

I use Ford Fulkerson algorithm as base algorithm and I use BFS to find an augmenting path.

Time complexity:

The time complexity for my solution is $O(VE^2)$, because every time we run BFS to search a augmenting path, it will reduce/revert an edge on residual graph, suppose that edge is $(v1 \rightarrow v2)$ and reverted to $(v2 \rightarrow v1)$. And originally the distance from source to $v1$ and $v2$ satisfies $d(\text{source}, v1) + 1 = d(\text{source}, v2)$. If during the loop we will need to revert the edge again (the augmenting path has $(v2 \rightarrow v1)$ in it), then we will have $d'(\text{source}, v2) + 1 = d'(\text{source}, v1)$. And because $d'(\text{source}, v1) = d'(\text{source}, v2) + 1 \geq d(\text{source}, v2) + 1 = d(\text{source}, v1) + 2$, so once we revert the edge again, the distance will at least increase by two. But the augmenting path's length is limited to $O(E)$, so the increase will not increase $O(E/2)$ times. And we have $O(V)$ vertices, Therefore the Ford Fulkerson algorithm with BFS will at most run $O(VE)$ times. And for each time we search for a augmenting path, we will need $O(V + E) = O(E)$ times to perform BFS, The overall time would be $O(VE^2)$.

Correctness Proof:

The Ford Fulkerson Algorithm is defined as A , and we assume there's a most similar algorithm A' but got a best result. And the better result has at least one pair of day, event pair differs from the A 's result. For that event $_j$, result A has $\text{day}_i \rightarrow \text{event}_j$, an result A' has $\text{day}_k \rightarrow \text{event}_j$.

Now we will construct an algorithm A'' by change the result A' of result from $\text{day}_k \rightarrow \text{event}_j$ to $\text{day}_i \rightarrow \text{event}_j$. And there will be two cases:

If day_i is not used in the result of A' , then then after the change, the maximum events can be attended remains same for A'' as the only difference is which day event_j will be visited. So A'' can get the best result but it is more similar to A than A' , which conflicts our assumption that A' is the most similar algorithm.

If day_i is noted in the result of A' , say $\text{day}_i \rightarrow \text{event}_a$. Now we repeat the above process, if on result A there's $\text{day}_g \rightarrow \text{event}_e$, we change the day_g on result A' to $\text{day}_g \rightarrow \text{event}_e$. And if there left a event_b again, we repeat the process. The process will be stop for two cases:

1. we end up have a day is unused in A' , which is same as we proofed above, the A'' will be similar to A than A' , our correctness holds.
2. we end up have a day_x that only can attend event_y in A' , and if we change day_x to connect to event_z according to A , event_y will not be attended. But this will never happen because if we have $\text{event}_x \rightarrow \text{event}_y$ and we did not revert the edge between $\text{event}_x \rightarrow \text{event}_y$, there will be a augmenting path from $\text{source} \rightarrow \text{event}_x \rightarrow \text{event}_y \rightarrow \text{sink}$, which contradicts that our algorithm will end if we cannot find a augmenting path. So our correctness holds

Therefore, Ford Fulkerson Algorithm is correct for this problem