Announcements

No class on Thursday (Fall break). Reminder: drop deadline is Friday.

Next written homework will be released later today or tomorrow.
The HW will be due after Fall break on Thursday October 15th.

Yesterday's public safety announcement was scary.
If you're interested in thinking about how computer science can be used to develop better gun policy, take my NETS 213 class "Crowdsourcing and Human Computation" next semester.
http://crowdsourcing-class.org/

http://algs4.cs.princeton.edu

2.4 PRIORITY QUEUES

- *API and elementary implementations*
- *binary heaps*
- *heapsort*
- *event-driven simulation*

Last updated on 10/6/15 8:15 AM

2.4 Priority Queues

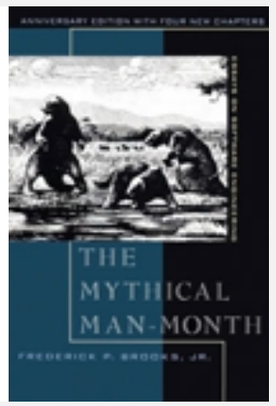
- *API and elementary implementations*
- *binary heaps*
- *heapsort*
- *event-driven simulation*

Collections

A collection is a data type that stores a group of items.

data type	core operations	data structure
stack	PUSH, POP	*linked list, resizing array*
queue	ENQUEUE, DEQUEUE	*linked list, resizing array*
priority queue	INSERT, DELETE-MAX	*binary heap*
symbol table	PUT, GET, DELETE	*binary search tree, hash table*
set	ADD, CONTAINS, DELETE	*binary search tree, hash table*

" Show me your code and conceal your data structures, and I shall continue to be mystified. Show me your data structures, and I won't usually need your code; it'll be obvious." — *Fred Brooks*



Priority queue

Collections. Insert and delete items. Which item to delete?

Stack. Remove the item most recently added.

Queue. Remove the item least recently added.

Randomized queue. Remove a random item.

Priority queue. Remove the largest (or smallest) item.

Generalizes: stack, queue, randomized queue.

operation	*argument*	*return value*
insert	P	
insert	Q	
insert	E	
remove max		Q
insert	X	
insert	A	
insert	M	
remove max		X
insert	P	
insert	L	
insert	E	
remove max		P

Priority queue API

Requirement. Items are generic; they must also be `Comparable`.

Key must be Comparable
(bounded type parameter)

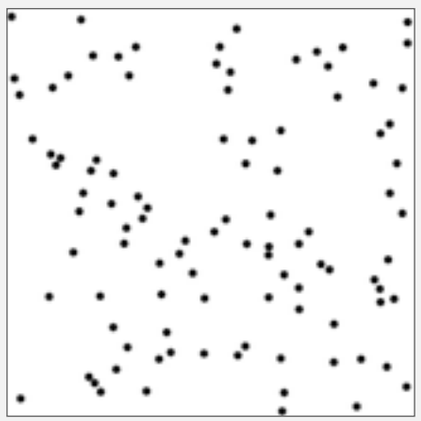
`public class MaxPQ<Key extends Comparable<Key>>`

	`MaxPQ()`	*create an empty priority queue*
	`MaxPQ(Key[] a)`	*create a priority queue with given keys*
`void`	`insert(Key v)`	*insert a key into the priority queue*
`Key`	`delMax()`	*return and remove a largest key*
`boolean`	`isEmpty()`	*is the priority queue empty?*
`Key`	`max()`	*return a largest key*
`int`	`size()`	*number of entries in the priority queue*

Note. Duplicate keys allowed; `delMax()` picks any maximum key.

Priority queue: applications

- Event-driven simulation. [customers in a line, colliding particles]
- Numerical computation. [reducing roundoff error]
- Discrete optimization. [bin packing, scheduling]
- Artificial intelligence. [A* search]
- Computer networks. [web cache]
- Operating systems. [load balancing, interrupt handling]
- Data compression. [Huffman codes]
- Graph searching. [Dijkstra's algorithm, Prim's algorithm]
- Number theory. [sum of powers]
- Spam filtering. [Bayesian spam filter]
- Statistics. [online median in data stream]



Priority queue: client example

Challenge. Find the largest M items in a stream of N items, where N>>M.

- Fraud detection: isolate $$ transactions.
- NSA monitoring: flag most suspicious documents.

N huge, M large

Constraint. Not enough memory to store N items.

Transaction data type is Comparable (ordered by $$)

use a min-oriented pq

```
MinPQ<Transaction> pq = new MinPQ<Transaction>();

while (StdIn.hasNextLine())
{
   String line = StdIn.readLine();
   Transaction transaction = new Transaction(line);
   pq.insert(transaction);
   if (pq.size() > M)
      pq.delMin();
}
```

pq now contains largest M items

Priority queue: client example

Challenge. Find the largest M items in a stream of N items, where N>>M.

implementation	time	space
sort	$N \log N$	N
elementary PQ	$M\,N$	M
binary heap	$N \log M$	M
best in theory	N	M

order of growth of finding the largest M in a stream of N items

Priority queue: unordered and ordered array implementation

operation	*argument*	*return value*	*size*	*contents (unordered)*							*contents (ordered)*						
insert	P		1	P							P						
insert	Q		2	P	Q						P	Q					
insert	E		3	P	Q	E					E	P	Q				
remove max		Q	2	P	E						E	P					
insert	X		3	P	E	X					E	P	X				
insert	A		4	P	E	X	A				A	E	P	X			
insert	M		5	P	E	X	A	M			A	E	M	P	X		
remove max		X	4	P	E	M	A				A	E	M	P			
insert	P		5	P	E	M	A	P			A	E	M	P	P		
insert	L		6	P	E	M	A	P	L		A	E	L	M	P	P	
insert	E		7	P	E	M	A	P	L	E	A	E	E	L	M	P	P
remove max		P	6	E	M	A	P	L	E		A	E	E	L	M	P	

A sequence of operations on a priority queue

Priority queue: implementations cost summary

Challenge. Implement all operations efficiently.

implementation	insert	del max	max
unordered array	1	N	N
ordered array	N	1	1
goal	$\log N$	$\log N$	$\log N$

order of growth of running time for priority queue with N items

http://algs4.cs.princeton.edu

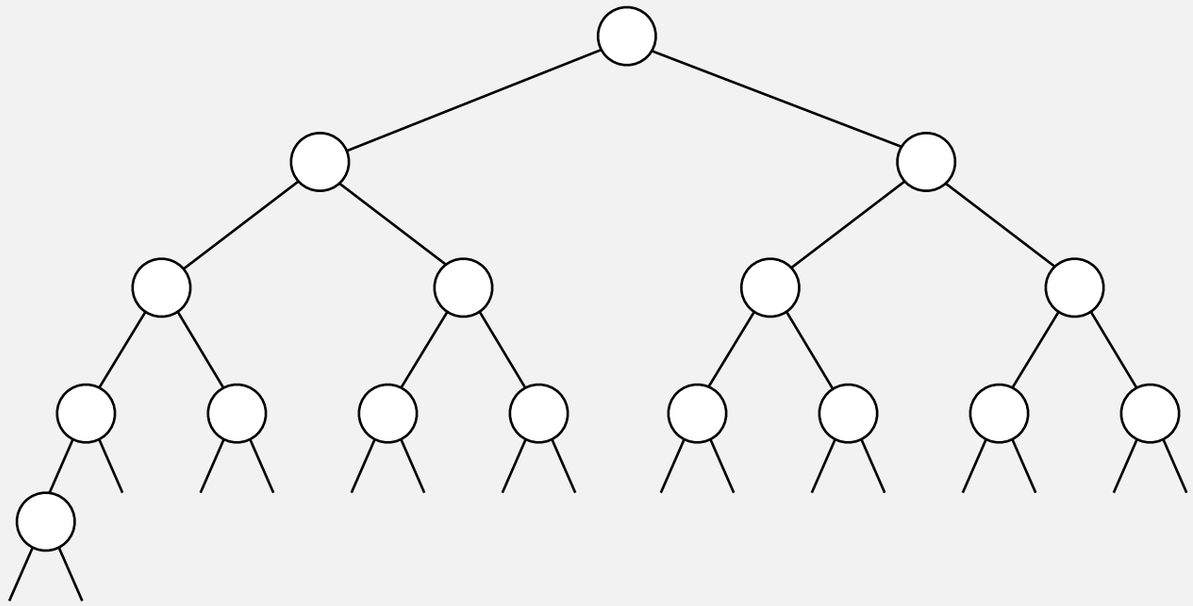
2.4 Priority Queues

- *API and elementary implementations*
- *binary heaps*
- *heapsort*
- *event-driven simulation*

Complete binary tree

Binary tree. Empty or node with links to left and right binary trees.

Complete tree. Perfectly balanced, except for bottom level.



complete binary tree with N = 16 nodes (height = 4)

Property. Height of complete binary tree with N nodes is $\lfloor \lg N \rfloor$.

Pf. Height increases only when N is a power of 2.

A complete binary tree in nature

Binary heap: representation

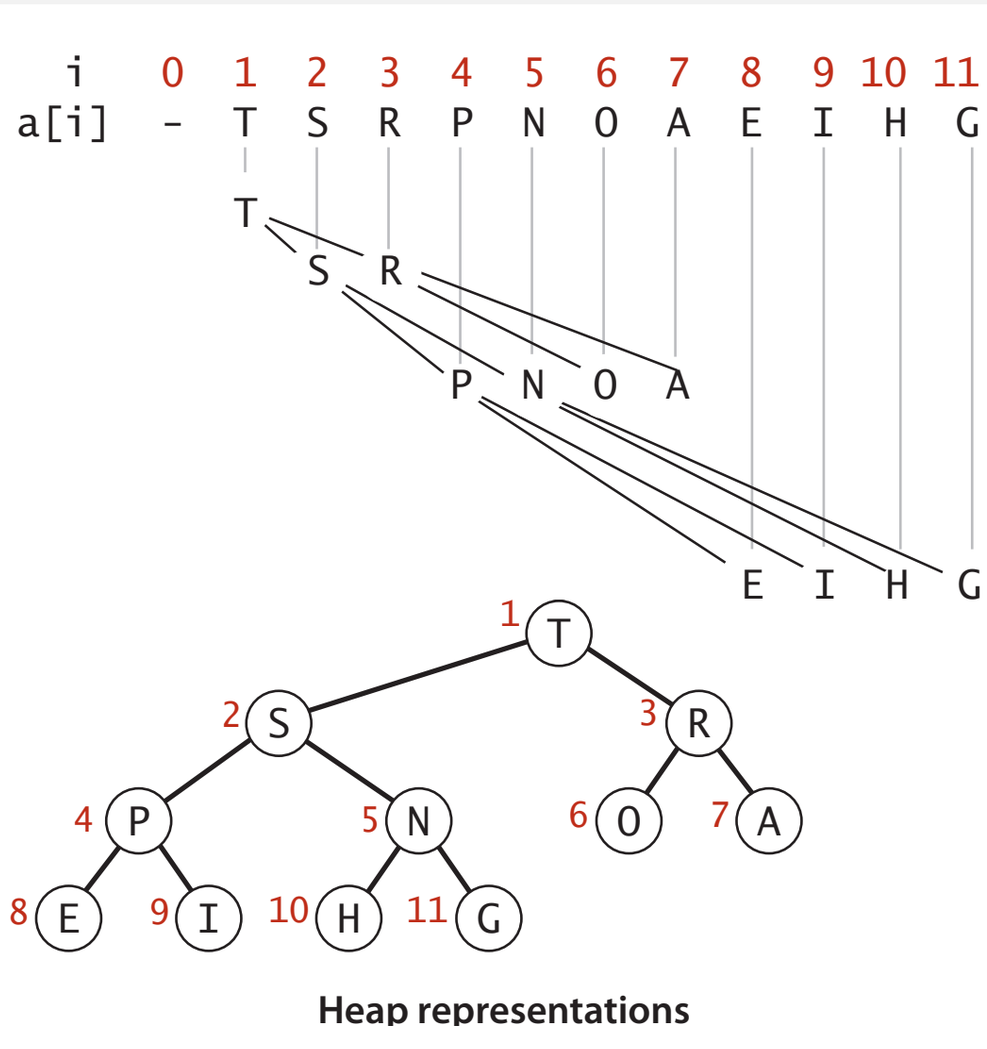
Binary heap. Array representation of a heap-ordered complete binary tree.

Heap-ordered binary tree.

- Keys in nodes.
- Parent's key no smaller than children's keys.

Array representation.

- Indices start at 1.
- Take nodes in level order.
- No explicit links needed!



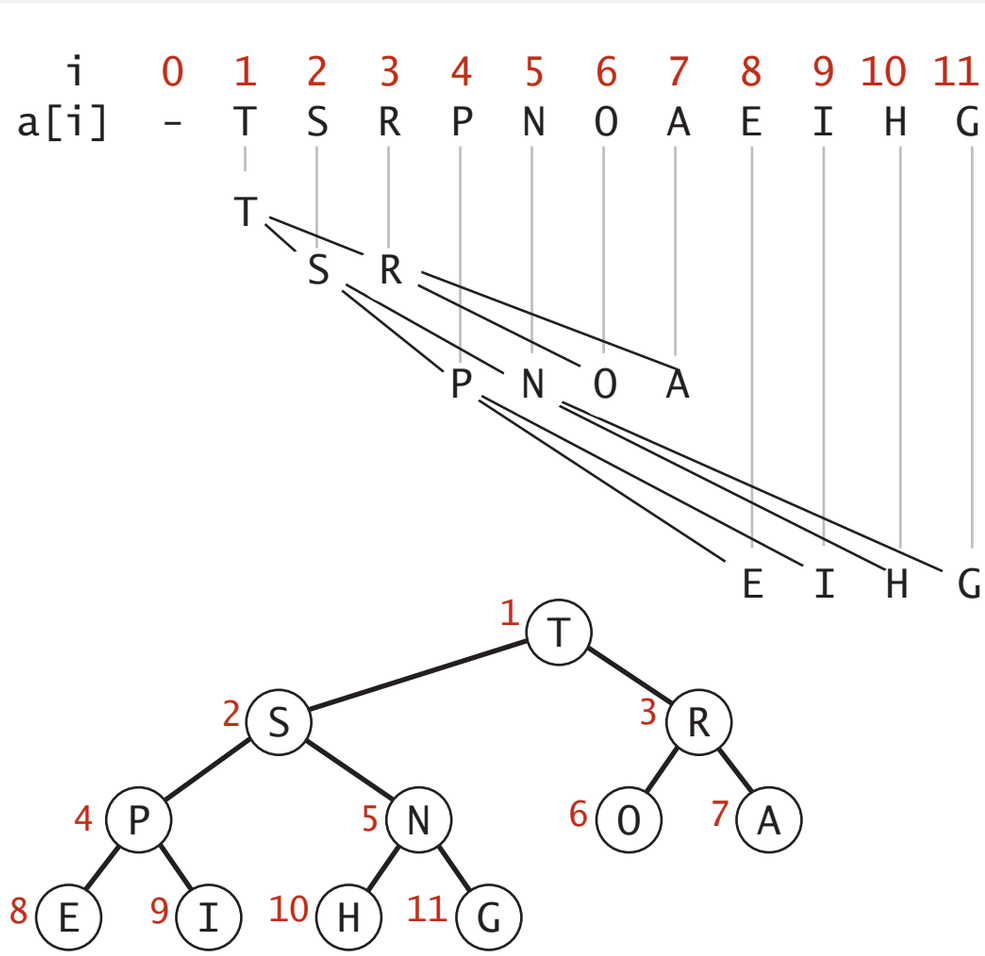
Heap representations

Binary heap: properties

Proposition. Largest key is `a[1]`, which is root of binary tree.

Proposition. Can use array indices to move through tree.

- Parent of node at `k` is at `k/2`.
- Children of node at `k` are at `2k` and `2k+1`.



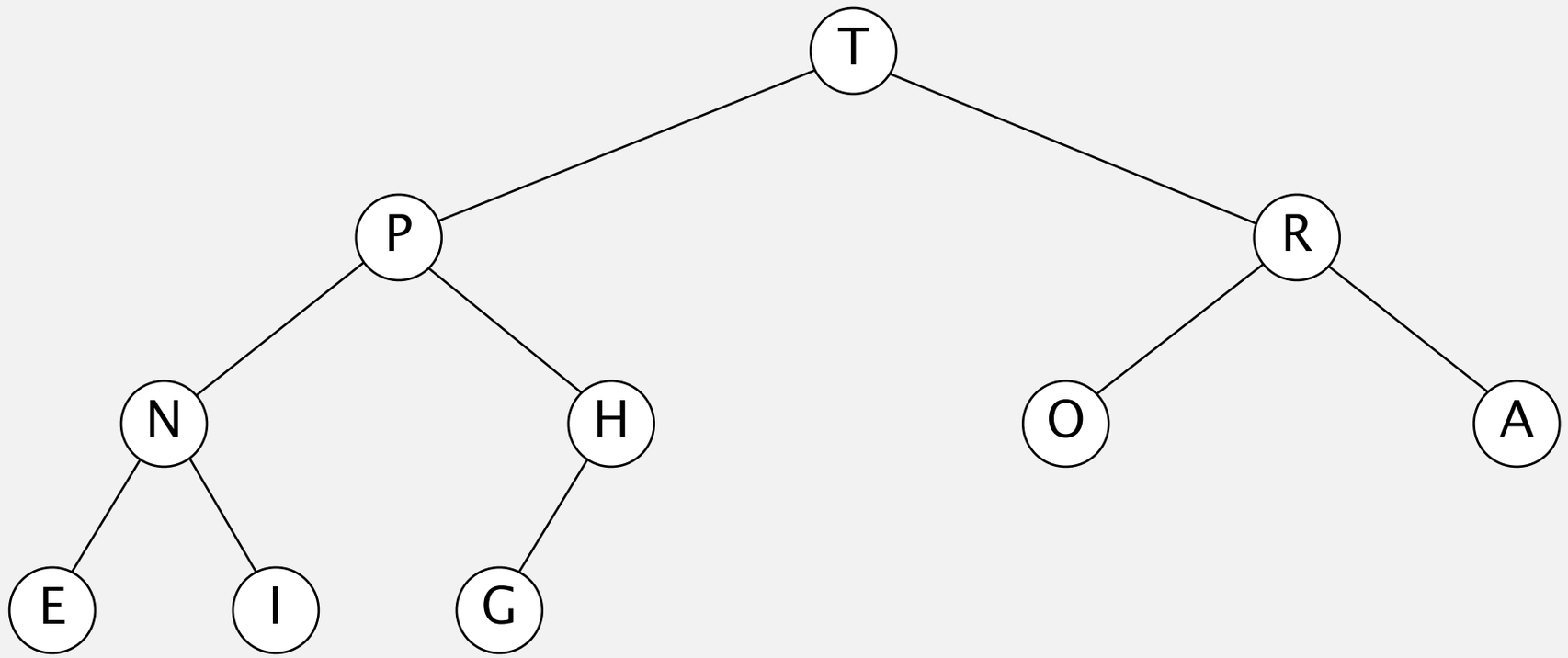
Heap representations

Binary heap demo

Insert. Add node at end, then swim it up.

Remove the maximum. Exchange root with node at end, then sink it down.

heap ordered



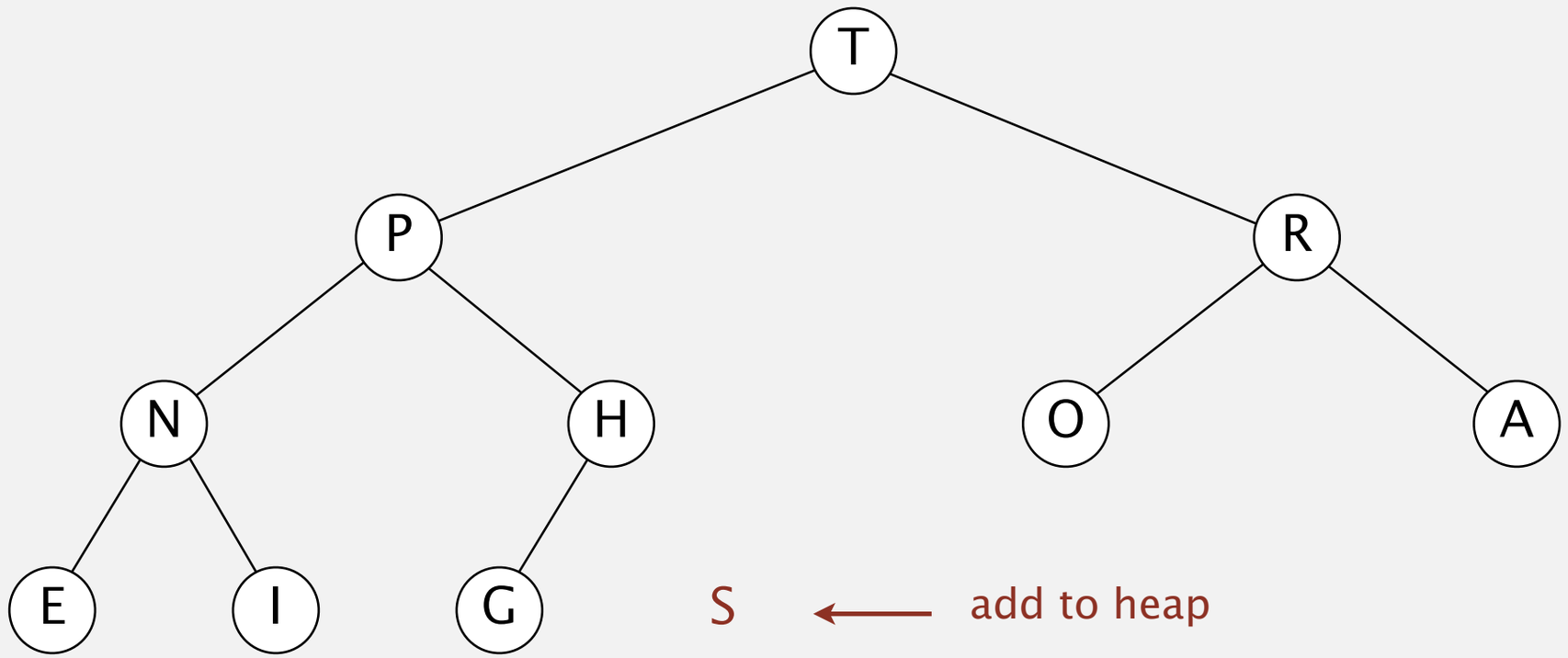
T	P	R	N	H	O	A	E	I	G	

Binary heap demo

Insert. Add node at end, then swim it up.

Remove the maximum. Exchange root with node at end, then sink it down.

insert S



T	P	R	N	H	O	A	E	I	G	

Binary heap demo

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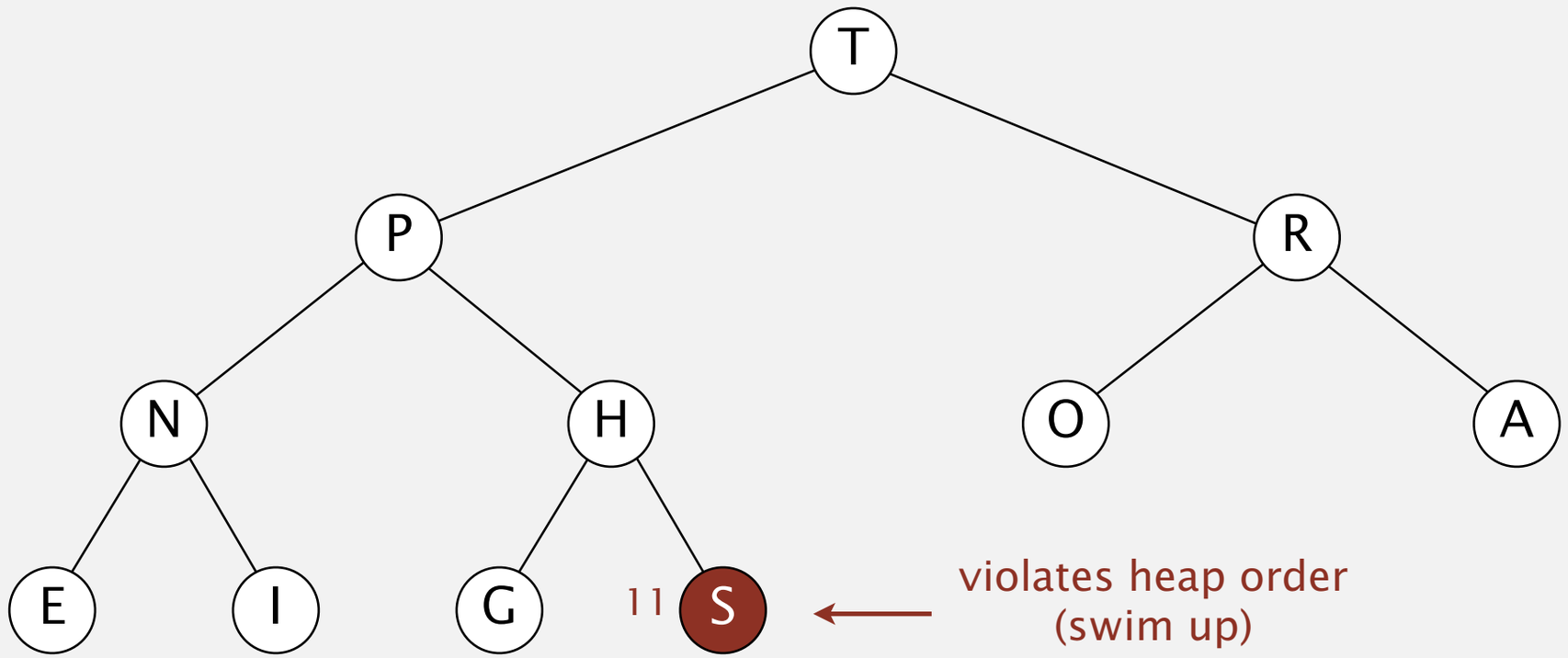

T	P	R	N	H	O	A	E	I	G	S
										11

Binary heap demo

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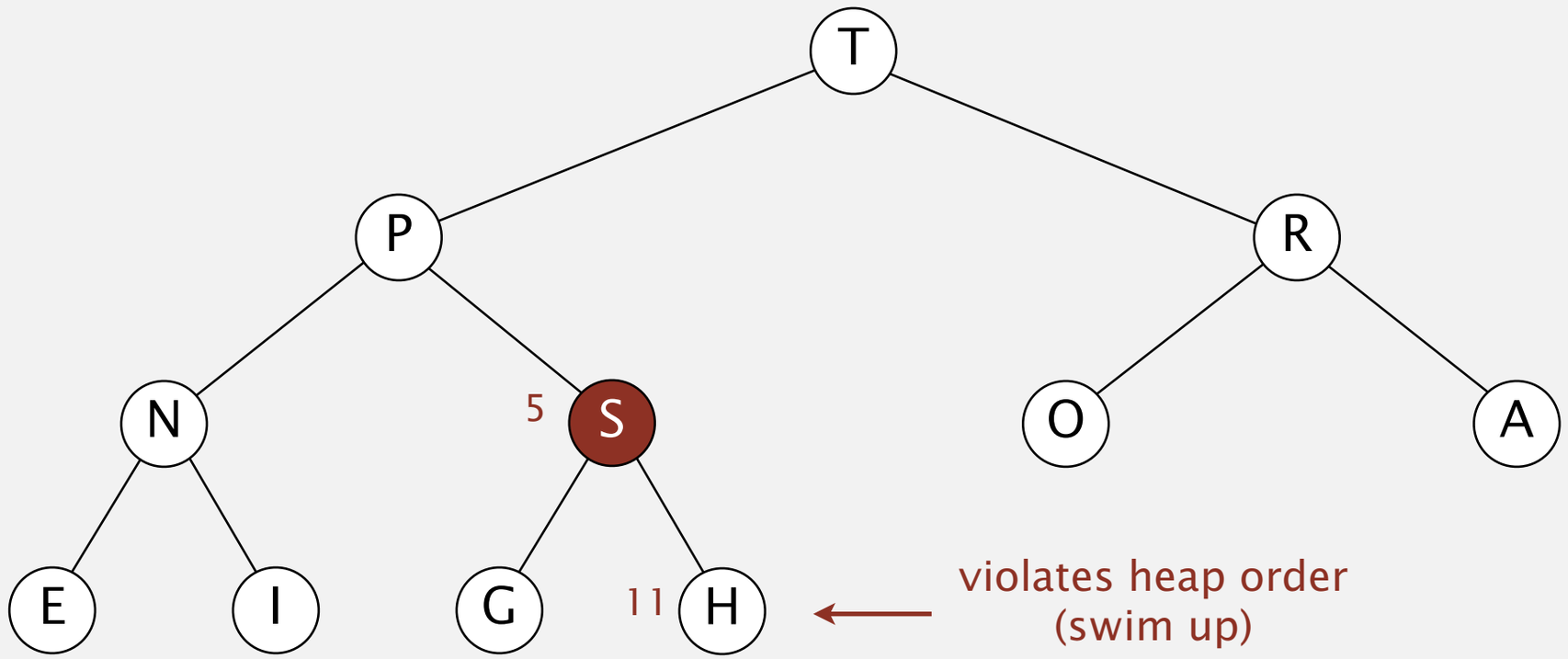
insert S



T	P	R	N	S	O	A	E	I	G	H
				5						11

Binary heap demo

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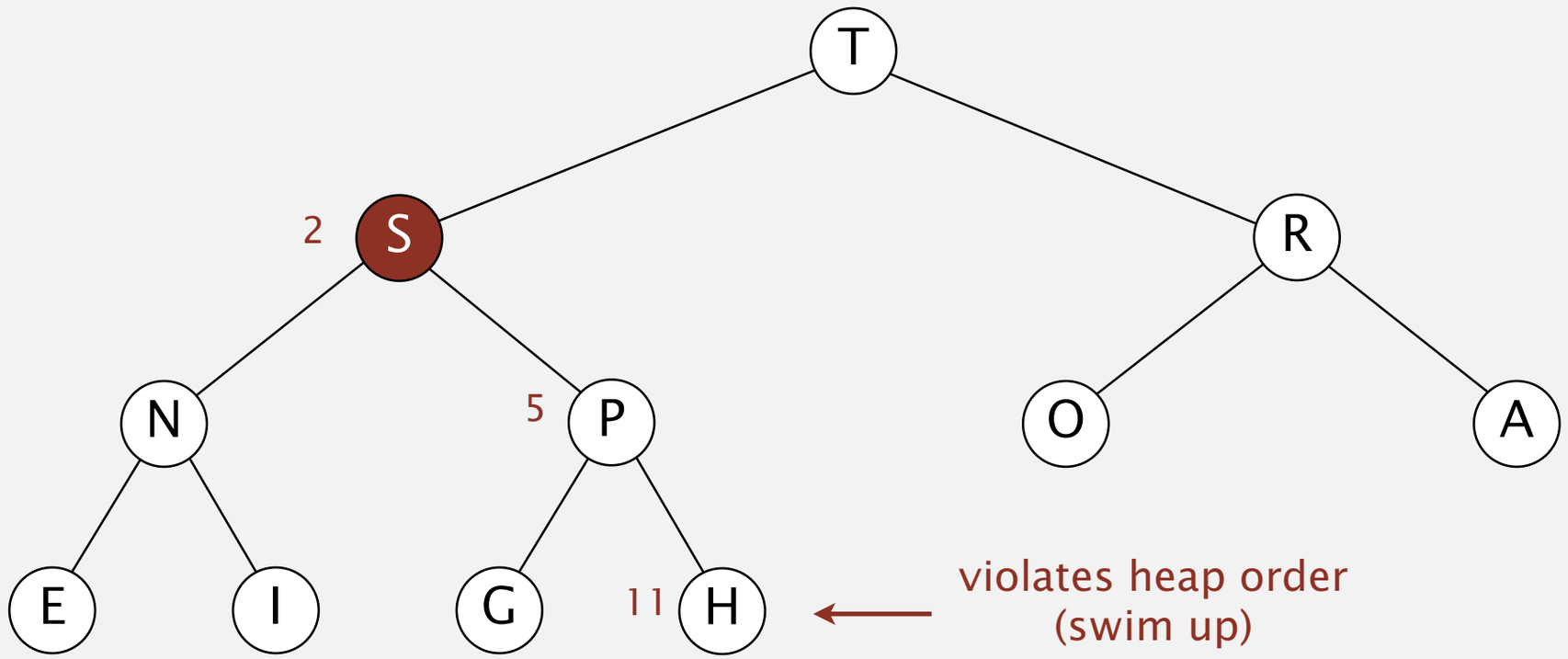
insert S



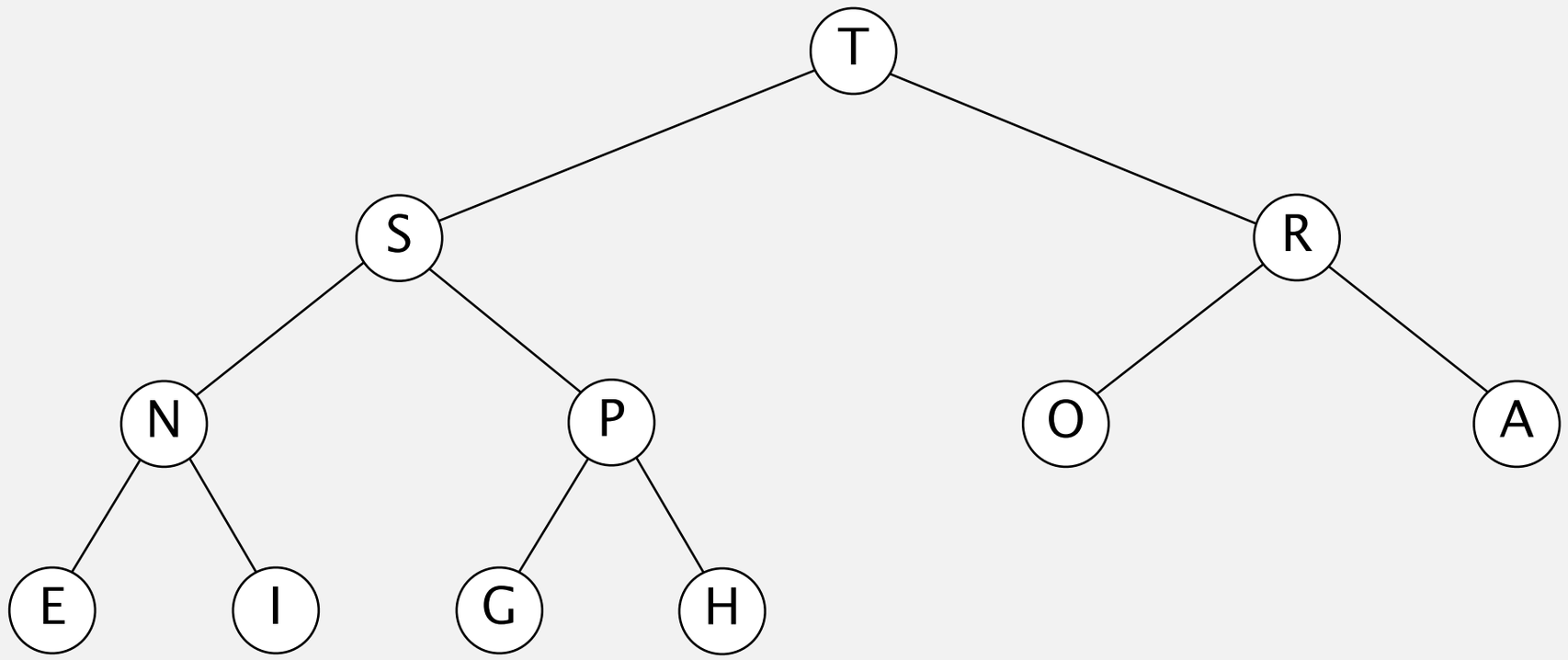
T	S	R	N	P	O	A	E	I	G	H
	2			5						11

Binary heap demo

Insert. Add node at end, then swim it up.

Remove the maximum. Exchange root with node at end, then sink it down.

heap ordered



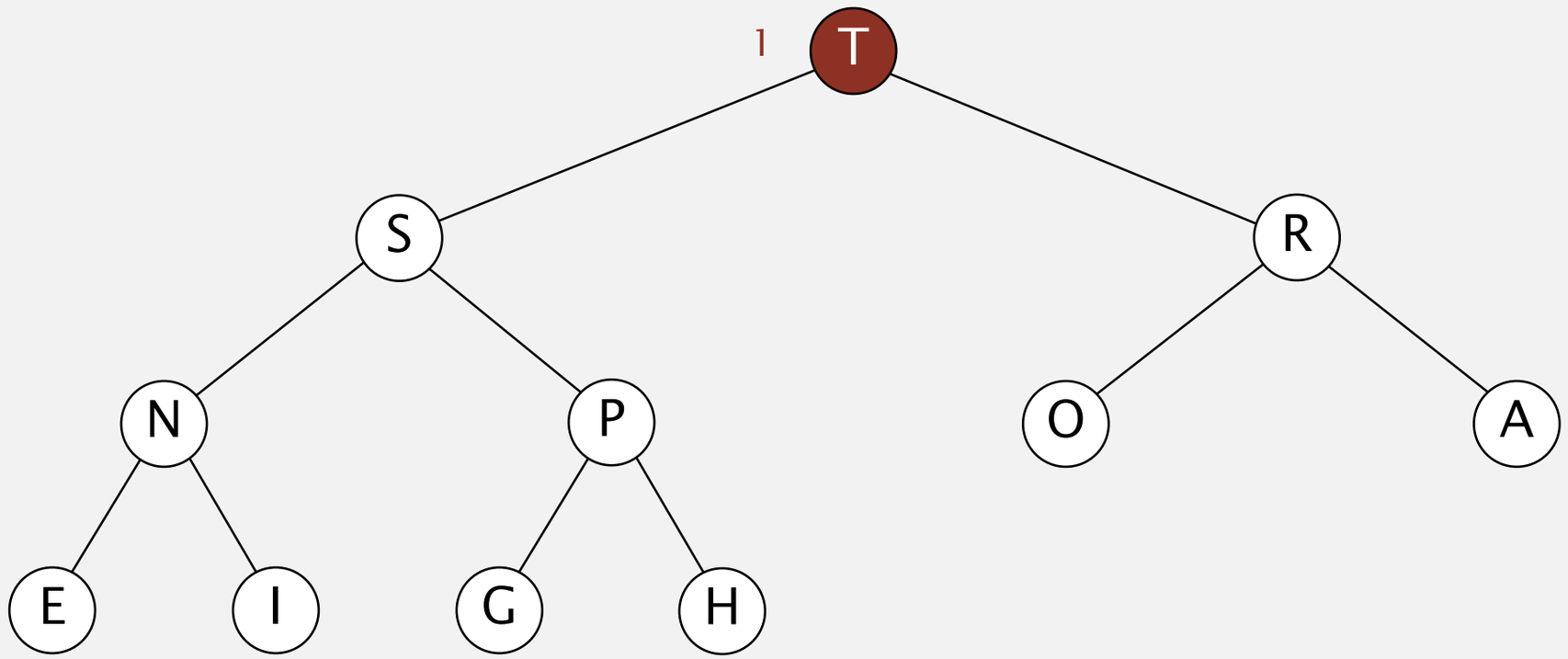
T	S	R	N	P	O	A	E	I	G	H

Binary heap demo

Insert. Add node at end, then swim it up.

Remove the maximum. Exchange root with node at end, then sink it down.

remove the maximum

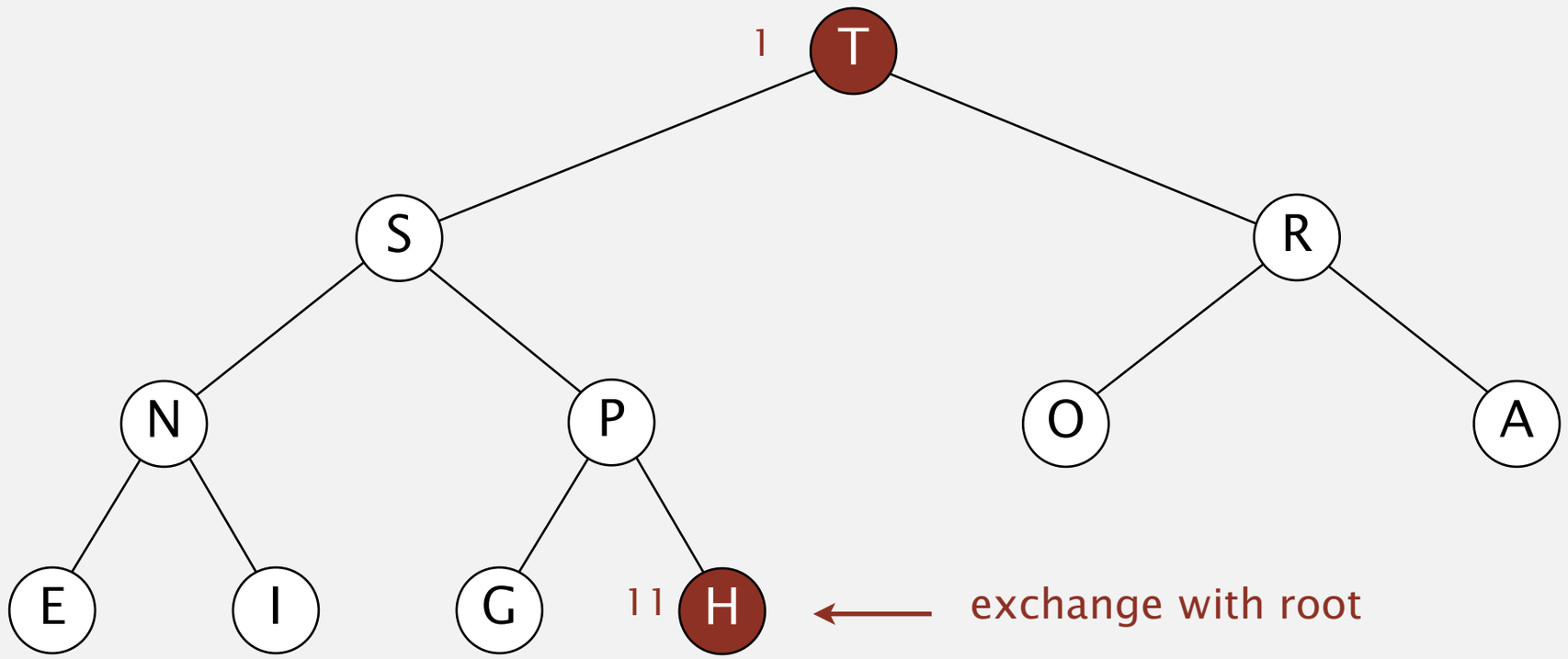


T	S	R	N	P	O	A	E	I	G	H
1										

Binary heap demo

Insert. Add node at end, then swim it up.

Remove the maximum. Exchange root with node at end, then sink it down.



