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# 5.5 Data Compression

- *introduction*
- *run-length coding*
- *Huffman compression*
- *LZW compression*


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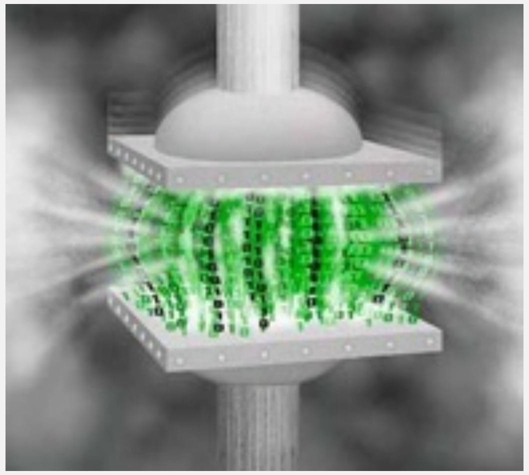
- *LZW compression*

# Data compression

Compression reduces the size of a file:

- To save space when storing it.
- To save time when transmitting it.
- Most files have lots of redundancy.



Who needs compression?

- Moore's law: # transistors on a chip doubles every 18–24 months.
- Parkinson's law: data expands to fill space available.
- Text, images, sound, video, …

> " *Everyday, we create 2.5 quintillion bytes of data—so much that 90% of the data in the world today has been created in the last two years alone.* " — *IBM report on big data (2011)*

Basic concepts ancient (1950s), best technology recently developed.

# Applications

## Generic file compression.

- Files: GZIP, BZIP, 7z.
- Archivers: PKZIP.
- File systems: NTFS, ZFS, HFS+, ReFS, GFS.

## Multimedia.

- Images: GIF, JPEG.
- Sound: MP3.
- Video: MPEG, DivX™, HDTV.

## Communication.

- ITU-T T4 Group 3 Fax.
- V.42bis modem.
- Skype, Google hangout.

**Databases.** Google, Facebook, NSA, ....

# Lossless compression and expansion

Message. Bitstream $B$ we want to compress.

Compress. Generates a "compressed" representation $C(B)$.

Expand. Reconstructs original bitstream $B$.

uses fewer bits
(you hope)


**Basic model for data compression**

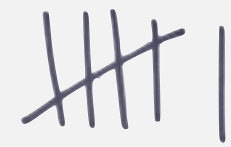
Compression ratio. Bits in $C(B)$ / bits in $B$.

Ex. 50–75% or better compression ratio for natural language.

# Food for thought

Data compression has been omnipresent since antiquity:

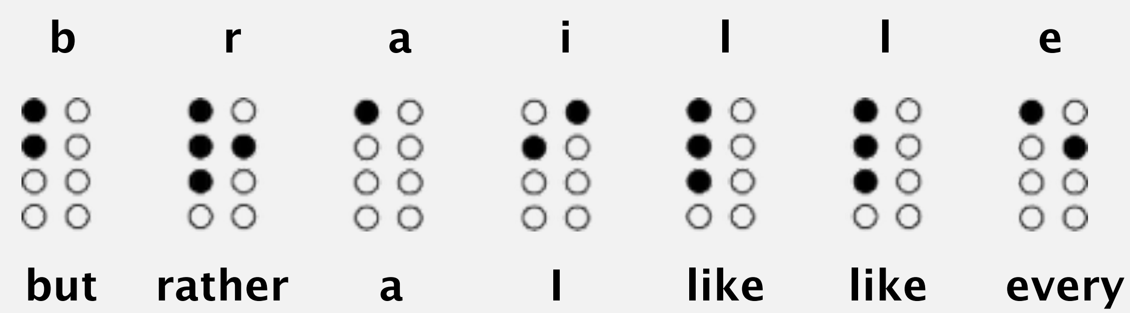
- Number systems.
- Natural languages.
- Mathematical notation.



$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

has played a central role in communications technology,

- Grade 2 Braille.
- Morse code.
- Telephone system.




and is part of modern life.

- JPEG.
- MP3.
- MPEG.

Q. What role will it play in the future?

# Data representation: genomic code

Genome. String over the alphabet { A, T, C, G }.

Goal. Encode an $N$-character genome: A T A G A T G C A T A G . . .

Standard ASCII encoding.

- $8$ bits per char.
- $8\,N$ bits.

| char | hex | binary |
|---|---|---|
| 'A' | 41 | 01000001 |
| 'T' | 54 | 01010100 |
| 'C' | 43 | 01000011 |
| 'G' | 47 | 01000111 |

Two-bit encoding.

- $2$ bits per char.
- $2\,N$ bits (25% compression ratio).

| char | binary |
|---|---|
| 'A' | 00 |
| 'T' | 01 |
| 'C' | 10 |
| 'G' | 11 |

Fixed-length code. $k$-bit code supports alphabet of size $2^k$.

Amazing but true. Some genomic databases in 1990s used ASCII.

# Reading and writing binary data

**Binary standard input.** Read bits from standard input.

```
public class BinaryStdIn
```

| | | |
|---|---|---|
| `boolean` | `readBoolean()` | *read 1 bit of data and return as a* `boolean` *value* |
| `char` | `readChar()` | *read 8 bits of data and return as a* `char` *value* |
| `char` | `readChar(int r)` | *read* `r` *bits of data and return as a* `char` *value* |
| *[similar methods for* `byte` *(8 bits);* `short` *(16 bits);* `int` *(32 bits);* `long` *and* `double` *(64 bits)]* | | |
| `boolean` | `isEmpty()` | *is the bitstream empty?* |
| `void` | `close()` | *close the bitstream* |

**Binary standard output.** Write bits to standard output

```
public class BinaryStdOut
```

| | | |
|---|---|---|
| `void` | `write(boolean b)` | *write the specified bit* |
| `void` | `write(char c)` | *write the specified 8-bit* `char` |
| `void` | `write(char c, int r)` | *write the* `r` *least significant bits of the specified* `char` |
| *[similar methods for* `byte` *(8 bits);* `short` *(16 bits);* `int` *(32 bits);* `long` *and* `double` *(64 bits)]* | | |
| `void` | `close()` | *close the bitstream* |

# Writing binary data

**Date representation.** Three different ways to represent 12/31/1999.

**A character stream (StdOut)**

```
StdOut.print(month + "/" + day + "/" + year);
```

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 00110001 | 00110010 | 00101111 | 00110111 | 00110001 | 00101111 | 00110001 | 00111001 | 00111001 | 00111001 |
| 1 | 2 | / | 3 | 1 | / | 1 | 9 | 9 | 9 |

*80 bits*

**Three ints (BinaryStdOut)**

```
BinaryStdOut.write(month);
BinaryStdOut.write(day);
BinaryStdOut.write(year);
```

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 00000000 | 00000000 | 00000000 | 00001100 | 00000000 | 00000000 | 00000000 | 00011111 | 00000000 | 00000000 | 00000111 | 11001111 |
| 12 | | | | 31 | | | | 1999 | | | |

*96 bits*

**A 4-bit field, a 5-bit field, and a 12-bit field (BinaryStdOut)**

```
BinaryStdOut.write(month, 4);
BinaryStdOut.write(day, 5);
BinaryStdOut.write(year, 12);
```

| | | |
|---|---|---|
| 11001111 | 10111110 | 01111000 |

12 | 31 | 1999

*21 bits ( + 3 bits for byte alignment at close)*

# Binary dumps

Q. How to examine the contents of a bitstream?

**Standard character stream**

```
% more abra.txt
ABRACADABRA!
```

**Bitstream represented as 0 and 1 characters**

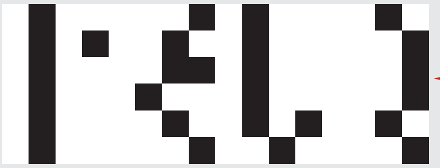
```
% java BinaryDump 16 < abra.txt
0100000101000010
0101001001000001
0100001101000001
0100010001000001
0100001001010010
0100000100100001
96 bits
```

**Bitstream represented with hex digits**

```
%  java HexDump 4 < abra.txt
41 42 52 41
43 41 44 41
42 52 41 21
12 bytes
```

**Bitstream represented as pixels in a `Picture`**

```
% java PictureDump 16 6 < abra.txt
```



*16-by-6 pixel window, magnified*

```
96 bits
```

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | NUL | SOH | STX | ETX | EOT | ENQ | ACK | BEL | BS | HT | LF | VT | FF | CR | SO | SI |
| 1 | DLE | DC1 | DC2 | DC3 | DC4 | NAK | SYN | ETB | CAN | EM | SUB | ESC | FS | GS | RS | US |
| 2 | SP | ! | “ | # | $ | % | & | ‘ | ( | ) | * | + | , | - | . | / |
| 3 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | : | ; | < | = | > | ? |
| 4 | @ | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
| 5 | P | Q | R | S | T | U | V | W | X | Y | Z | [ | \ | ] | ^ | _ |
| 6 | ` | a | b | c | d | e | f | g | h | i | j | k | l | m | n | o |
| 7 | p | q | r | s | t | u | v | w | x | y | z | { | \| | } | ~ | DEL |

**Hexadecimal to ASCII conversion table**

# Universal data compression

ZeoSync. Announced 100:1 lossless compression of random data using Zero Space Tuner™ and BinaryAccelerator™ technology.

SCIENCE : DISCOVERIES

## Firm Touts 'Perfect Compression'

Declan McCullagh 01.16.02

WASHINGTON -- Physicists do not question the laws of thermodynamics. Chemistry researchers unwaveringly cite Boyle's Law to describe the relationship between gas pressure and temperature.

Computer scientists also have their own fundamental laws, perhaps not as well known, but arguably even more solid. One of those laws says a perfect compression mechanism is impossible.

A slightly expanded version of that law says it is mathematically impossible to write a computer program that can compress all files by at least one bit. Sure, it's possible to write a program to compress *typical* data by far more than one bit -- that assignment is commonly handed to computer science sophomores, and the technique is used in .jpg and .zip files.

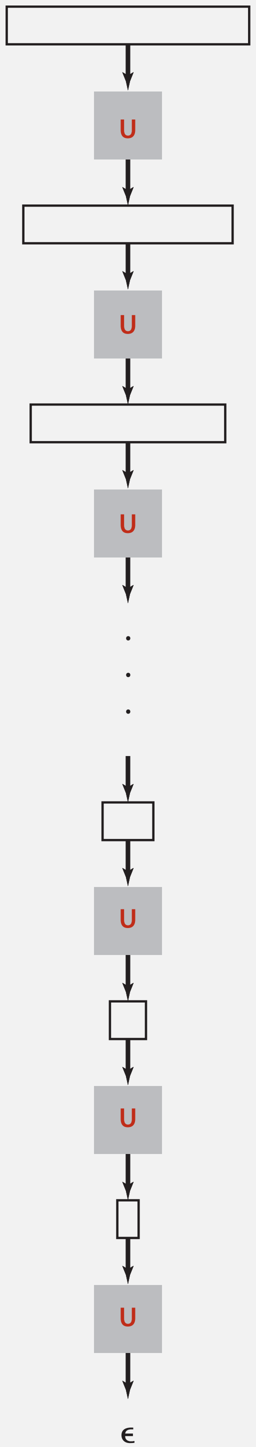
But those general techniques, while useful, don't work on all files; otherwise, you could repeatedly compress a .zip, .gzip or .sit file to nothingness. Put another way, compression techniques can't work with random data that follow no known patterns.

So when a little-known company named ZeoSync announced last week it had achieved perfect compression -- a breakthrough that would be a bombshell roughly as big as $e=mc^2$ -- it was greeted with derision. Their press release was roundly mocked for having more trademarks than a Walt Disney store, not to mention the more serious sin of being devoid of any technical content or evidence of peer review.

**ZeoSync corporation folds after issuing $40 million in private stock**

# Universal data compression

**Proposition.** No algorithm can compress every bitstring.




Universal data compression?

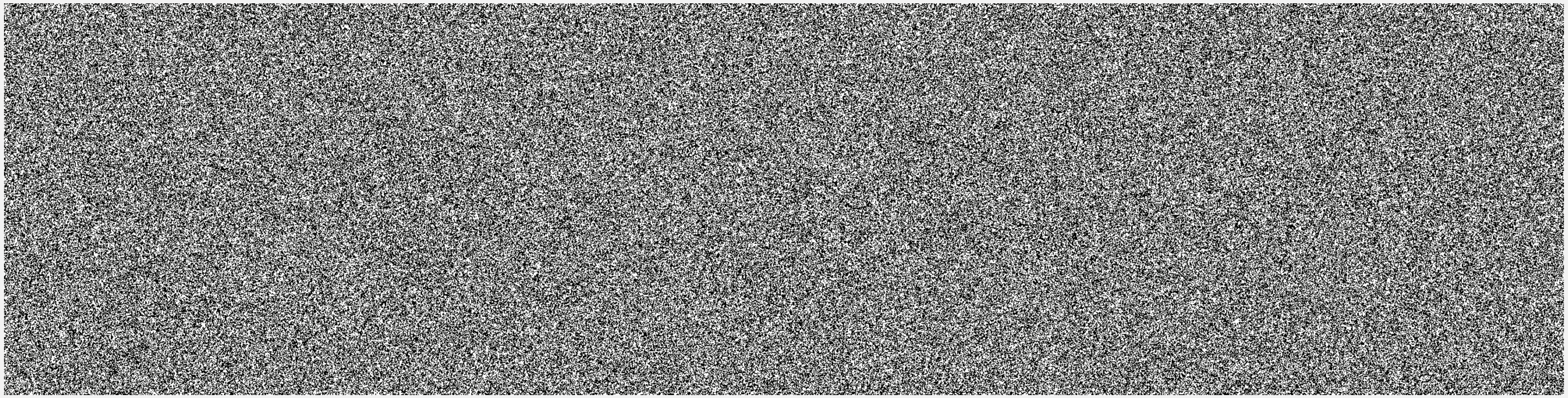
**Pf 1.** [by contradiction]


**Pf 2.** [by counting]

- Suppose your algorithm that can compress all $1{,}000$-bit strings.
- $2^{1000}$ possible bitstrings with $1{,}000$ bits.
- Only $1 + 2 + 4 + \ldots + 2^{998} + 2^{999}$ can be encoded with $\leq 999$ bits.
- Similarly, only $1$ in $2^{499}$ bitstrings can be encoded with $\leq 500$ bits!

