Ahern-Dissertation-AppendixC

November 22, 2015

1 Acquisition

This section details the simulations of the variational learning model. We begin by defining the parameters of the linguistic environment. We then define the other components of the model and simulate individual trajectories.

The linguistic environment is defined by the distribution over grammars, and their relative independent evidence.

```
In [61]: # Set parameters for linguistic environment
         p = 0.5  # Probability of grammar G_2
         a1 = .1  # Independent evidence for G1
         a2 = .5  # Independent evidence for G2
         # Expected probability of G_2 in the limit
         E_p = p*a2/((1-p)*a1 + p*a2)
```

The

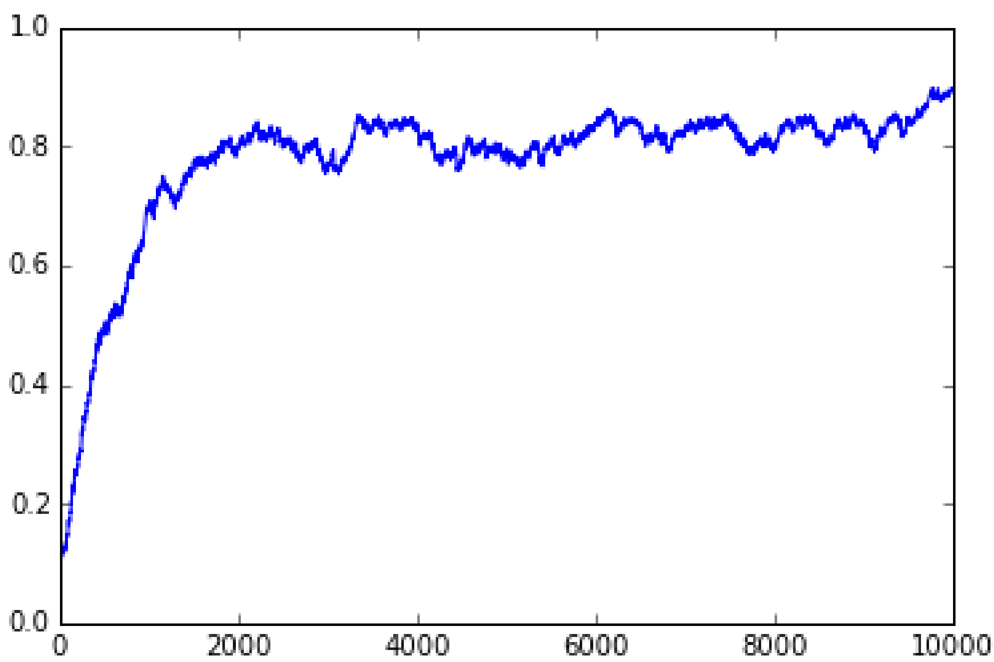
```
In [62]: # Import models
         import numpy as np
         import random
         from scipy import stats

         # Create randomv variable to simulate environment
         xk = np.arange(3) # There are three potential outcomes
         # 1. s_0 : token is comptable with G1 only
         # 2. s_1 : token is compatibile with G1 and G2
         # 3. s_2 : token is compatible with G2 onle
         pk = ((1-p)*a1, (1-p)*(1-a1) + p*(1-a2), p*a2)
         custm = stats.rv_discrete(name='custm', values=(xk, pk))
         # Run simulation for number of steps
         Nturns=10000
         Nagents=2000
         p=np.zeros((Nturns, Nagents))
         # Construct vector to hold probability
         gamma=0.005 # Set learning parameter
         p[0,:] = .1 * np.ones((1, Nagents))
         # Generate token from environment
         s_env = custm.rvs(size=Nturns)
```

```
In [63]: for i in range(Nagents):
             #print s_env
             s_env = custm.rvs(size=Nturns)
             for n in range(1,Nturns):
                 if random.random() < p[n-1, i]:
```

```
                # Learner picks G2
                if s_env[n] == 0:
                    # Token is only compatible with G1
                    # ---> Punish G2
                    p[n, i] = (1-gamma)*p[n-1, i]
                else:
                    # Token in compatbile with G2
                    # ---> Reward G2
                    p[n, i] = p[n-1, i] + (1 - p[n-1, i])*gamma
            else:
                # Learner picks G1
                if s_env[n] == 2:
                    # Token is only compatible with G2
                    # ---> Punish G1
                    p[n, i] = p[n-1, i] + (1 - p[n-1, i])*gamma
                else:
                    # Token is compatible with G1
                    p[n, i] = (1-gamma)*p[n-1, i]
```