T	S	R	N	P	O	A	E	I	G	H
1										11

Binary heap demo

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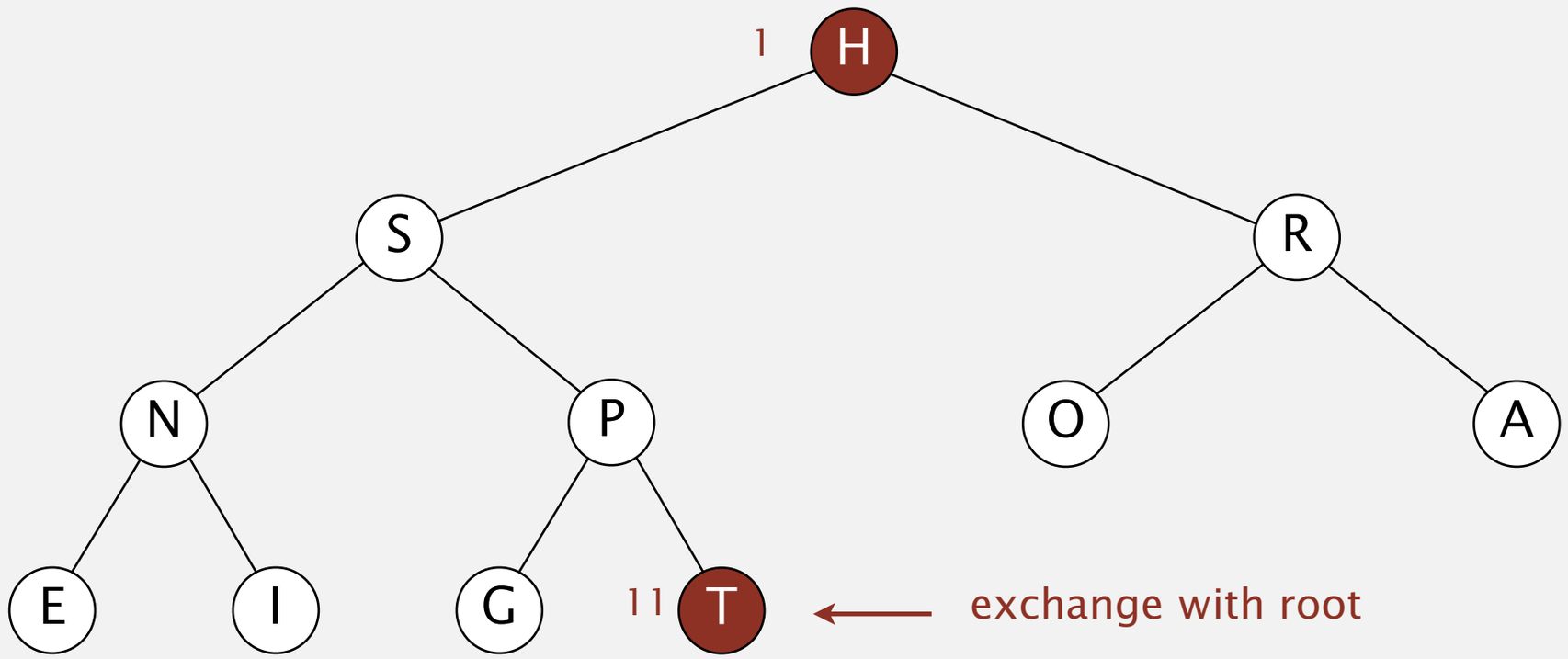

H	S	R	N	P	O	A	E	I	G	T
1										11

Binary heap demo

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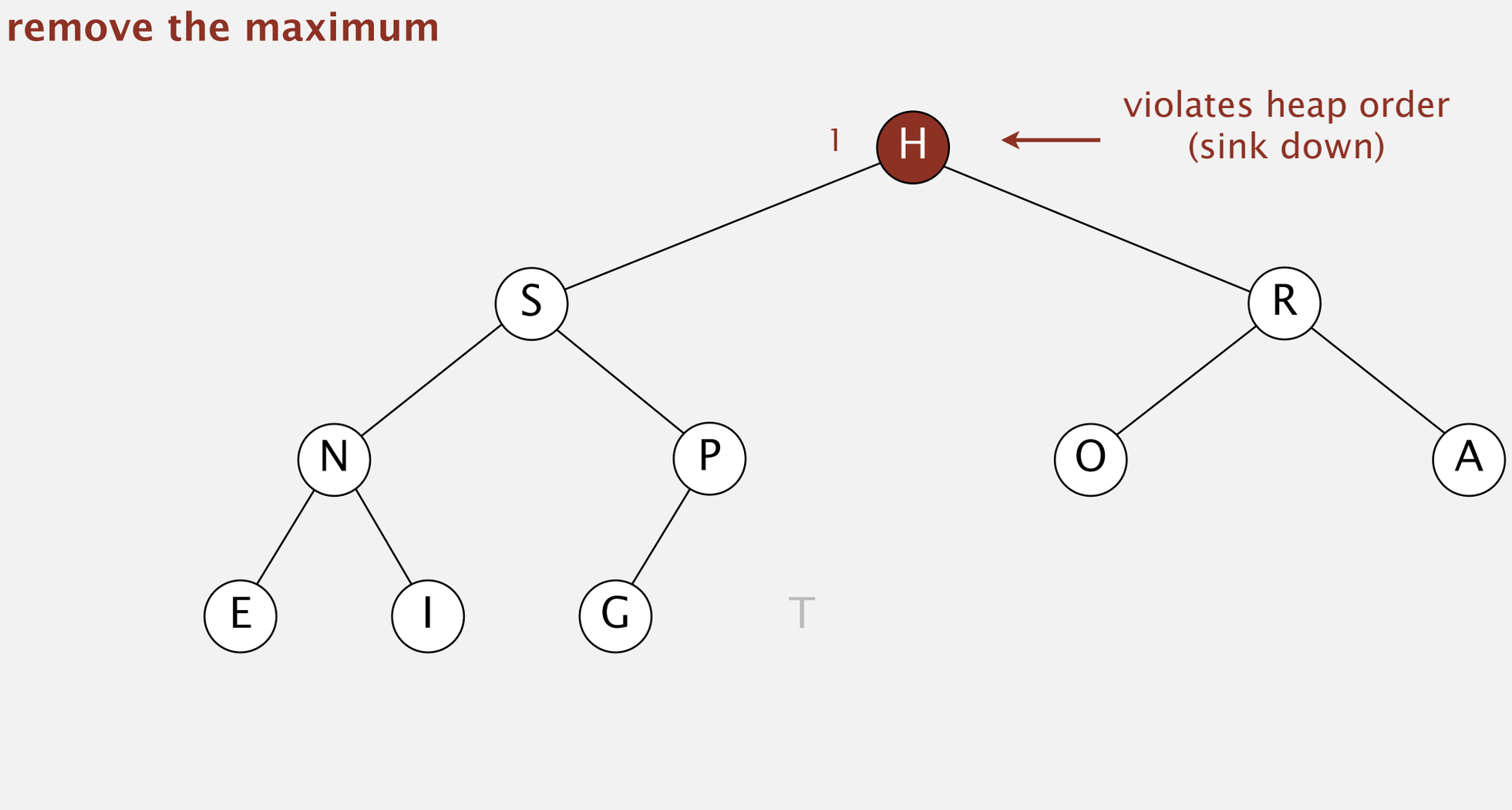

H	S	R	N	P	O	A	E	I	G	T
1										

Binary heap demo

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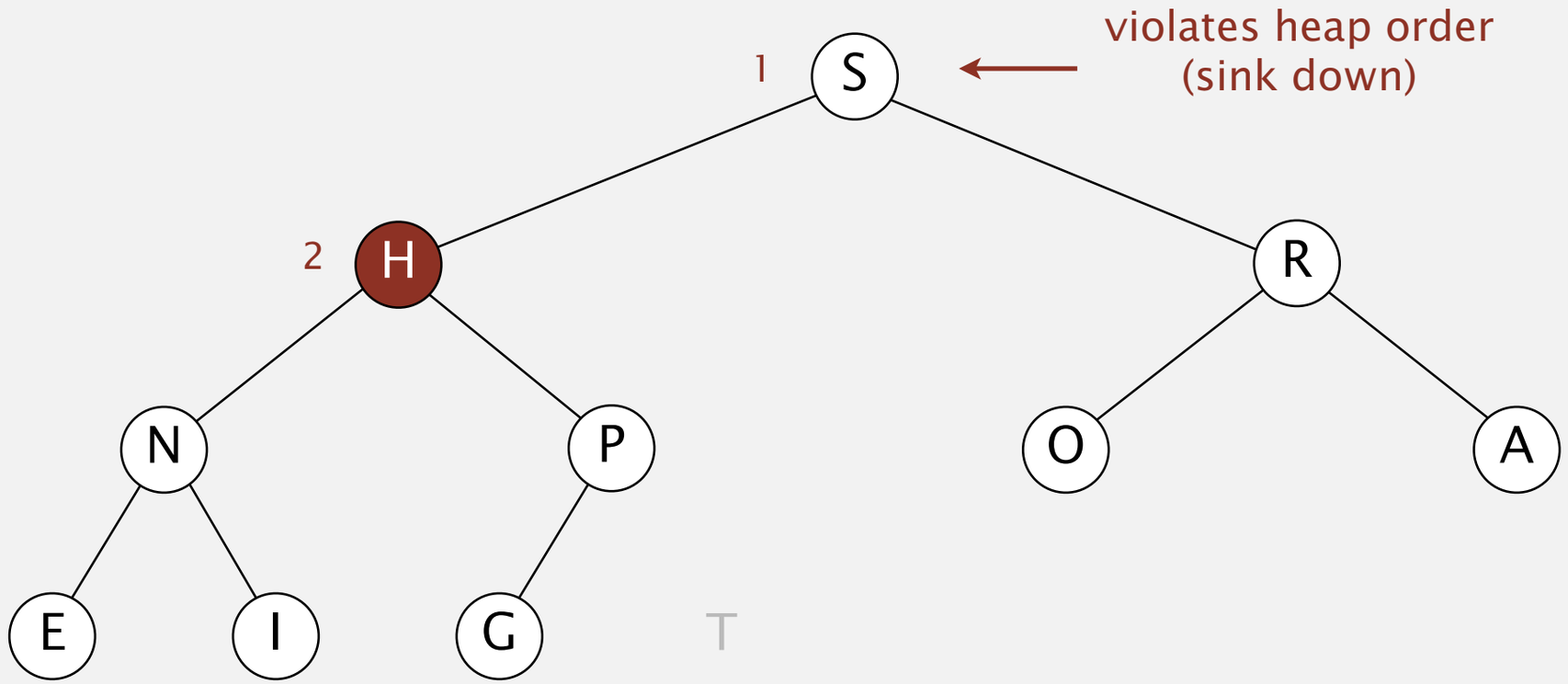
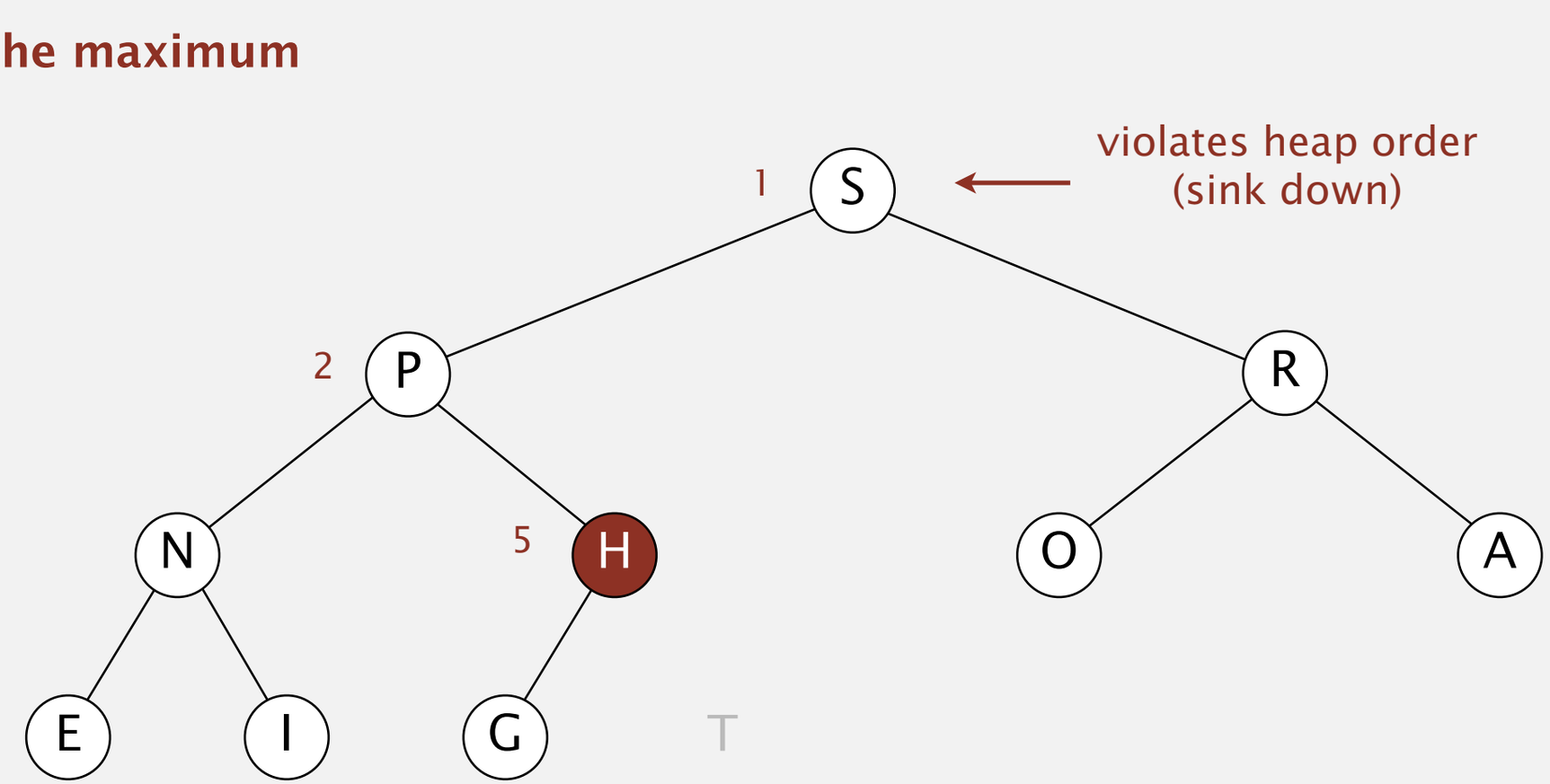


S	H	R	N	P	O	A	E	I	G	T
1	2									

Binary heap demo

Insert. Add node at end, then swim it up.

Remove the maximum. Exchange root with node at end, then sink it down.



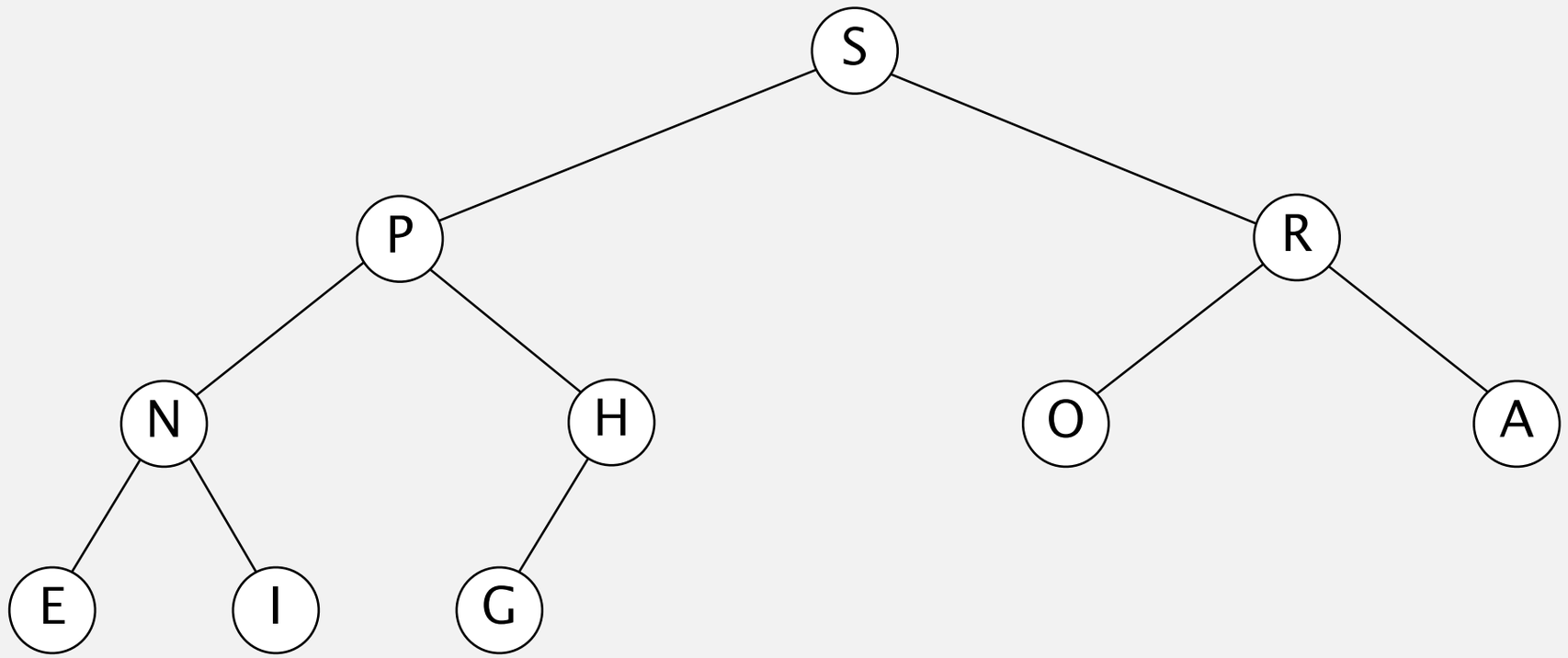
S	P	R	N	H	O	A	E	I	G	T
1	2			5						

Binary heap demo

Insert. Add node at end, then swim it up.

Remove the maximum. Exchange root with node at end, then sink it down.

heap ordered



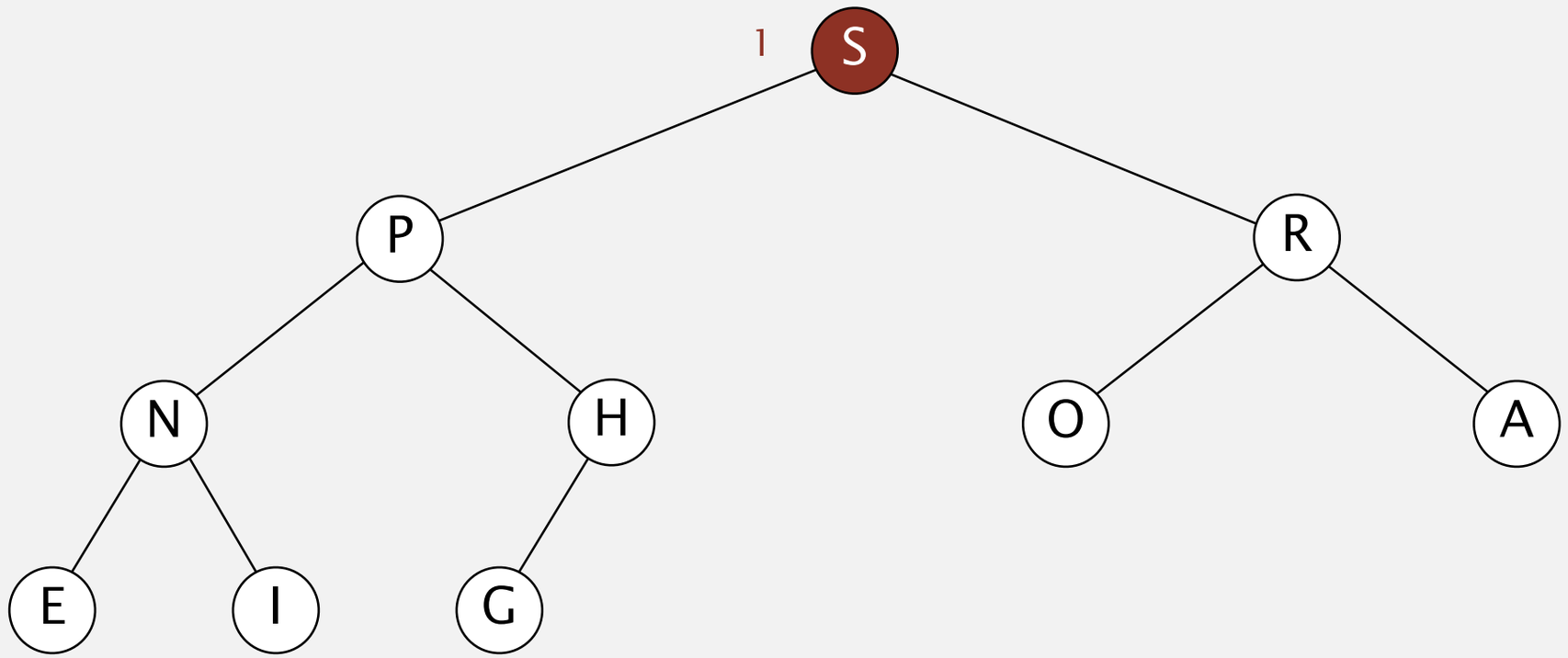
S	P	R	N	H	O	A	E	I	G	

Binary heap demo

Insert. Add node at end, then swim it up.

Remove the maximum. Exchange root with node at end, then sink it down.

remove the maximum



S	P	R	N	H	O	A	E	I	G	
1										

Binary heap demo

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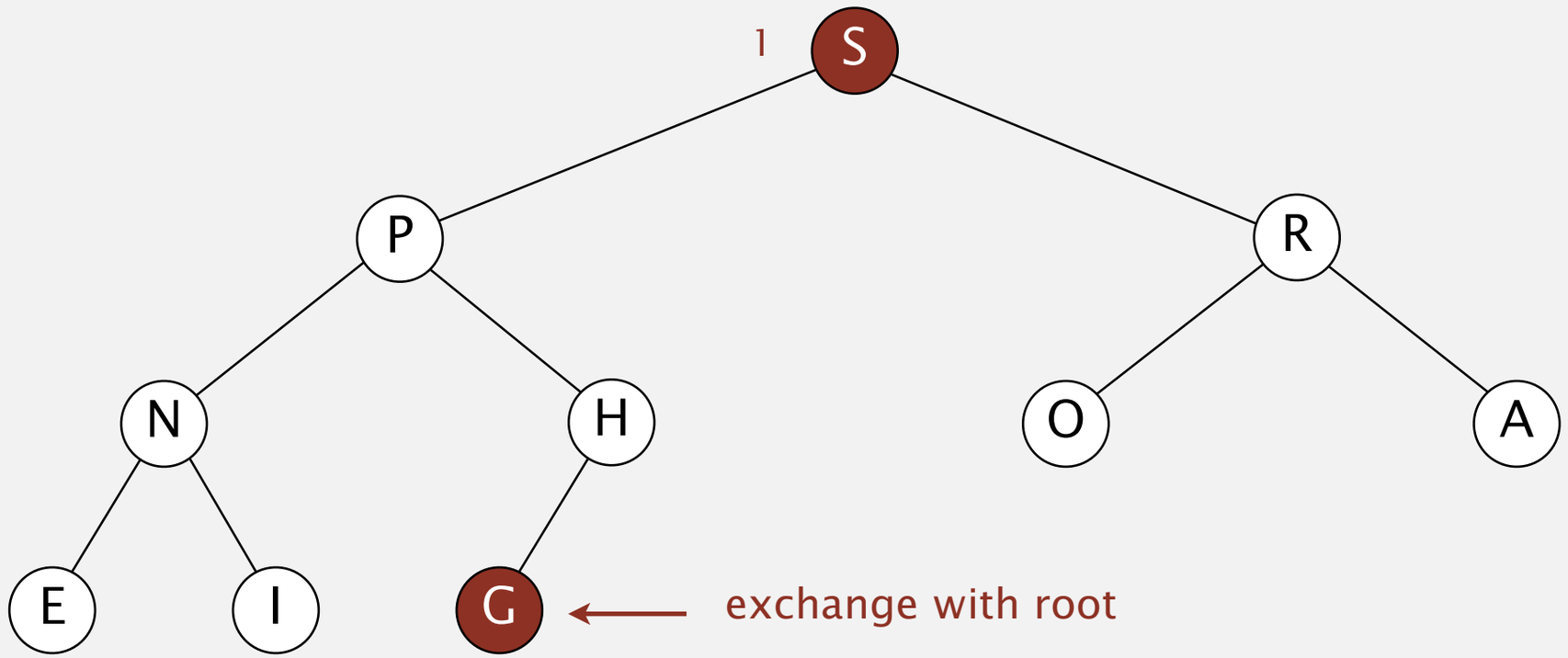
Remove the maximum. Exchange root with node at end, then sink it down.



S	P	R	N	H	O	A	E	I	G	
1										

Binary heap demo

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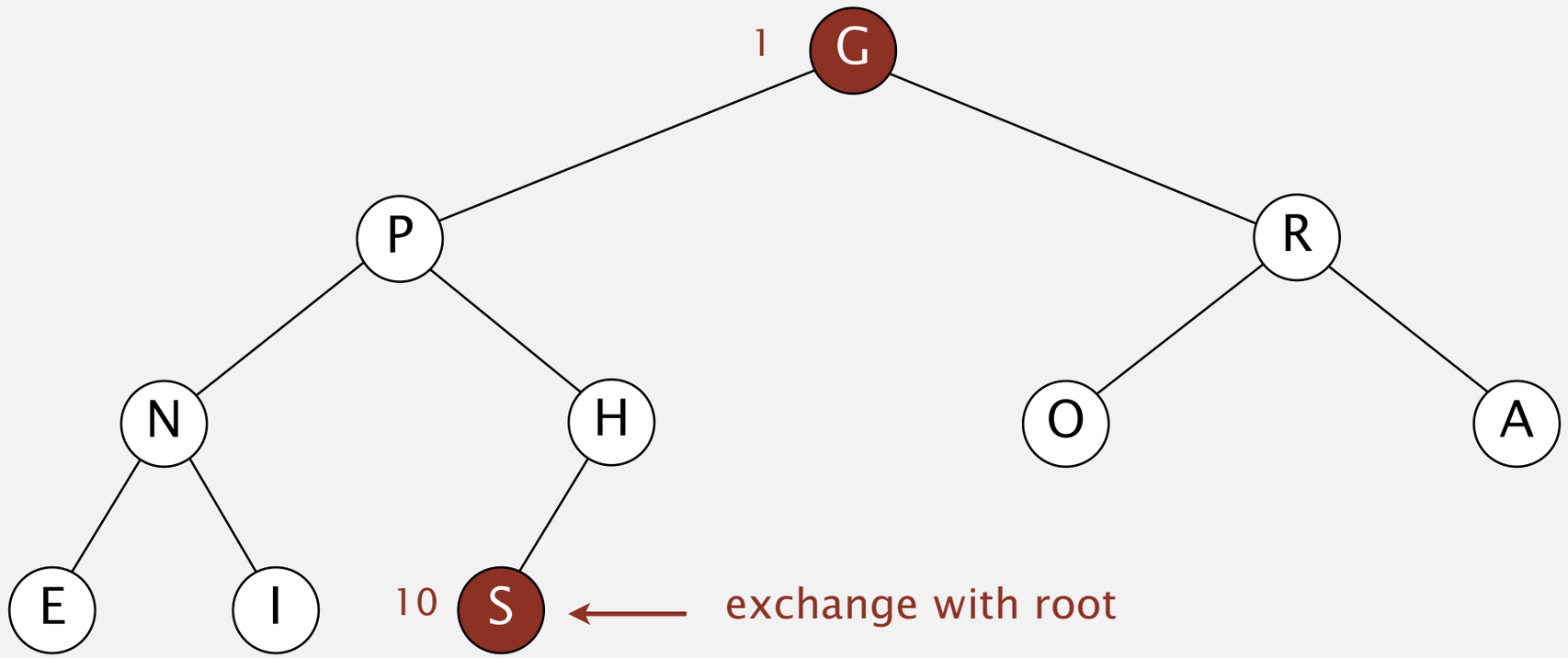
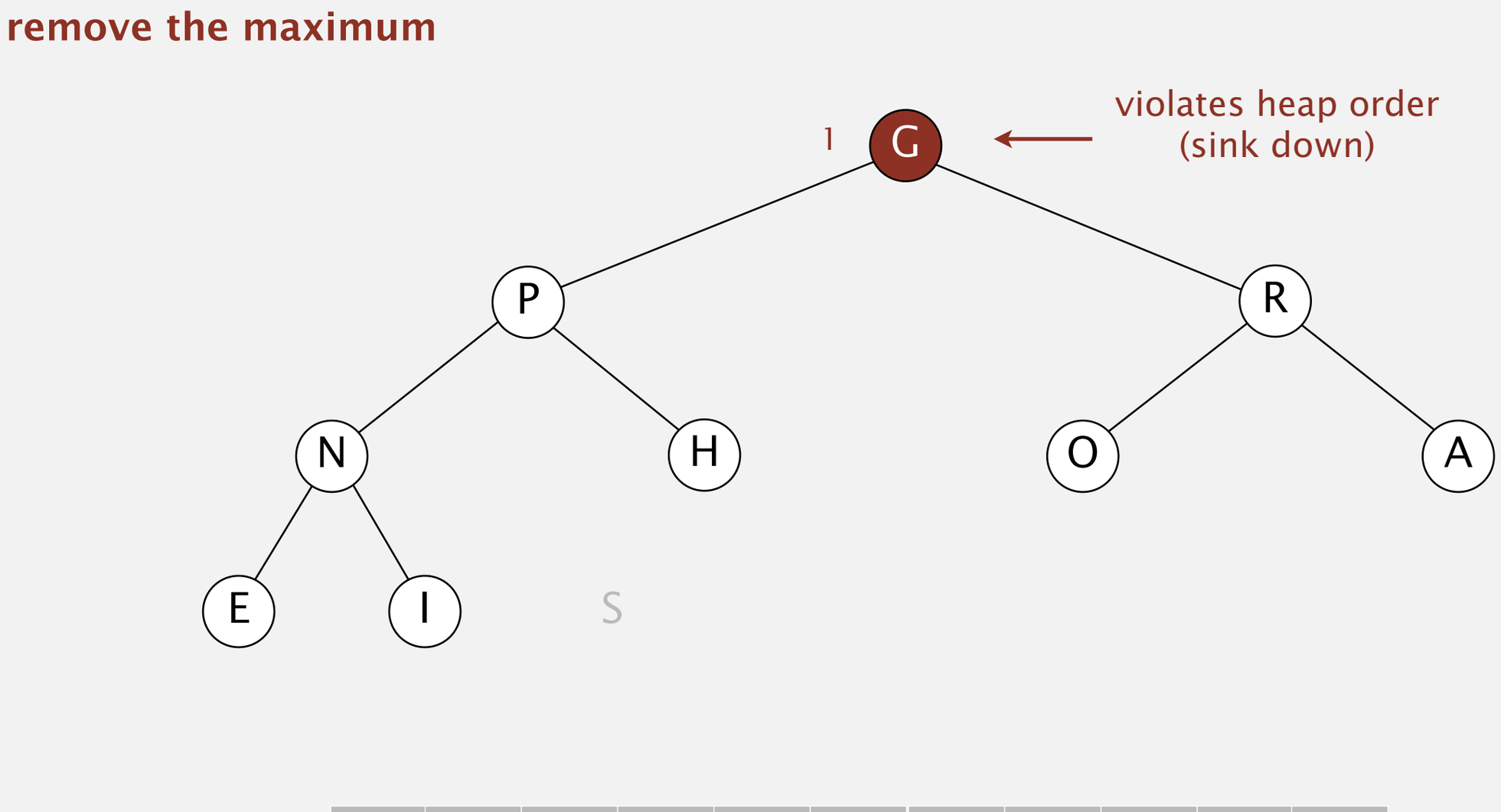


G	P	R	N	H	O	A	E	I	S	
1									10	

Binary heap demo

Insert. Add node at end, then swim it up.

Remove the maximum. Exchange root with node at end, then sink it down.



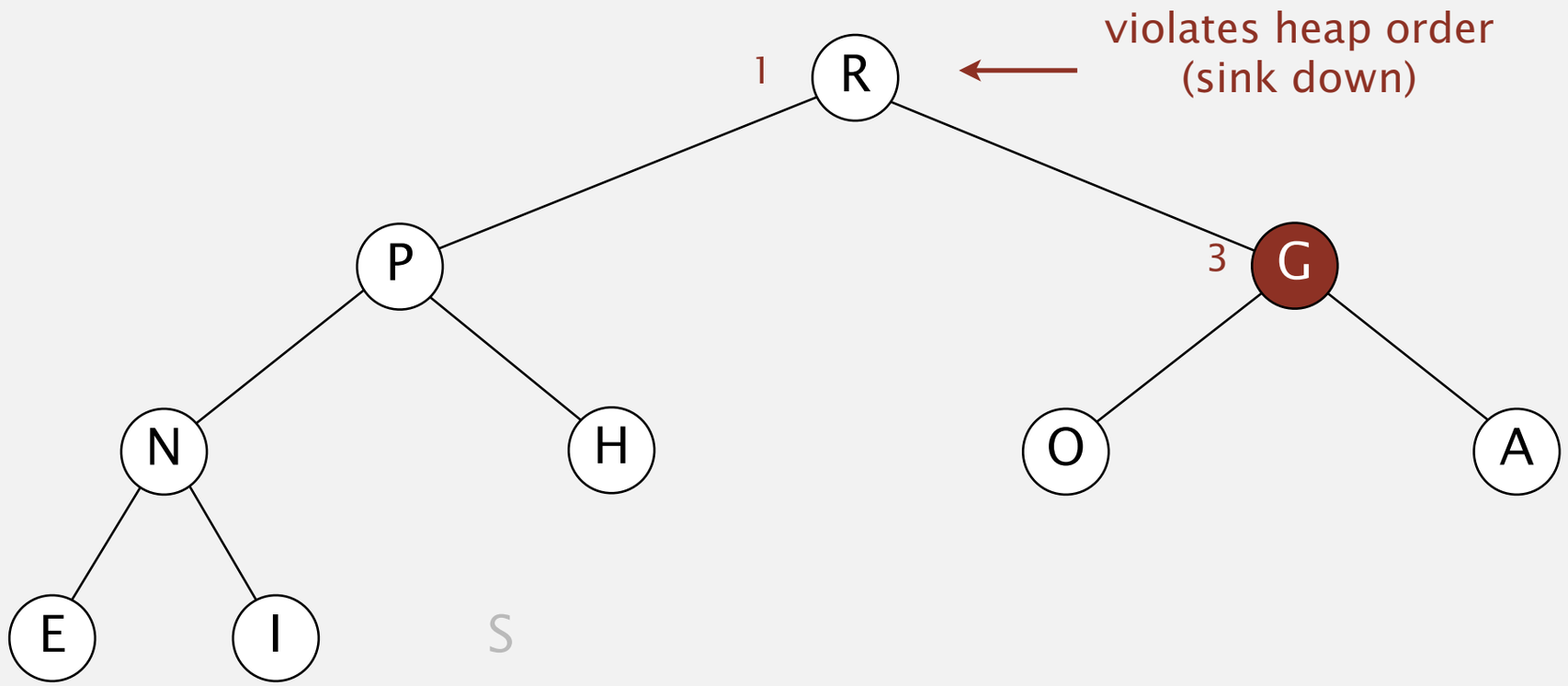
G	P	R	N	H	O	A	E	I	S	
1										

Binary heap demo

Insert. Add node at end, then swim it up.

Remove the maximum. Exchange root with node at end, then sink it down.

remove the maximum

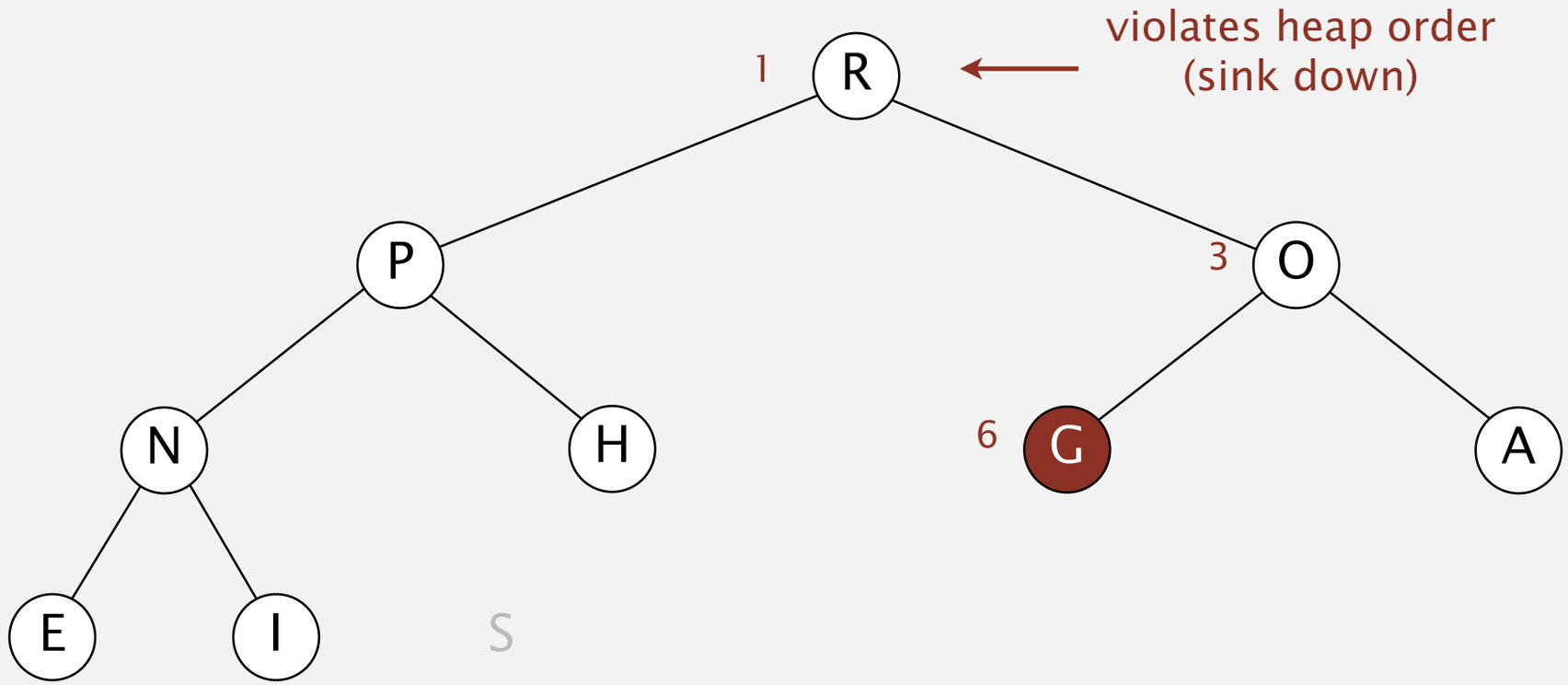


R	P	G	N	H	O	A	E	I	S	
1		3								

Binary heap demo

Insert. Add node at end, then swim it up.

Remove the maximum. Exchange root with node at end, then sink it down.



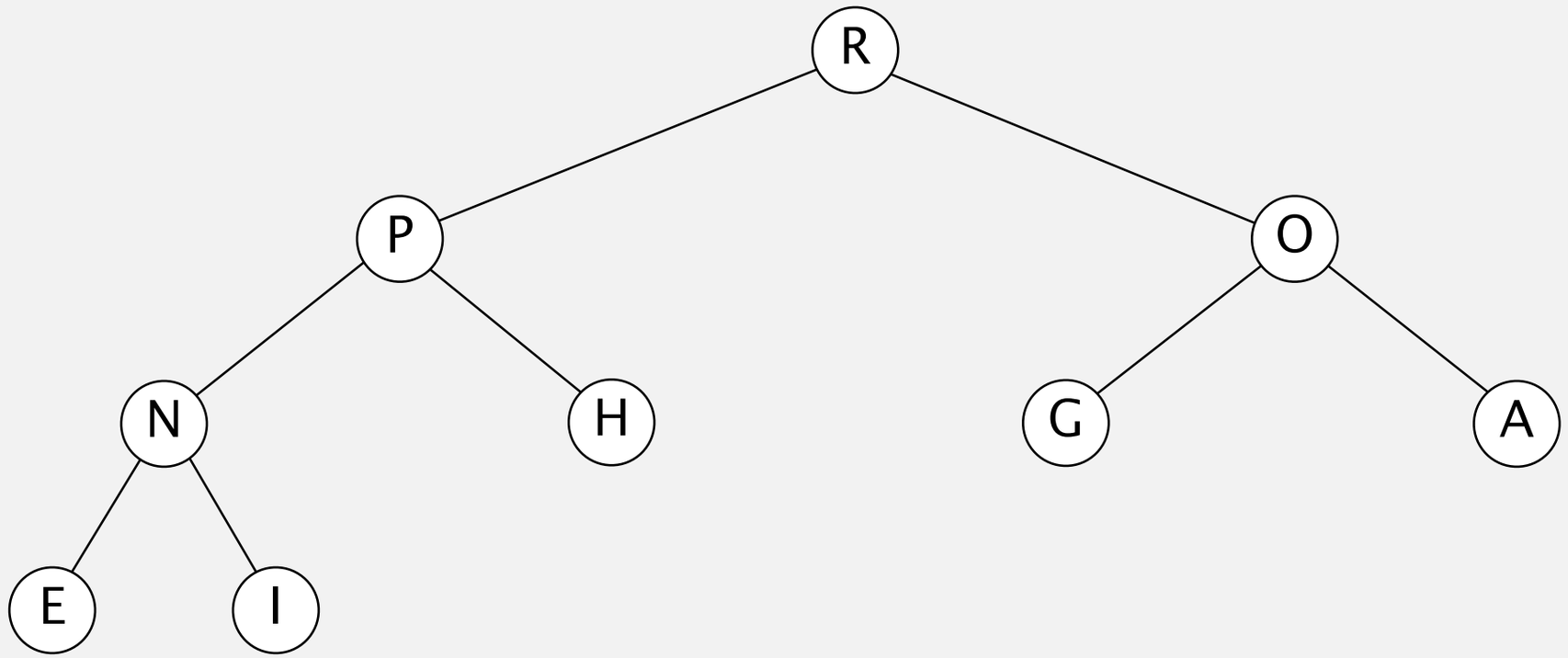
R	P	O	N	H	G	A	E	I	S	
1		3			6					

Binary heap demo

Insert. Add node at end, then swim it up.

Remove the maximum. Exchange root with node at end, then sink it down.

heap ordered



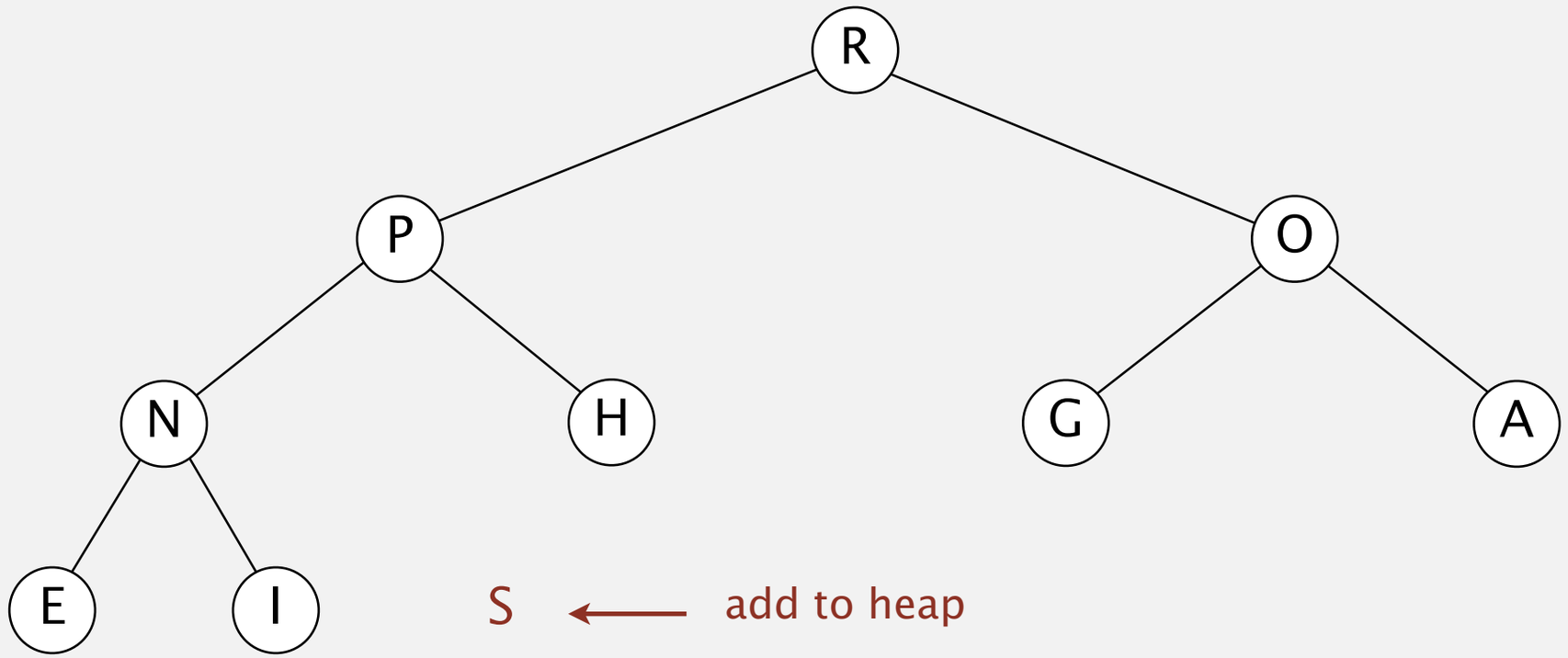
R	P	O	N	H	G	A	E	I		

Binary heap demo

Insert. Add node at end, then swim it up.

