

Programming Challenges (GB21802)

Week 7 - String Manipulation

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String Problems

The manipulations of string is a common task in real life applications such as:

- Analysis of Bioinformatics Gene Data;
- Pre-processing/wrangling, of API data (ex: JSON)
- Text processing from human interfaces (natural language)

Characteristics of String Problems

- "Parsing" of inputs with special rules;
- Using Dynamic Programming for finding patterns;
- Special data structures for storing patterns;

Topics for this week

We will study the following topics this week

- String basics and ad-hoc problems
 - Basic string libraries in C++ and Java;
 - Examples of Ad-hoc problems;
- String Matchign Algorithm;
 - Naive search;
 - KMP;
 - Z-Algorithm;
- Strings algorithms with DP;
 - Edit Distance
 - Common substring
 - Palindromes
- Suffix Tree and Suffix Array;

String Basic Operations

String Representation

```
// C/C++ (ends with '\\0')  
char[100] str;
```

```
#include<string>  
str s;
```

```
// JAVA  
String str;
```

```
// JAVA strings are immutable!  
// Modifying them = new object
```

Data Input

```
// Word  
scanf("%s",&str); cin >> str;
```

```
// Line  
gets(str);  
fgets(str,1000,stdin);  
getline(cin,str);
```

```
// Word  
Scanner sc = new  
    Scanner(System.in);  
str = sc.next();
```

```
// Line  
str = sc.nextLine();
```

String Basic Operations

Testing Two Strings for Equality

```
// C/C++  
result = strcmp(str, "test");  
result = (str == "test");
```

```
// JAVA  
result = str.equals("test");
```

Combining Two or More Strings

```
strcpy(str, "hello");  
strcat(str, " world");  
str = "hello";  
str.append(" world");
```

```
str = "hello";  
str += " world";  
// Careful!  
// Creates new strings
```

Editing/Testing single characters in a string

```
#include <ctype.h>  
for (int i=0;str[i];i++)  
    str[i] = toupper(str[i])
```

```
// Java Strs are immutable  
// create a new string  
// or use StringBuffer
```

String Basic Operations

String Tokenizer – Separates a string based on a character

```
// C/C++
#include <string.h>
for (char *p=strtok(str, " ");
     p; p=strtok(NULL, " "))
    printf("%s",p)

#include <sstream>
stringstream p(str);
while (!p.eof()) {
    string token;
    p >> token;
}
```

```
// JAVA
import java.util.*;
StringTokenizer st = new
    StringTokenizer(str, " ");
while (st.hasMoreTokens())
    System.out.println(
        st.nextToken());
```

String Basic Operations

Finding a Substring

```
// C/C++  
char *p=strstr(str,substr);  
if (p) printf("%d",p-str-1);  
  
int pos=str.find(substr);  
if (pos!=string::npos)  
    cout << pos-1 << endl;
```

```
// JAVA  
int pos =  
    str.indexOf(substr);  
if (pos != -1)  
    System.out.println(pos);
```

Sorting Characters in a string

```
#include <algorithm>  
sort(s, s+(int)strlen(s));  
sort(s.begin(),s.end());
```

```
//Immutable, break the  
//string using  
//toCharArray()
```

Ad-hoc String Problems

Let's see some general string problems that can be solved using the string library functions that we just reviewed.

If you have difficulty in these problems, try using the **Complete Search** approach on them first!

Immediate Decodability

Problem Outline

Given a set of **2 to 8 binary words**, of length between **1 and 10**, decide if the set is **immediately decodable**.

Immediate decodable means **no word is a prefix of another word**.

Input example 1 (Decodable)

- 001
- 110
- 10101
- 01101
- 100

Input example 2 (not decodable)

- **001**
- 10101
- **00101**
- 11011
- 1011

QUIZ: How do you solve this problem?

Immediate Decodability

Hints

Input example 1 (Decodable)

- 001
- 110
- 10101
- 01101
- 100

Input example 2 (not decodable)

- **001**
- 10101
- **00101**
- 11011
- 1011

- A simple way to solve is to test every pair of strings, to see if one is a prefix of another;
 - What is the difference between prefix and substring?
 - How many steps this algorithm takes?
- You can improve this algorithm if you reduce the number of comparisons;
 - How can you prune the algorithm?
 - Does the order of the strings matter?

