FRE – GY6273 : Valuation Theory
Module 5: Relative Valuation and Structural Models

Professor Philips, Spring 2022

Agenda – Module 5

- Foundations of Relative Valuation
- Applying Relative Valuation Techniques to Our Tech Companies
- The P/B-ROE Model
- The P/S-Net Margin Model
- Robust Regression Revisited
- Summary and Closing Thoughts

How Can We Perform Valuations in the Absence of Data?

- All the valuation models we have studied so far require a fair amount of data
  - Some of the data is specific to the firm (cash flows)
  - Other data is endogenous (discount rates etc.)
- But what if we do not have access to all the data we need for our valuation?
  - Private firms or firms with complex financials
  - Firms that do not make their financials available on the web
  - Firms in countries where accounting rules require relatively little disclosure
- We need valuation shortcuts that work without fine-grained firm level data!
- Relative valuation is the most widely used valuation shortcut

Relative Valuation – I

- To estimate the value of a private firm, compare it to an *identical* firm that is publicly traded

- By the law of one price, the values of the two firms must be identical as well!
  - This is essentially how options and futures are priced (replication)

- Problem: we can almost never find an identical firm
  - Practical solution: find a *similar* firm, their values should be similar as well

- This trick is not restricted to private firms! It can be used to:
  - Value divisions within public firms
  - Find overvalued or undervalued firms
  - Allow cross-border comparisons

- Most important use case when it is invaluable: Venture Capital!

Relative Valuation – II

- Start by identifying a “sufficiently similar” firm (or a collection of firms)
  - Modify financial statements of both firms (if needed) to enhance comparability
  - Express both firms’ financials with a common denominator, then solve for value
    - Can sometimes scale in non-financial ways as well (e.g. number of users, viewers, visitors)

- Some dimensions along which the firms ought to be as similar as possible
  - Sector / Sub-sector / Industry / Sub-industry
  - Products or services
  - Capital Structure / Risk
  - Growth and market opportunities / value drivers

- Some important dimensions along which the firms can be very different
  - Market Cap / Firm Value / Book Value / Revenue
  - Earnings / EBIT / Abnormal Earnings / Profit Margins
  - Dividends / Free Cash Flow

Relative Valuation – III

- Firm A trades on the NYSE, has FCFE of $1, -$1, $3 in years 1, 2 and 3 and later
  - Firm A's market cap is $20

- Firm B is private, has FCFE of $10, -$10, $30 in years 1, 2, and 3 and later
  - What should Firm B's market cap be?

- Observe that Firm B's FCFE is exactly 10x Firm A's FCFE in every year
  - Firm A's market cap is 20x its cash flow this year
  - Firms A and B have identical risk and return profiles
  - Firm B's market cap should be 20x its cash flow this year as well: $V_B =$ $200!

- Another approach: Firms must have identical discount rates
  - Can show using Excel's Solver that the discount rate for firm A is 20.84%
  - Firm B's risk appears identical to that of firm A
    - Firm B's cost of equity ought to be identical to that of firm A!
  - Firm B's value can now be computed by discounting its FCFE at 20.84%
  - Firm B's value is $200!

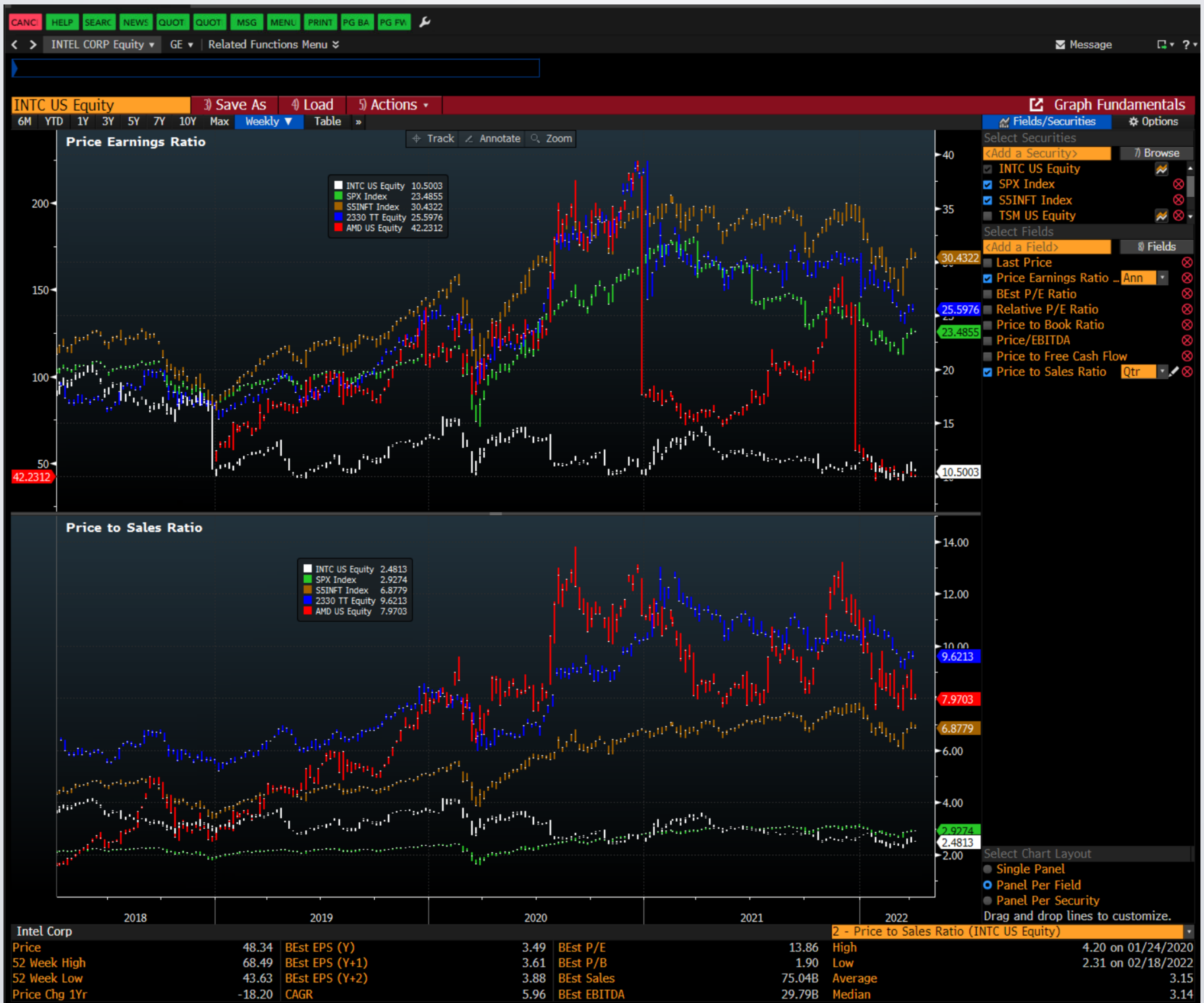
Relative Valuation – IV

- Approach 1 is easier than approach 2 – just apply a multiple to FCFE!
  - Avoids the need for lots of messy calculations
  - Could also apply a multiple to Book Value, Earnings, EBIT, Sales, Abnormal E, etc.

- Powerful technique, but easy to abuse and to get lead astray
  - The two firms may be only superficially similar!
  - Their markets and the quality of their management teams could be very different
  - One could be smaller than the other – and may have better growth opportunities

Relative Valuation – V

- INTC vs. S&P 500, S&P 500 Info Tech, AMD and TSMC (P/E and P/S)



Relative Valuation – VI

- Intel's valuation ratios vs. its competitors – observe implied prices on RHS

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INTEL CORP Equity | EQRV | Related Functions Menu — Message

INTC US $ ↓48.355 -0.845 Q48.35/48.36Q 15x45

At 11:10 d Vol 6,617,200 O 48.83K H 48.98P L 48.285D Val 321.902M

INTC US Equity — Export — Settings — Equity Relative Valuation

Comp Source: Custom Comps containing INTC US — Name: Intel Corp Comp Set — Curr: USD

vs Comps | Group Dynamics | vs Self

Analysis of INTC US Multiples - Premium to Comps — 3M 6M 1Y 2Y 5Y

Metric	Current vs 2Y Average Historical Premium: Current	Hist Avg	Diff	# SD	3M Trend	2Y Historical Premium Range: Low	Range	High	Implied @ Hist Avg: Multiple	Price (USD)
Current Price							Current / Hist Avg			48.39
1) BF P/E	-39%	-55%	16%	2.3		-66%		-36%	10.2x	35.96
2) BF EV/EBITDA	-50%	-54%	4%	0.6		-63%		-33%	6.1x	44.42
3) BF EV/EBIT	-30%	-48%	17%	2.3		-62%		-30%	9.0x	36.06
4) BF EV/Rev	-55%	-50%	-5%	-0.4		-65%		-20%	2.9x	53.39
5) LF P/BV	-65%	-46%	-19%	-1.1		-67%		-5%	3.2x	74.10

View All Comps Below

Summary of Current Multiples

	Name	2Y Corr	Mkt Cap (USD)	BF P/E	BF EV/EBITDA	BF EV/EBIT	BF EV/Rev	LF P/BV
11)	Intel Corp		197.81B	13.7x	6.7x	12.0x	2.7x	2.1x
	Current Premium to Com...			-39%	-50%	-30%	-55%	-65%
	Mean (Including INTC U...		175.92B	22.5x	13.3x	17.2x	5.9x	5.9x
12)	NVIDIA Corp	0.44	663.02B	44.9x	37.4x	38.1x	18.2x	24.9x
13)	Taiwan Semiconductor M...	0.09	532.65B	18.5x	10.3x	16.2x	7.1x	7.0x
14)	Broadcom Inc	0.53	252.38B	16.7x	13.8x	14.4x	8.7x	13.2x
15)	Advanced Micro Devices ...	0.31	175.75B	25.6x	21.2x	21.9x	6.7x	17.5x
16)	QUALCOMM Inc	0.48	165.89B	12.1x	9.5x	10.2x	3.9x	14.6x
17)	Marvell Technology Inc	0.45	58.71B	28.9x	26.6x	28.4x	10.1x	3.7x
18)	Microchip Technology Inc	0.54	39.36B	13.6x	12.9x	13.9x	6.2x	6.8x
19)	Lattice Semiconductor Co...	0.49	7.76B	38.0x	32.6x	36.8x	12.4x	18.8x
20)	Synaptics Inc	0.43	7.22B	13.3x	11.6x	11.2x	4.1x	6.8x
21)	Silicon Laboratories Inc	0.45	5.57B	52.4x	18.8x	22.2x	3.5x	2.5x
22)	SiTime Corp	0.35	4.89B	60.1x	42.1x	47.1x	13.7x	7.7x

Relative Valuation – VII

- Intel's credit quality vs. its competitors

	Name	S&P Rating	Moody's Rating	Fitch Rating	Total Debt/ EBITDA	Total Debt/ Book Cap	EBITDA minus CAPEX/	EBITDA/ Interest
	Median				1.20	29.78%	11.78	21.22
101)	INTEL CORP	A+	A1	A+	1.19	28.80%	11.78	33.47
102)	TAIWAN SEMICONDUCTOR MANUFAC	AA-	Aa3	NR	0.70	25.83%	42.82	198.04
103)	ADVANCED MICRO DEVICES	A-	A3	NR	0.18	8.90%	112.50	122.24
104)	NVIDIA CORP	A	A2	WD	0.98	30.78%	44.10	50.97
105)	LATTICE SEMICONDUCTOR CORP	BB-	--	--	1.35	30.76%	45.04	49.40
106)	QUALCOMM INC	A	A2	--	1.21	62.09%	17.33	21.22
107)	SYNAPTICS INC	BB-	Ba2	BB	2.24	48.61%	9.07	13.76
108)	BROADCOM INC	BBB-	Baa3	BBB-	2.50	61.71%	7.54	9.11
109)	MICROCHIP TECHNOLOGY INC	NR	Baa2	BBB	2.86	62.95%	7.90	8.27
110)	MARVELL TECHNOLOGY INC	BBB-	Baa3	BBB-	4.40	23.14%	5.67	8.05
111)	SILICON LABORATORIES INC	--	--	--	0.43	1.30%	0.24	1.25
112)	SITIME CORP	--	--	--	0.18	1.21%	--	--

Relative Valuation – VIII

- Analysts' estimates of earnings for Intel and its competitors

	Name	Exp Report Date	BEst Sales (Cur Qtr)	BEst EBITDA (Cur Qtr)	BEst P/E (Cur Qtr)	BEst EPS Mean (Cur Qtr)	BEst EPS Low (Cur Qtr)	BEst EPS High (Cu... Qtr)	Number of Ests (Cur Qtr)	BEst EPS 1mo % Chg (Cur	BEst EPS 3mo % Chg (Cur
	Median		3.70B	1.28B	22.96	0.85	0.80	0.90	22.50	0.00%	13.61%
101)	INTEL CORP	04/28/22	18.31B	6.89B	14.17	0.80	0.65	0.83	30.00	0.38%	-6.12%
102)	TAIWAN SEMICONDUCTOR MANUFAC	04/14/22	16.37B	10.91B	19.20	0.25	0.24	0.26	27.00	2.47%	12.16%
103)	QUALCOMM INC	04/27/22	10.61B	4.33B	12.09	2.93	2.87	3.12	23.00	0.00%	18.63%
104)	NVIDIA CORP	05/25/22	8.10B	3.57B	46.36	1.30	1.23	1.37	34.00	-0.15%	11.50%
105)	BROADCOM INC	06/03/22	7.90B	4.97B	16.68	8.71	8.55	8.85	26.00	0.31%	10.06%
106)	ADVANCED MICRO DEVICES	04/27/22	5.58B	1.62B	26.73	0.91	0.84	0.97	8.00	1.11%	32.22%
107)	MICROCHIP TECHNOLOGY INC	05/06/22	1.82B	943.50M	13.75	1.25	1.25	1.27	22.00	0.00%	4.59%
108)	MARVELL TECHNOLOGY INC	06/07/22	1.43B	522.86M	29.95	0.51	0.51	0.53	28.00	0.00%	4.27%
109)	SYNAPTICS INC	05/06/22	470.56M	176.00M	12.77	3.55	3.53	3.58	10.00	0.00%	32.36%
110)	SILICON LABORATORIES INC	04/28/22	226.22M	41.23M	55.38	0.64	0.62	0.67	9.00	0.00%	49.07%
111)	LATTICE SEMICONDUCTOR CORP	05/04/22	146.18M	53.23M	39.75	0.33	0.33	0.34	11.00	0.00%	25.19%
112)	SITIME CORP	05/04/22	65.09M	20.60M	64.23	0.76	0.75	0.79	7.00	0.00%	15.06%

Relative Valuation – IX

- A disciplined relative valuation approach proceeds as follows
  - Identify a set of potential comparable firms
  - Collect historical AND forward looking information for the firm and its comps
  - Identify the key characteristic that drive the valuation (e,g. risk, growth, profitability, required investment, credit quality etc.) of these firms
  - Assess comparability of financial statements, make adjustments as needed
  - Select the appropriate characteristic for comparison (e.g. Revenues, Earnings)
  - **Compute the ratio of market value to the chosen characteristic for each comparable firm (**P/E_{t+1}, P/OE_t, P/S, P/OCF, P/B, P/ Abnormal E, P/ EBO Value**)**
    - P/FCF is not often used as FCF can be lumpy, PE/g is not well justified by theory
  - Compute robust statistics + a range for these ratios over the universe of comps
  - Apply a robust multiplier to the relevant characteristic for your chosen firm
  - Assess the range and consistency of possible market values from your calculations

Relative Valuation – X

- Suppose a firm has many divisions, each in a different line of business
- We can break it up into segments by business line
- Value each segment separately by comparing it against pure plays
- Add the segment values to get the value of the firm

Relative Valuation – XI

- Simple exercise: look at valuation metrics for our 6 tech firms
  - P/OE has the lowest volatility and interquartile range as a % of the HL Mean

- There's a surprising amount of variation across these 6 tech firms!
  - P/B, P/A and P/S seem to be the most variable (Std. Deviation / HL Mean)