In [64]:
```
plt.plot(p[:,4])
plt.ylim(0,1)
plt.show()
```



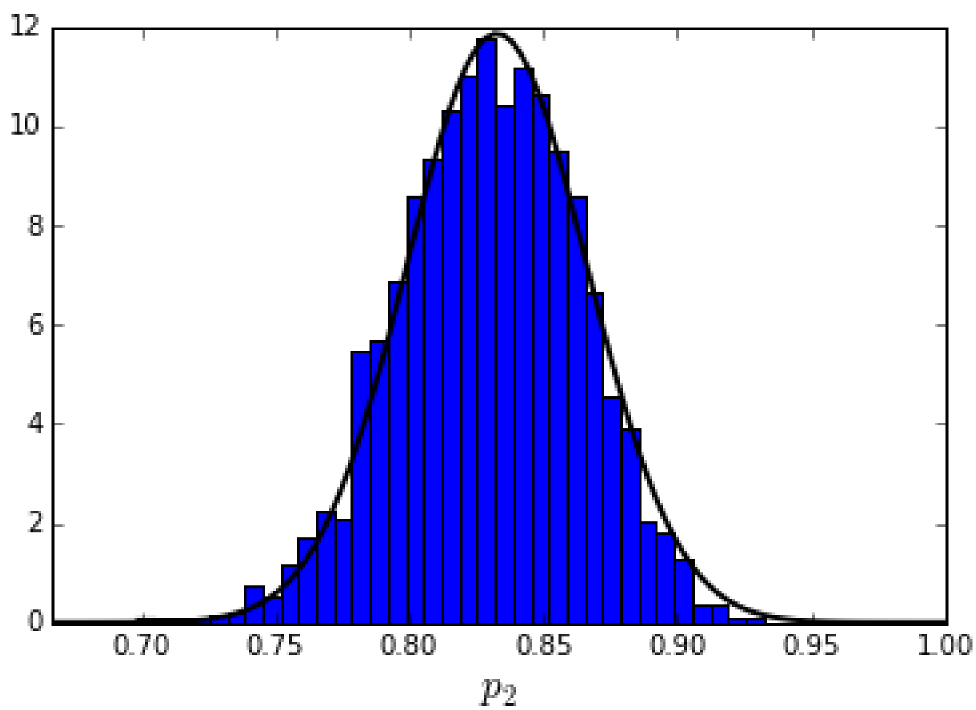
In [65]:
```
final = p[-1,:]
np.arange(min(final), max(final), gamma)
```

Out[65]:
```
array([ 0.70175164,  0.70675164,  0.71175164,  0.71675164,  0.72175164,
        0.72675164,  0.73175164,  0.73675164,  0.74175164,  0.74675164,
        0.75175164,  0.75675164,  0.76175164,  0.76675164,  0.77175164,
        0.77675164,  0.78175164,  0.78675164,  0.79175164,  0.79675164,
```

```
        0.80175164,  0.80675164,  0.81175164,  0.81675164,  0.82175164,
        0.82675164,  0.83175164,  0.83675164,  0.84175164,  0.84675164,
        0.85175164,  0.85675164,  0.86175164,  0.86675164,  0.87175164,
        0.87675164,  0.88175164,  0.88675164,  0.89175164,  0.89675164,
        0.90175164,  0.90675164,  0.91175164,  0.91675164,  0.92175164,
        0.92675164,  0.93175164])
```

```
In [66]: from scipy.stats import norm
```

```
In [96]: final = p[-1,:]
         plt.hist(final, bins=35, normed=True, align='left')
         xmin, xmax = plt.xlim(2*E_p-1, 1)
         mu, std = norm.fit(final)
         x = np.linspace(xmin, xmax, 100)
         pr = norm.pdf(x, mu, std)
         plt.plot(x, pr, 'k', linewidth=2)
         #
         plt.xlabel(r'$p_2$', fontsize=15)
         plt.savefig("lrp-dist.png", format='png', dpi=1000)
         plt.show()
```