Remove the maximum. Exchange root with node at end, then sink it down.

insert S



R	P	O	N	H	G	A	E	I	S	

Binary heap demo

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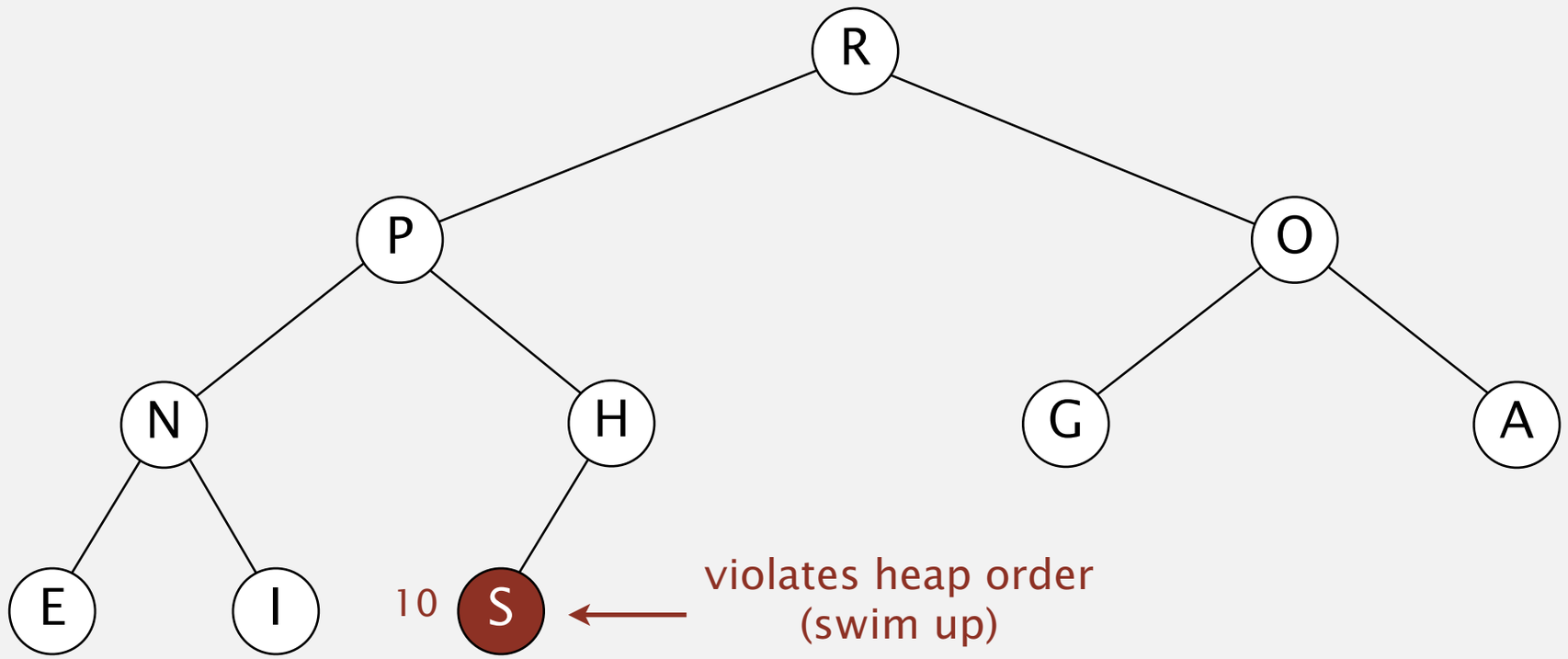
insert S



R	P	O	N	H	G	A	E	I	S	
									10	

Binary heap demo

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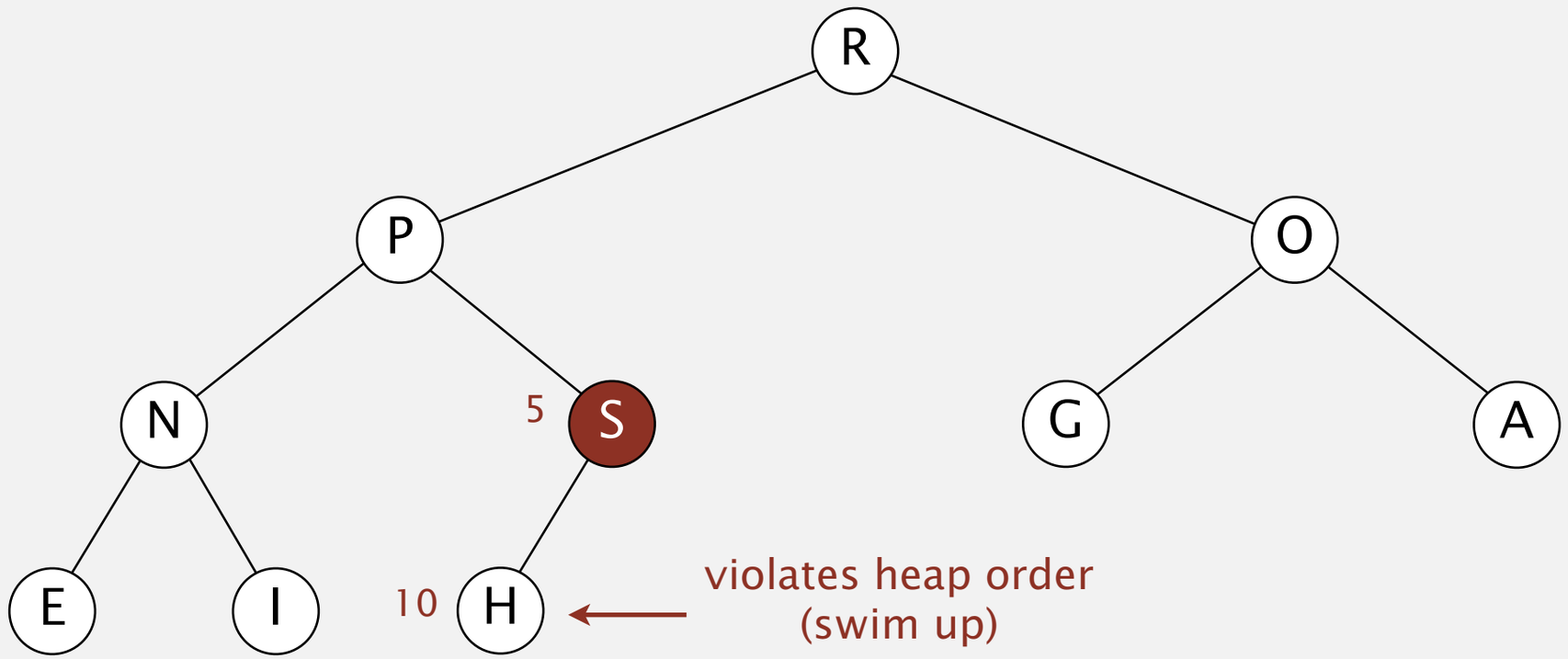
insert S



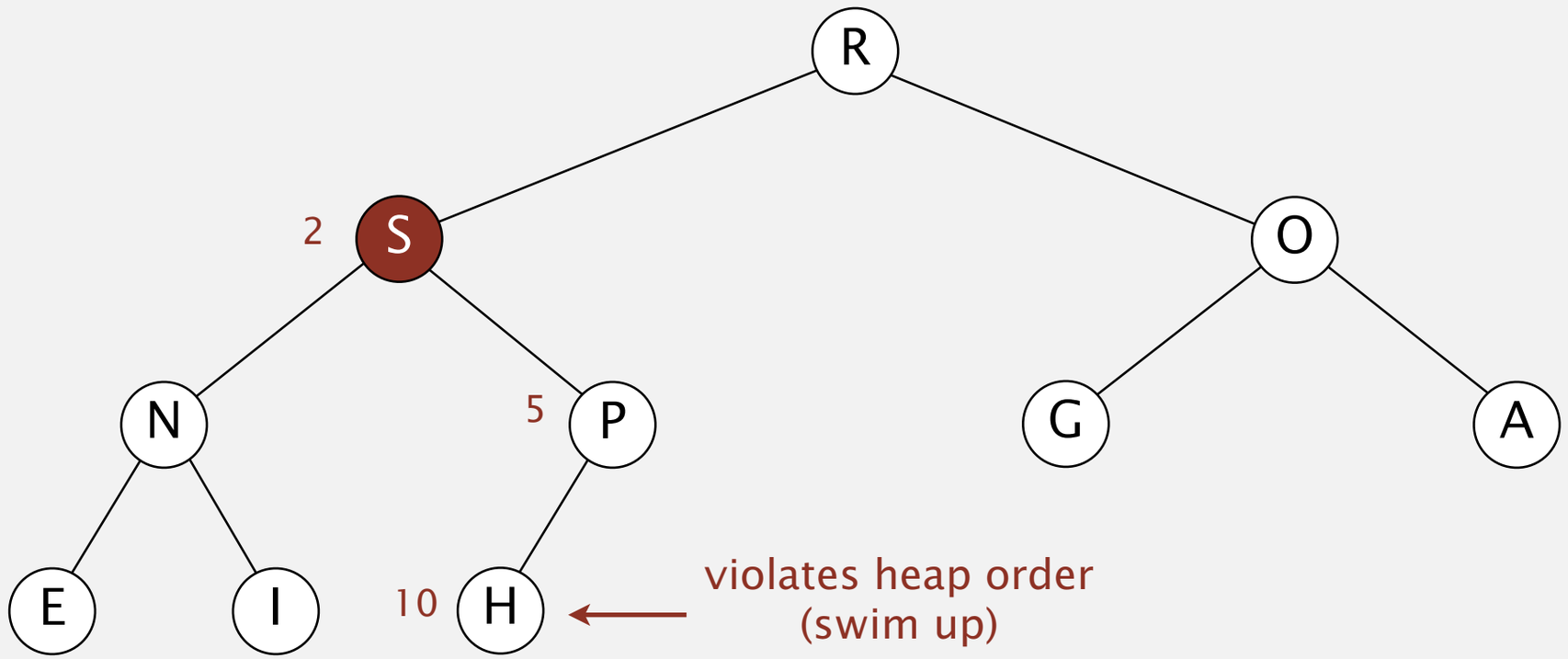
R	P	O	N	S	G	A	E	I	H	
				5					10	

Binary heap demo

Insert. Add node at end, then swim it up.

Remove the maximum. Exchange root with node at end, then sink it down.

insert S



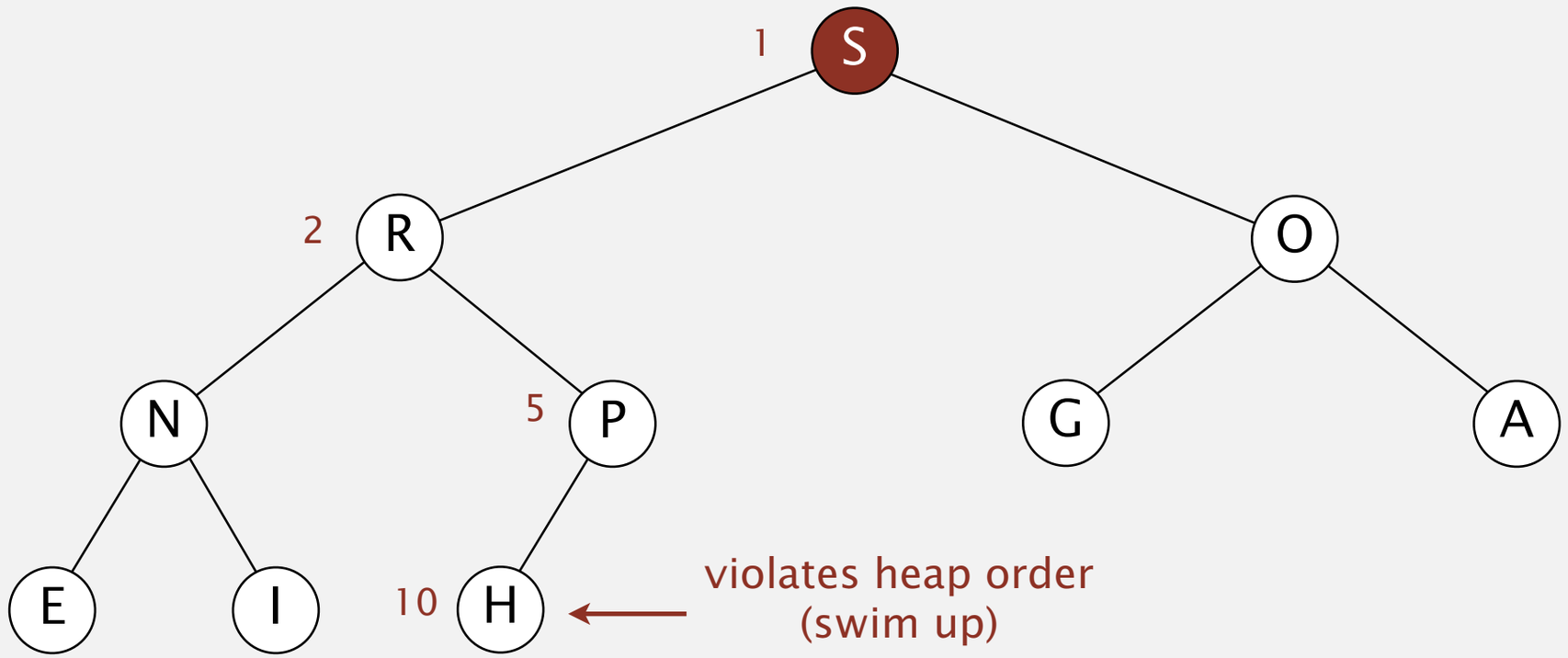
R	S	O	N	P	G	A	E	I	H	
	2			5					10	

Binary heap demo

Insert. Add node at end, then swim it up.

Remove the maximum. Exchange root with node at end, then sink it down.

insert S



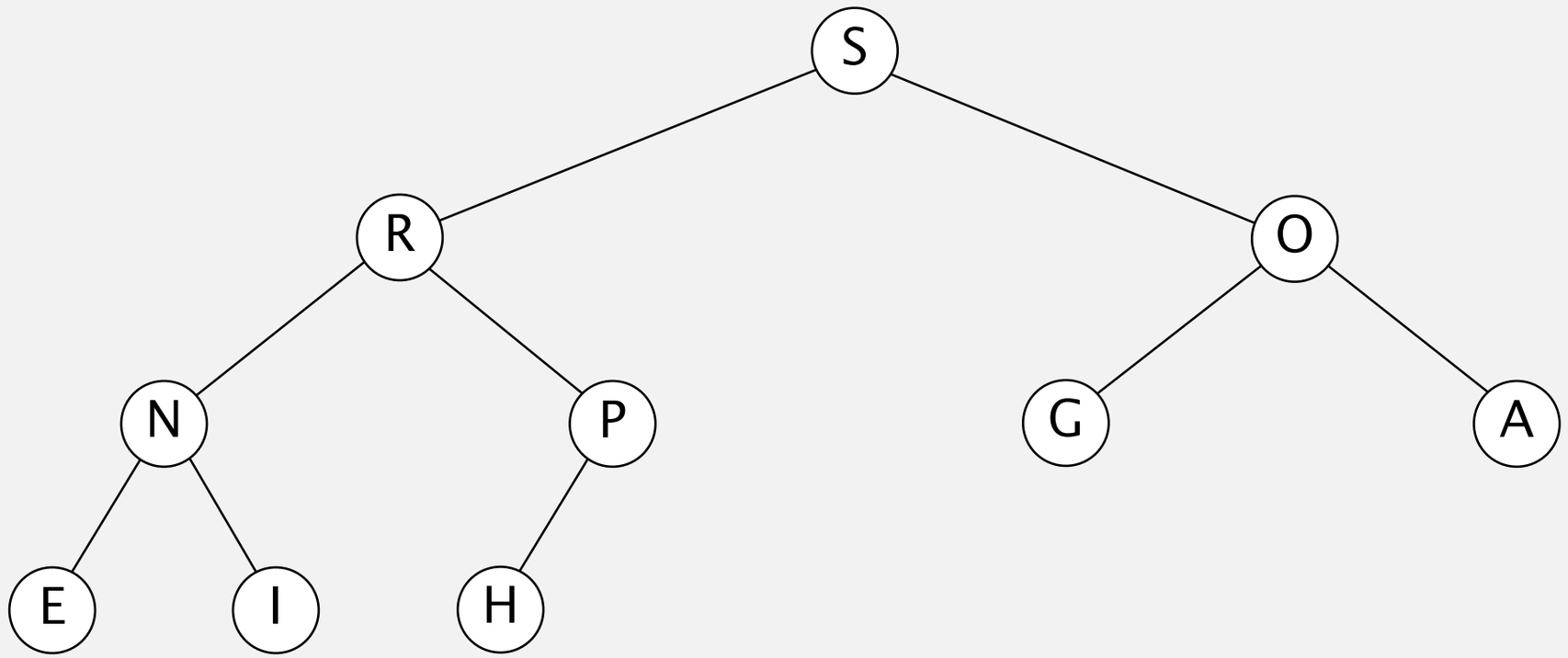
S	R	O	N	P	G	A	E	I	H	
1	2			5					10	

Binary heap demo

Insert. Add node at end, then swim it up.

Remove the maximum. Exchange root with node at end, then sink it down.

heap ordered



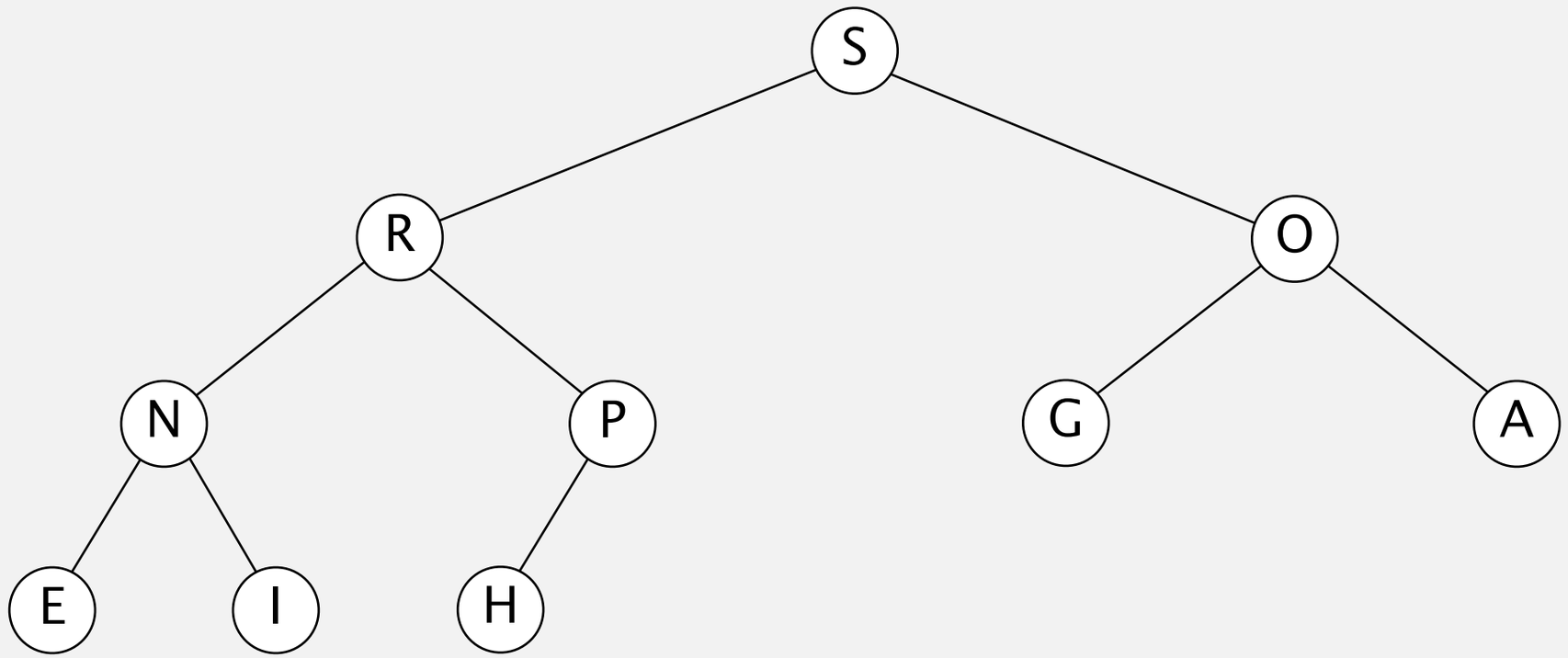
S	R	O	N	P	G	A	E	I	H	

Binary heap demo

Insert. Add node at end, then swim it up.

Remove the maximum. Exchange root with node at end, then sink it down.

heap ordered



S	R	O	N	P	G	A	E	I	H	

Binary heap: promotion

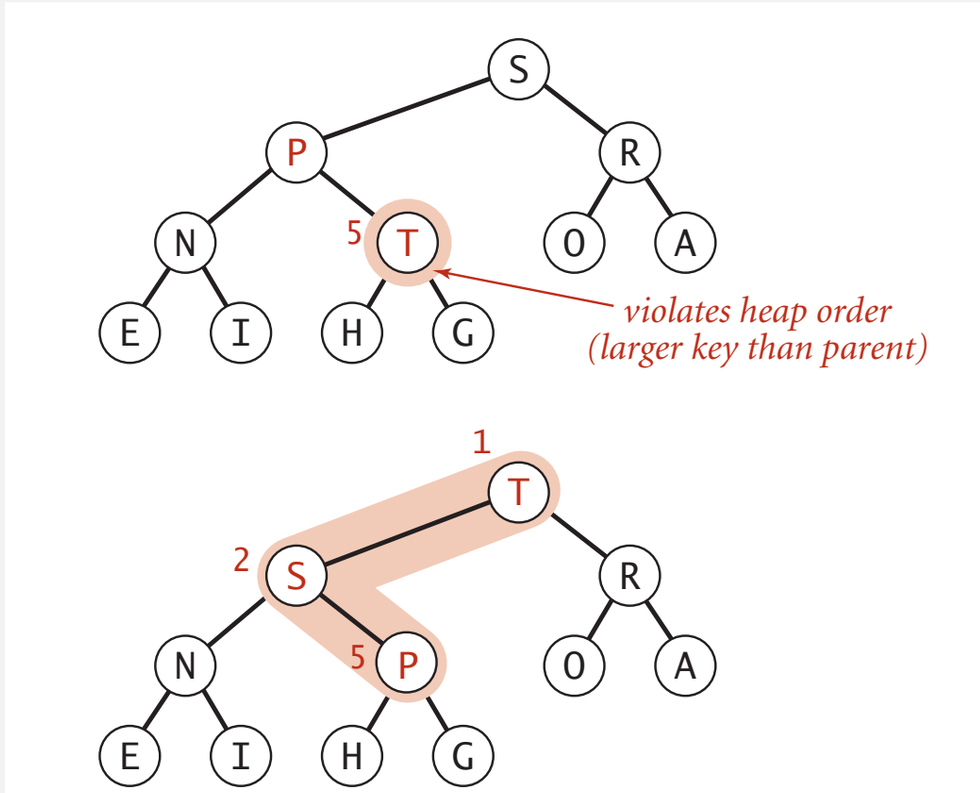
Scenario. A key becomes larger than its parent's key.

To eliminate the violation:

- Exchange key in child with key in parent.
- Repeat until heap order restored.

```
private void swim(int k)
{
   while (k > 1 && less(k/2, k))
   {
      exch(k, k/2);
      k = k/2;
   }
}
```

parent of node at k is at k/2

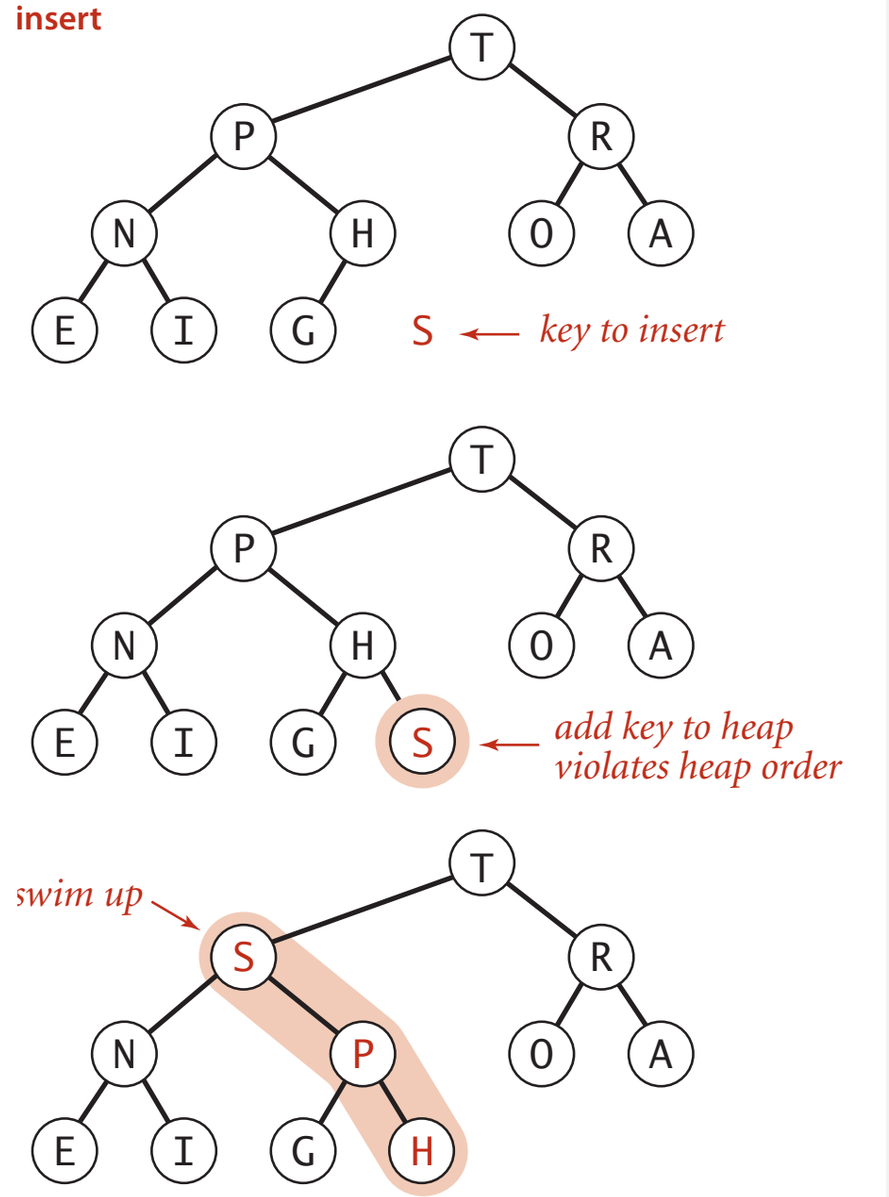


Binary heap: insertion

Insert. Add node at end, then swim it up.

Cost. At most $1 + \lg N$ compares.

```
public void insert(Key x)
{
   pq[++N] = x;
   swim(N);
}
```



Binary heap: demotion

Scenario. A key becomes smaller than one (or both) of its children's.

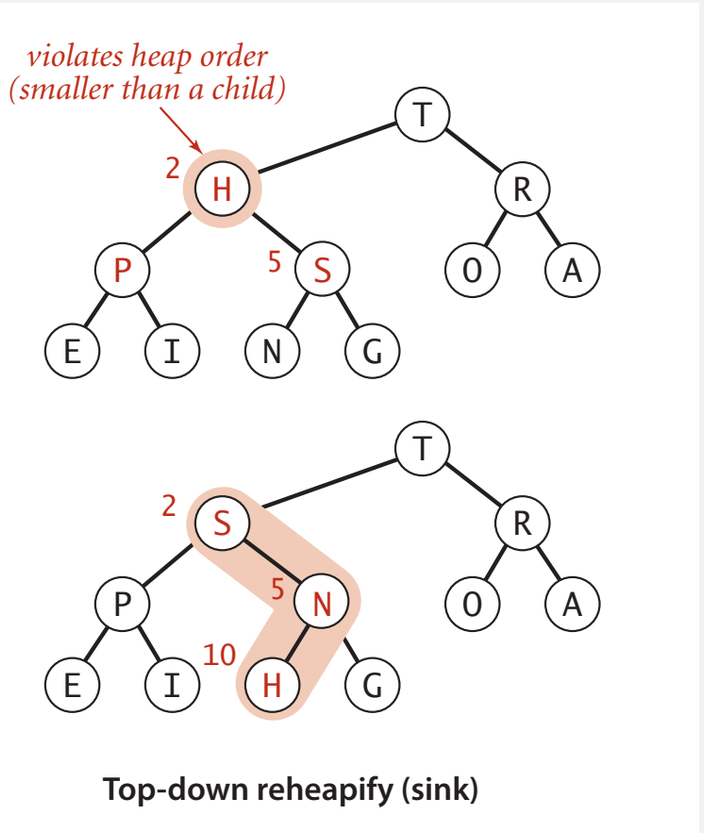
To eliminate the violation:

why not smaller child?

- Exchange key in parent with key in larger child.
- Repeat until heap order restored.

```
private void sink(int k)
{
   while (2*k <= N)
   {
      int j = 2*k;
      if (j < N && less(j, j+1)) j++;
      if (!less(k, j)) break;
      exch(k, j);
      k = j;
   }
}
```

children of node at k are 2*k and 2*k+1



Top-down reheapify (sink)

Power struggle. Better subordinate promoted.

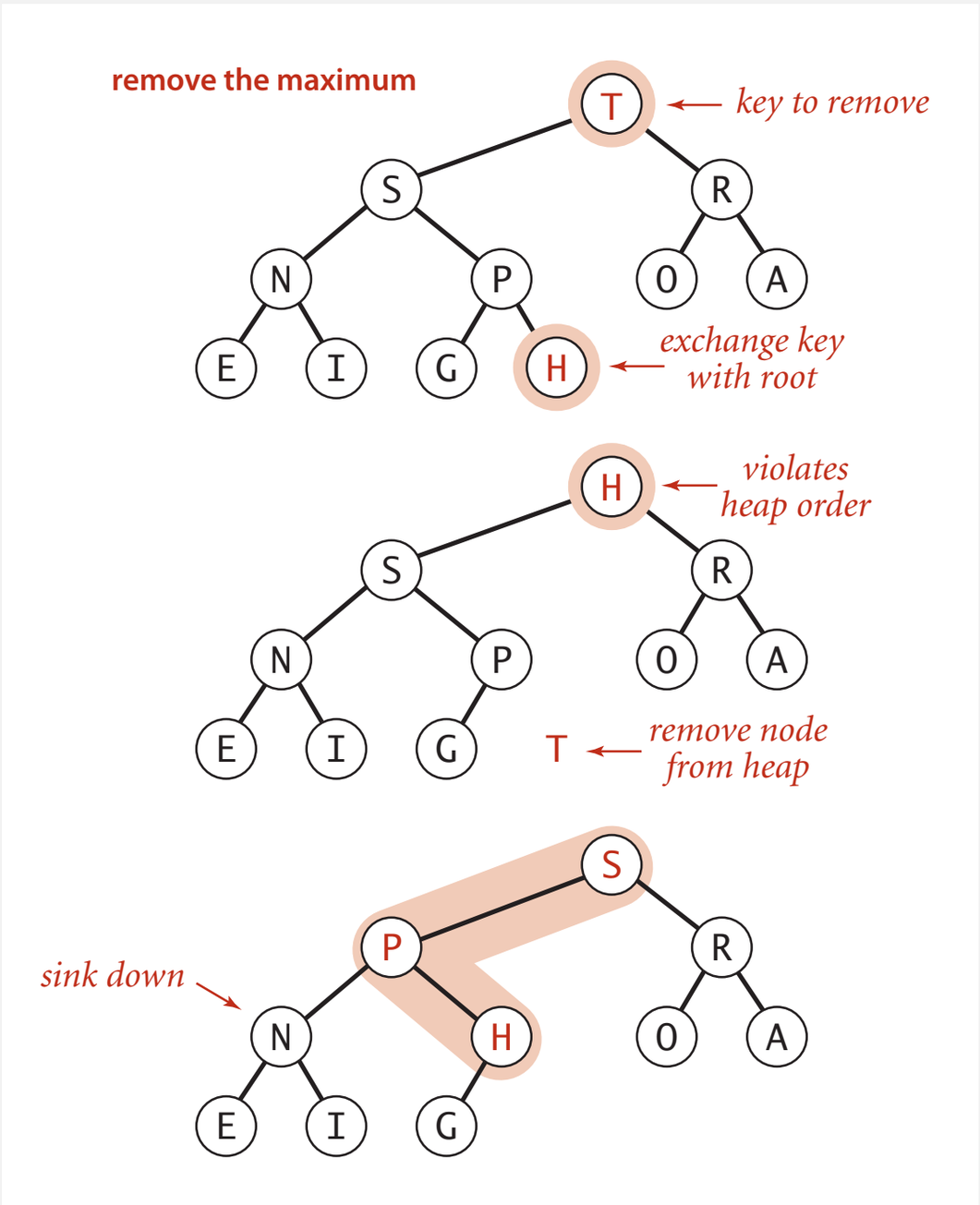
Binary heap: delete the maximum

Delete max. Exchange root with node at end, then sink it down.

Cost. At most $2 \lg N$ compares.

```
public Key delMax()
{
   Key max = pq[1];
   exch(1, N--);
   sink(1);
   pq[N+1] = null;
   return max;
}
```

prevent loitering



Binary heap: Java implementation

```
public class MaxPQ<Key extends Comparable<Key>>
{
   private Key[] pq;
   private int N;

   public MaxPQ(int capacity)
   {  pq = (Key[]) new Comparable[capacity+1];  }

   public boolean isEmpty()
   {  return N == 0;  }
   public void insert(Key key)  // see previous code
   public Key delMax()          // see previous code

   private void swim(int k)     // see previous code
   private void sink(int k)     // see previous code

   private boolean less(int i, int j)
   {  return pq[i].compareTo(pq[j]) < 0;  }
   private void exch(int i, int j)
   {  Key t = pq[i]; pq[i] = pq[j]; pq[j] = t;  }
}
```

fixed capacity (for simplicity)

PQ ops

heap helper functions

array helper functions

Priority queue: implementations cost summary

implementation	insert	del max	max
unordered array	1	N	N
ordered array	N	1	1
binary heap	$\log N$	$\log N$	1

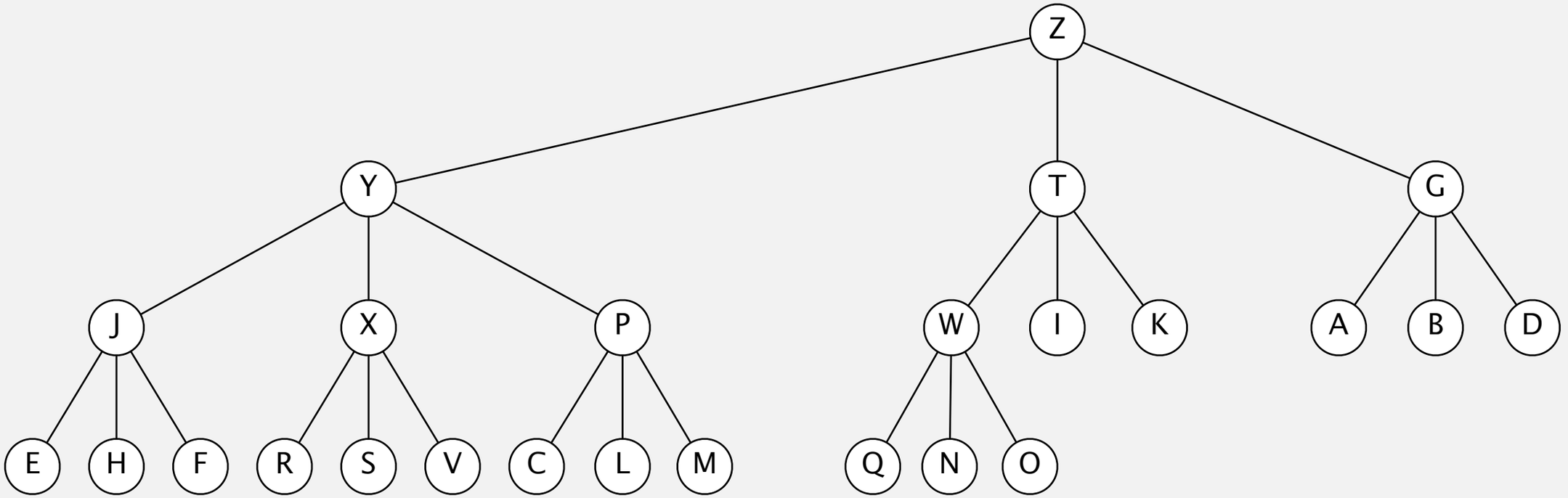
order-of-growth of running time for priority queue with N items

Binary heap: practical improvements

Multiway heaps.

- Complete d-way tree.
- Parent's key no smaller than its children's keys.

Fact. Height of complete d-way tree on N nodes is $\sim \log_d N$.



3-way heap

Priority queue: implementation cost summary

implementation	insert	del max	max	
unordered array	1	N	N	
ordered array	N	1	1	
binary heap	$\log N$	$\log N$	1	
d-ary heap	$\log_d N$	$d \log_d N$	1	← sweet spot: d = 4
Fibonacci	1	$\log N$ †	1	
Brodal queue	1	$\log N$	1	
impossible	1	1	1	← why impossible?

† amortized

order-of-growth of running time for priority queue with N items

Binary heap: considerations

Underflow and overflow.

- Underflow: throw exception if deleting from empty PQ.
- Overflow: add no-arg constructor and use resizing array.

leads to log N
amortized time per op
(how to make worst case?)

Minimum-oriented priority queue.

- Replace `less()` with `greater()`.
- Implement `greater()`.

Other operations.

- Remove an arbitrary item.
- Change the priority of an item.

can implement efficiently with `sink()` and `swim()`
[stay tuned for Prim/Dijkstra]

Immutability of keys.

- Assumption: client does not change keys while they're on the PQ.
- Best practice: use immutable keys.

Immutability: implementing in Java

Data type. Set of values and operations on those values.

Immutable data type. Can't change the data type value once created.

```
public final class Vector {
   private final int N;
   private final double[] data;

   public Vector(double[] data) {
      this.N = data.length;
      this.data = new double[N];
      for (int i = 0; i < N; i++)
         this.data[i] = data[i];
   }
   ⋮
}
```

instance variables `private` and `final` (neither necessary nor sufficient, but good programming practice)

defensive copy of mutable instance variables

instance methods don't change instance variables

Immutable. `String`, `Integer`, `Double`, `Color`, `Vector`, `Transaction`, `Point2D`.

Mutable. `StringBuilder`, `Stack`, `Counter`, Java array.

Immutability: properties

Data type. Set of values and operations on those values.

Immutable data type. Can't change the data type value once created.

Advantages.

- Simplifies debugging.
- Simplifies concurrent programming.
- More secure in presence of hostile code.
- Safe to use as key in priority queue or symbol table.

Disadvantage. Must create new object for each data type value.

> *" Classes should be immutable unless there's a very good reason to make them mutable.... If a class cannot be made immutable, you should still limit its mutability as much as possible. "*
>
> *— Joshua Bloch (Java architect)*

http://algs4.cs.princeton.edu

2.4 Priority Queues

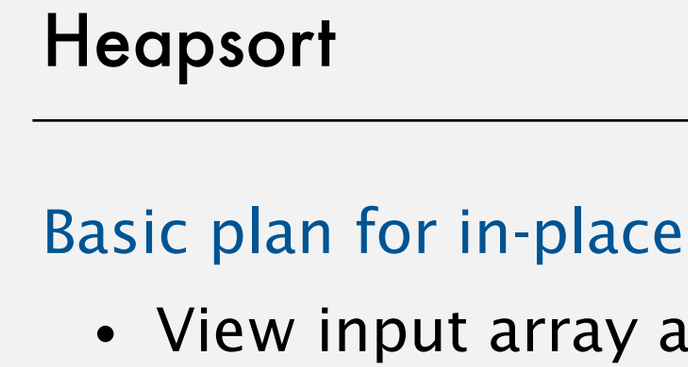
- *API and elementary implementations*
- *binary heaps*
- *heapsort*
- *event-driven simulation*

Heapsort

Basic plan for in-place sort.

- View input array as a complete binary tree.
- Heap construction: build a max-heap with all N keys.
- Sortdown: repeatedly remove the maximum key.

keys in arbitrary order



1	2	3	4	5	6	7	8	9	10	11
S	O	R	T	E	X	A	M	P	L	E

build max heap (in place)

1	2	3	4	5	6	7	8	9	10	11
X	T	S	P	L	R	A	M	O	E	E

sorted result (in place)

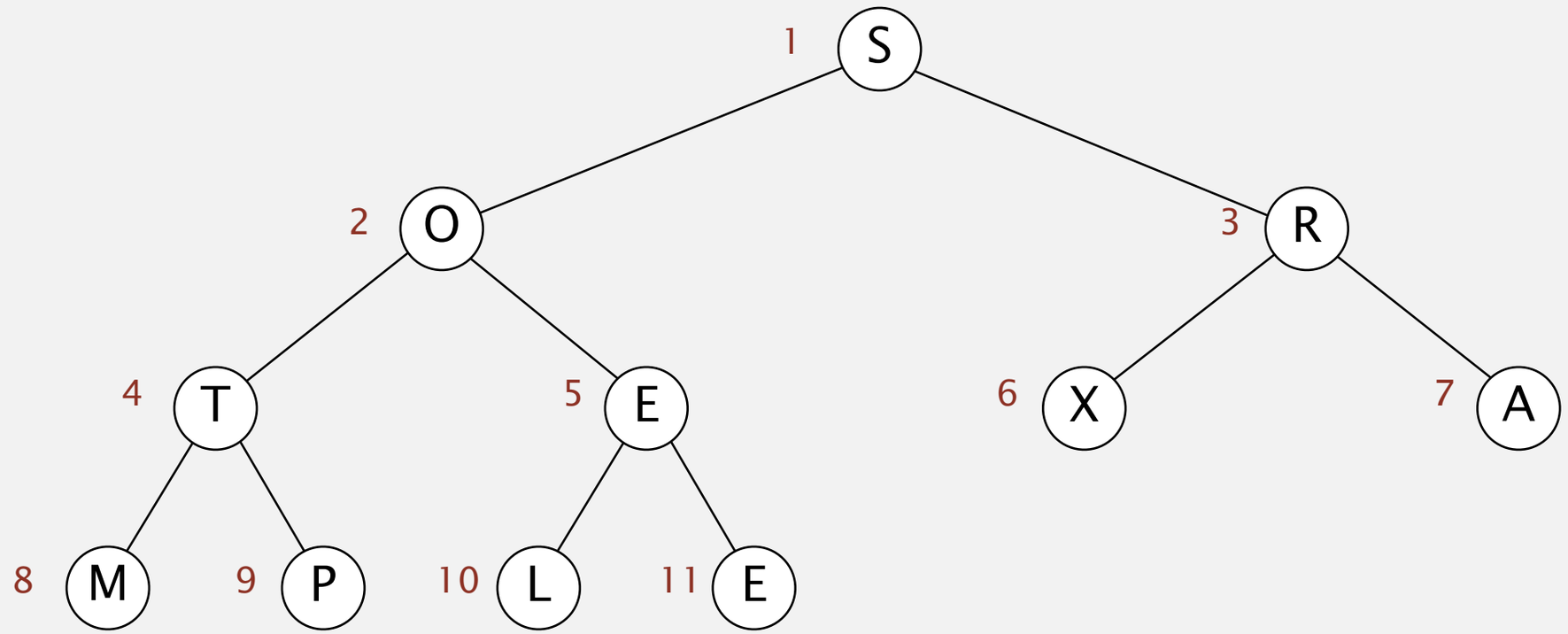
1	2	3	4	5	6	7	8	9	10	11
A	E	E	L	M	O	P	R	S	T	X

Heapsort demo

Heap construction. Build max heap using bottom-up method.

we assume array entries are indexed 1 to N

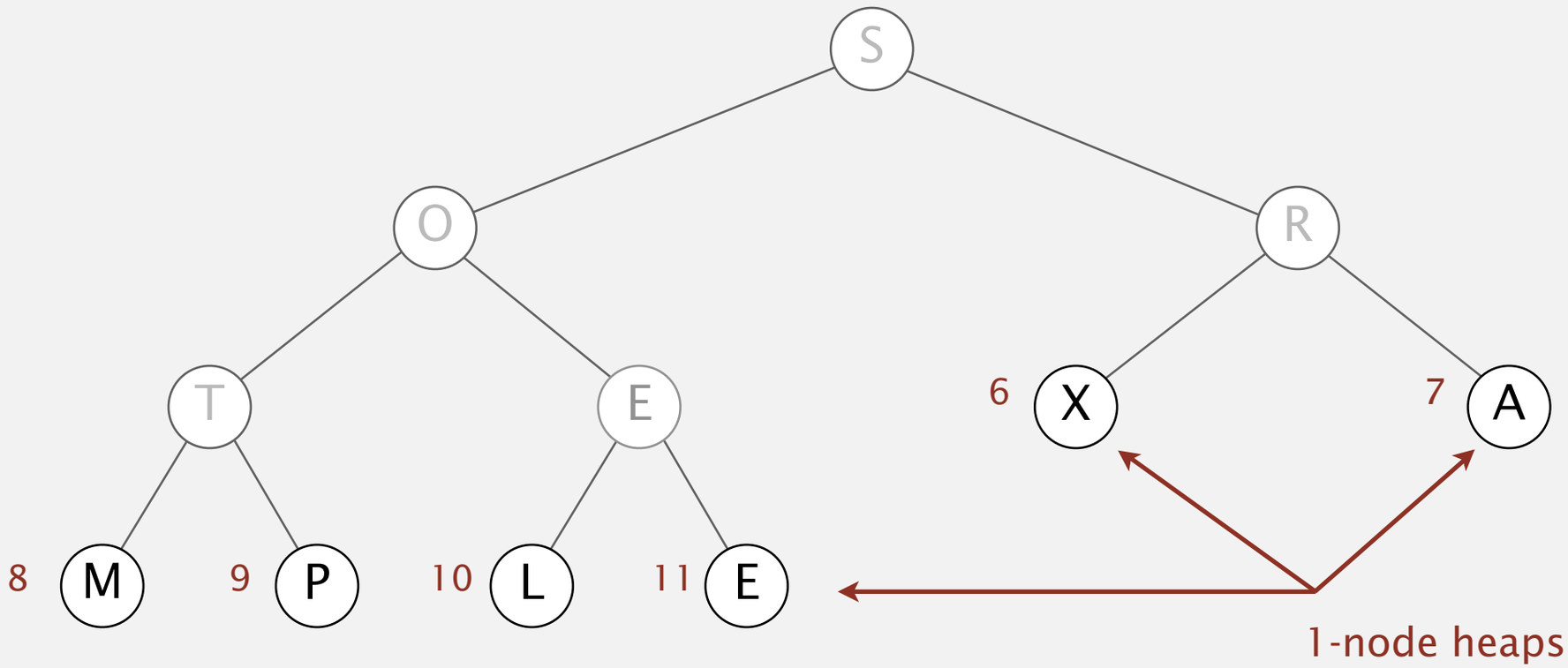
array in arbitrary order



S	O	R	T	E	X	A	M	P	L	E
1	2	3	4	5	6	7	8	9	10	11

Heapsort demo

Heap construction. Build max heap using bottom-up method.