Ticker	P/B	P/S	P/OCF	P/OE	P/E	P/Assets	HL Mean P/B Exc. Self	HL Mean P/S Exc. Self	HL Mean P/OCF Exc. Self	HL Mean P/OE Exc. Self	HL Mean P/E Exc. Self	HL Mean P/Assets Exc. Self
GOOG	7.61	7.44	20.90	24.34	25.20	5.33	20.22	6.65	16.75	19.58	21.88	4.06
AAPL	38.25	6.60	23.20	22.15	25.49	6.88	10.84	7.38	16.75	19.58	21.88	3.67
INTC	2.19	2.64	6.96	10.73	10.51	1.24	22.62	7.44	20.32	21.96	24.77	5.52
FB	7.33	7.76	15.87	19.58	23.25	5.52	20.22	6.60	18.65	19.93	21.88	4.06
MSFT	14.35	12.12	26.54	29.13	33.24	6.10	20.22	6.48	15.87	18.44	20.74	4.06
ORCL	37.62	5.53	14.09	14.72	16.29	1.71	10.84	7.38	19.53	21.96	24.37	5.52
Average	17.89	7.01	17.93	20.11	22.33	4.46	17.49	6.99	17.98	20.24	22.59	4.48
Median	10.98	7.02	18.39	20.87	24.23	5.42	20.22	7.01	17.70	19.76	21.88	4.06
Hodges-Lehmann Mean	19.90	7.02	18.39	19.93	23.25	4.29	16.73	6.99	18.10	20.20	22.56	4.59
Std. Dev.	16.00	3.11	7.07	6.64	7.93	2.38	5.24	0.45	1.78	1.42	1.60	0.82
75th percentile	31.80	7.68	22.62	23.79	25.42	5.96	20.22	7.38	19.31	21.45	23.75	5.15
25th percentile	7.40	5.80	14.54	15.93	18.03	2.61	13.18	6.61	16.75	19.58	21.88	4.06
Std. Dev. / HL Mean	0.80	0.44	0.38	0.33	0.34	0.55	0.31	0.06	0.10	0.07	0.07	0.18
IQR / HL Mean	1.23	0.27	0.44	0.39	0.32	0.78	0.42	0.11	0.14	0.09	0.08	0.24

Relative Valuation – XII

- Small differences in profitability, big differences in growth and asset intensity

Operating Margin	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
GOOG	30.0%	27.7%	25.0%	25.8%	26.3%	23.6%	20.1%	21.1%	22.6%	30.6%
AAPL	35.3%	28.7%	28.7%	30.5%	27.8%	26.8%	26.7%	24.6%	24.1%	29.8%
INTC	27.4%	23.3%	27.5%	25.3%	22.1%	28.8%	32.9%	30.6%	30.4%	24.6%
FB	10.6%	35.6%	40.1%	34.7%	45.0%	49.7%	44.6%	33.9%	38.0%	39.6%
MSFT	29.5%	34.4%	32.0%	19.4%	28.6%	30.1%	31.8%	34.1%	37.0%	41.6%
ORCL	36.9%	39.5%	38.6%	36.3%	34.0%	34.2%	33.7%	34.3%	35.6%	37.6%

Revenue Growth	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
GOOG		20.6%	18.9%	13.6%	20.4%	22.8%	23.4%	18.3%	12.8%	41.2%
AAPL		9.2%	7.0%	27.9%	-7.7%	6.3%	15.9%	-2.0%	5.5%	33.3%
INTC		-1.2%	6.0%	-0.9%	7.3%	5.7%	12.9%	1.6%	8.2%	1.5%
FB		54.7%	58.4%	43.8%	54.2%	47.1%	37.4%	26.6%	21.6%	37.2%
MSFT		5.6%	11.5%	7.8%	-2.6%	5.9%	14.3%	14.0%	13.6%	17.5%
ORCL		0.2%	2.9%	-0.1%	-3.1%	2.0%	4.2%	0.3%	-1.1%	3.6%

NOA / Sales	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
GOOG	57%	55%	67%	66%	56%	42%	43%	45%	43%	38%
AAPL	-2%	-4%	-5%	-9%	-10%	-8%	-6%	-3%	-5%	-1%
INTC	86%	95%	96%	102%	117%	124%	121%	125%	117%	132%
FB	71%	51%	200%	144%	108%	80%	77%	65%	77%	65%
MSFT	23%	25%	34%	23%	15%	28%	23%	32%	32%	42%
ORCL	81%	85%	86%	96%	96%	122%	99%	103%	106%	108%

- Ideally, a relative valuation will draw from all three financial statements
  - Focus on value drivers, primarily future earnings and growth

Relative Valuation – XIII

- Individual errors can be large, even within a set of technology stocks!
  - But averaging gets us within +/-50% of market (except for INTC and ORCL)
  - *It's really important to get the right peer group and to use valid metrics*!
  - Robust estimators are also very important – outliers can distort the mean

- Remarkably accurate for a relatively informationless technique!
  - But a comprehensive financial model will usually do better
  - Often misused by lazy analysts - no robust averaging or business insights!

Pricing Error Caused by Using the Hodges-Lehmann Mean of Each Valuation Ratio (Excluding Self)

Ticker	P/B	P/S	P/OCF	P/OE	P/E	P/Assets	Avg. % Error	Median % Error
GOOG	166%	-11%	-20%	-20%	-13%	-24%	13%	-13%
AAPL	-72%	12%	-28%	-12%	-14%	-47%	-27%	-14%
INTC	933%	181%	192%	105%	136%	345%	315%	181%
FB	176%	-15%	17%	2%	-6%	-26%	25%	2%
MSFT	41%	-46%	-40%	-37%	-38%	-34%	-26%	-38%
ORCL	-71%	33%	39%	49%	50%	223%	54%	39%
Average % Error	195%	26%	27%	15%	19%	73%		
Average |% Error|	243%	50%	56%	37%	43%	116%		
Median % Error	103%	1%	-1%	-5%	-10%	-25%		
Median % Error	124%	30%	33%	26%	28%	60%		

Relative Valuation – XIV

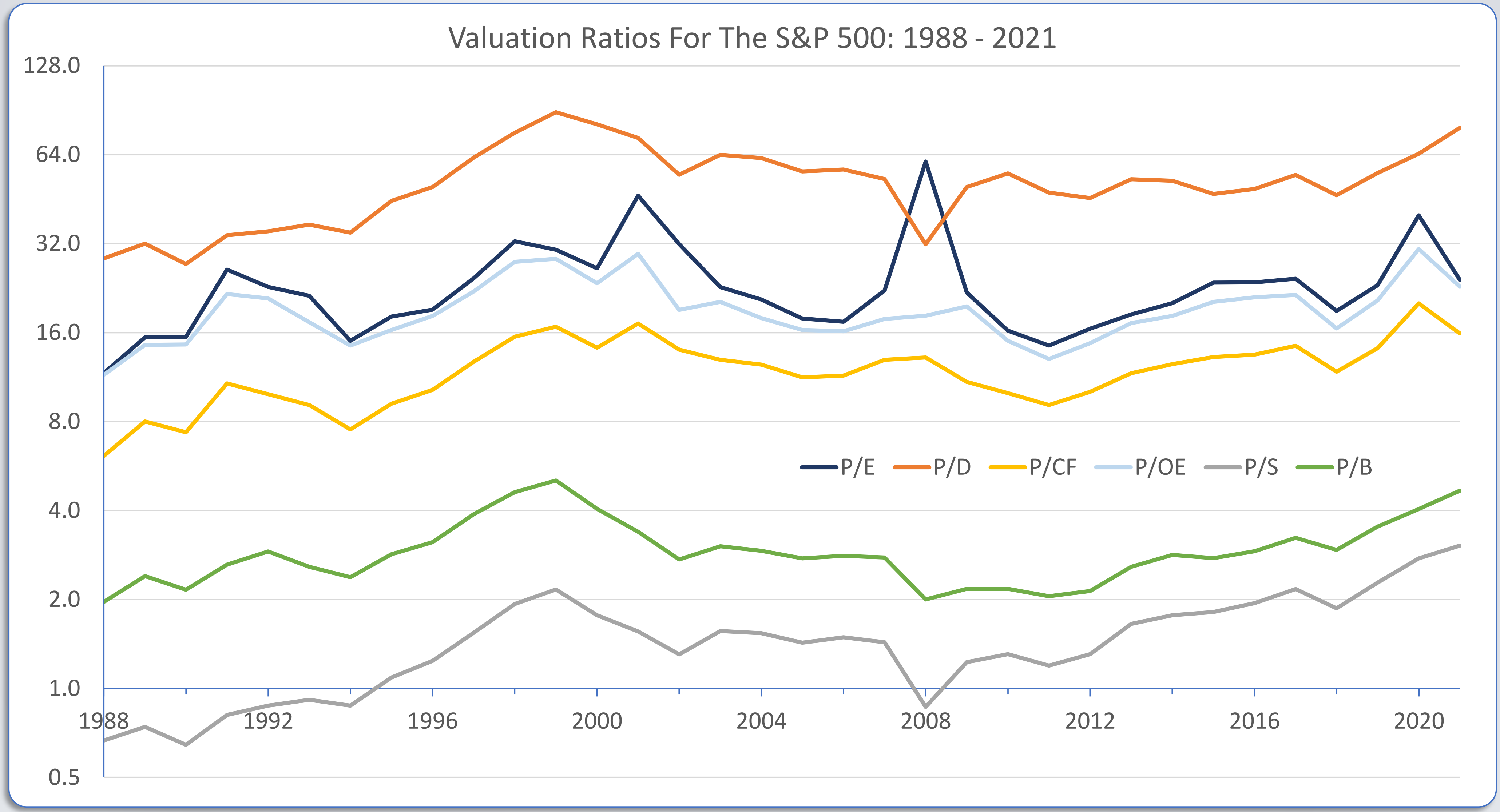
- We sometimes use non-accounting based measures in the numerator
  - Cable and cell phone companies: number of subscribers, or pops (population that is potentially reachable by the firm's products and services)
  - Retail companies: square feet of retail space
  - Websites: unique visitors

- These measures tend to be unique, and work only in certain industries because they are all correlated very highly with revenues and earnings
  - Number of subscribers x Revenue / Subscriber = Revenue
  - Population x Subscribers / Population x Revenue / Subscriber = Revenue
  - Retail area x Revenue / Square foot = Revenue
  - Unique visitors x Ad clicks / Visitor x Revenue / Click = Revenue

- Metrics can be very different for firms in superficially similar industries
  - E.g Apple Store has sales of ~$5,500 / sq. ft. vs. retail average of $300 / sq. ft.
    - Note: Sales / sq. ft. may or may not translate to earnings / sq. ft.!
  - And these metrics can change over time (e.g. mall sales have declined sharply)

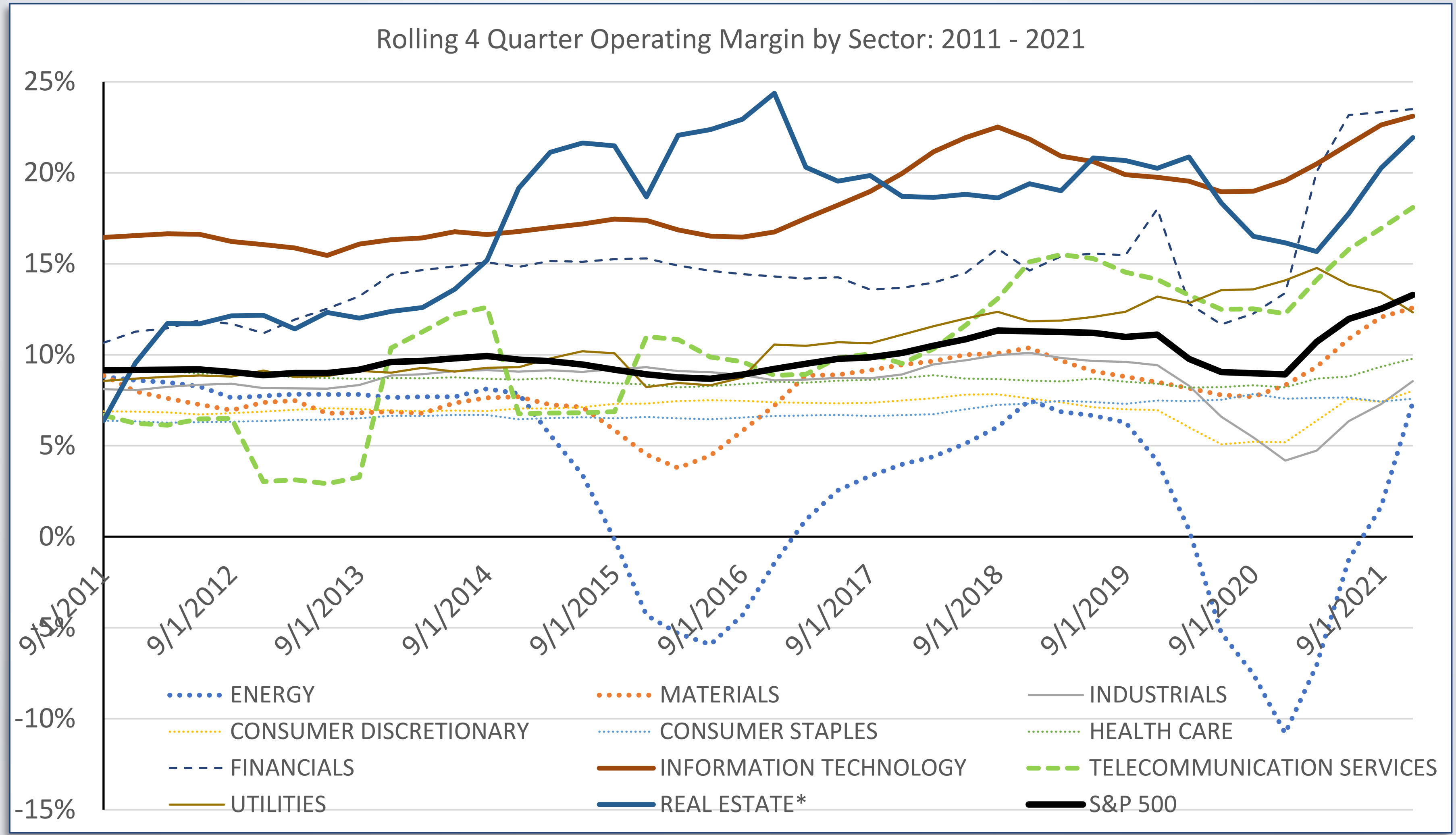
Relative Valuation – XV

- Market multiples vary over time!



