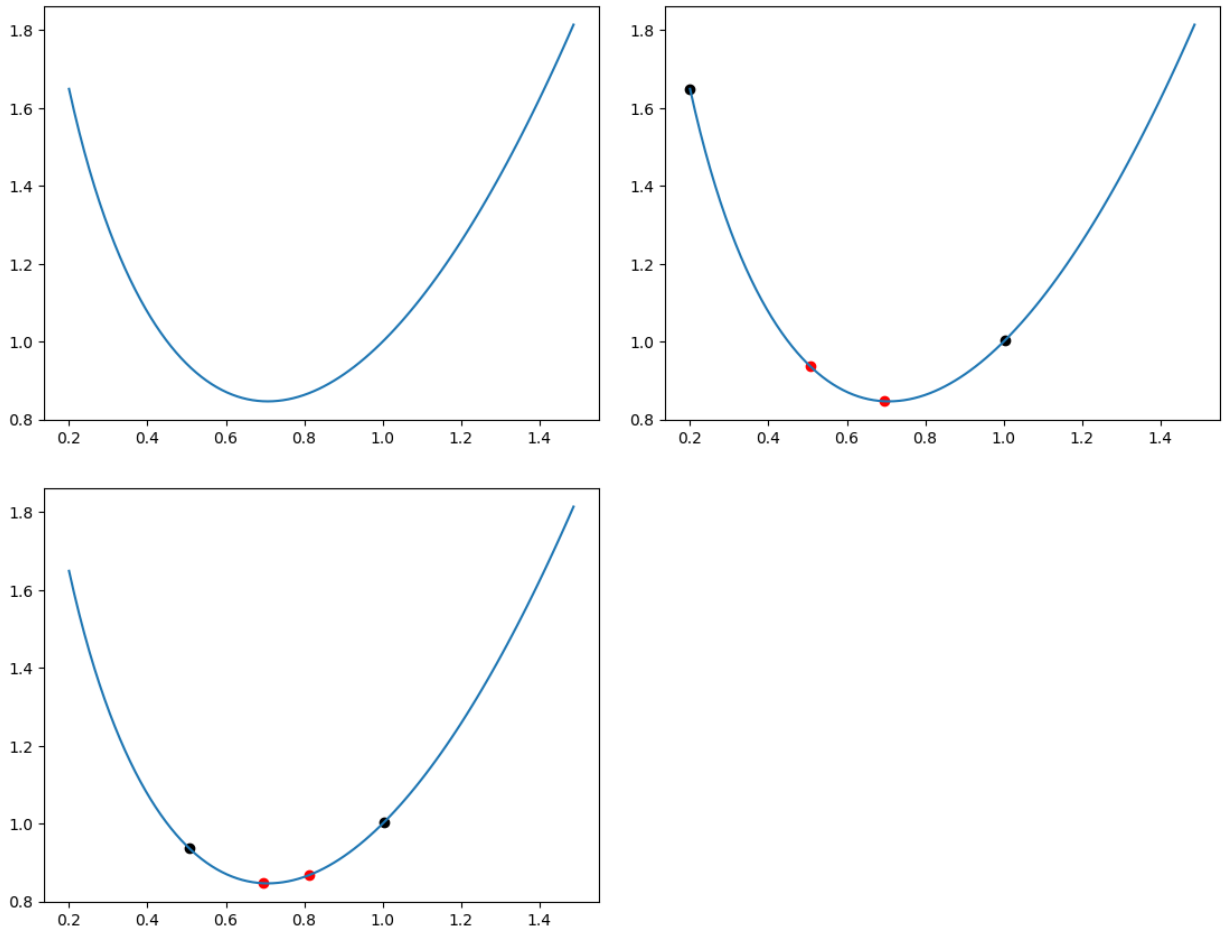
Math3016 Assignment Report

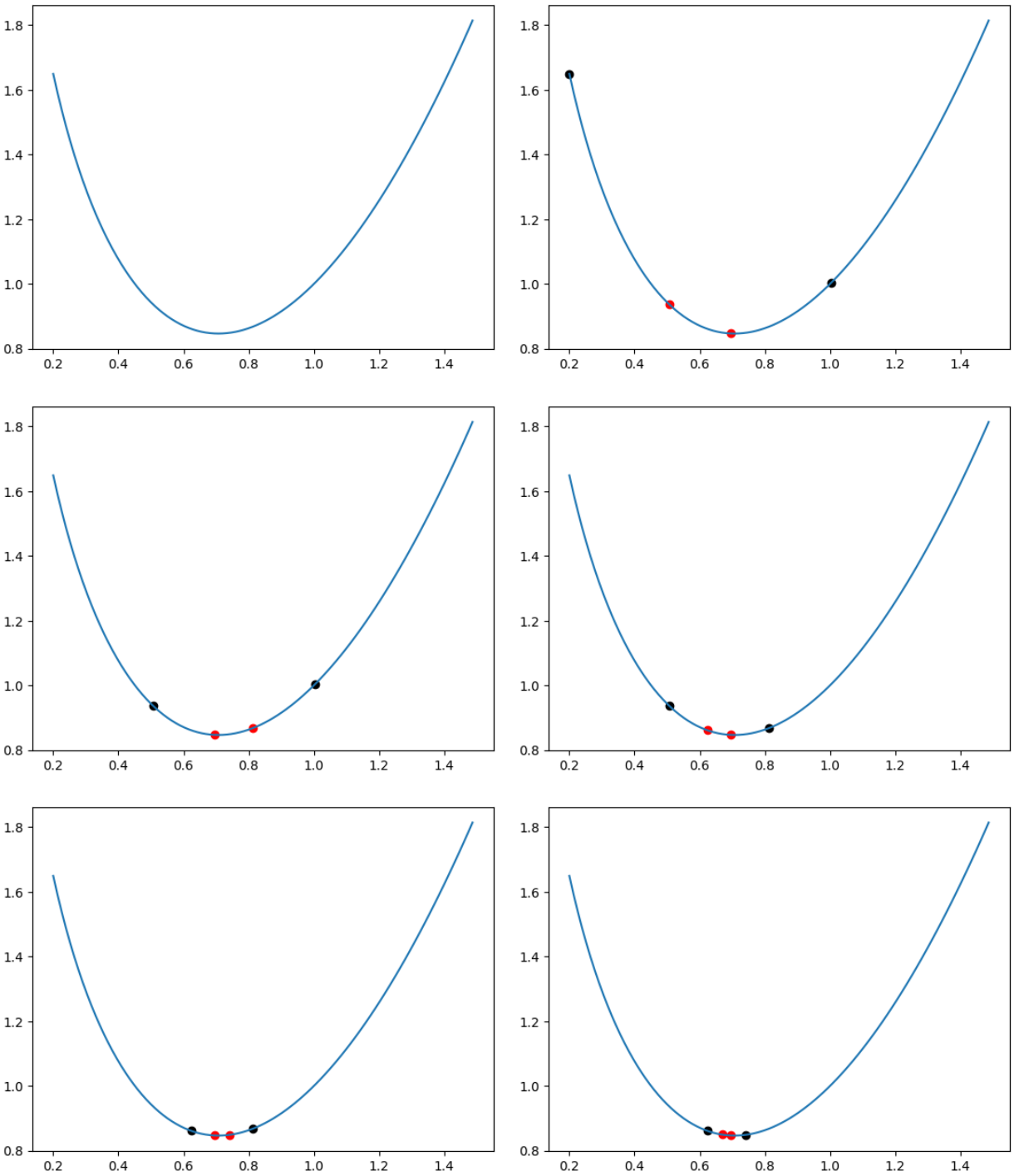