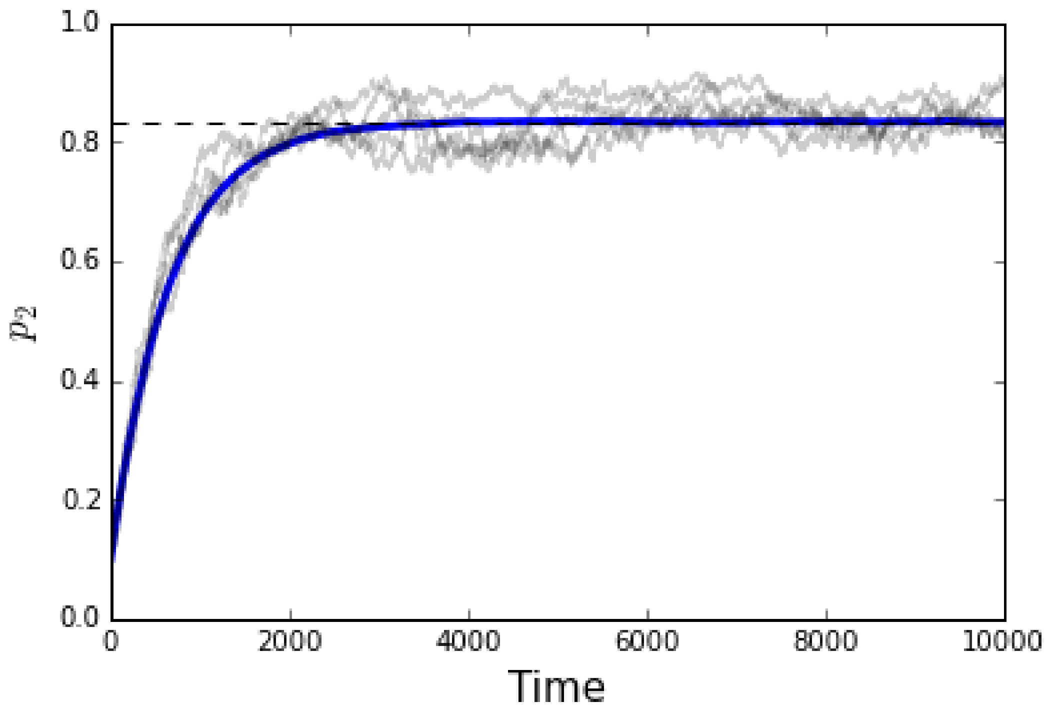



```
In [95]: p_mean = np.mean(p, axis=1)
         final.mean()
```

```
Out[95]: 0.8326857077507871
```

```
In [6]: import matplotlib.pyplot as plt
        %matplotlib inline
```

In [92]:
```
hfont = {'fontname':'Helvetica'}
plt.plot(p_mean, linewidth=3, color='b')
for i in range(5):
    plt.plot(p[:,i], color='k', alpha=.2)
plt.ylim(0,1)
plt.axhline(y=E_p, ls='dashed', color='k')
plt.ylabel(r"$p_2$", fontsize=15, **hfont)
plt.xlabel("Time", fontsize=15, **hfont)
plt.savefig("lrp-learning.png", format='png', dpi=1000)
plt.show()
```



2 Dynamics

In [5]:
```
from sympy import symbols, diff
```

In [6]:
```
p, r, s = symbols('p r s')
lrp_diff = p*(1-p)*s/(1 - s*(1-p))

print diff(lrp_diff, p).subs(p,0)
print diff(lrp_diff, p).subs(p,1)
```

```
s/(-s + 1)
-s
```

The first model we want to fit is derived from Yang's (2002) variational model of syntactic learning. The form is logistic-like, where r acts as the growth parameter. In reality, it represents the ratio of evidence a child learner has for two competing grammars. In this case, these are the grammars underlying the forms of negation "ne" and "ne... not".

