# Programming Challenges (GB21802)

Week 7 - String Manipulation

Claus Aranha
caranha@cs.tsukuba.ac.jp

University of Tsukuba, Department of Computer Sciences

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## String Problems

The manipulations of string is a common task in real life applications such as:

- Analysis of Bioinformatics Gene Data;
- Pre-processing/wrangling, of API data (ex: JSON)
- Text processing from human interfaces (natural language)

### Characteristics of String Problems

- "Parsing" of inputs with special rules;
- Using Dynamic Programming for finding patterns;
- Special data structures for storing patterns;

## Topics for this week

We will study the following topics this week

- String basics and ad-hoc problems;
  - Basic string library functions in C++ and Java;
  - Examples of Ad-hoc string problems;
- String Matching Algorithm;
  - Naive search;
  - KMP;
  - Z-Algorithm;
- Strings algorithms with DP;
  - Edit Distance
  - Common substring
  - Palindromes
- Suffix Tree and Suffix Array;

## Part I: Basics

## Basic String Operations (C++ and Java)

### String Representation

```
// C/C++ (string ends with '\0')
char[100] str;

#include<string> // C++
str s;
```

```
// JAVA
String str;

// JAVA strings are imutable!
// Modifying them creates a new object
```

### Data Input

```
// Reading one Word
scanf("%s",&str); cin >> str;

// Reading one Line
gets(str);
fgets(str,1000,strdin);
getline(cin,str);
```

```
// Reading one Word
Scanner sc = new
       Scanner(System.in);
str = sc.next();

// Reading one Line
str = sc.nextLine();
```

# Basic String Operations (C++ and Java)

## Testing if two strings are equal

```
// C/C++                                  // JAVA
result = strcmp(str,"test");              result = str.equals("test");
result = (str == "test");
```

## Combining Two or More Strings

```
strcpy(str,"hello");  // Option 1         str = "hello";
strcat(str," world");                     str += " world";
str = "hello";        // Option 2         // Careful! This creates new strings!
str.append(" world");
```

## Editing a single character in a string

```
#include <ctype.h>                        // Java Strs are immutable
str[i] = toupper(str[i])                  // create a new string
                                          // or use StringBuffer
```

## String Basic Operations

### String Tokenizer – Separates a string based on a character

```
// C/C++
#include <string.h>
for (char *p.strtok(str," ");
     p;
     p=strtok(NULL," "))
   printf("%s",p)

#include <sstream>
stringstream p(str);
while (!p.eof()) {
  string token;
  p >> token;
}
```

```
// JAVA
import java.util.*;
StringTokenizer st = new
  StringTokenizer(str," ");
while (st.hasMoreTokens())
  System.out.println(st.nextToken());
```

## String Basic Operations

### Finding the index of a Substring

```
// C/C++
char *p = strstr(str,substr);
if (p) printf("%d",p - str - 1);

int pos = str.find(substr);
if (pos != string::npos)
  cout << pos-1 << endl;
```

```
// JAVA
int pos =
  str.indexOf(substr);
if (pos != -1)
  System.out.println(pos);
```

### Sorting Characters in a string

```
#include <algorithm>
sort(s, s+(int)strlen(s));
sort(s.begin(),s.end());
```

```
// Immutable, break the string
// using toCharArray(), then
// sort the Array with Collections
```

## Ad-hoc String Problems

In the next slides, we will see some "generic" string problems.

You can solve these problems by some thinking, and using the string manipulation functions we mentioned before.

If you have difficulty finding a solution for these problems, first try to create a **complete search** solution, and then prune if TLE.

## Immediate Decodability

### Problem Outline

Given a set of **2 to 8 binary words**, of length between **1 and 10**, decide if the set is **immediately decodable.**

Immediate decodable means **no word is a prefix of another word.**

**Input example 1 (Decodable)**

- 001
- 110
- 10101
- 01101
- 100

**Input example 2 (not decodable)**

- **001** <- prefix of 3
- 10101
- **00101**
- 11011
- 1011

**QUIZ:** How do you solve this problem?

## Immediate Decodability

### Hints

**Input example 1 (Decodable)**

- 001
- 110
- 10101
- 01101
- 100

**Input example 2 (not decodable)**

- **001**
- 10101
- **00101**
- 11011
- 1011

- Complete Search: For every pair $s_1$, $s_2$, test if one is prefix of another.
  - What is the difference between prefix and substring?
  - How many steps this algorithm takes?
- Improve the complete search by pruning comparisons.
  - Does the order of the strings matter?

## Ad-hoc Problem 2 – Caesar Cypher

### Problem Outline

A **rotational cypher** transforms *plaintext* to *cyphertext* by adding a constant value "k" to every character (including spaces).

