Sunday, April 24, 2022 6:46 PM

$f(n) = n^{1.01}$; $g(n) = n(\log n)^2$

The answer is $f = O(g)$

consider $\frac{f(n)}{g(n)} = \frac{n^{1.01}}{n(\log n)^2} = \frac{n^{0.01}}{(\log n)^2}$

Because $\lim_{n\to+\infty} n^{0.01} = +\infty$ $\lim_{n\to+\infty} (\log n)^2 = +\infty$

and both are differentiable in $(0, +\infty]$

Than we could use Hospital rule,

And for simplification, we consider $\log n$ based on e

so $(\log n)' = \frac{1}{n}$

$$\lim_{n\to+\infty} \frac{n^{1.01}}{n(\log n)^2} = \lim_{n\to+\infty} \frac{n^{0.01}}{(\log n)^2} = \lim_{n\to+\infty} \frac{(n^{0.01})'}{[(\log n)^2]'} = \lim_{n\to+\infty} \frac{0.01 n^{-0.99}}{2 \log n \frac{1}{n}} = \lim_{n\to+\infty} \frac{0.01 n^{0.01}}{2 \log n}$$

$$= \lim_{n\to+\infty} \frac{1}{200} \frac{(n^{0.01})'}{(\log n)'} = \lim_{n\to+\infty} \frac{1}{200} \cdot \frac{0.01 n^{-0.99}}{\frac{1}{n}} = \lim_{n\to+\infty} \frac{n^{0.01}}{20000} = +\infty$$

So $f(n)$ dominates $g(n)$

$f = O(g)$