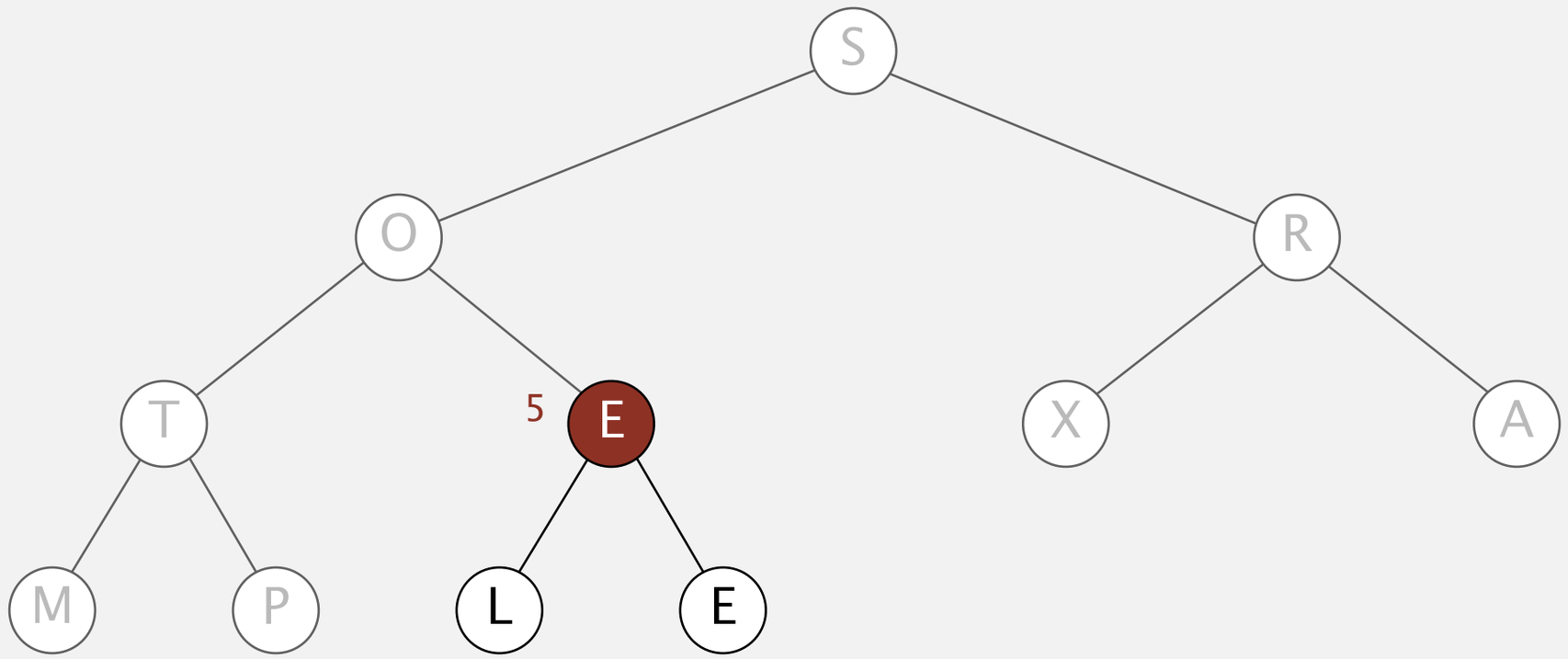


S	O	R	T	E	X	A	M	P	L	E
					6	7	8	9	10	11

Heapsort demo

Heap construction. Build max heap using bottom-up method.

sink 5

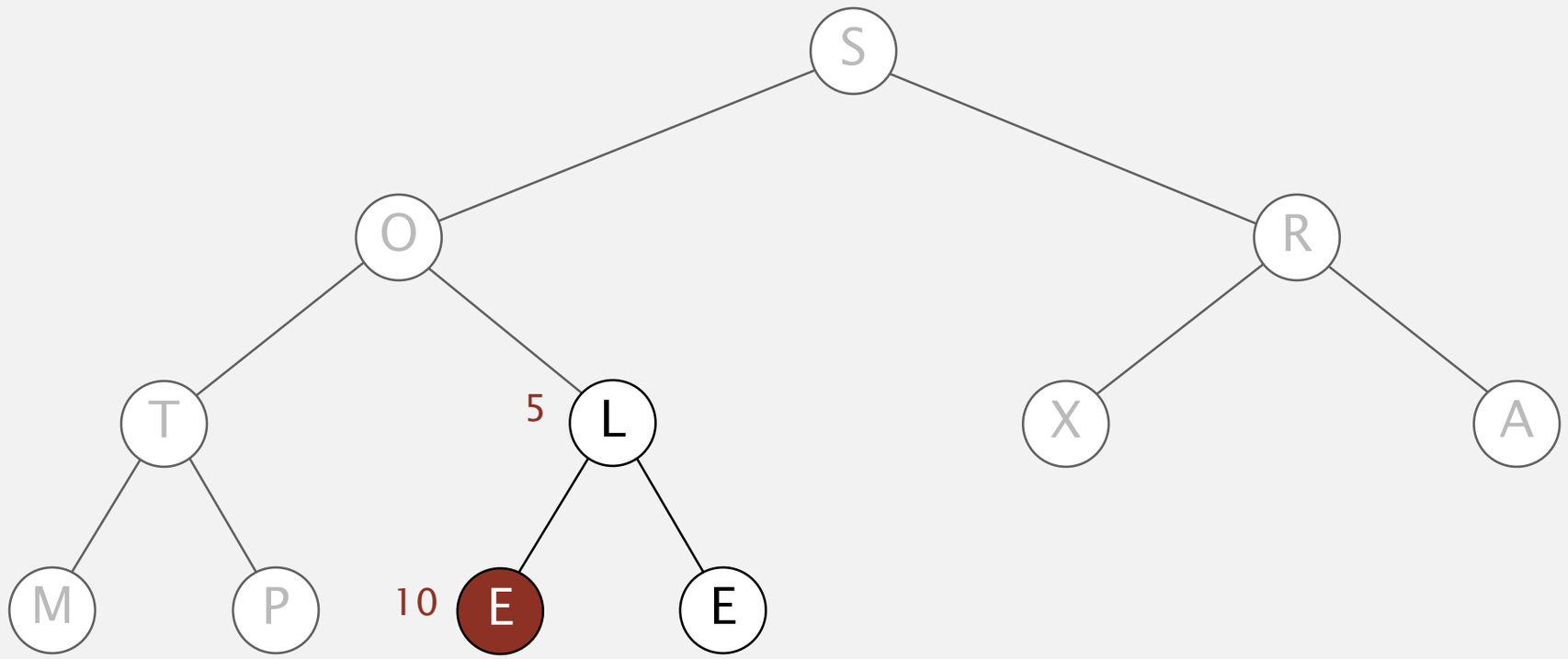


S	O	R	T	E	X	A	M	P	L	E
				5						

Heapsort demo

Heap construction. Build max heap using bottom-up method.

sink 5

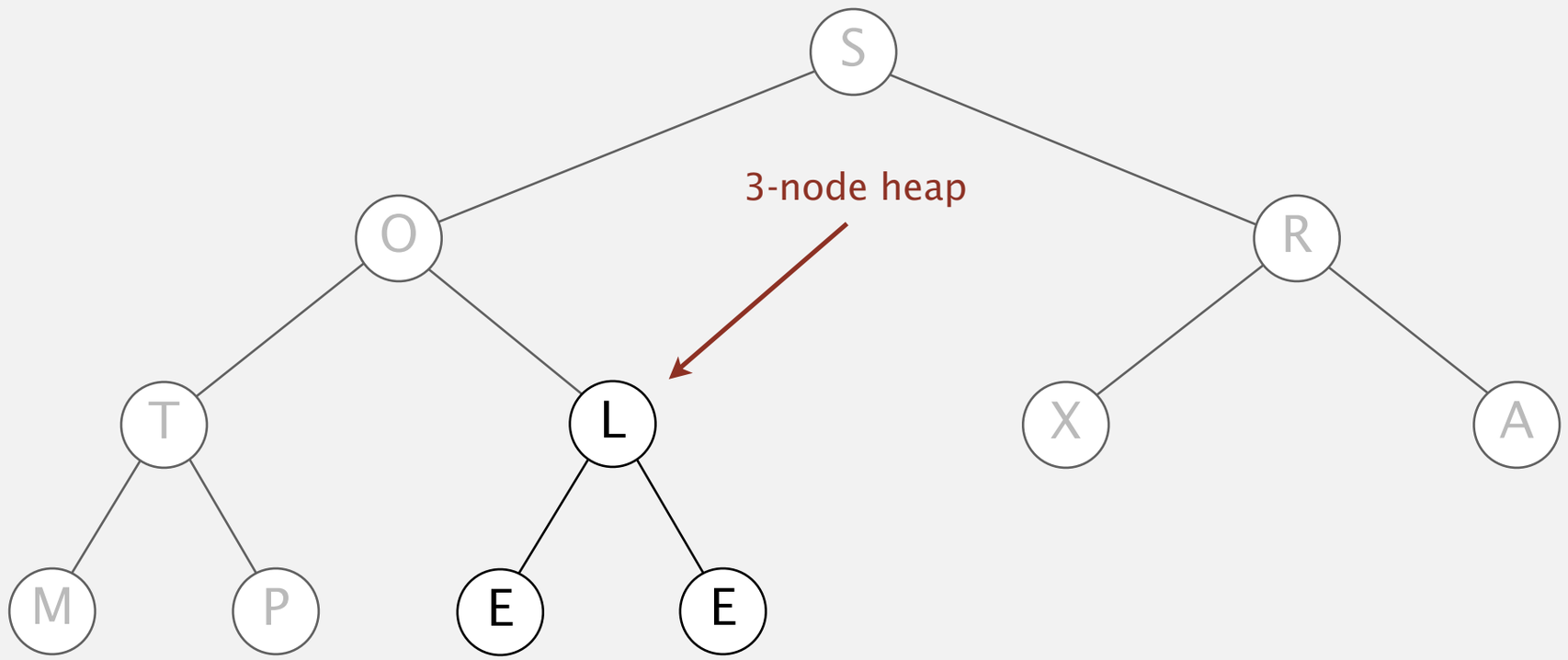


S	O	R	T	L	X	A	M	P	E	E
				5					10	

Heapsort demo

Heap construction. Build max heap using bottom-up method.

sink 5

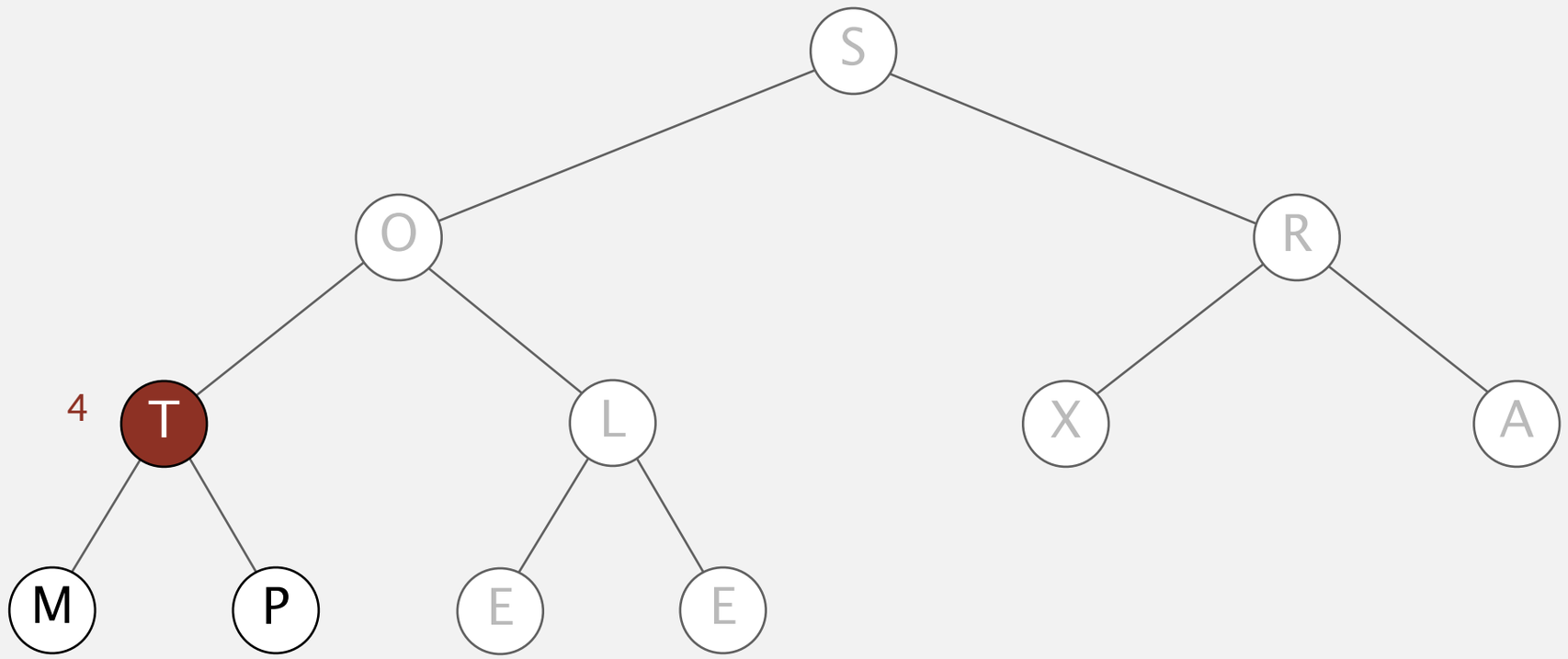


S	O	R	T	L	X	A	M	P	E	E

Heapsort demo

Heap construction. Build max heap using bottom-up method.

sink 4

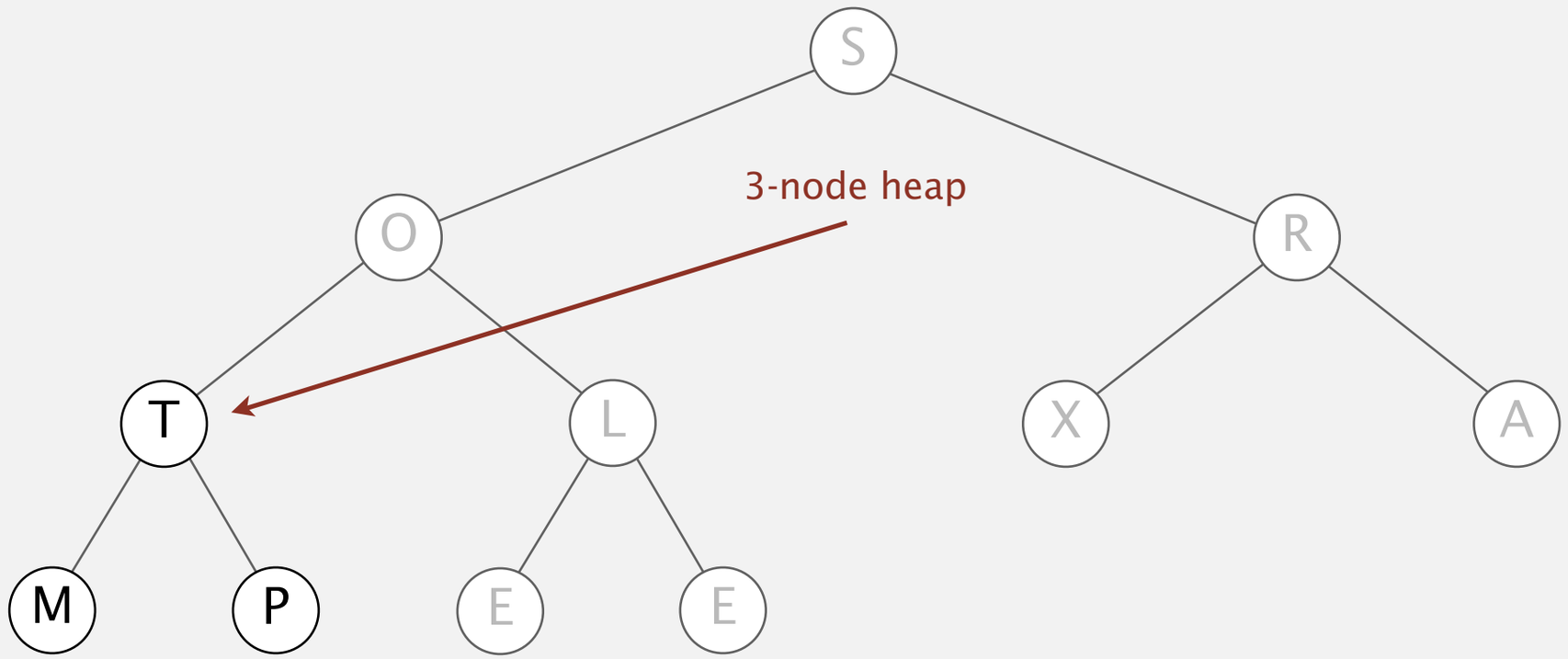


S	O	R	T	L	X	A	M	P	E	E
			4							

Heapsort demo

Heap construction. Build max heap using bottom-up method.

sink 4

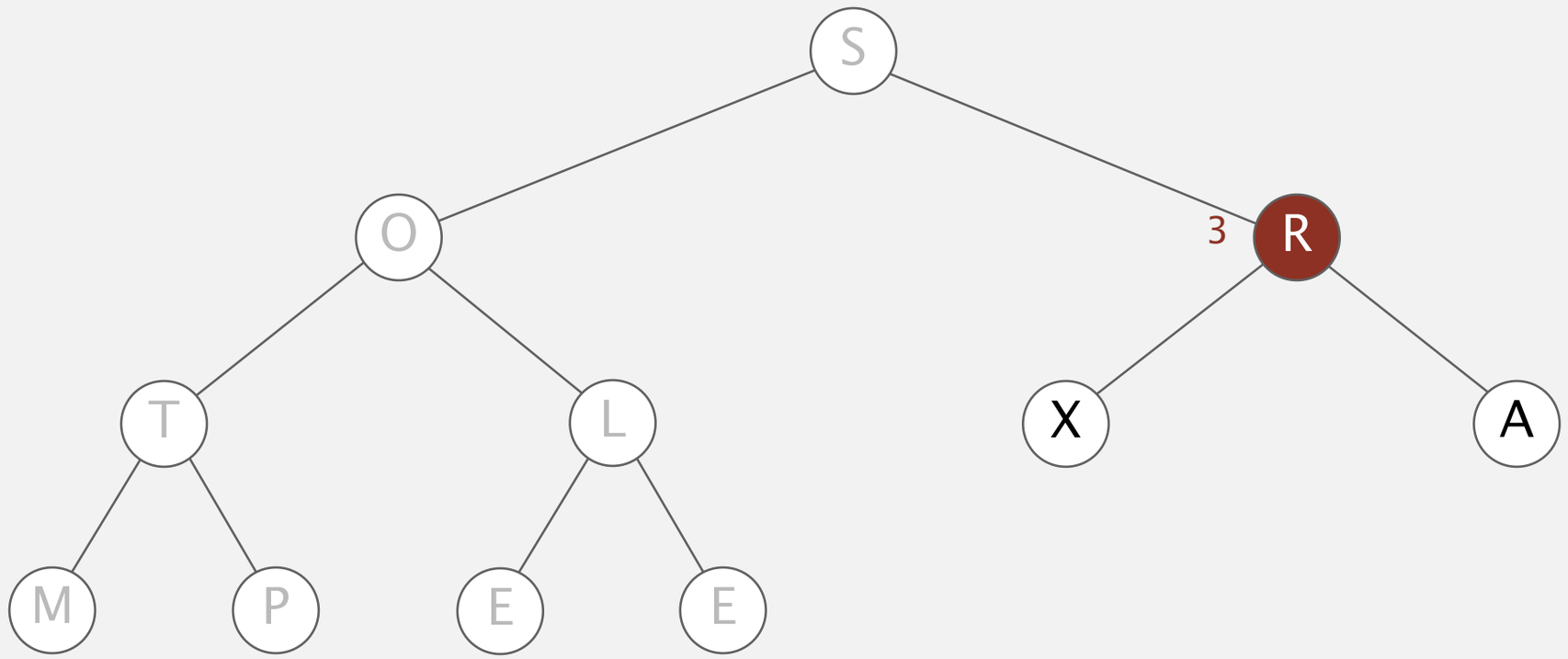


S	O	R	T	L	X	A	M	P	E	E

Heapsort demo

Heap construction. Build max heap using bottom-up method.

sink 3

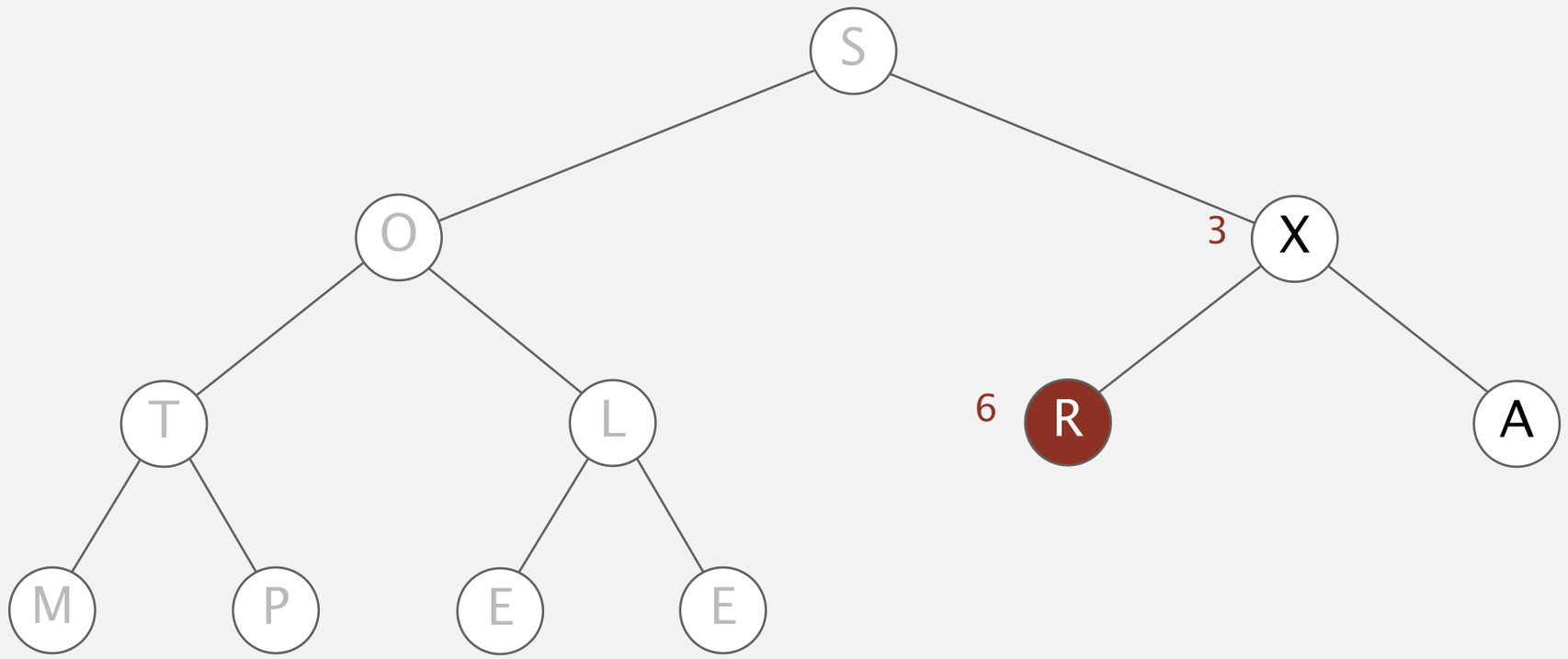


S	O	R	T	L	X	A	M	P	E	E
		3								

Heapsort demo

Heap construction. Build max heap using bottom-up method.

sink 3

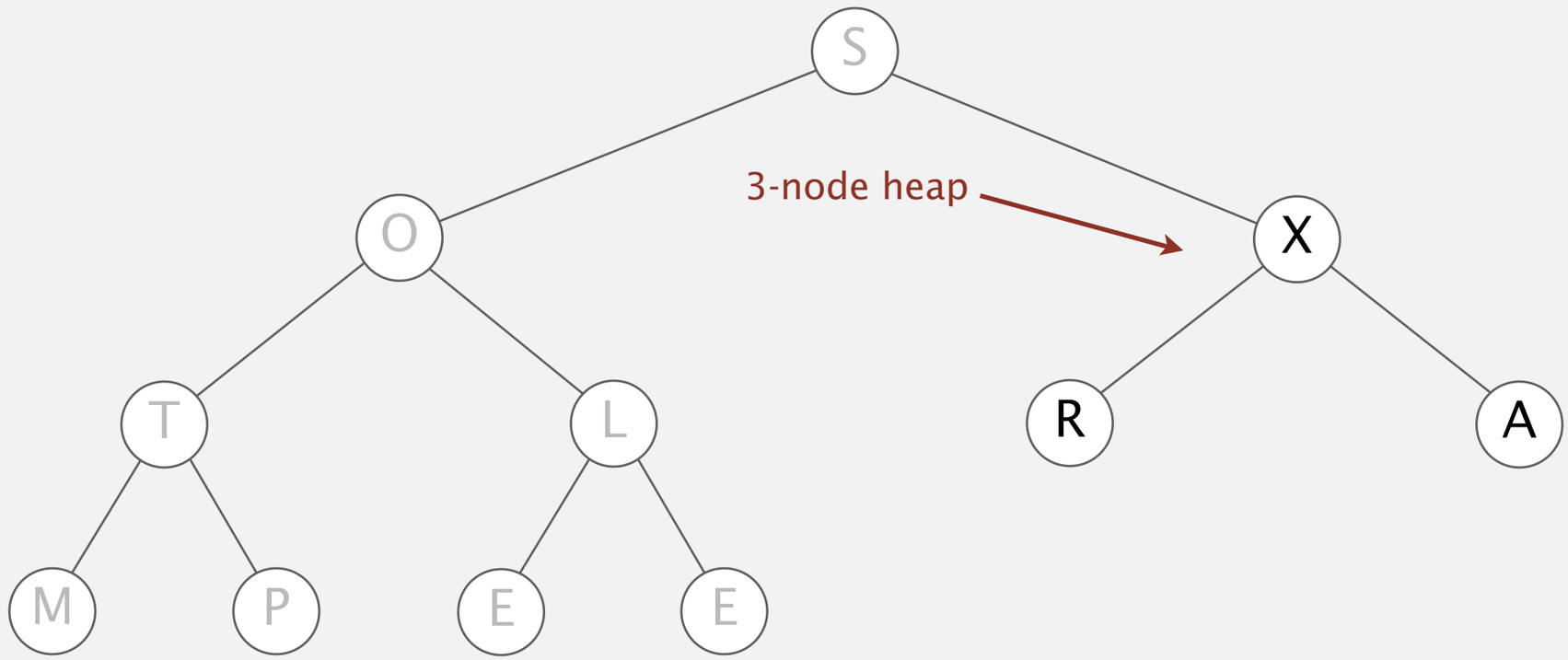


S	O	X	T	L	R	A	M	P	E	E
		3			6					

Heapsort demo

Heap construction. Build max heap using bottom-up method.

sink 3

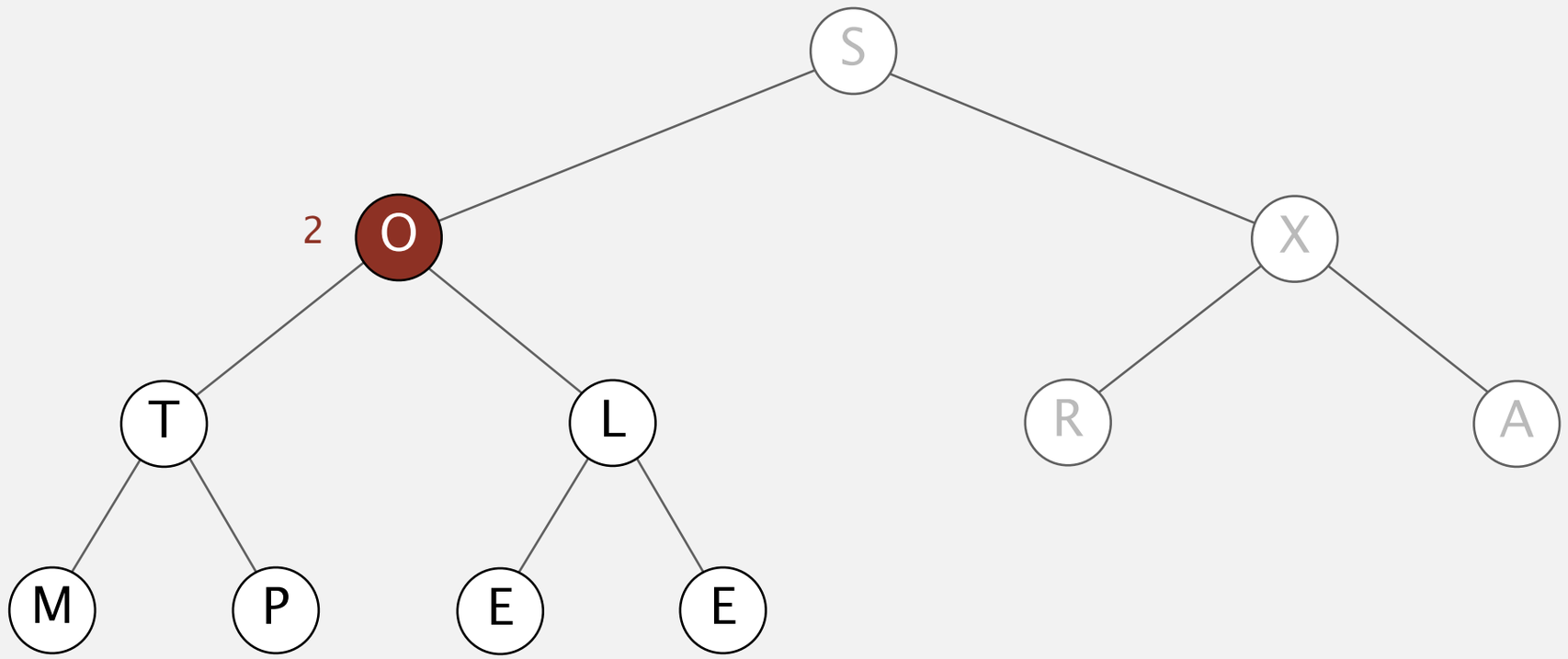


S	O	X	T	L	A	A	M	P	E	E

Heapsort demo

Heap construction. Build max heap using bottom-up method.

sink 2

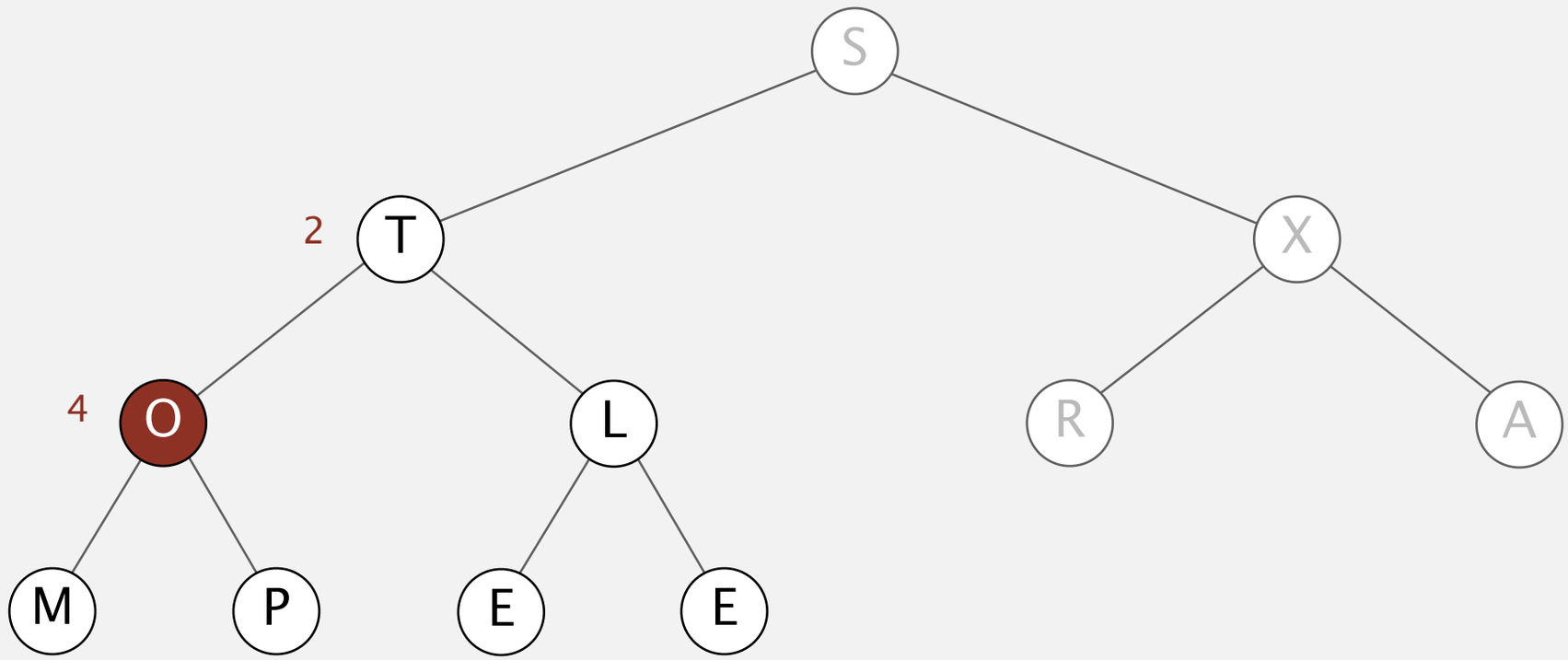


S	O	X	T	L	R	A	M	P	E	E
	2									

Heapsort demo

Heap construction. Build max heap using bottom-up method.

sink 2

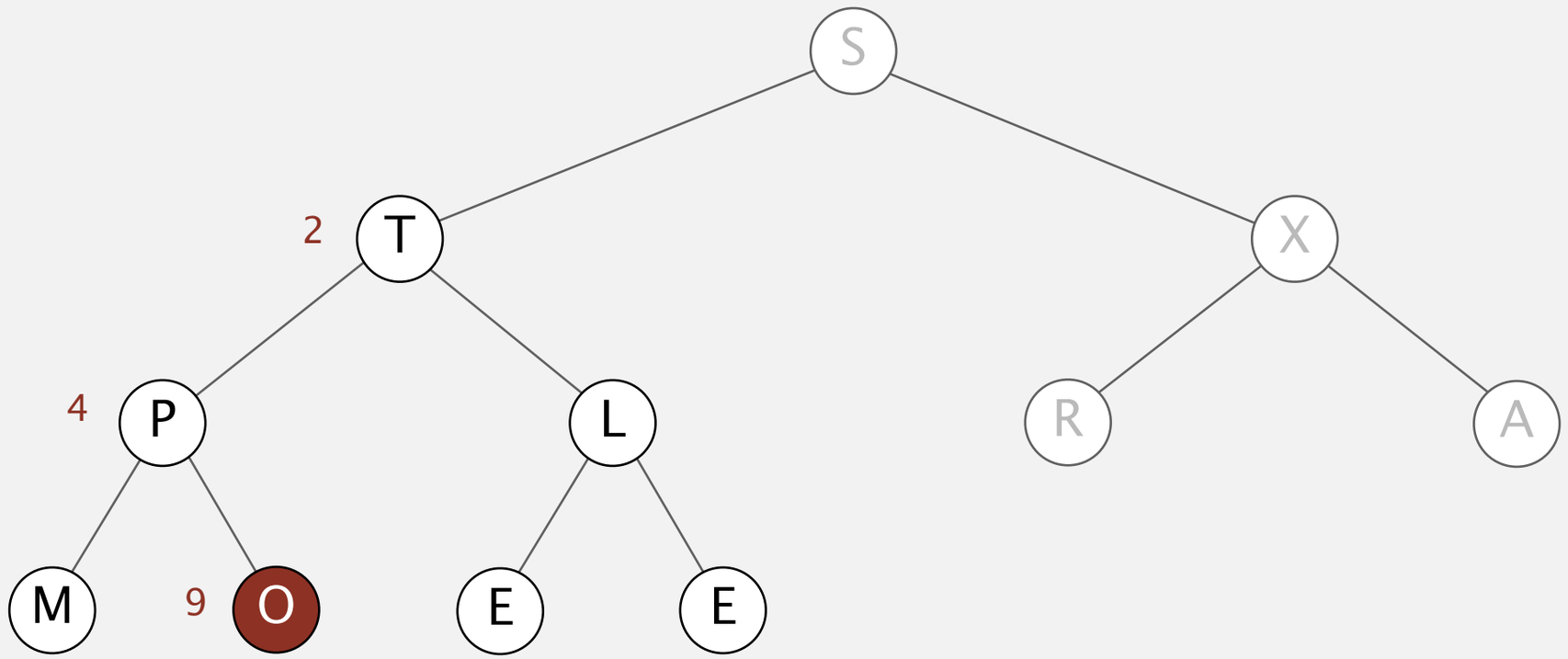


S	T	X	O	L	R	A	M	P	E	E
	2		4							

Heapsort demo

Heap construction. Build max heap using bottom-up method.

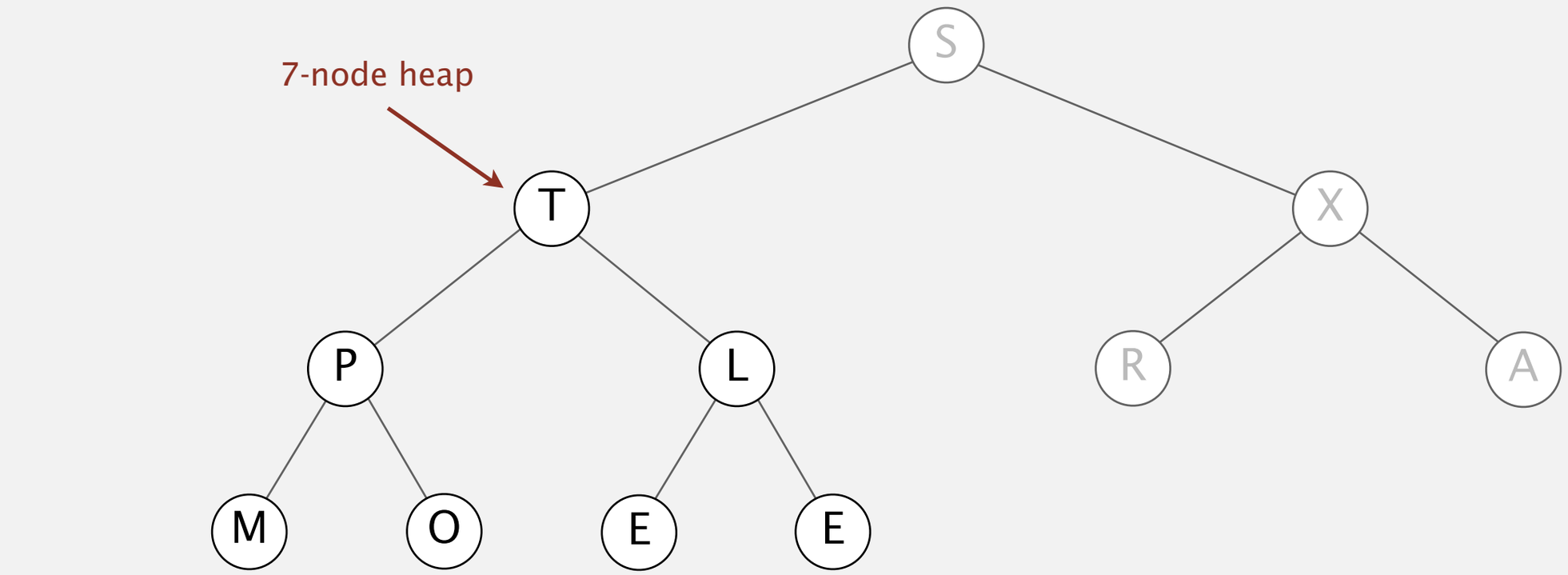
sink 2



S	T	X	P	L	R	A	M	O	E	E
	2		4					9		

Heapsort demo

Heap construction. Build max heap using bottom-up method.

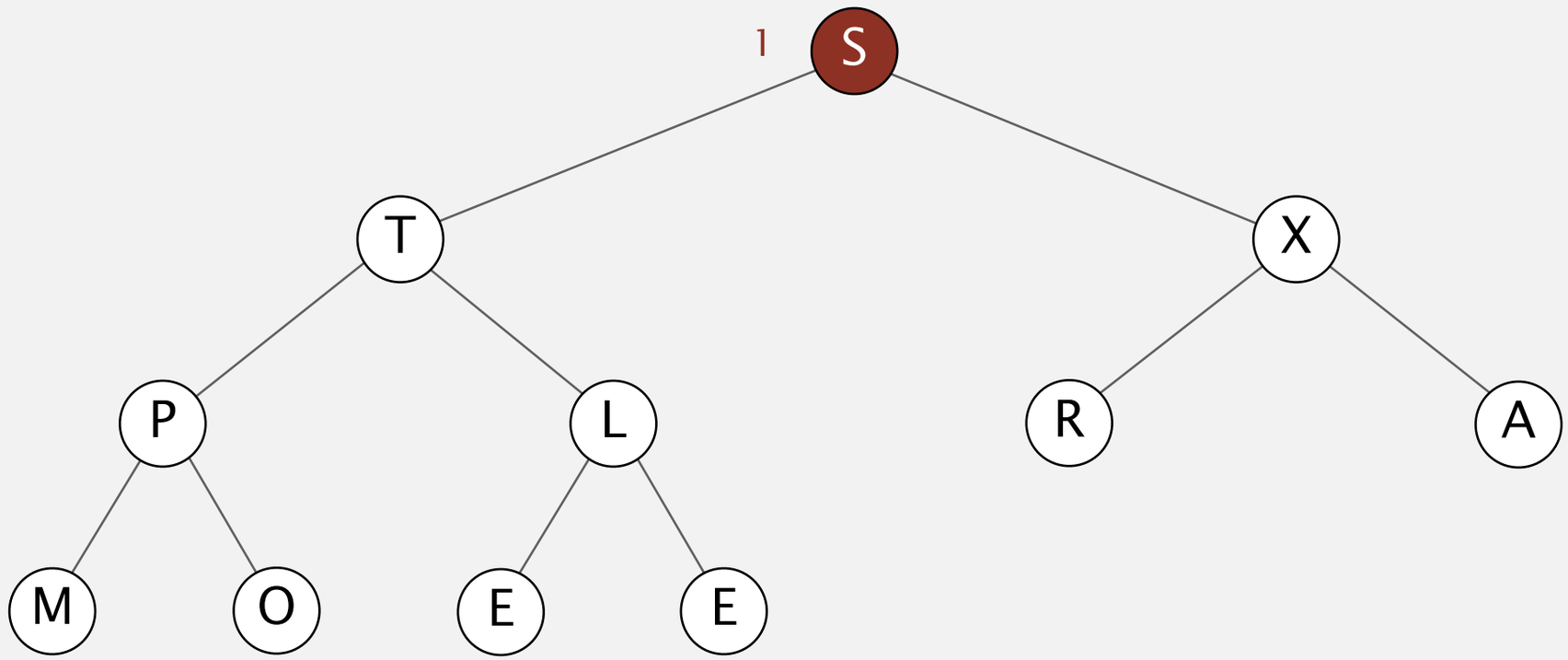


S	T	X	P	L	R	A	M	O	E	E

Heapsort demo

Heap construction. Build max heap using bottom-up method.

sink 1

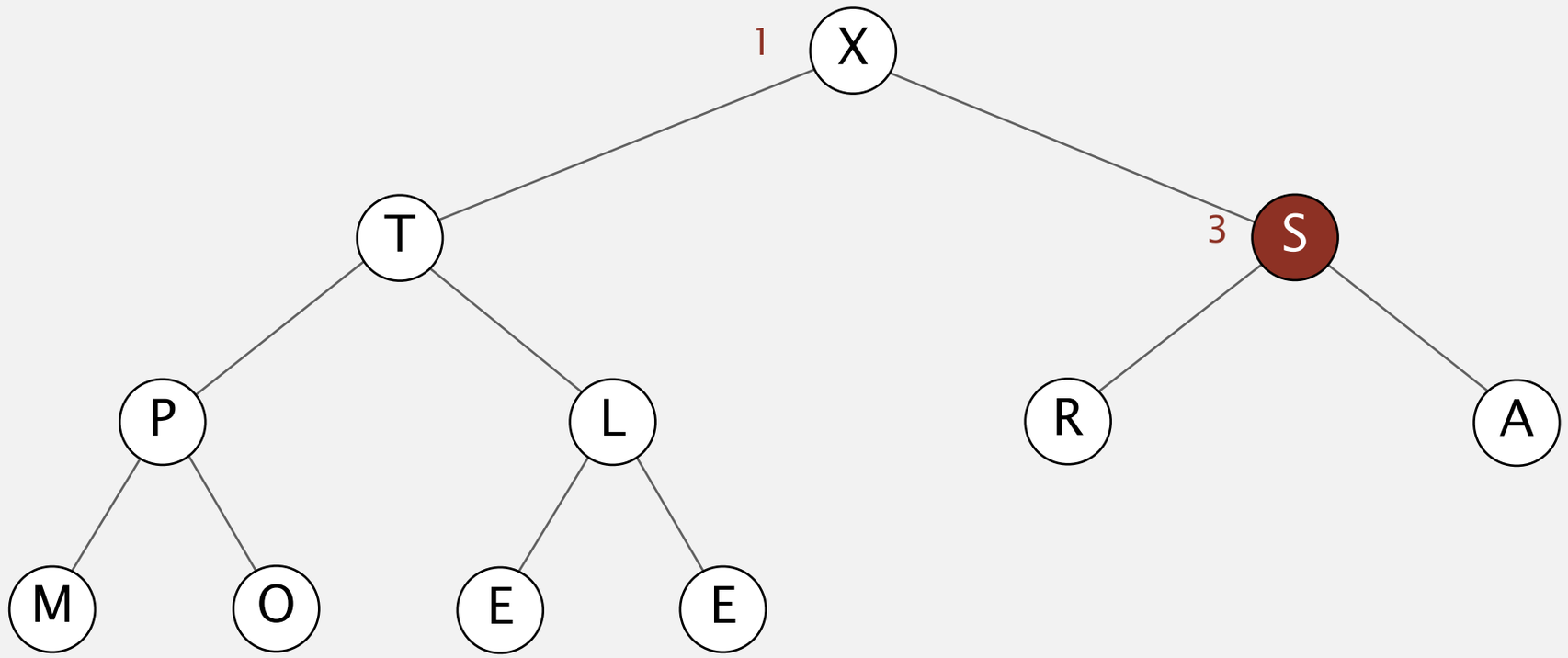


S	T	X	P	L	R	A	M	O	E	E
1										

Heapsort demo

Heap construction. Build max heap using bottom-up method.