Example: I LOVE YOU + $(k = 3) \rightarrow$ LCORYHCARY

Given a dictionary of plaintext, find the best translation of the cyphertext.

```
THIS  DAWN  THAT    || INPUT:  BUUBDLA PSSPABUAEBXO
ZORRO OTHER AT      || OUTPUT: ATTACK ZORRO AT DAWN
THING THE
```

**QUIZ**: How do we solve this problem?

## Ad-hoc Problem 2 – Caesar Cypher

```
THIS  DAWN  THAT    || INPUT:  BUUBDLA PSSPABUAEBXO
ZORRO OTHER AT      || OUTPUT: ATTACK ZORRO AT DAWN
THING THE
```

- Our objective is to find the rotation that fits the largest number of words in the dictionary.
- **Complete Searc**: Try every rotation, for each rotation see if the words are substrings.
- This is a very slow approach. Can it be faster?

## Part II: String Matching

## The String Matching Problem

**Definition**

Given a string $T$ (also called **text**), we want to test if the substring $P$ (also called **pattern**) exists in $T$.

If $P$ exists in $T$, we want to know the **index** of the start of $P$ in $T$.

Example:

```
T: STEVEN EVENT

P: EVE              indexes: 2 and 7
P: EVENT            indexes: 7
P: EVENING          indexes: -1 or NULL
```

## String Matching and Libraries

### How do we solve string matching problems?

**Use your language's string library!**

- In C/C++: strstr(T,P) or T.find(P)
- In Java: T.indexOf(P)

No bugs! Usually very efficient!

But there are some special cases where you want to program by hand.

- Maybe you have a specific matching function (1 equals l)
- Maybe your string changes over time;
- Maybe you have to match multiple strings at the same time;
- Maybe you have to string match in a graph;
- etc...

## String Matching: Complete Search

For every character $T_i$, test if $P$ begins at that position.

```
for (int i = 0; i < |T|; i++)
  bool match = true;
  for (int j = 0; j < |P| && match; j++)
    if (i+j >= |T| || P[j] != T[i+j])
      match = false;
  if (match)
    printf("Match P at index %d\n", i);
```

**Number of Steps**:

- Average case: $O(|T|)$ – For natural T and small P;
- Worst case: $O(|T| \times |P|)$;
  - T = AAAAAAAAAAAAB
  - P = AAAAAAAB

## The Knuth-Morris-Pratt (KMP) Algorithm

- Complete Search can be very expensive if the prefix of *P* happens many times in *T*.
- In 1977, Knuth, Morris and Pratt developed an algorithm that **uses these prefixes** to realize fast string matching.

### Basic Idea

- The KMP algorithm identifies "borders" in the partial match between *P* and *T*.
- These borders are characterized by identical prefixes and sufixes in the T-P match.
- The algorithm uses these matches to advance the indexes of *T* and *P*, greatly reducing the number of comparisons.

The KMP algorithm is O(P+T).

## KMP Algorithm – Simulation

```
              1         2         3         4         5
    012345678901234567890123456789012345678901234567890
T = I DO NOT LIKE SEVENTY SEV BUT SEVENTY SEVENTY SEVEN
P = SEVENTY SEVEN
// for i from 0 to 13, KMP works like full search

                  SEVENTY SEVEN
// Here, the collision is at i=25, j = 11, But because "SEV" is
// a "border", i stays the same and j is rewinded to 3

                                  SEVENTY SEVEN
// Here we find a match with i=43, j=13; SEVEN is a border, so j
// is rewinded to 5, and i is kept the same. The algorithm
// continues matching at i=44, j=5 ("T")

                                          SEVENTY SEVEN
// KMP finds a second match
```

## KMP Algorithm – Rewind Array

To avoid repeated matches, the KMP algorithm builds a **rewind table** $b$ (back).

```
      0  1  2  3  4  5  6  7  8  9  0  1  2  3
P =   S  E  V  E  N  T  Y     S  E  V  E  N  \0
b =  -1  0  0  0  0  0  0  0  0  1  2  3  4  5
```

Following the table $b$, we know that if we find a mismatch at $j = 11$, then we need to rewing $j$ to $b[11] = 3$ to continue matching.

The text index $i$, on the other hand, will stay the same, and go forward by 1 if $b[j] = -1$.

## KMP Algorithm – PseudoCode

```
char T[MAX_N], P[MAX_N];    int b[MAX_N], n, m;

void kmpPreprocess() {                          // Create the Back Array
  int i = 0, j = -1; b[0] = -1;
  while (i < m) {
     while (j >= 0 && P[i] != P[j]) j = b[j];
     i++; j++;
     b[i] = j; }}

void kmpSearch() {                              // Search the substring
  int i = 0, j = 0;
  while (i < n) {
     while (j >= 0 && T[i] != P[j]) j = b[j];
     i++; j++;
     if (j == m) {
        printf("P is found at index %d in T\n", i - j);
        j = b[j]; }}}
```

## String Matching with the Z-Algorithm

Another alogirthm that performs string matcfhing in linear time is the **Z algorithm**.