$$\dot{y} = y(1-y)\frac{1-r}{(1-y)r+y} \tag{1}$$

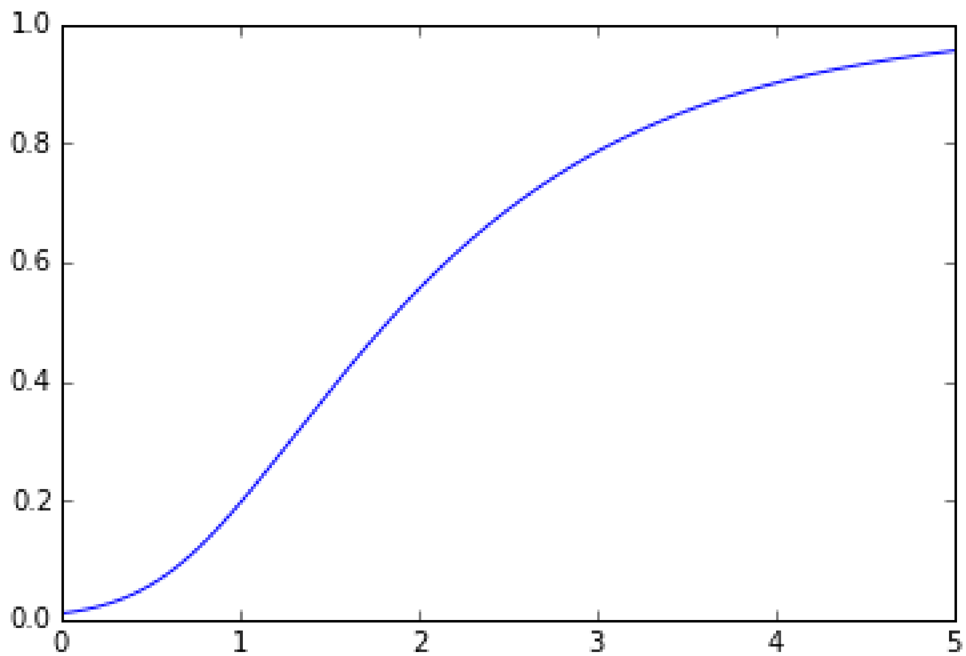
```
In [97]: def y_diff(y, t, params):
             s = params[0] # unpack parameters
             return y*(1-y)*s/(1 - s*(1-y))
```

We can solve this equation numerically for any initial condition y_0 and rate of growth r.

```
In [98]: from scipy.integrate import odeint
```

```
In [99]: import matplotlib.pyplot as plt
         %matplotlib inline
```

```
In [100]: t = np.linspace(0, 5, num=376)
          y_sol = odeint(y_diff, 0.01, t, args=([.8],))
          plt.plot(t, y_sol)
          plt.ylim(0,1)
          plt.show()
```



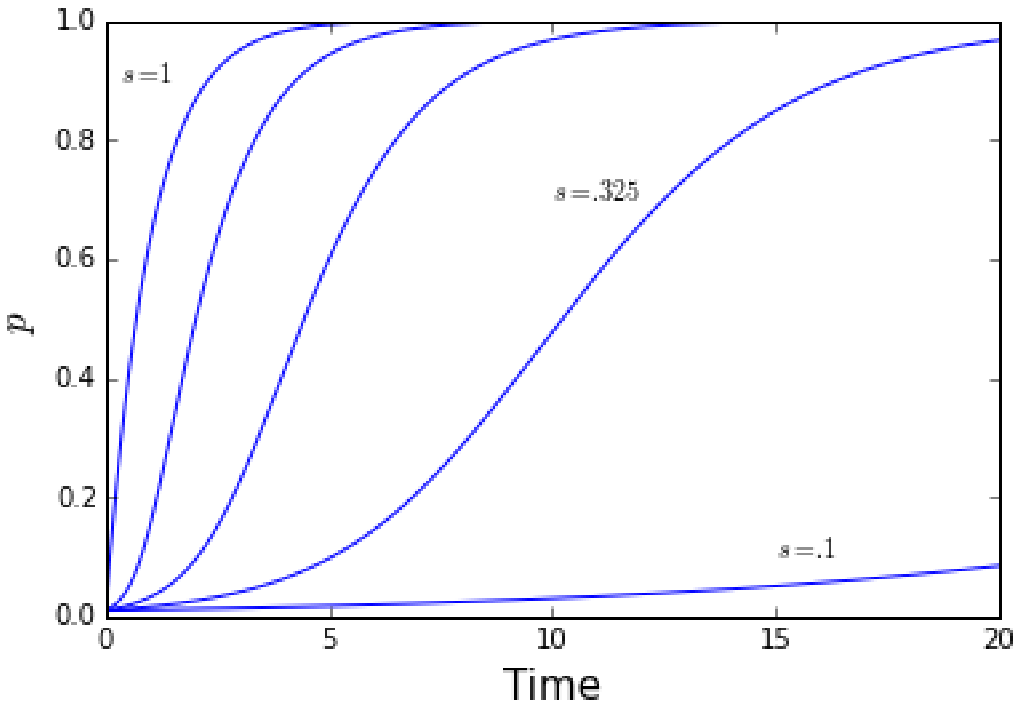
In fact, we can vary r and see the result. As $r \to 0$ the curve gets steeper and as $r \to 1$ it flattens out.

```
In [116]: np.linspace(.1,1, num=5)
```

```
Out[116]: array([ 0.1  ,  0.325,  0.55 ,  0.775,  1.   ])
```

