Market Timing With a Robust Moving Average

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Abstract

In this paper we entertain a method of finding the most robust moving average weighting scheme to use for the purpose of timing the market. Robustness of a weighting scheme is defined its ability to generate sustainable performance under all possible market scenarios regardless of the size of the averaging window. The method is illustrated using the long-run historical data on the Standard and Poor's Composite stock price index. We find the most robust moving average weighting scheme, demonstrates its advantages, and discuss its practical implementation.

Key words: technical analysis, market timing, moving average, robustness

JEL classification: G11, G17.

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1 Introduction

Starting from the mid 2000s, there have been an explosion in the academic literature on technical analysis of financial markets (Park and Irwin (2007)). Since that time, market timing with moving averages has been the subject of substantial interest on the part of academics and investors alike.[1] This interest developed because over the course of the last 15 years, especially over the decade of 2000s, many trading rules based on moving averages outperformed the market by a large margin.

Yet despite recent numerous academic studies, the situation with practical implementation of market timing strategies remains rather complicated due to the following reasons. There have been proposed many technical trading rules based on moving averages of prices calculated on a fixed size data window. The main examples are: the momentum rule, the price-minus-moving-average rule, the change-of-direction rule, and the double-crossover method. In addition, there are several popular types of moving averages: simple (or equally-weighted) moving average, linearly-weighted moving average, exponentially-weighed moving average, etc. In order to time the market a trader needs to choose: (1) a trading rule, (2) a moving average weighting scheme, and (3) a size of the averaging window. This choice is very complicated because there exists a huge number of potential combinations of trading rules with moving average weighting schemes and sizes of the averaging window.

In practice, in order to find the best combination of a trading rule with a moving average weighting scheme and a size of the averaging window, using the historical data a trader performs the test of all possible combinations and selects the combination with the best observed performance. Even though this approach to selecting the best trading combination is termed as "data-mining", this approach works and the only real issue with this approach is that it systematically overestimates how well the trading combination will perform in the future (Aronson (2006), Zakamulin (2014)).

The results of the recent study by Zakamulin (2015) allows a trader to simplify dramatically the selection of the best combination of a trading rule with a moving average weighting scheme. Specifically, Zakamulin (2015) revealed that the computation of all technical trading indicators

[1] Some examples are: Brock, Lakonishok, and LeBaron (1992), Okunev and White (2003), Moskowitz, Ooi, and Pedersen (2012), Faber (2007), Gwilym, Clare, Seaton, and Thomas (2010), Kilgallen (2012), Clare, Seaton, Smith, and Thomas (2013), Zakamulin (2014).

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based on moving averages of prices can equivalently be interpreted as the computation of the moving average of price changes. The straightforward use of this result might be as follows. Instead of testing various combinations of a trading rule with a moving average weighing scheme and a size of the averaging window, a trader needs only to test various combinations of a weighting scheme (used to compute the moving average of price changes) and a size of the averaging window, and then select the combination with the best performance in a back test. Yet, the empirical study performed in Zakamulin (2015) suggests that this approach to selecting the best trading combination has two potentially very serious flaws. In particular, Zakamulin (2015) found that there is no single optimal size of the averaging window. On the contrary, there are substantial time-variations in the optimal size of the averaging window for each weighting scheme. In addition, Zakamulin (2014) and Zakamulin (2015) demonstrated that the performance of a market timing strategy, relative to that of its passive counterpart, is highly uneven over time. Therefore, the issue of outliers is of concern. This is because in the presence of outliers (extraordinary good or bad performance over a rather short historical period) the long-run performance of a trading combination does not reflect its typical performance. As a result of these two issues, the best performing trading combination in the past might not perform well in the near future.

In this paper we entertain a novel approach to selecting the trading rule (specified by a particular moving average weighting scheme) to use for the purpose of timing the market. The motivation for this approach is twofold. First, we acknowledge that there is no single optimal size of the averaging window. Second, we acknowledge that the performance of a trading rule is highly uneven through time and over some relatively short particular historical episodes the performance might be unusually far from that over the rest of the dataset. Based on these premises, we find the most robust moving average weighting scheme. By robustness of a weighting scheme we mean not only its robustness to outliers. Robustness of a weighting scheme is also defined as its ability to generate sustainable performance under all possible market scenarios regardless of the size of the averaging window. Our approach is illustrated using the long-run historical data on the Standard and Poor's Composite stock price index.

The rest of the paper is organized as follows. Section 2 presents the market timing rules and moving average weighting schemes. The data for our study is presented in Section 3. Section 4 describes our methodology for finding a robust moving average. Section 5 presents the most robust moving average weighting scheme and demonstrates its advantages. Section 6 discusses the practical implementation of the most robust moving average. Finally, Section 7 concludes the paper.

2 Market Timing Rules and Moving Average Weighting Schemes

A moving average of prices is calculated using a fixed size data "window" that is rolled through time. Denote by P_t the period t closing price of a stock market index. Furthermore, denote by $MA_t(k)$ the general weighted moving average at period-end t with k lagged prices. The general weighted moving average is computed using the following formula:

$$MA_t(k) = \frac{w_t P_t + w_{t-1} P_{t-1} + w_{t-2} P_{t-2} + \ldots + w_{t-k} P_{t-k}}{w_t + w_{t-1} + w_{t-2} + \ldots + w_{t-k}} = \frac{\sum_{j=0}^{k} w_{t-j} P_{t-j}}{\sum_{j=0}^{k} w_{t-j}},$$

where w_{t-j} is the weight of price P_{t-j} in the computation of the weighted moving average. There are many types of moving averages; the most popular ones are: equally-weighted, linearly-weighted, and exponentially-weighted. Yet all of them are calculated using the same general formula and the only real difference between the various types of moving averages lies in the weighting scheme given by $\{w_t, w_{t-1}, \ldots, w_{t-k}\}$.