The Z algorithm first makes a **Search String** $S = P +' S' + T$.
The Z algorithm next constructs a **Z array** of "prefix lengths".
For every index $i \in S$, $Z[i]$ is the size of the prefix of $S$ that begins in $i$.

```
T = AASABAABAAT, P = AAB, S = P + '$' + T

... Build Z Array ...
S    = AAB$AASABAABAAT
Z[S]= X10021010310210
                ^
                String matched here. Z[i] = Len(P)
```

## Z-Algorithm – Pseudocode

```
void Zarray(string S, int Z[]) {
    int n = S.length(); int L, R, k;
    L = R = 0;                  // Prefix counters
    for (int i = 1; i < n; i++) {
        if (i > R) {            // Full search of prefix
            L = R = i;
            while (R < n && S[R] == S[R-L]) R++;
            Z[i] = R-L; R--;
        } else {                // Inside prefix candidate
            k = i-L;
            if (Z[k] < R-i+1) Z[i] = Z[k]; // no extension
            else {                         // prefix extension
                L = i;
                while (R < n && S[R] == S[R-L]) R++;
                Z[i] = R-L; R--;
}   }   }   }
```

**Simulation:** `https://personal.utdallas.edu/~besp/demo/John2010/z-algorithm.htm`

## Z algorithm or KMP algorithm?

Should you use the Z algorithm or the KMP algorithm?

- Both algorithms have the same time complexity: $O(T + P)$
- Which algorithm is easier to understand?
    - KMP calculates a recursive suffix state machine for P;
    - Z-algorithm calculates a substring size array for T;

## Part III: Strings and DP

## String Algorithms with Dynamic Programming

Some string problems can be described as a **search problem.** In this section, we will introduce two string tasks that can be solved with DP algorithms:

- String Alignment/Edit Distance
- Longest Common Subsequence

It is interesting to note that substring matching is also a search problem, and that KMP / Z-algorithms can be seen as a kind of memoization.

## String DP: String Alignment

The **String Alignment**[1] problem is defined as follows. Align two strings, A and B, with the maximum "alignment score":

- Character A[i] and B[i] match: do nothing, score +2
- Character A[i] and B[i] mismatch: replace A[i], score -1
- Insert a space in A[i]: score -1 (equals to delete B[i])
- Insert a space in B[i]: score -1 (equals to delete A[i])

```
   Original     non-optimal     optimal
A: ACAATCC    | A_CAATCC      | A_CAATCC
B: AGCATGC    | AGCATGC_      | AGCA_TGC
score:        | 2-22--2- = 4  | 2-22-2-2 = 7
```

[1] Also called Edit Distance or Levenhstein Distance, used by spellchecking algorithms!

## String Alignment: Bottom Up DP

The **Complete Search** approach requires recursively testing each of the three options for each A[i] (Total cost: $O(3^n)$).

We can solve this in $O(n^2)$ using DP:

- $V(i,j)$: optimal score for prefix $A[1..i], B[1..j]$
- Start condition:
  - $V(0,0) = 0$ (Do nothing)
  - $V(i,0) = -1 \times i$, $V(0,j) = -1 \times j$ (delete A or B)
- Recurrence: $V(i,j) = \max(C_1, C_2, C_3)$, where
  - $C_1 = V(i-1, j-1) + \text{score}(A[i], B[j])$ score of match or mismatch;
  - $C_2 = V(i-1, j) + \text{score}(A[i], \_)$ delete $A[i]$;
  - $C_3 = V(i, j-1) + \text{score}(\_, B[j])$ delete $B[j]$;

## String Alignment: Bottom Up DP

### Simulation Matching AGCATGC and ACAATCC

- Recurrence: $V(i,j) = \max(C_1, C_2, C_3)$, where
  - $C_1 = V(i-1, j-1) + \text{score}(A[i], B[j])$ score of match or mismatch;
  - $C_2 = V(i-1, j) + \text{score}(A[i], \_)$ delete $A[i]$;
  - $C_3 = V(i, j-1) + \text{score}(\_, B[j])$ delete $B[j]$;

| | _ | A | G | C | A | T | G | C |
|---|---|---|---|---|---|---|---|---|
| _ | 0 | −1 | −2 | −3 | −4 | −5 | −6 | −7 |
| A | −1 | | | | | | | |
| C | −2 | | | | | | | |
| A | −3 | | | | | | | |
| A | −4 | | | | | | | |
| T | −5 | | | | | | | |
| C | −6 | | | | | | | |
| C | −7 | | | | | | | |