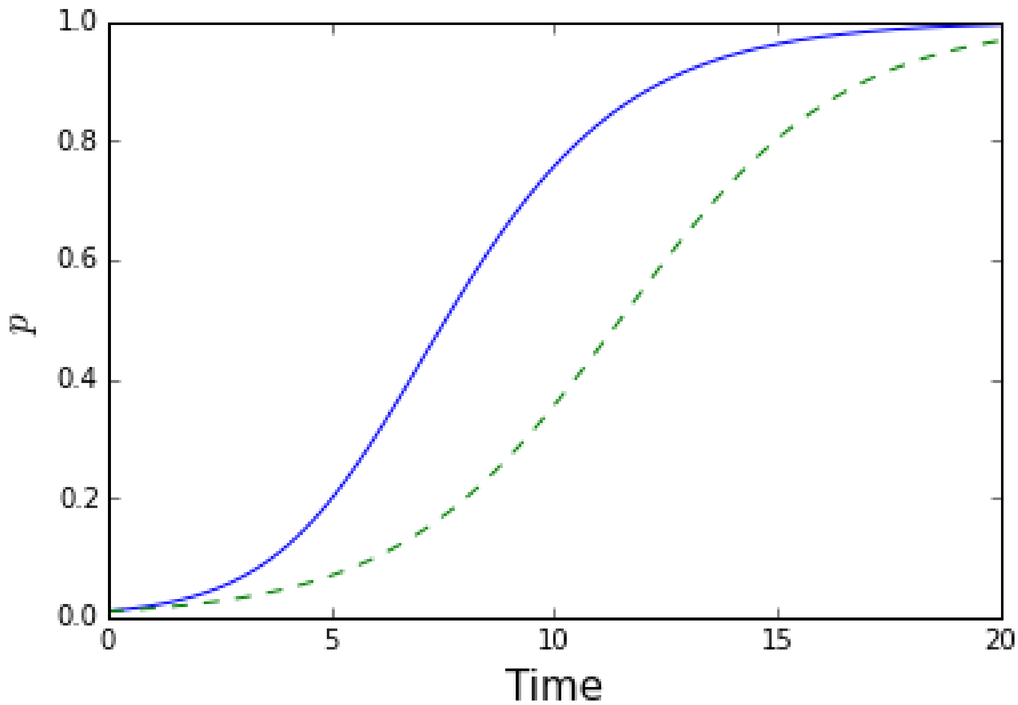
```
In [138]: hfont = {'fontname':'Helvetica'}
          t = np.linspace(0, 20, num=376)
          for i in np.linspace(.1,1, num=5):#, endpoint=False):
              y_sol = odeint(y_diff, .01, t, args=([i],))
              plt.plot(t, y_sol, 'b')
```

```
plt.ylim(0,1)
plt.xlabel('Time', fontsize=15, **hfont)
plt.ylabel(r'$p$', fontsize=15, **hfont)
plt.text(.3, .9, r'$s=1$')
plt.text(10, .7, r'$s=.325$')
plt.text(15, .1, r'$s=.1$')
plt.savefig("lrp-gain.eps", format='eps', dpi=1000)
plt.show()
```




```
In [125]: def log_diff(y, t, params):
              s = params[0] # unpack parameters
              return y*(1-y)*s

In [142]: hfont = {'fontname':'Helvetica'}
          t = np.linspace(0, 20, num=376)
          s = .4
          y1_sol = odeint(y_diff, .01, t, args=([s],))
          y2_sol = odeint(log_diff, .01, t, args=([s],))
          plt.plot(t, y1_sol)
          plt.plot(t, y2_sol, 'g--')
          plt.ylim(0,1)
          plt.xlabel('Time', fontsize=15, **hfont)
          plt.ylabel(r'$p$', fontsize=15, **hfont)
          plt.savefig("lrp-log.png", format='png', dpi=1000)
          plt.show()
```



3 Modeling

Now that we have defined the differential equation and know that we can solve it, we'd like to fit it to the data we described above. We want to fit three parameters:

- y_0 : the initial state of the differential equation
- r : the growth rate of the differentital equation
- t : the time scale of the equation