Relative Valuation – XVI

- Sectors are remarkably heterogenous in their profitability!
  - Industry margins appear to mean revert around some baseline level



Relative Valuation – XVII

- **Item** | **Potential Sources of Problems**

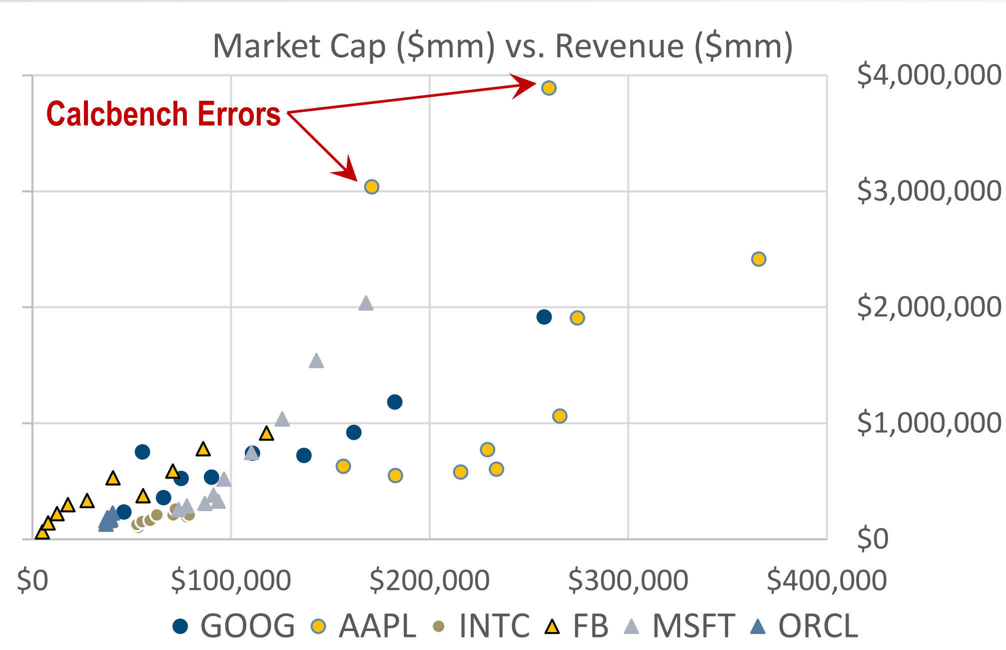
Item	Potential Sources of Problems
P/ E_{t+1}	Earnings can be volatile (accounting rules), analysts are often optimistic
P/ OE_t	Operating Earnings ignore Interest Expense, impact of capital structure
P/ Filtered E	Optimal filtering strategy can be very industry dependent
P/S	Margins vary across industries
P/ EBO Value	When does growth in abnormal earnings cease
PE/ g	Does not seem to flow naturally out of any valuation model
P/ FCF	Free cash flow is lumpy, estimates depend on methodology
P/ B or P/ NOA	Some firms are asset intensive, book value can be biased

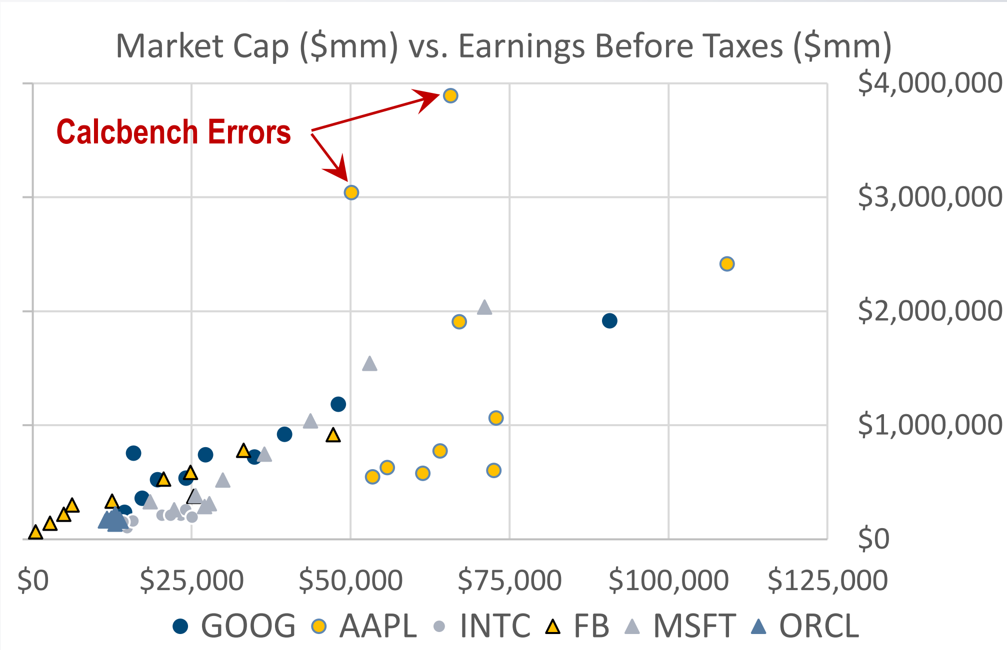
- Pragmatic solution: Value firm using many multiples, then average
  - Use the median or the Hodges-Lehmann mean to protect against outliers
  - Don’t mix time-series and cross-sectional models (k_e is time varying)

- *Use multiple univariate models instead of one multivariate model*

Relative Valuation – XVIII

- A better approach: plot market cap against each of the explanatory variables
  - Run robust regressions – *data can have huge outliers* (see AAPL @ \$3 and \$4 trn)
  - *Can easily include non-linear transformations of the data (e.g. quadratic term)*





- Best approach: Value firm using many multiples, take a robust average
- *Very powerful technique – use whenever data is scarce (e.g. VC, turnarounds)*

Relative Valuation – XIX

- Theoretical justification for relative valuation

- The Gordon Growth model gives us simple expressions for P/B and F/NOA:

$$\frac{V_t}{B_t} = \frac{ROE - g}{k_e - g} \qquad (2)$$

$$\frac{F_t}{NOA_t} = \frac{ROIC - g}{WACC - g} \qquad (3)$$

- These can be transformed to give us simple expressions for P/E and F/NOPAT

$$\frac{V_t}{E_{t+1}} = \frac{V_t}{B_t} \cdot \frac{1}{ROE} = \frac{ROE - g}{k_e - g} \cdot \frac{1}{ROE} \qquad (4)$$

$$\frac{F_t}{NOPAT_{t+1}} = \frac{F_t}{NOA_t} \cdot \frac{1}{ROIC} = \frac{ROIC - g}{WACC - g} \cdot \frac{1}{ROIC} \qquad (5)$$

Relative Valuation – XX

- Near the end of Module 3, we derived the following equations
  - Excess P/B is proportional to the *level* of abnormal earnings

$$\frac{V_t}{B_t} = 1 + \sum_{i\geq 0} \frac{B_{t+i} \cdot (ROE_{t+i+1} - k)}{B_t \cdot (1+k)^{i+1}} \qquad (6)$$

  - P/E is roughly proportional to *growth* in abnormal earnings

$$\frac{V_t}{E_t} = \frac{1+k}{k} \cdot \left[1 + \sum_{i\geq 1} \frac{E_{t+i}^{Abnormal} - E_{t+i-1}^{Abnormal}}{E_t \cdot (1+k)^{i}}\right] - p_t \qquad (7)$$

  - Cannot justify PE/g as a valuation metric, even with simplifying assumptions in (7)
  - P/E is roughly equal to $\frac{1}{k}$ (first two terms of EBO equation)

$$\frac{V_t}{E_{t+1}} \approx \frac{1}{k} \qquad (8)$$

- Since Earnings = Revenues x Net Margin, P/S is related to fundamentals too!
  - Note: growth rates, expected life and asset intensity can be very different

Some Thoughts on Relative Valuation

- Relative valuation usually scales single items from all three financial statements:
  - Income statement (P/E)
  - Balance Sheet (P/B)
  - Statement of Cash Flows (P/OCF)

- But it can also scale combinations of items from one financial statement…
  - Revenue Weighted CAPE
    - Both items from Income Statement

- As well as combinations of items from different financial statements…
  - P / Abnormal Earnings
    - One item from Income Statement, one from Balance Sheet
  - P/ EBO Value
    - Items from income statement and balance sheet + exogeneous estimate of the cost of capital

- But financial statements articulate and must therefore evolve over time
  - We can exploit this fact to build a *dynamic* relative valuation model

Static vs. Dynamic Valuation Models

- A static model evaluates price at a point in time
  - Estimate inputs at fixed points in time, discount back to get today's price
  - Examples: DDM, DCF, EBO

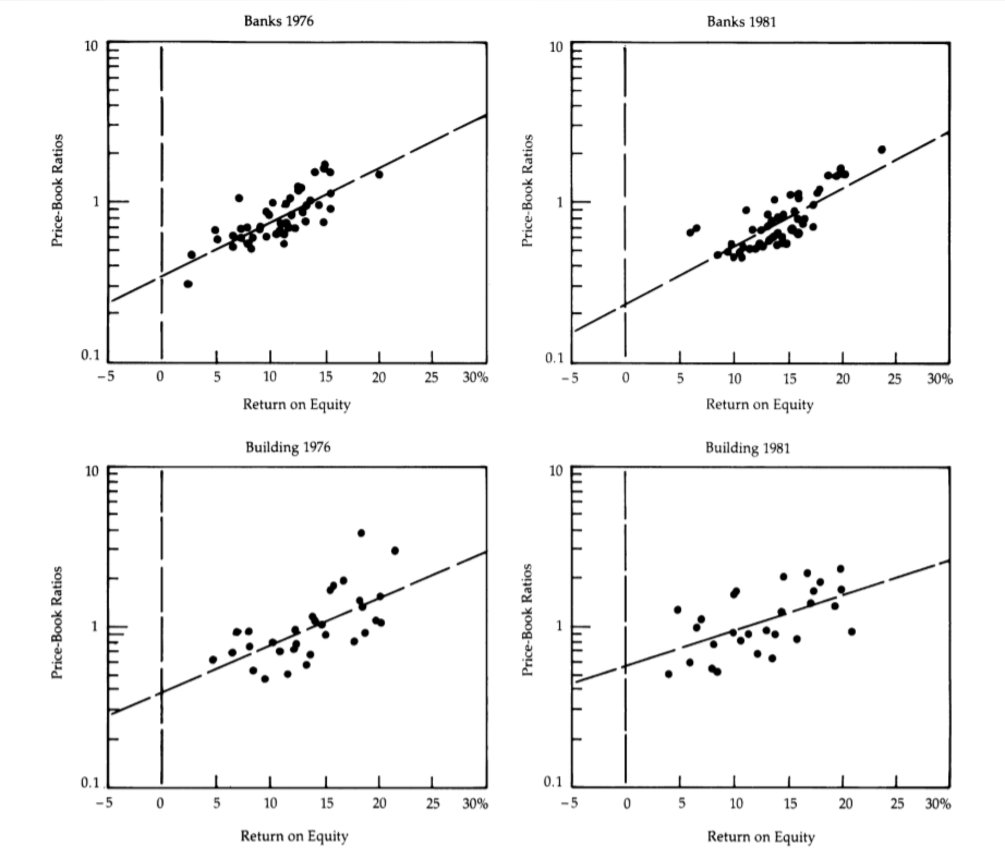
- A dynamic model evolves some function of price over time
  - Trajectory must be consistent with the model: a hint of continuous time!
    - Most famous example: Options (Black-Scholes)!
    - Bond price trajectory must be consistent with the yield curve

- Both static and dynamic models can have the same intellectual roots
  - Both ultimately give us a fix on today's price
  - Choice of one over the other is empirical – what works best in practice?

- We next develop a dynamic valuation model: the P/B – ROE Model
  - Evolves Valuation Ratios to give us a fix on value
  - Exploits the fact that the Balance Sheet and Income Statement must articulate

The P/B–ROE Model – I

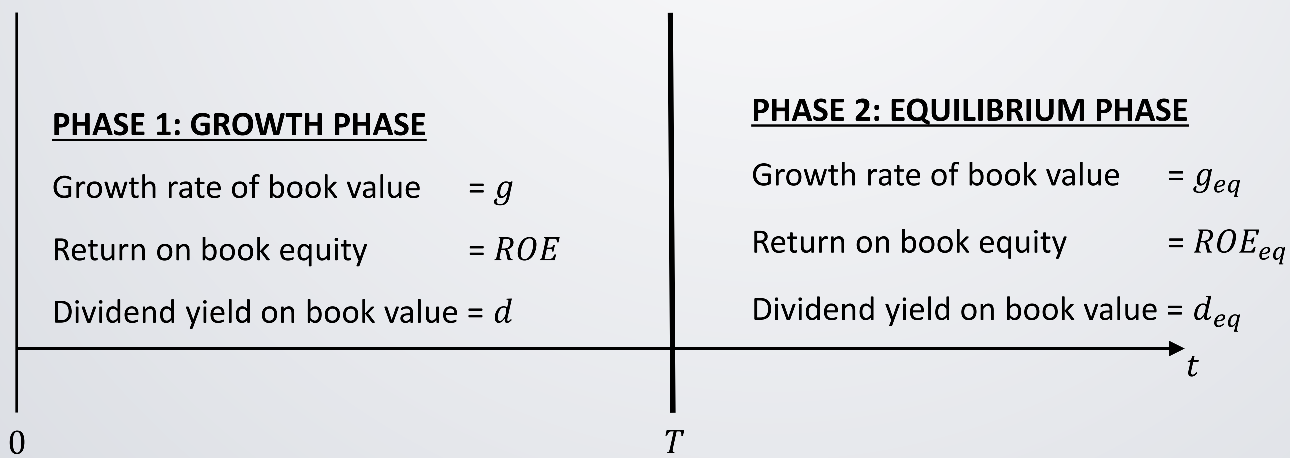
- Developed by Jarrod Wilcox in the mid 1970s, first published in 1984
  - While working as a management consultant, plots $ln(P/B)$ vs. ROE for Polaroid
  - Sees a log-linear pattern in the data, then starts playing around with other firms
  - Finally builds a model that explains this picture



The P/B–ROE Model – II

- Assume that a firm has two phases – a growth phase and an equilibrium phase
  - Growth Phase: $t < T$, Equilibrium Phase: $t \geq T$
  - Distinct growth rates, ROE's and dividend yields in these two phases
  - Capital structure is time-invariant (rational dividend policy)
  - Cost of capital k_e is exogenously determined and is time-invariant

- Similar in spirit (but not identical to) to an EBO model
  - Evolution of the firm is described by a differential equation

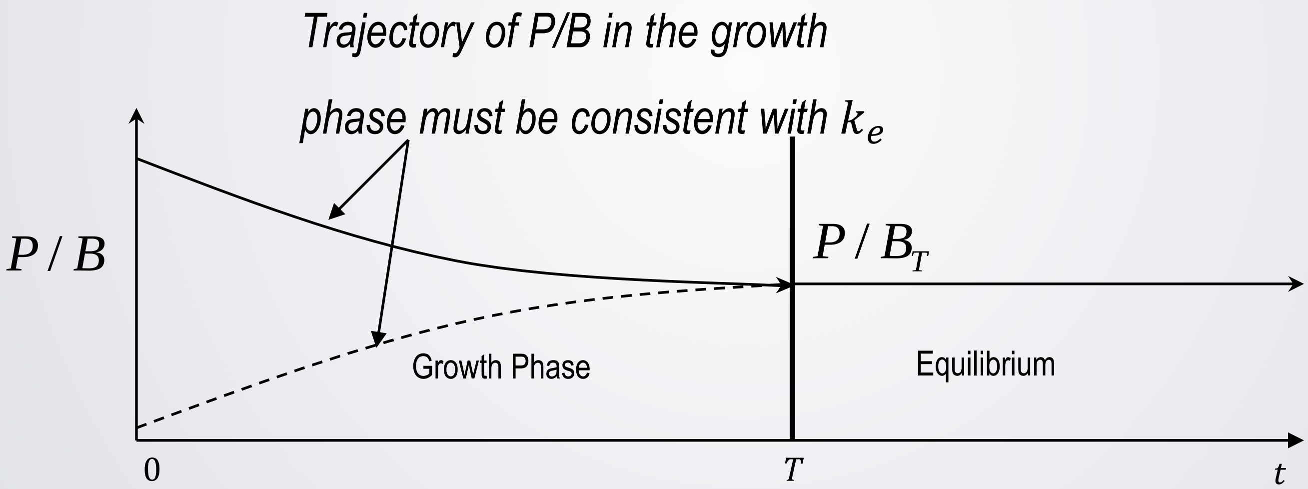


The P/B–ROE Model – III

- Notation (assume that we start at time $t = 0$)
  - P_t: Price at time t
  - B_t: Book Value at time t
  - P/B_t: Price-to-Book ratio at time t
  - T: Time horizon over which the firm can earn a constant abnormal ROE
  - g_t: Growth rate of Book Value at time t
    - $g_t = g$ for $t < T$
    - $g_t = g_{eq}$ for $t \geq T$
  - D_t: Cumulative dividend process at time t
  - d_t: *Rate* of dividend payments at time t divided by B_t (dividend yield on *book*)
    - $d_t = d$ for $t < T$
    - $d_t = d_{eq}$ for $t \geq T$
  - $ROE_t = g_t + d_t$: Instantaneous ROE at time t
    - $ROE_t = ROE$ for $t < T$
    - $ROE_t = ROE_{eq}$ for $t \geq T$
  - Cost of capital k_e is exogenously determined and is time-invariant

The P/B–ROE Model – IV

- Look at the evolution of the balance sheet and income statement over time
  - At each point in time, their trajectory must be consistent with the cost of equity
  - In particular, ROE, B_t and P/B_t must all be consistent with k_e
  - Assume that realized returns = expected returns, both on price and on book



- If ROE is high, P/B_0 must be high, and must decline to its terminal value
- If ROE is low, P/B_0 must be low, and must increase to its terminal value
- From these stylized facts, we can determine P/B_0 and P_0

The P/B–ROE Model – V

- Total Return = Price Return + Dividend Income
- If all parameters are time invariant, Total Return $= k_e$
- Total Return $= \dfrac{\frac{\partial P_t}{\partial t} + \frac{\partial D_t}{\partial t}}{P_t} = \dfrac{1}{P_t} \cdot \dfrac{\partial P_t}{\partial t} + \dfrac{1}{P_t} \cdot \dfrac{\partial D_t}{\partial t} = k_e$ (9)
- But $P_t = B_t \cdot P/B_t$, so that

$$\frac{1}{P_t} \cdot \frac{\partial P_t}{\partial t} = \frac{1}{B_t} \cdot \frac{\partial B_t}{\partial t} + \frac{1}{P/B_t} \cdot \frac{\partial P/B_t}{\partial t} \tag{10}$$

- It follows that

$$k_e = \frac{1}{B_t} \cdot \frac{\partial B_t}{\partial t} + \frac{1}{P/B_t} \cdot \frac{\partial P/B_t}{\partial t} + \frac{1}{P_t} \cdot \frac{\partial D_t}{\partial t}$$

$$= \frac{1}{B_t} \cdot \frac{\partial B_t}{\partial t} + \frac{1}{P/B_t} \cdot \frac{\partial P/B_t}{\partial t} + \frac{1}{P/B_t} \cdot \frac{1}{B_t} \cdot \frac{\partial D_t}{\partial t} \tag{11}$$

The P/B–ROE Model – VI

- But $\frac{1}{B_t} \cdot \frac{\partial D_t}{\partial t} = d_t$, the dividend yield on book value and $\frac{1}{B_t} \cdot \frac{\partial B_t}{\partial t} = g_t$
- It follows that

$$k_e = g_t + \frac{1}{P/B_t} \cdot \left(\frac{\partial P/B_t}{\partial t} + d_t\right) \quad (12)$$

$$\Longrightarrow \frac{\partial P/B_t}{\partial t} = P/B_t \cdot (k_e - g_t) - d_t \quad (13)$$

- This is a first order ordinary differential equation!
  - In the steady state, when $d_t = d_{eq}\ \forall t \geq T$, the left-hand side is 0. It follows that

$$P/B_{eq} = \frac{d_{eq}}{k_e - g_{eq}} \quad (14)$$

- Sanity check: this is exactly what we got from the Gordon Growth Model!