## Problem 2: Longest Common Subsequence in Strings

### Problem Definition

Given strings *A* and *B*, what is their longest common subsequence?

```
A   :    'ACAATCC'      - A_CAAT_CC
B   :    'AGCATGC'      - AGCA_TGC_
LCS:      AC AT C       - A_CA_T_C_ : ACATC
```

- We can solve LCS using a modification of String Aligment;
- Use String Alignment DP, with different scores:
  - Cost of Mismatch: $-\infty$
  - Cost of insert/deletion: 0
  - Cost of Matching: 1

## Problem 3: Longest Palindrome

### Problem Description

A **palindrome** is a string $S$ where $S = \text{rev}(S)$. For example: MADAM.

Given a string $T$, what is the **longest palindrome** that you can create by deleting characters from $T$?

Examples:

- ADA**M** – ADA
- MADAM – MADAM
- NEVERODDOREVEN**ING** – NEVERODDOREVEN
- RACE**F1**CAR**FAST** – RACECAR

**QUIZ:** Can you solve with Full Search? String Alignment DP? Others?

## Longest Palindrome

**Problem Description**

Given a string $S$ of size up to $N = 1000$ characters, what is the longest palindrome that you can make by deleting characters from $S$?

DP Solution:

- State Table:
  - len(i,j) - The largest palindrome found between $i$ and $j$
- Start Conditions:
  - If $l = r$ then $\text{len}(l, r) = 1$.
  - If $r = l + 1$ and $S[l] = S[r]$, $\text{len}(l, r) = 2$, else $\text{len}(l, r) = 1$.
- Transition:
  - If $S[l] = S[r]$, then $\text{len}(l, r) = 2 + \text{len}(l + 1, r - 1)$;
  - else $\text{len}(l, r) = \max(\text{len}(l + 1, r), \text{len}(l, r - 1))$

This DP has complexity $O(n^2)$

## Longest Palindrome

### Longest Palindrome DP: Diagonal Table Top Down

len(l,r)
final state

| | R | A | C | E | F | 1 | C | A | R |
|---|---|---|---|---|---|---|---|---|---|
| R | 1 | 1 | 1 | 1 | 1 | 1 | 3 | 5 | 7 |
| A | | 1 | 1 | 1 | 1 | 1 | 3 | 5 | 5 |
| C | | | 1 | 1 | 1 | 1 | 3 | 3 | 3 |
| E | | | | 1 | 1 | 1 | 1 | 1 | 1 |
| F | | | | | 1 | 1 | 1 | 1 | 1 |
| 1 | | | | | | 1 | 1 | 1 | 1 |
| C | | | | | | | 1 | 1 | 1 |
| A | | | | | | | | 1 | 1 |
| R | | | | | | | | | 1 |

len(l,r)
initial state

| | R | A | C | E | F | 1 | C | A | R |
|---|---|---|---|---|---|---|---|---|---|
| R | 1 | 1 | | | | | | | |
| A | | 1 | 1 | | | | | | |
| C | | | 1 | 1 | | | | | |
| E | | | | 1 | 1 | | | | |
| F | | | | | 1 | 1 | | | |
| 1 | | | | | | 1 | 1 | | |
| C | | | | | | | 1 | 1 | |
| A | | | | | | | | 1 | 1 |
| R | | | | | | | | | 1 |

```
  transition:
- If A[l] == A[r]: len(diag)+2
- If A[l] != A[r]: max(left,down)
```

## Part IV: Suffix Tree, Array

## Suffix Trie: Definition

Data structure used to find matching suffixes of multiple strings.

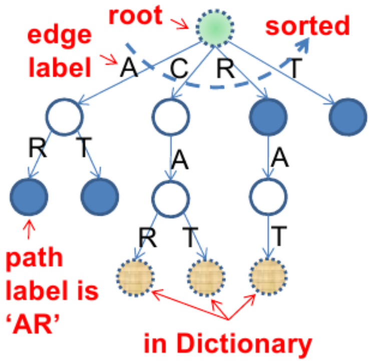
Suffix Trie for {'CAR','CAT','RAT'}

All Suffixes

1. CAR
2. AR
3. R
4. CAT
5. T
6. RAT
7. AT
8. T

Sorted, Unique Suffixes

1. AR
2. AT
3. CAR
4. CAT
5. R
6. RAT
7. T (x2)



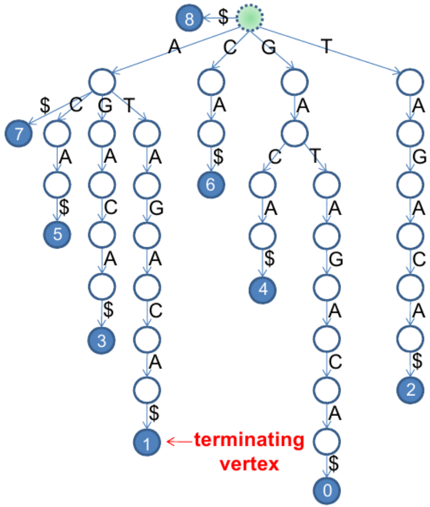
## Suffix Trie: Counting the number of substrings of GATAGACA