While, the first two parameters have clear interpretations, the last is equally as important. Since the solution to the differential equation evolves along an arbitrary time scale. Fitting the solution to data requires a specification of how those arbitrary units map to years, months, or days. In fact, we can fit this parameter to the data along with the other ones, but we have to be careful that there aren't multiple solutions.

```
In [13]: import pandas as pd
         import numpy as np

In [14]: df1 = pd.read_csv("./first_curve_data.csv")

In [15]: def error_function1(params):
             """Find the SSE of model"""
             # Unpack the parameters
             y0 = params[0]
             s = params[1]
             #k = params[2]
             # Solve y_diff given y0, r, k
             y_sol = odeint(y_diff, y0, np.linspace(0, 75, num=376), args=([s],))
```

```
    # Get data indices
    indices = np.subtract(df1.year.unique(), min(df1.year))
    # Get count of tokens for indices
    counts = df1.groupby('year').size() #.loc[index + 1125]
    # Get count of value==1 for indices
    values = df1.groupby('year').aggregate(np.sum)
    # Initialize RSS value
    RSS = 0
    # Loop over indices
    for index in indices:
        zero_count = counts.loc[index + min(df1.year)] - values.loc[index + min(df1.year), 'va
        one_count = values.loc[index + min(df1.year), 'value']
        #
        RSS += zero_count*(0 - y_sol[index])**2 + one_count*(1 - y_sol[index])**2
    return RSS
```

In [16]:
```
df2 = pd.read_csv("./second_curve_data.csv")
df2 = df2[df2.year >= 1300]
```

In [17]:
```
def error_function2(params):
    """Find the RSS of model"""
    # Unpack the parameters
    y0 = params[0]
    r = params[1]
    #k = params[2]
    # Solve y_diff given y0, r, k
    y_sol = odeint(y_diff, y0, np.linspace(0, 40, num=201), args=([r],))
    # Get data indices
    indices = np.subtract(df2.year.unique(), min(df2.year))
    # Get count of tokens for indices
    counts = df2.groupby('year').size() #.loc[index + 1125]
    # Get count of value==1 for indices
    values = df2.groupby('year').aggregate(np.sum)
    # Initialize RSS value
    RSS = 0
    # Loop over indices
    for index in indices:
        zero_count = counts.loc[index + min(df2.year)] - values.loc[index + min(df2.year), 'va
        one_count = values.loc[index + min(df2.year), 'value']
        #
        RSS += zero_count*(0 - y_sol[index])**2 + one_count*(1 - y_sol[index])**2
    return RSS
```

We want to minimize the error function within certain bounds. That is, we want $y_0 \in [0, 1]$, $r \in [0, 1]$, and probably $t < 375$. We'll start off with an initial guess that roughly matches those parameters. Note that this can take a minute or two, so be patient.

In [18]:
```
from scipy.optimize import minimize
```

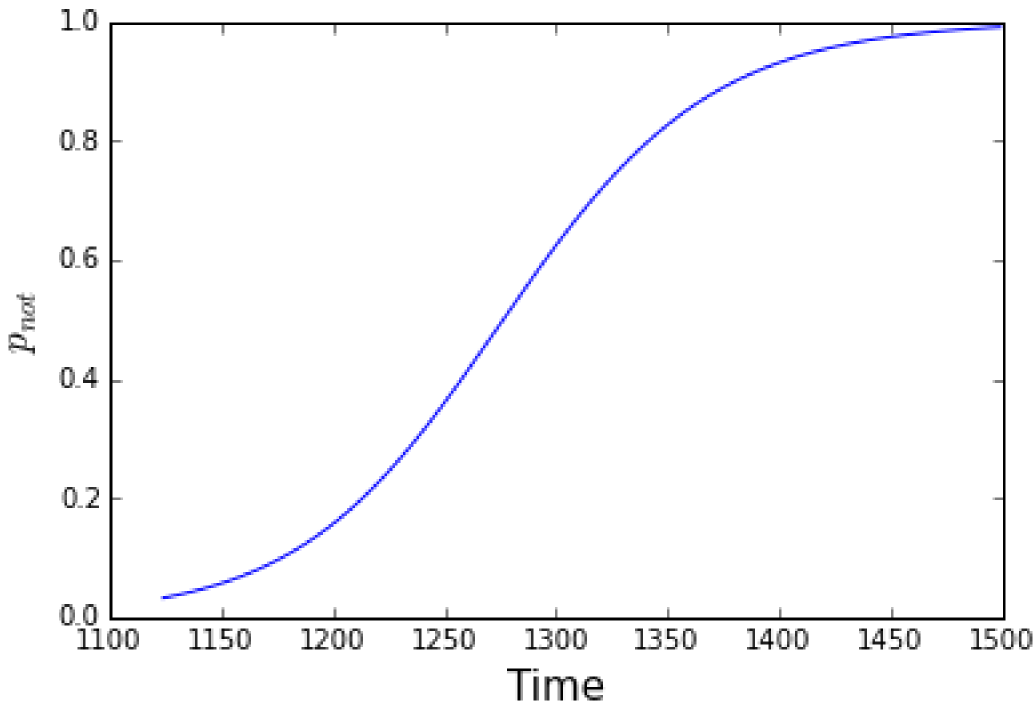
In [19]:
```
# First transition
guess = np.array([.01, .1])
bnds = ((0, 1), (0, 1))
res1 = minimize(error_function1, x0=guess, method="SLSQP", bounds=bnds)
res1
```

```
Out[19]:   status: 0
          success: True
             njev: 13
             nfev: 62
              fun: array([ 472.32994975])
                x: array([ 0.03137994,  0.10291529])
          message: 'Optimization terminated successfully.'
              jac: array([ 0.01625061,  0.02018738,  0.        ])
              nit: 13
```

```
In [21]: # First transition
         guess = np.array([.01, .1])
         bnds = ((0, 1), (0, 1))
         res1 = minimize(error_function1, x0=guess, method="COBYLA", bounds=bnds)
         res1
```

```
Out[21]:   status: 1
             nfev: 112
            maxcv: 0.0
          success: True
              fun: 472.33364035784126
                x: array([ 0.03193982,  0.10262013])
          message: 'Optimization terminated successfully.'
```

```
In [22]: # Plot of fitted parametrs
         y1_sol = odeint(y_diff, res1.x[0], np.linspace(0, 75, num=376), args=([res1.x[1]],))
         plt.plot(range(1124,1500), y1_sol)
         plt.ylim(0,1)
         plt.xlim(1100, 1500)
         plt.xlabel('Time', fontsize=15, **hfont)
         plt.ylabel(r'$p_{not}$', fontsize=15, **hfont)
         plt.savefig("lrp-first.eps", format='eps', dpi=1000)
         plt.show()
```