Chenyang Wu, Student ID:32588925

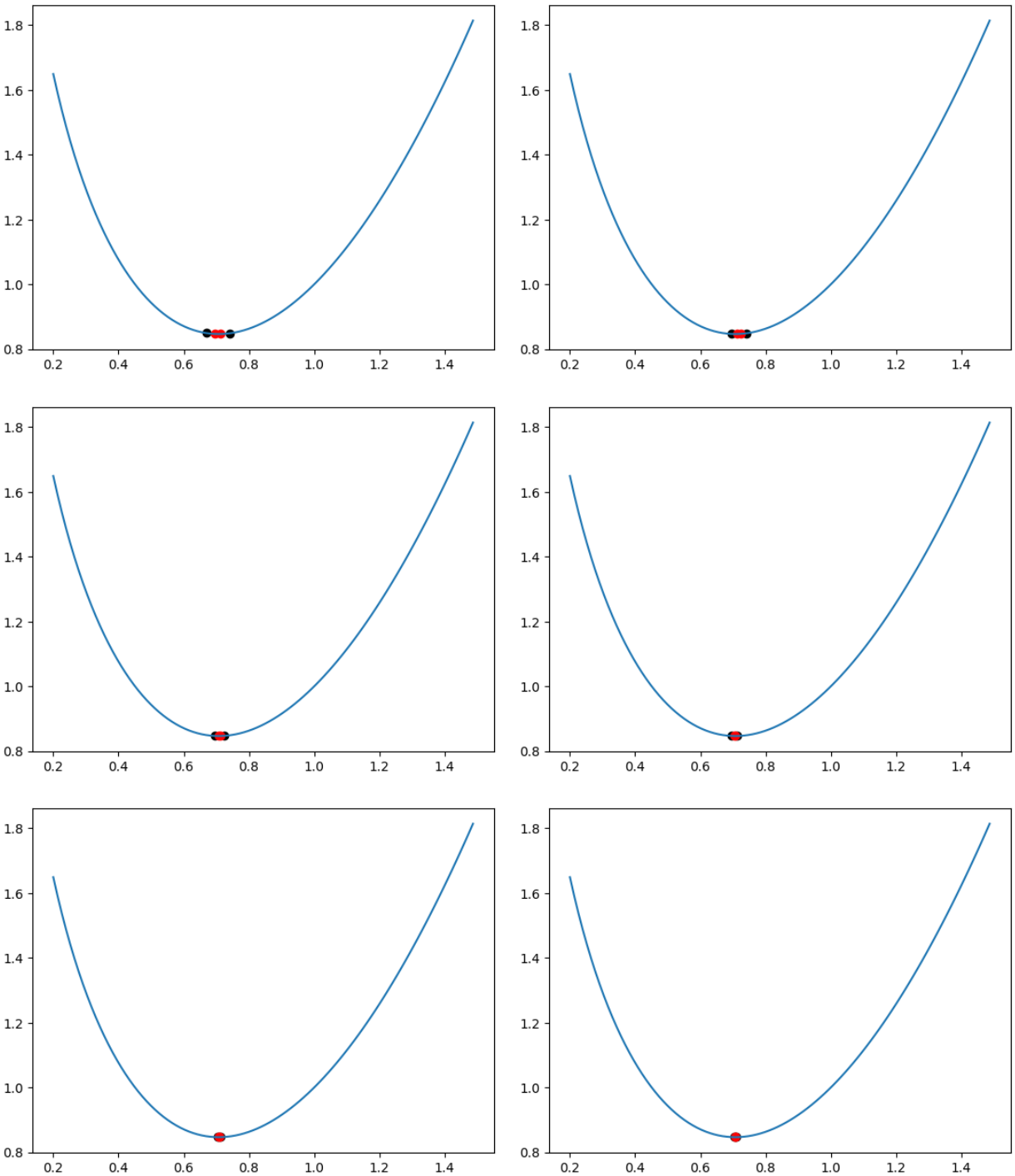