Ad-hoc Problem 2 – Caesar Cypher

Problem Outline

A **rotational cypher** transforms *plaintext* to *cyphertext* by adding a constant value "k" to every character.

Example: I LOVE YOU + ($k = 3$) → LCORYHCARY

Given a dictionary of plaintext, find the best translation of the cyphertext.

```
THIS   DAWN   THAT   || INPUT:  BUUBDLA PSSPABUAEBXO
ZORRO  OTHER  AT      || OUTPUT:  ATTACK ZORRO AT DAWN
THING  THE
```

QUIZ: How do we solve this problem?

Ad-hoc Problem 2 – Caesar Cypher

```
THIS   DAWN   THAT   || INPUT:  BUUBDLA PSSPABUAEBXO
ZORRO  OTHER  AT      || OUTPUT: ATTACK ZORRO AT DAWN
THING  THE
```

- Our objective is to find the rotation that fits the largest number of words in the dictionary.
- Try every rotation, for each rotation see if the words are substrings.
- This is a very slow approach. Can it be faster?

Discussion of Ad-hoc problems

Problem 3 – Ensuring Truth

Given a boolean formula in the following format, is the formula **satisfiable**?

$$(x_1 \wedge \hat{x}_2 \wedge \dots \wedge x_n) \vee (x_i \wedge \hat{x}_j \wedge \dots) \vee \dots$$

Examples:

<code>(a&b&c) (a&b) (a)</code>	<code><--- Satisfiable;</code>
<code>(x&~x)</code>	<code><--- Not satisfiable;</code>

- A big part of the program is to build a function to read a string with size over 5000 in the right format.
- SAT is a very hard problem, but for this particular string format, is there a simple way to calculate satisfiability?

String Matching

Many String problems include some form of **string matching**

Find a substring P inside of string T .

- $P = \text{OBEY}$
 - $T = \text{ASPBOBEBLEOLBOBEY EYBEOLBEAY}$
-
- The easiest solution: Use `strstr` from the standard library!
 - But... what if the search has special conditions?
 - **example:** O and 0 are the same character
 - Let's study how to make a string matching algorithm!

First: What is the complexity of a Complete search?

String Matching: Naive Algorithm

Complete search approach: For every character $n[i]$, test if substring m begins there.

```
int naiveMatching() {  
    for (int i = 0; i < n; i++)  
        bool found = true;  
        for (int j = 0; j < m && found; j++)  
            if (i+j >= n || M[j] != N[i+j])  
                found = false;  
        if (found)  
            printf("Found at index %d\n", i)
```

- Average case: $O(n)$
- Worst case: $O(mn)$

$M = \text{AAAAB}$, $N = \text{AAAAAAAAAAAAAAAAAAAAAB}$

- Why is this case bad?

The Knuth-Morris-Pratt (KMP) Algorithm

Basic Idea

The KMP algorithm will never re-match a character in M that was matched in N .

If KMP finds a mismatch, it will skip n to $m + 1$, and rewind m to the appropriate value to continue the match.

$N =$ COLIN COMBWE LL CALLED A CO-CO-CO-COMBO BREAKER

$M =$ CO-COMBO

COXXXX

COXXXXXXXX

CXXXXXXXX

CO-COX

CO-COX

CO-COMBO

The Knuth-Morris-Pratt (KMP) Algorithm

How it works

The KMP needs to construct a “M rewinded table” b . When a mismatch happens, the substring M is rewinded, but the N index always moves forward.

M =	0	1	2	3	4	5	6	7		<-- Substring index
	C	O	-	C	O	M	B	O		
b =	-1	0	0	0	1	2	0	0		<-- M "rewind" index

If a miss happens at $m = 5$ (M), the algorithm will return the M counter (j) to $j = b[5] = 2$.