Create all *n* suffixes:

| i | suffix |
|---|---|
| 0 | GATAGACA$ |
| 1 | ATAGACA$ |
| 2 | TAGACA$ |
| 3 | AGACA$ |
| 4 | GACA$ |
| 5 | ACA$ |
| 6 | CA$ |
| 7 | A$ |
| 8 | $ |

Number of repeats of substring *m*:

- 'A': 4 subtrees
- 'GA': 2 subtrees
- 'AA': 0 subtrees

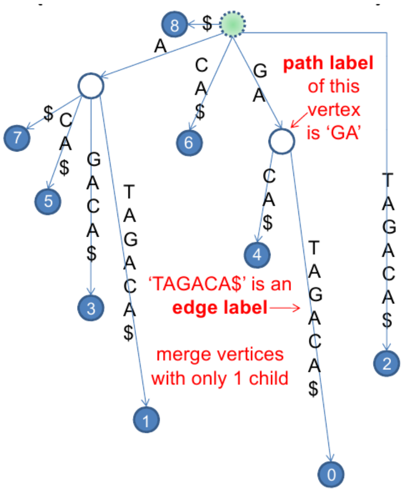


## Suffix Trie: Counting the number of substrings of GATACA

You can make the Suffix Tree better by merging the nodes that have a single child.

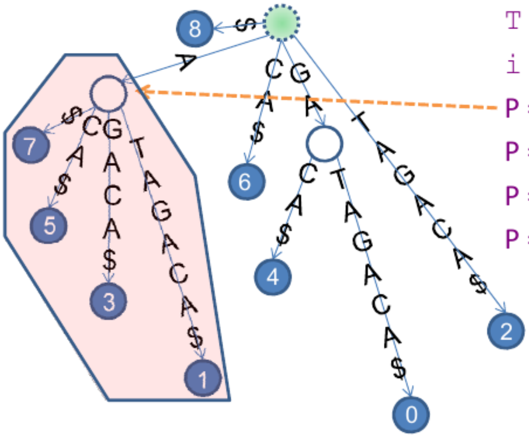
This data structure is useful for many algorithms.

| i | suffix |
|---|---|
| 0 | GATAGACA$ |
| 1 | ATAGACA$ |
| 2 | TAGACA$ |
| 3 | AGACA$ |
| 4 | GACA$ |
| 5 | ACA$ |
| 6 | CA$ |
| 7 | A$ |
| 8 | $ |


8
$
A
CA$
GA
TAGACA$
$
CA$
GACA$
TAGACA$
CA$
TAGACA$
7
5
3
1
6
4
0
2
path label of this vertex is 'GA'
'TAGACA$' is an edge label
merge vertices with only 1 child

## Uses of a Suffix Tree 1: String Matching

Assuming that we have the Suffix Tree already built, we can find all occurrences of substring $m$ in $T$ in time $O(m + \text{occ})$, where occ is the number of occurrences.



T = 'GATAGACA$'

i = '012345678'

P = 'A' → Occurrences: 7, 5, 3, 1

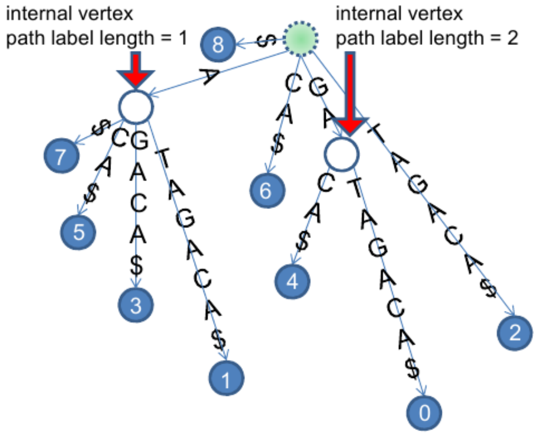
P = 'GA' → Occurrences: 4, 0

P = 'T' → Occurrences: 2

P = 'Z' → Not Found

## Uses of a Suffix Tree 2: Longest Repeated Substring

- The LRS is the longest substring with number of occurrences $> 2$;
- The LRS is the deepest **internal node** in the tree;

