(t) = \sqrt{f(0)^2 + \frac{2\kappa}{\phi\mu}(P_0 - P_{15})t} \; . \tag{1.4}$$

For the parameters listed above, the front will be at position $f = 9.6$ m at $t = 50$ s. This solution is only valid for zero capillary suction. A nonzero suction function will tend to diffuse the sharp front.

With coarse meshes it is impossible to simulate a sharp front, of course, since the front is spread over at least one element.

Figure 1.2 shows the results from a MOOSE simulation with a uniform mesh of size 0.1 m. At $t = 50$ s the front in this simulation sits at $x = 9.6$ m as desired. However, the simulation takes 10 seconds to complete due to the very low values of saturation obtained for van Genuchten $\alpha = 10^{-3}\,\mathrm{Pa}^{-1}$. Other simulations give similar results but run much faster. For instance, the test suite uses the van-Genuchten parameter $\alpha = 10^{-4}\,\mathrm{Pa}^{-1}$, but the front diffuses a little into the unsaturated region (the front sits between $x = 9.7\,\mathrm{m}$ and $x = 10.4\,\mathrm{m}$ at $t = 50\,\mathrm{s}$).

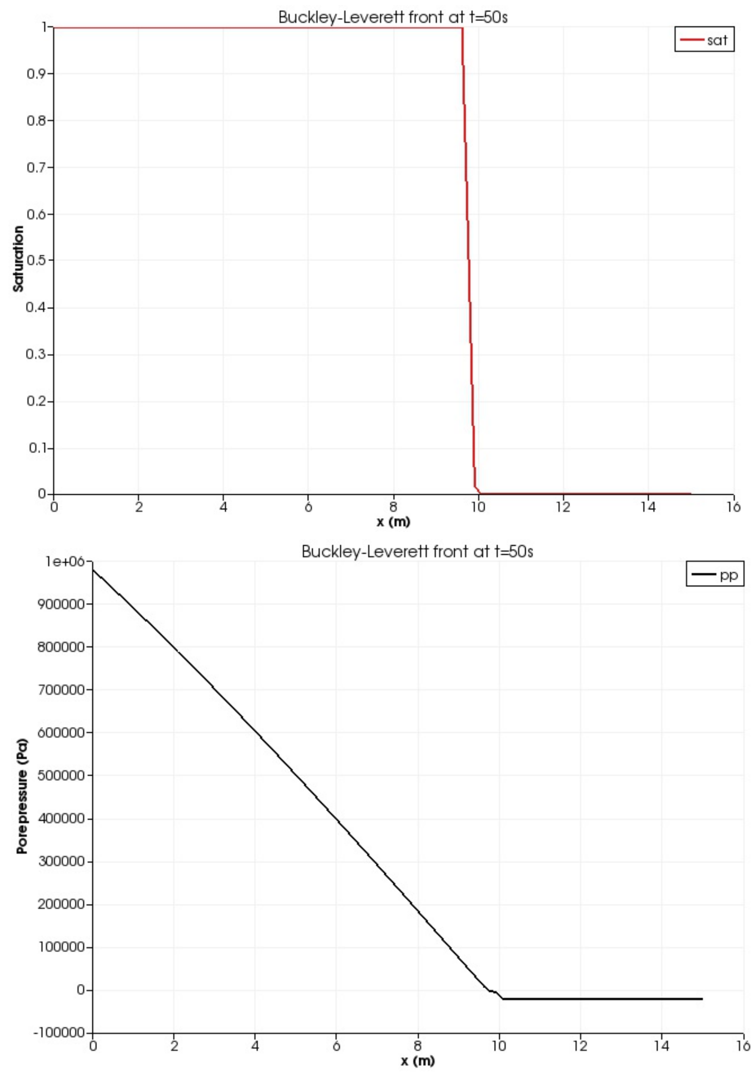

Figure 1.2: The MOOSE solution of the Buckley-Leverett problem at $t = 50\,\mathrm{s}$. Top: saturation. Bottom: porepressure. The front sits between $x = 9.6\,\mathrm{m}$ as expected from the analytical solution.