December 10, 2023

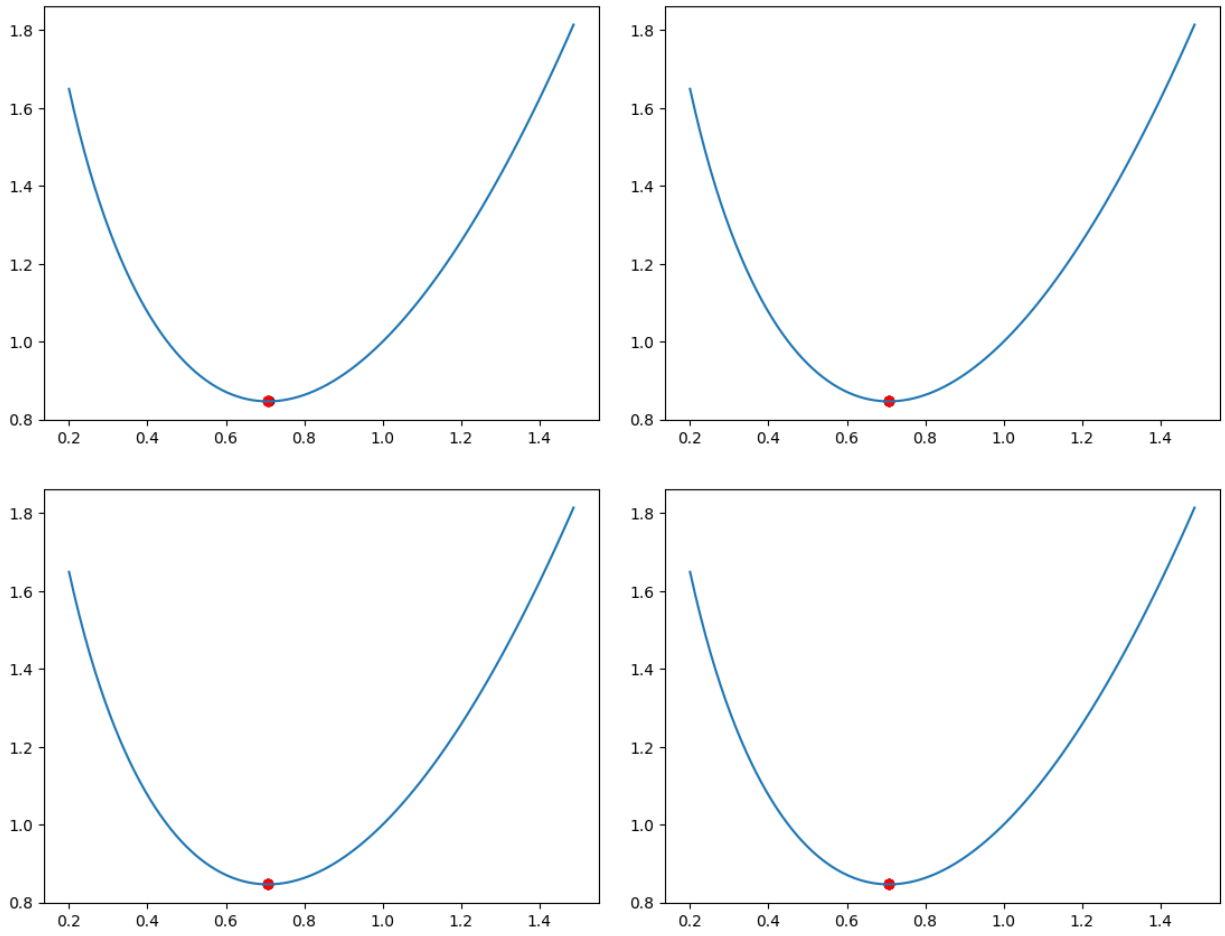
Question 1
(i)



Minimum of this function between the interval of 0.2 and 1.5 with epsilon = 0.5 is 0.7551494000000001
(ii)

