

$$3^{1500} = 3^{(1011011100)_2}$$

$$= 3^{2^{10}} \cdot 3^{2^8} \cdot 3^{2^7} \cdot 3^{2^6} \cdot 3^{2^4} \cdot 3^{2^3} \cdot 3^{2^2}$$

$$3^{2^1} \bmod 11 = 9$$

$$3^{2^2} \equiv (9)^2 \bmod 11 = 4$$

$$3^{2^3} \equiv (4)^2 \bmod 11 = 5$$

$$3^{2^4} \equiv (5)^2 \bmod 11 = 3$$

$$3^{2^5} \equiv (3)^2 \bmod 11 = 9$$

$$3^{2^6} \equiv (9)^2 \bmod 11 = 4$$

$$3^{2^7} \equiv (4)^2 \bmod 11 = 5$$

$$3^{2^8} \equiv (5)^2 \bmod 11 = 3$$

$$3^{2^9} \equiv (3)^2 \bmod 11 = 9$$

$$3^{2^{10}} \equiv (9)^2 \bmod 11 = 4$$

$$\therefore 3^{1500} \equiv 4 \times 3 \times 5 \times 4 \times 3 \times 5 \times 4 \bmod 11$$

$$= 1$$