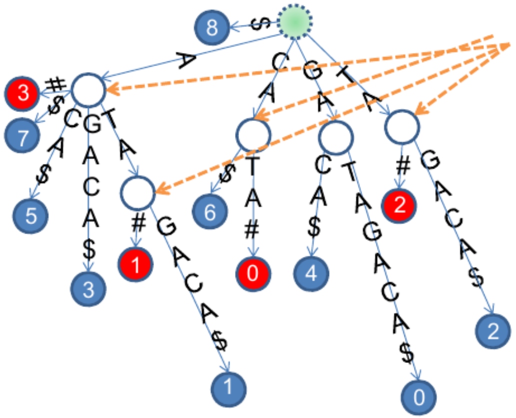


e.g. T = 'GATAGACA$'
The longest repeated substring is 'GA' with path label length = 2

The other repeated substring is 'A', but its path label length = 1

## Uses of a Suffix Tree 3: Longest Common Substring

- Make a Suffix Tree of *M* and *N* combined, with a different ending character to each.
- The LCS is the deepest **internal node** that includes both ending characters.



These are the internal vertices representing suffixes from both strings

The deepest one has path label 'ATA'

## Suffix Array

- The algorithms in previous slides are very efficient...
  ... if you have the suffix tree
- The suffix tree can be built in $O(n)$...
  ... but implementation is rather complex;
- In this course, we will see the Suffix Array;
- The Suffix Array is built in $O(n \log n)$...
  ... but the implementation is very simple!

I encourage you to study the implementation of the suffix tree by yourself!

## Suffix Array Implementation Idea

- To make a Suffix array, make an array of all possible suffixes of $T$, and sort it;
- The order of the suffix array is the visit in preorder of the suffix tree;
- We can adapt all algorithms accordingly;

| i | suffix |
|---|---|
| 0 | GATAGACA$ |
| 1 | ATAGACA$ |
| 2 | TAGACA$ |
| 3 | AGACA$ |
| 4 | GACA$ |
| 5 | ACA$ |
| 6 | CA$ |
| 7 | A$ |
| 8 | $ |

Sort $\rightarrow$

| i | SA[i] | suffix |
|---|---|---|
| 0 | 8 | $ |
| 1 | 7 | A$ |
| 2 | 5 | ACA$ |
| 3 | 3 | AGACA$ |
| 4 | 1 | ATAGACA$ |
| 5 | 6 | CA$ |
| 6 | 4 | GACA$ |
| 7 | 0 | GATAGACA$ |
| 8 | 2 | TAGACA$ |

## Suffix Array: Slow Implementation

### Simple Implementation

```
#include <algorithm>
#include <cstdio>
#include <cstring>
using namespace std;
char T[MAX_N]; int SA[MAX_N],i,n;

bool cmp(int a, int b) { return strcmp(T+a, T+b) < 0; }
// O(n)

int main() {
  n = (int) strlen (gets(T));
  for (int i = 0; i < n; i++) SA[i] = i;
  sort (SA, SA+n, cmp); // O(n^2 log n) }
```

This implementation is too slow for strings bigger than 1000 characters.

## Suffix Array: Better Implementation (1)

O(n log n) implementation using “ranking pairs/radix sort”

```
char T[MAX_N]; int n; int c[MAX_N];
int RA[MAX_N], tempRA[MAX_N], SA[MAX_N], tempSA[MAX_N];

void countingSort(int k) {
  int i, sum, maxi = max(300,n);                      //255 ASCII chars or n
  memset(c, 0, sizeof(c));
  for (i = 0; i < n; i++) c[i+k<n? RA[i+k] : 0]++
  for (i = sum = 0; i < maxi; i++)
    { int t = c[i]; c[i] = sum; sum += t; }           //frequency
  for (i = 0; i < n; i++)
    tempSA[c[SA[i]+k < n ? RA[SA[i]+k] : 0]++] = SA[i];
  for (i = 0; i < n; i++)                             // update suffix array
    SA[i] = tempSA[i];
}

// ... continues next slide
```

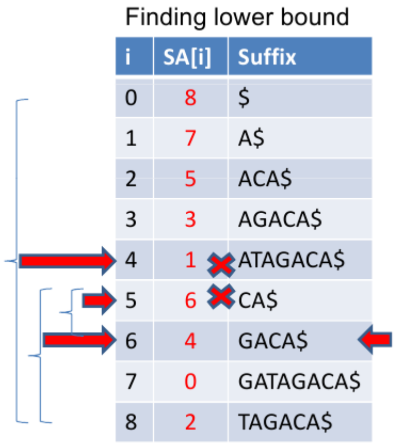
## Suffix Array: Better Implementation (2)