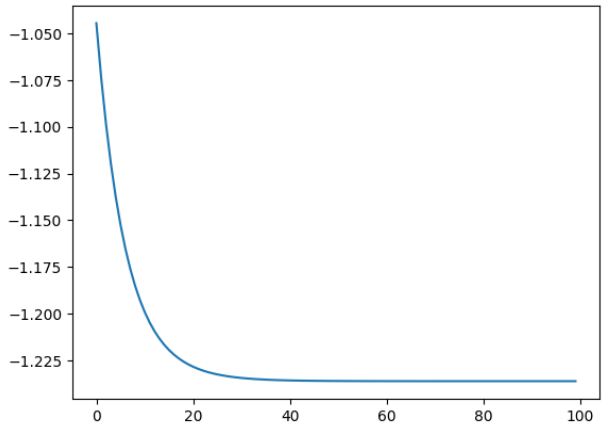

1.8
1.6
1.4
1.2
1.0
0.8
0.2
0.4
0.6
0.8
1.0
1.2
1.4


1.8
1.6
1.4
1.2
1.0
0.8
0.2
0.4
0.6
0.8
1.0
1.2
1.4

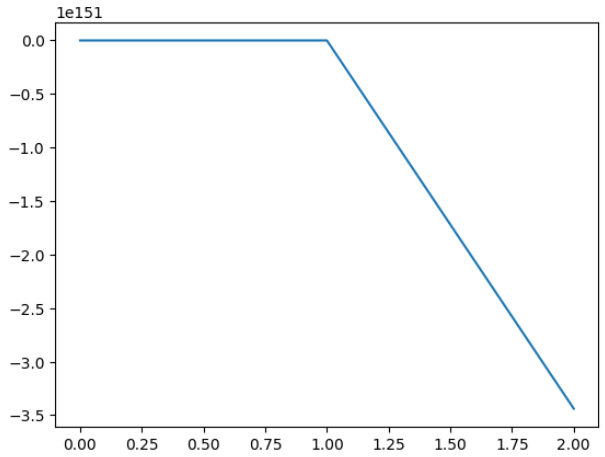


Minimum of this function between the interval of 0.2 and 1.5 with epsilon = 0.001 is 0.7070158367975996
Hence by minimizing epsilon, we can find a more accurate solution than the one found in part i
==========

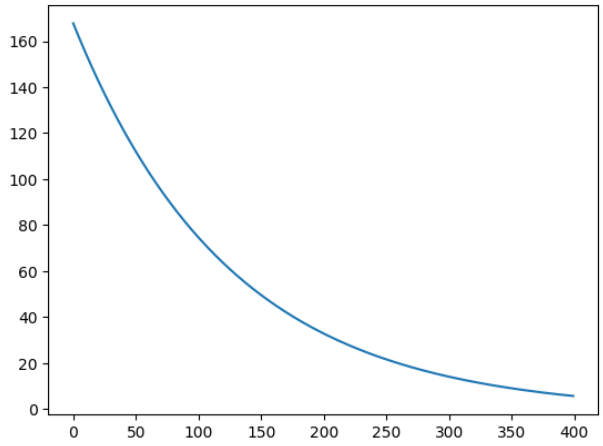
Question 2
(i)



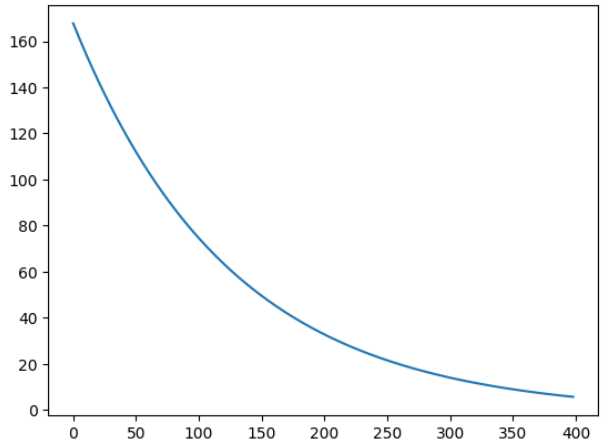
```
Solution found by the gradient descent algorithm with fixed step size with start
point -2, 100 iterations and step size 0.1 is -1.2931977413210847
(ii)

C:\Users\12530\AppData\Local\Temp\ipykernel_2916\2661283232.py:5:
RuntimeWarning: overflow encountered in exp
  return np.sin(x)-np.exp(x)
C:\Users\12530\AppData\Local\Temp\ipykernel_2916\2661283232.py:8:
RuntimeWarning: overflow encountered in exp
  return np.cos(x)-np.exp(x)
C:\Users\12530\AppData\Local\Temp\ipykernel_2916\2661283232.py:5:
RuntimeWarning: invalid value encountered in sin
  return np.sin(x)-np.exp(x)
C:\Users\12530\AppData\Local\Temp\ipykernel_2916\2661283232.py:8:
RuntimeWarning: invalid value encountered in cos
  return np.cos(x)-np.exp(x)
```