sink 1

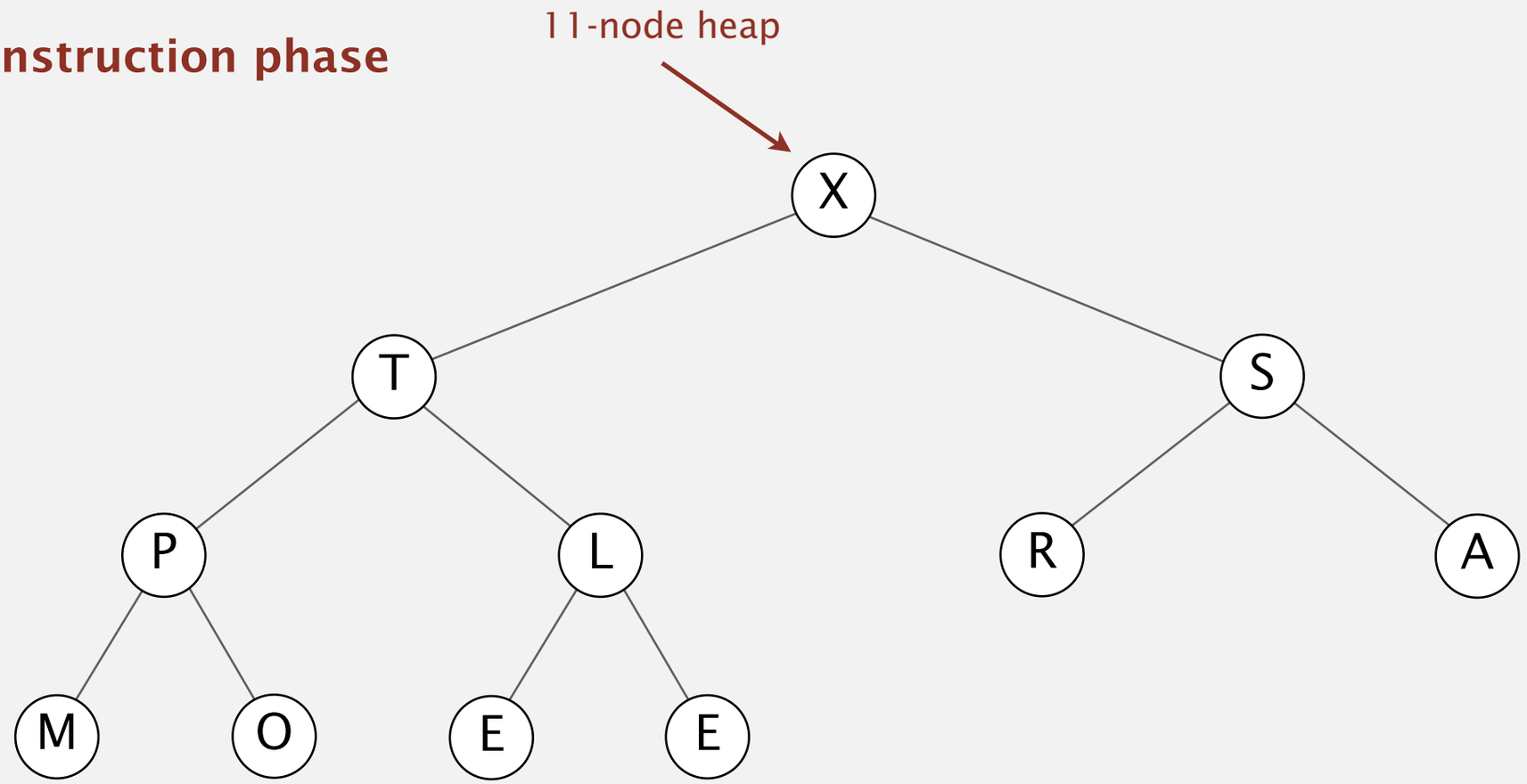


X	T	S	P	L	R	A	M	O	E	E
1		3								

Heapsort demo

Heap construction. Build max heap using bottom-up method.