The P/B–ROE Model – VII

- In equilibrium, $ROE_{eq} = g_{eq} + d_{eq}$, and it follows that

$$P/B_{eq} = \frac{ROE_{eq} - g_{eq}}{k_e - g_{eq}} \tag{15}$$

- Now solve the ODE with boundary condition $P/B_T = \frac{ROE_{eq} - g_{eq}}{k_e - g_{eq}}$
- The solution to this ODE is

$$P/B_t = P/B_0 \cdot e^{(k_e - g)t} + \frac{d}{k_e - g}\left(1 - e^{(k_e - g)t}\right) \tag{16}$$

- Sanity check:

$$\frac{\partial P/B_t}{\partial t} = (k_e - g) \cdot \frac{P}{B_0} \cdot e^{(k_e - g)t} - d \cdot e^{(k_e - g)t}$$

$$= (k_e - g) \cdot \left[\frac{P}{B_0} \cdot e^{(k_e - g)t} + \frac{d}{k_e - g}\left(1 - e^{(k_e - g)t}\right)\right] - d \;\text{☑} \tag{17}$$

The P/B–ROE Model – VIII

- Next, allow the balance sheet to evolve with time. At time T we have

$$P/B_T = P/B_0 \cdot e^{(k_e-g)T} + \frac{d}{k_e-g}\left(1 - e^{(k_e-g)T}\right) \quad (18)$$

$$\approx P/B_0 \cdot e^{(k_e-g)T} - \frac{d \cdot (k_e-g)T}{k_e-g}$$

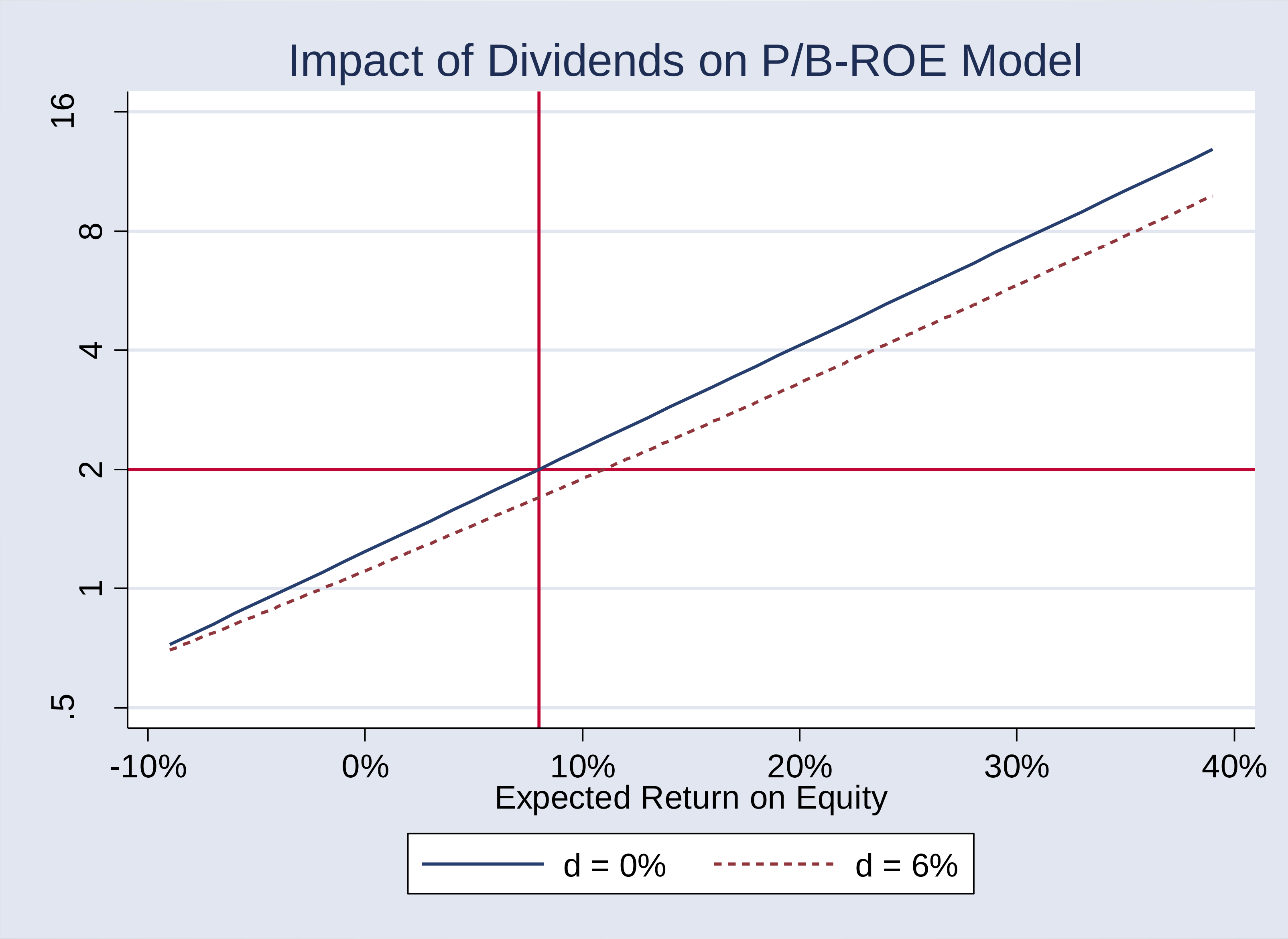
$$= P/B_0 \cdot e^{\left(k_e-(ROE-d)\right)T} - d \cdot T \quad (19)$$

- If d, the dividend yield on book value $\longrightarrow 0$, the second term disappears, and

$$P/B_T \longrightarrow P/B_0 \cdot e^{(k_e-ROE)T} \quad (20)$$

- This is a reasonable approximation, as we are not just discarding the last term
  - We account for its removal by suitably adjusting the exponent in the first term
  - If $P/B_0 = 1$, the approximation is almost exact

The P/B–ROE Model – IX



The P/B–ROE Model – X

- Finally, take logs on both sides and rearrange terms to get

$$ln(P/B_0) = ln(P/B_T) + (ROE - k_e) \cdot T \tag{21}$$

or equivalently

$$ln(P/B_0) = (ln(P/B_T) - k_e \cdot T) + ROE \cdot T \tag{22}$$

- These equations are known as Wilcox's P/B - ROE model
  - Links the income statement and the balance sheet
  - Gives us a fundamentally different way of thinking about relative valuation
  - We will later extend this to revenues and net margins

The P/B–ROE Model – XI

- Equations (21) and (22) can be estimated cross-sectionally using robust regression!

$$ln\left(P/B_i^0\right) = \alpha + \beta \cdot (ROE_i - k_i^e) + \varepsilon_i \quad (23)$$

$$ln\left(P/B_i^0\right) = \gamma + \delta \cdot ROE_i + \varepsilon_i \quad (24)$$

- Use (23) if you have data on ROE and have individual estimates of k_e
  - E.g. Estimate k_e using k_d + Equity premium vs. corporate bonds
  - α is an estimate of $ln(P/B_T)$, while β is an estimate of T

- Use (24) if you have data on ROE and cannot get an estimate of k_e
  - γ is harder to interpret, while δ is still an estimate of T

- If assets are priced at replacement cost, P/B_T should be 1
  - P/B_T was $\approx$ 1 when the model was first developed in the late 1970s, so $ln(P/B_T)$=0
  - In this case, we can estimate k_e (overall) using (24) as $k_e \approx -\frac{\gamma}{\delta}$

The P/B–ROE Model – XII

- Results of regressions from 1988-2002 for Value Line Universe
  - The pooled slope was 3.66 years.
  - For companies with very stable *ROE*s it rose to about 9 years.
  - A stable *ROE* allows projecting recent values further into the future.
- Competition has increased since 2002, but S&P 500 ROE seems persistently high
  - Particularly true among growth companies
- More worrying, industry level regressions across show *lower* slopes
  - Might indicate that the model's utility is fading
- Even so, it illustrates how models are built, tested and used
  - Better than blindly doing relative valuation without thinking!

The P/B–ROE Model – XIII

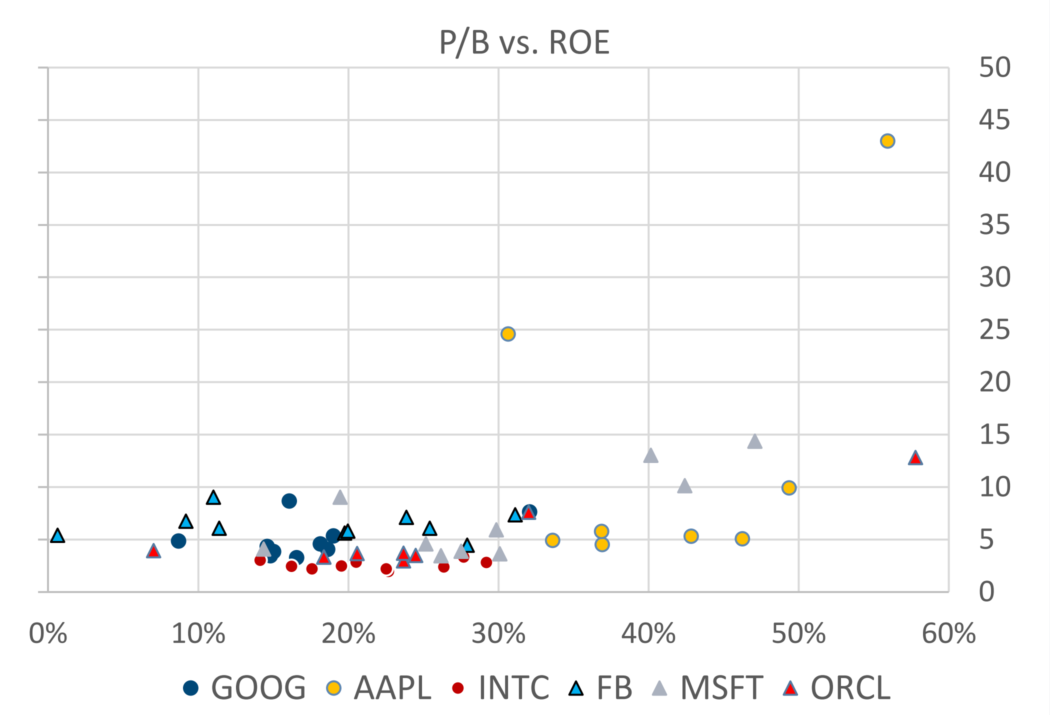
- The model has been in existence for 40 years – is it still useful?
  - Yes it is – though it is less useful than it once was!

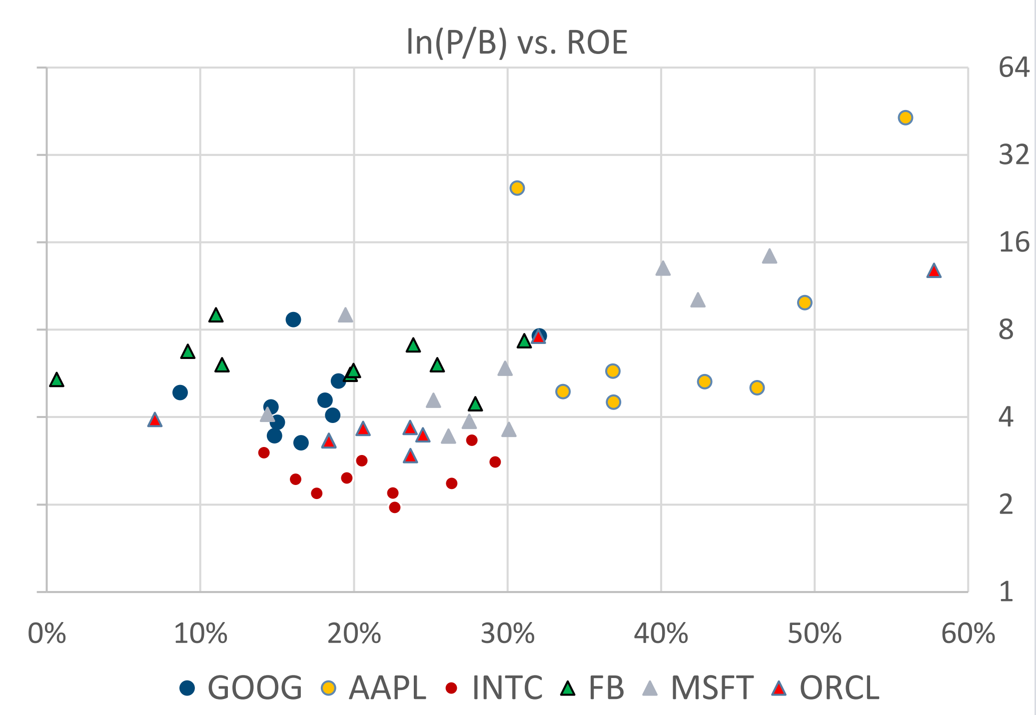
- We test it for our 6 tech companies and at the industry level in 2021
  - GOOG, AAPL, FB, INTC, MSFT, ORCL
  - Get data for P/B and ROE for industries from Professor Damodaran's website at http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datacurrent.html#multiples
  - You can get data for the US, EU, JP, EM, CN, IN, Global

- Data for US, EU, JP, EM (All Emerging Markets), CN, IN

- Plot P/B and ln(P/B) and vs. ROE
  - Do OLS and robust regressions to determine the relationship
  - Compute R^2 and robust R^2 to compare models

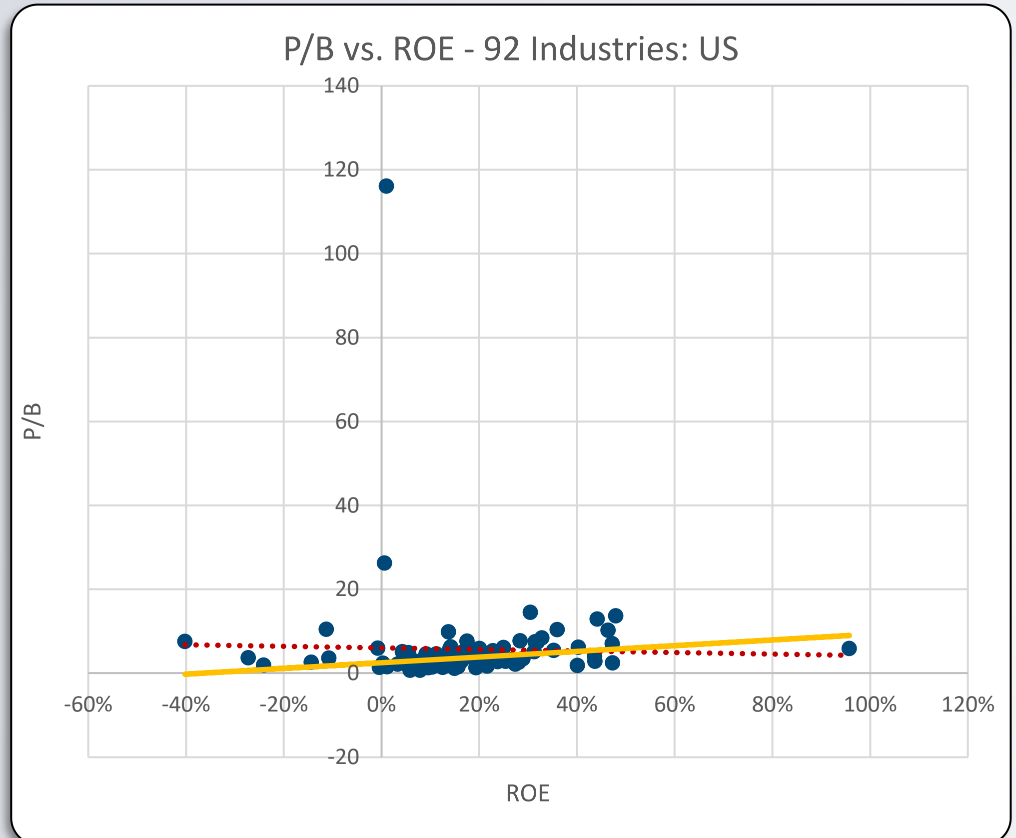
P/B vs. ROE and ln(P/B) vs. ROE: Six Tech Companies