O(n log n) implementation using “ranking pairs/radix sort”

```
// ... continued from last slide
void constructSuffixArray() {
  int i, k, r;
  for (i = 0; i < n; i++) { RA[i] = T[i]; SA[i] = i;}
  for (k = 1; k < n; k <<=1) {
    countingSort(k); countingSort(0); tempRA[SA[0]] = r = 0;
    for (i = 1; i < n; i++)
      tempRA[SA[i]] =
          (RA[SA[i]] == RA[SA[i-1]] && RA[SA[i]+k] == RA[SA[i-1]+k]) ?
           r : ++r;
    for (i = 0; i < n; i++)
      RA[i] = tempRA[i];
    if (RA[SA[n-1]] == n-1) break;
  }
}
```

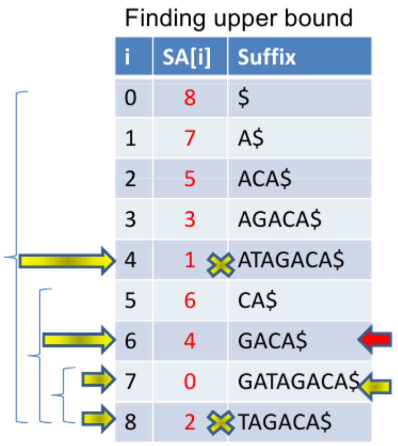
## Using Suffix Array for String Matching:

- Do binary search two times: One to find the lower bound, one to find the upper bound;



Finding lower bound

| i | SA[i] | Suffix |
|---|---|---|
| 0 | 8 | $ |
| 1 | 7 | A$ |
| 2 | 5 | ACA$ |
| 3 | 3 | AGACA$ |
| 4 | 1 | ATAGACA$ |
| 5 | 6 | CA$ |
| 6 | 4 | GACA$ |
| 7 | 0 | GATAGACA$ |
| 8 | 2 | TAGACA$ |

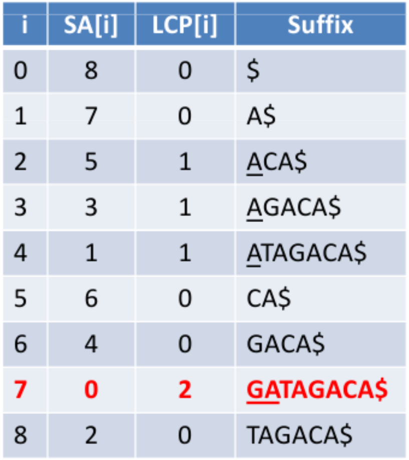


Finding upper bound

| i | SA[i] | Suffix |
|---|---|---|
| 0 | 8 | $ |
| 1 | 7 | A$ |
| 2 | 5 | ACA$ |
| 3 | 3 | AGACA$ |
| 4 | 1 | ATAGACA$ |
| 5 | 6 | CA$ |
| 6 | 4 | GACA$ |
| 7 | 0 | GATAGACA$ |
| 8 | 2 | TAGACA$ |

## Using Suffix Array for Longest Repeated Substring

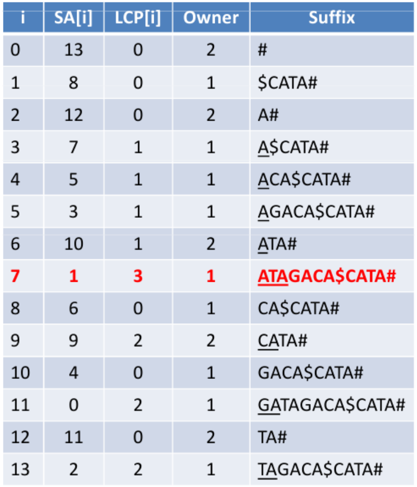
Find the pair of indexes $i$ and $i + i$ with longest common prefix.



| i | SA[i] | LCP[i] | Suffix |
|---|---|---|---|
| 0 | 8 | 0 | $ |
| 1 | 7 | 0 | A$ |
| 2 | 5 | 1 | ACA$ |
| 3 | 3 | 1 | AGACA$ |
| 4 | 1 | 1 | ATAGACA$ |
| 5 | 6 | 0 | CA$ |
| 6 | 4 | 0 | GACA$ |
| **7** | **0** | **2** | **GATAGACA$** |
| 8 | 2 | 0 | TAGACA$ |

## Using Suffix Array for Longest Common Substring

Append strings *M* and *N* with different endings, and find LCS



| i | SA[i] | LCP[i] | Owner | Suffix |
|---|---|---|---|---|
| 0 | 13 | 0 | 2 | # |
| 1 | 8 | 0 | 1 | $CATA# |
| 2 | 12 | 0 | 2 | A# |
| 3 | 7 | 1 | 1 | A$CATA# |
| 4 | 5 | 1 | 1 | ACA$CATA# |
| 5 | 3 | 1 | 1 | AGACA$CATA# |
| 6 | 10 | 1 | 2 | ATA# |
| **7** | **1** | **3** | **1** | **ATAGACA$CATA#** |
| 8 | 6 | 0 | 1 | CA$CATA# |
| 9 | 9 | 2 | 2 | CATA# |
| 10 | 4 | 0 | 1 | GACA$CATA# |
| 11 | 0 | 2 | 1 | GATAGACA$CATA# |
| 12 | 11 | 0 | 2 | TA# |
| 13 | 2 | 2 | 1 | TAGACA$CATA# |