The Knuth-Morris-Pratt (KMP) Algorithm – Code

```
char N[MAX_N], M[MAX_N];
int b[MAX_N], n, m;

void kmpPreprocess() {
    int i = 0, j = -1; b[0] = -1;
    while (i < m) {
        while (j >= 0 && M[i] != M[j]) j = b[j];
        i++; j++;
        b[i] = j; }}

void kmpSearch() {
    int i = 0, j = 0;
    while (i < n) {
        while (j >= 0 && N[i] != M[j]) j = b[j];
        i++; j++;
        if (j == m) {
            printf("M is found at index %d in N\n", i-j);
            j = b[j]; }}}}
```

String Processing with Dynamic Programming

Some string problems can be explained as a **search** problem. In this case, we can solve them using Dynamic Programming.

- String Alignment/Edit Distance
- Longest Common Subsequence

String DP: Edit Distance

The **Edit Distance**, **String Alignment** or **Levenshtein Distance**, consists of measuring how many spaces are needed to **minimize the difference between two strings**.

```
S1: ACAATCC -> A_CAATCC -> A_CAATCC
S2: AGCATGC -> AGCATGC_ -> AGCA_TGC
Diff:         -> +.++--+. -> +.+++.+-+
```

To maximise score, we want to avoid **letter mismatches**.

Uses

- Finding similar words.
- Identifying Misspellings.

String DP: Edit Distance

Score Maximization Problem

Align two strings, A and B , with the maximum alignment score. For each pair of characters, we have three choices:

- $A[i]$ and $B[i]$ are the same character ('+' : +2 score)
- $A[i]$ and $B[i]$ are diff character ('-' : -1 score)
- Add a space to $A[i]$ or $B[i]$ ('.' : -1 score)

S1: ACAATCC	-> A_CAATCC	-> A_CAATCC
S2: AGCATGC	-> AGCATGC_	-> AGCA_TGC
Diff:	-> 2-22--2- = +4	-> 2-22-2-2 = +7

Trying all combinations: ($O(3^n)$). Let's try DP.

Edit Distance: Bottom Up DP Approach – Setup

State table

The state table V has dimensions $size(A)$ by $size(B)$

$V[i][j]$ is the maximum score for matching *substrings* $A[1..i]$, $B[1..j]$.

Initial Conditions

- $V(0, 0) = 0$ – Empty Strings
- $V(i, 0) = i * -1$ – Fill "B" with "_"
- $V(0, j) = j * -1$ – Fill "A" with "_"

Edit Distance: Bottom Up DP Approach – Update

State table

The state table V has dimensions $size(A)$ by $size(B)$

$V[i][j]$ is the maximum score for matching *substrings* $A[1..i]$, $B[1..j]$.

Transition Rule:

$Score(C_1, C_2)$ is the score of matching characters C_1 and C_2 .

Update: $V(i, j) = \max(option1, option2, option3)$

- option1 = $V(i-1, j-1) + Score(A[i], B[j])$ // Match or mismatch
- option2 = $V(i-1, j) + Score(A[i], _)$ // Delete $A[i]$
- option3 = $V(i, j-1) + Score(_, B[j])$ // Insert $B[j]$

Edit Distance: Bottom Up DP Approach – Example

Match ACAATCC and AGCATGC with a table.

Problem 2 – Longest Common Subsequence

Problem Definition

Given two strings A and B , what is the longest common subsequence between them?

Example:

String A: 'ACAATCC' – A_CAAT_CC

String B: 'AGCATGC' – AGCA_TGC_

Longest Common Subsequence: A_CA_T_C_

LCS: ACATC

- The LCS problem is similar to the String Alignment problem;
- The same DP algorithm presented before can be used;
- Set cost of Mismatch to $-\infty$, the cost of insert/deletion to 0, and the cost of matching to 1;

Longest Palindrome

Problem Description

Given a string S of size up to $N = 1000$ characters, what is the longest palindrome that you can make by deleting characters from S ?

Examples

- ADAM – ADA
- MADAM – MADAM
- NEVERODDOREVENING – NEVERODDOREVEN
- RACEF1CARFAST – RACECAR

Longest Palindrome

Problem Description

Given a string S of size up to $N = 1000$ characters, what is the longest palindrome that you can make by deleting characters from S ?