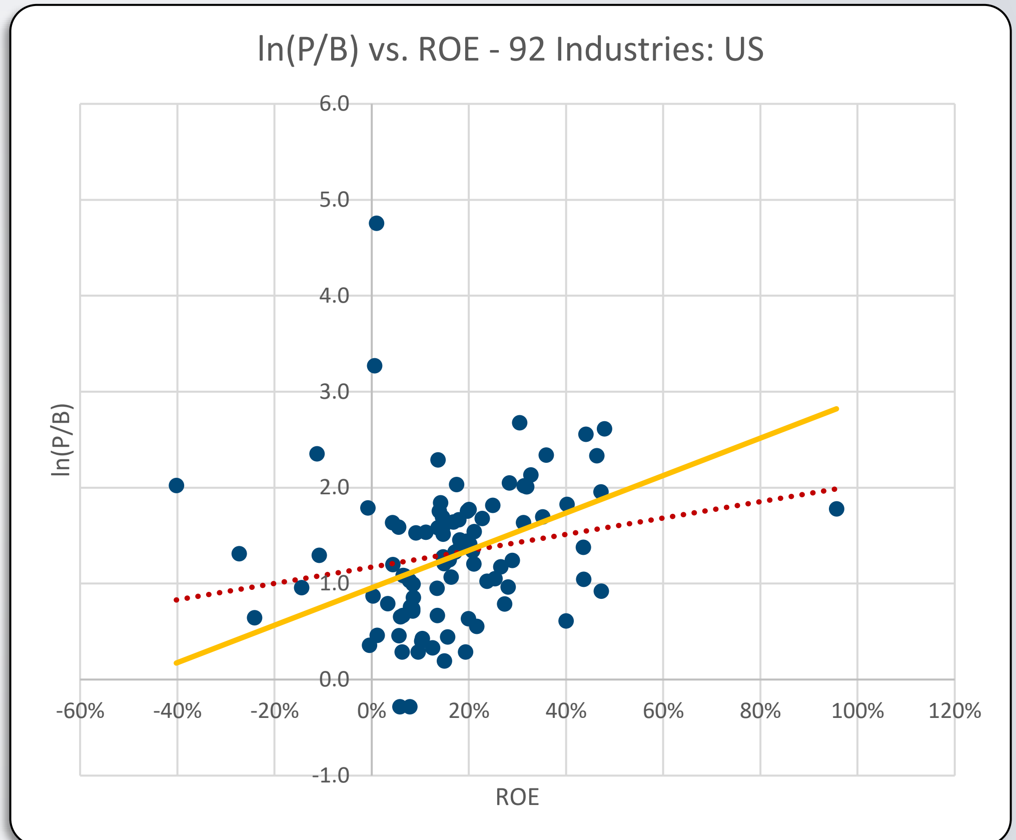




		ln(P/B) = a + b x ROE			ROE = a + b x ln(P/B)			R^2
Regression	**Variables**	**Slope**	**Intercept**		**Slope**	**Intercept**		
OLS	**PB vs. ROE**	1.95	1.13		0.249	-0.127		48.5%
Robust	**PB vs. ROE**	1.91	1.04		0.116	0.084		22.3%

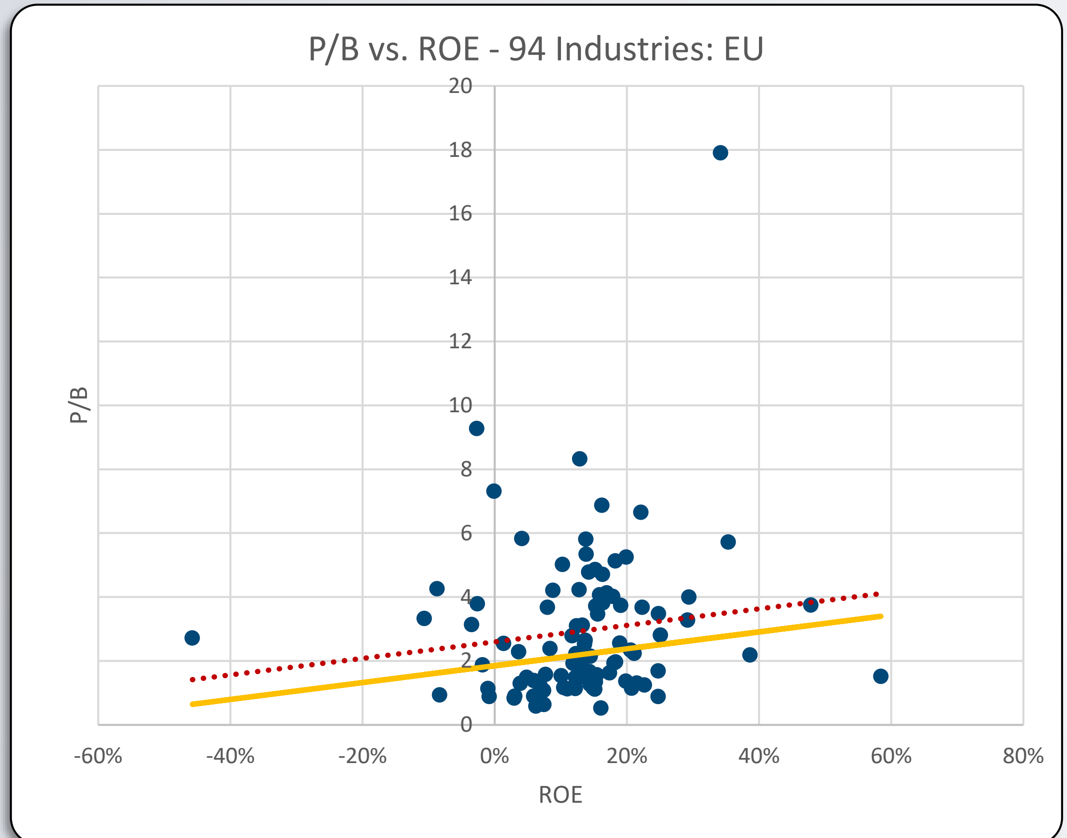
The P/B–ROE Model : US Industries

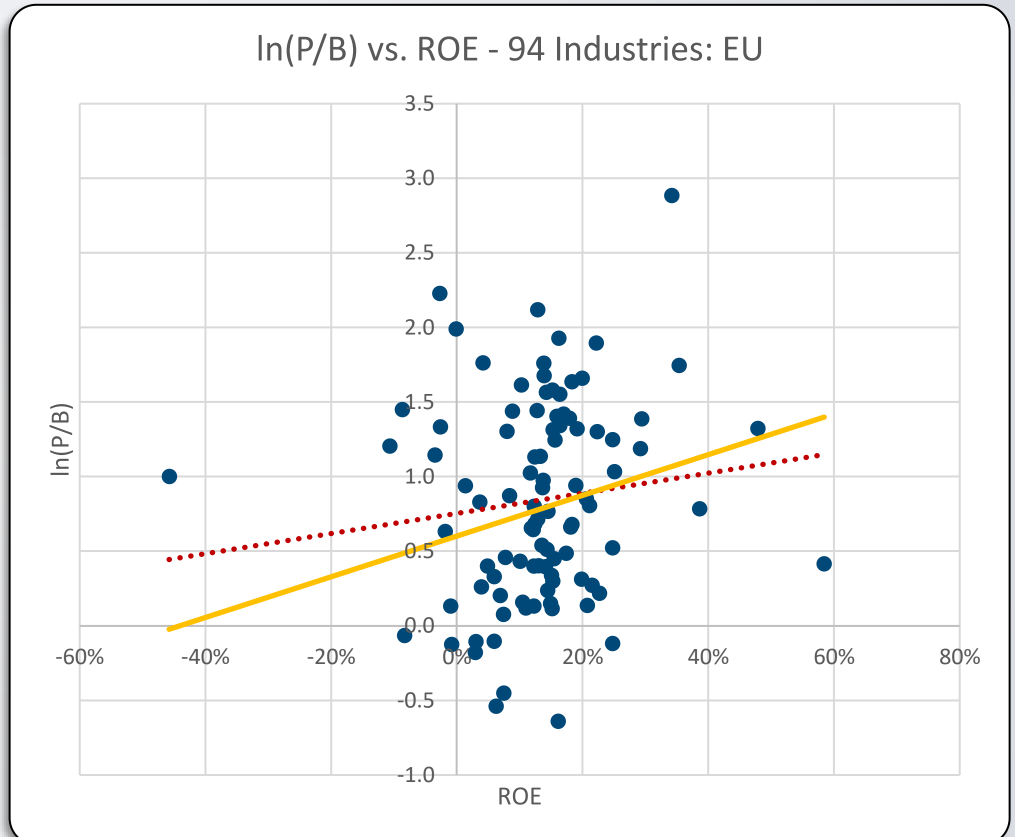




		PB or ln(PB) = a + b x ROE			ROE = a + b x PB or ln(PB)			R^2
Regression	Variables	Slope	Intercept		Slope	Intercept		
OLS	PB vs. ROE	-1.84	6.03		0.000	0.167		0.1%
Robust	PB vs. ROE	6.75	2.51		0.023	0.066		15.4%
OLS	ln PB vs ROE	0.85	1.17		0.047	0.104		4.0%
Robust	ln PB vs ROE	1.95	0.95		0.082	0.057		15.9%

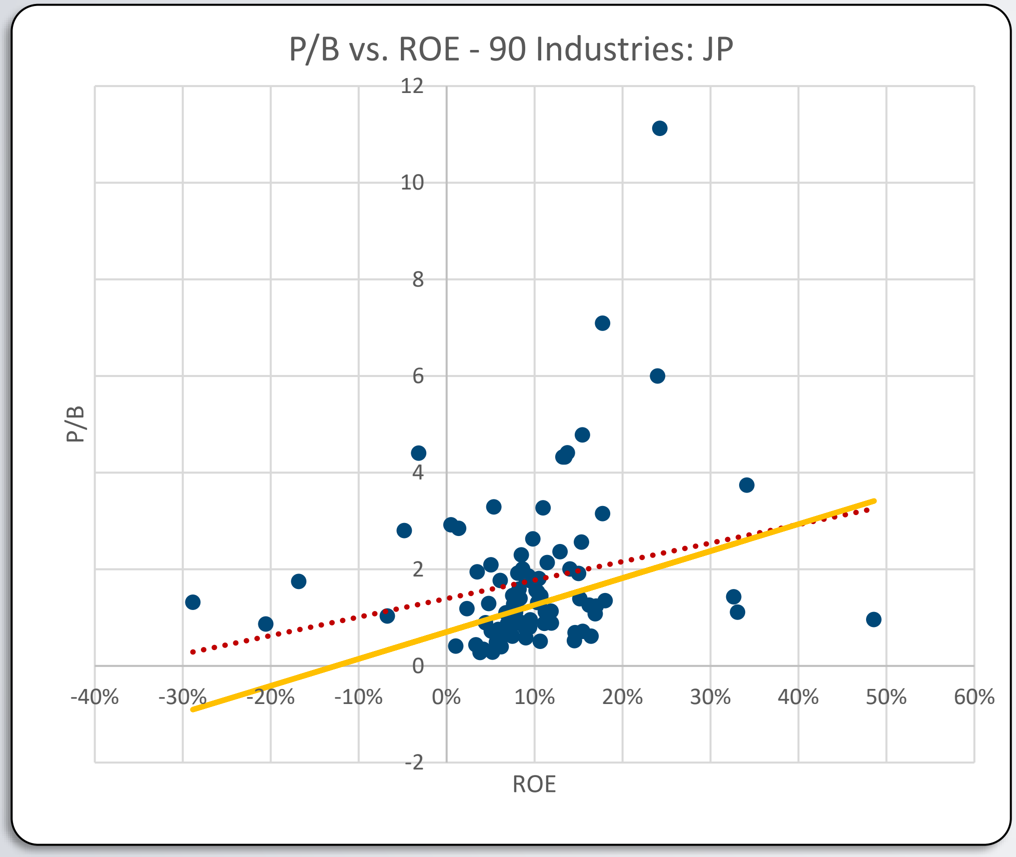
The P/B–ROE Model : EU Industries

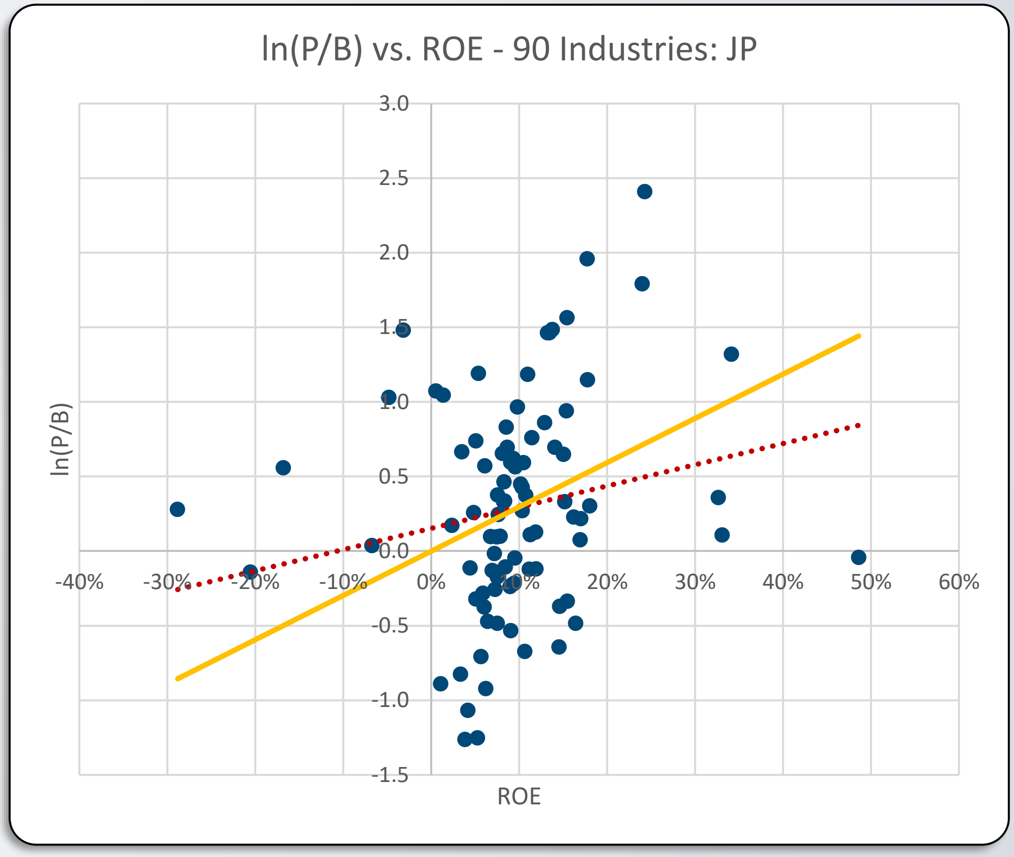




		PB or ln(PB) = a + b x ROE			ROE = a + b x PB or ln(PB)			R^2
Regression	**Variables**	**Slope**	**Intercept**		**Slope**	**Intercept**		
OLS	**PB vs. ROE**	2.58	2.61		0.007	0.111		1.8%
Robust	**PB vs. ROE**	2.64	1.85		0.009	0.110		2.5%
OLS	**ln PB vs ROE**	0.67	0.75		0.022	0.113		1.5%
Robust	**ln PB vs ROE**	1.36	0.60		0.027	0.112		3.6%