end of construction phase

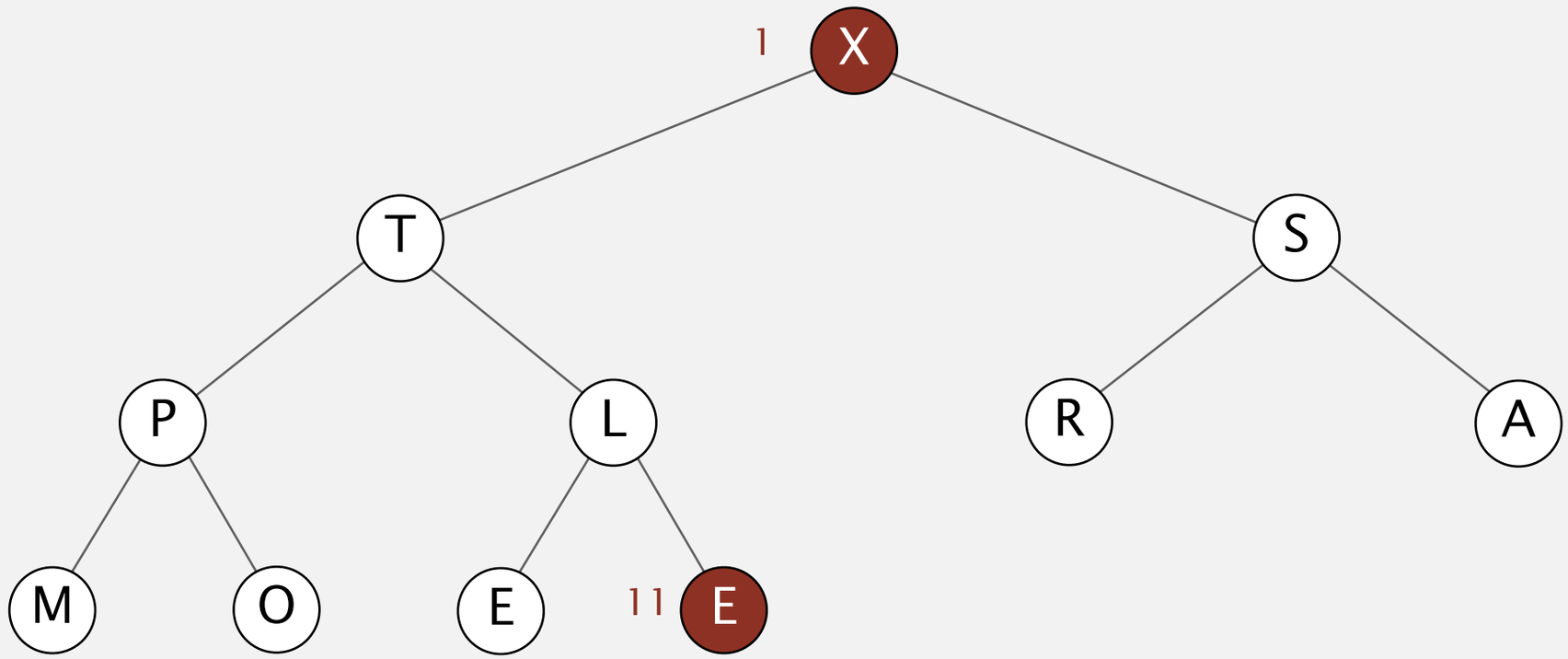


X	T	S	P	L	R	A	M	O	E	E

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

exchange 1 and 11

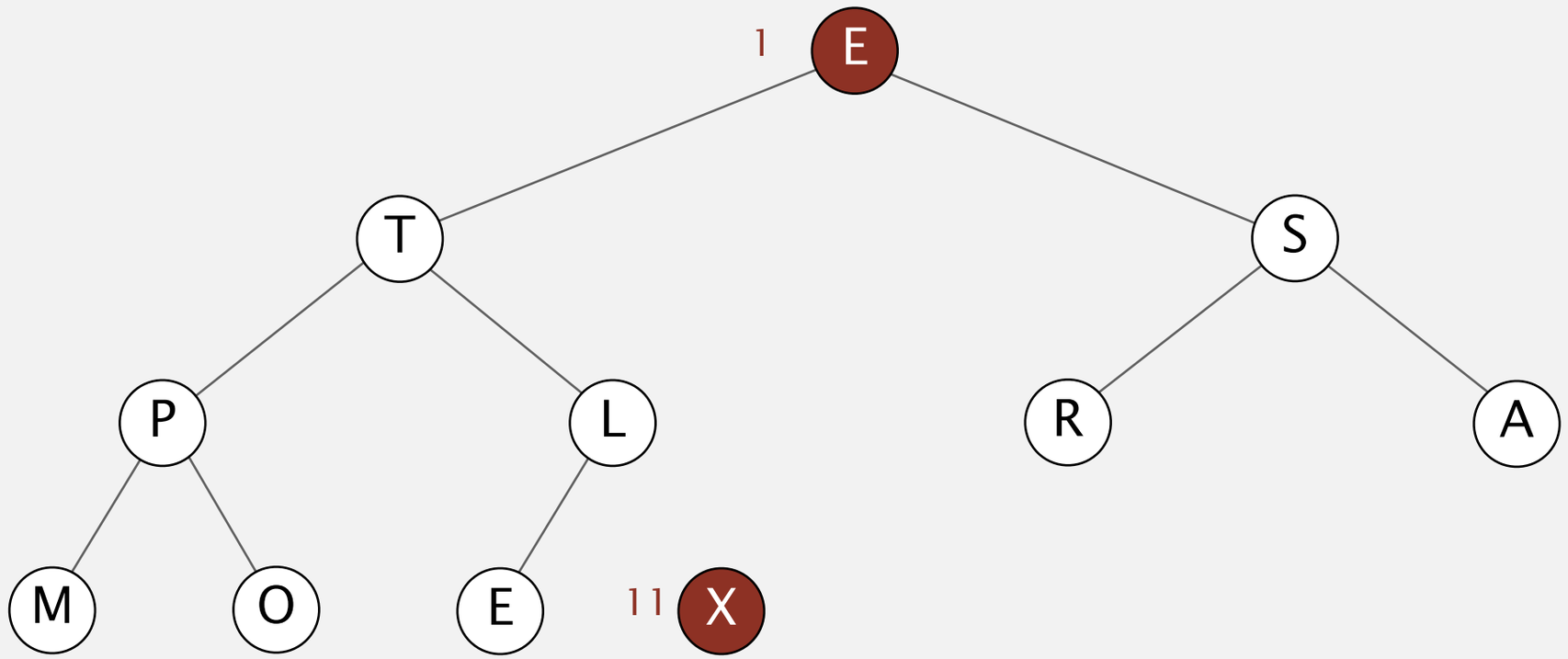


X	T	S	P	L	R	A	M	O	E	E
1										11

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

exchange 1 and 11

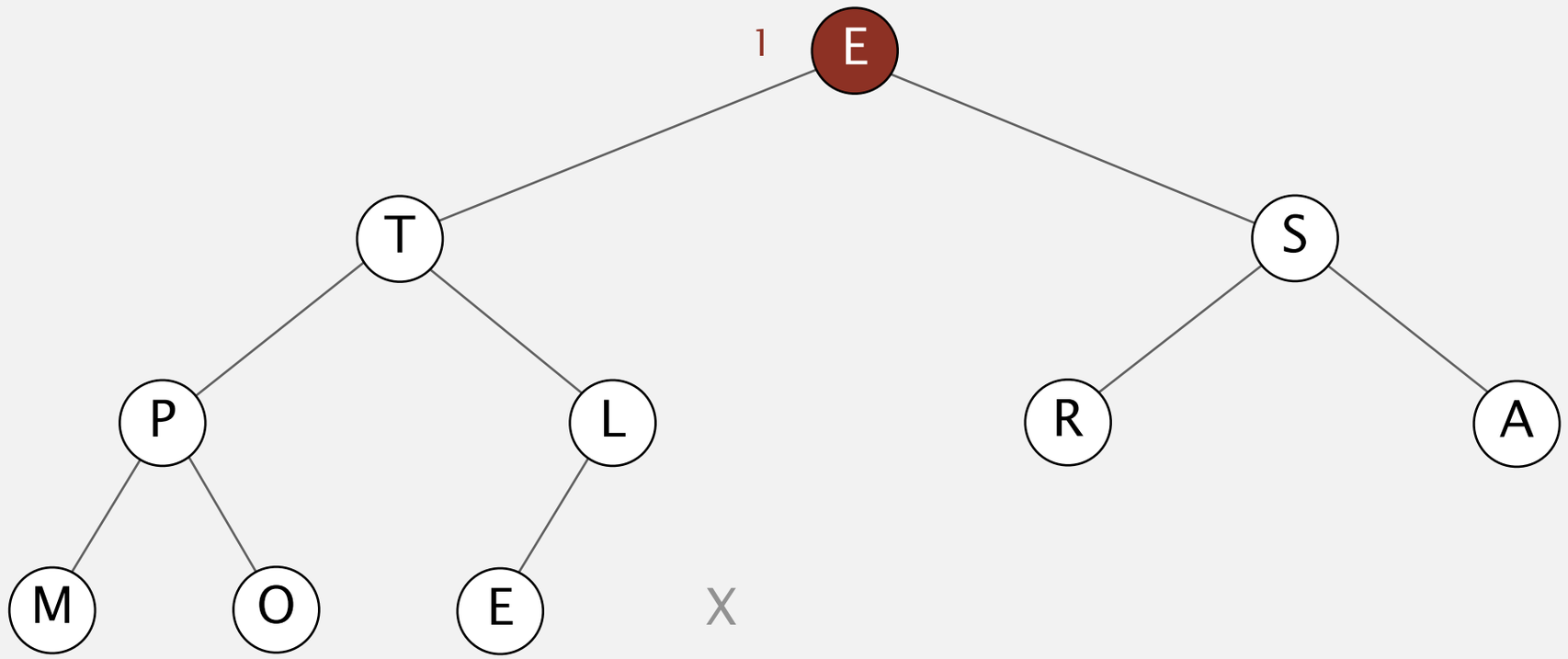


E	T	S	P	L	R	A	M	O	E	X
1										11

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

sink 1

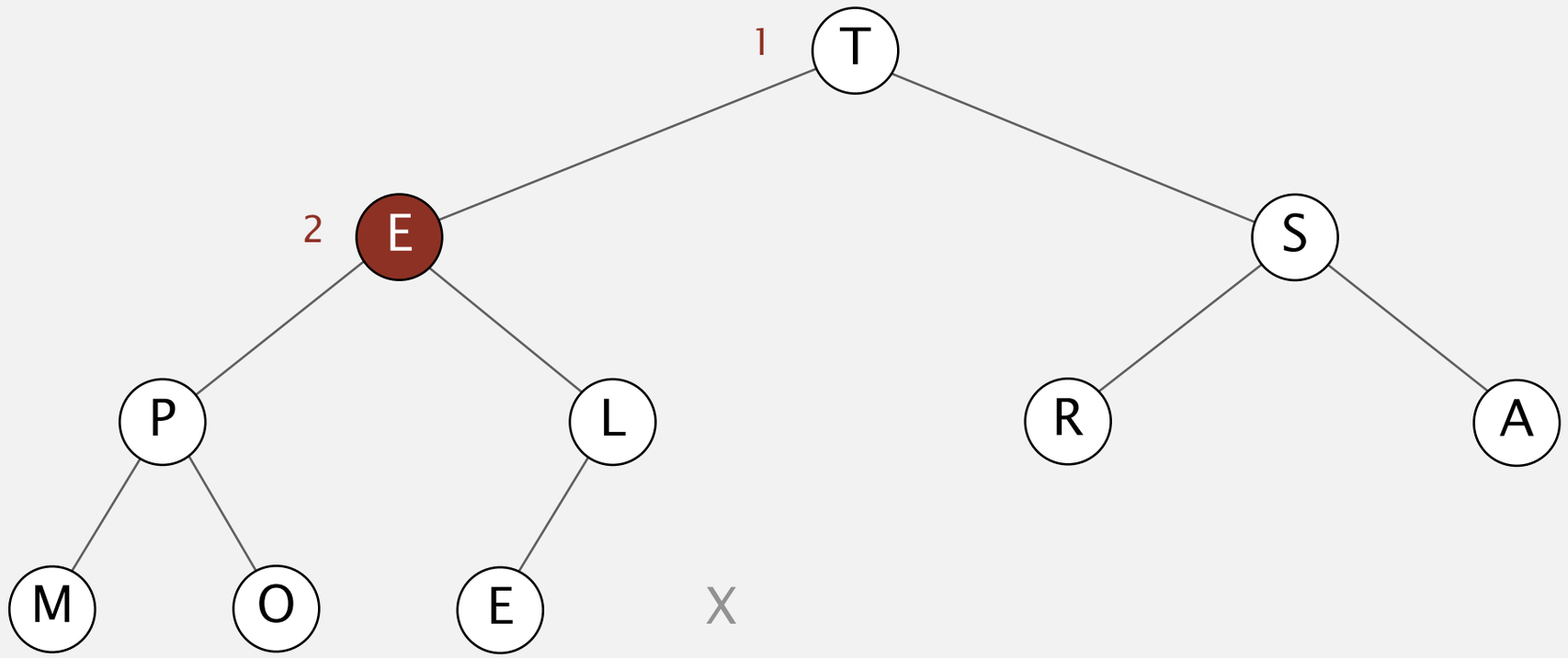


E	T	S	P	L	R	A	M	O	E	X
1										

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

sink 1

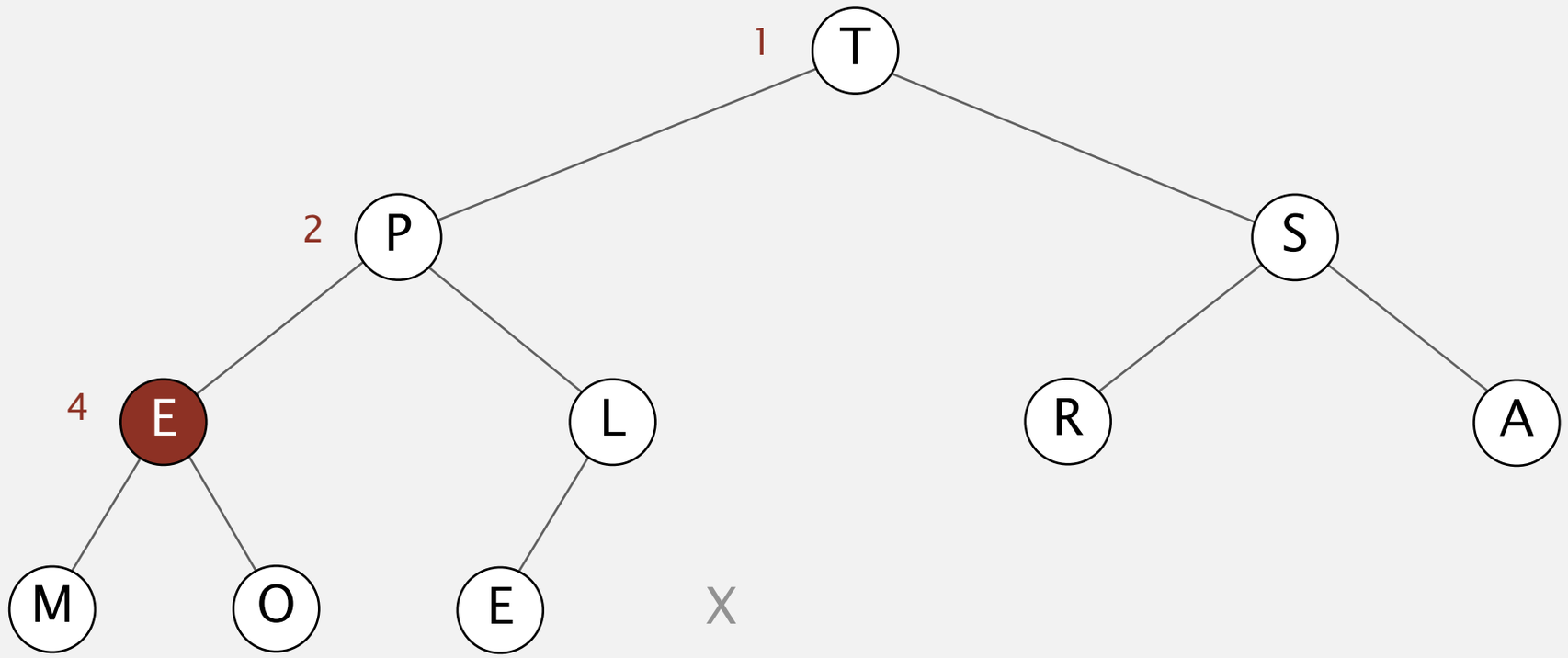


T	E	S	P	L	R	A	M	O	E	X
1	2									

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

sink 1

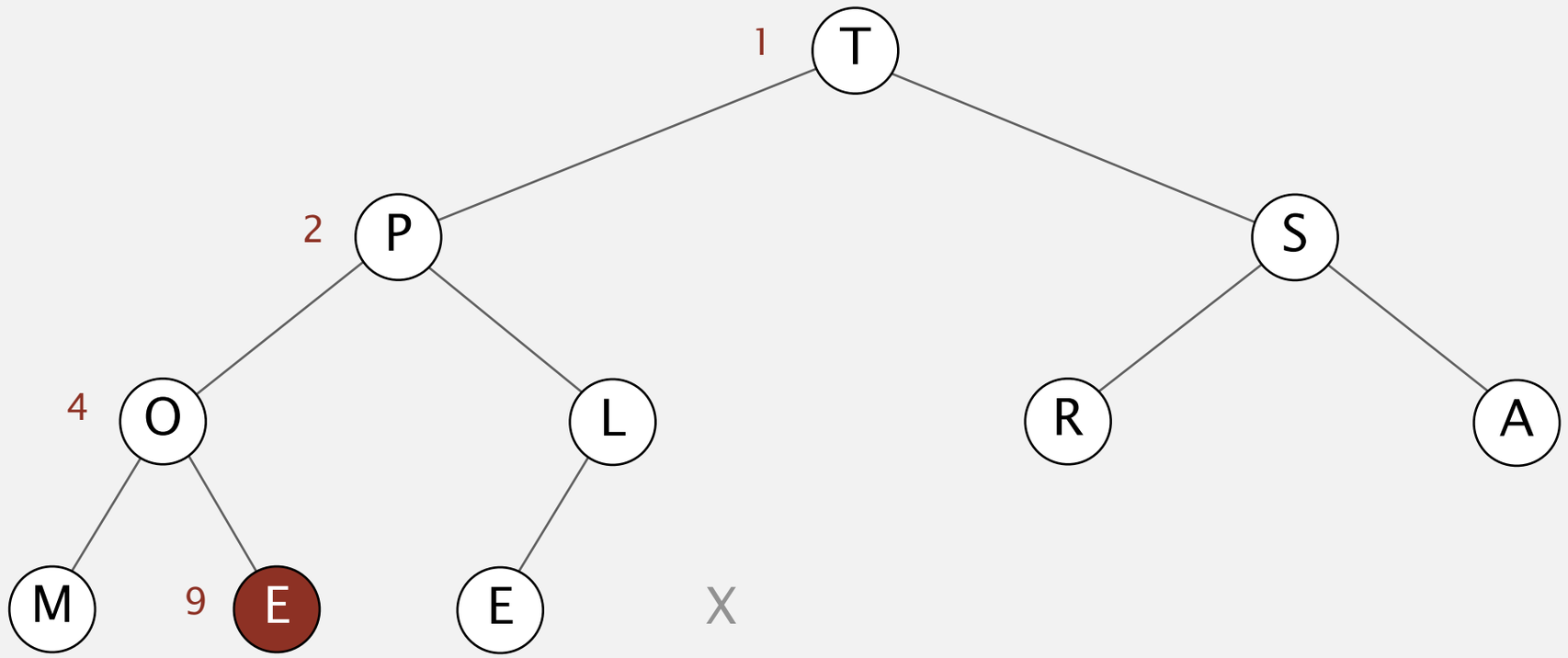


T	P	S	E	L	R	A	M	O	E	X
1	2		4							

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

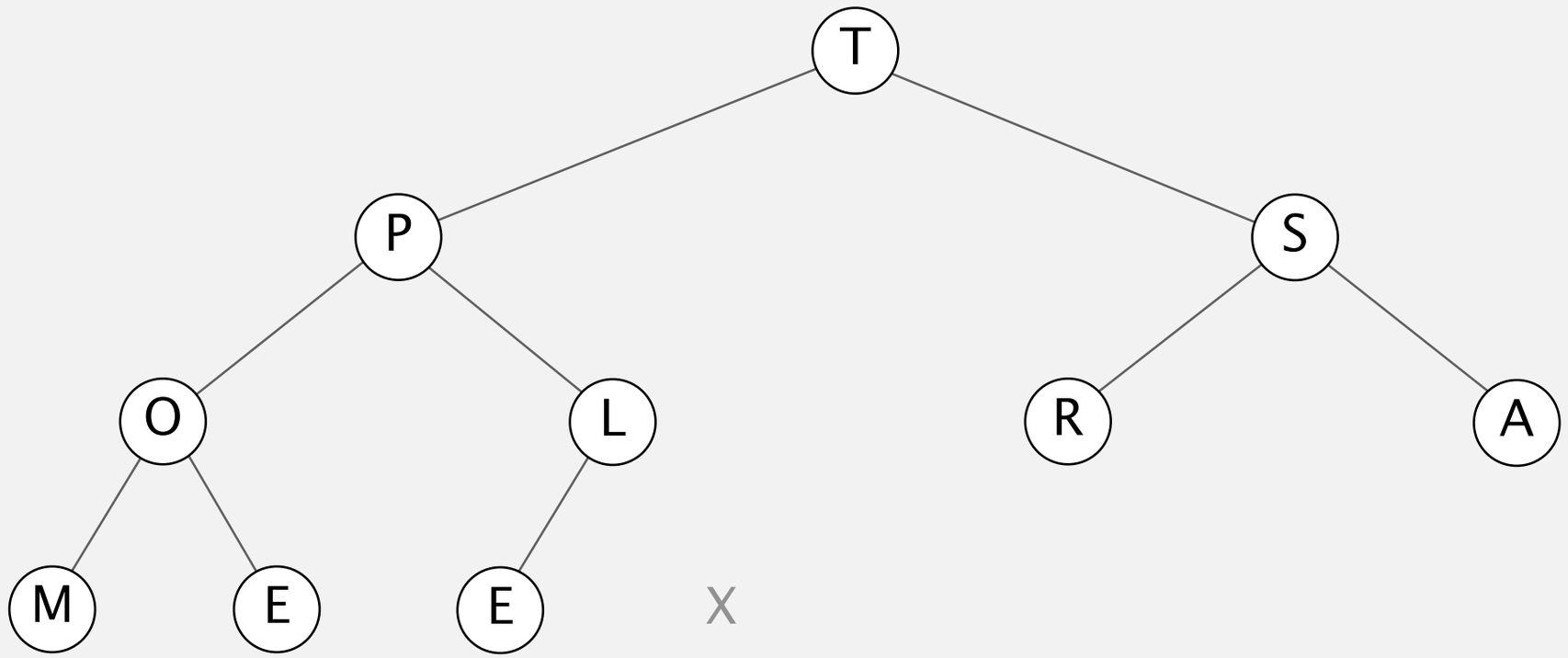
sink 1



T	P	S	O	L	R	A	M	E	E	X
1	2		4					9		

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

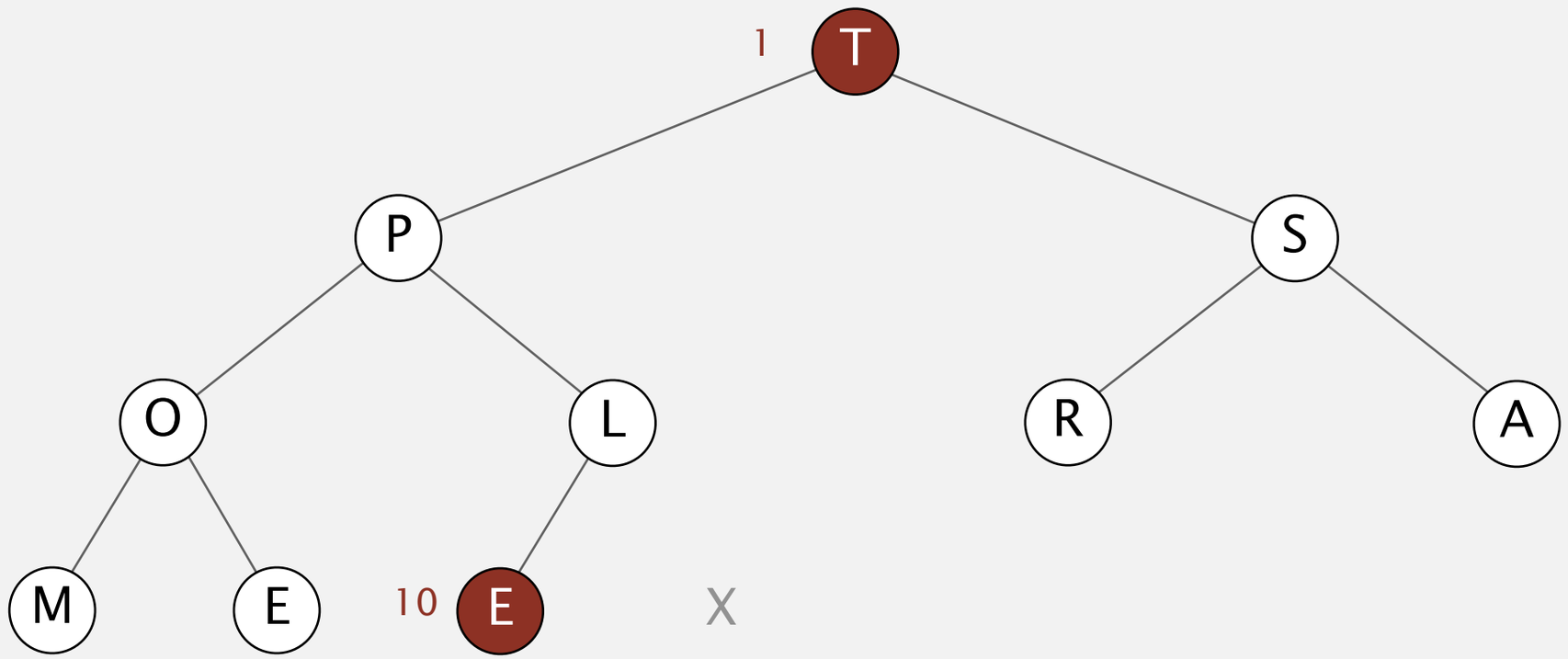


T	P	S	O	L	R	A	M	E	E	X

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

exchange 1 and 10

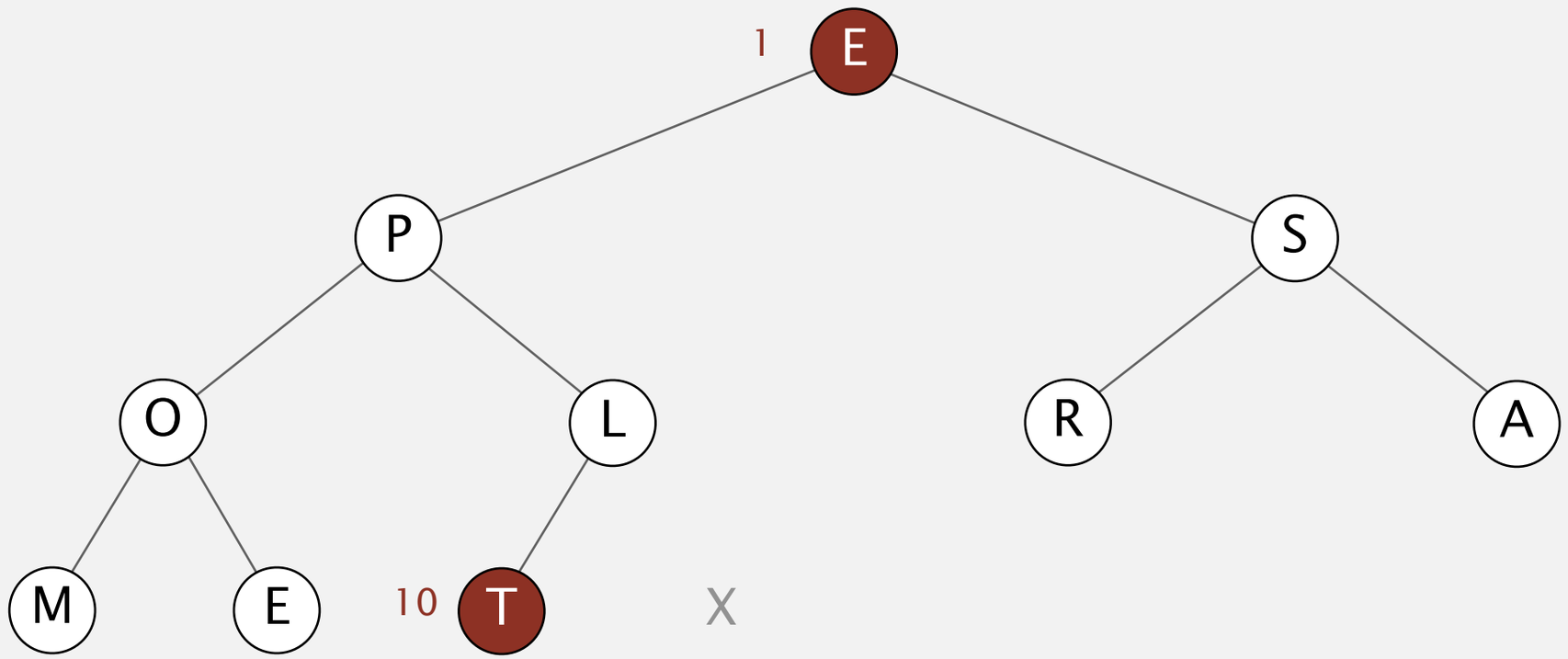


T	P	S	O	L	R	A	M	E	E	X
1									10	

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

exchange 1 and 10

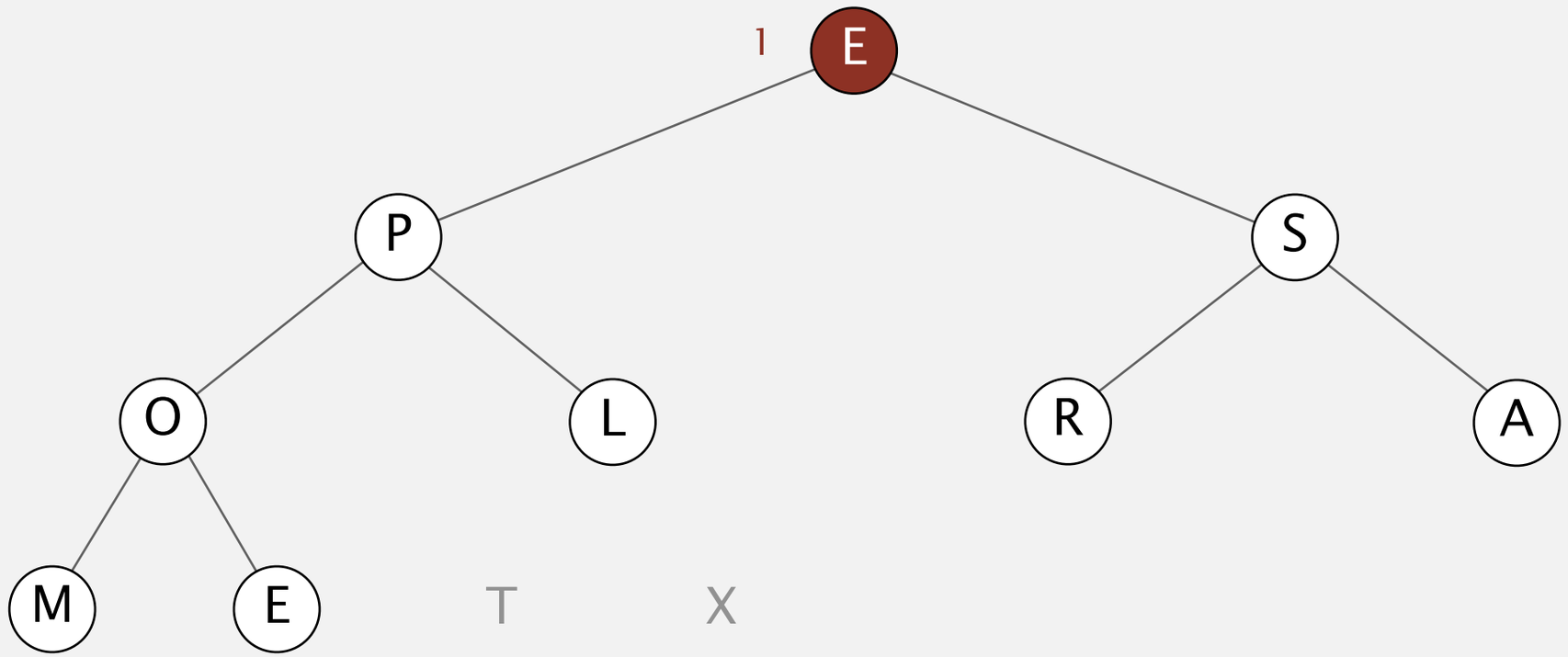


E	P	S	O	L	R	A	M	E	T	X
1									10	

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

sink 1

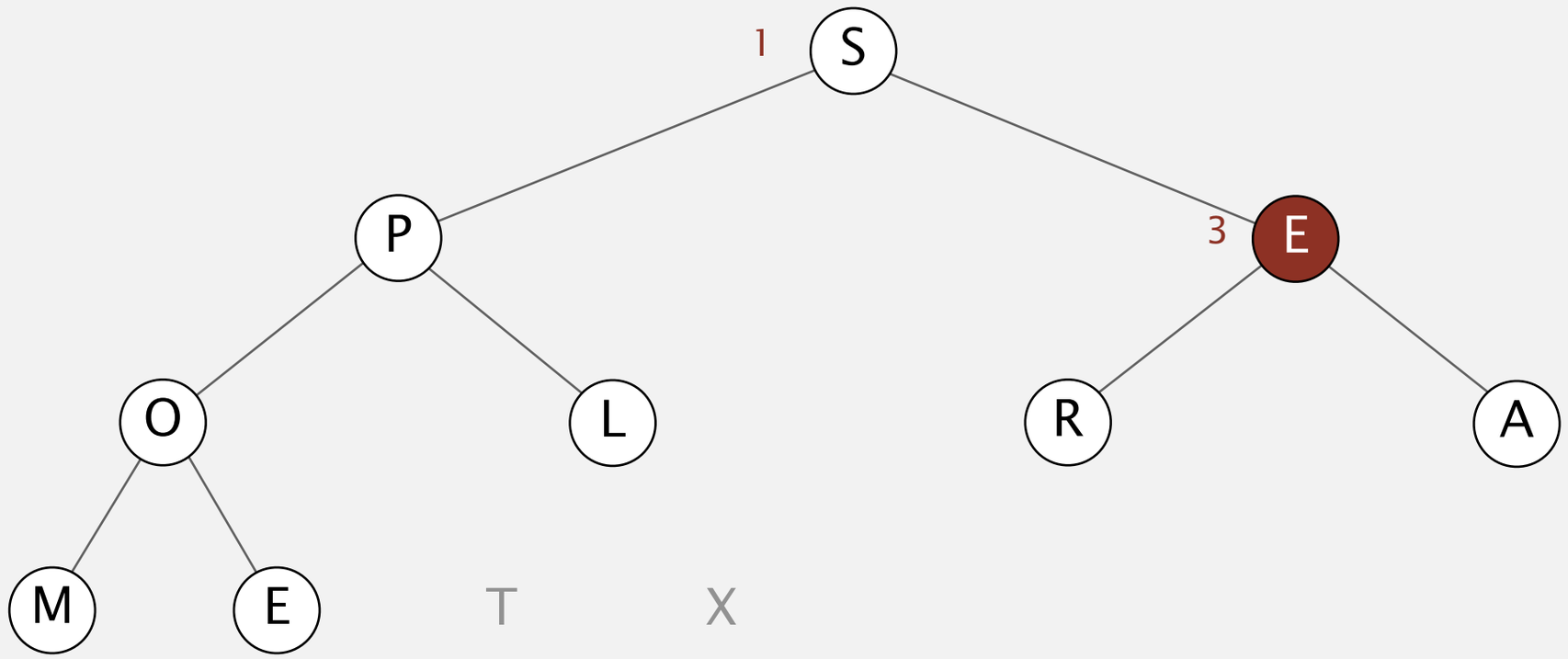


E	P	S	O	L	R	A	M	E	T	X
1										

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

sink 1

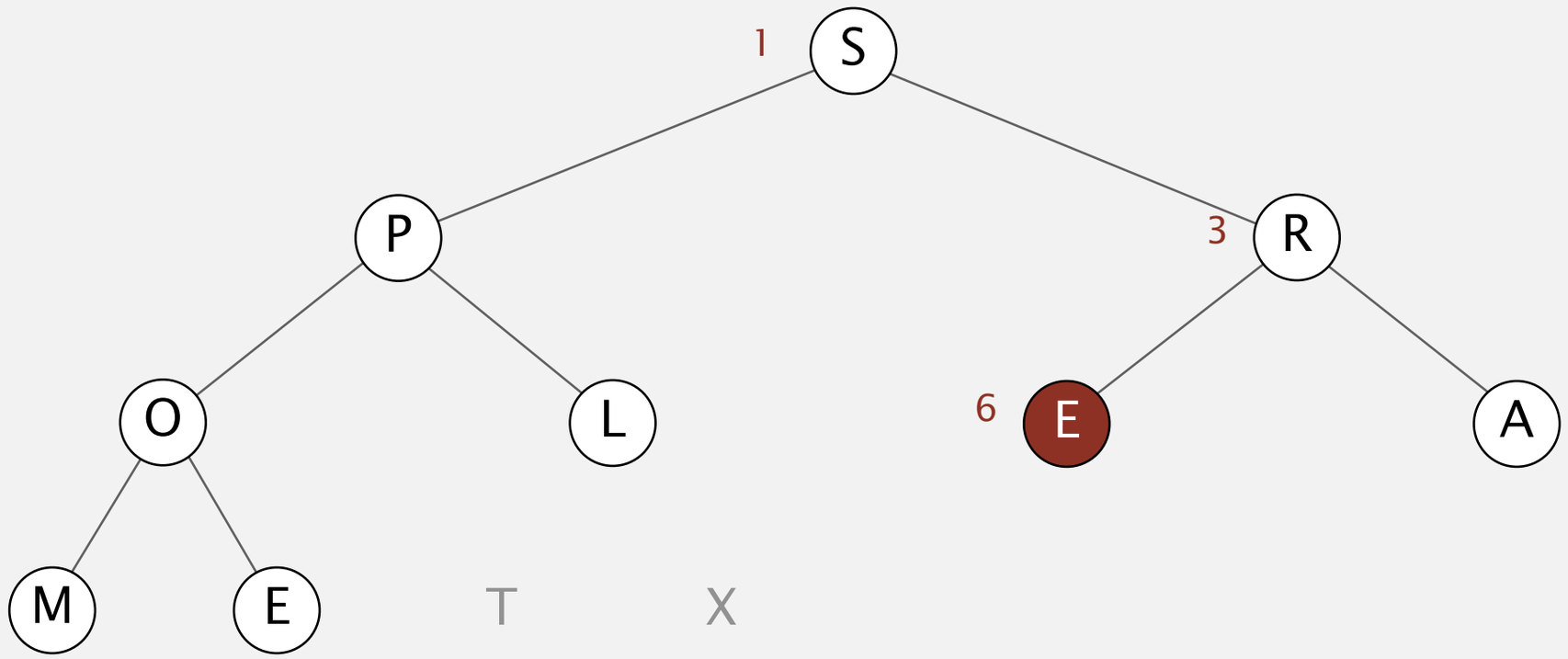


S	P	E	O	L	R	A	M	E	T	X
1		3								

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

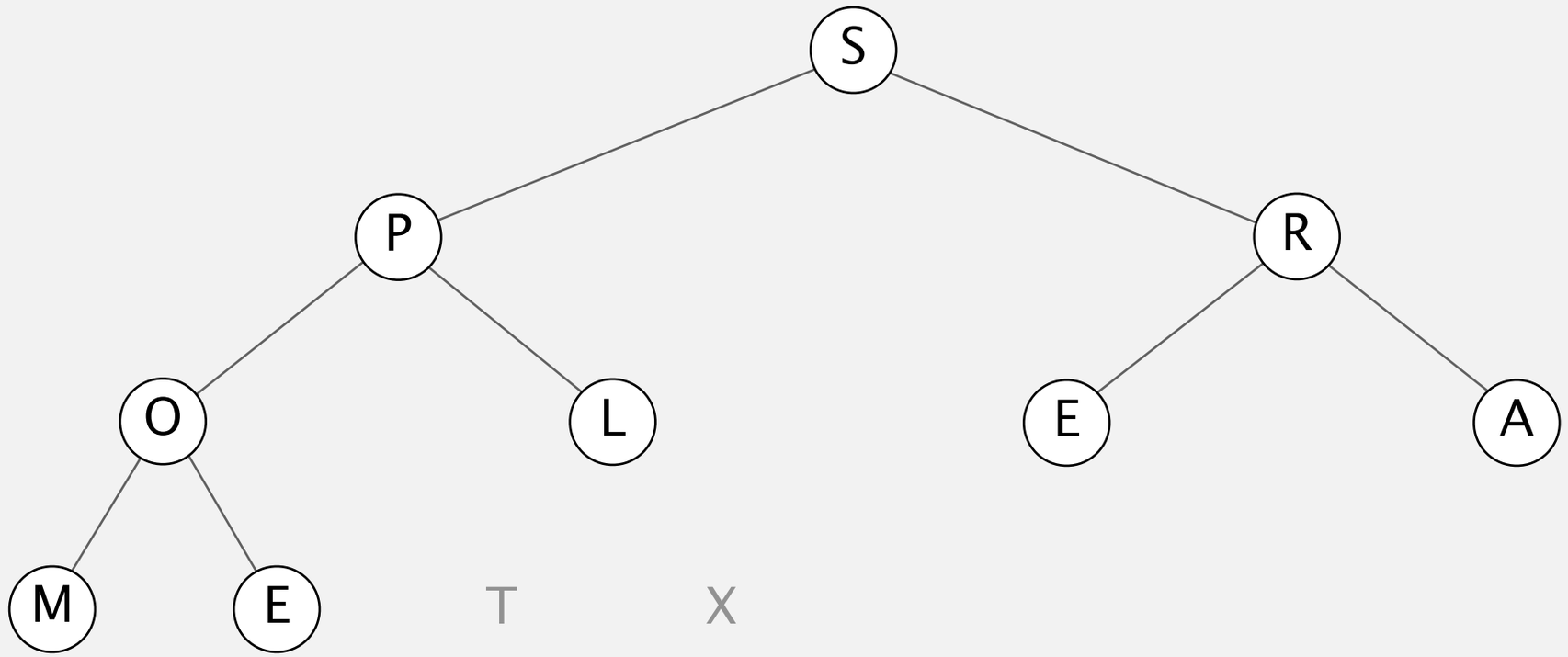
sink 1



S	P	R	O	L	E	A	M	E	T	X
1		3			6					

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

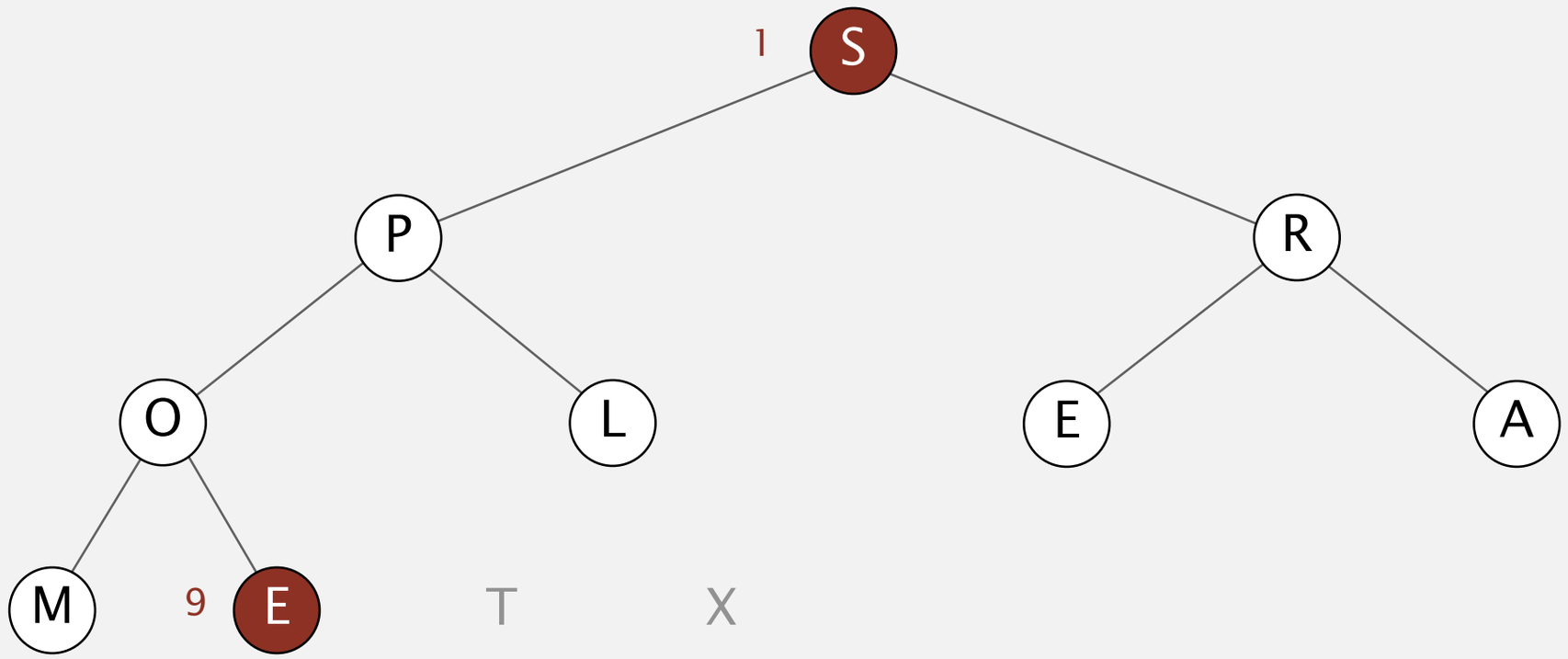


S	P	R	O	L	E	A	M	E	T	X

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

exchange 1 and 9

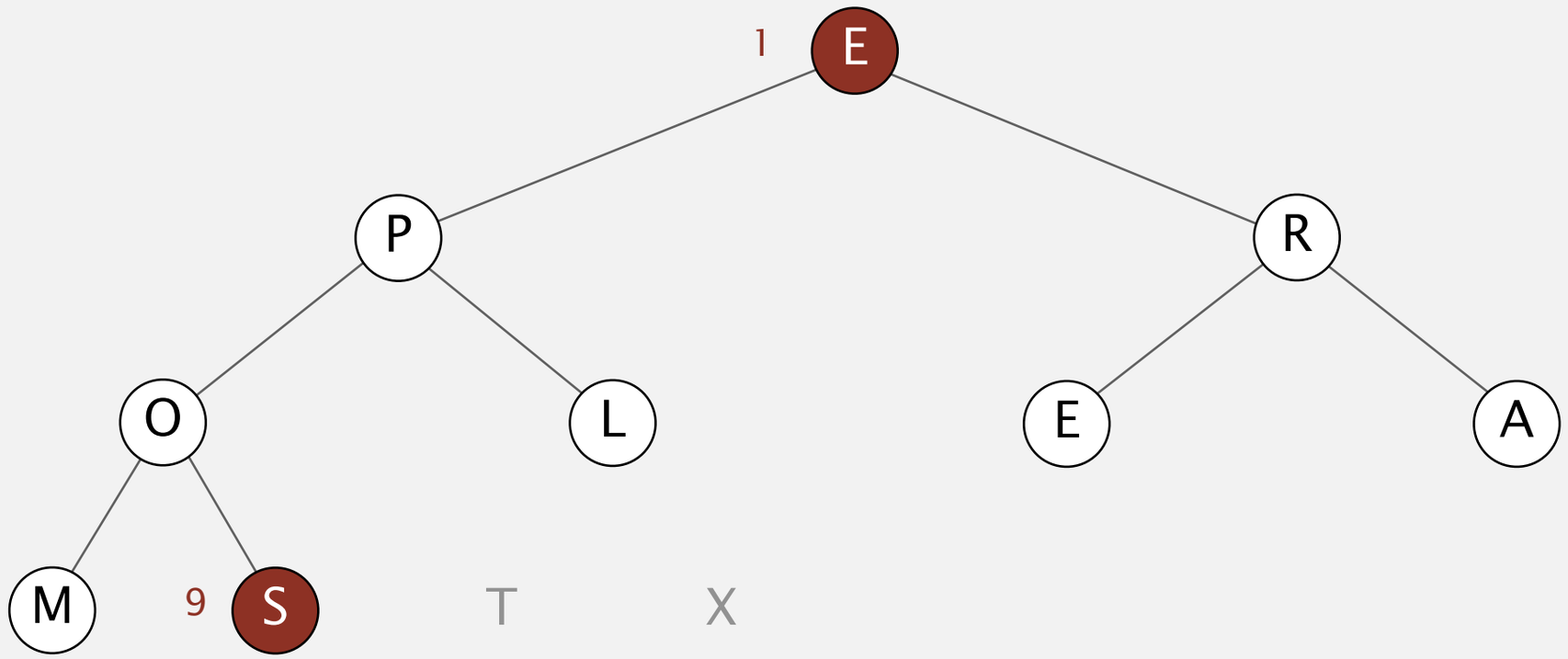


S	P	R	O	L	E	A	M	E	T	X
1								9		

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

exchange 1 and 9

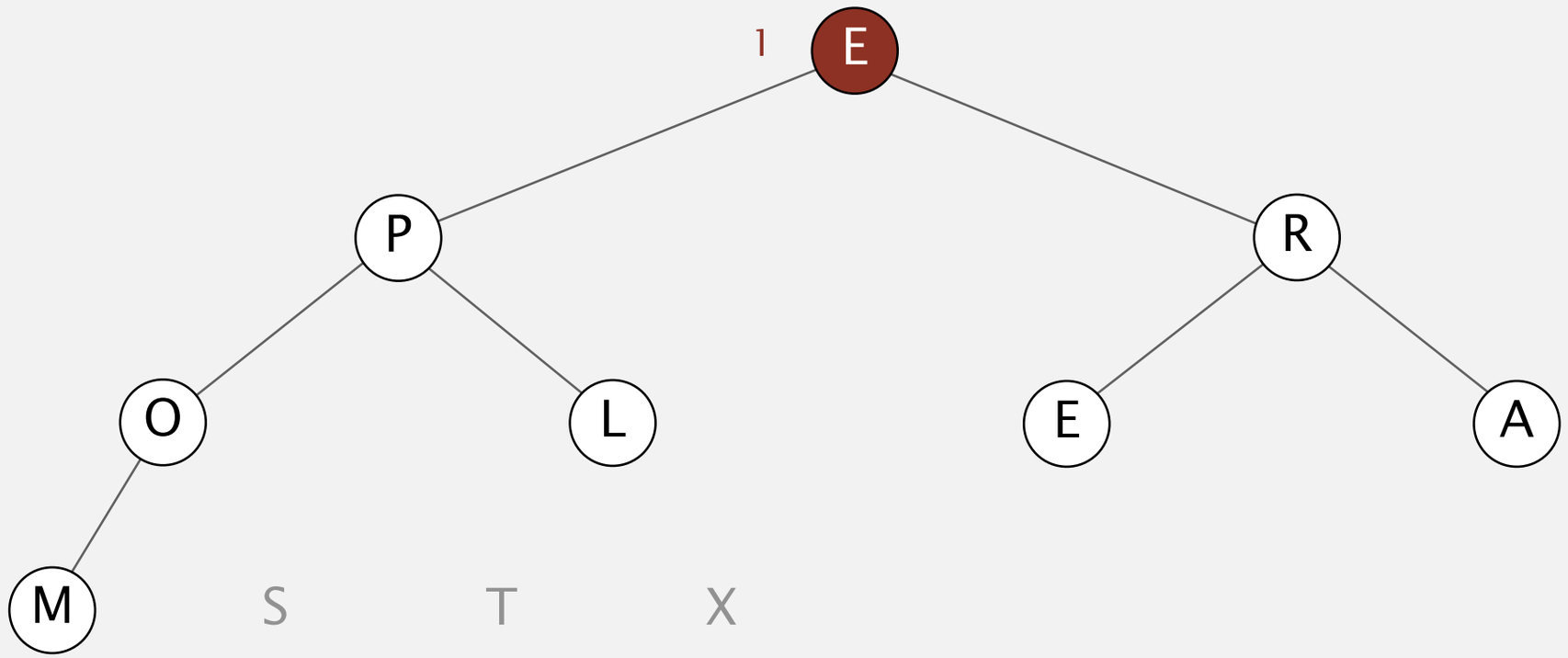


E	P	R	O	L	E	A	M	S	T	X
1								9		

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

sink 1

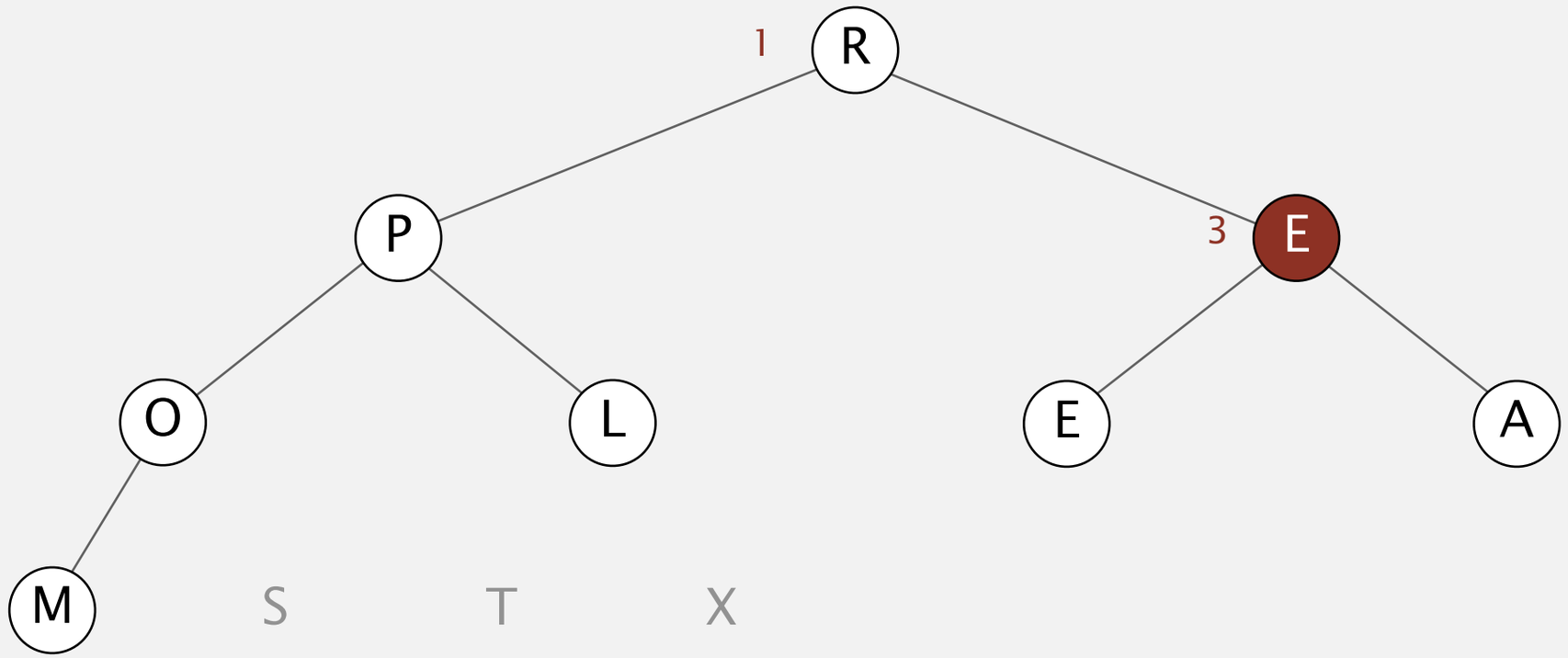


E	P	R	O	L	E	A	M	S	T	X
1										

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

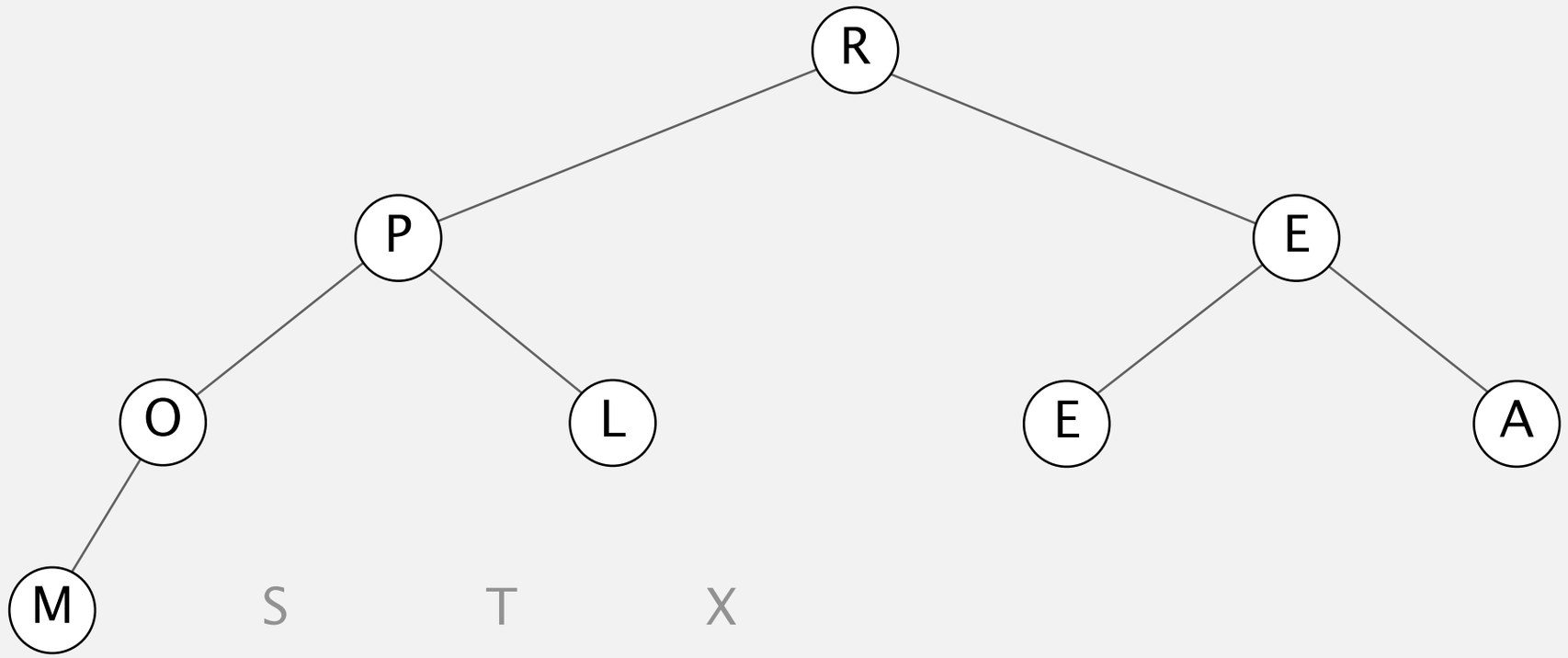
sink 1



R	P	E	O	L	E	A	M	S	T	X
1		3								

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

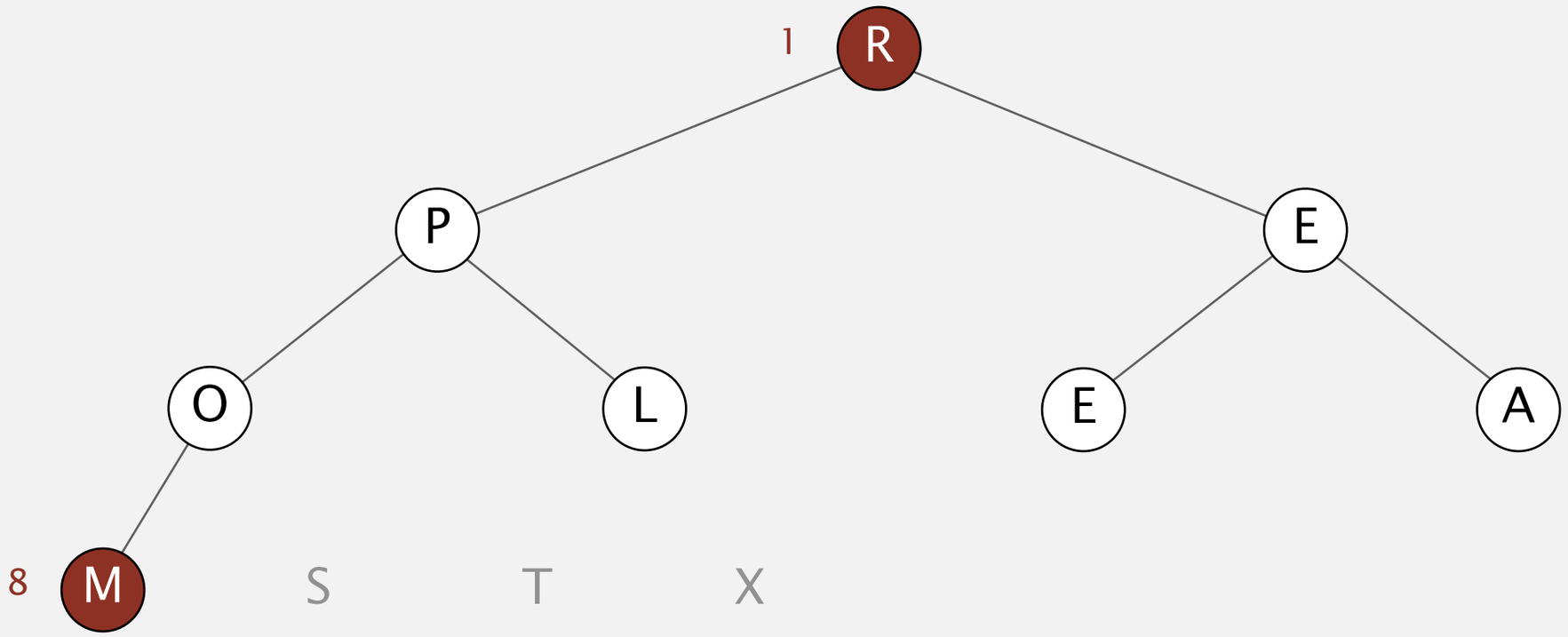


R	P	E	O	L	E	A	M	S	T	X

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

exchange 1 and 8

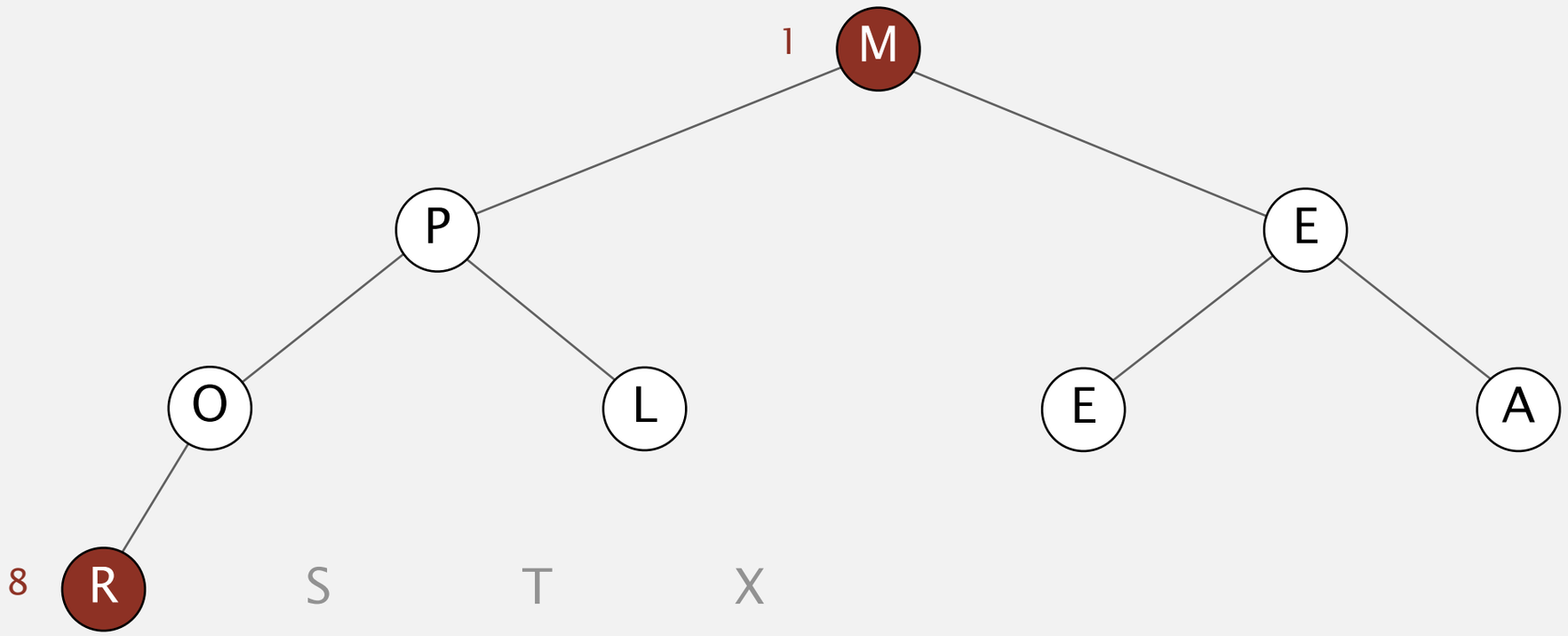


R	P	E	O	L	E	A	M	S	T	X
1							8			

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

exchange 1 and 8

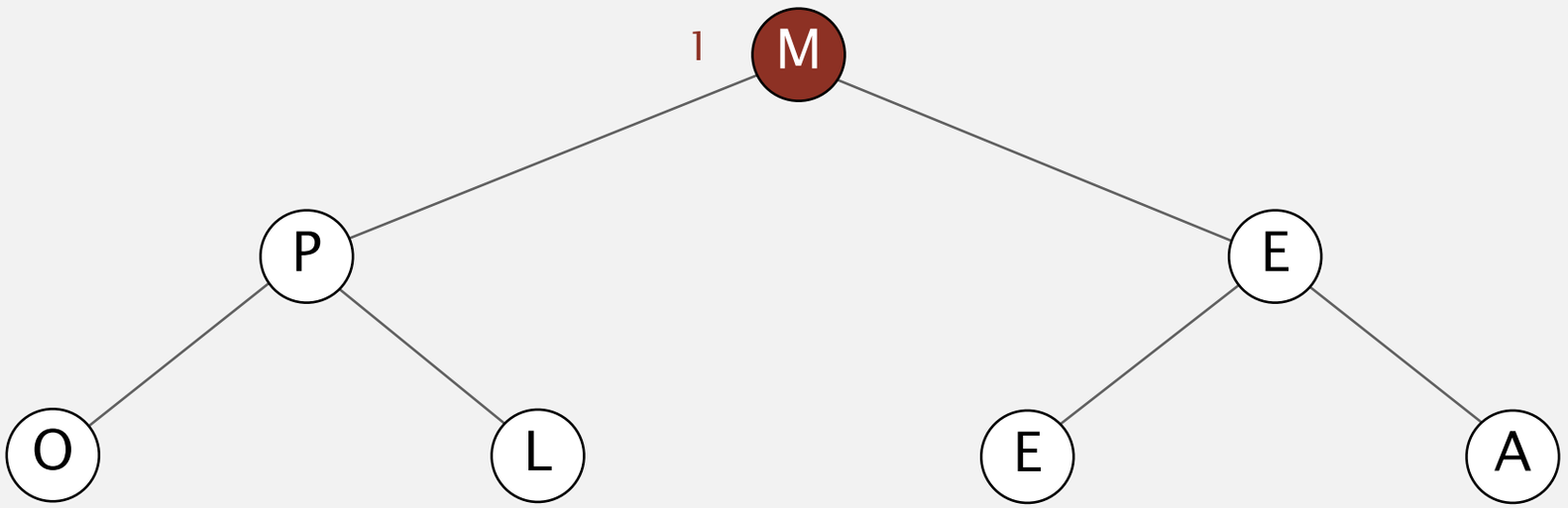


M	P	E	O	L	E	A	R	S	T	X
1							8			

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

sink 1



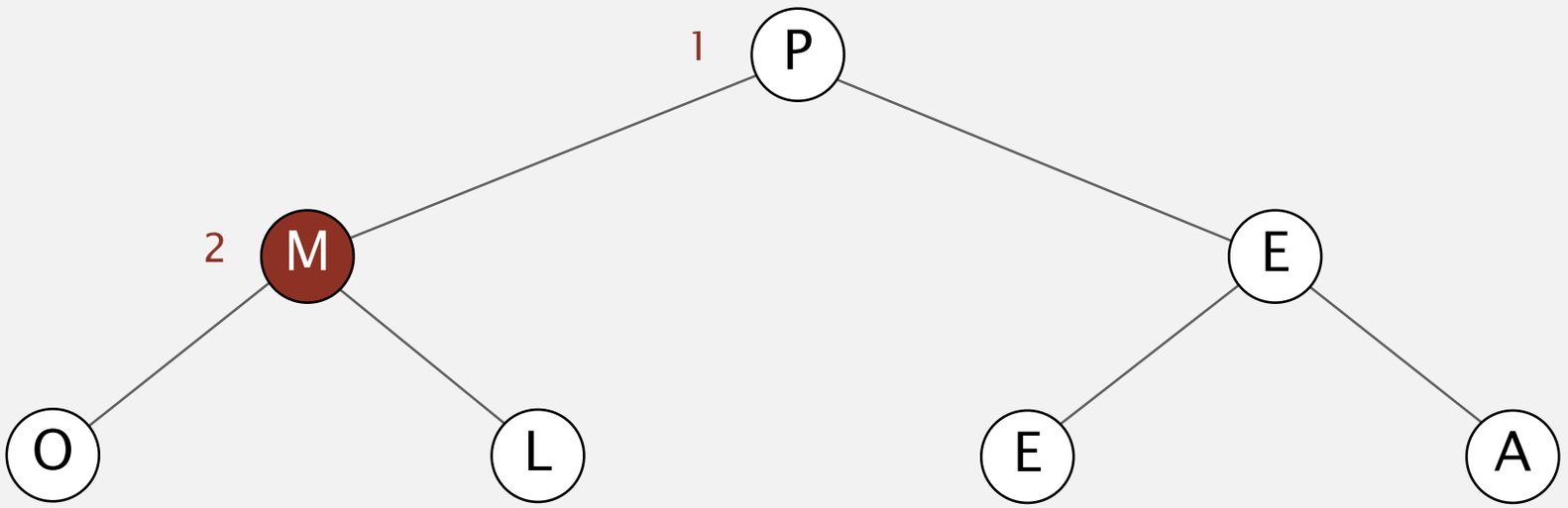
R S T X

M	P	E	O	L	E	A	R	S	T	X
1										

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

sink 1



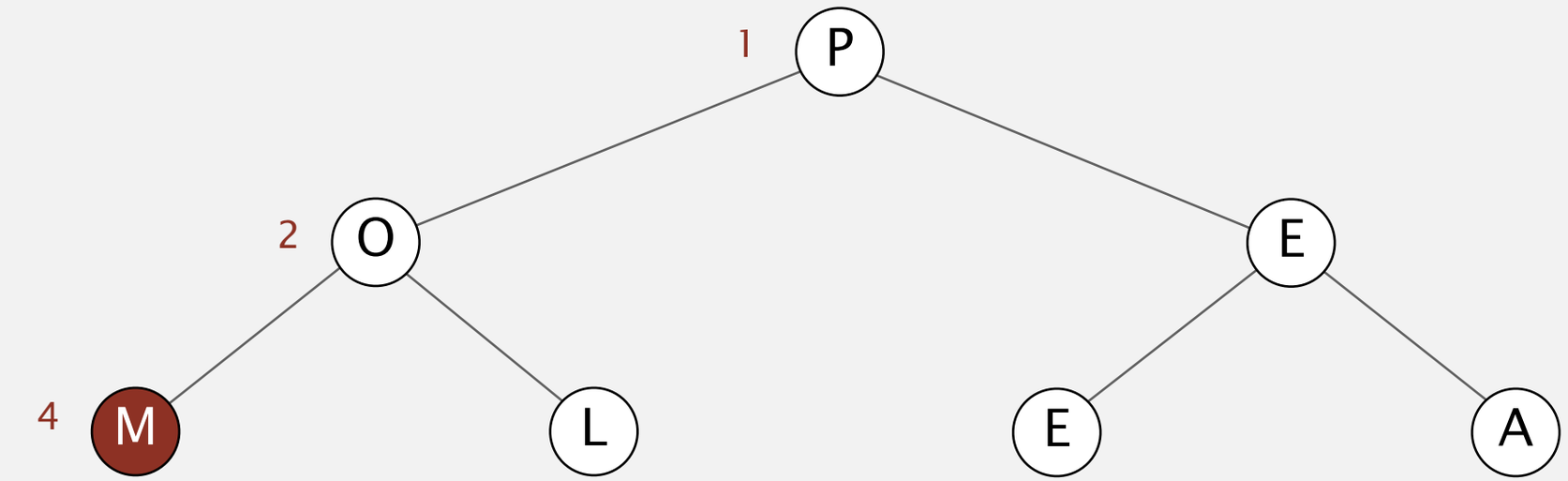
