Newton-Cooling Tests

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August 9, 2016

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1 Newton cooling from a bar

These tests demonstrate that MOOSE behaves correctly when a simulation contains a sink. The sink is a piecewise linear function of pressure.

Darcy's equation for (single-phase) flow through a fully saturated medium without gravity and without sources is

$$\frac{\partial}{\partial t}\phi\rho = \nabla_i \left(\frac{\rho\kappa_{ij}}{\mu} \nabla_j P \right) \ , \tag{1.1}$$

with the following notation:

- ϕ is the medium's porosity;
- ρ is the fluid density;
- κ_{ij} is the permeability tensor;
- μ is the fluid viscosity;
- $\partial/\partial t$ and ∇_i denote the time and spatial derivatives, respectively.

Using $\rho \propto \exp(P/B)$, where B is the fluid bulk modulus, Darcy's equation becomes

$$\frac{\partial}{\partial t}\rho = \nabla_i \alpha_{ij} \nabla_j \rho \ , \tag{1.2}$$

with

$$\alpha_{ij} = \frac{\kappa_{ij} B}{\mu\phi} \ . \tag{1.3}$$

Here the porosity and bulk modulus are assumed to be constant in space and time.

Consider the one-dimensional case where a bar sits between $x = 0$ and $x = L$ with initial pressure distribution so $\rho(x, t = 0) = \rho_0(x)$. Maintain the end $x = 0$ at constant pressure, so that $\rho(x = 0, t) = \rho_0(0)$. At the end $x = L$, prescribe a sink flux

$$\left.\frac{\partial\rho}{\partial x}\right|_{x=L} = -C\left(\rho - \rho_e\right)_{x=L} \ , \tag{1.4}$$

where ρ_e is a fixed quantity ("e" stands for "external"), and C is a constant conductance. This corresponds to the flux

$$\left.\frac{\partial P}{\partial x}\right|_{x=L} = -CB\left(1 - e^{(P_e - P)/B}\right)_{x=L} \ , \tag{1.5}$$

which can easily be coded into a MOOSE input file: the flux is $\rho\kappa\nabla P/\mu = -CB\kappa(e^{P/B} - e^{P_e/B})/\mu$, and this may be represented by a piecewise linear function of pressure.

The solution of this problem is well known and is

$$\rho(x,t) = \rho_0(0) - \frac{\rho_0(0) - \rho_e}{1+LC}Cx + \sum_{n=1}^{\infty} a_n \sin\frac{k_n x}{L} e^{-k_n^2\alpha t/L^2} \ , \tag{1.6}$$

where k_n is the n^{th} positive root of the equation $LC\tan k + k = 0$ (k_n is a little bigger than $(2n-1)\pi/2$), and a_n is determined from

$$a_n \int_0^L \sin^2\frac{k_n x}{L}\,\mathrm{d}x = \int_0^L \left(\rho_0(x) - \rho_0(0) + \frac{\rho_0(0)-\rho_e}{1+LC}Cx\right)\sin\frac{k_n x}{L}\,\mathrm{d}x \ , \tag{1.7}$$

which may be solved numerically (Mathematica is used to generate the solution in Figure 1.1).

The problem is solved in MOOSE using the following parameters:

Bar length	100 m
Bar porosity	0.1
Bar permeability	10^{-15} m^2
Gravity	0
Water density	1000 kg.m^{-3}
Water viscosity	0.001 Pa.s
Water bulk modulus	1 MPa
Initial porepressure P_0	2 MPa
Environmental pressure P_e	0
Conductance C	0.05389 m^{-1}

This conductance is chosen so at steadystate $\rho(x = L) = 2000\,\mathrm{kg.m}^{-3}$.

The problem is solved using 1000 elements along the x direction ($L = 100\,\mathrm{m}$), and using 100 time-steps of size 10^6 s. Using fewer elements or fewer timesteps means the agreement with the theory is marginally poorer. Two tests are performed: one with transient flow, and one using the steadystate solver. In this case the initial condition is $P = 2 - x/L\,\mathrm{MPa}$, since the uniform $P = 2\,\mathrm{MPa}$ does not converge. The results are shown in Figure 1.1.