# Undecidability

```
% java RandomBits | java PictureDump 2000 500
```



```
1000000 bits
```

**A difficult file to compress: one million (pseudo-) random bits**

```
public class RandomBits
{
   public static void main(String[] args)
   {
      int x = 11111;
      for (int i = 0; i < 1000000; i++)
      {
         x = x * 314159 + 218281;
         BinaryStdOut.write(x > 0);
      }
      BinaryStdOut.close();
   }
}
```

# Rdenudcany in Enlgsih lnagugae

Q. How much redundancy in the English language?

A. Quite a bit.

> *" ... randomising letters in the middle of words [has] little or no effect on the ability of skilled readers to understand the text. This is easy to denmtrasote. In a pubiltacion of New Scnieitst you could ramdinose all the letetrs, keipeng the first two and last two the same, and reibadailty would hadrly be aftcfeed. My ansaylis did not come to much beucase the thoery at the time was for shape and senqeuce retigcionon. Saberi's work sugsegts we may have some pofrweul palrlael prsooscers at work. The resaon for this is suerly that idnetiyfing coentnt by paarllel prseocsing speeds up regnicoiton. We only need the first and last two letetrs to spot chganes in meniang. "* — Graham Rawlinson

The gaol of data cmperisoson is to inetdify rdenudcany and epxloit it.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- run-length coding
- Huffman compression
- LZW compression

# Run-length encoding

Simple type of redundancy in a bitstream. Long runs of repeated bits.

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1

40 bits

Representation. 4-bit counts to represent alternating runs of 0s and 1s: 15 0s, then 7 1s, then 7 0s, then 11 1s.

$\underbrace{1\,1\,1\,1}_{15}\ \underbrace{0\,1\,1\,1}_{7}\ \underbrace{0\,1\,1\,1}_{7}\ \underbrace{1\,0\,1\,1}_{11}$ ← 16 bits (instead of 40)

Q. How many bits to store the counts?

A. We typically use 8 (but 4 in the example above for brevity).

Q. What to do when run length exceeds max count?

A. Intersperse runs of length 0.

Applications. JPEG, ITU-T T4 Group 3 Fax, ...

# Run-length encoding: Java implementation

```
public class RunLength
{
   private final static int R   = 256;
   private final static int lgR = 8;

   public static void compress()
   {  /* see textbook */  }

   public static void expand()
   {
      boolean bit = false;
      while (!BinaryStdIn.isEmpty())
      {
         int run = BinaryStdIn.readInt(lgR);
         for (int i = 0; i < run; i++)
            BinaryStdOut.write(bit);
         bit = !bit;
      }
      BinaryStdOut.close();
   }

}
```

maximum run-length count

number of bits per count

read 8-bit count from standard input

write 1 bit to standard output

pad 0s for byte alignment


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# 5.5 Data Compression

- introduction
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- LZW compression

David Huffman

# Variable-length codes

Use different number of bits to encode different chars.

Ex. Morse code: • • • – – – • • •

Issue. Ambiguity.

S O S ?

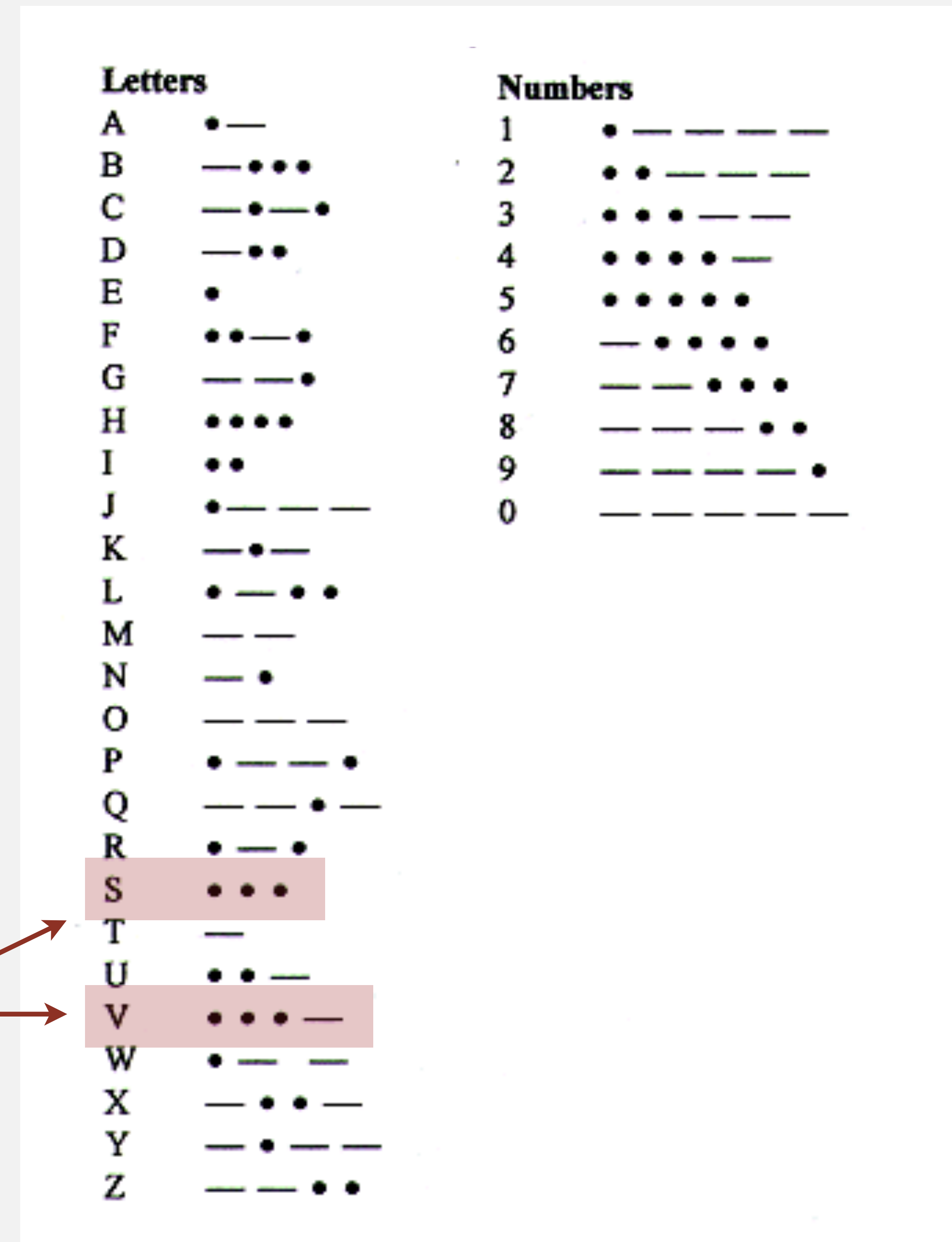
V 7 ?

I A M I E ?

E E W N I ?

In practice. Use a medium gap to separate codewords.

codeword for S is a prefix of codeword for V



| Letters | | Numbers | |
|---|---|---|---|
| A | •— | 1 | •———— |
| B | —••• | 2 | ••——— |
| C | —•—• | 3 | •••—— |
| D | —•• | 4 | ••••— |
| E | • | 5 | ••••• |
| F | ••—• | 6 | —•••• |
| G | ——• | 7 | ——••• |
| H | •••• | 8 | ———•• |
| I | •• | 9 | ————• |
| J | •——— | 0 | ————— |
| K | —•— | | |
| L | •—•• | | |
| M | —— | | |
| N | —• | | |
| O | ——— | | |
| P | •——• | | |
| Q | ——•— | | |
| R | •—• | | |
| S | ••• | | |
| T | — | | |
| U | ••— | | |
| V | •••— | | |
| W | •—— | | |
| X | —••— | | |
| Y | —•—— | | |
| Z | ——•• | | |

# Variable-length codes

Q. How do we avoid ambiguity?

A. Ensure that no codeword is a prefix of another.

Ex 1. Fixed-length code.

Ex 2. Append special stop characater to each codeword.

Ex 3. General prefix-free code.

**Codeword table**

| *key* | *value* |
|---|---|
| ! | 101 |
| A | 0 |
| B | 1111 |
| C | 110 |
| D | 100 |
| R | 1110 |

**Compressed bitstring**

011111110011001000111111100101 ← *30 bits*

A B RA CA DA B RA !

**Codeword table**

| *key* | *value* |
|---|---|
| ! | 101 |
| A | 11 |
| B | 00 |
| C | 010 |
| D | 100 |
| R | 011 |

**Compressed bitstring**

11000111101011100110001111101 ← *29 bits*

A B R A C A D A B R A !

# Prefix-free codes: trie representation

Q. How to represent the prefix-free code?

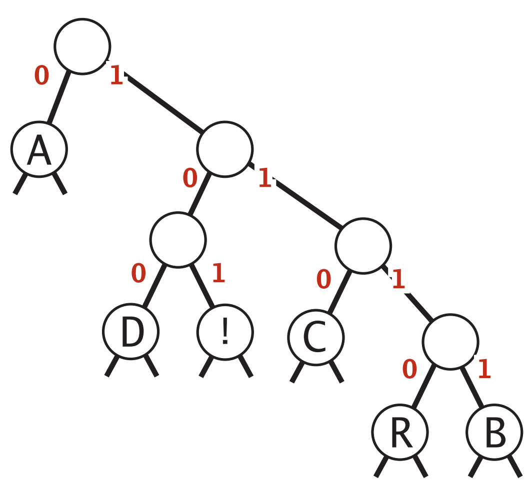
A. A binary trie!

- Characters in leaves.
- Codeword is path from root to leaf.

**Codeword table**

| *key* | *value* |
|---|---|
| ! | 101 |
| A | 0 |
| B | 1111 |
| C | 110 |
| D | 100 |
| R | 1110 |

**Trie representation**




**Compressed bitstring**

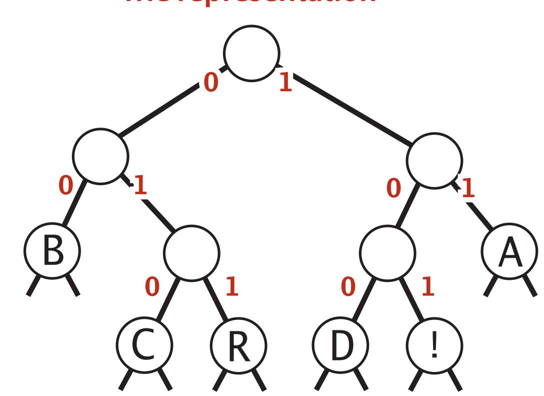
011111110011001000111111100101 ← *30 bits*

A B RA CA DA B RA !