The most popular trading rules used for timing the market are: the momentum rule (MOM), the price-minus-moving-average rule (P-MA), the moving-average-change-of-direction rule (ΔMA), and the double-crossover method (DCM). The technical trading indicators in these rules are computed as

$$\begin{aligned}
\text{Momentum rule:} \quad & \text{Indicator}_t = P_t - P_{t-k}, \\
\text{Price-minus-moving-average rule:} \quad & \text{Indicator}_t = P_t - MA_t(k), \\
\text{Moving-average-change-of-direction rule:} \quad & \text{Indicator}_t = MA_t(k) - MA_{t-1}(k), \\
\text{Double-crossover method:} \quad & \text{Indicator}_t = MA_t(s) - MA_t(k),
\end{aligned}$$

where $s < k$ defines the size of a shorter window. In all these market timing rules, the Buy signal is generated when the value of a technical trading indicator is positive. Otherwise, the Sell signal is generated.

Zakamulin (2015) demonstrates that despite being computed seemingly differently at the first sight, all technical trading indicators presented above are computed in the same general manner. In particular, the computation of every technical trading indicator can equivalently be interpreted as the computation of the weighted moving average of price changes. Specifically, every technical trading indicator can be equivalently computed using the following formula:

$$\text{Indicator}_t = \frac{\sum_{i=1}^{k} y_{t-i} \Delta P_{t-i}}{\sum_{i=1}^{k} y_{t-i}},$$

where $\Delta P_{t-i} = P_{t-i+1} - P_{t-i}$ denotes the price change over the period from $t-i$ to $t-i+1$, and y_{t-i} is the weight of the price change ΔP_{t-i} in the computation of the moving average of price changes. The weights y_{t-i} are computed using the weights $\{w_t, w_{t-1}, \dots, w_{t-k}\}$ that specify how the moving average of prices is computed. In particular,

$$y_{t-i} = f(w_t, w_{t-1}, \dots, w_{t-k}),$$

where $f(\cdot)$ is some function specified by the underlying trading rule (Zakamulin (2015)).

Even though there are various trading rules based on moving averages of prices and various types of moving averages, there are basically only three types of the shape of function $f(\cdot)$: equal weighting of price changes (as in the MOM rule), underweighting the most old price changes (as in the P-MA rule or in the most ΔMA rules), and underweighting both the most recent and the most old price changes (as in the DCM). In order to generate these shapes, we will employ three types of Exponential Moving Average (EMA) weighting schemes: convex EMA, concave EMA, and hump-shaped EMA.

The ConVex EMA (CV-EMA) is the most common type of EMA. The value of the trading indicator in this case is computed as

$$\text{Indicator}_t(\text{CV-EMA}) = \frac{\sum_{i=1}^{k} \lambda^{i-1} \Delta P_{t-i}}{\sum_{i=1}^{k} \lambda^{i-1}},$$

where $0 \leq \lambda \leq 1$ is a decay factor. This weighting scheme for price changes corresponds to that in the Exponential-Moving-Average-Change-of-Direction (ΔEMA) trading rule (Zakamulin (2015)). When $\lambda = 1$, the CV-EMA weighting scheme reduces to the simple (or equally-weighted) moving average of price changes (the same as in the MOM rule). When $0 < \lambda < 1$, the CV-EMA assigns greater weights to the most recent price changes. By varying the value of λ, one is able to adjust the weighting to give greater or lesser weights to the most recent price changes. When $\lambda = 0$, the CV-EMA reduces to the value of the most recent price change. Figure 1, Panel A, illustrates the CV-EMA weighting scheme for two arbitrary values of λ. Note that function λ^{i-1} is a convex exponential function with respect to i.

The ConCave EMA (CC-EMA) also underweights the most old price changes. The value of the trading indicator in this case is computed as

$$\text{Indicator}_t(\text{CC-EMA}) = \frac{\sum_{i=1}^{k} \left(1 - \lambda^{k-i+1}\right) \Delta P_{t-i}}{\sum_{i=1}^{k} \left(1 - \lambda^{k-i+1}\right)}.$$

This weighting scheme for price changes corresponds to that in the Price-minus-Reverse-Exponential-Moving-Average (P-REMA) trading rule (Zakamulin (2015)). In contrast to the CV-EMA weighting scheme where the degree of underweighting decreases as the lag of the price change increases, in the CC-EMA weighting scheme the degree of underweighting increases as the lag of the price change increases. When $\lambda = 0$, the weighting scheme reduces to the simple moving average of price changes (as in MOM rule). When $\lambda \to 1$, the CC-EMA reduces to the linear moving average of price changes (see the Appendix for a proof). It is worth noting that the use of the linear moving average of price changes corresponds to the use of the most popular Price-minus-Simple-Moving-Average (P-SMA) trading rule (Zakamulin (2015)). Figure 1, Panel B, illustrates the CC-EMA weighting scheme for two arbitrary values of λ. Observe that function $1 - \lambda^{k-i}$ is a concave exponential function with respect to i.