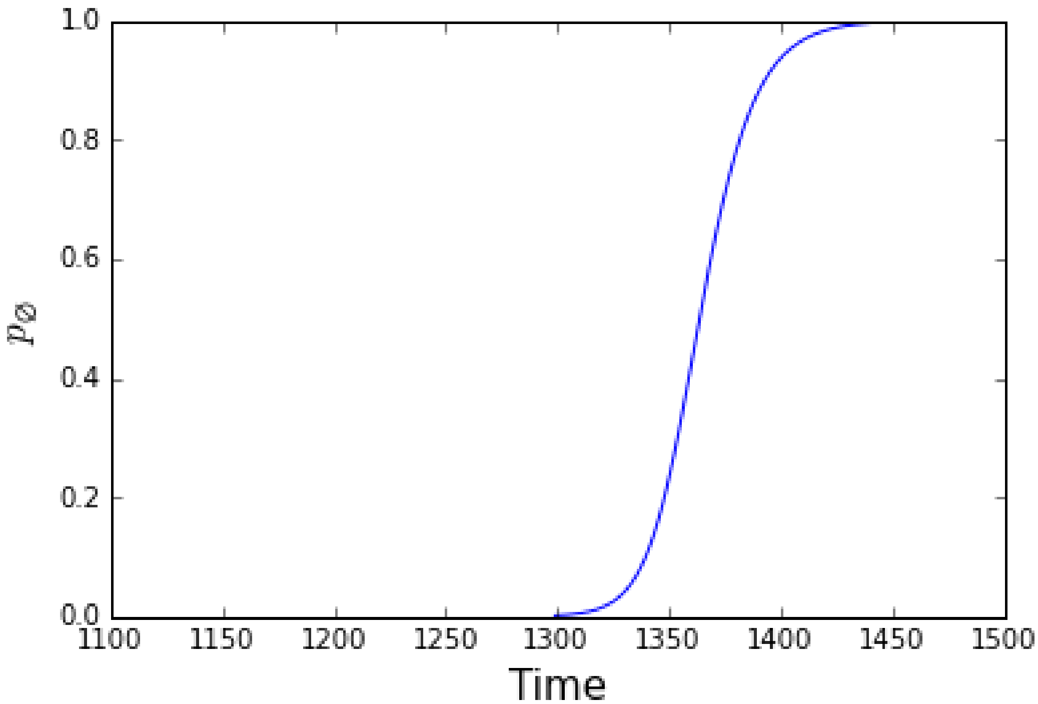
In [23]:
```
# Second transition
guess = np.array([.01, .5])
bnds = ((0, 1), (0, 1))
res2 = minimize(error_function2, x0=guess, method="SLSQP", bounds=bnds)
res2
```