**Codeword table**

| *key* | *value* |
|---|---|
| ! | 101 |
| A | 11 |
| B | 00 |
| C | 010 |
| D | 100 |
| R | 011 |

**Trie representation**




**Compressed bitstring**

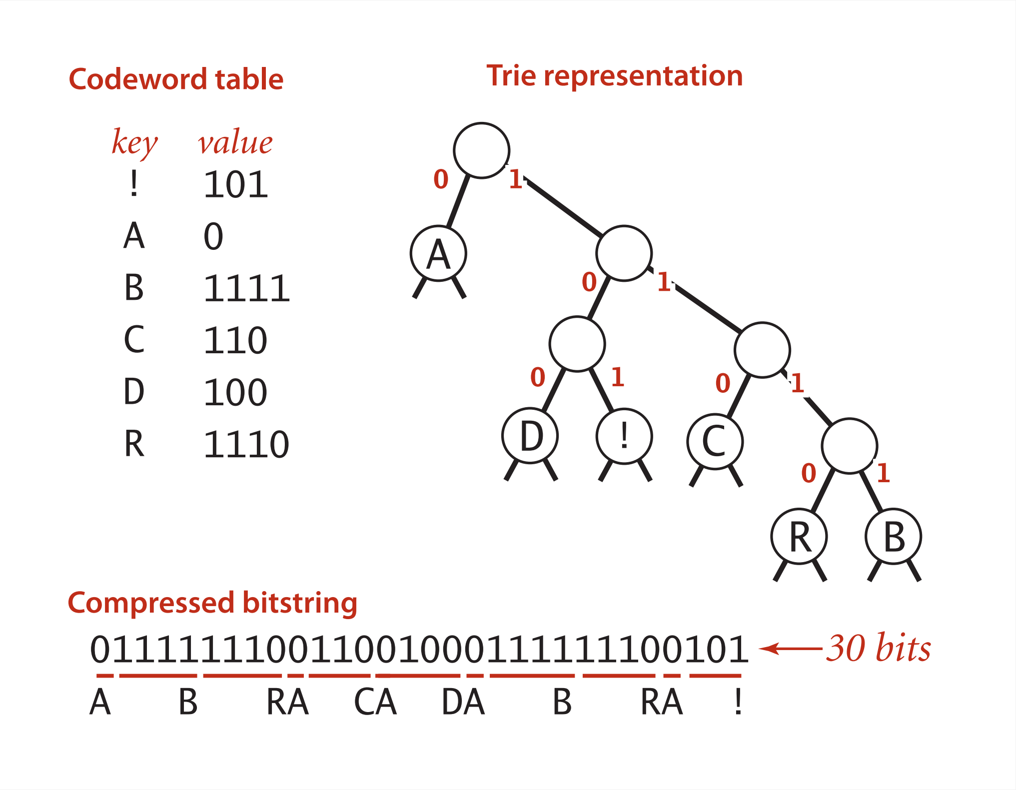
11000111101011100110001111101 ← *29 bits*

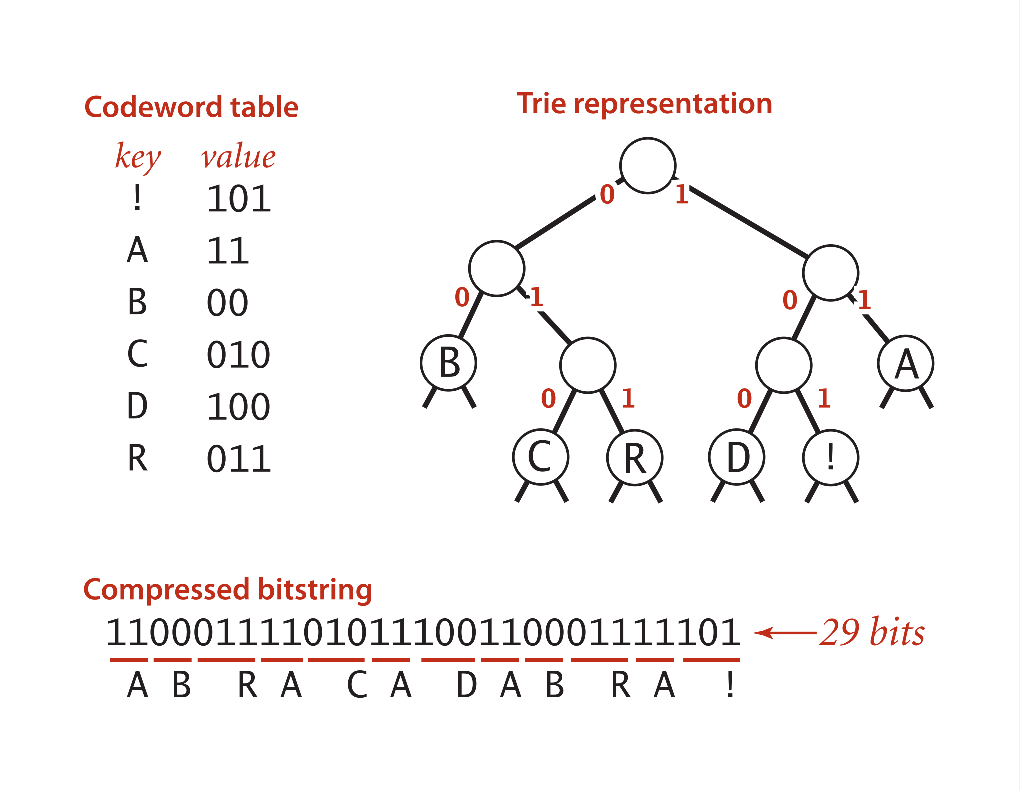
A B R A C A D A B R A !

# Prefix-free codes: expansion

## Expansion.

- Start at root.
- Go left if bit is 0; go right if 1.
- If leaf node, write character; return to root node; repeat.

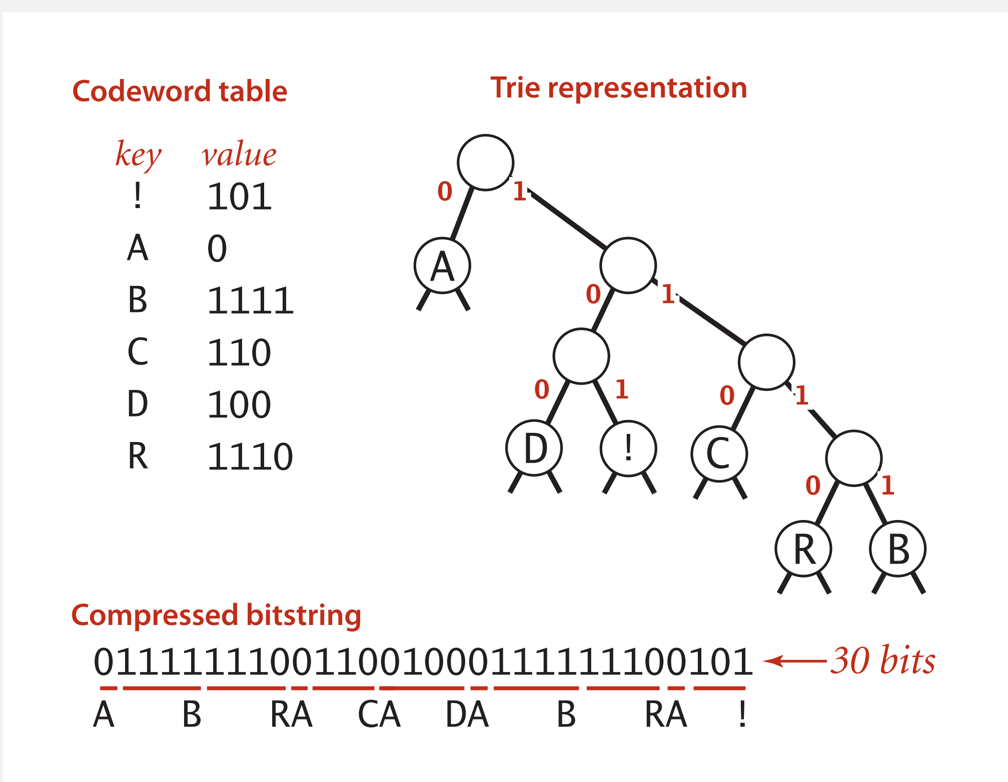


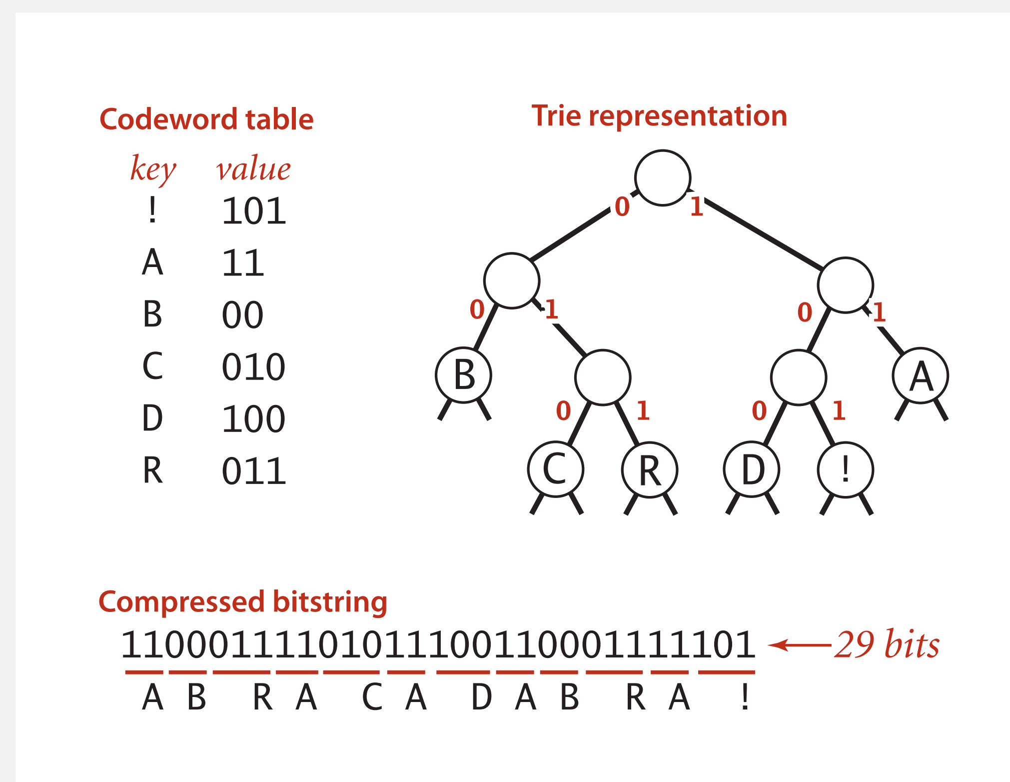



# Prefix-free codes: compression

## Compression.

- Method 1: start at leaf; follow path up to the root; print bits in reverse.
- Method 2: create ST of key-value pairs.



# Huffman coding overview

Static model. Use the same prefix-free code for all messages.

Dynamic model. Use a custom prefix-free code for each message.

Compression.

- Read message.
- Build best prefix-free code for message. How? [ahead]
- Write prefix-free code (as a trie).
- Compress message using prefix-free code.

Expansion.

- Read prefix-free code (as a trie) from file.
- Read compressed message and expand using trie.

# Huffman trie node data type

```
private static class Node implements Comparable<Node>
{
   private final char ch;   // used only for leaf nodes
   private final int freq;  // used only by compress()
   private final Node left, right;

   public Node(char ch, int freq, Node left, Node right)
   {
      this.ch    = ch;
      this.freq  = freq;
      this.left  = left;
      this.right = right;
   }

   public boolean isLeaf()
   {  return left == null && right == null; }

   public int compareTo(Node that)
   {  return this.freq - that.freq;  }
}
```

initializing constructor

is Node a leaf?

compare Nodes by frequency (stay tuned)

# Prefix-free codes: expansion

```
public void expand()
{
   Node root = readTrie();
   int N = BinaryStdIn.readInt();

   for (int i = 0; i < N; i++)
   {
      Node x = root;
      while (!x.isLeaf())
      {
         if (!BinaryStdIn.readBoolean())
            x = x.left;
         else
            x = x.right;
      }
      BinaryStdOut.write(x.ch, 8);
   }
   BinaryStdOut.close();
}
```

read in encoding trie

read in number of chars

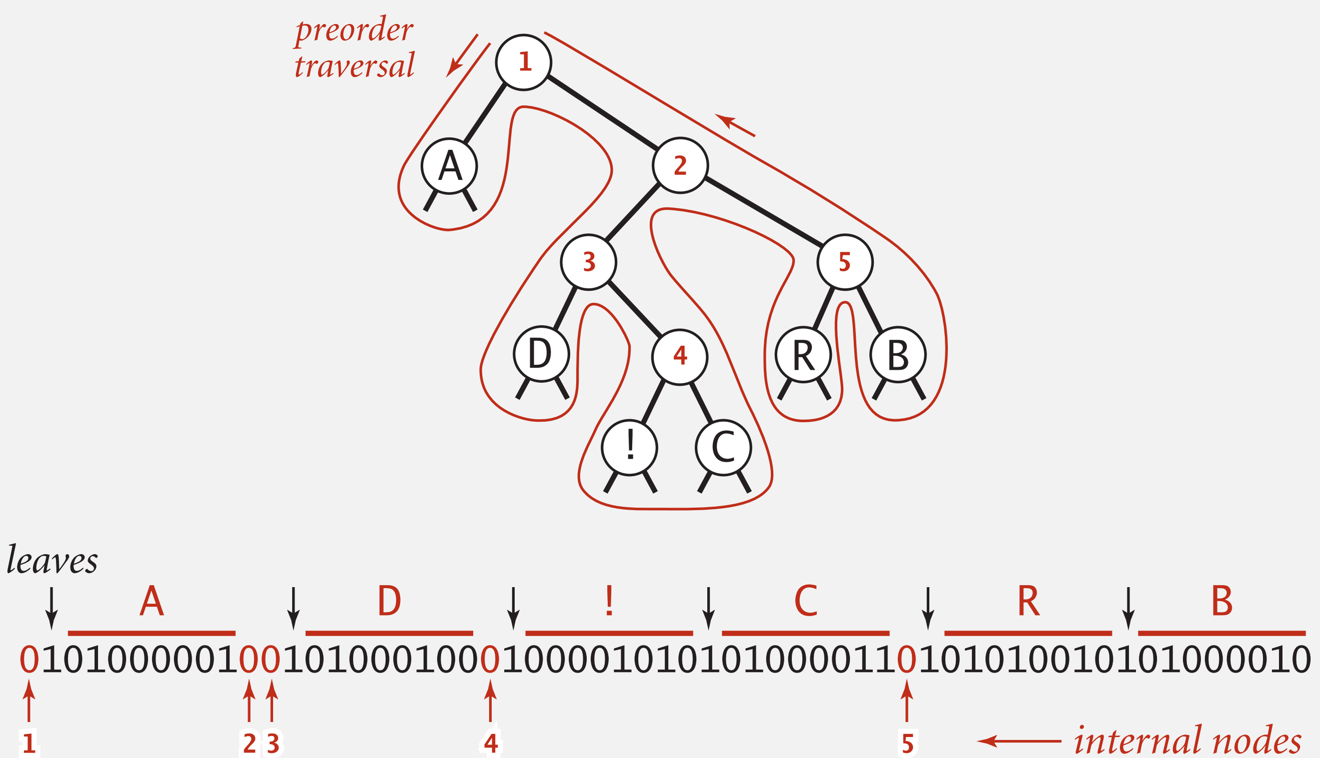
expand codeword for $i^{th}$ char

Running time. Linear in input size $N$.