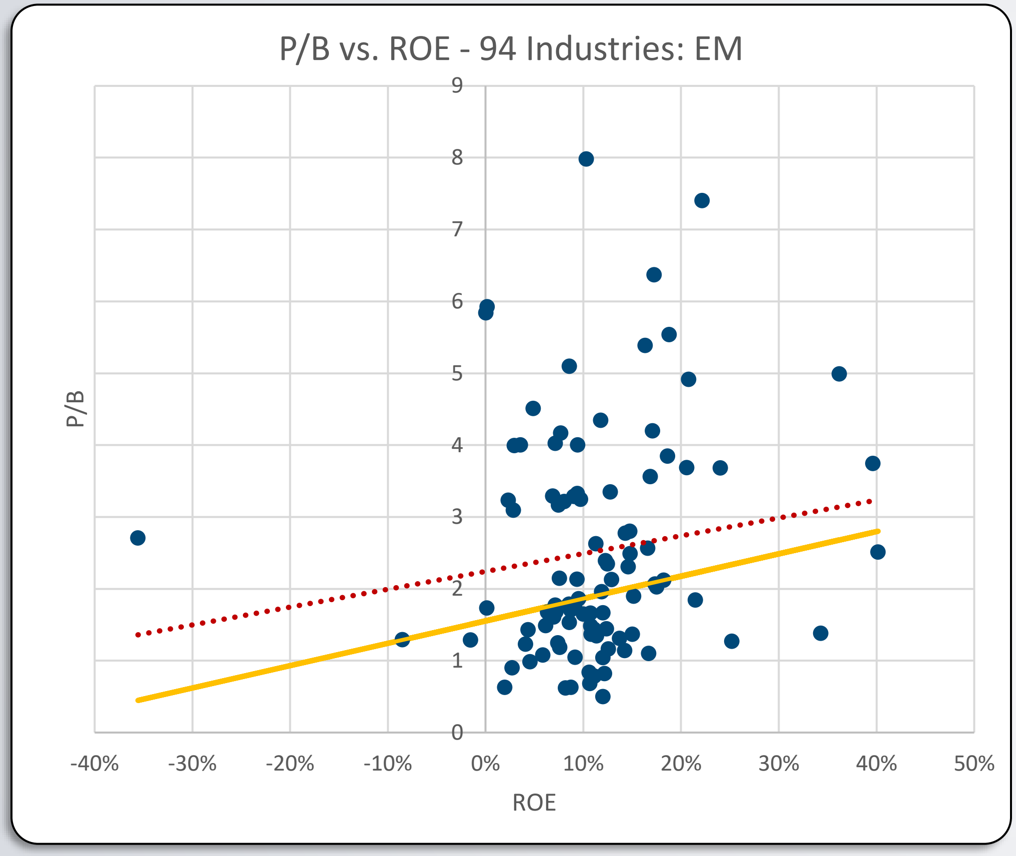
The P/B–ROE Model : JP Industries

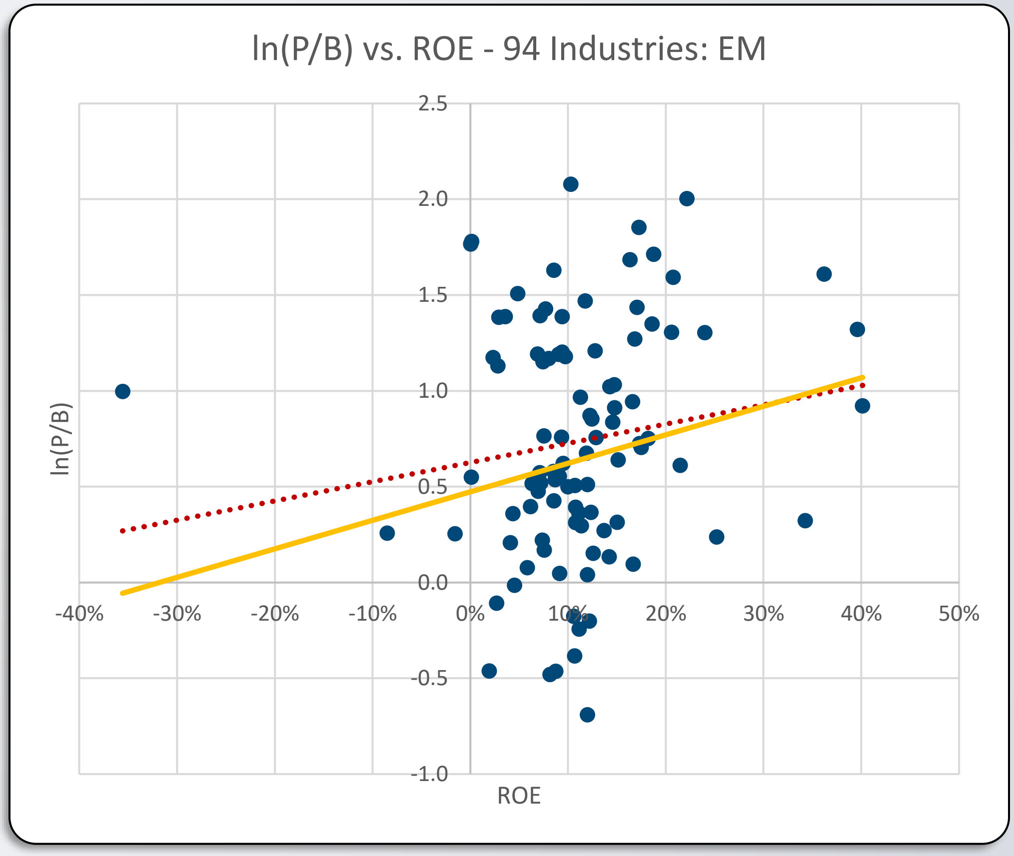




		PB or ln(PB) = a + b x ROE			ROE = a + b x PB or ln(PB)			R^2
Regression	Variables	Slope	Intercept		Slope	Intercept		
OLS	PB vs. ROE	3.83	1.40		0.014	0.069		5.5%
Robust	PB vs. ROE	5.58	0.70		0.019	0.055		10.6%
OLS	ln PB vs ROE	1.42	0.15		0.027	0.087		3.8%
Robust	ln PB vs ROE	2.97	0.00		0.027	0.080		8.1%

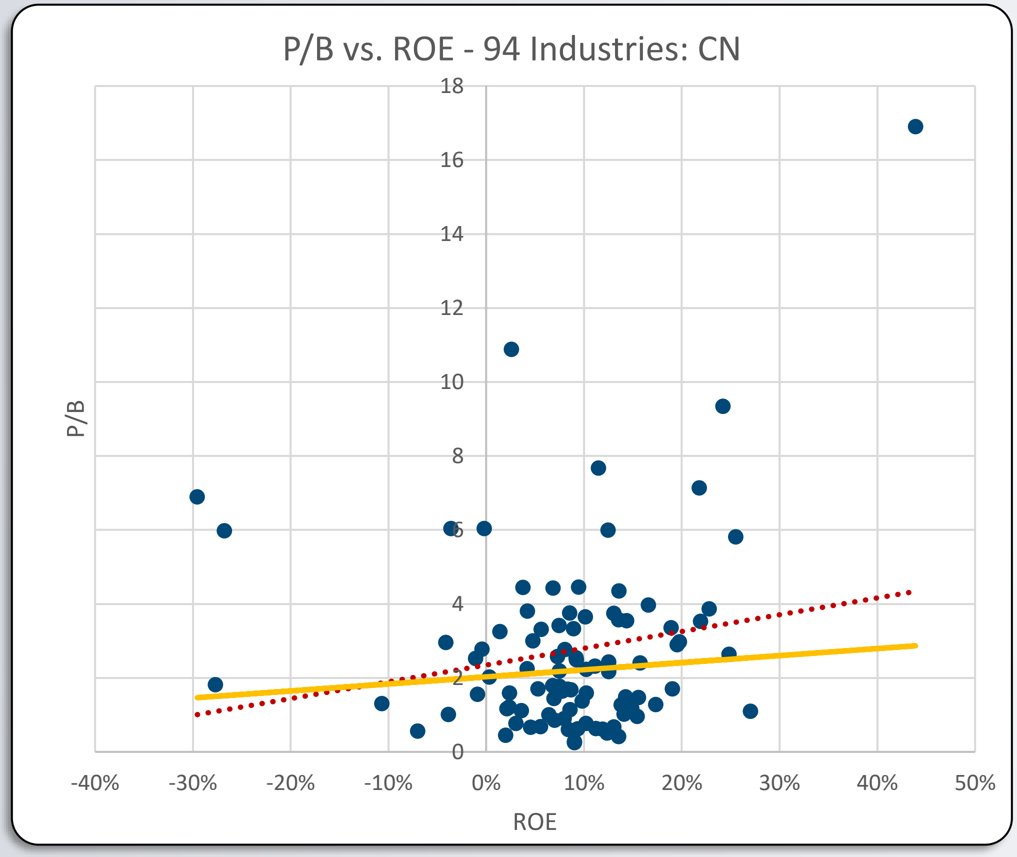
The P/B–ROE Model : EM Industries

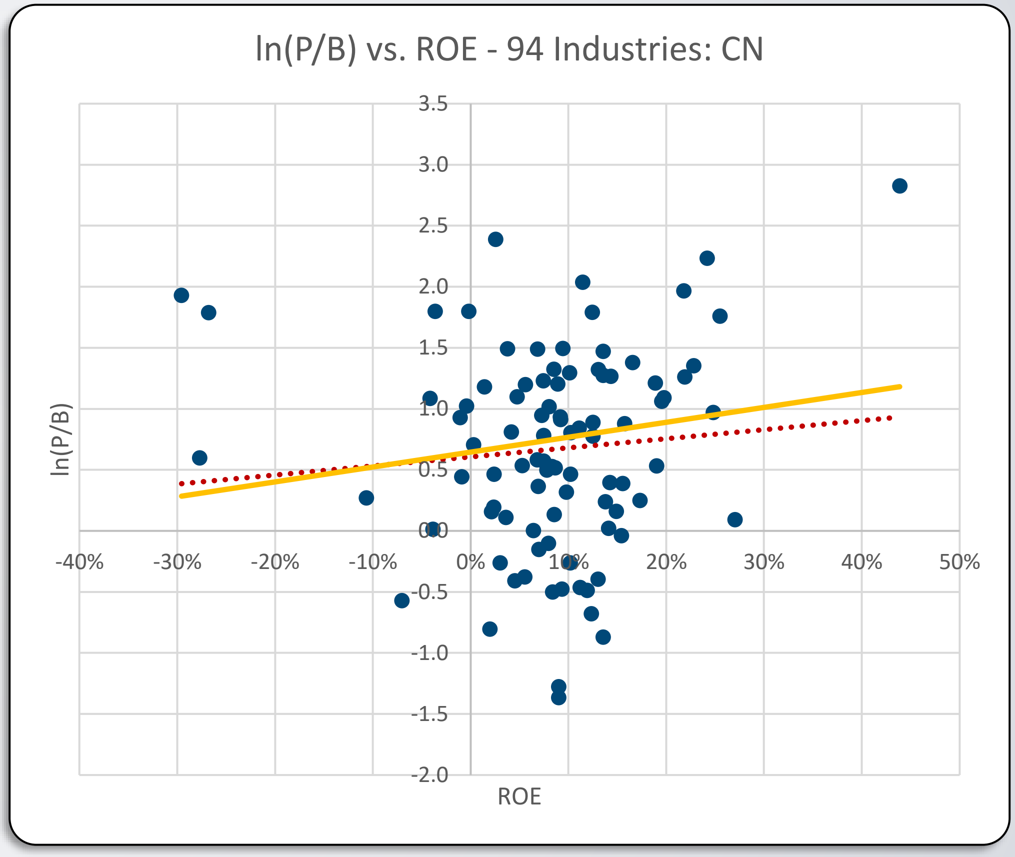




		PB or ln(PB) = a + b x ROE			ROE = a + b x PB or ln(PB)			R^2
Regression	**Variables**	**Slope**	**Intercept**		**Slope**	**Intercept**		
OLS	**PB vs. ROE**	2.49	2.26		0.009	0.090		2.1%
Robust	**PB vs. ROE**	3.11	1.55		0.009	0.087		2.8%
OLS	**ln PB vs ROE**	1.01	0.63		0.023	0.095		2.3%
Robust	**ln PB vs ROE**	1.49	0.47		0.021	0.095		3.0%

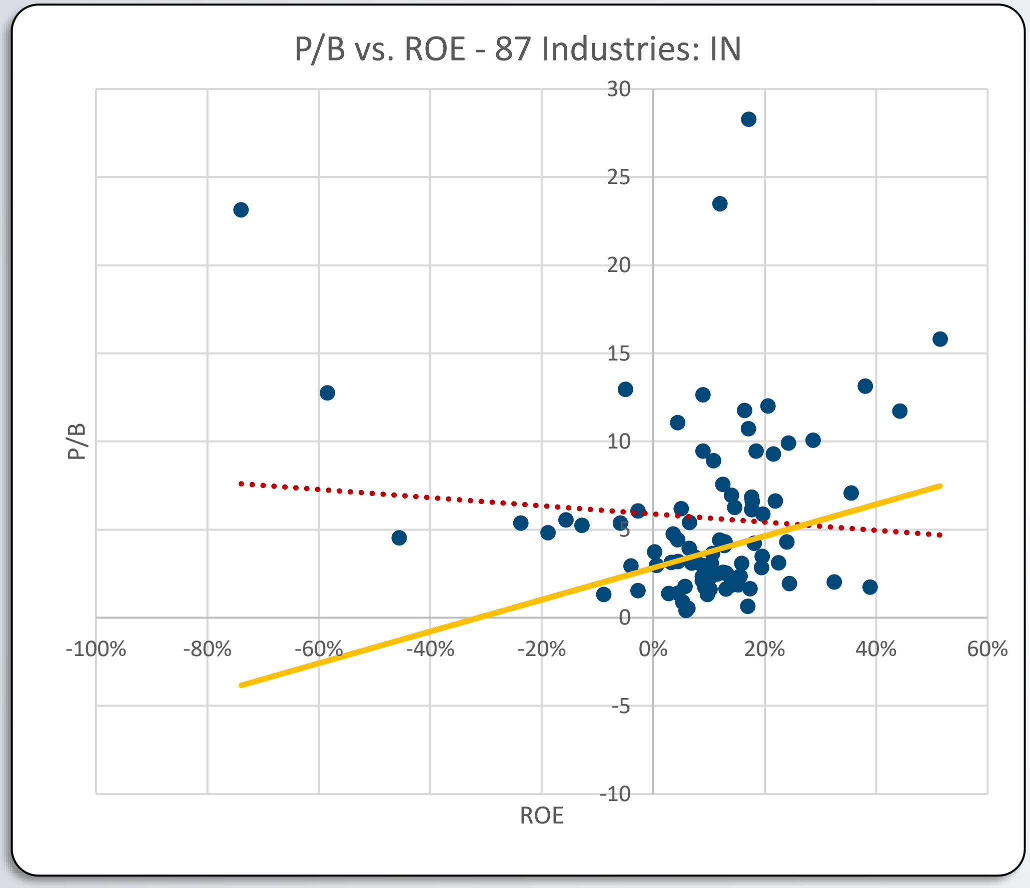
The P/B–ROE Model : CN Industries

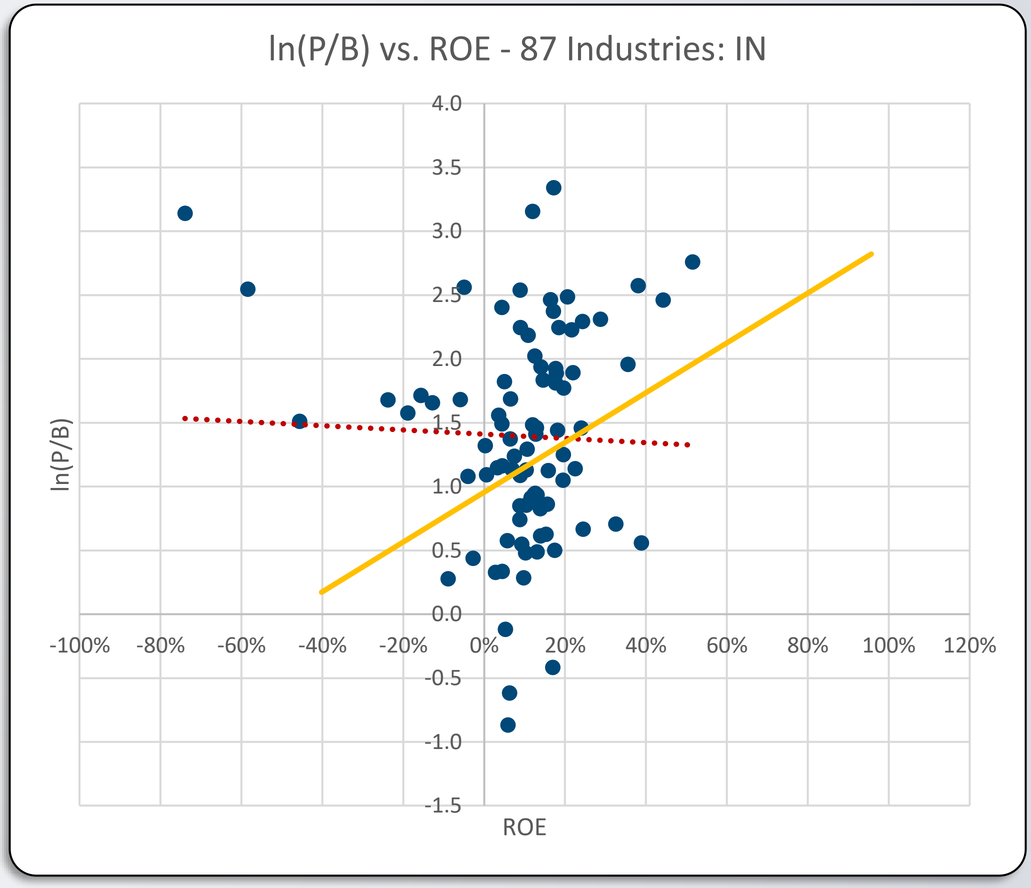




		PB or ln(PB) = a + b x ROE		ROE = a + b x PB or ln(PB)		R^2
Regression	**Variables**	**Slope**	**Intercept**	**Slope**	**Intercept**	
OLS	**PB vs. ROE**	4.50	2.33	0.008	0.064	3.5%
Robust	**PB vs. ROE**	1.92	2.03	0.005	0.080	0.9%
OLS	**ln PB vs ROE**	0.73	0.60	0.012	0.078	0.8%
Robust	**ln PB vs ROE**	1.22	0.64	0.012	0.081	1.4%