R S T X

P	M	E	O	L	E	A	R	S	T	X
1	2									

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

sink 1

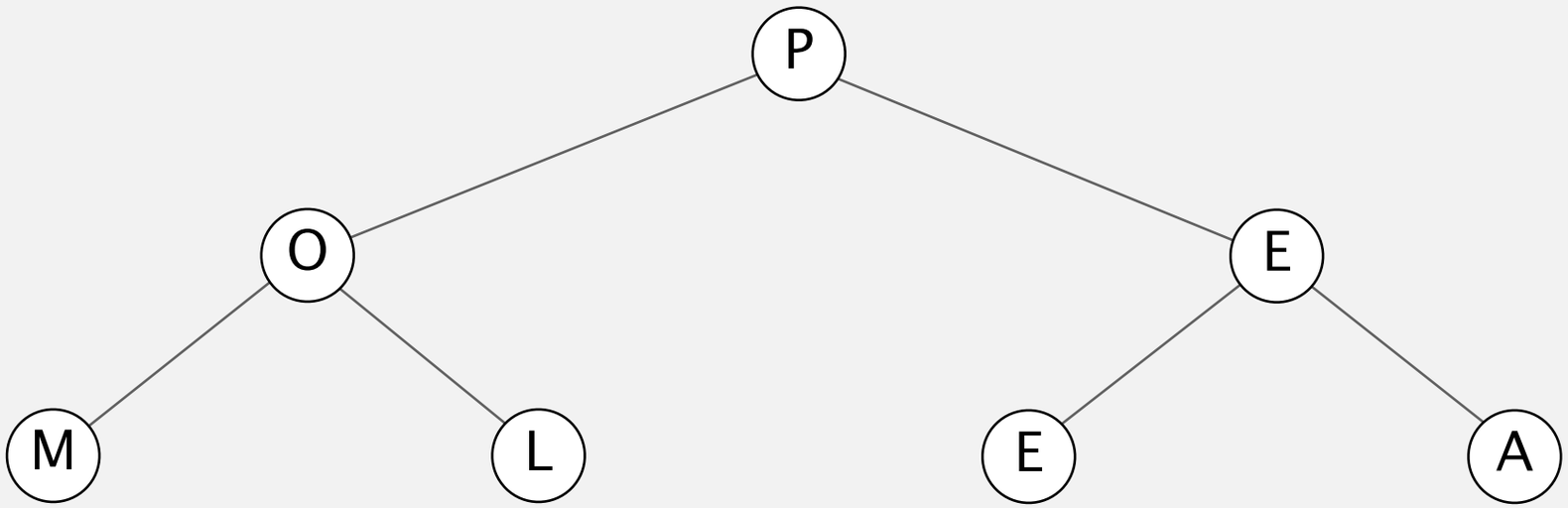


R S T X

P	O	E	M	L	E	A	R	S	T	X
1	2		4							

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

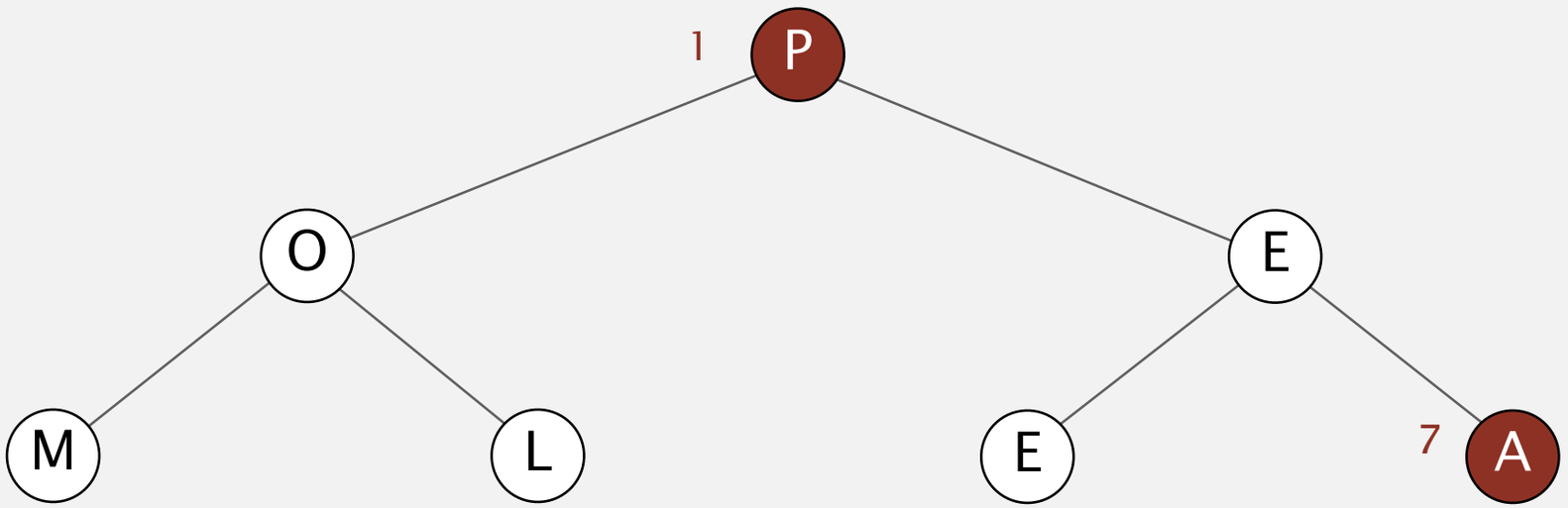


P	O	E	M	L	E	A	R	S	T	X

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

exchange 1 and 7



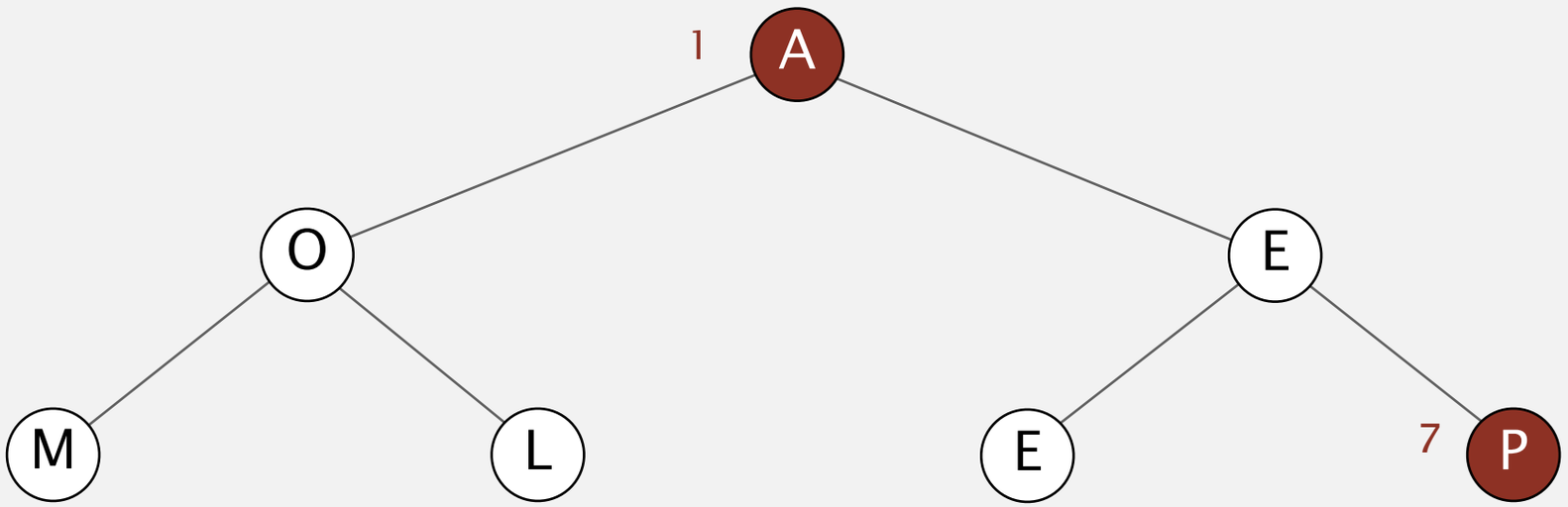
R S T X

P	O	E	M	L	E	A	R	S	T	X
1						7				

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

exchange 1 and 7



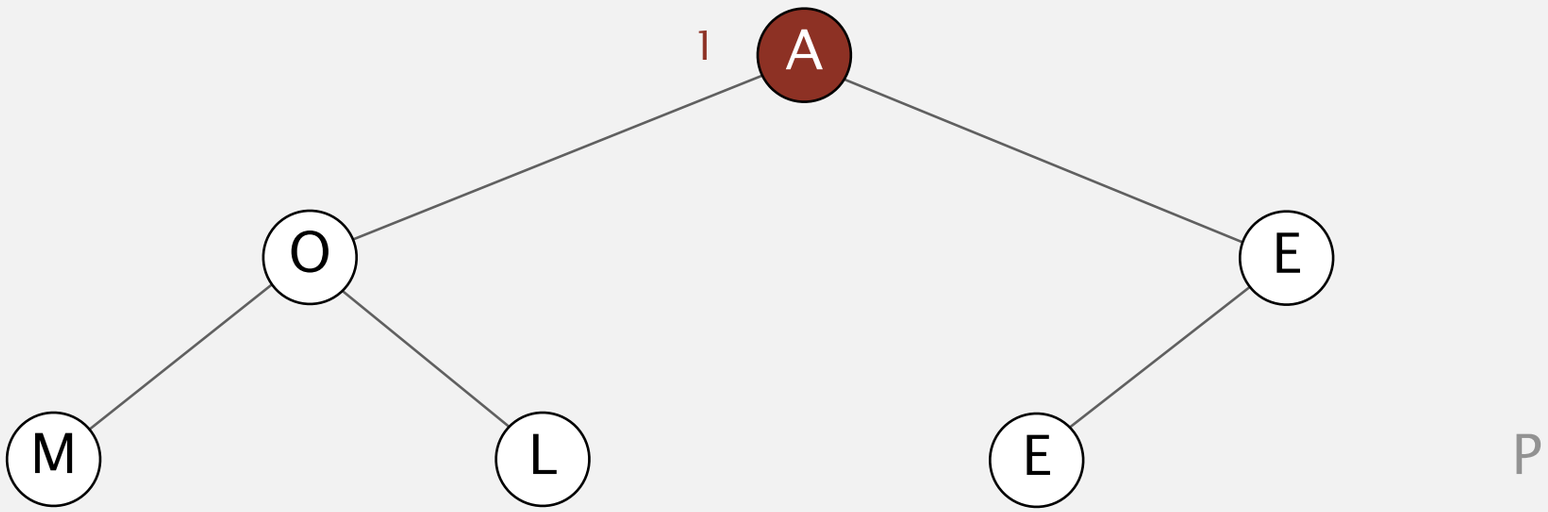
R S T X

A	O	E	M	L	E	P	R	S	T	X
1						7				

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

sink 1

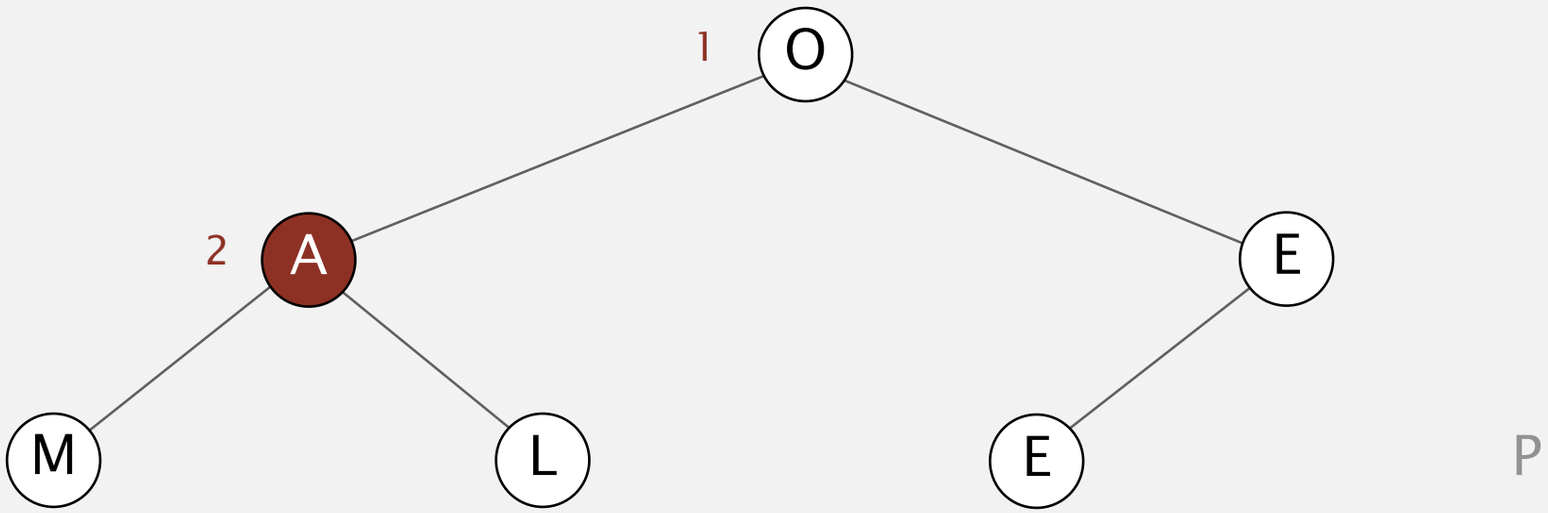


A	O	E	M	L	E	P	R	S	T	X
1										

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

sink 1

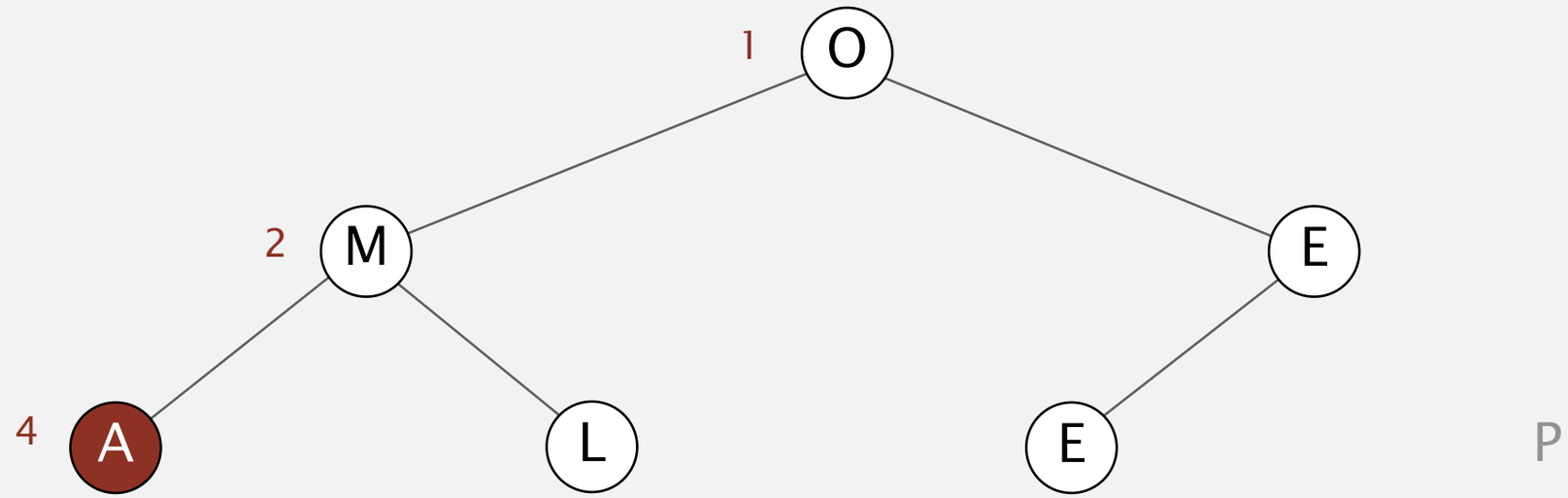


O	A	E	M	L	E	P	R	S	T	X
1	2									

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

sink 1

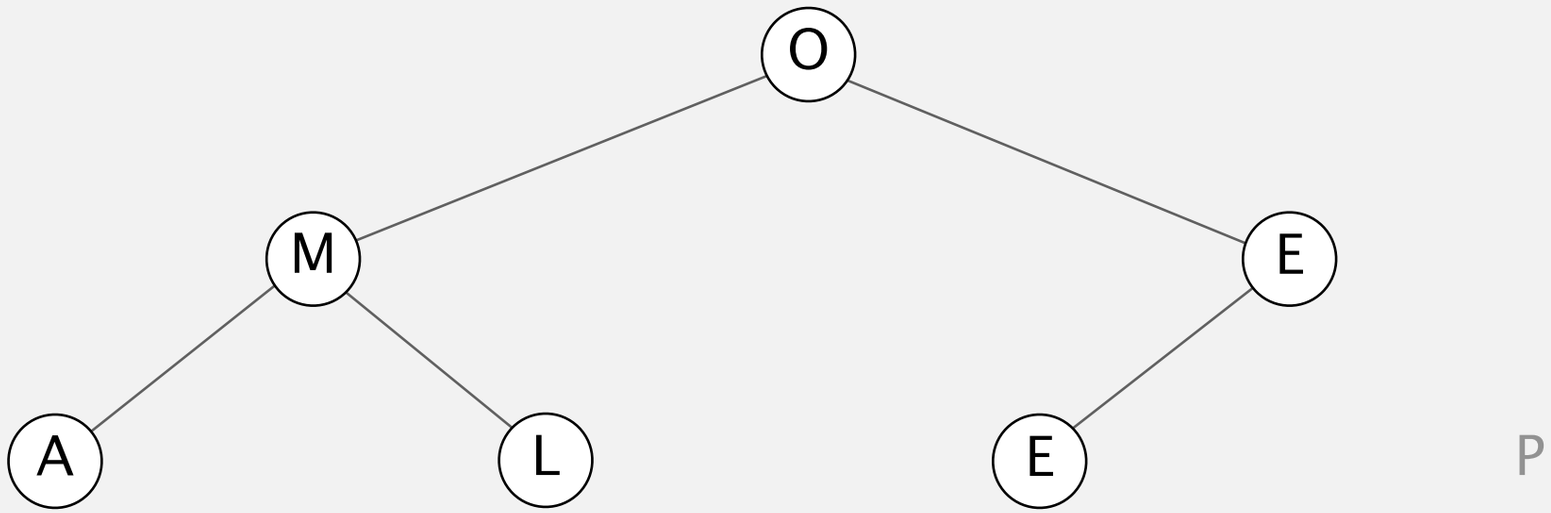


O	M	E	A	L	E	P	R	S	T	X
1	2		4							

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

sink 1

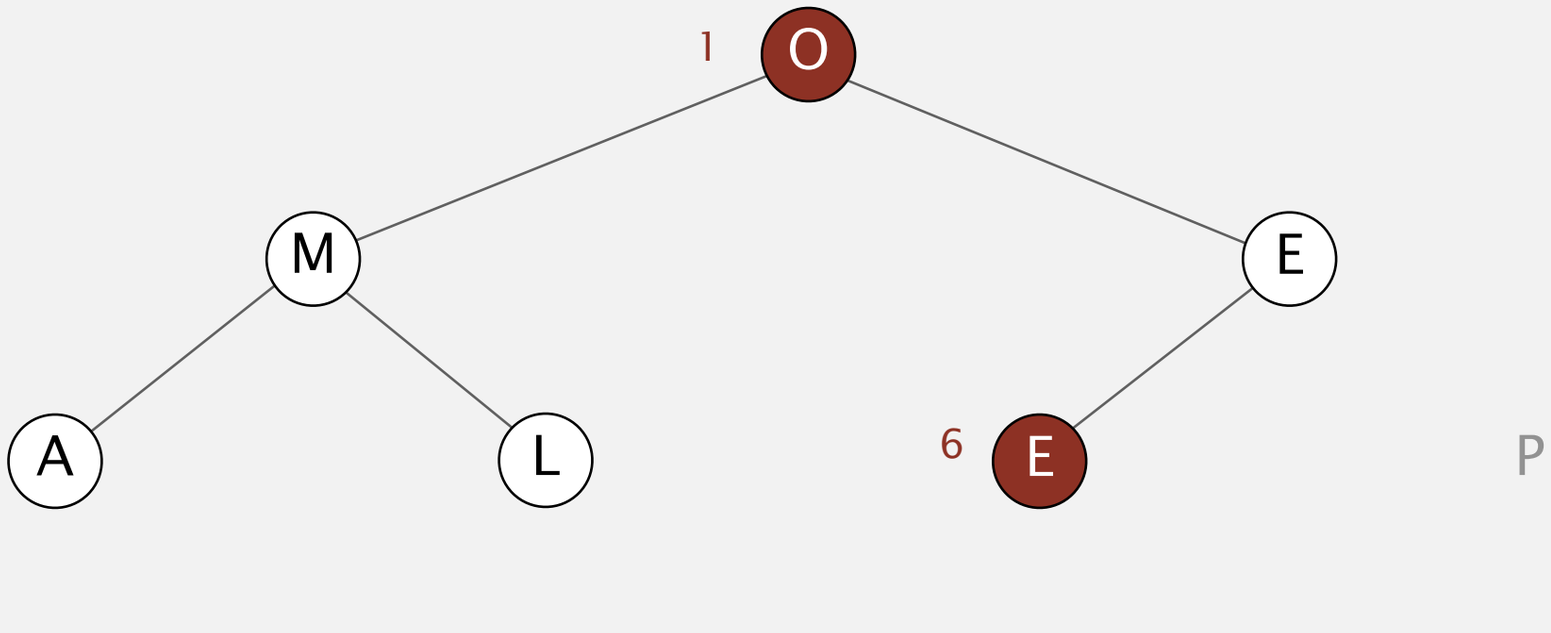


O	M	E	A	L	E	P	R	S	T	X

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

exchange 1 and 6

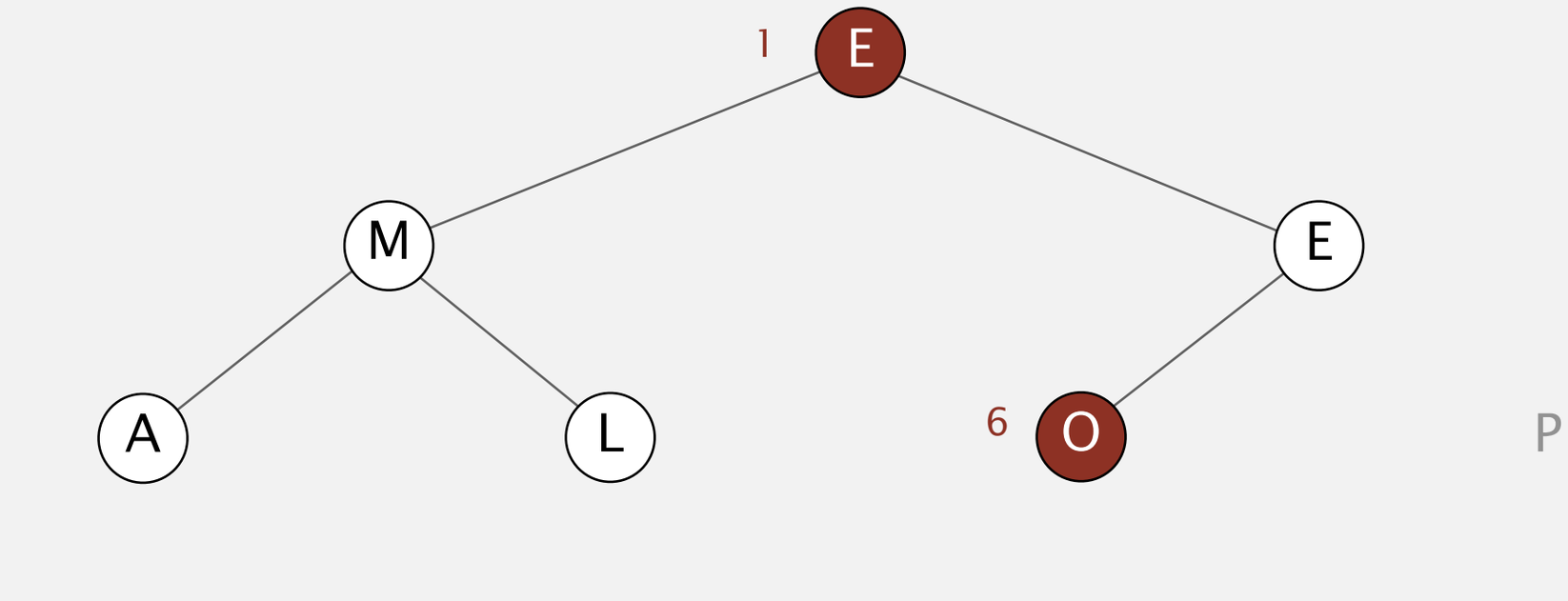


O	M	E	A	L	E	P	R	S	T	X
1					6					

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

exchange 1 and 6

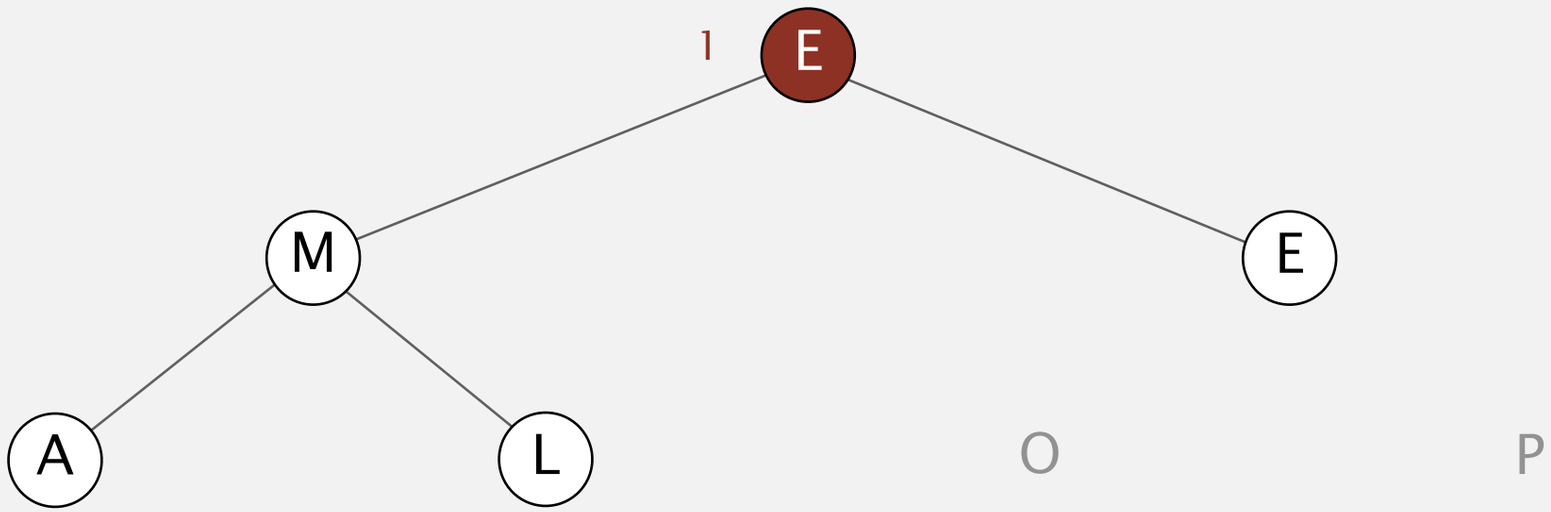


E	M	E	A	L	O	P	R	S	T	X
1					6					

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

sink 1

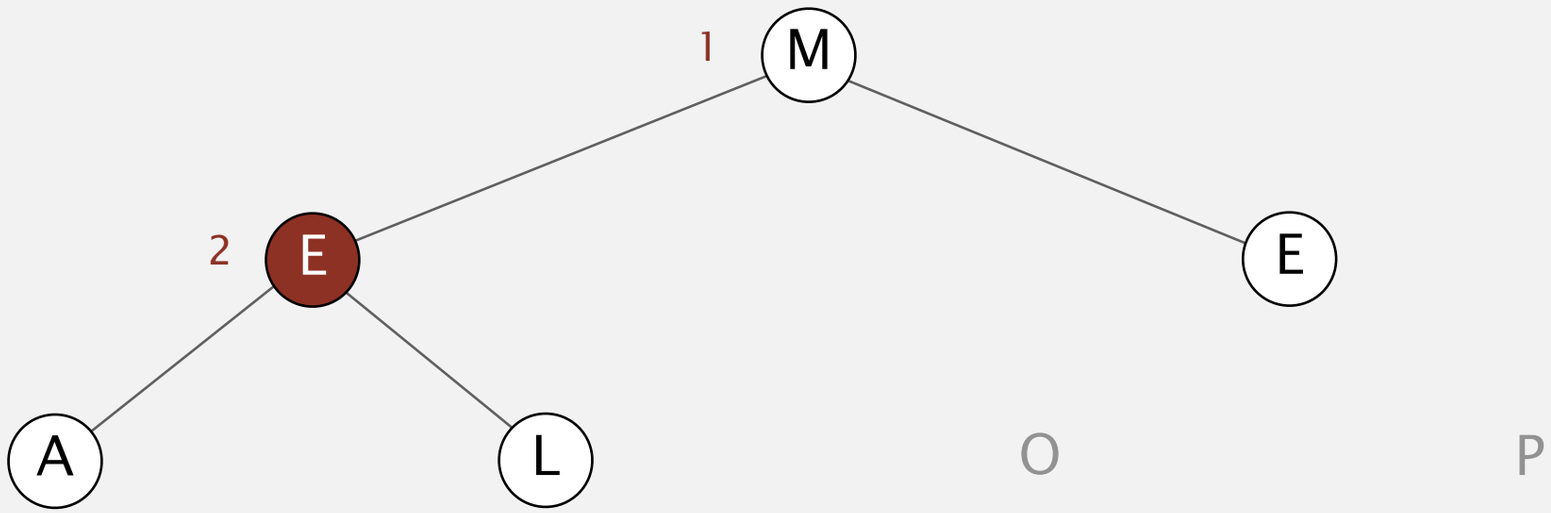


E	M	E	A	L	O	P	R	S	T	X
1										

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

sink 1

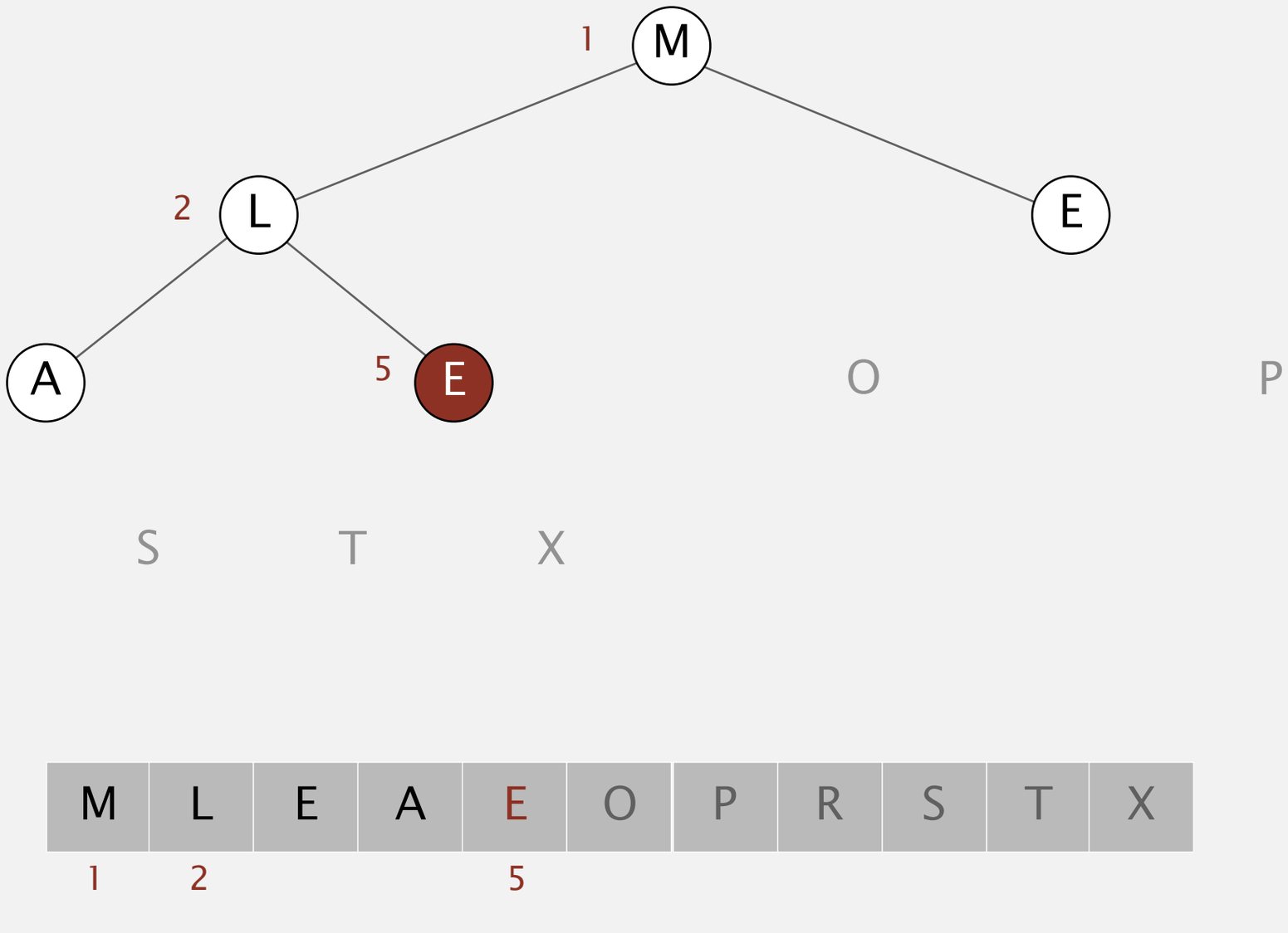


M	E	E	A	L	O	P	R	S	T	X
1	2									

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

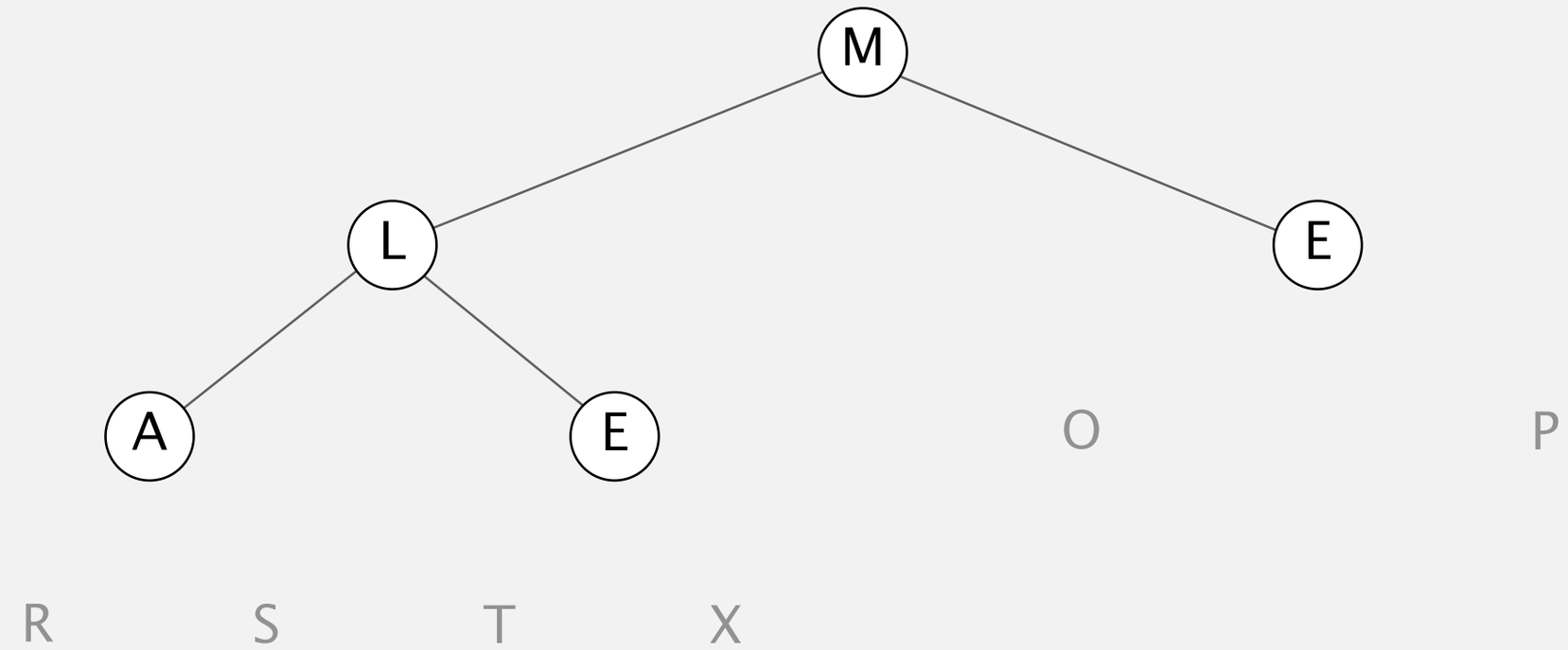
sink 1



M	L	E	A	E	O	P	R	S	T	X
1	2			5						

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

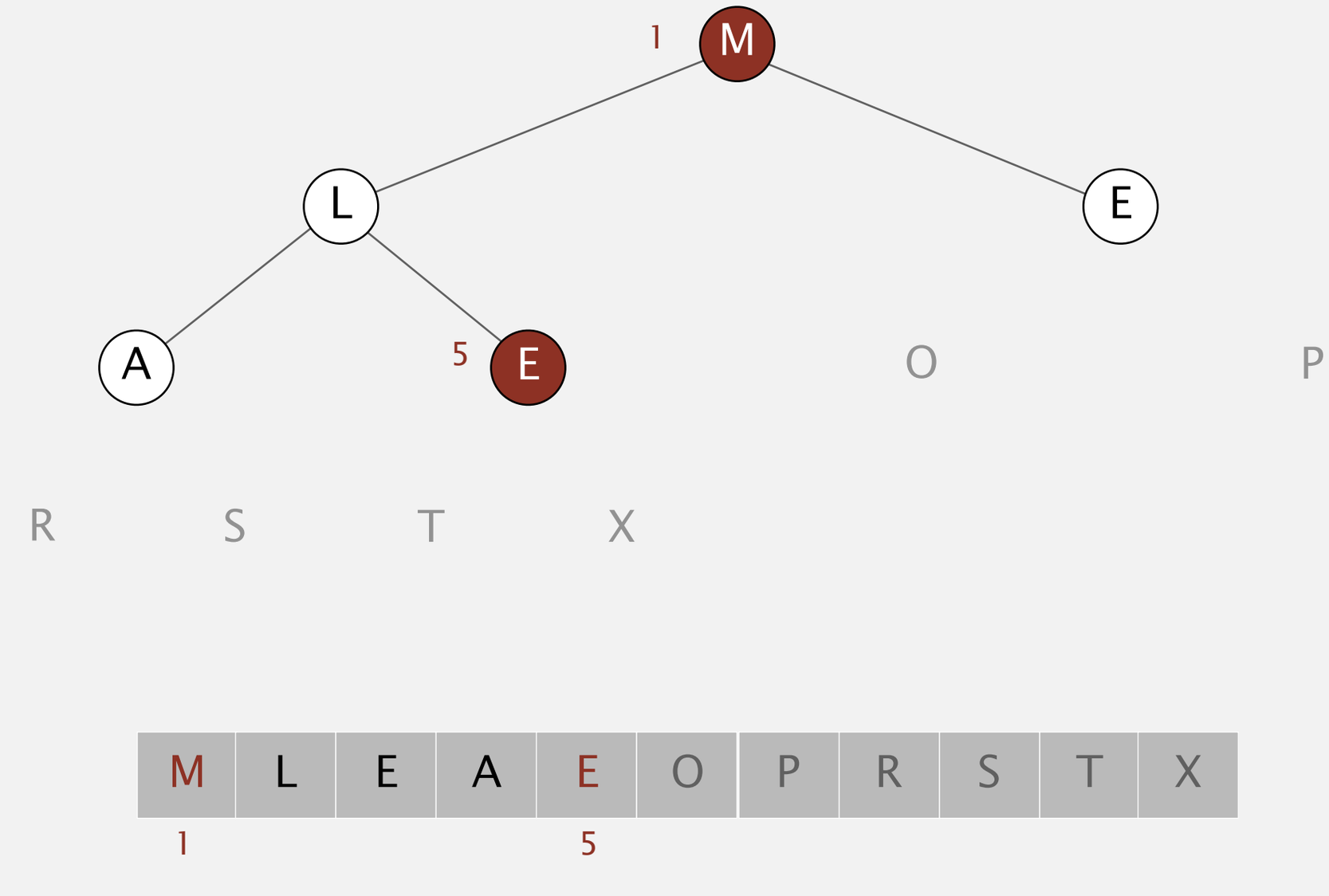


M	L	E	A	E	O	P	R	S	T	X

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

exchange 1 and 5

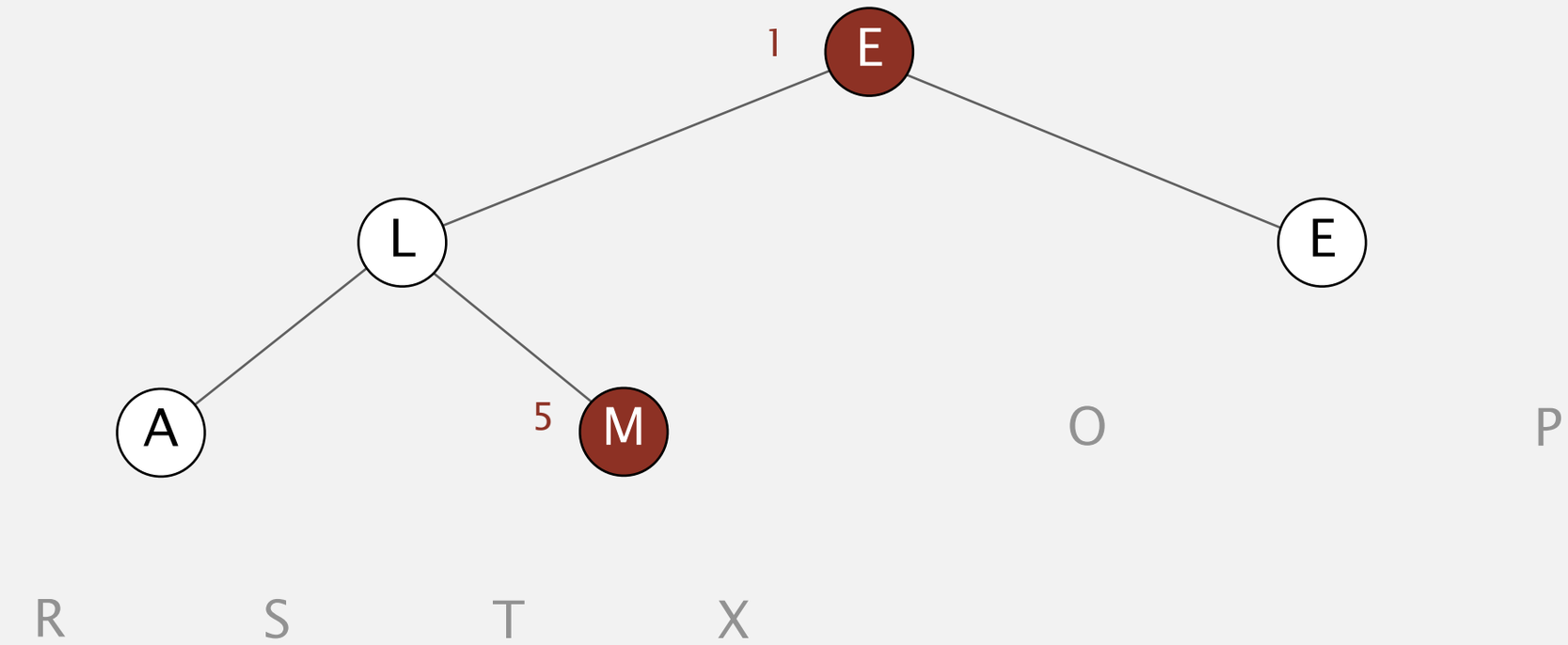


M	L	E	A	E	O	P	R	S	T	X
1				5						

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

exchange 1 and 5

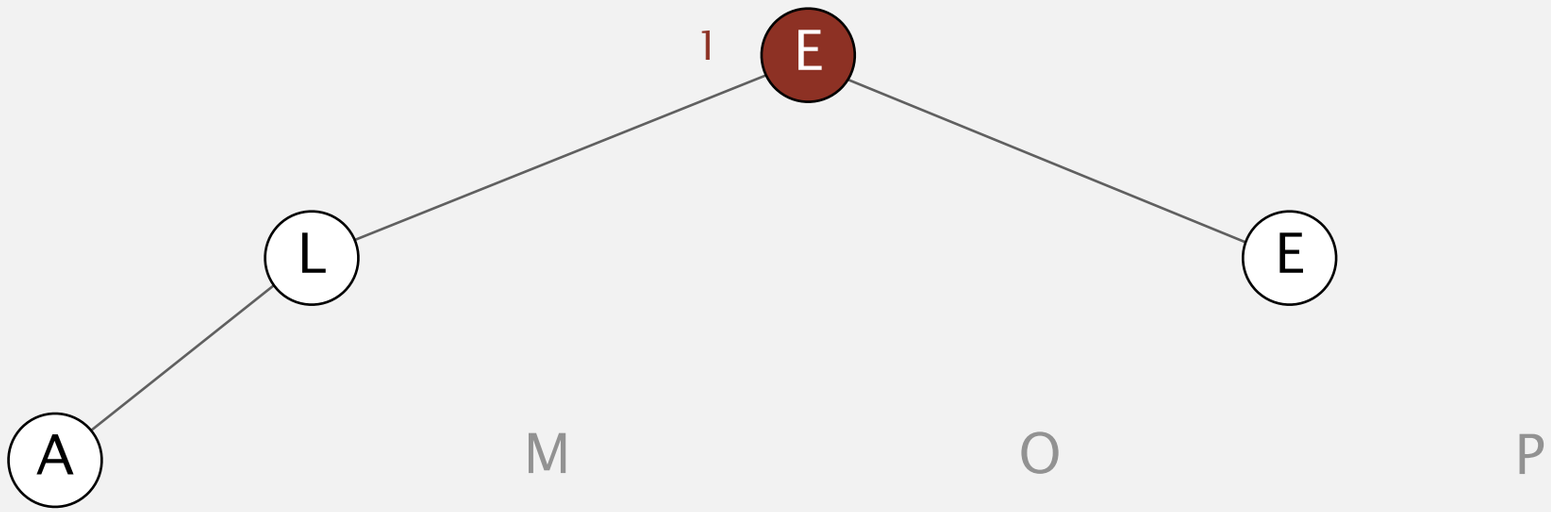


E	L	E	A	M	O	P	R	S	T	X
1				5						

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

sink 1

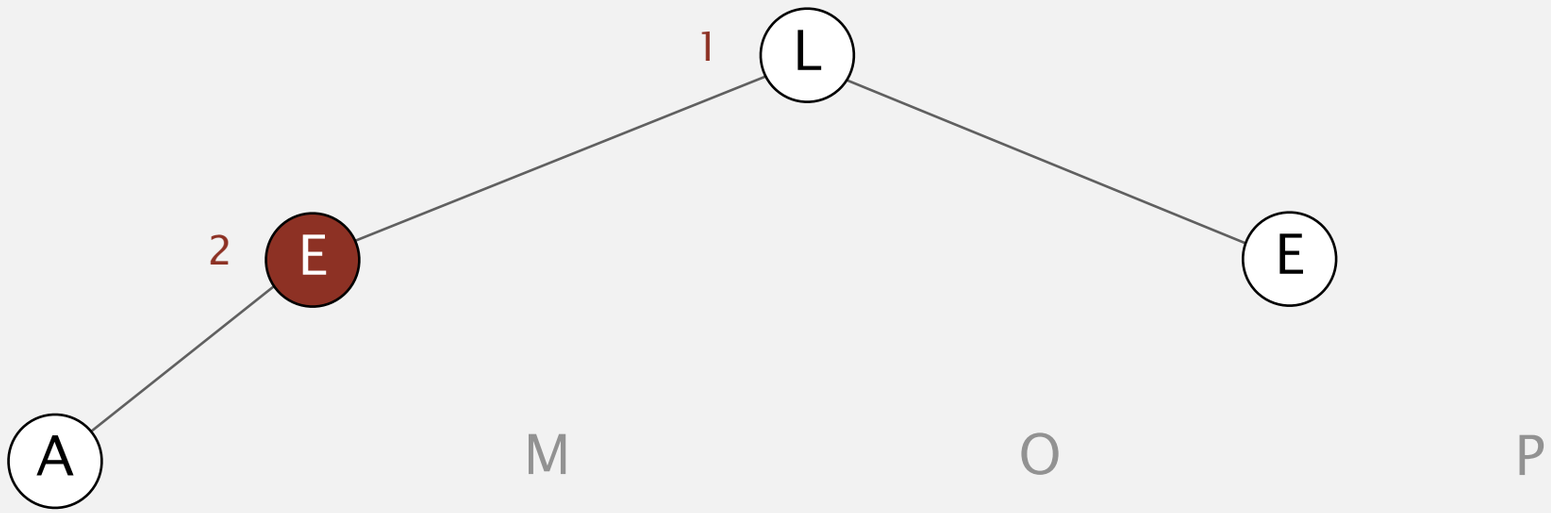


E	L	E	A	M	O	P	R	S	T	X
1										

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

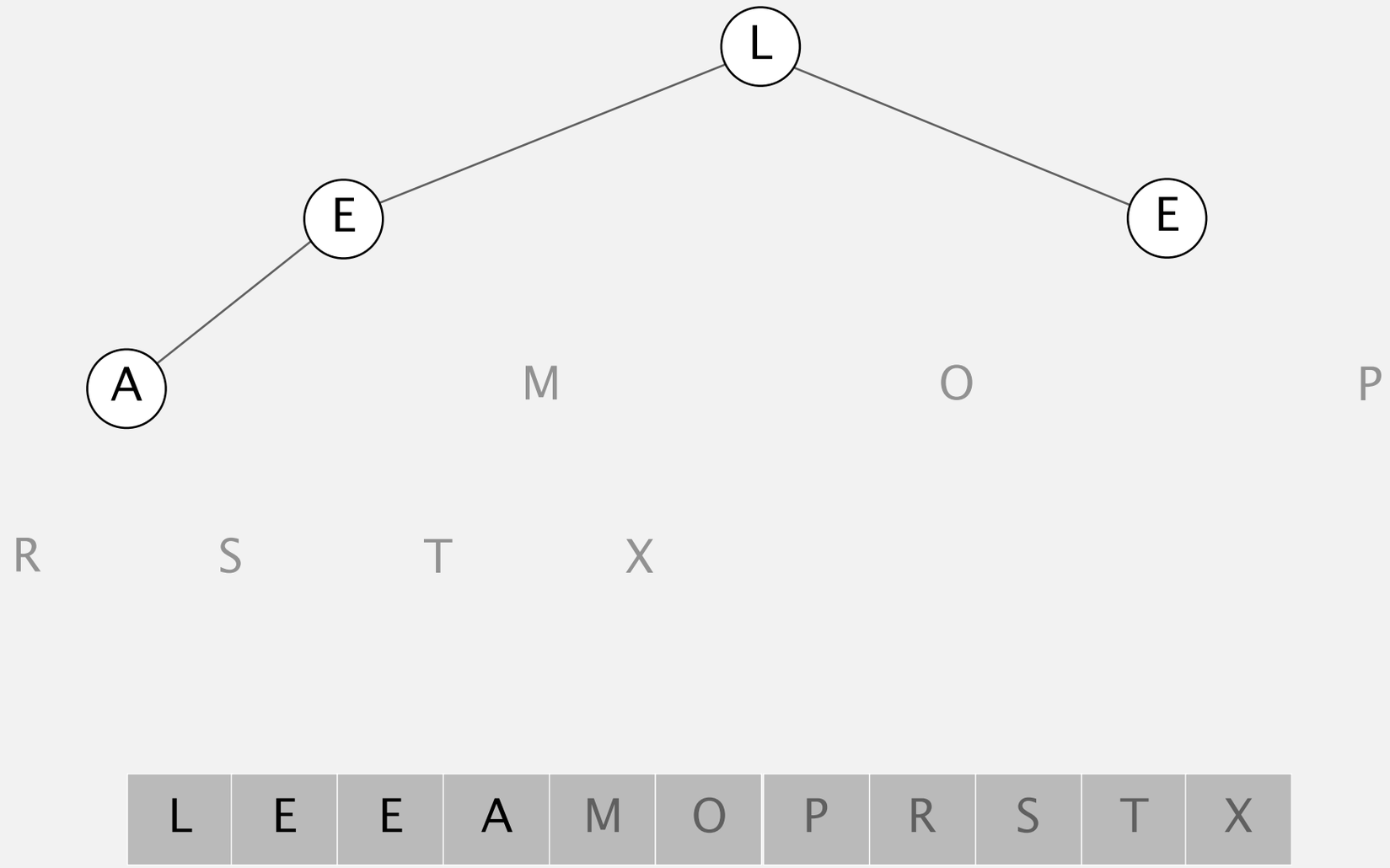
sink 1



L	E	E	A	M	O	P	R	S	T	X
1	2									

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

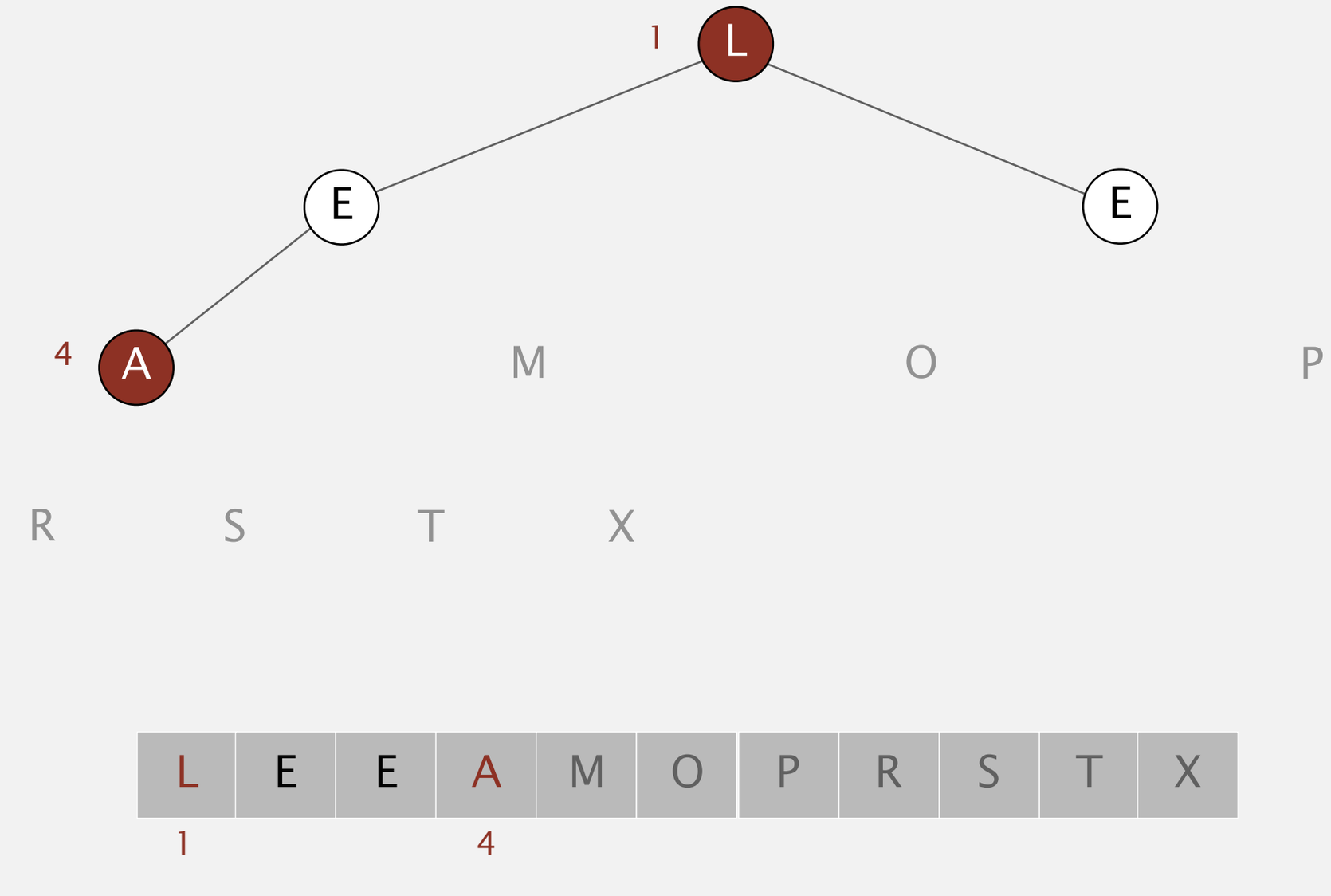


L	E	E	A	M	O	P	R	S	T	X

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

exchange 1 and 4

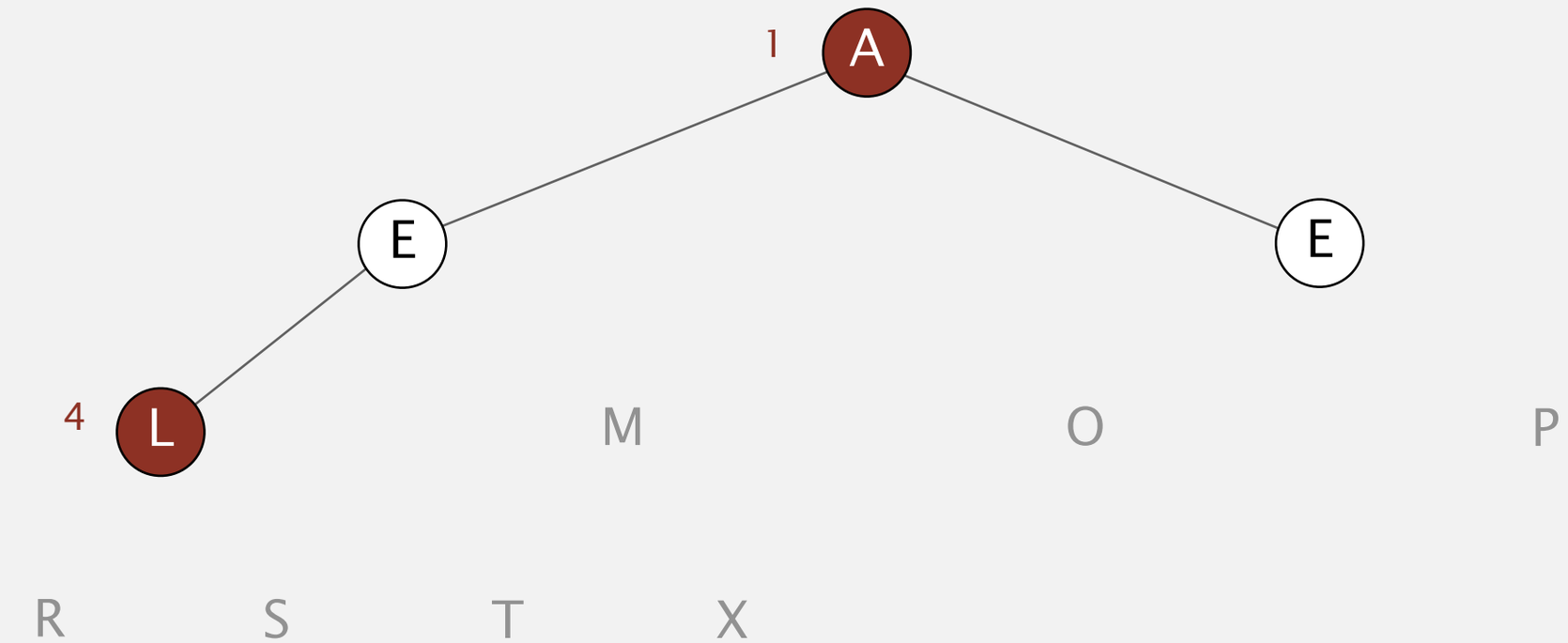


L	E	E	A	M	O	P	R	S	T	X
1			4							

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

exchange 1 and 4

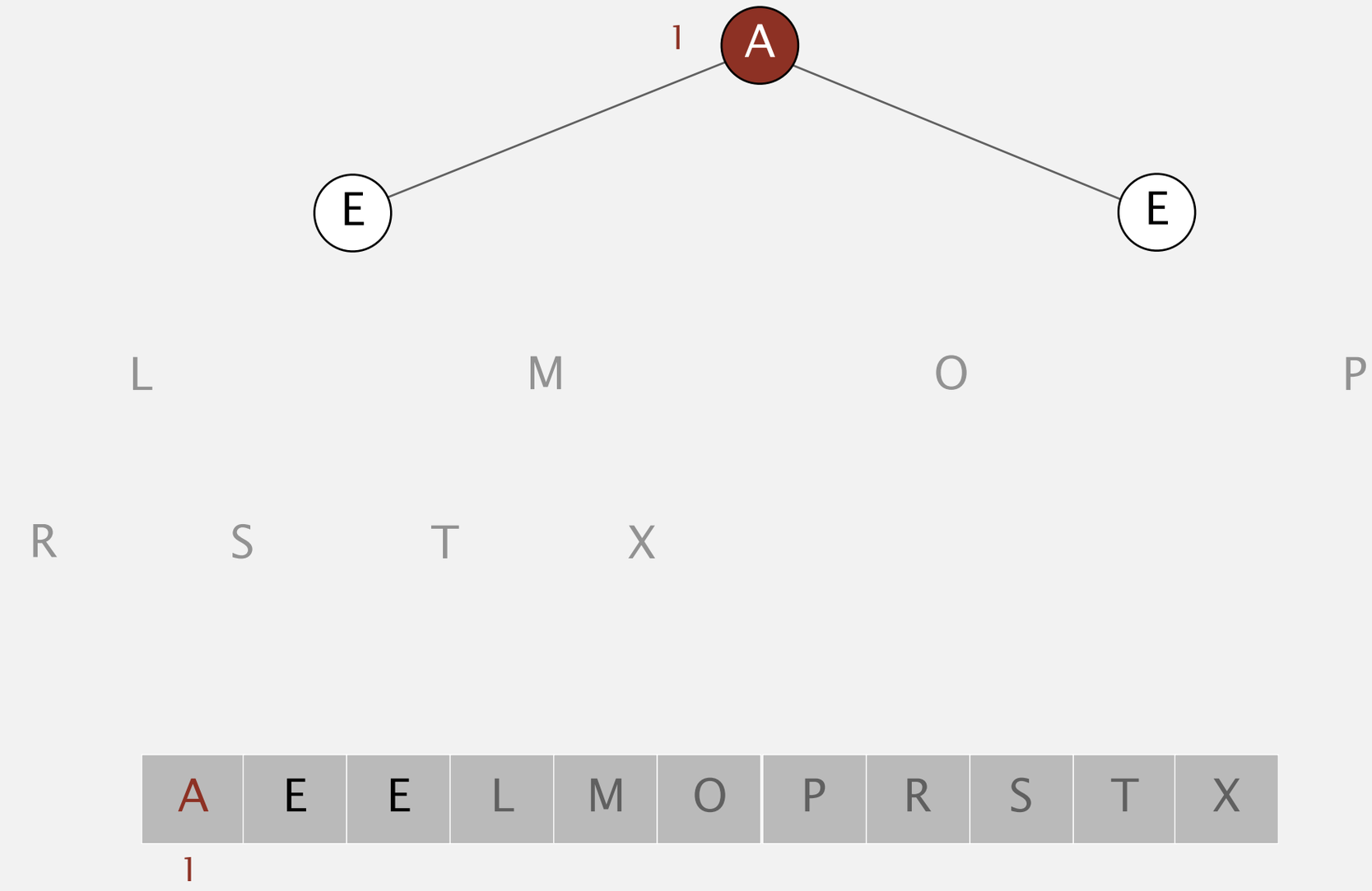


A	E	E	L	M	O	P	R	S	T	X
1			4							

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

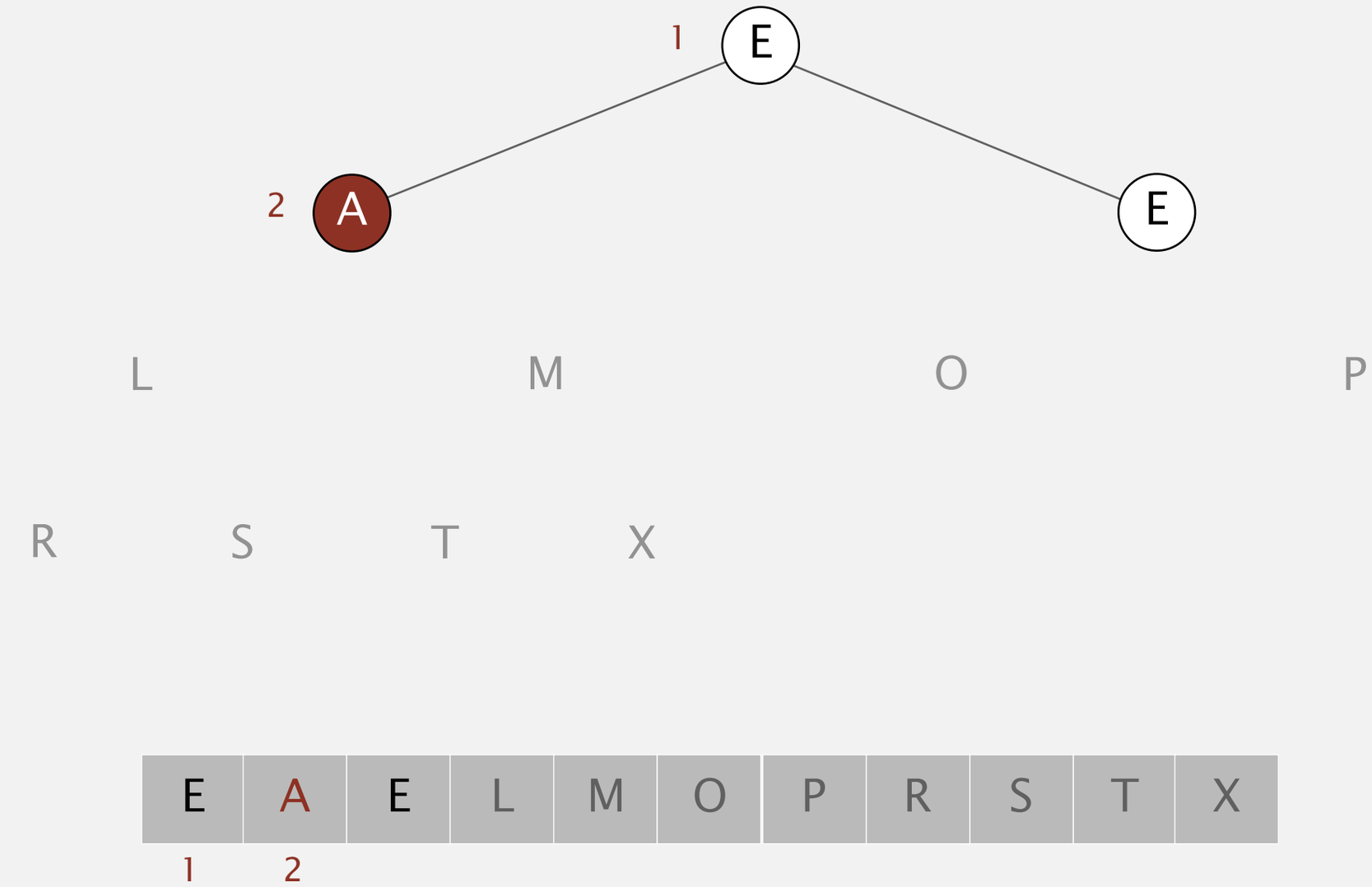
sink 1



Heapsort demo