DP Solution:

- State Table:
 - $\text{len}(i,j)$ - The largest palindrome found between i and j
- Start Conditions:
 - If $l = r$ then $\text{len}(l, r) = 1$.
 - If $r = l + 1$ and $S[l] = S[r]$, $\text{len}(l, r) = 2$, else $\text{len}(l, r) = 1$.
- Transition:
 - If $S[l] = S[r]$, then $\text{len}(l, r) = 2 + \text{len}(l + 1, r - 1)$;
 - else $\text{len}(l, r) = \max(\text{len}(l + 1, r), \text{len}(l, r - 1))$

This DP has complexity $O(n^2)$

Longest Palindrome

Longest Palindrome DP: Diagonal Table Top Down

len(l,r)
final state

R A C E F 1 C A R

R	1	1	1	1	1	1	3	5	7
A		1	1	1	1	1	3	5	5
C			1	1	1	1	3	3	3
E				1	1	1	1	1	1
F					1	1	1	1	1
1						1	1	1	1
C							1	1	1
A								1	1
R									1

len(l,r)
initial state

R A C E F 1 C A R

R	1	1							
A		1	1						
C			1	1					
E				1	1				
F					1	1			
1						1	1		
C							1	1	
A								1	1
R									1

transition:
- $A[l] == A[r]$?
 $\text{len}(l+1, r-1) + 2$
- $A[l] != A[r]$?
 $\max(\text{left}, \text{down})$

Suffix Trie: Definition

Definition

Data structure used to find matching suffixes of multiple strings.

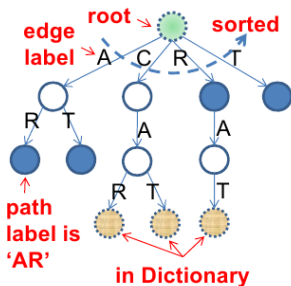
Suffix Trie for {'CAR','CAT','RAT'}

All Suffixes

- 1 CAR
- 2 AR
- 3 R
- 4 CAT
- 5 T
- 6 RAT
- 7 AT
- 8 T

Sorted, Unique
Suffixes

- 1 AR
- 2 AT
- 3 CAR
- 4 CAT
- 5 R
- 6 RAT
- 7 T



Suffix Trie: Using it for a single, long string

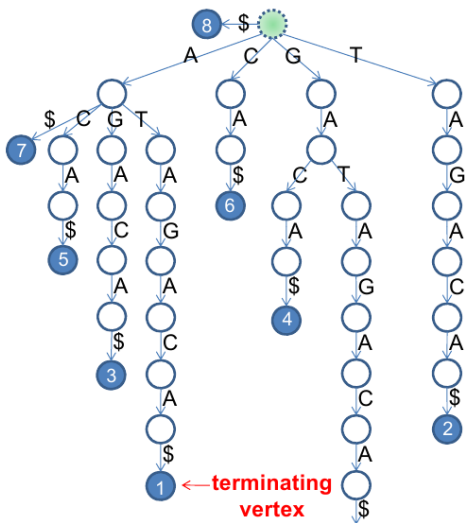
Suffix Trie ($T = \text{'GATAGACA\$'}$)

Create all n suffixes:

i	suffix
0	GATAGACA\$
1	ATAGACA\$
2	TAGACA\$
3	AGACA\$
4	GACA\$
5	ACA\$
6	CA\$
7	A\$
8	\$

Count the occurrence of substring m :

- 'A': 4 times
- 'GA': 2 times
- 'AA': 0 times



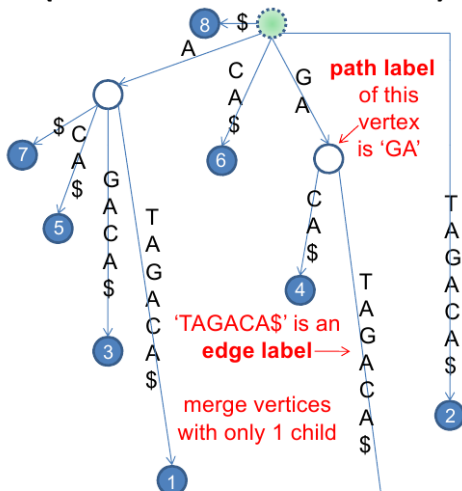
Suffix Trie: Suffix Tree

Suffix Trie (T='GATAGACA\$')

Compress single child nodes to obtain “Suffix Tree”