# Prefix-free codes: how to transmit

Q. How to write the trie?

A. Write preorder traversal of trie; mark leaf and internal nodes with a bit.




Using preorder traversal to encode a trie as a bitstream

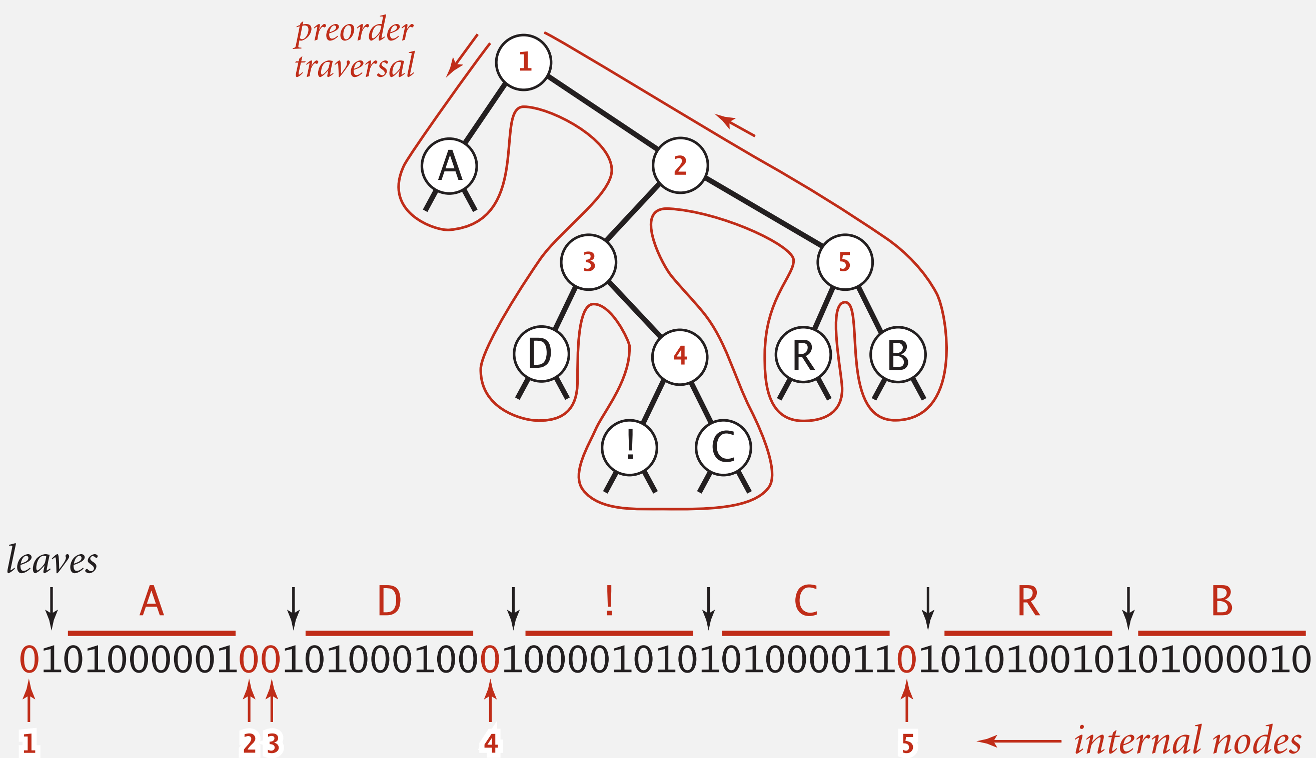
```
private static void writeTrie(Node x)
{
   if (x.isLeaf())
   {
      BinaryStdOut.write(true);
      BinaryStdOut.write(x.ch, 8);
      return;
   }
   BinaryStdOut.write(false);
   writeTrie(x.left);
   writeTrie(x.right);
}
```

Note. If message is long, overhead of transmitting trie is small.

# Prefix-free codes: how to transmit

Q. How to read in the trie?

A. Reconstruct from preorder traversal of trie.




Using preorder traversal to encode a trie as a bitstream

```
private static Node readTrie()
{
   if (BinaryStdIn.readBoolean())
   {
      char c = BinaryStdIn.readChar(8);
      return new Node(c, 0, null, null);
   }
   Node x = readTrie();
   Node y = readTrie();
   return new Node('\0', 0, x, y);
}
```

arbitrary value
(value not used with internal nodes)

# Huffman codes

Q. How to find best prefix-free code?

Huffman algorithm:

- Count frequency `freq[i]` for each char `i` in input.
- Start with one node corresponding to each char `i` (with weight `freq[i]`).
- Repeat until single trie formed:
  - select two tries with min weight `freq[i]` and `freq[j]`
  - merge into single trie with weight `freq[i] + freq[j]`

Applications:

# Huffman coding demo

- Count frequency for each character in input.

| char | freq | encoding |
|---|---|---|
| A | | |
| B | | |
| C | | |
| D | | |
| R | | |
| ! | | |

**input**

A B R A C A D A B R A !

# Huffman coding demo

- Count frequency for each character in input.

| char | freq | encoding |
|---|---|---|
| A | 5 | |
| B | 2 | |
| C | 1 | |
| D | 1 | |
| R | 2 | |
| ! | 1 | |

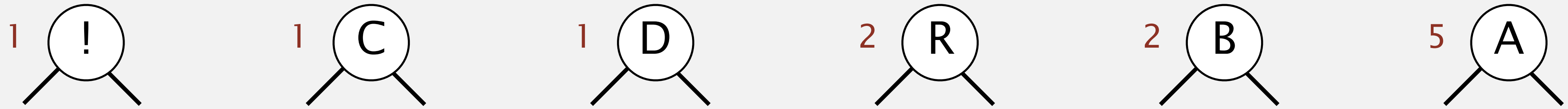
**input**

A B R A C A D A B R A !

# Huffman coding demo

- Start with one node corresponding to each character with weight equal to frequency.

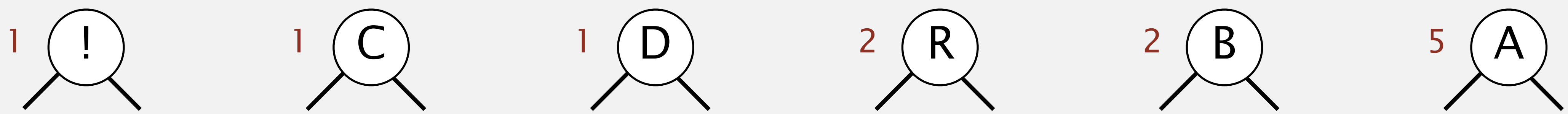
| char | freq | encoding |
|---|---|---|
| A | 5 | |
| B | 2 | |
| C | 1 | |
| D | 1 | |
| R | 2 | |
| ! | 1 | |



# Huffman coding demo

- Select two tries with min weight.
- Merge into single trie with cumulative weight.

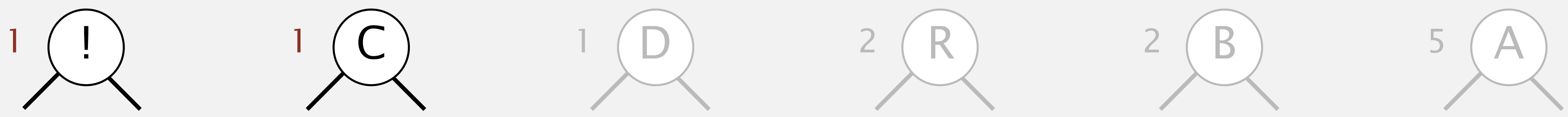
| char | freq | encoding |
|---|---|---|
| A | 5 | |
| B | 2 | |
| C | 1 | |
| D | 1 | |
| R | 2 | |
| ! | 1 | |



# Huffman coding demo

- Select two tries with min weight.
- Merge into single trie with cumulative weight.

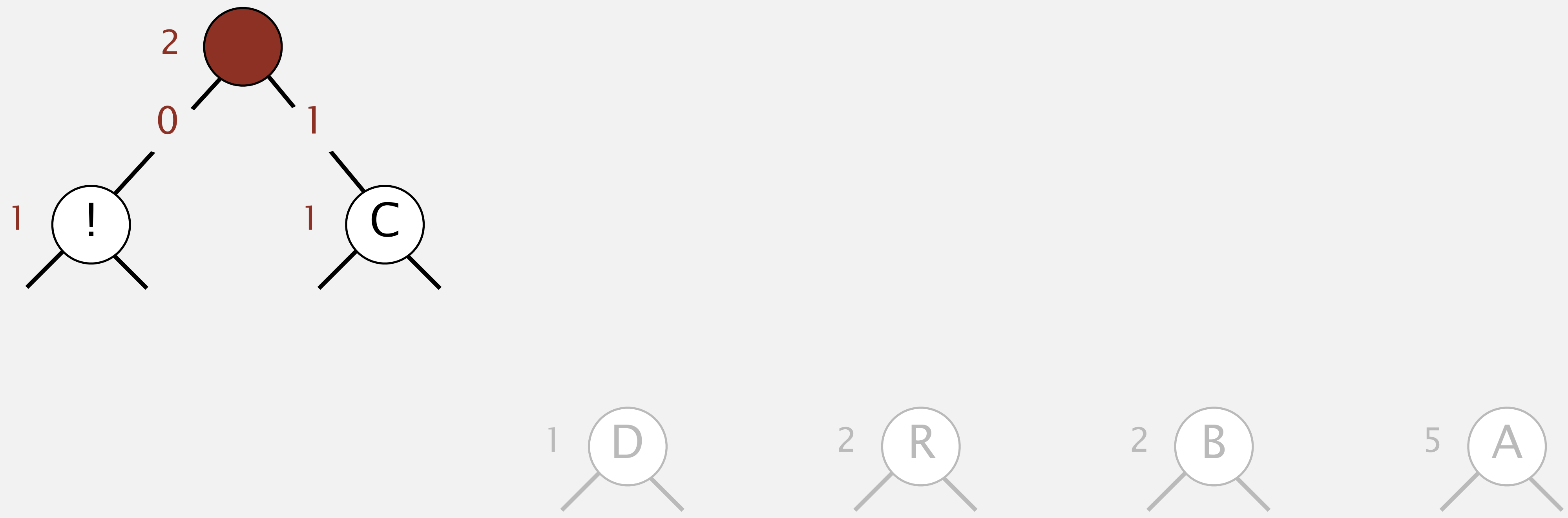
| char | freq | encoding |
|---|---|---|
| A | 5 | |
| B | 2 | |
| C | 1 | |
| D | 1 | |
| R | 2 | |
| ! | 1 | |



# Huffman coding demo

- Select two tries with min weight.
- Merge into single trie with cumulative weight.

| char | freq | encoding |
|---|---|---|
| A | 5 | |
| B | 2 | |
| C | 1 | 1 |
| D | 1 | |
| R | 2 | |
| ! | 1 | 0 |



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- Select two tries with min weight.
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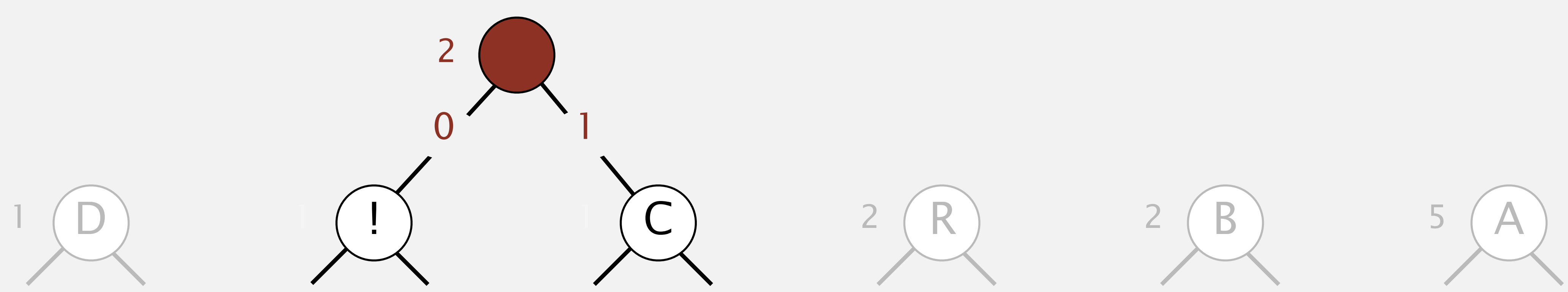