The Hump-Shaped EMA (HS-EMA) underweights both the most recent and the most old price changes. The use of the HS-EMA weighting scheme for price changes corresponds to the use of the popular DCM trading rule based on EMA in both short and long moving averages. Zakamulin (2015) demonstrates that the value of the trading indicator for the DCM in this case can equivalently be computed as

$$\text{Indicator}_t(\text{HS-EMA}) = \frac{\sum_{i=1}^{k} \left(\lambda^{i} - \lambda^{k+1}\right) \Delta P_{t-i}}{1 - \lambda^{k+1}} - \frac{\sum_{i=1}^{s} \left(\lambda^{i} - \lambda^{s+1}\right) \Delta P_{t-i}}{1 - \lambda^{s+1}}.$$

There is an uncertainly about the proper choice of the size of the shorter window s. Since the most popular combination in practice is to use a 200-day long window and a 50-day short window, we set $s = \frac{1}{4}k$ for all values of k. Figure 1, Panel C, illustrates the HS-EMA weighting scheme for two arbitrary values of λ.

For some fixed number of price change lags k, the shape of each moving average weighting scheme depends on the value of the decay factor λ. In order to generates many different shapes of the weighting function $f(\cdot)$, in each trading rule we vary the value of $\lambda \in \{0.00, 0.99\}$ with a step of $\Delta\lambda = 0.01$. As a result, for each type of the EMA we get 100 different shapes. Since we have three different types of the EMA, the total number of generated shapes amounts to 300.

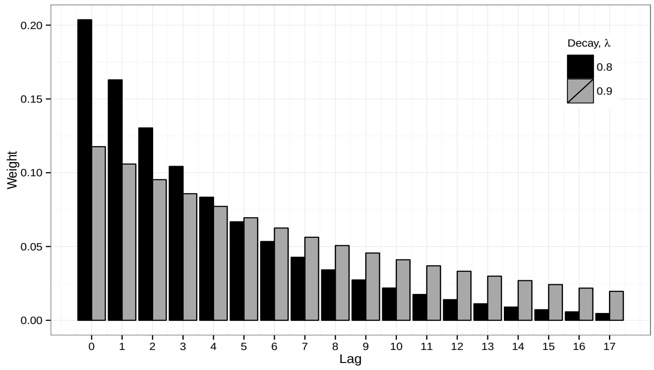
3 Data

We use the same dataset as that in the study by Zakamulin (2015). This dataset comes at the monthly frequency and consists of the capital appreciation and total returns on the Standard and Poor's Composite stock price index, as well as the risk-free rate of return proxied by the Treasury Bill rate. The sample period begins in January 1857, ends in December 2014, and covers 158 full years (1896 monthly observations). The data on the S&P Composite index comes from two sources. The returns for the period January 1857 to December 1925 are provided by William Schwert.[2] The returns for the period January 1926 to December 2014 are computed from the closing monthly priced of the S&P Composite index and corresponding dividend data provided by Amit Goyal.[3] The Treasury Bill rate for the period January 1920 to December 2014 is also provided by Amit Goyal. The Treasury Bill rate for the period January 1857 to December 1919 is estimated using the monthly data for the Commercial Paper Rates for New York. The method of estimation is described in all details in Welch and Goyal (2008).

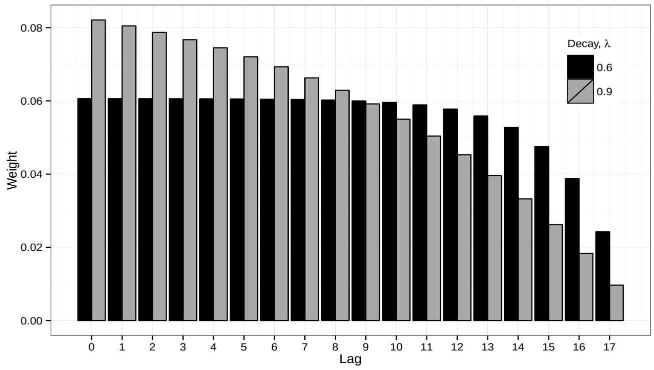
4 The Methodology for Finding a Robust Moving Average

We say that a moving average weighting scheme is robust if it is able to generate sustainable performance under all possible market scenarios regardless of the size of the averaging window. Consequently, in order to find a robust weighting scheme, we need to evaluate the performances of all different trading rules (where each rule is specified by a particular shape of the weighting function), using all feasible sizes of the averaging window, and over all possible

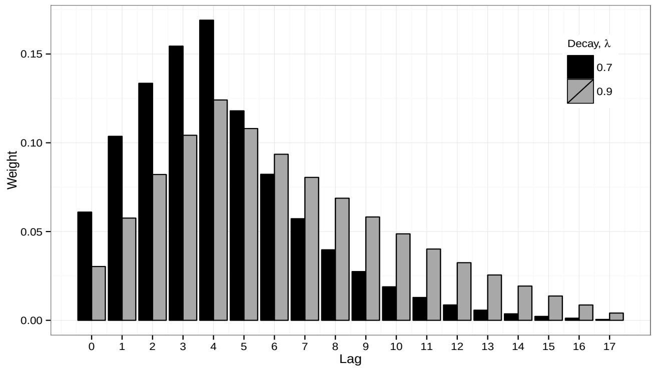
[2] `http://schwert.ssb.rochester.edu/data.htm`
[3] `http://www.hec.unil.ch/agoyal/`



Panel A: Convex EMA weighting scheme



Panel B: Concave EMA weighting scheme



Panel C: Hump-shaped EMA weighting scheme

Figure 1: The types of the moving average weighting schemes used in our study. Panel A illustrates the convex exponential moving average weighting scheme. Panel B illustrates the concave exponential moving average weighting scheme. Panel C illustrates the hump-shaped exponential moving average weighting scheme. $\boldsymbol{\lambda}$ denotes the decay factor. In all illustrations the number of price changes $k = 18$. **Lag** denotes the weight of the lag ΔP_{t-i}, where Lag0 denotes the most recent price change ΔP_{t-1} and Lag17 denotes the most oldest price change ΔP_{t-18}.

market scenarios. Then we need to compare the performances and select a trading rule with the most stable performance.

