

$$f(n) = \frac{n^2}{(\log n)} ; g(n) = n(\log n)^2$$

Answer: $f = O(g)$

$$\text{Consider } \frac{f(n)}{g(n)} = \frac{\frac{n^2}{\log n}}{n(\log n)^2} = \frac{n}{(\log n)^3}$$

Because $\lim_{n \rightarrow \infty} n = \infty$, $\lim_{n \rightarrow \infty} (\log n)^3 = \infty$

and at $(0, \infty)$ both are differentiable, so we could use L'Hopital rule.

For simplification, $\log n$ based on e , so $(\log n)' = \frac{1}{n}$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{n}{(\log n)^3} \stackrel{\frac{\infty}{\infty}}{=} \lim_{n \rightarrow \infty} \frac{1}{3 \frac{1}{n} \log n} = \lim_{n \rightarrow \infty} \frac{n}{3 \log n}$$

$$\stackrel{\frac{\infty}{\infty}}{=} \lim_{n \rightarrow \infty} \frac{1}{3 \frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{n}{3} = \infty$$

So $f(n)$ dominates $g(n)$

$$f = O(g)$$