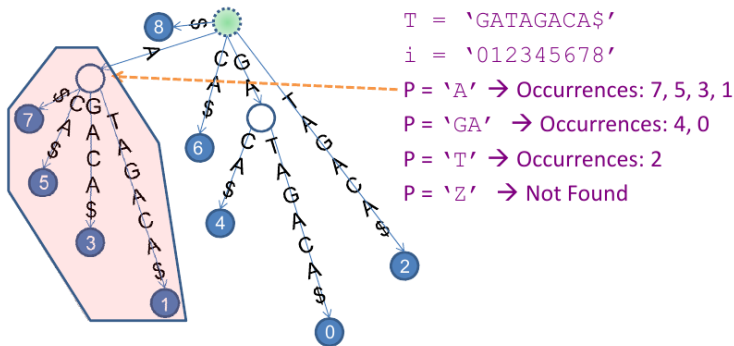
i	suffix
0	GATAGACA\$
1	ATAGACA\$
2	TAGACA\$
3	AGACA\$
4	GACA\$
5	ACA\$
6	CA\$
7	A\$
8	\$

With the suffix tree, many algorithms become faster.



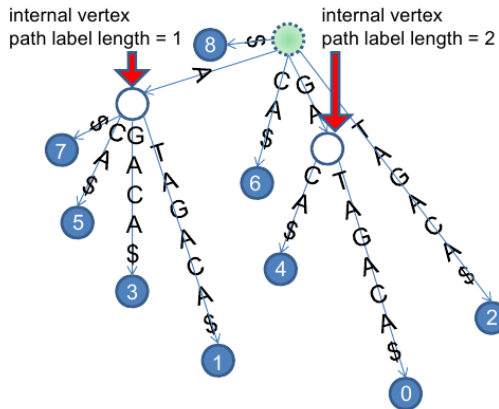
Uses of a Suffix Tree 1: String Matching

Assuming that we have the Suffix Tree already built, we can find all occurrences of substring m in T in time $O(m + \text{occ})$, where occ is the number of occurrences.



Uses of a Suffix Tree 2: Longest Repeated Substring

- The LRS is the longest substring with number of occurrences > 2 ;
- The LRS is the deepest internal node in the tree;



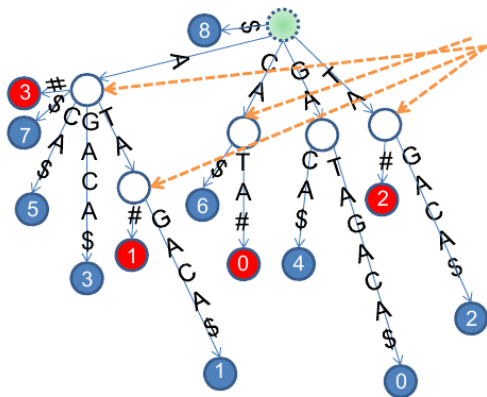
e.g. $T = \text{'GATAGACA\$'}$

The longest repeated substring is 'GA' with path label length = 2

The other repeated substring is 'A', but its path label length = 1

Uses of a Suffix Tree 3: Longest Common Substring

- We can find the common substring of M and N by making a combined Suffix Tree. Each string has a different ending character.
- The common substring is the deepest node that has both characters.



These are the internal vertices representing suffixes from both strings

The deepest one has path label 'ATA'

Suffix Trie: Suffix Array (1)

- The algorithms in previous slides are very efficient...
... if you have the suffix tree
- The suffix tree can be built in $O(n)$...
... but implementation is rather complex;
- In this course, we will see the Suffix Array;
- The Suffix Array is built in $O(n \log n)$...
... but the implementation is very simple!

I encourage you to study the implementation of the suffix tree by yourself!