The P/B–ROE Model : IN Industries





		PB or ln(PB) = a + b x ROE			ROE = a + b x PB or ln(PB)			R^2
Regression	**Variables**	**Slope**	**Intercept**		**Slope**	**Intercept**		
OLS	**PB vs. ROE**	-2.31	5.89		-0.003	0.110		0.7%
Robust	**PB vs. ROE**	9.02	2.83		0.008	0.069		7.6%
OLS	**ln PB vs ROE**	-0.16	1.41		-0.008	0.105		0.1%
Robust	**ln PB vs ROE**	2.39	1.06		0.038	0.062		9.0%

The P/B–ROE Model for the S&P 500 – I

- Use Calcbench to get P/B and ROE for the S&P 500 in 2021
  - S&P 500 Tickers can be obtained from Calcbench

```
tickers = calcbench.tickers(index = "SP500")
```

- Run OLS and Robust regressions of ln(P/B) vs. ROE

```
#Theil-Sen parameters are input as (y,x). OLS parameters are put in as (x,y)
robPB_ROE_res = stats.theilslopes( lnPB_masked, ROE_masked )
olsPB_ROE_res = stats.linregress( ROE_masked, lnPB_masked )
```

- Robust slope = 2.83, OLS slope = 0.11
  - Robust $R^2 = 31\%$, OLS $R^2 = 7\%$

- Interesting results:
  - Robust slope = 2.83 is lower than it was 20 years ago – economy is more competitive!
  - OLS slope is ridiculously low
  - Robust R^2 is noticeably higher than OLS R^2

- Central question for investors: Can the model’s residuals predict future returns?

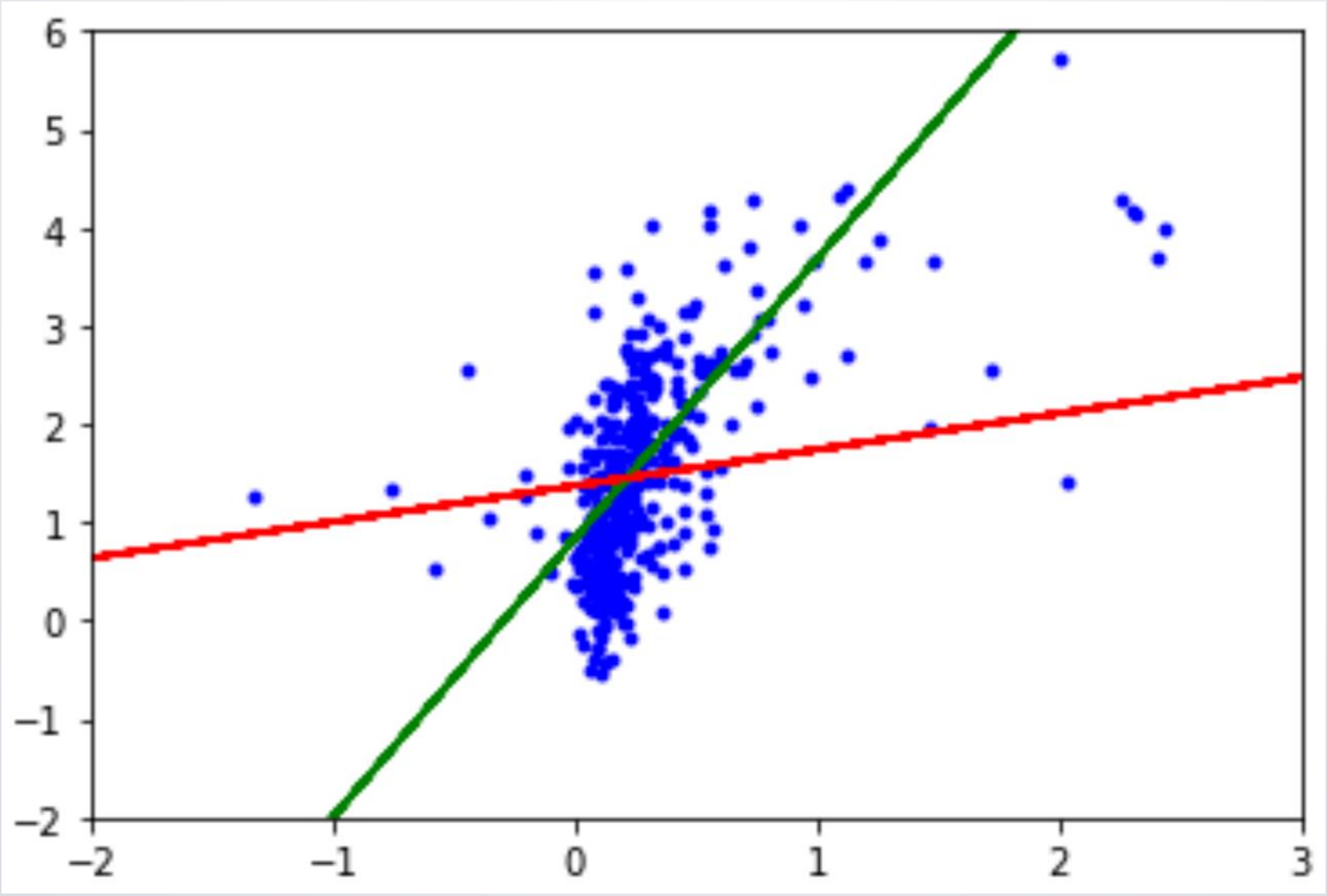
The P/B–ROE Model for the S&P 500 – II

PB-ROE_calcbench_test.py*

```python
# At CONDA command prompt, run the following commands
# conda install git
# conda install pip
# pip install git+git://github.com/calcbench/python_api_client.git
# github page at https://github.com/calcbench/python_api_client
# Documentation at http://calcbench.github.io/python_api_client/html/index.html
# Examples at https://github.com/calcbench/notebooks
import matplotlib.pyplot as plt
import numpy as np
import calcbench as cb

# Get the version of scipy.stats that explicitly allows for masked arrays, as there are missing values, NaNs etc. in the data
import scipy.stats.mstats as stats

# Create a file named calcbenchCredentials.py with two lines in the same directory as the code to hold your userid and password
# calcbench_userid   = "yourcalcbenchuserid"
# calcbench_password = "yourcalcbenchpassword"
import calcbenchCredentials
cb.set_credentials( calcbenchCredentials.calcbench_userid, calcbenchCredentials.calcbench_password )

# For a list of metrics, visit https://www.calcbench.com/home/standardizedmetrics
# Obtain a Pandas dataframe with all the data needed to run the P/B-ROE model for the S&P 500.
# The dataframe is indexed by a MultiIndex, as it has 3 dimensions: data items, securities and time
# see https://pandas.pydata.org/pandas-docs/stable/user_guide/advanced.html# for details
tickers = cb.tickers(index = "SP500")
dataItems = ["ROE", "StockholdersEquity", "MarketCapAtEndOfPeriod"]
data = cb.normalized_data(tickers, dataItems, start_year=2021, end_year=2021, period_type='annual')
lnPB = np.log( data.MarketCapAtEndOfPeriod / data.StockholdersEquity )

# Create masked arrays to identify all the missing / invalid data items
lnPB_masked = np.ma.masked_array( lnPB, mask = np.isnan(lnPB) )
ROE_masked  = np.ma.masked_array( data.ROE,  mask = np.isnan(data.ROE) )

# Theil-Sen parameters are entered as (y,x), OLS parameters are entered as (x,y)
# Regress y on x and x on y to compute a robust R squared
robPB_ROE_fit = stats.theilslopes( lnPB_masked, ROE_masked  )
olsPB_ROE_fit = stats.linregress(  ROE_masked,  lnPB_masked )
robROE_PB_fit = stats.theilslopes( ROE_masked,  lnPB_masked )
olsROE_PB_fit = stats.linregress(  lnPB_masked, ROE_masked  )

robR2_PB_ROE = robPB_ROE_fit[0] * robROE_PB_fit[0]
olsR2_PB_ROE = olsPB_ROE_fit[0] * olsROE_PB_fit[0]

print("\nln(P/B) vs. ROE: Robust R2=", robR2_PB_ROE, "OLS R2=", olsR2_PB_ROE)
print( robPB_ROE_fit )
print( olsPB_ROE_fit )

print("\nROE vs. ln(P/B): Robust R2=", robR2_PB_ROE, "OLS R2=", olsR2_PB_ROE)
print( robROE_PB_fit )
print( olsROE_PB_fit )

fig = plt.figure()
axes = fig.add_subplot( 111 )
axes.set_xlim( [-2, 3] )
axes.set_ylim( [-2, 6] )
axes.plot( ROE_masked[0], lnPB_masked[0], 'b.')
axes.plot( ROE_masked[0], (robPB_ROE_fit[1] + robPB_ROE_fit[0] * ROE_masked)[0], 'g-' )
axes.plot( ROE_masked[0], (olsPB_ROE_fit[1] + olsPB_ROE_fit[0] * ROE_masked)[0], 'r-' )
plt.show()
```

The P/B–ROE Model: Python and Calcbench – III

- Green line with slope 2.83 is robust fit, red line with slope .11 is OLS
  - Robust $R^2 = 31\%$, OLS $R^2 = 7\%$
  - Notice the huge impact of outliers and leverage points!
  - *Always use robust methods when working with statistical models*



The P/B–ROE Model – Conclusions

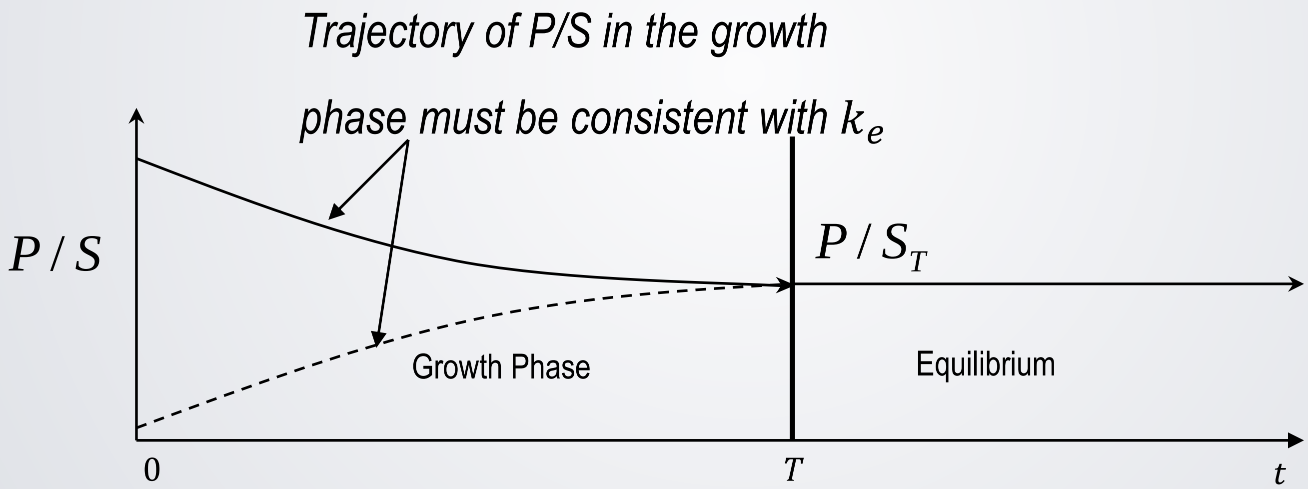
- Seems to work quite well across regions at the industry level, and also for stocks
- Robust regression consistently does better than OLS
- How can we improve measurements on both axes?
  - Have to think about distortions to book value
  - Use Operating Earnings, Filtered Earnings, or Earnings Before Extraordinary Items instead of As-Reported Earnings when measuring ROE
  - How can we filter noise uniformly across all firms
- But this needs data from databases, and lots of data cleansing…

The P/S–Net Margin Model – I

- The P/B-ROE model jointly evolved the income statement and the balance sheet
- We next create a dynamic model that evolves only the income statement
- Additional Notation (we continue to assume that we start at time $t = 0$)
  - S_t: Revenues (or Sales) / period at time t
  - P/S_t: Price-to-Sales ratio at time t
  - g_t: Growth rate of revenues at time t
    - $g_t = g$ for $t < T$
    - $g_t = g_{eq}$ for $t \geq T$
  - E_t: Cumulative earnings process at time t
  - d'_t: *Rate* of dividend payments at time t divided by S_t (dividend yield on *revenues*)
    - $d'_t = d'$ for $t < T$
    - $d'_t = d'_{eq}$ for $t \geq T$
  - e_t: Instantaneous net margin at time t
    - $e_t = e$ for $t < T$
    - $e_t = e_{eq}$ for $t \geq T$

The P/S–Net Margin Model – II

- Consider the evolution of the income statement over time
  - At each point in time, it must be consistent with the cost of equity
  - In particular, e_t, g_t and P/S_t must all be consistent with k_e
  - Assume that realized returns = expected returns, both on price and on revenues



- If e_t is high, P/S_0 must be high, and must decline to its terminal value
- If e_t is low, P/S_0 must be low, and must increase to its terminal value
- From these stylized facts, we can determine P/S_0 and P_0

The P/S–Net Margin Model – III

- Total Return = Price Return + Dividend Income
- If all parameters are time invariant, Total Return $= k_e$
- Total Return $= \frac{\frac{\partial P_t}{\partial t}+\frac{\partial D_t}{\partial t}}{P_t} = \frac{1}{P_t} \cdot \frac{\partial P_t}{\partial t} + \frac{1}{P_t} \cdot \frac{\partial D_t}{\partial t} = k_e$ (25)
- But $P_t = S_t \cdot P/S_t$, so that

$$\frac{1}{P_t} \cdot \frac{\partial P_t}{\partial t} = \frac{1}{S_t} \cdot \frac{\partial S_t}{\partial t} + \frac{1}{P/S_t} \cdot \frac{\partial P/S_t}{\partial t} \tag{26}$$

- It follows that

$$k_e = \frac{1}{S_t} \cdot \frac{\partial S_t}{\partial t} + \frac{1}{P/S_t} \cdot \frac{\partial P/S_t}{\partial t} + \frac{1}{P_t} \cdot \frac{\partial D_t}{\partial t}$$

$$= \frac{1}{S_t} \cdot \frac{\partial S_t}{\partial t} + \frac{1}{P/S_t} \cdot \frac{\partial P/S_t}{\partial t} + \frac{1}{P/S_t} \cdot \frac{1}{S_t} \cdot \frac{\partial D_t}{\partial t} \tag{27}$$