Every market timing rule prescribes investing in the stocks (that is, the market) when a Buy signal is generated and moving to cash when a Sell signal is generated. Thus, the time t return to a market timing strategy is given by

$$r_t = \delta_{t|t-1} r_{Mt} + \left(1 - \delta_{t|t-1}\right) r_{ft},$$

where r_{Mt} and r_{ft} are the month t returns on the stock market (including dividends) and the risk-free asset respectively, and $\delta_{t|t-1} \in \{0, 1\}$ is a trading signal for month t (0 means Sell and 1 means Buy) generated at the end of month $t-1$.

By performance we mean a risk-adjusted performance. Our main measure of performance is the Sharpe ratio which is a reward-to-total-risk performance measure. We compute the Sharpe ratio using the methodology presented in Sharpe (1994). Specifically, the computation of the Sharpe ratio starts with computing the excess returns, $R_t = r_t - r_{ft}$. Then the Sharpe ratio is computed as the ratio of the mean excess returns to the standard deviation of excess returns. Because the Sharpe ratio is often criticized on the grounds that the standard deviation appears to be an inadequate measure of risk, we also use the Sortino ratio (due to Sortino and Price (1994)) as an alternative performance measure.

In practice, the most typical recommended size of the averaging window amounts to 10-12 months (see, among others, Brock et al. (1992), Faber (2007), Moskowitz et al. (2012), and Clare et al. (2013)). However, as demonstrated in Zakamulin (2015), there are large time-variations in the optimal size of the averaging window for each trading rule (in a back test over a rolling horizon of 20 years). Therefore, we require that a robust moving average weighting scheme must generate a sustainable performance over a broader manifold of horizons, from 4 to 18 months. That is, to find a robust moving average we vary $k \in [4, 18]$. Note that the number of alternative sizes of the averaging window amounts to $m = 15$.

Technical analysis is based on a firm belief that there are recurrent regularities, or patterns, in the stock price dynamics. In other words, "history repeats itself". Based on the paradigm of historic recurrence, we expect that in the subsequent future time period the stock price dynamics (one possible market scenario) will represent a repetition of already observed stock price dynamics over a past period of the same length.[4] The problem is that we do not know what part of the history will repeat in the nearest future. Therefore we want that a robust moving average weighting scheme generates a sustainable performance over all possible historical realizations of the stock price dynamics. We follow the most natural and straightforward idea and split the total sample of historical data into n smaller blocks of data. These blocks of historical data are considered as possible variants of the future stock price dynamics.

We need to make a choice of a suitable block length that should preferably include at least one bear market. Our choice is to use the block length $l = 120$ months (10 years) and is partly motivated by the results reported by Lunde and Timmermann (2003). In particular, these authors studied the durations of bull and bear markets using virtually the same dataset as ours. The bull and bear markets are determined as a filter rule θ_1/θ_2 where θ_1 is a percentage defining the threshold of the movements in stock prices that trigger a switch from a bear to a bull market, and θ_2 is the percentage for shifts from a bull to a bear market. Using a 15/15 filter rule, Lunde and Timmermann find that the mean durations of the bull and bear markets are 24.5 and 7.7 months respectively. Therefore with the block length of 10 years we are almost guaranteed to cover a few alternating bull and bear markets. To increase the number of blocks of data and to decrease the performance dependence on the choice of the split points between the blocks of data, we use 10-year blocks with a 5-year overlap between the blocks. Specifically, the first block of data covers the 10-year period from January 1860 to December 1869; the second block of data covers the 10-year period from January 1865 to December 1874; etc. As a result of this partition, the number of 10-year blocks amounts to $n = 30$.

The choice of the most robust moving average weighting scheme is made using the following method. We fix the size of the averaging window and simulate all trading strategies over the total sample. Each trading strategy is specified by a particular shape of the moving average weighting scheme. Subsequently, we measure and record the performance of every moving average weighting scheme over each block of data. In each block of data, we then rank the performances of all alternative moving average weighting schemes. In particular, the weighting scheme with the best performance in a block of data is assigned rank 1 (highest), the one with

[4] It is worth noting that very popular nowadays block-bootstrap methods of resampling the historical data are based on the same historic recurrence paradigm. Specifically, block-bootstrap is a non-parametric method of simulating alternative historical realizations of the underlying data series that are supposed to preserve all relevant statistical properties of the original data series. In this method the simulated data series are generated using blocks of historical data. For a review of bootstrapping methods, see Berkowitz and Kilian (2000).

the next best performance is assigned rank 2, and then down to rank 300 (lowest). After that, we change the size of the averaging window, k, and repeat the procedure all over again. In the end, each moving average weighting scheme receives $n \times k = 30 \times 15 = 450$ ranks; each of these ranks is associated with the weighting scheme's performance for some specific block of data and some specific size of the averaging window. Finally we compute the median rank for each moving average weighting scheme. We assume that the most robust moving average weighting scheme has the *highest median rank*. That is, the most robust moving average weighting scheme is that one that has the highest median performance rank across different historical sub-periods and different sizes of the averaging window. Note that since we use the median rank instead of the average rank, and since we use ranks instead of performances, we avoid the outliers issue (when an extraordinary good performance in some specific historical period influences the overall performance).

5 Empirical Results

Rank	**Weighting scheme**	**Decay, λ**
1	CV-EMA	0.87
2	CV-EMA	0.88
3	CV-EMA	0.89
4	CV-EMA	0.90
5	CC-EMA	0.99
6	CC-EMA	0.73
7	CV-EMA	0.86
8	CV-EMA	0.91
9	CC-EMA	0.79
10	CV-EMA	0.85

Table 1: Top 10 most robust moving average weighting schemes out of total 300 tested. **CV-EMA** denotes the convex exponential moving average weighting scheme where the weight of ΔP_{t-i} is given by λ^{i-1}. **CC-EMA** denotes the concave exponential moving average weighting scheme where the weight of ΔP_{t-i} is given by $1 - \lambda^{k-i+1}$.

