

$$T(n) = \sqrt{n} T(\sqrt{n}) + n$$

let  $n = 2^k$ , we get

$$T(2^k) = 2^{\frac{k}{2}} T(2^{\frac{k}{2}}) + 2^k$$

divide  $2^k$  for both side.

$$\frac{T(2^k)}{2^k} = \frac{T(2^{\frac{k}{2}})}{2^{\frac{k}{2}}} + 1$$

$$\text{let } G(k) = \frac{T(2^k)}{2^k}$$

$$\text{Then } G(k) = G(\frac{k}{2}) + 1$$

According to Master Theorem.

$$a=1 \quad b=2 \quad f(n)=1, \log_b a = \log_2 1 = 0$$

$$\text{case 2: } f(n) = n^0 = \theta(n^0 \log^0 n)$$

$$\therefore G(k) = \theta(\log k)$$

$$\therefore \frac{T(2^k)}{2^k} = \log k \quad \because k = \log_2 n$$

$$\frac{T(n)}{n} = \log \log_2 n$$

$$T(n) = \theta(n \cdot \log \log n)$$