The P/S–Net Margin Model – IV

- But $\frac{1}{S_t} \cdot \frac{\partial D_t}{\partial t} = d'_t$, the dividend yield on revenues and $\frac{1}{S_t} \cdot \frac{\partial S_t}{\partial t} = g_t$
- It follows that

$$k_e = g_t + \frac{1}{P/S_t} \cdot \left(\frac{\partial P/S_t}{\partial t} + d'_t\right) \quad (28)$$

$$\Longrightarrow \frac{\partial P/S_t}{\partial t} = P/S_t \cdot (k_e - g_t) - d'_t \quad (29)$$

- This is a first order ordinary differential equation!
  - In the steady state, when $d'_t = d'_{eq} \forall t \geq T$, the left-hand side is 0. It follows that

$$P/S_{eq} = \frac{d'_{eq}}{k_e - g_{eq}} \quad (30)$$

- Empirically, P/S tends to vary between 0.5 and 2.5 for the US stock market

The P/S–Net Margin Model – V

- Sanity check 1: this looks identical what we got from the P/B-ROE Model!

$$P/B_{eq} = \frac{d_{eq}}{k_e - g_{eq}} \tag{31}$$

- The solution to this ODE is

$$P/S_t = P/S_0 \cdot e^{(k_e-g)t} + \frac{d'}{k_e-g}\left(1 - e^{(k_e-g)t}\right) \tag{32}$$

- Sanity check 2:

$$\frac{\partial P/S_t}{\partial t} = (k_e - g) \cdot \frac{P}{S_0} \cdot e^{(k_e-g)t} - d' \cdot e^{(k_e-g)t}$$

$$= (k_e - g) \cdot \left[\frac{P}{S_0} \cdot e^{(k_e-g)t} + \frac{d'}{k_e-g}\left(1 - e^{(k_e-g)t}\right)\right] - d' \; ☑ \tag{33}$$

The P/S–Net Margin Model – VI

- Next, consider what happens as we hit the boundary at T:

$$P/S_T = P/S_0 \cdot e^{(k_e - g)T} + \frac{d'}{k_e - g}\left(1 - e^{(k_e - g)T}\right) \tag{34}$$

$$\approx P/S_0 \cdot e^{(k_e - g)T} - \frac{d' \cdot (k_e - g)T}{k_e - g} \tag{35}$$

- We now have to switch gears from the P/B-ROE model
  - Assume that revenue growth is proportional to retained earnings $g_t = c \cdot (e_t - d_t')$ for some constant c

- In the P/B-ROE model $c = 1$ by definition
  - Each dollar of retained earnings grows book value by exactly \$1
  - But this is not necessarily true for revenues!
  - The ratio of marginal sales to retained earnings must be determined empirically

The P/S–Net Margin Model – VII

- If $d' = 0$, i.e. if the firm retains all its earnings,

  $g_t = c \cdot e_t$, and

  $$P/S_T \longrightarrow P/S_0 \cdot e^{(k_e - c \cdot e_t)T} \qquad (36)$$

- This is a reasonable approximation, as we are not just discarding the last term
  - We account for its removal by suitably adjusting the exponent in the first term

- Finally, take logs on both sides and rearrange terms to get

  $$ln(P/S_0) = ln(P/S_T) + (g_t - k_e) \cdot T \qquad (37)$$

  or equivalently

  $$ln(P/S_0) = (ln(P/S_T) - k_e \cdot T) + c \cdot e_t \cdot T \qquad (38)$$

The P/S–Net Margin Model – VIII

- Equations (35) and (36) can be estimated cross-sectionally using linear regression!

$$ln\left(P/S_i^0\right) = \alpha + \beta \cdot (g_i - k_i^e) + \varepsilon_i \quad (39)$$

$$ln\left(P/S_i^0\right) = \gamma + \delta \cdot e_i + \varepsilon_i \quad (40)$$

- Use (37) if you have individual estimates of k_e and g_i
  - E.g. Estimate k_e using k_d + Equity premium vs. corporate bonds
  - Also, estimate g_i using time series models or get analysts forecasts of revenue growth
  - α is an estimate of $ln(P/S_T)$, while β is an estimate of T

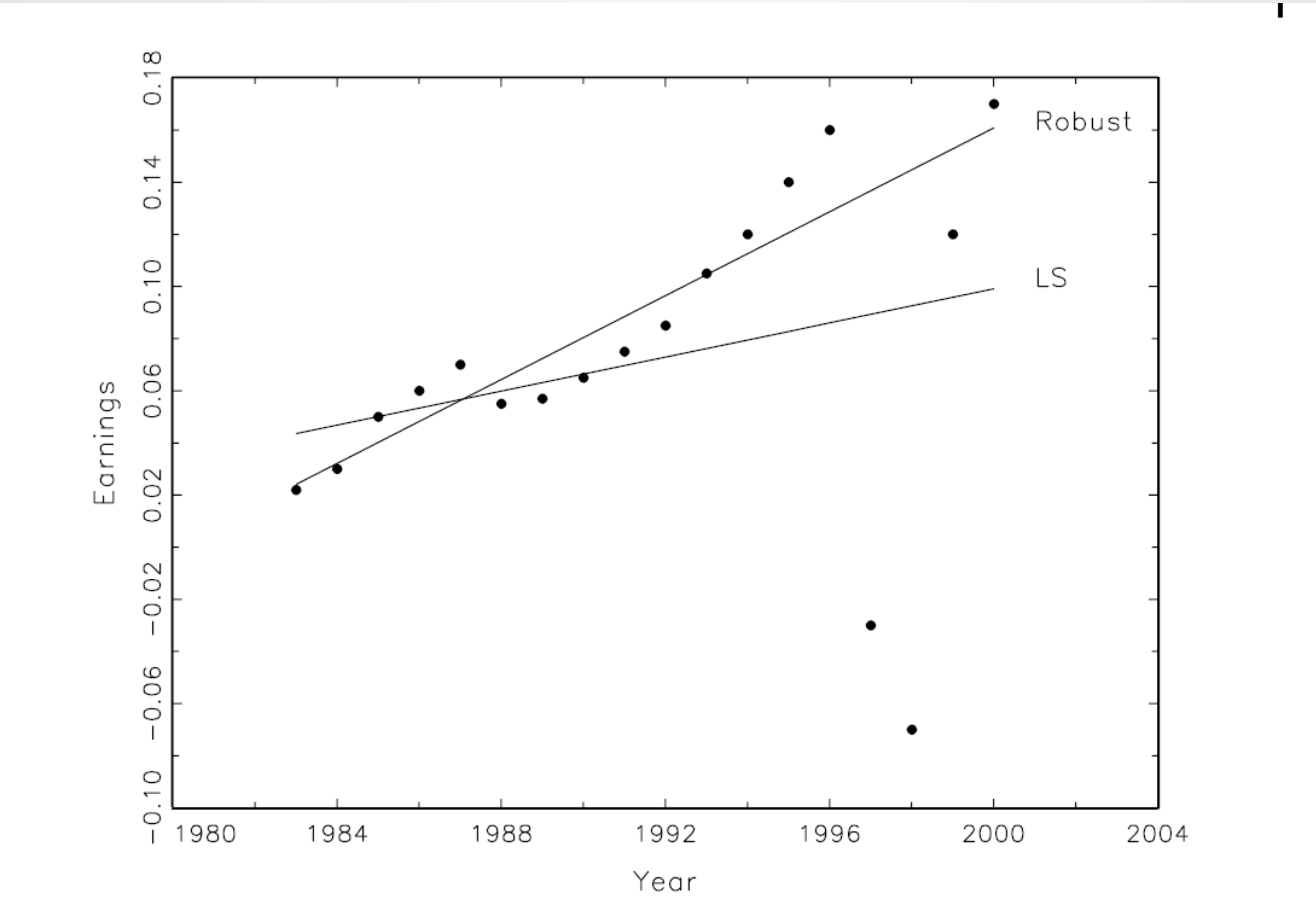
- Use (38) if you have data on e_i and cannot get an estimate of k_e
  - γ is harder to interpret, while δ is now an estimate of $c \cdot T$

The P/S–Net Margin Model – IX

- Get data for P/S and Net Margin for industries from Professor Damodaran's website at
  http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datacurrent.html#multiples
  - You can get data for the US, EU, JP, EM, CN, IN, Global
- Fewer insights than P/B-ROE, but still allows us to do relative valuations
- These valuation models are completely orthogonal to the standard valuation models presented in most textbooks, and are particularly useful for private companies where only limited financial data is available

A Brief Summary of Robust Regression – I

- IVENSYS per-share earnings vs. time
  - Two outliers greatly reduce the estimated growth rate of earnings!
  - We saw similar impacts on IBM's β in 1987, and S&P 500 returns vs Operating Earnings



A Brief Summary of Robust Regression – II

- OLS identifies an optimal $\boldsymbol{\beta}$ that minimizes $\sum \epsilon_i^2$ (sum of squared errors)
  - $\widehat{\boldsymbol{\beta}} = \underset{\boldsymbol{\beta}}{\operatorname{argmin}} \sum_i (y_i - \boldsymbol{X}_i \boldsymbol{\beta})^2$ (41)
  - $\boldsymbol{X}_i$ is the i^{th} row of $\boldsymbol{X}$, the matrix of explanatory variables whose first column $= \mathbf{1}$

- Differentiate (41) w.r.t. each component of $\boldsymbol{\beta}$ and equate the gradient to **0** to get
  - $\sum_i \boldsymbol{x}_i \left(y_i - \boldsymbol{X}_i \widehat{\boldsymbol{\beta}}\right) = \mathbf{0}$ (42)

- Robust regression minimizes $\sum \rho(y_i - \boldsymbol{X}_i \boldsymbol{\beta})$ (sum of $\rho(\text{errors})$, ρ must be even)
  - $\widehat{\boldsymbol{\beta}} = \underset{\boldsymbol{\beta}}{\operatorname{argmin}} \sum_i \rho(y_i - \boldsymbol{X}_i \boldsymbol{\beta})$ (43)

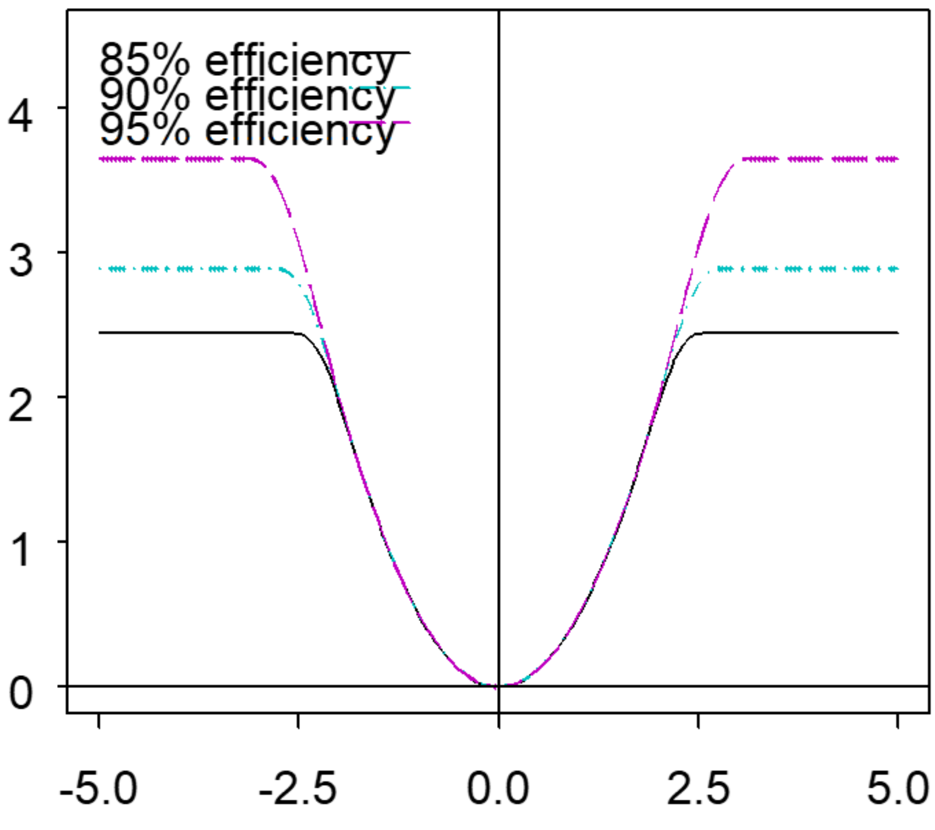
- Differentiate (43) w.r.t. each component of $\boldsymbol{\beta}$ and equate the gradient to **0** to get
  - $\sum_i \boldsymbol{x}_i \, \psi\left(y_i - \boldsymbol{X}_i \widehat{\boldsymbol{\beta}}\right) = \mathbf{0}$ (44)
  - $\psi = \rho'$ (45)

- The magic of robust regression lies in choosing ρ appropriately!

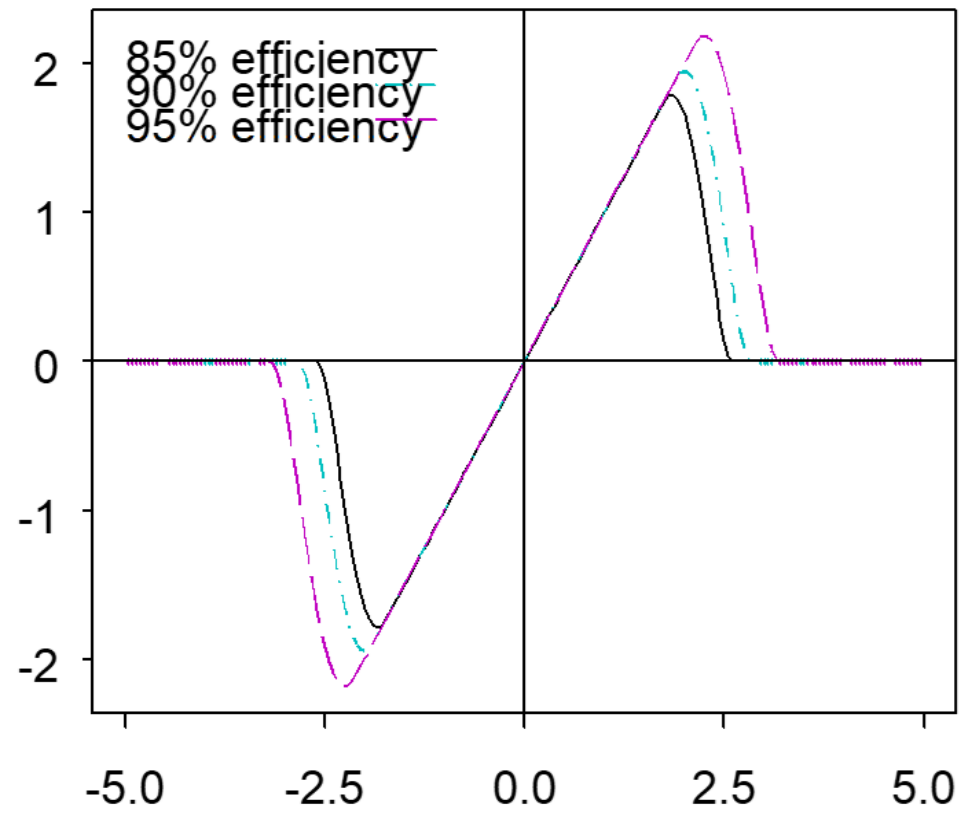
A Brief Summary of Robust Regression – III

- There is an optimal ρ (Yohai and Zamar (1997)), but its derivation is complex
  - Looks quadratic near 0, but levels off smoothly as we move away from the center
  - Such estimators are known as M-estimators (Maximum Likelihood Type Estimators)

Optimal ρ



Optimal $\psi = \frac{\partial \rho}{\partial \epsilon}$



Summary

- Relative valuation models are not very precise, but are very widely used
  - Even so, they can be useful, especially when used within an industry or sector
- Often used sloppily, with inappropriate comparables and classical statistics
  - Choose comps with care, use robust statistics, allow nonlinear relationships!
  - Combine outputs from univariate models, don't use multivariate models
- The P/B-ROE and P/S-Net margin models are dynamic models
  - Not as static as DCF, not as dynamic as the Brownian motions used in option pricing
- In spite of the many assumptions they make, they give us useful insights
  - At one time, they were powerful enough to identify mispriced securities!
- Markets have now become much more efficient
  - Makes them more useful as valuation models than as stock selection models