# Huffman coding demo

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| char | freq | encoding |
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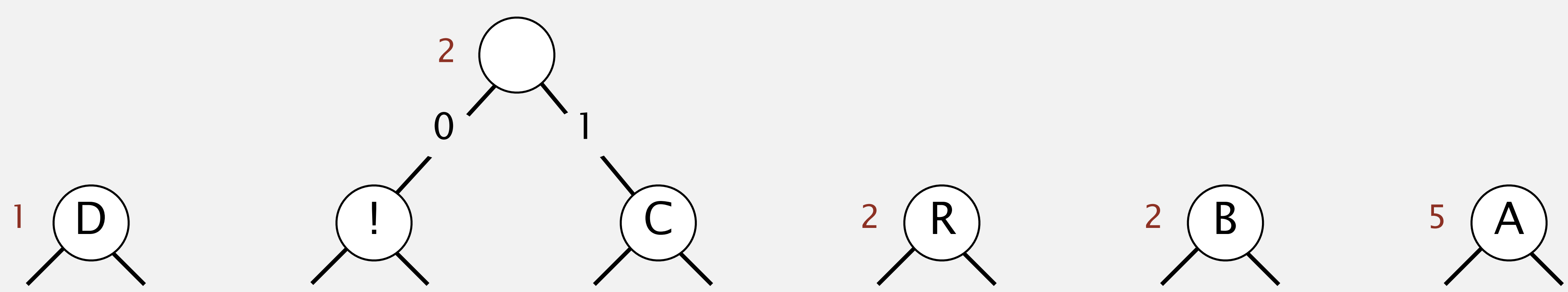

# Huffman coding demo

- Select two tries with min weight.
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| char | freq | encoding |
|---|---|---|
| A | 5 | |
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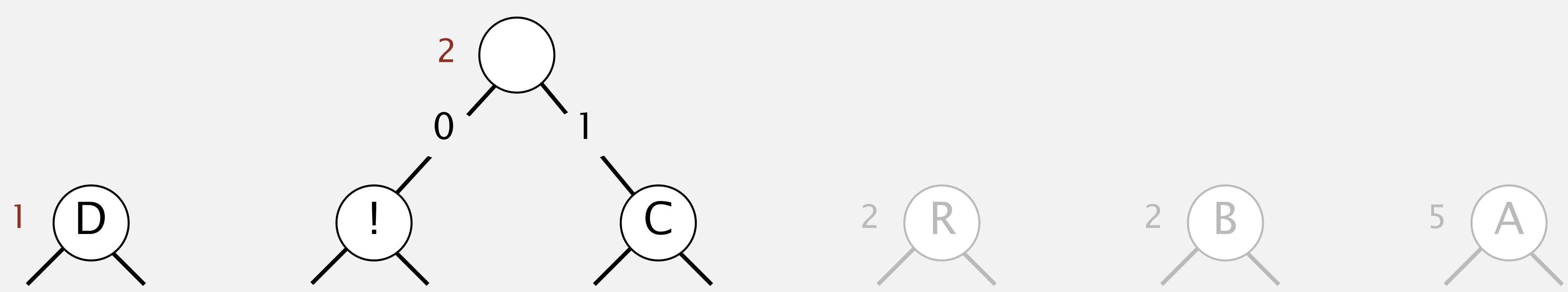
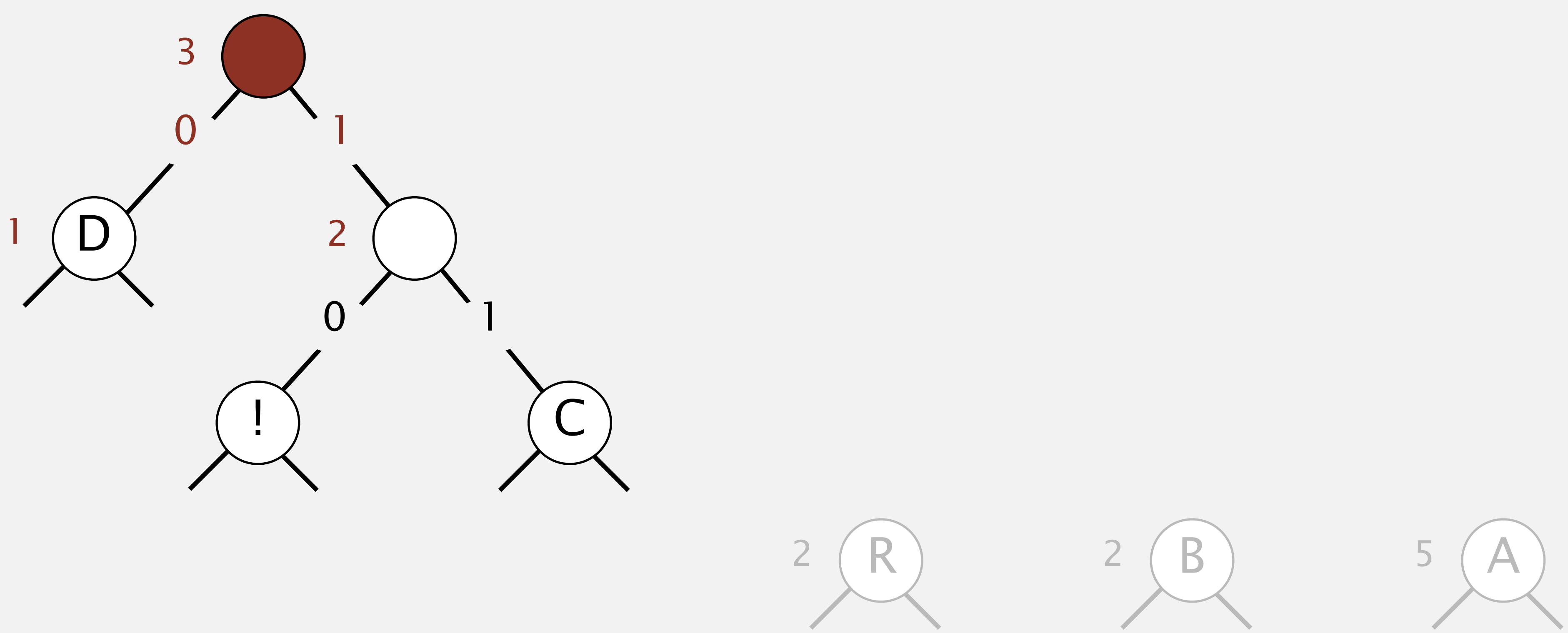


# Huffman coding demo

- Select two tries with min weight.
- Merge into single trie with cumulative weight.

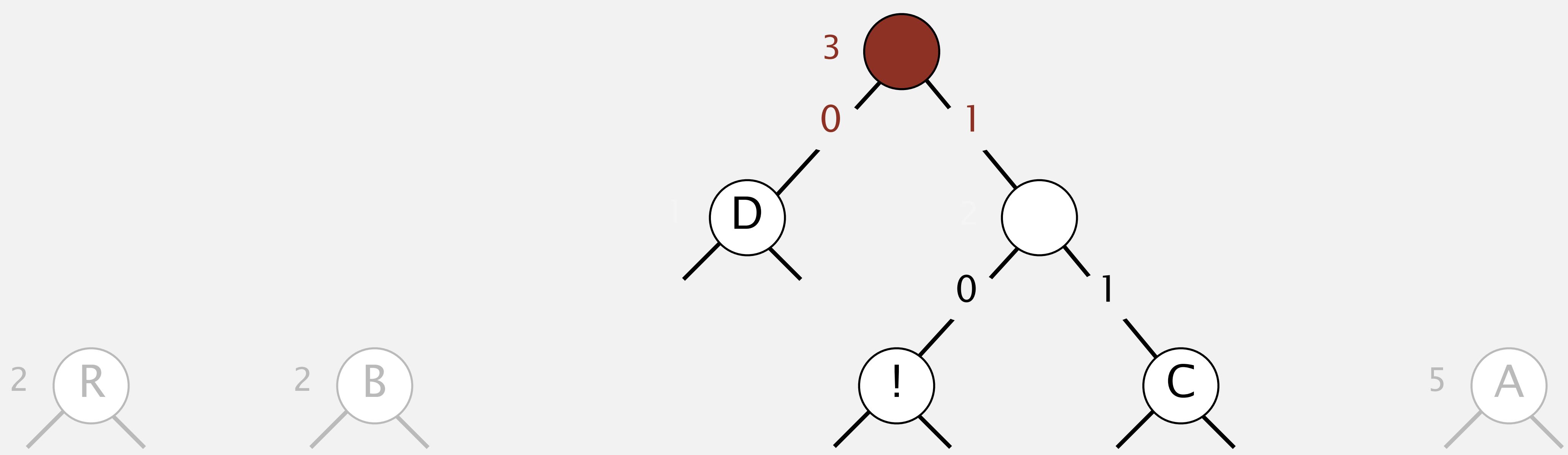
| char | freq | encoding |
|---|---|---|
| A | 5 | |
| B | 2 | |
| C | 1 | 1 1 |
| D | 1 | 0 |
| R | 2 | |
| ! | 1 | 1 0 |



# Huffman coding demo

- Select two tries with min weight.
- Merge into single trie with cumulative weight.

| char | freq | encoding |
|---|---|---|
| A | 5 | |
| B | 2 | |
| C | 1 | 1 1 |
| D | 1 | 0 |
| R | 2 | |
| ! | 1 | 1 0 |



# Huffman coding demo

- Select two tries with min weight.
- Merge into single trie with cumulative weight.

| char | freq | encoding |
|---|---|---|
| A | 5 | |
| B | 2 | |
| C | 1 | 1 1 |
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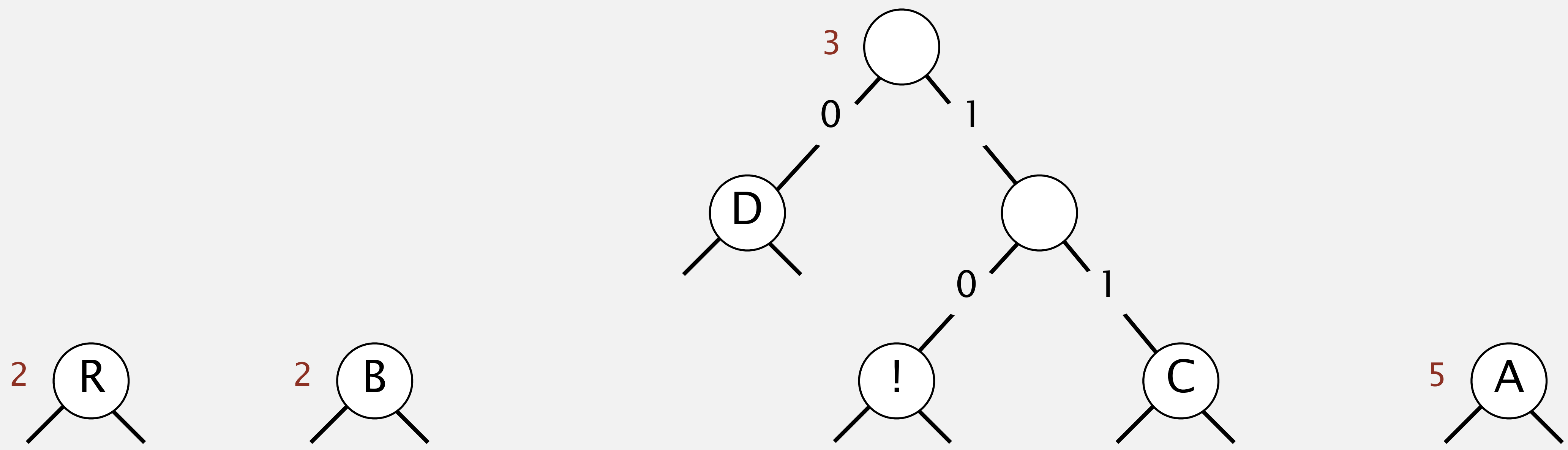
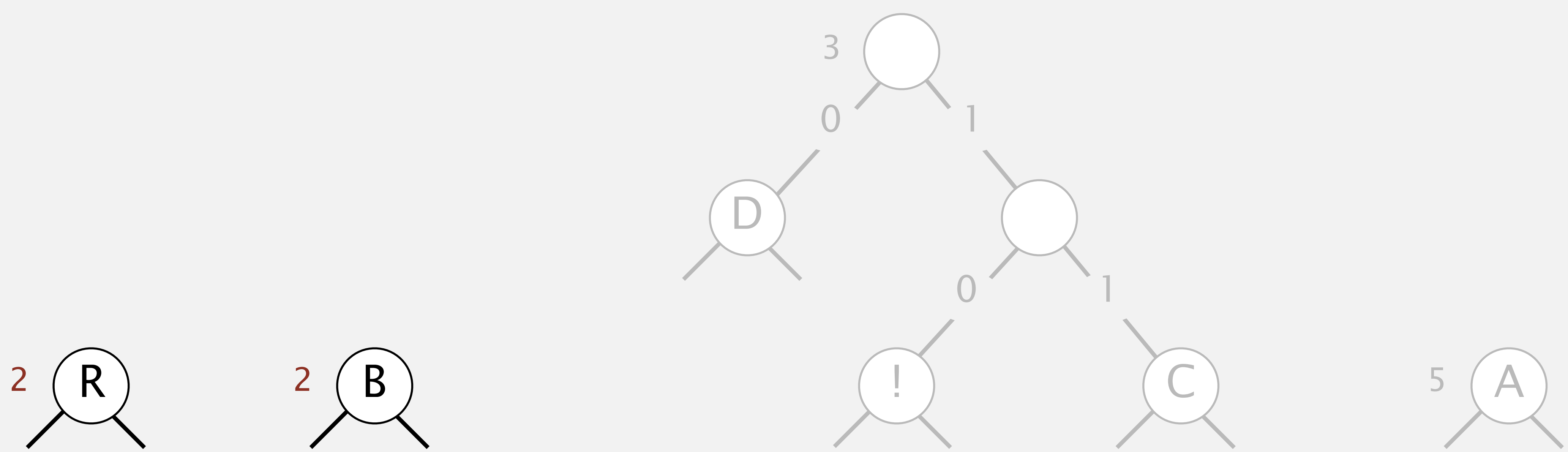


# Huffman coding demo

- Select two tries with min weight.
- Merge into single trie with cumulative weight.

