$T(n) = 2T(\frac{2}{3}n) + n^2$

Approach 1.

$$
\begin{aligned}
T(n) &= 2T(\tfrac{2}{3}n) + n^2 \\
&= 2\cdot(2\cdot T(\tfrac{4}{9}n) + \tfrac{4}{9}n^2) + n^2 \\
&= 4T(\tfrac{4}{9}n) + (1 + \tfrac{8}{9})n^2 \\
&= 8T(\tfrac{8}{27}n) + (1 + \tfrac{8}{9} + \tfrac{64}{81})n^2 \\
&= 16T(\tfrac{16}{81}n) + (1 + \tfrac{8}{9} + \tfrac{64}{81} + \tfrac{512}{729})n^2 \\
&\cdots \\
&= 2^{\log_{\frac{3}{2}} n} T(1) + 9\cdot(1 - (\tfrac{8}{9})^{\log_{\frac{3}{2}} n - 1})n^2 \\
&\leq 2^{\frac{\log_2 n}{\log_2 \frac{3}{2}}} T(1) + 9n^2 \\
&= O(n^2)
\end{aligned}
$$

Approach 2.

$$T(n) = 2T(\tfrac{2}{3}n) + n^2$$

$a=2 \quad b=\frac{3}{2} \quad f(n)=n^2$

$\log_b a = \log_{\frac{3}{2}} 2$

For case 3 in Master Therom.

$f(n) = \Omega(n^{\log_{\frac{3}{2}} 2 + \varepsilon})$ when $0 < \varepsilon < 2 - \log_{\frac{3}{2}} 2$

so $f(n)$ grows polynomially faster than $n^{\log_{\frac{3}{2}} 2}$

and $2f(\frac{2}{3}n) \leq c f(n)$

$\Leftrightarrow \quad \frac{8}{9} n^2 \leq cn^2$ when $1 > c \geq \frac{8}{9}$

Therefore, $T(n) = \theta(f(n)) = \theta(n^2) = O(n^2)$