Table 1 reports the top 10 most robust moving average weighting schemes in our study. 7 out of 10 top most robust weighting schemes belong to the family of the CV-EMA where the decay factor $\lambda \in [0.85, 0.91]$ with a step of 0.01. The most robust weighting scheme is the CV-EMA with $\lambda = 0.87$. The other 3 out of 10 top most robust weighting schemes belong to the family of the CC-EMA where the decay factor $\lambda \in \{0.99, 0.73, 0.79\}$. It is worth noting that the use of the CC-EMA weighting scheme for price changes with $\lambda = 0.99$ is virtually identical to the use of the most popular among practitioners P-SMA trading rule.[5] Thus, the P-SMA rule employs a robust moving average which belongs to the top 5 most robust moving average weighting schemes in our study.

The most robust weighting scheme in our study is also "robust" with respect to the performance measure used, the segmentation of the total historical sample into blocks of data, and the amount of transaction costs. Specifically, we used the Sortino ratio instead of the Sharpe ratio and obtained the same results. We also tried different segmentations of blocks of data: used 5- and 10-year non-overlapping blocks, used 5-year blocks with 2- and 3-year overlap. We varied the amount of one-way proportional transaction costs in the range 0.0-0.5%. In each case we arrived to the same most robust moving average weighting scheme.

In order to demonstrate the advantages of the robust moving average, we compare its performance with that of 4 benchmarks. The first benchmark is the passive buy-and-hold strategy. The other 3 benchmarks are the active trading strategies that use the MOM rule, the P-SMA rule, and the DCM. Table 2 reports the annualized Sharpe ratios of the passive market and active trading strategies versus the size of the moving average window. The active strategies are simulated over the period from January 1860 to December 2014. The size of the averaging window is varied from 4 months to 18 months.

Our first observation is that the trading rule with the (most) robust moving average showed the best performance only for 4 out of 15 alternative sizes of the averaging window. The P-SMA rule scored the best for 6 out of 15 sizes of the averaging window. However, the robust moving average generates the best median and mean performances.[6]

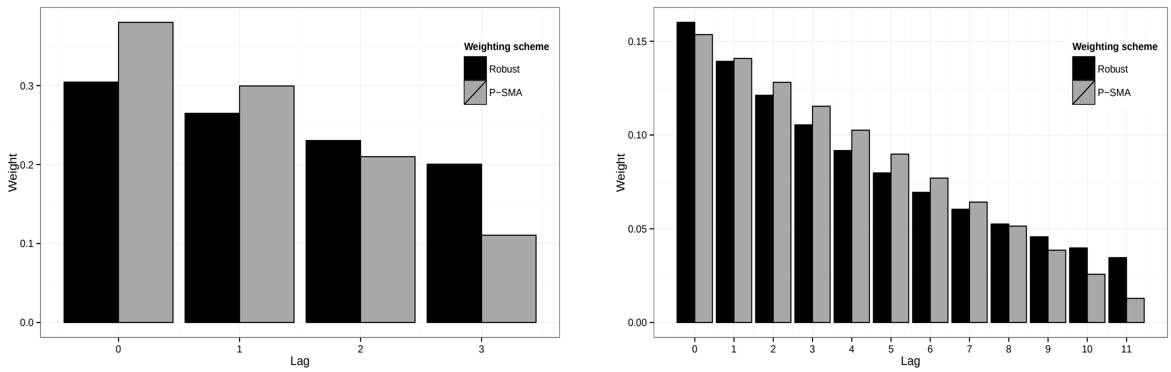
Our second observation is that the MOM rule generates a good performance only when the size of the averaging window is relatively short. Specifically, when $k \in [4, 5]$ the MOM rule generates the best performance; when $k \in [6, 10]$ the performance of the MOM rule is rather good. However, when the size of the averaging window increases beyond 10 months, the performance of the MOM rule starts to deteriorate. In contrast, the performance of the robust moving average and the P-SMA rule remains stable when the size of the averaging window

[5] In the Appendix we prove that, when $\lambda \to 1$, the CC-EMA weighting scheme reduces to the linear moving average.

[6] In Table 2, due to rounding the value of a Sharpe ratio to a number with 2 digits after the decimal delimiter, sometimes we do not see the difference in performances. Yet the bold text indicates the trading rule with the best performance.

Window, months	**Market**	**Weighting scheme** **Robust**	**MOM**	**P-SMA**	**DCM**
4	0.38	0.47	**0.50**	0.44	0.44
5	0.38	0.55	**0.57**	0.51	0.48
6	0.38	0.48	0.49	0.50	**0.52**
7	0.38	**0.55**	0.53	0.52	0.49
8	0.38	**0.54**	0.50	0.52	0.48
9	0.38	**0.55**	0.50	0.54	0.49
10	0.38	0.53	**0.55**	0.54	0.48
11	0.38	0.54	0.49	**0.56**	0.50
12	0.38	0.53	0.48	**0.53**	0.51
13	0.38	**0.53**	0.45	0.53	0.52
14	0.38	0.51	0.43	**0.54**	0.50
15	0.38	0.50	0.44	**0.54**	0.49
16	0.38	0.53	0.41	**0.54**	0.50
17	0.38	0.52	0.38	**0.52**	0.49
18	0.38	**0.55**	0.38	0.50	0.48
Median	0.38	**0.53**	0.49	0.53	0.49
Mean	0.38	**0.53**	0.47	0.52	0.49

Table 2: Annualized Sharpe ratios of the passive market and active trading strategies versus the size of the moving average window. **Market** denotes the passive market strategy. **Robust** denotes the CV-EMA weighting scheme with $\lambda = 0.87$. **MOM** denotes the momentum rule. **P-SMA** denotes the price-minus-simple-moving-average rule. **DCM** denotes the double-crossover method where the moving averages in both the short and long window are computed using the CV-EMA with $\lambda = 0.9$. The active strategies are simulated over the period from January 1860 to December 2014. For each size of the averaging window, bold text indicates the weighting scheme with the best performance.