Suffix Trie: Suffix Array (2)

- To make a Suffix array, make an array of all possible suffixes of T , and sort it;
- The order of the suffix array is the **visit in preorder** of the suffix tree;
- We can adapt all algorithms accordingly;

i	suffix		i	SA[i]	suffix
0	GATAGACA\$	Sort →	0	8	\$
1	ATAGACA\$		1	7	A\$
2	TAGACA\$		2	5	ACA\$
3	AGACA\$		3	3	AGACA\$
4	GACA\$		4	1	ATAGACA\$
5	ACA\$		5	6	CA\$
6	CA\$		6	4	GACA\$
7	A\$		7	0	GATAGACA\$
8	\$		8	2	TAGACA\$

Suffix Array: Implementation (1)

Simple Implementation

```
#include <algorithm>
#include <cstdio>
#include <cstring>
using namespace std;
char T[MAX_N]; int SA[MAX_N], i, n;

bool cmp(int a, int b) { return strcmp(T+a, T+b) < 0; }
// O(n)

int main() {
    n = (int) strlen (gets(T));
    for (int i = 0; i < n; i++) SA[i] = i;
    sort (SA, SA+n, cmp); // O(n^2 log n) }
```

This implementation is too slow for strings bigger than 1000 characters.

Suffix Array: Implementation (2.1)

O(n log n) implementation using “ranking pairs/radix sort”

```
char T[MAX_N]; int n; int c[MAX_N];
int RA[MAX_N], tempRA[MAX_N], SA[MAX_N], tempSA[MAX_N];

void countingSort(int k) {
    int i, sum, maxi = max(300,n); //255 ASCII chars or n
    memset(c, 0, sizeof(c));
    for (i = 0; i < n; i++) c[i+k<n? RA[i+k] : 0]++;
    for (i = sum = 0; i < maxi; i++)
        { int t = c[i]; c[i] = sum; sum += t; } //frequency
    for (i = 0; i < n; i++)
        tempSA[c[SA[i]+k < n ? RA[SA[i]+k] : 0]++] = SA[i];
    for (i = 0; i < n; i++) // update suffix array
        SA[i] = tempSA[i];
}

// ... continues next slide
```

Suffix Array: Implementation (2.2)

$O(n \log n)$ implementation using “ranking pairs/radix sort”

// ... continued from last slide

```
void constructSA() {
    int i, k, r;
    for (i = 0; i < n; i++) { RA[i] = T[i]; SA[i] = i; }
    for (k = 1; k < n; k <= 1) {
        countingSort(k); countingSort(0);
        tempRA[SA[0]] = r = 0;
        for (i = 1; i < n; i++) tempRA[SA[i]] =
            (RA[SA[i]] == RA[SA[i-1]] &&
             RA[SA[i]+k] == RA[SA[i-1]+k]) ? r : ++r;
        for (i = 0; i < n; i++)
            RA[i] = tempRA[i];
        if (RA[SA[n-1]] == n-1) break;
    }
}
```

Suffix Array: Using Suffix Array (1)

String Matching: Finding 'GA'

- Do a binary search once to find the lower bound;
- Do a binary search once to find the upper bound;

Finding lower bound

i	SA[i]	Suffix
0	8	\$
1	7	A\$
2	5	ACA\$
3	3	AGACA\$
4	1	ATAGACA\$
5	6	CA\$
6	4	GACA\$
7	0	GATAGACA\$
8	2	TAGACA\$

Finding upper bound

i	SA[i]	Suffix
0	8	\$
1	7	A\$
2	5	ACA\$
3	3	AGACA\$
4	1	ATAGACA\$
5	6	CA\$
6	4	GACA\$
7	0	GATAGACA\$
8	2	TAGACA\$

Suffix Array: Using Suffix Array (2)

Longest Repeated Substring

Find the longest common prefix between suffix i and $i + 1$

i	SA[i]	LCP[i]	Suffix
0	8	0	\$
1	7	0	A\$
2	5	1	<u>A</u> CA\$
3	3	1	<u>A</u> GACA\$
4	1	1	<u>A</u> TAGACA\$
5	6	0	CA\$
6	4	0	GACA\$
7	0	2	<u>G</u>ATAGACA\$
8	2	0	TAGACA\$

Suffix Array: Using Suffix Array (3)

Longest Common Substring

- Create Suffix Array for appended strings *MN*;
- Find the longest common prefix that has both string enders;

i	SA[i]	LCP[i]	Owner	Suffix
0	13	0	2	#
1	8	0	1	\$CATA#
2	12	0	2	A#
3	7	1	1	<u>A</u> \$CATA#
4	5	1	1	<u>ACA</u> \$CATA#
5	3	1	1	<u>AGACA</u> \$CATA#
6	10	1	2	<u>ATA</u> #
7	1	3	1	<u>ATAGACA</u> \$CATA#
8	6	0	1	CA\$CATA#
9	9	2	2	<u>CATA</u> #
10	4	0	1	GACA\$CATA#
11	0	2	1	<u>GATAGACA</u> \$CATA#
12	11	0	2	TA#