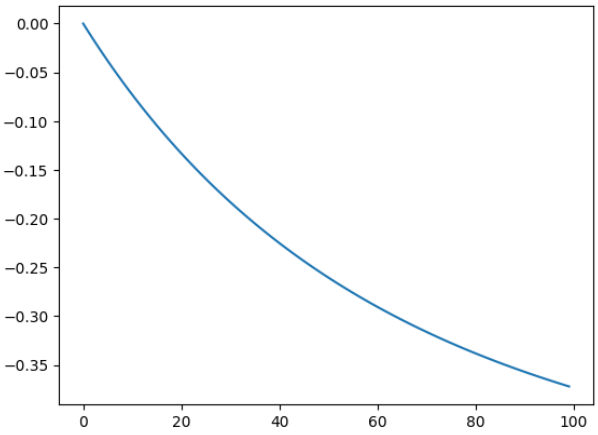

```
Solution found by the gradient descent algorithm with fixed step size with start
point -2, 100 iterations and step size 10 is nan
Hence due to high alpha, it diverges
============================================================

Question 3
(i)
```



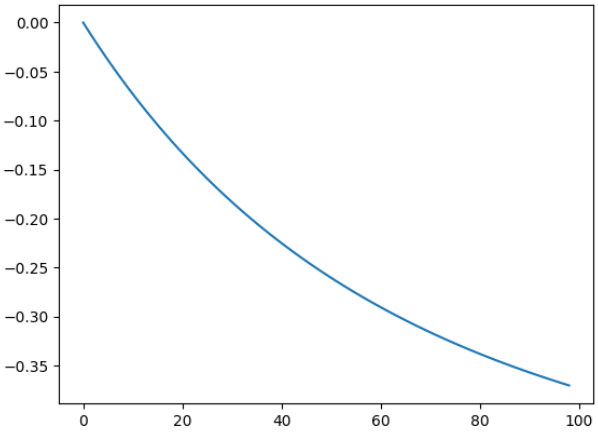
Solution found by the gradient descent algorithm with fixed step size with start point [-8 4], 400 iterations and step size 0.001 is [-2.73052209 -1.10376821] function value: 48.39843167207197



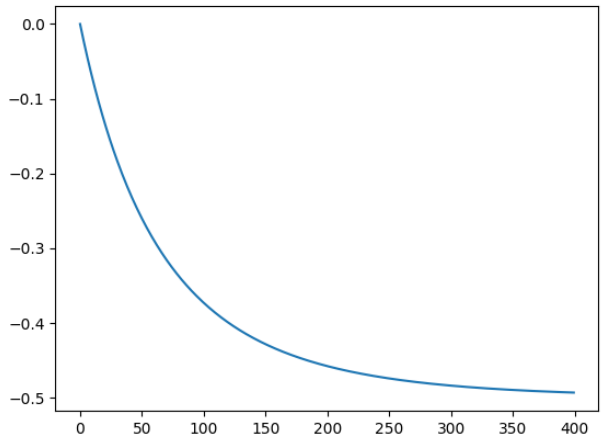
Solution found by the accelerated gradient descent algorithms with start point [-8 4], 400 iterations and step size 0.001 is [-2.73606588 -1.09876278] function value: 48.80523010895042
In this case, gradient descent with fixed step size performs better as it get a lower value than the other algorithm.
(ii)