Out[23]:
```
  status: 0
 success: True
    njev: 23
    nfev: 109
     fun: array([ 318.1133742])
       x: array([ 0.00174836,  0.34127753])
 message: 'Optimization terminated successfully.'
     jac: array([-1.45677567, -0.08760452,  0.        ])
     nit: 25
```

In [28]:
```
# Second transition
guess = np.array([.01, .1])
bnds = ((0, 1), (0, 1))
res2 = minimize(error_function2, x0=guess, method="COBYLA", bounds=bnds)
res2
```

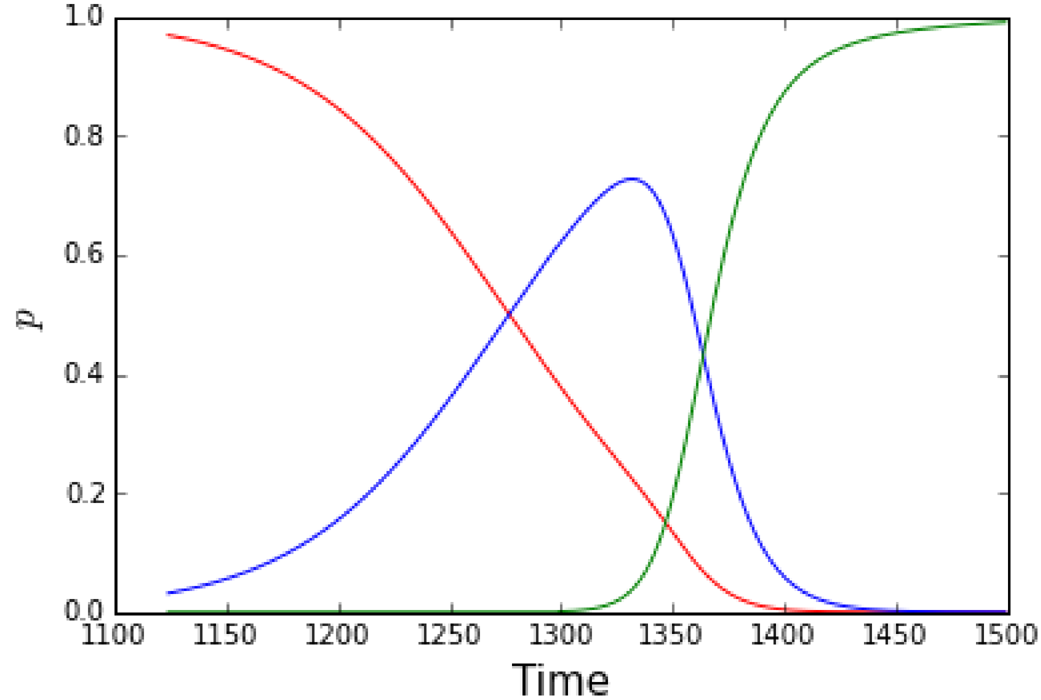
Out[28]:
```
  status: 1
    nfev: 867
   maxcv: 0.0
 success: True
     fun: 2168818.4800749901
       x: array([ -5.35554894e-06,   8.16376378e-01])
 message: 'Optimization terminated successfully.'
```

In [270]:
```
# Plot of fitted parametrs
y2_sol = odeint(y_diff, res2.x[0], np.linspace(0, 40, num=201), args=([res2.x[1]],))
plt.plot(range(1299,1500), y2_sol)
plt.ylim(0,1)
#plt.ylim(-.1,1.1)
plt.xlim(1100,1500)
plt.xlabel('Time', fontsize=15, **hfont)
plt.ylabel(r'$p_\varnothing$', fontsize=15, **hfont)
plt.savefig("lrp-second.eps", format='eps', dpi=1000)
plt.show()
```



In [271]:
```
p_spec = y1_sol
p_head = np.append(np.zeros((376 - 201, 1)), y2_sol).reshape(376,1)

plt.plot(range(1124,1500), np.multiply((1-p_spec), (1-p_head)), 'r')
plt.plot(range(1124,1500), np.multiply(p_spec, (1-p_head)), 'b')
plt.plot(range(1124,1500), np.multiply(p_spec, p_head), 'g')
plt.xlim(1100,1500)
plt.xlabel('Time', fontsize=15, **hfont)
plt.ylabel(r'$p$', fontsize=15, **hfont)
plt.savefig("lrp-combined.eps", format='eps', dpi=1000)
plt.show()
```


1.0
0.8
0.6
0.4
0.2
0.0
p
1100
1150
1200
1250
1300
1350
1400
1450
1500
Time