Panel A: Averaging window of 4 months

Panel B: Averaging window of 12 months

Figure 2: The shape of the robust moving average weighting scheme versus the shape of the weighting scheme in the P-SMA trading rule. Panel A illustrates the shapes when the size of the averaging window amounts to $k = 4$ months. Panel B illustrates the shapes when the size of the averaging window amounts to $k = 12$ months. **Lag** denotes the weight of the lag ΔP_{t-i}, where Lag0 denotes the most recent price change ΔP_{t-1}.

increases. All this suggests that indeed, as many analysts argue, the most recent stock prices (or price changes) contain more relevant information on the future direction of the stock price than earlier stock prices. We conjecture that there are probably substantial time-variations in the optimal size of the moving averaging window and the optimal weighting scheme. It is quite probable that the MOM rule allows a trader to generate the best performance when the trader knows the optimal size of the averaging window. But because there is a big uncertainly about the optimal window size, underweighting the most old prices makes the moving average to be robust. That is, underweighting the most old prices allows the weighting scheme to generate sustainable performance even if the size of the averaging window is way above the optimal size. In principle, either in the robust moving average or in the P-SMA rule we can extend the size of averaging window beyond 18 months without any noticeable performance deterioration, because the weights of the old prices diminish quite fast and approach zero as the size of the averaging window increases.

It is worth emphasizing that the shape of the robust moving average weighting scheme differs from the shape of the weighting scheme in the P-SMA trading rule mainly when the size of the averaging window is short. Figure 2 illustrates the shape of the robust moving average weighting scheme versus the shape of the weighting scheme in the P-SMA trading rule for two different sizes of the averaging window, 4 and 12 months. When the size of the averaging window is 12 months, there are only marginal differences between the two weighting schemes. In contrast, when the size of the averaging window is 4 months, the shape of the robust weighting scheme is somewhere in between the shapes of the weighting schemes in the MOM and P-SMA rules. That is, when the size of the averaging window is rather short, the robust weighting scheme underweights older price changes to a lesser degree as compared with that in the P-SMA rule.

To further demonstrate the advantages of the robust moving average, Table 3 reports the rank of the robust moving average weighting scheme together with the ranks of the 3 active benchmark strategies for each 10-year period out of 30 overlapping periods. The active benchmark strategies are the same as above: the MOM rule (given by the CC-EMA with $\lambda = 0.00$), the P-SMA rule (proxied by the CC-EMA with $\lambda = 0.99$), and the DCM (given by the HS-EMA with $\lambda = 0.90$). We remind the reader that in our study there are totally 300 alternative weighting schemes. As a result, the rank of a weighting scheme can be any integer number from 1 to 300. To compute the ranks in this table, we use the size of the averaging window of 10 months. It is worth noting that with this window size the best overall performance, among 4 competing moving averages (see Table 2), is generated by the MOM rule; the second best by the P-SMA rule; the robust moving average scores 3rd; the DCM has the worst performance. However, the robust weighting scheme has the highest median rank and the second highest mean rank. Even though the MOM rule generates the best performance over the total historical sample, its median rank over all sub-periods, and especially the mean rank, is noticeable below those of the robust moving average. Specifically, the mean rank of the MOM rule is higher than its median rank. This tells us that the distribution of the performances of the MOM rule over sub-periods is right-skewed. Apparently, the superior performance of the MOM rule tends to be generated mainly over a few historical sub-periods. In contrast, for the robust moving average the mean rank is virtually identical to the median rank. This tells us that the distribution of the performances of the robust moving average over sub-periods is symmetrical. Finally, we observe that out of 4 competing rules, the P-SMA rule most often outperforms the other rules in sub-periods. Specifically, it is the best performing rule in 11 out of 30 sub-periods. Besides, the P-SMA rule has the highest mean rank. Yet, the robust moving average has the highest median rank over all sub-periods.

Period	Weighting scheme			
	Robust	MOM	P-SMA	DCM
1860 - 1869	206	164	76	**30**
1865 - 1874	169	**5**	290	285
1870 - 1879	137	**5**	228	271
1875 - 1884	112	**102**	114	143
1880 - 1889	**51**	109	127	103
1885 - 1894	113	108	172	**36**
1890 - 1899	198	**62**	216	152
1895 - 1904	133	164	**77**	278
1900 - 1909	**91**	218	129	100
1905 - 1914	125	**108**	253	114
1910 - 1919	160	221	**156**	209
1915 - 1924	**3**	124	25	193
1920 - 1929	**2**	107	63	229
1925 - 1934	93	**27**	130	255
1930 - 1939	116	**26**	109	246
1935 - 1944	273	199	**62**	127
1940 - 1949	286	171	**150**	296
1945 - 1954	61	126	**5**	143
1950 - 1959	19	181	**1**	101
1955 - 1964	58	249	**7**	210
1960 - 1969	124	212	**119**	268
1965 - 1974	108	88	**46**	168
1970 - 1979	92	206	38	**12**
1975 - 1984	158	256	41	**3**
1980 - 1989	79	189	**42**	145
1985 - 1994	**2**	64	152	127
1990 - 1999	90	**14**	186	97
1995 - 2004	112	**63**	116	125
2000 - 2009	**14**	111	18	154
2005 - 2014	103	237	**63**	121
Median	**110**	117.5	111.5	144
Mean	109.6	130.5	**107.0**	158.0