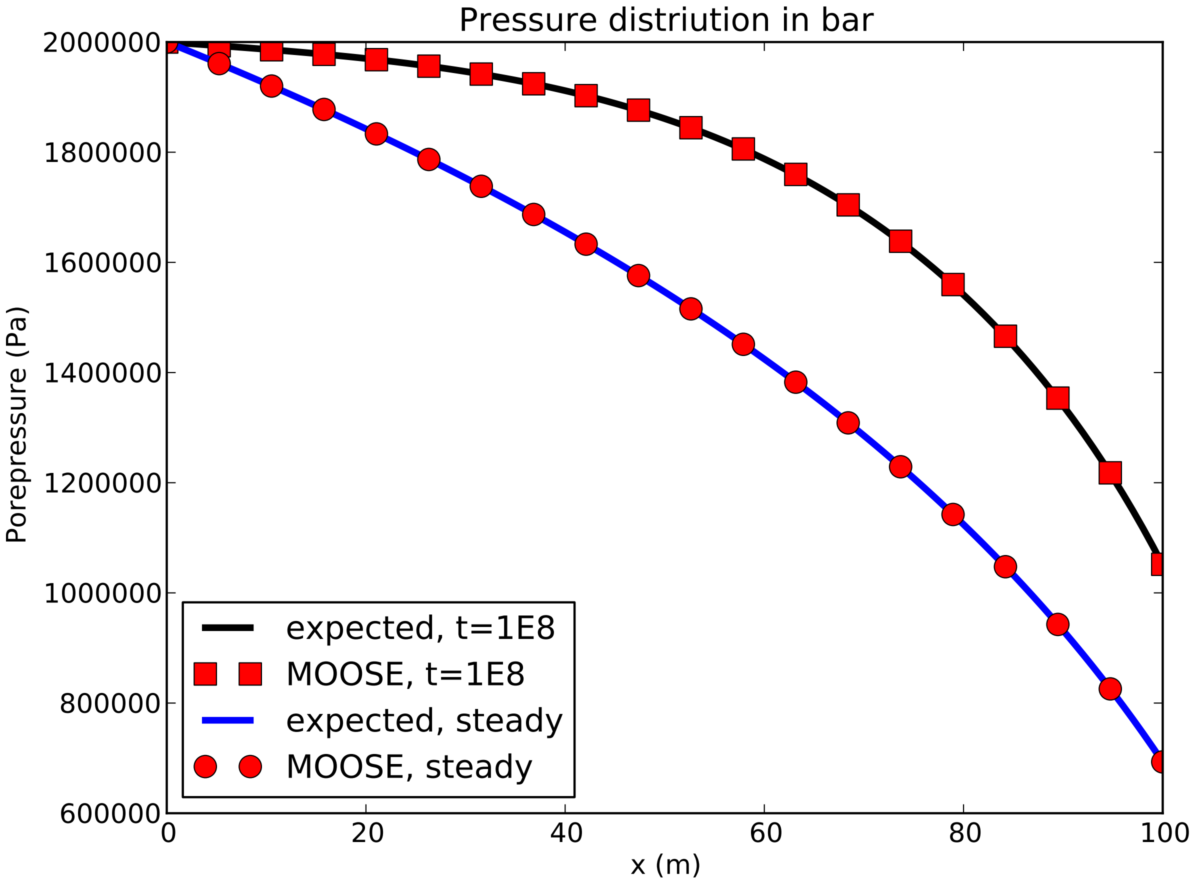

Figure 1.1: The porepressure in the bar at $t = 10^8$ s, and at steadystate. The pressure at $x = 0$ is held fixed, while the sink is applied at $x = 100$ m. MOOSE agrees well with theory demonstrating that piecewise-linear sinks/sources and single-phase Darcy fluid flow are correctly implemented in MOOSE.