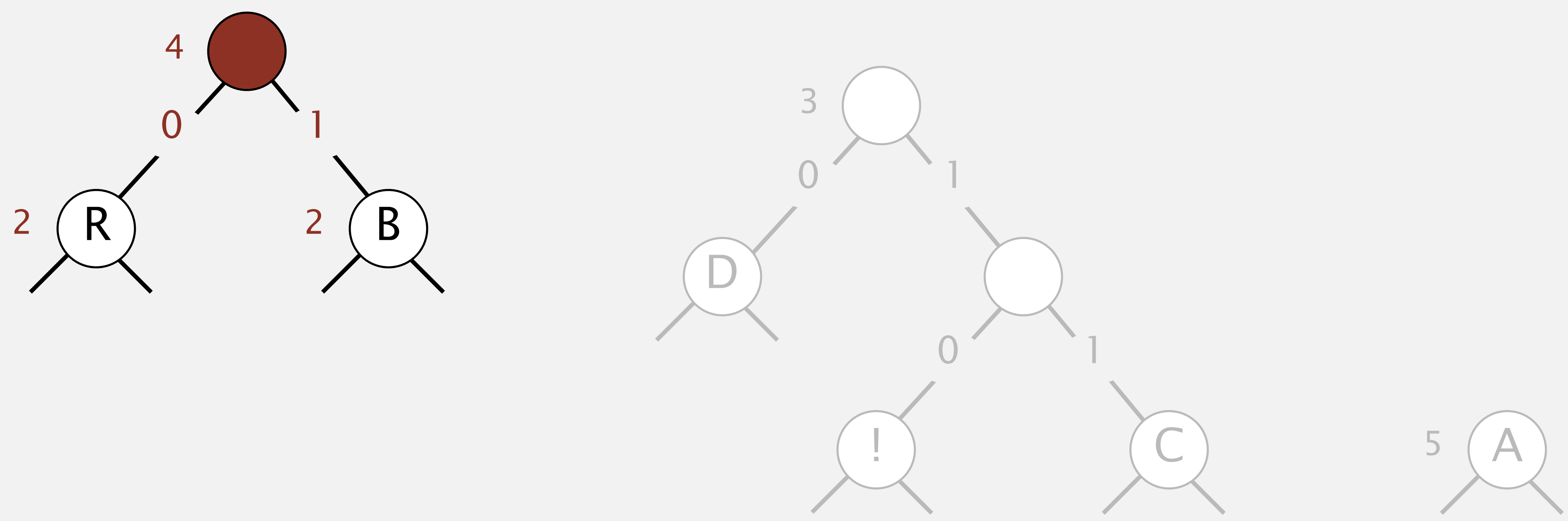
| char | freq | encoding |
|---|---|---|
| A | 5 | |
| B | 2 | |
| C | 1 | 1 1 |
| D | 1 | 0 |
| R | 2 | |
| ! | 1 | 1 0 |



# Huffman coding demo

- Select two tries with min weight.
- Merge into single trie with cumulative weight.

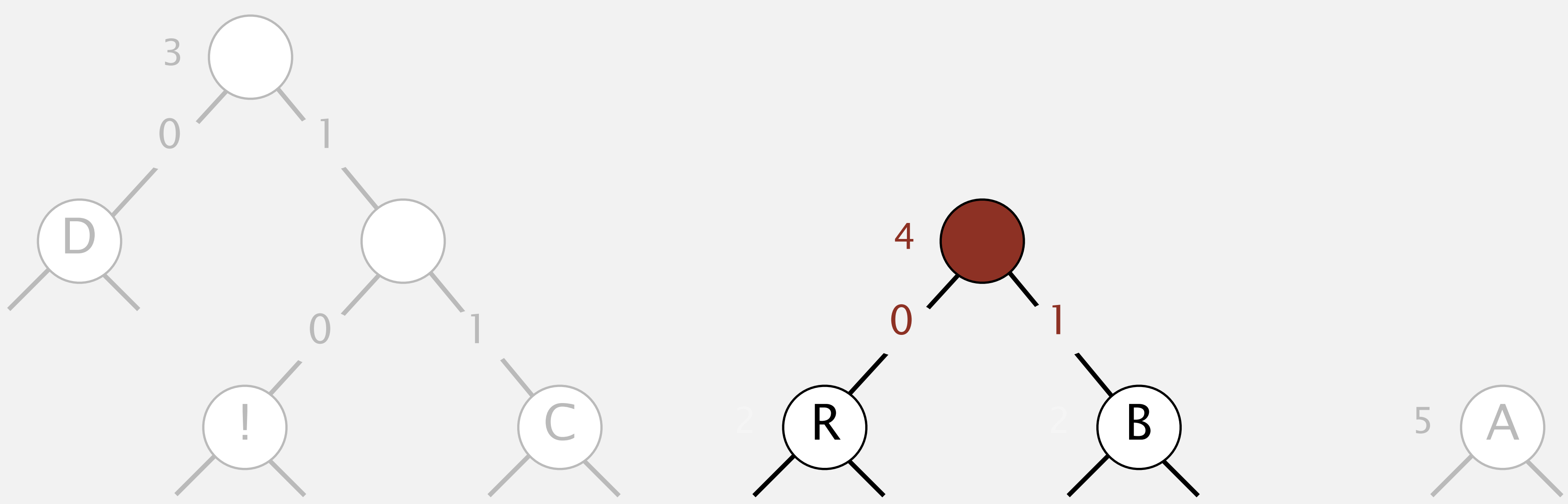
| char | freq | encoding |
|---|---|---|
| A | 5 | |
| B | 2 | 1 |
| C | 1 | 1 1 |
| D | 1 | 0 |
| R | 2 | 0 |
| ! | 1 | 1 0 |



# Huffman coding demo

- Select two tries with min weight.
- Merge into single trie with cumulative weight.

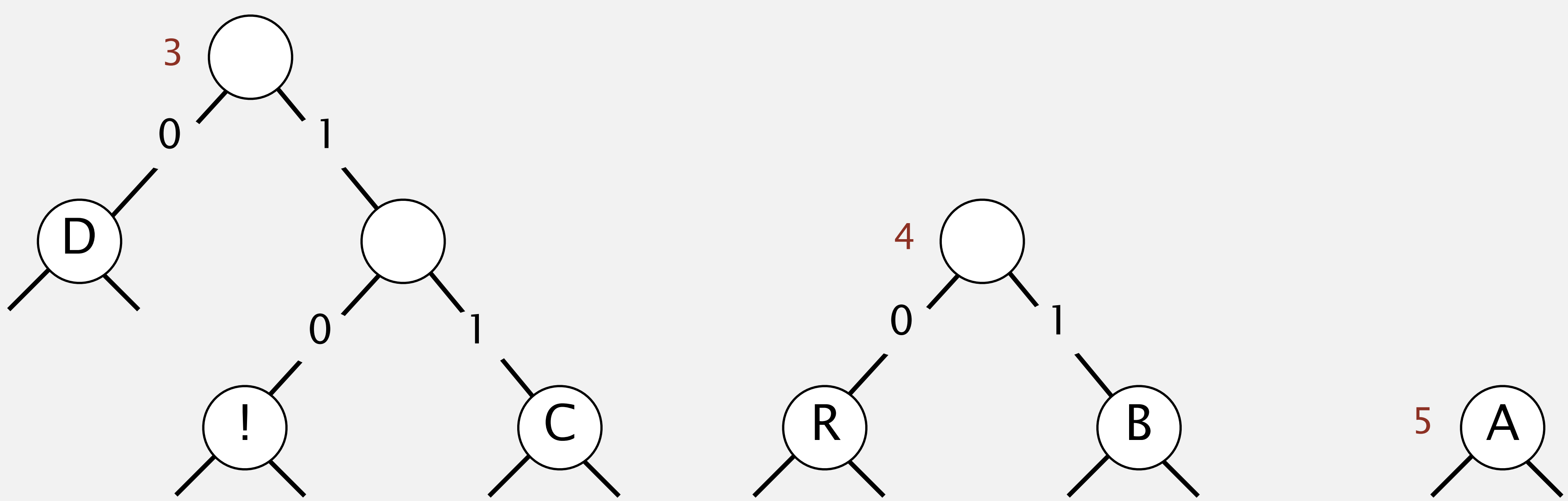
| char | freq | encoding |
|---|---|---|
| A | 5 | |
| B | 2 | 1 |
| C | 1 | 1 1 |
| D | 1 | 0 |
| R | 2 | 0 |
| ! | 1 | 1 0 |



# Huffman coding demo

- Select two tries with min weight.
- Merge into single trie with cumulative weight.

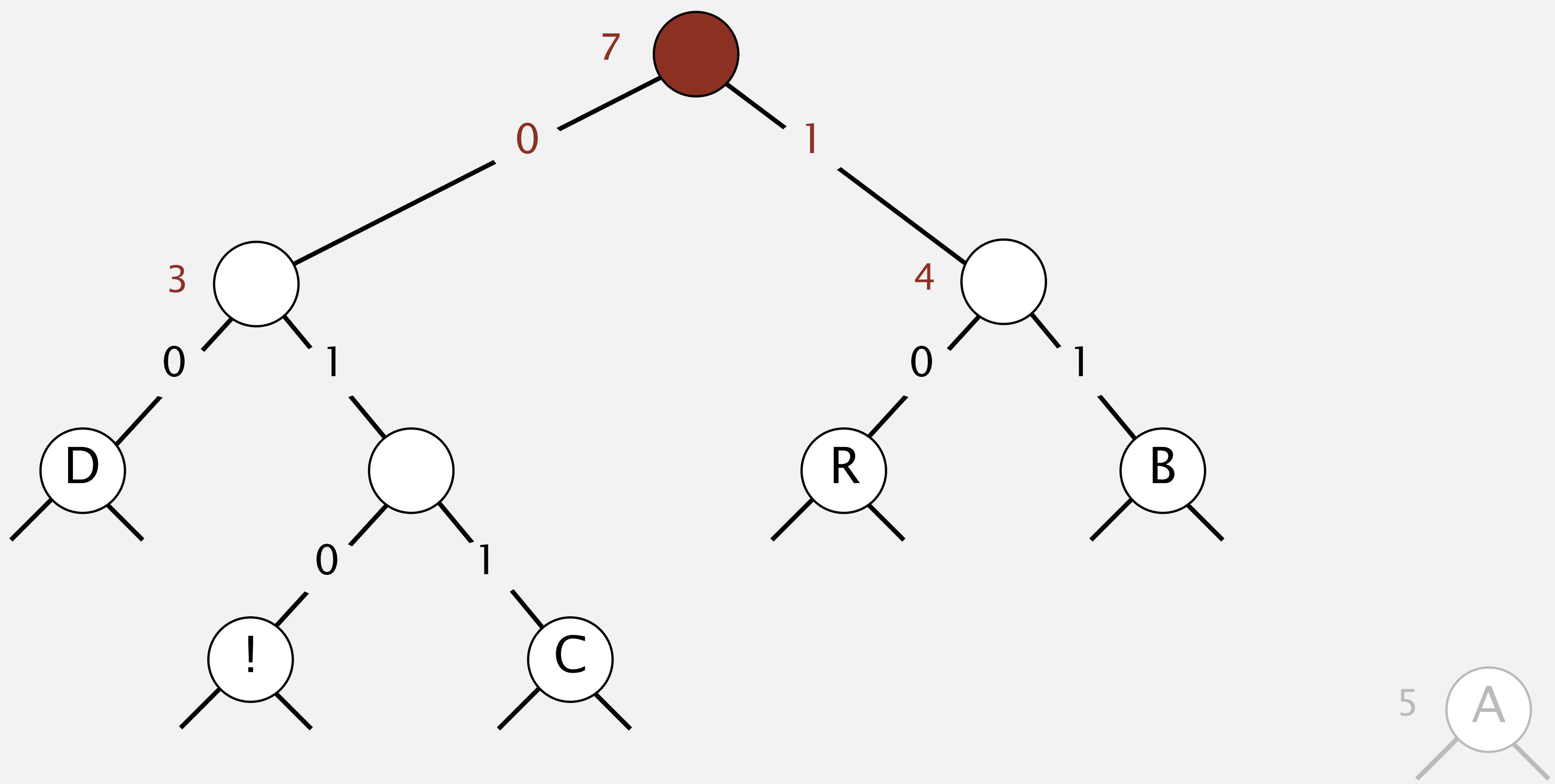
| char | freq | encoding |
|---|---|---|
| A | 5 | |
| B | 2 | 1 |
| C | 1 | 1 1 |
| D | 1 | 0 |
| R | 2 | 0 |
| ! | 1 | 1 0 |



# Huffman coding demo

- Select two tries with min weight.
- Merge into single trie with cumulative weight.