## Part OMAKE: Problem hints

## Problems for this Week

- Immediate Decodability
- Caesar Cypher
- Power Strings
- Where's Waldorf
- Extend to Palindrome
- String Partition
- Prince and Princess
- Power Strings
- Life Forms

## Immediate Decodability

### Outline

A set of tokens is decodable if it is **impossible** to write a string that can be parsed in more than one way.

Decodability is detected by checking if a token is a prefix of another.

```
Input:                   Output:
 01                      decodable
 10                      (no string is a prefix)
 0001
 00101

 001                     not decodable
 0100                    (001 is a prefix of 00101)
 00101
 01101
```

## Caesar Cypher

**Outline**

A **k-rotation cypher** replaces every symbol $N$ with symbol $N + k$, including spaces (which are symbol 0).

- **Input:** A list of correct words, and an encrypted text
- **Task:** Find the shift that matches the maximum number of words in the dictionary. Output the decrypted text

Notes about the problem:

- Input: Small, No case, no symbols, spaces
- Crypto text may contain words not in dictionary
- Output requirements (linebreak at 60 characters)

```
THIS      DAWN      THAT      || BUUBDLA PSSPABUAEBXO
THE       ZORRO     OTHER     || ATTACK ZORRO AT DAWN
AT        THING               ||
```

## Power Strings

**Problem Outline**

You are given a string $s$, and you must find the smallest string $s'$, so that $s = s' + s' + s' + \ldots = (s')^n$. This is equal to finding $s'$ with maximum $n$.

Example Input and Output:

```
INPUT          MINIMUM STRING   N
abcd           abcd             1: abcd
abababab       ab               4: ab + ab + ab + ab
kallakalla     kalla            2: kalla + kalla
abababa        abababa          1: abababa
```

This is a mixture of search and string matching. If your search is not very good, you may face TLE, so write your search carefully.

## Where is Waldorf?

### Problem Outline

This the traditional magazine challenge: Find words inside a cube of letters. Pay attention:

- Words can be vertical, horizontal or diagonal;
- Words can be backwards;
- Search is **not case sensitive**

```
abcDEFGhigg          Words:
hEbkWalDork          Waldorf -- 2 5
FtyAwaldORm          Bambi   -- 2 3
FtsimrLqsrc          Betty   -- 1 2
byoArBeDeyv          Dagbert -- ? ?
Klcbqwikomk
strEBGadhrb
yUiqlxcnBjf
```

## Extend to Palindrome

### Problem Outline

You receive a word as input, and you must add the smallest number of letters at the end to make it a palindrome. Examples:

```
alert:      alertrela
abcba:      abcba
aaaalll:    aaaalllaaaa
```

Hints:

- Which letters do you add to a word to make it a palindrome;
- How do you decide if you add a letter or not?
- Can you modify the KLM algorithm to help you make the decision?

## String Partition

### Problem Outline

You must break a large string of digits into smaller numbers (max size: 32 bit signed integer), so that the sum of the numbers is the largest.

**Hints:**

- The max number of digits in the string is N=500;
- Start with a search on the breaking points;
- The total sum can be bigger than signed int;

```
INPUT                           OUTPUT
1234554321                      1234554321
5432112345                      543211239
000                             0
111111111111111111111111111111  3333333333
```

## Prince and Princess

**Problem outline**

The Prince and the princess make different paths through the same $n * n$ grid. Both paths start and end at the same square.

Your task is to make both paths identical by eliminating steps, and print the size of the common path.

```
Input                      Output
1 7 5 4 8 3 9              4
1 4 3 5 6 2 8 9            (Common path: 1,5,8,9)

1 3 4 2 5 8 7 10           5
1 5 8 9 3 2 7 10           (Common path: 1,5,8,7,10)
```

## Life Forms

### Problem Outline

Given a set of strings, find the **largest common substring** that is shared by more than half of the strings.

Hints:

- Generalization of LCS, but for multiple strings;
- If there are multiple substrings of the same size, output all of them;

```
INPUT          OUTPUT
abcdefg        bcdefg
bcdefgh        cdefgh
cdefghi
```

## About these Slides

## Image Credits I

[Page 37] Suffix Tree/Array images from Steven Halim, "Competitive Programming 3", chapter 6.6