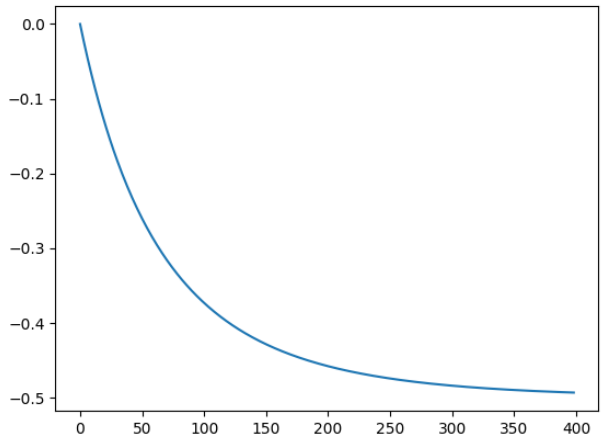
Solution found by the gradient descent algorithm with fixed step size with start point [-1 1], 100 iterations and step size 0.001 is [-0.8677291 0.8677291] function value: -0.37202876139455343



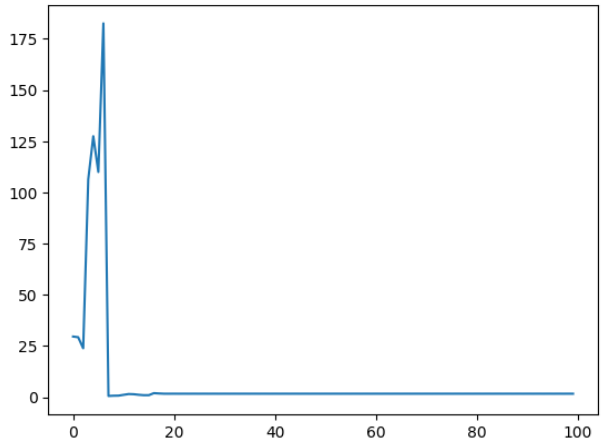
Solution found by the accelerated gradient descent algorithms with start point [-1 1], 100 iterations and step size 0.001 is [-0.86861331 0.86861331] function value: -0.3704706154032733
In this case, accelerated gradient descent performs better as it get a lower value than the other algorithm.
(iii)
Modify the starting point and iterations



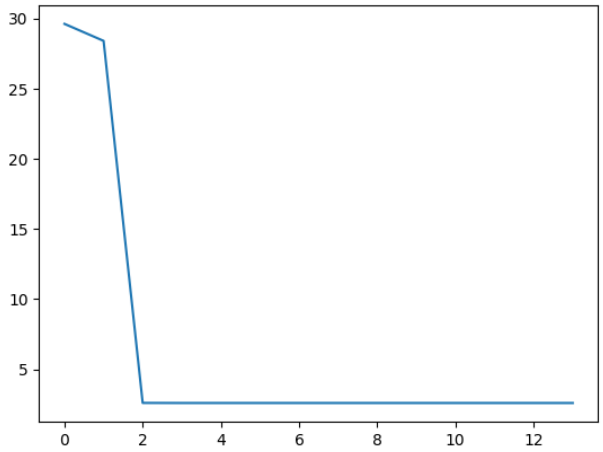

```
Solution found by the gradient descent algorithm with fixed step size with start
point [ 1 -1], 400 iterations and step size 0.001 is [ 0.74570839 -0.74570839]
function value: -0.4937098420841769
```



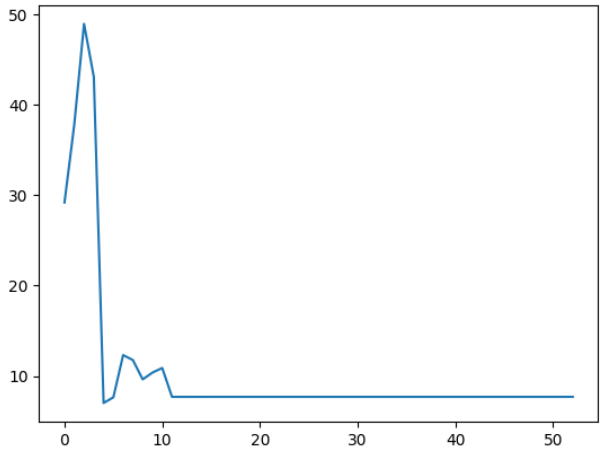

```
Solution found by the accelerated gradient descent algorithms with start point [
1 -1], 400 iterations and step size 0.001 is [ 0.74587646 -0.74587646]
function value: -0.49365348182761104
In this case, different starting points and increased iterations might lead to
finding a different local minimum.
============================================================

Question 4
```