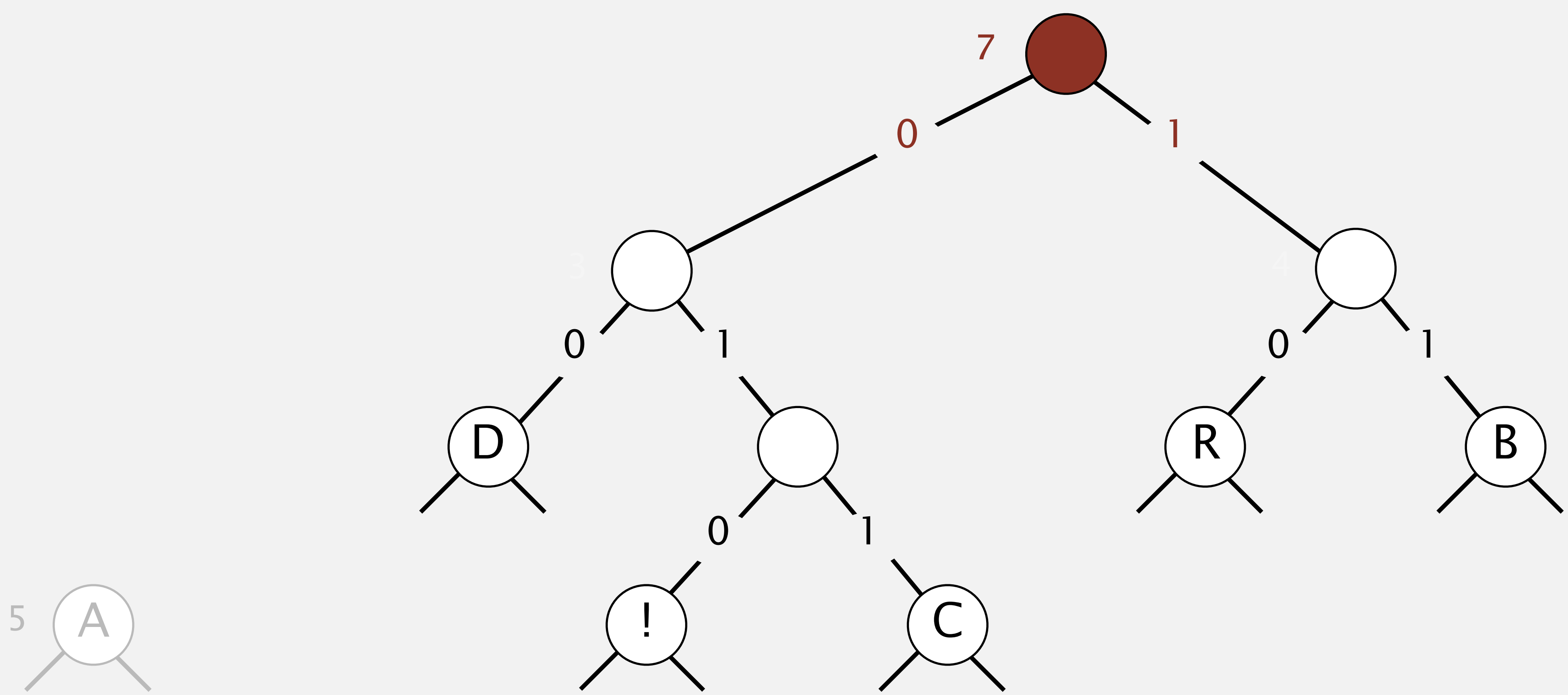
| char | freq | encoding |
|---|---|---|
| A | 5 | |
| B | 2 | 1 1 |
| C | 1 | 0 1 1 |
| D | 1 | 0 0 |
| R | 2 | 1 0 |
| ! | 1 | 0 1 0 |



# Huffman coding demo

- Select two tries with min weight.
- Merge into single trie with cumulative weight.

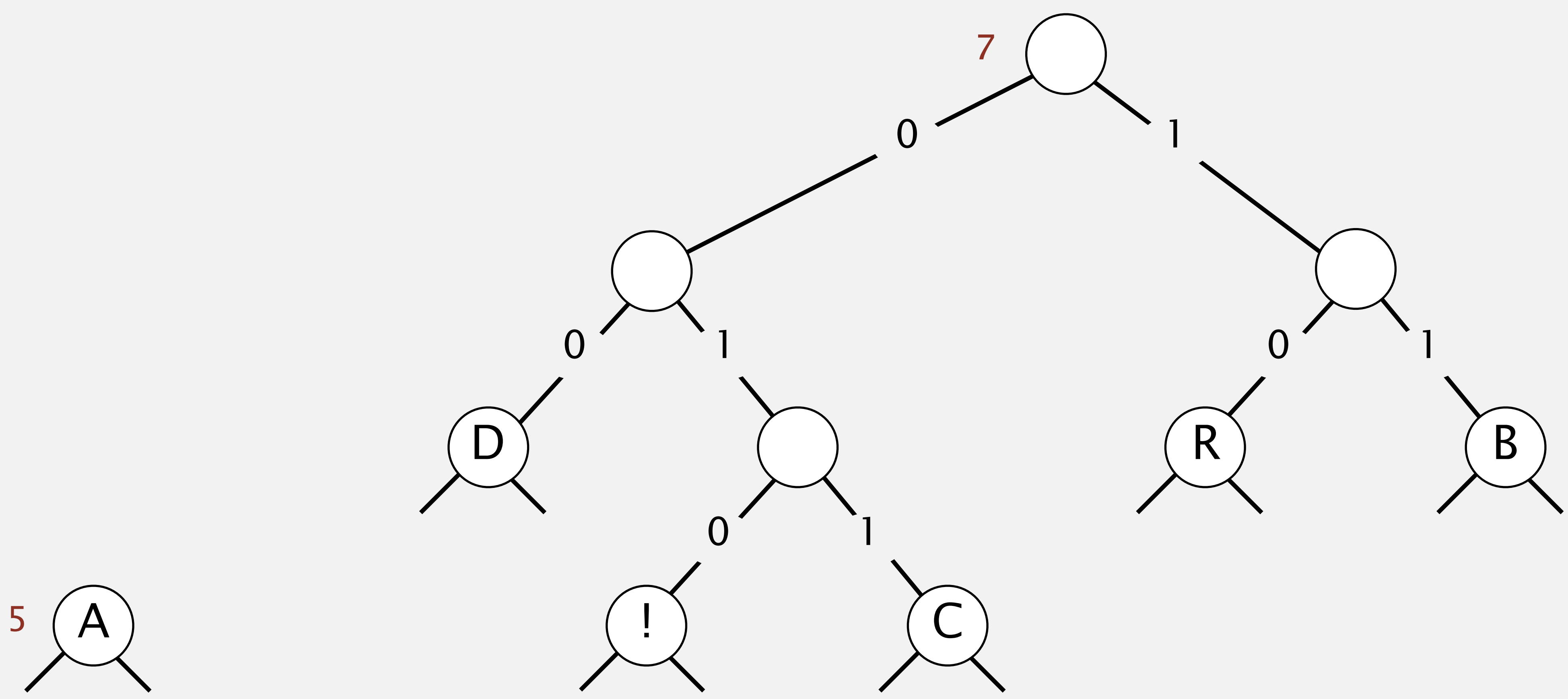
| char | freq | encoding |
|---|---|---|
| A | 5 | |
| B | 2 | 1 1 |
| C | 1 | 0 1 1 |
| D | 1 | 0 0 |
| R | 2 | 1 0 |
| ! | 1 | 0 1 0 |



# Huffman coding demo

- Select two tries with min weight.
- Merge into single trie with cumulative weight.

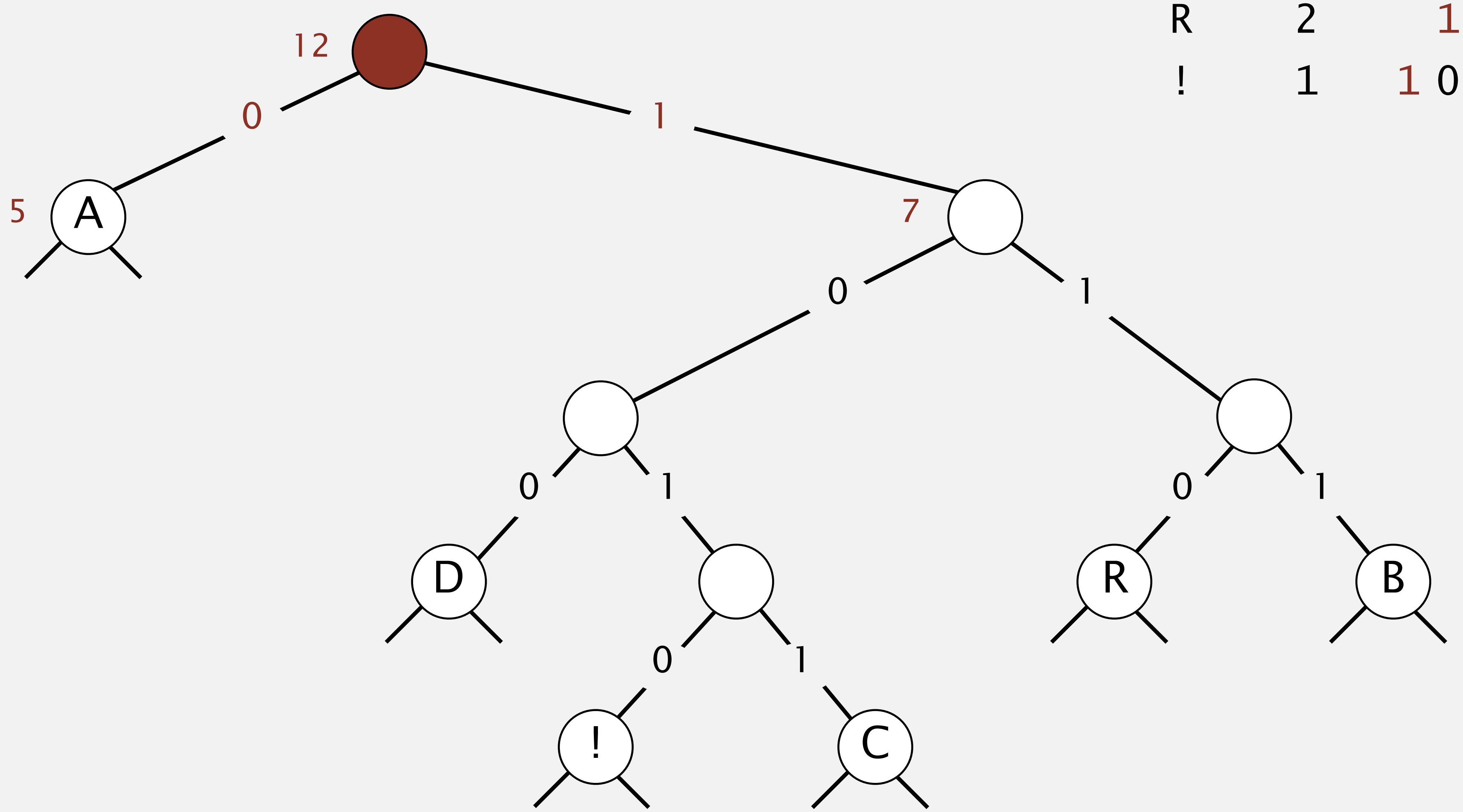
| char | freq | encoding |
|---|---|---|
| A | 5 | |
| B | 2 | 1 1 |
| C | 1 | 0 1 1 |
| D | 1 | 0 0 |
| R | 2 | 1 0 |
| ! | 1 | 0 1 0 |



# Huffman coding demo

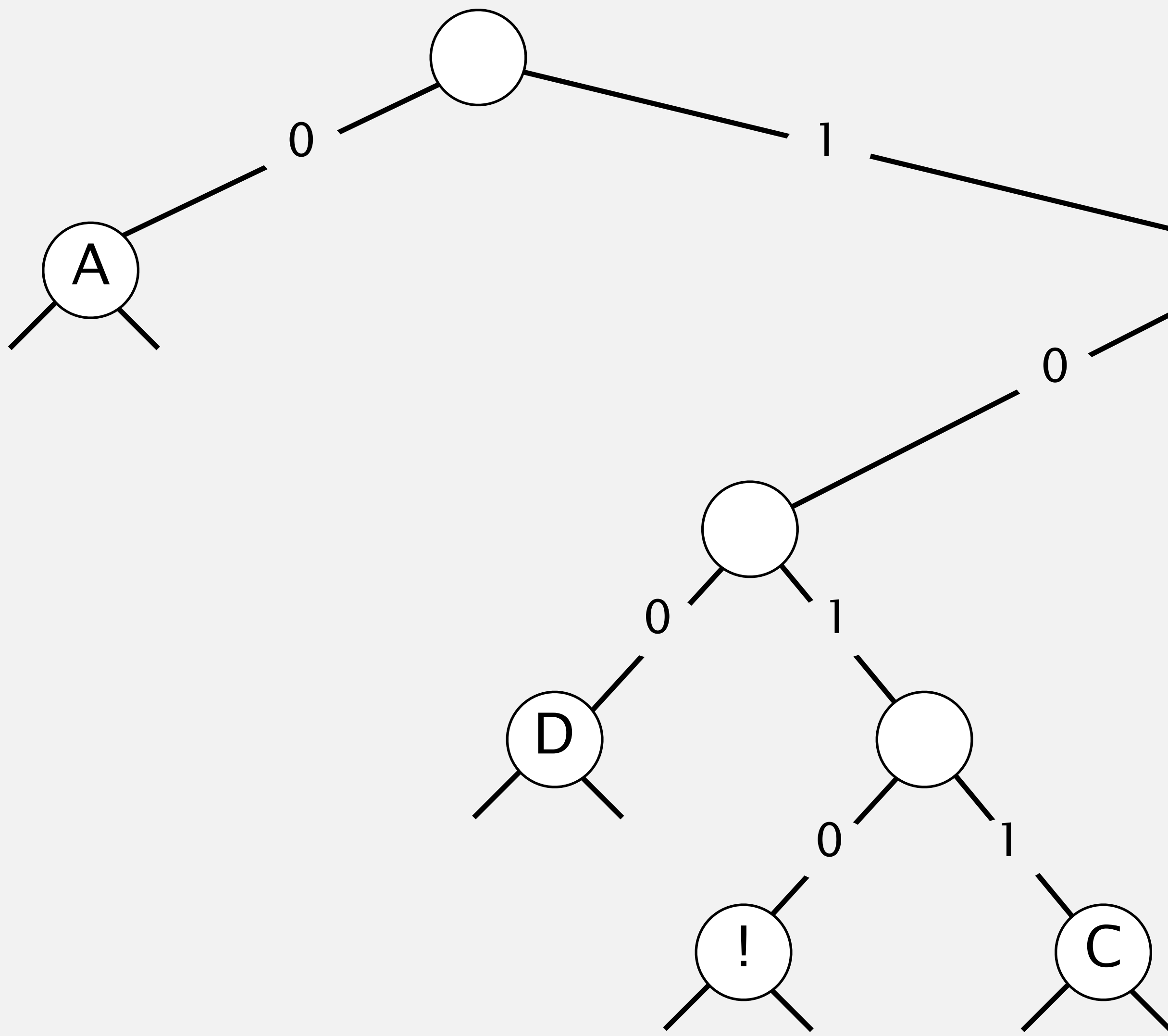
- Select two tries with min weight.
- Merge into single trie with cumulative weight.