Table 3: Ranks of the four alternative trading rules over 10-year historical periods with 5-year overlap. The total number of tested rules amounts to 300. As a result, the rank of a trading rule can take any integer number from 1 to 300. The trading rules are ranked according to their performance; the best performing rule is assigned the 1st rank, the worst performing rule is assigned the 300th rank. In all trading rules the size of averaging window amounts to $k = 10$ months. **Robust** denotes the CV-EMA weighting scheme with $\lambda = 0.87$. **MOM** denotes the momentum rule. **P-SMA** denotes the price-minus-simple-moving-average rule. **DCM** denotes the double-crossover method where the moving averages in both the short and long window are computed using the CV-EMA with $\lambda = 0.9$. For each sub-period, bold text indicates the weighting scheme with the highest rank (i.e., best performance) among the 4 alternative weighting schemes.

6 Practical Implementation of the Robust Moving Average

To implement the trading with the robust moving average, the trader can use any available trading software that is able to compute the exponential moving average (EMA) of prices over a fixed size data window. The formula for the computation of the EMA at month-end t with k lagged prices is given by

$$EMA_t(k) = \frac{P_t + \lambda P_{t-1} + \lambda^2 P_{t-2} + \ldots + \lambda^k P_{t-k}}{1 + \lambda + \lambda^2 + \ldots + \lambda^k} = \frac{\sum_{j=0}^{k} \lambda^j P_{t-j}}{\sum_{j=0}^{k} \lambda^j}.$$

The trading signal is computed as the exponential-moving-average-change-of-direction rule with $k-1$ lagged prices:

$$\begin{aligned}\text{Indicator}_t(\Delta\text{EMA}) = EMA_t(k-1) - EMA_{t-1}(k-1) &= \frac{\sum_{j=0}^{k-1} \lambda^j P_{t-j}}{\sum_{j=0}^{k-1} \lambda^j} - \frac{\sum_{j=0}^{k-1} \lambda^j P_{t-1-j}}{\sum_{j=0}^{k-1} \lambda^j} \\ = \frac{\sum_{j=0}^{k-1} \lambda^j \left(P_{t-j} - P_{t-1-j}\right)}{\sum_{j=0}^{k-1} \lambda^j} &= \frac{\sum_{i=1}^{k} \lambda^{i-1} \Delta P_{t-i}}{\sum_{i=1}^{k} \lambda^{i-1}},\end{aligned}$$

where $i = j+1$. Consequently, to compute the trading signal of the most robust moving average using the averaging window of, say, 10 months, the trader needs to compute the change in the value of the rolling EMA(9). Specifically, this rolling EMA is computed using the last price and $k = 9$ lagged prices. The application of the EMA(9) to the S&P 500 index and the resulting trading signal, over the period from January 1995 to December 2014, is illustrated in Figure 3.

In principle, when the size of the averaging window, k, is rather large such that $\lambda^k \approx 0$, then the trading signal of the robust moving average can also be computed using the price-minus-exponential-moving-average (P-EMA) rule. In particular, Zakamulin (2015) shows that the trading signal for this rule can equivalently be computed as:

$$\text{Indicator}_t(\text{P-EMA}) = \frac{\sum_{i=1}^{k} \left(\lambda^{i-1} - \lambda^k\right) \Delta P_{t-i}}{\sum_{i=1}^{k} \left(\lambda^{i-1} - \lambda^k\right)}.$$

When $\lambda^k \approx 0$, this trading signal reduces to that of the convex EMA weighting scheme for price changes.

When it comes to the choice of the size of the averaging window, according to our results