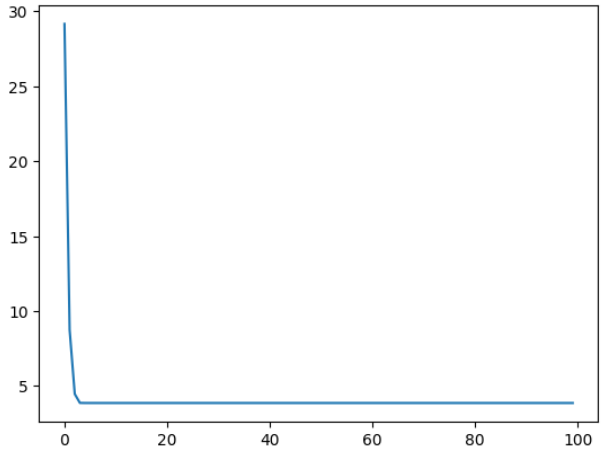

```
Solution of Newton method of Bohachevsky function with start point [-5  2] and
100 iterations is [-0.36070816 -0.97787629]
```



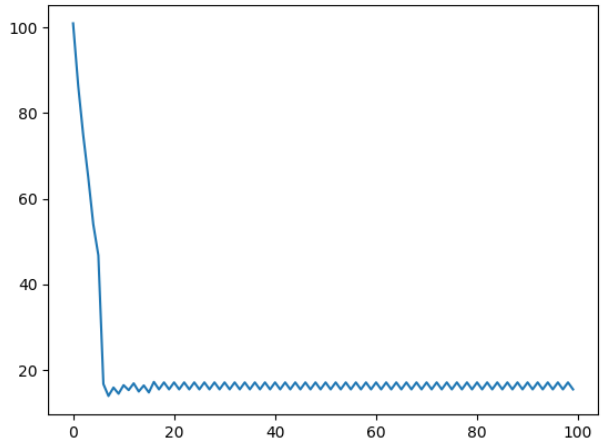

```
Solution of Newton method with line search of Bohachevsky function with start
point [-5  2] and 14 iterations is [-0.61861207 -1.45102235]
```



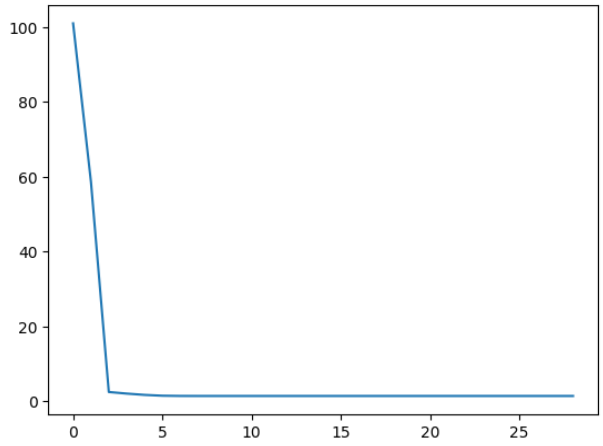

```
Solution of Newton method of Bohachevsky function with start point [-4.9  2.1]
and 53 iterations is [1.22249478 2.34578393]
```




```
Solution of Newton method with line search of Bohachevsky function with start
point [-4.9  2.1] and 100 iterations is [5.35200912e-09 1.93030743e+00]
```




```
Solution of Newton method of Bohachevsky function with start point [-10   1] and
100 iterations is [-3.72041569 -0.94676734]
```




```
Solution of Newton method with line search of Bohachevsky function with start
point [-10   1] and 29 iterations is [ 0.61861207 -0.96851918]
```

```
============================================================

Question 5
The solution of linear conjugate gradient algorithm with start point [0 0]
matrix [[ 3 -1]
 [-1  3]] and vector [0 1] is [0.125 0.375] with 2 iterations.
 The minimum of this function is 6.8125
```