| char | freq | encoding |
|---|---|---|
| A | 5 | 0 |
| B | 2 | 1 1 1 |
| C | 1 | 1 0 1 1 |
| D | 1 | 1 0 0 |
| R | 2 | 1 1 0 |
| ! | 1 | 1 0 1 0 |



# Huffman coding demo

| char | freq | encoding |
|---|---|---|
| A | 5 | 0 |
| B | 2 | 1 1 1 |
| C | 1 | 1 0 1 1 |
| D | 1 | 1 0 0 |
| R | 2 | 1 1 0 |
| ! | 1 | 1 0 1 0 |



# Constructing a Huffman encoding trie: Java implementation

```java
private static Node buildTrie(int[] freq)
{
    MinPQ<Node> pq = new MinPQ<Node>();
        for (char i = 0; i < R; i++)
           if (freq[i] > 0)
              pq.insert(new Node(i, freq[i], null, null));

    while (pq.size() > 1)
    {
       Node x = pq.delMin();
       Node y = pq.delMin();
       Node parent = new Node('\0', x.freq + y.freq, x, y);
       pq.insert(parent);
    }

    return pq.delMin();
}
```

initialize PQ with singleton tries

merge two smallest tries

not used for internal nodes

total frequency

two subtries

# Huffman compression summary

Proposition. Huffman's algorithm produces an optimal prefix-free code.

Pf. See textbook.

no prefix-free code
uses fewer bits

Two-pass implementation (for compression).

- Pass 1: tabulate character frequencies; build trie.
- Pass 2: encode file by traversing trie (or symbol table).

Running time (for compression). Using a binary heap $\Rightarrow$ $N + R \log R$.

Running time (for expansion). Using a binary trie $\Rightarrow$ $N$.

input size

alphabet size

Q. Can we do better? [stay tuned]


http://algs4.cs.princeton.edu

# 5.5 Data Compression

- introduction
- run-length coding
- Huffman compression
- *LZW compression*

Abraham Lempel

Jacob Ziv

# Statistical methods

Static model. Same model for all texts.

- Fast.
- Not optimal: different texts have different statistical properties.
- Ex: ASCII, Morse code.

Dynamic model. Generate model based on text.

- Preliminary pass needed to generate model.
- Must transmit the model.
- Ex: Huffman code.

Adaptive model. Progressively learn and update model as you read text.

- More accurate modeling produces better compression.
- Decoding must start from beginning.
- Ex: LZW.

# LZW compression demo

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *input* | A | B | R | A | C | A | D | A | B | R | A | B | R | A | B | R | A |
| *matches* | A | B | R | A | C | A | D | A B | | R A | | B R | | A B R | | | A |
| *value* | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | | 83 | | 82 | | 88 | | | 41 80 |

**LZW compression for A B R A C A D A B R A B R A B R A**

| key | value |
|---|---|
| ⋮ | ⋮ |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| ⋮ | ⋮ |

| key | value |
|---|---|
| AB | 81 |
| BR | 82 |
| RA | 83 |
| AC | 84 |
| CA | 85 |
| AD | 86 |

| key | value |
|---|---|
| DA | 87 |
| ABR | 88 |
| RAB | 89 |
| BRA | 8A |
| ABRA | 8B |
| | |

**codeword table**

# Lempel-Ziv-Welch compression

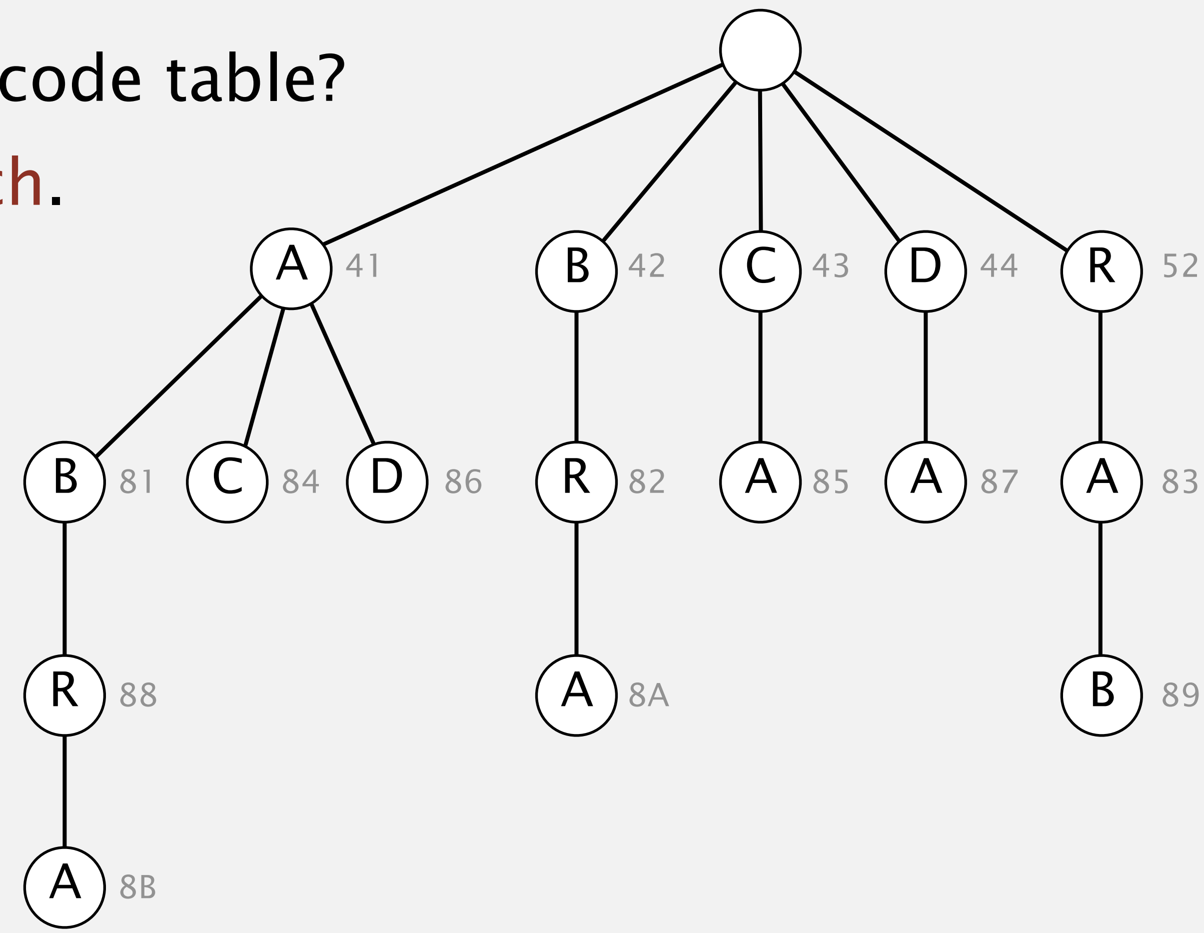
LZW compression.

- Create ST associating $W$-bit codewords with string keys.
- Initialize ST with codewords for single-character keys.
- Find longest string $s$ in ST that is a prefix of unscanned part of input.
- Write the $W$-bit codeword associated with $s$.
- Add $s + c$ to ST, where $c$ is next character in the input.

longest prefix match

Q. How to represent LZW compression code table?

A. A trie to support longest prefix match.



# LZW expansion demo

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *value* | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | 83 | 82 | 88 | 41 | 80 |
| *output* | A | B | R | A | C | A | D | A B | R A | B R | A B R | A | |

**LZW expansion for 41 42 52 41 43 41 44 81 83 82 88 41 80**

| key | value | key | value | key | value |
|---|---|---|---|---|---|
| ⋮ | ⋮ | 81 | AB | 87 | DA |
| 41 | A | 82 | BR | 88 | ABR |
| 42 | B | 83 | RA | 89 | RAB |
| 43 | C | 84 | AC | 8A | BRA |
| 44 | D | 85 | CA | 8B | ABRA |
| ⋮ | ⋮ | 86 | AD | | |

**codeword table**

# LZW expansion

LZW expansion.

- Create ST associating string values with $W$-bit keys.
- Initialize ST to contain single-character values.
- Read a $W$-bit key.
- Find associated string value in ST and write it out.
- Update ST.

Q. How to represent LZW expansion code table?

A. An array of length $2^W$.

| key | value |
| --- | --- |
| ⋮ | ⋮ |
| 65 | A |
| 66 | B |
| 67 | C |
| 68 | D |
| ⋮ | ⋮ |
| 129 | AB |
| 130 | BR |
| 131 | RA |
| 132 | AC |
| 133 | CA |
| 134 | AD |
| 135 | DA |
| 136 | ABR |
| 137 | RAB |
| 138 | BRA |
| 139 | ABRA |
| ⋮ | ⋮ |

# LZW tricky case: compression

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *input* | A | B | A | B | A | B | A | |
| *matches* | A | B | A B | | A B A | | | |
| *value* | 41 | 42 | 81 | | 83 | | | 80 |

**LZW compression for ABABABA**

| key | value |
|---|---|
| ⋮ | ⋮ |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| ⋮ | ⋮ |

| key | value |
|---|---|
| AB | 81 |
| BA | 82 |
| ABA | 83 |
| | |
| | |
| | |

**codeword table**

# LZW tricky case: expansion

| *value* | 41 | 42 | 81 | 83 | 80 |
|---|---|---|---|---|---|
| *output* | A | B | A B | A B A | |

need to know code for 83 before it is in codeword table!

we can deduce that the code for 83 is ABx for some character x

now, we have deduced x!

**LZW expansion for 41 42 81 83 80**

| key | value |
|---|---|
| ⋮ | ⋮ |
| 41 | A |
| 42 | B |
| 43 | C |
| 44 | D |
| ⋮ | ⋮ |

| key | value |
|---|---|
| 81 | AB |
| 82 | BA |
| 83 | ABA |
| | |
| | |
| | |

**codeword table**

# LZW implementation details

How big to make ST?

- How long is message?
- Whole message similar model?
- [many other variations]

What to do when ST fills up?

- Throw away and start over. [GIF]
- Throw away when not effective. [Unix compress]
- [many other variations]

Why not put longer substrings in ST?

- [many variations have been developed]

# LZW in the real world

Lempel-Ziv and friends.

- LZ77.
- LZ78.
- LZW.
- Deflate / zlib = LZ77 variant + Huffman.

Unix compress, GIF, TIFF, V.42bis modem: LZW. ← previously under patent

zip, 7zip, gzip, jar, png, pdf: deflate / zlib.

iPhone, Wii, Apache HTTP server: deflate / zlib. ← not patented (widely used in open source)

# Lossless data compression benchmarks

| year | scheme | bits / char |
|---|---|---|
| 1967 | **ASCII** | 7 |
| 1950 | **Huffman** | 4.7 |
| 1977 | **LZ77** | 3.94 |
| 1984 | **LZMW** | 3.32 |
| 1987 | **LZH** | 3.3 |
| 1987 | **move-to-front** | 3.24 |
| 1987 | **LZB** | 3.18 |
| 1987 | **gzip** | 2.71 |
| 1988 | **PPMC** | 2.48 |
| 1994 | **SAKDC** | 2.47 |
| 1994 | **PPM** | 2.34 |
| 199b | **Burrows-Wheeler** | 2.29 |
| 1997 | **BOA** | 1.99 |
| 1999 | **RK** | 1.89 |

**data compression using Calgary corpus**

# Data compression summary

Lossless compression.

- Represent fixed-length symbols with variable-length codes.  [Huffman]
- Represent variable-length symbols with fixed-length codes.  [LZW]

Lossy compression.  [not covered in this course]

- JPEG, MPEG, MP3, …
- FFT/DCT, wavelets, fractals, …

$$X_k = \sum_{n=0}^{N-1} x_n \cos\left[\frac{\pi}{N}\left(n + \frac{1}{2}\right) k\right]$$

Theoretical limits on compression.  Shannon entropy: $H(X) = -\sum_{i}^{n} p(x_i) \lg p(x_i)$

Practical compression.  Exploit extra knowledge whenever possible.