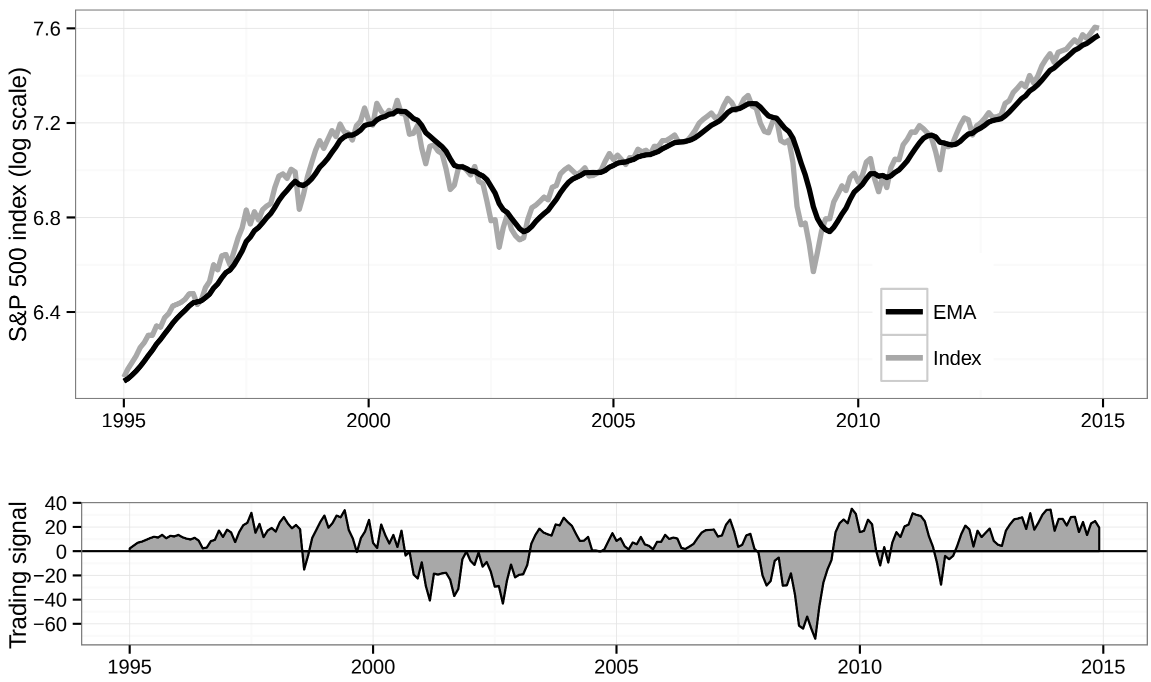


Figure 3: The application of the EMA(9) to the S&P 500 index and the resulting trading signal over the period from January 1995 to December 2014.

the robust moving average delivers a rather stable performance when the size of the window is greater than 4 months. The robust moving average shows the best performance (relative to its benchmarks) when the size of the averaging window $k \in [7, 9]$ months. For shorter windows ($k < 7$), one can probably consider implementing the equally-weighted moving average instead of the robust moving average. For longer windows ($k > 10$), one can safely use the linear moving average (the standard P-SMA rule).

7 Conclusions

Resent research on the performance of market timing strategies based on moving averages of prices has revealed the following two important features. First, there are substantial time-variations in the optimal moving average weighting scheme and the optimal size of the averaging window. As an immediate result, there is no particular moving average weighting scheme coupled with some particular size of the averaging window that produces the best performance under all market scenarios. Second, the performance of the market timing strategy is highly uneven over time; the long-run performance is often substantially influenced by untypical performance over some relatively short historical episode(s). Both of these features significantly complicate the choice of a reliable market timing strategy.

In this paper we proposed and implemented the novel method of selection the moving average weighting scheme to use for the purpose of timing the market. The criterion of selection is to choose the most robust moving average. Robustness of a moving average is defined as its insensitivity to outliers and its ability to generate sustainable performance under all possible market scenarios regardless of the size of the averaging window. We performed a search over 300 different shapes of the weighting scheme using 15 feasible sizes of the averaging window and many alternative segmentations of the historical stock price data. Our results suggest that the convex exponential moving average with the decay factor of 0.87 (for monthly data) represents the most robust weighting scheme. We also found that the popular price-minus-simple-moving-average trading rule belongs to the top 5 most robust moving averages in our study.

One of the main implications of our study is that, in order to be robust, the weighting scheme has to overweight the most recent price changes. But it is not because the last price change is more important than the next to last price change. It is because the price changes in some distant past are not important at all. Therefore it would be probably more correct to say instead "the weighting scheme has to underweight the most old price changes". It is quite possible that equal weighting of price changes over some time-varying window size produces the best performance. But because a trader never knows the current optimal window size, underweighting the older price changes reduces the performance dependence on the size of the averaging window.

Appendix

In this technical appendix we prove that the concave EMA weighting scheme, given by

$$\text{Indicator(CC-EMA)}_t = \frac{\sum_{i=1}^{k} \left(1-\lambda^{k-i+1}\right) \Delta P_{t-i}}{\sum_{i=1}^{k} \left(1-\lambda^{k-i+1}\right)},$$

reduces to the linear moving average weighting scheme when $\lambda \to 1$.

The first step in the proof is to derive the approximate expression for λ^{k-i+1} when $\lambda \to 1$.

We introduce $h = 1 - \lambda$. Therefore

$$\lim_{\lambda \to 1} \lambda^{k-i+1} = \lim_{h \to 0} (1-h)^{k-i+1}.$$

We approximate the value of $(1-h)^{k-i+1}$ using a one-term Taylor series expansion:

$$(1-h)^{k-i+1} \approx 1 - (k-i+1)h\,.$$

As a result, when h is rather small, the weight of ΔP_{t-i} can be approximated by

$$1 - \lambda^{k-i+1} \approx (k-i+1)h\,.$$

The second and final step in the proof is to set this weight into the original formula for the concave EMA and obtain the following approximation for a rather small h:

$$\frac{\sum_{i=1}^{k} \left(1 - \lambda^{k-i+1}\right) \Delta P_{t-i}}{\sum_{i=1}^{k} \left(1 - \lambda^{k-i+1}\right)} \approx \frac{\sum_{i=1}^{k} (k-i+1) h \Delta P_{t-i}}{\sum_{i=1}^{k} (k-i+1) h}.$$

Observe that the fraction on the right-hand-side of the approximation does not depend on the value of h because it is a common factor for both the numerator and denominator of the fraction. Therefore in the limit the concave EMA weighting scheme converges to

$$\begin{aligned}\lim_{\lambda \to 1} \left(\frac{\sum_{i=1}^{k} \left(1 - \lambda^{k-i+1}\right) \Delta P_{t-i}}{\sum_{i=1}^{k} \left(1 - \lambda^{k-i+1}\right)} \right) &= \frac{\sum_{i=1}^{k} (k-i+1) \Delta P_{t-i}}{\sum_{i=1}^{k} (k-i+1)} \\ &= \frac{k \Delta P_{t-1} + (k-1) \Delta P_{t-2} + (k-2) \Delta P_{t-3} + \ldots + 2 \Delta P_{t-k+1} + \Delta P_{t-k}}{k + (k-1) + (k-2) + \ldots + 2 + 1},\end{aligned}$$

which is an easily recognizable linear moving average of price changes.

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