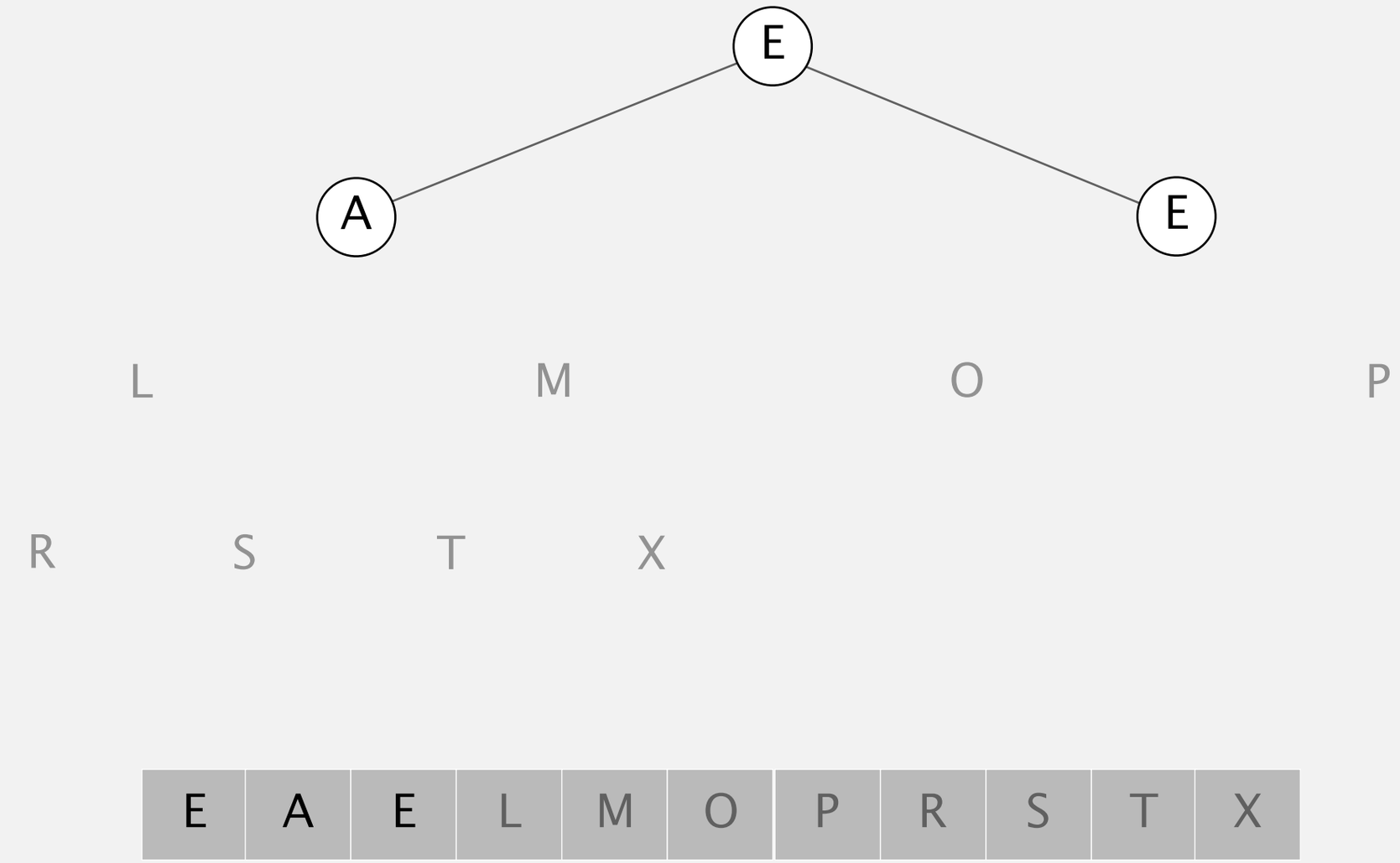
Sortdown. Repeatedly delete the largest remaining item.

sink 1



Heapsort demo

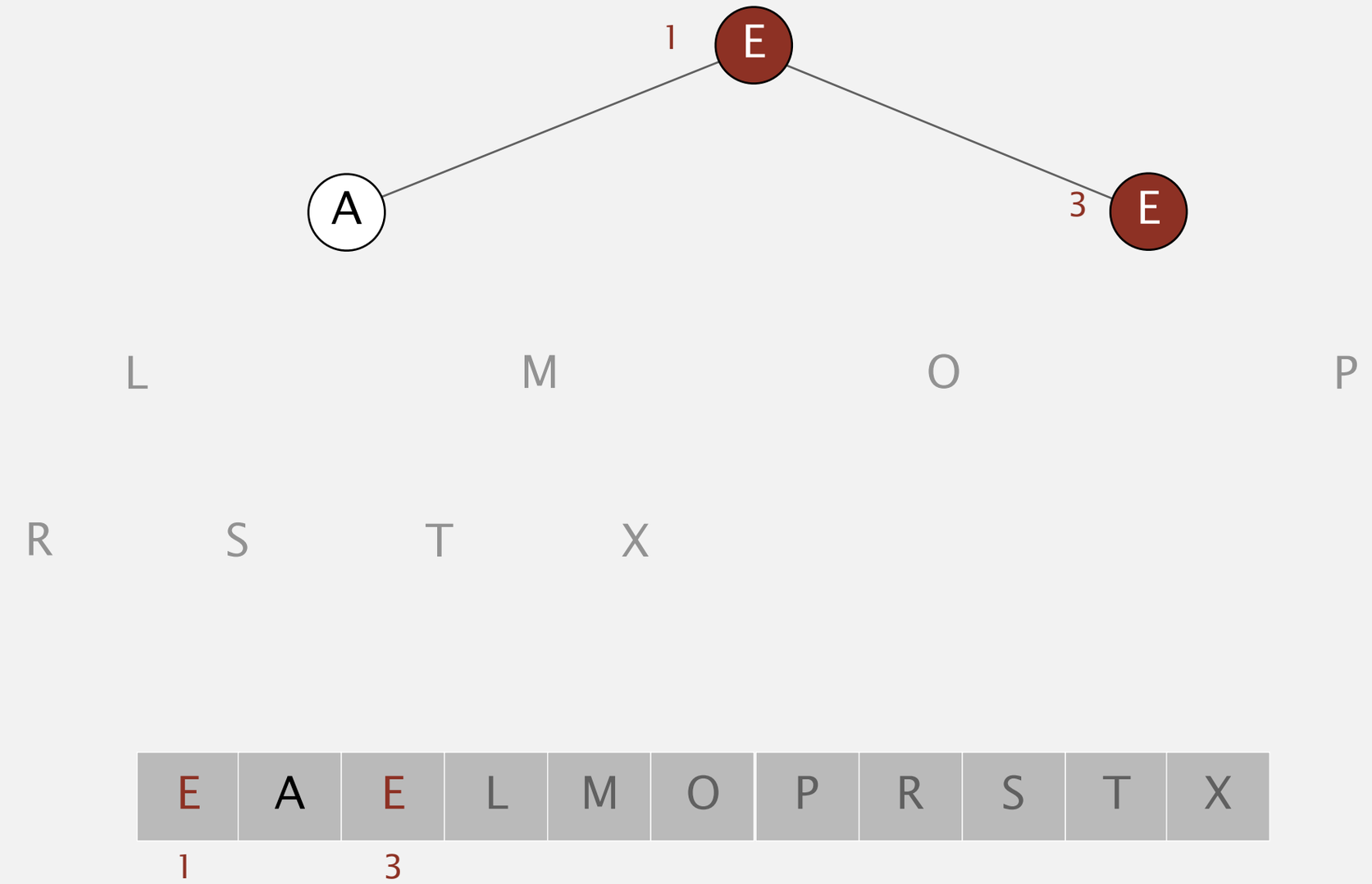
Sortdown. Repeatedly delete the largest remaining item.



Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

exchange 1 and 3

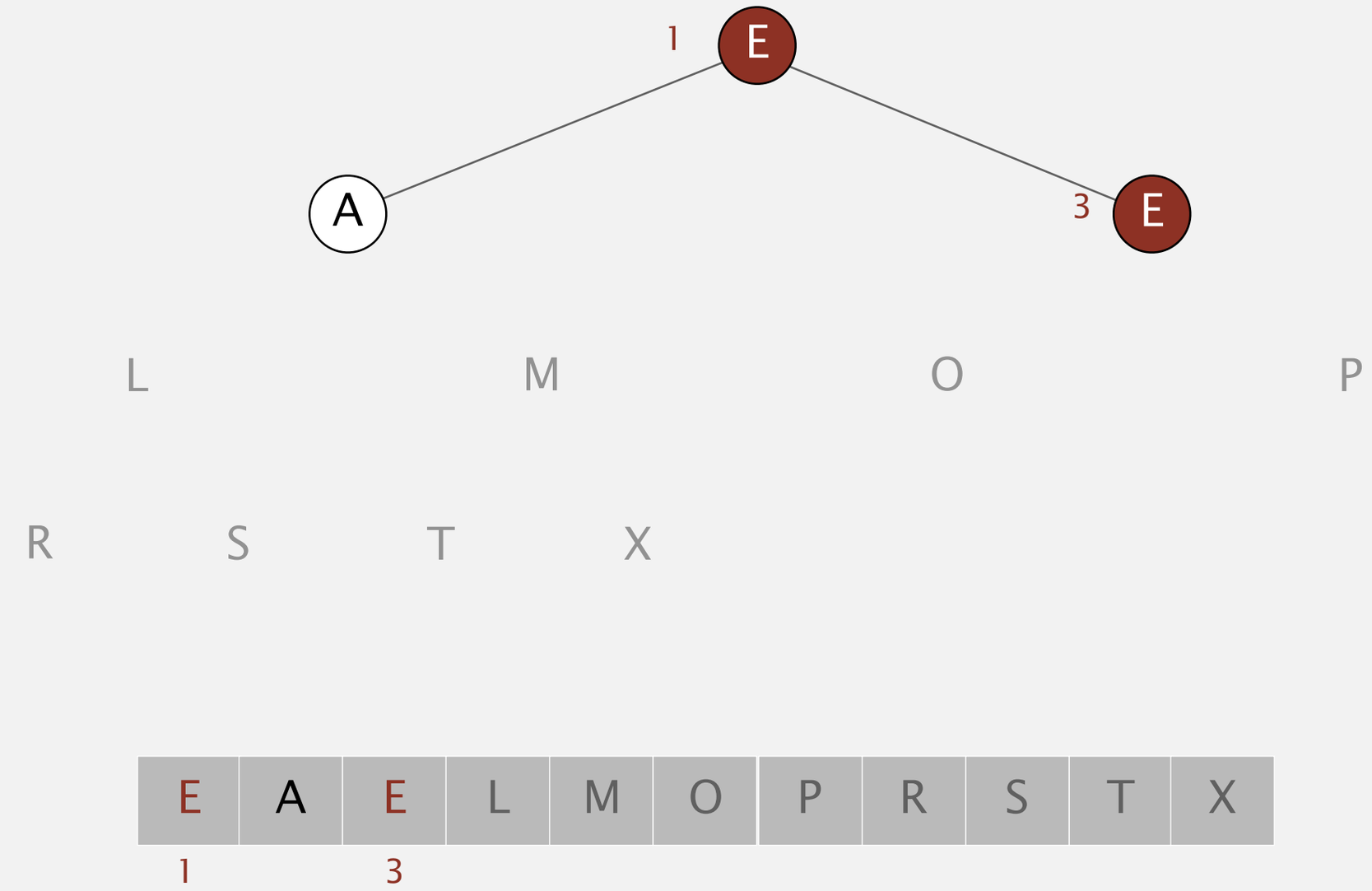


E	A	E	L	M	O	P	R	S	T	X
1		3								

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

exchange 1 and 3

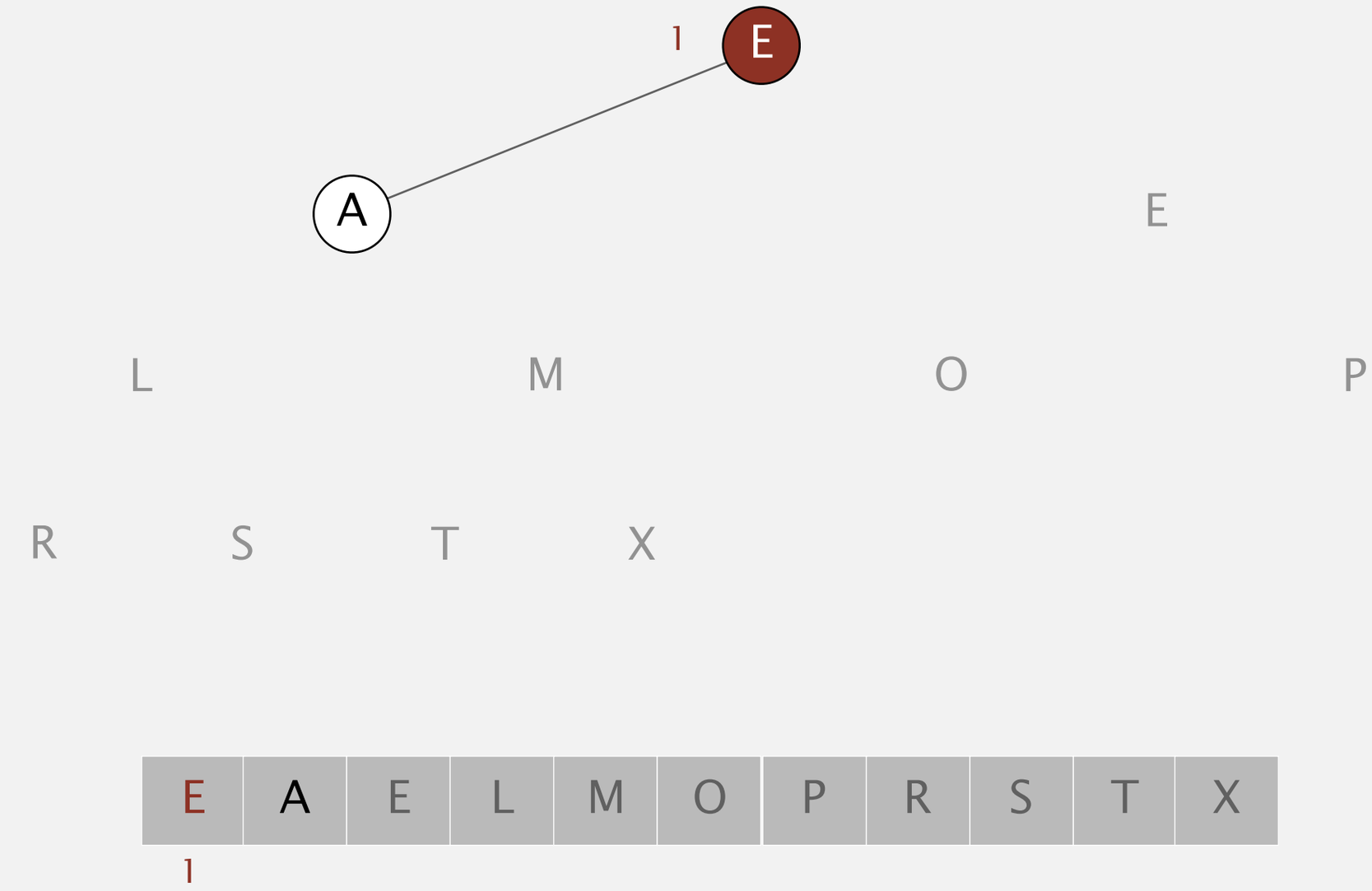


E	A	E	L	M	O	P	R	S	T	X
1		3								

Heapsort demo

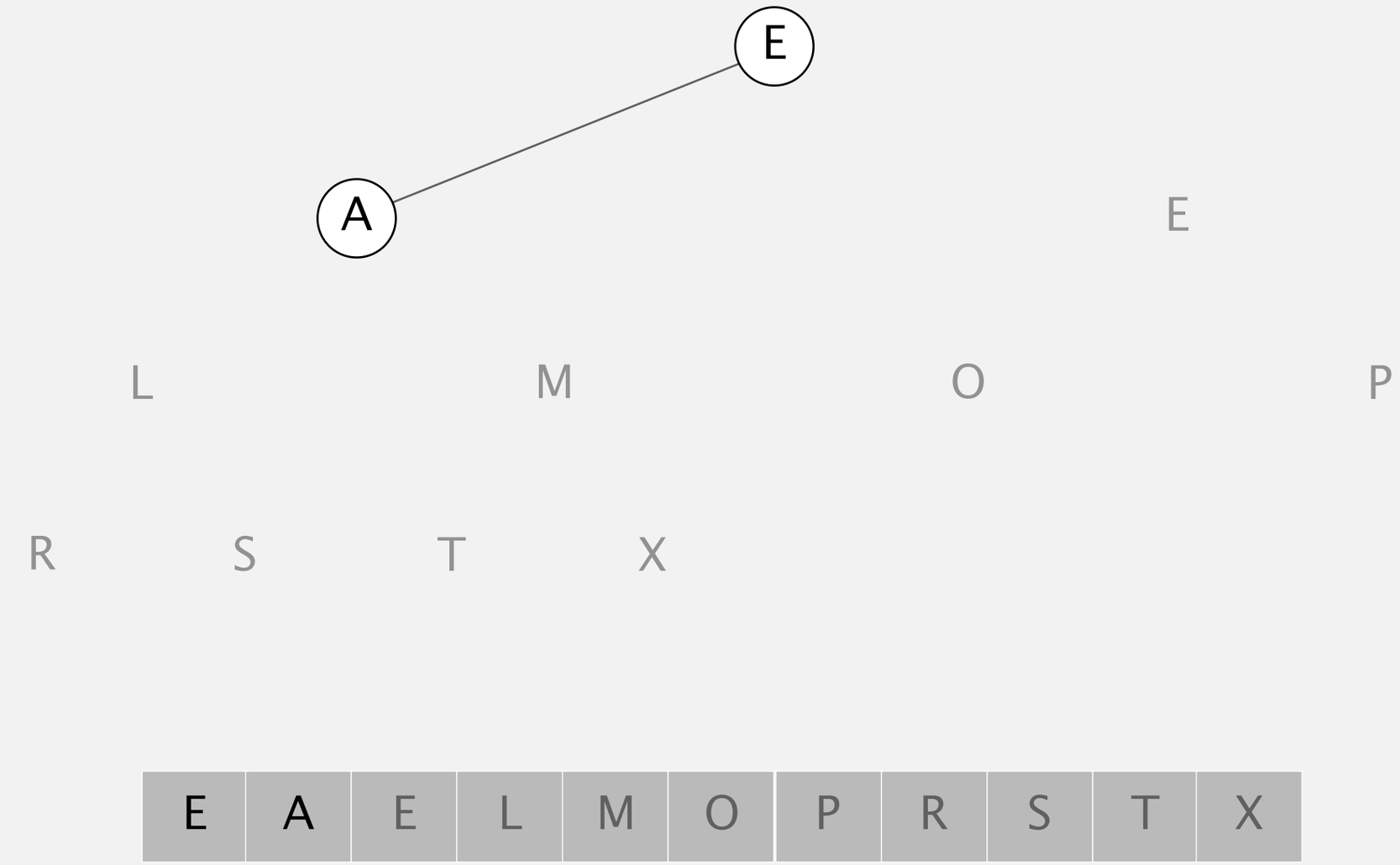
Sortdown. Repeatedly delete the largest remaining item.

sink 1



Heapsort demo

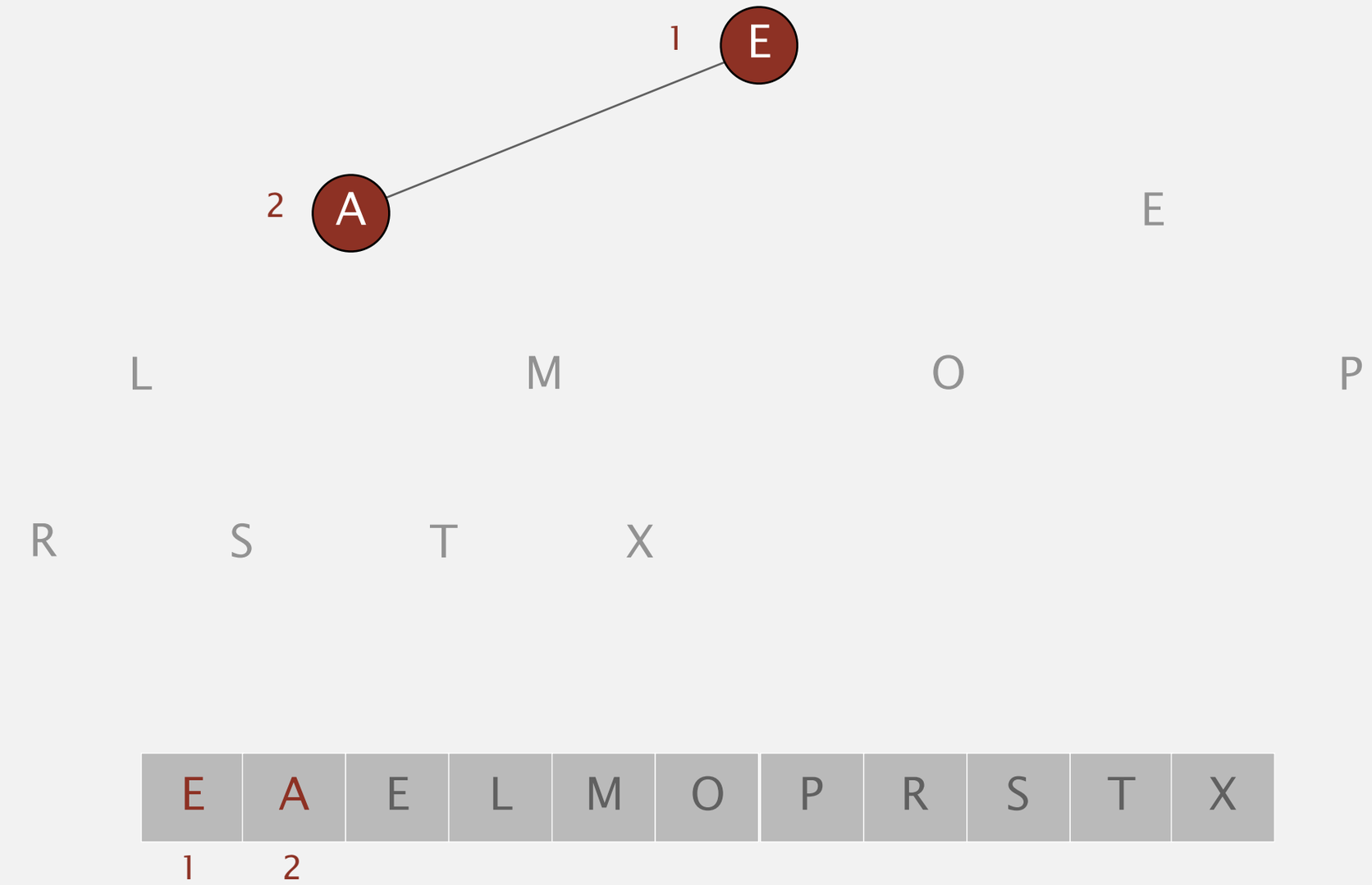
Sortdown. Repeatedly delete the largest remaining item.



Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

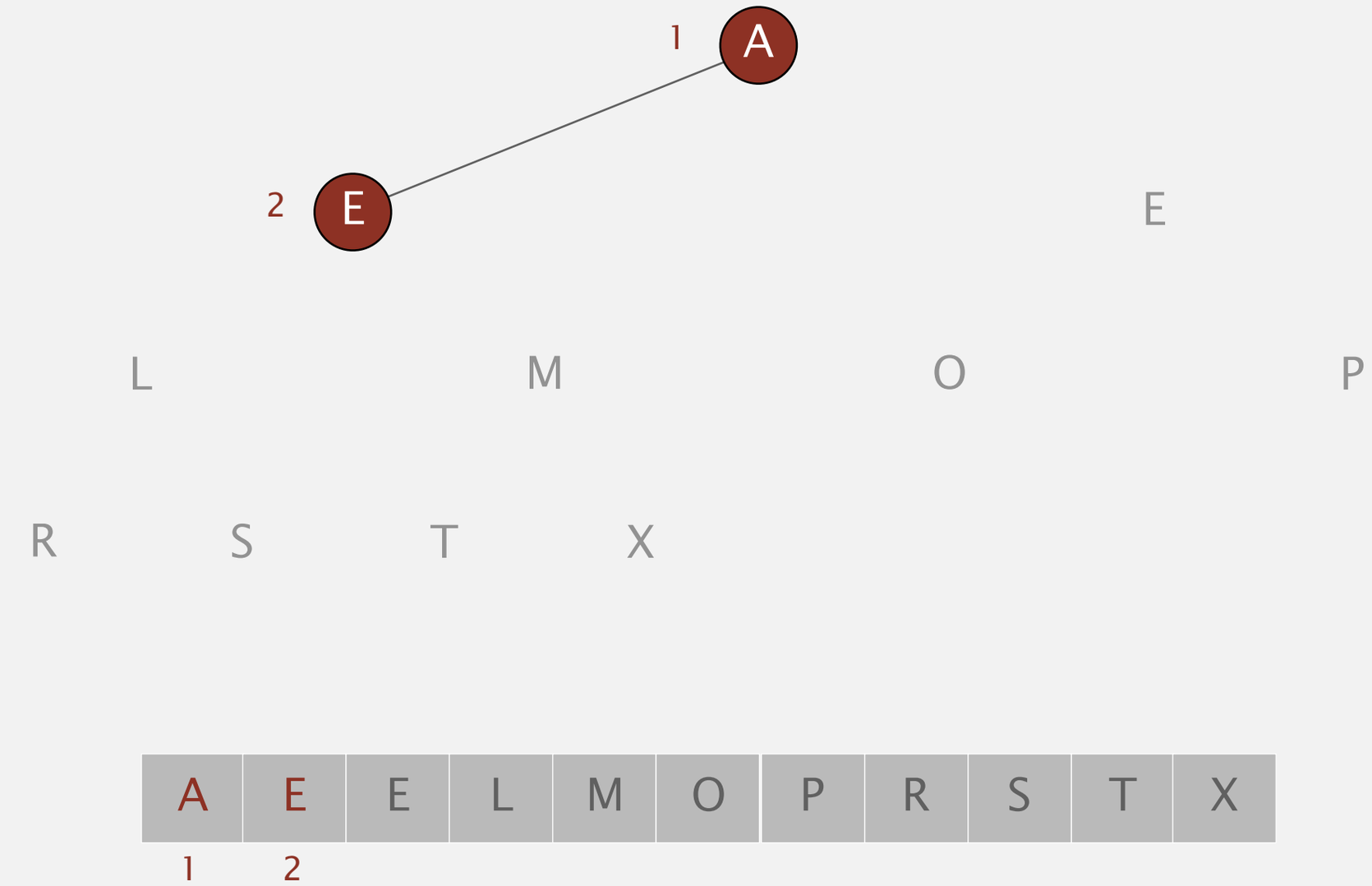
exchange 1 and 2



Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

exchange 1 and 2



A	E	E	L	M	O	P	R	S	T	X
1	2									

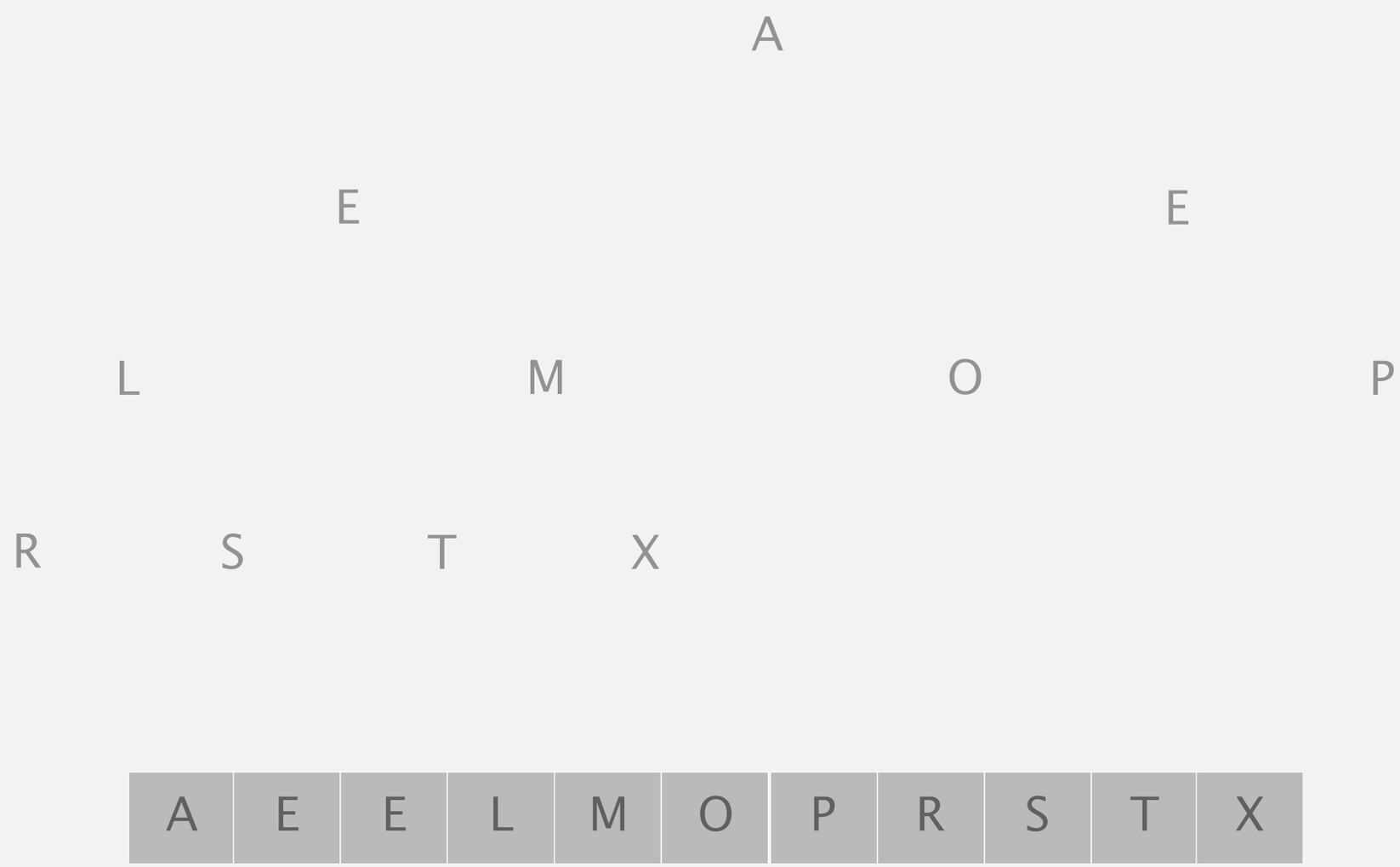
Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

Heapsort demo

Sortdown. Repeatedly delete the largest remaining item.

end of sortdown phase



Heapsort demo

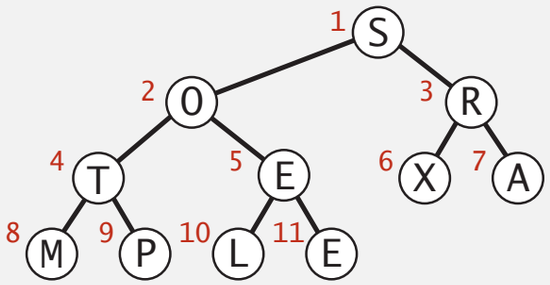
Sortdown. Repeatedly delete the largest remaining item.

array in sorted order

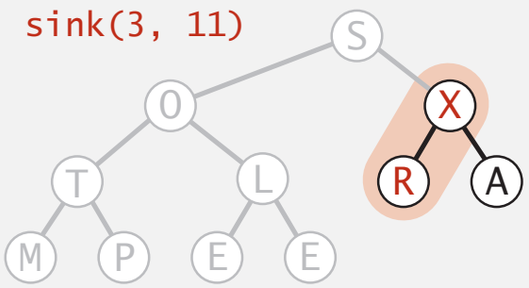
Heapsort: heap construction

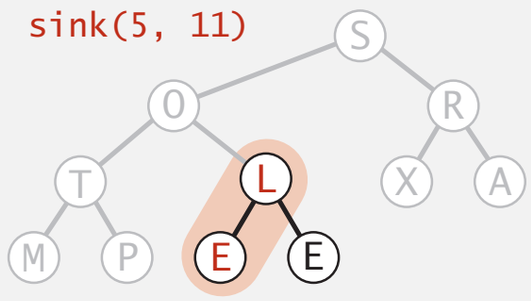
First pass. Build heap using bottom-up method.

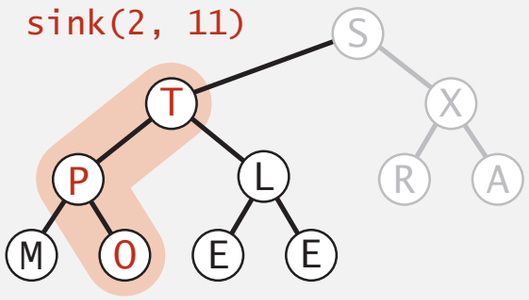
```
for (int k = N/2; k >= 1; k--)
   sink(a, k, N);
```

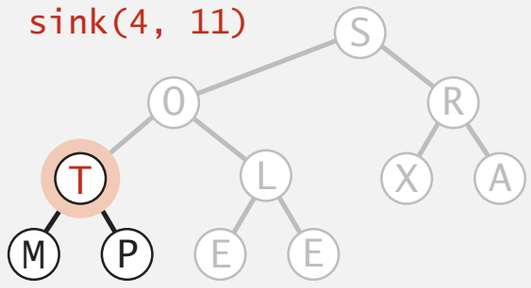


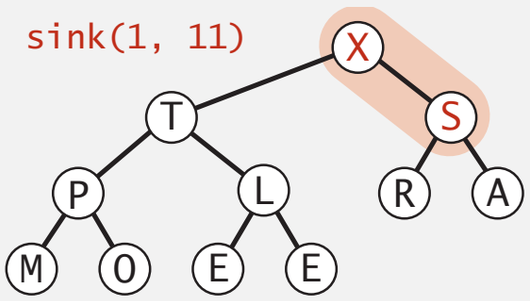
starting point (arbitrary order)











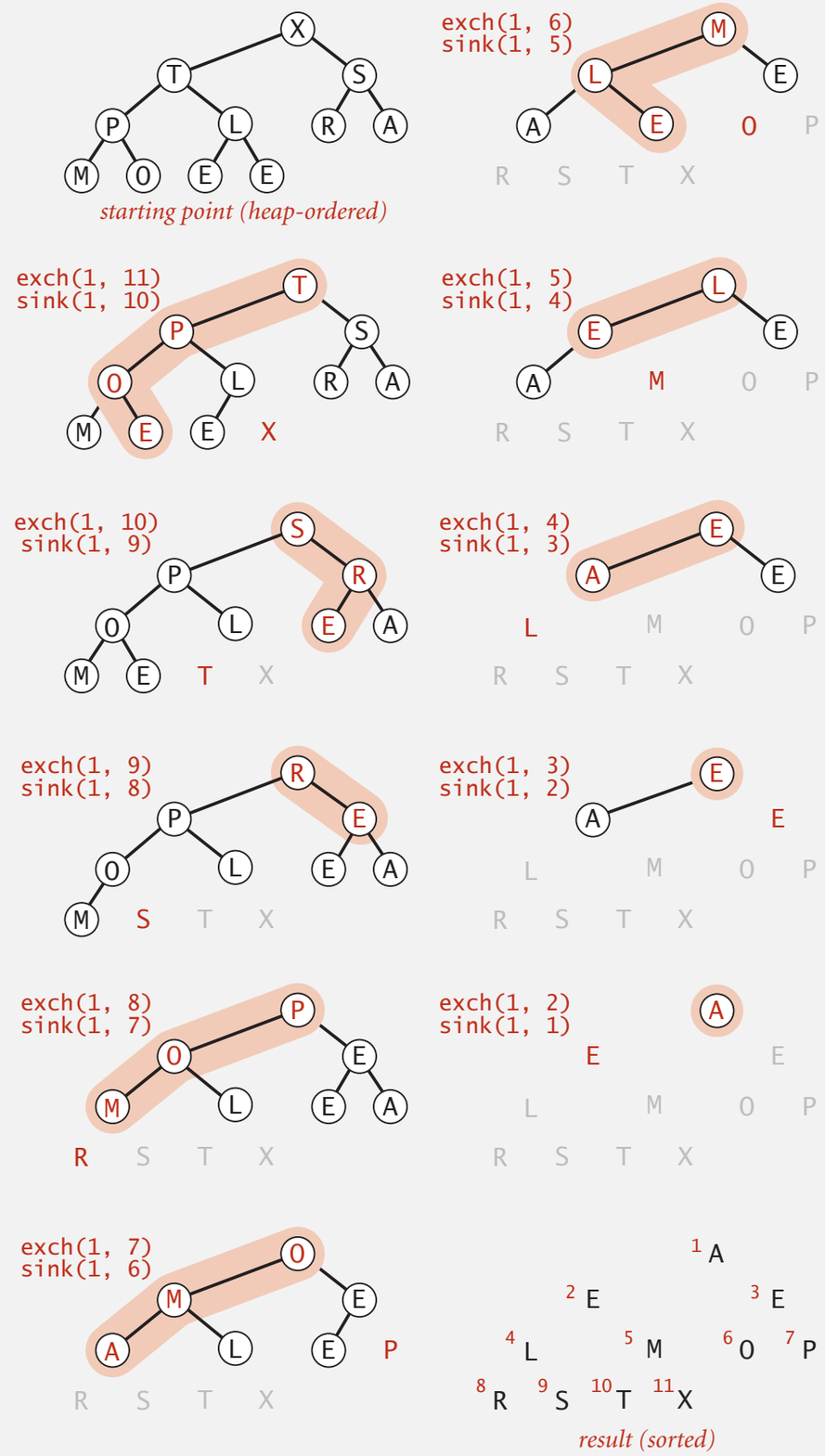
result (heap-ordered)

Heapsort: sortdown

Second pass.

- Remove the maximum, one at a time.
- Leave in array, instead of nulling out.

```
while (N > 1)
{
   exch(a, 1, N--);
   sink(a, 1, N);
}
```



Heapsort: Java implementation

```
public class Heap
{
   public static void sort(Comparable[] a)
   {
      int N = a.length;
      for (int k = N/2; k >= 1; k--)
         sink(a, k, N);
      while (N > 1)
      {
         exch(a, 1, N);
         sink(a, 1, --N);
      }
   }

   private static void sink(Comparable[] a, int k, int N)
   {  /* as before */  }

   private static boolean less(Comparable[] a, int i, int j)
   {  /* as before */  }

   private static void exch(Object[] a, int i, int j)
   {  /* as before */  }

}
```

but make static (and pass arguments)

but convert from 1-based indexing to 0-base indexing

Heapsort: trace

		a[i]											
N	k	0	1	2	3	4	5	6	7	8	9	10	11
initial values			S	O	R	T	E	X	A	M	P	L	E
11	5		S	O	R	T	L	X	A	M	P	E	E
11	4		S	O	R	T	L	X	A	M	P	E	E
11	3		S	O	X	T	L	R	A	M	P	E	E
11	2		S	T	X	P	L	R	A	M	O	E	E
11	1		X	T	S	P	L	R	A	M	O	E	E
heap-ordered			X	T	S	P	L	R	A	M	O	E	E
10	1		T	P	S	O	L	R	A	M	E	E	X
9	1		S	P	R	O	L	E	A	M	E	T	X
8	1		R	P	E	O	L	E	A	M	S	T	X
7	1		P	O	E	M	L	E	A	R	S	T	X
6	1		O	M	E	A	L	E	P	R	S	T	X
5	1		M	L	E	A	E	O	P	R	S	T	X
4	1		L	E	E	A	M	O	P	R	S	T	X
3	1		E	A	E	L	M	O	P	R	S	T	X
2	1		E	A	E	L	M	O	P	R	S	T	X
1	1		A	E	E	L	M	O	P	R	S	T	X
sorted result			A	E	E	L	M	O	P	R	S	T	X

Heapsort trace (array contents just after each sink)