Problems for this Week

- Immediate Decodability
- Caesar Cypher
- Power Strings
- Where's Waldorf
- Extend to Palindrome
- String Partition
- Prince and Princess
- Power Strings
- Life Forms

Immediate Decodability

Outline

A set of tokens is decodable if it is **impossible** to write a string that can be parsed in more than one way.

Decodability is detected by checking if a token is a prefix of another.

Input :

01
10
0001
00101

Output :

decodable
(no string is a prefix)

001
0100
00101
01101

not decodable
(001 is a prefix of 00101)

Caesar Cypher

Outline

A **k-rotation cypher** replaces every symbol N with symbol $N + k$, including spaces (which are symbol 0).

- **Input:** A list of correct words, and an **encrypted text**
- **Task:** Find the shift that matches the maximum number of words in the dictionary. Output the **decrypted text**

Notes about the problem:

- Input: Small, No case, no symbols, spaces
- Crypto text may contain words not in dictionary
- Output requirements (linebreak at 60 characters)

THIS	DAWN	THAT		BUUBDLA PSSPABUAEBXO
THE	ZORRO	OTHER		ATTACK ZORRO AT DAWN
AT	THING			

Power Strings

Problem Outline

You are given a string s , and you must find the smallest string s' , so that $s = s' + s' + s' + \dots = (s')^n$. This is equal to finding s' with maximum n .

Example Input and Output:

INPUT	MINIMUM STRING	N
abcd	abcd	1: abcd
abababab	ab	4: ab + ab + ab + ab
kallakalla	kalla	2: kalla + kalla
abababa	abababa	1: abababa

This is a mixture of search and string matching. If your search is not very good, you may face TLE, so write your search carefully.

Where is Waldorf?

Problem Outline

This the traditional magazine challenge: Find words inside a cube of letters. Pay attention:

- Words can be vertical, horizontal or diagonal;
- Words can be backwards;
- Search is **not case sensitive**

abcDEFghigg
hEbkWalDork
FtyAwaldORm
FtsimrLqsrc
byoArBeDeyv
Klcbqwikomk
strEBGadhrb
yUiqlxcnBjf

Words:

Waldorf -- 2 5

Bambi -- 2 3

Betty -- 1 2

Dagbert -- ? ?

Extend to Palindrome

Problem Outline

You receive a word as input, and you must add the smallest number of letters at the end to make it a palindrome. Examples:

alert:	alertrela
abcba:	abcba
aaaalll:	aaaalllaaaa

Hints:

- Which letters do you add to a word to make it a palindrome;
- How do you decide if you add a letter or not?
- Can you modify the KLM algorithm to help you make the decision?

String Partition

Problem Outline

You must break a large string of digits into smaller numbers (max size: 32 bit signed integer), so that the sum of the numbers is the largest.

Hints:

- The max number of digits in the string is $N=500$;
- Start with a search on the breaking points;
- The total sum can be bigger than signed int;

INPUT

1234554321

5432112345

000

111111111111111111111111111111111111

OUTPUT

1234554321

543211239

0

3333333333

Prince and Princess

Problem outline

The Prince and the princess make different paths through the same $n * n$ grid. Both paths start and end at the same square.

Your task is to make both paths identical by eliminating steps, and print the size of the common path.

Input

1 7 5 4 8 3 9

1 4 3 5 6 2 8 9

1 3 4 2 5 8 7 10

1 5 8 9 3 2 7 10

Output

4

(Common path: 1, 5, 8, 9)

5

(Common path: 1, 5, 8, 7, 10)

Life Forms

Problem Outline

Given a set of strings, find the **largest common substring** that is shared by more than half of the strings.

Hints:

- Generalization of LCS, but for multiple strings;
- If there are multiple substrings of the same size, output all of them;

INPUT

abcdefg

bcdefgh

cdefghi

OUTPUT

bcdefg

cdefgh

About these Slides

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