Name:
Date:
Counting worksheet

1. $12 * 7 = 84$
2. $8 * 6 = 48$
3. $16 * 16 = 256$
4. $\frac{92}{4} = 23$
5. $\frac{56}{7} = 8$
6. $2^0 = 1$
7. $2^1 = 2$
8. $2^5 = 32$
9. $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
10. $2^8 = 256$ (patterns a byte can take on)
11. $2^{10} = 1024$
12. $2^3 * 2^5 = 2^8$
13. $3^4 = (3^2)^2 = 9^2 = 81$
14. $6^2 * 6^5 = 6^7$
15. $6^2 * 7^3 = 36 * 343 = 12,348$
16. $\frac{2^{16}}{2^{12}} = 2^4 = 16$
17. $\sqrt[3]{x^4} = x^{\frac{4}{3}}$
18. $(3^2)^4 = 3^8 = (3^4)^2$
19. $\frac{3}{4} + \frac{1}{3} = \frac{13}{12}$
20. $\frac{2}{3} * \frac{3}{5} = \frac{2}{5}$
21. $\frac{3}{4} - \frac{2}{3} = \frac{1}{12}$

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$ pronounced as 'n choose k', these are combinations

1. $\binom{2}{1} = 2$
2. $\binom{2}{2} = 1$
3. $\binom{4}{4} = 1$
4. $\binom{4}{3} = 4$
5. $\binom{4}{2} = 6$
6. $\binom{4}{1} = 4$
7. $\binom{4}{0} = 1$
8. $\binom{7}{2} = 21$
9. $\binom{7}{5} = 21$

${}^nP_k = \frac{n!}{(n-k)!}$ pronounced as 'k-permutations of n'
Note that $\binom{n}{k} = \frac{{}^nP_k}{{}^kP_k}$

1. ${}^2P_1 = 2$
2. ${}^2P_2 = 2$
3. ${}^4P_4 = 24$
4. ${}^4P_3 = 4! = 24$
5. ${}^4P_2 = 12$
6. ${}^4P_1 = 4$
7. ${}^4P_0 = 1$
8. ${}^7P_2 = 42$
9. ${}^7P_5 = \frac{7!}{2!} = 2520$

Product rule

1. A restaurant has 1 drink option and 2 food options. How many drink-food combinations are there? 2
2. A restaurant has 3 drink options and 4 food options. How many drink-food combinations are there? 12

Sum rule

1. A restaurant has 1 drink option and 2 food options. You want to get either a drink or a food item. How many different ways might you order? 3

2. A restaurant has 3 drink options and 4 food options. You want to get either a drink or a food item. How many different ways might you order? 7

3. A restaurant has 5 predefined meals, 3 drink options and 4 food options. You want to get either a predefined meal, or a drink and a food option. How many different ways might you order? 5 + 12 = 17

4. I have 8 shirts, one of which is gray, one of the others is red. I have 3 pairs of pants, one of which is gray. How many shirt-pants combinations can I make? 24

5. I will either wear a red shirt or a gray shirt. How many shirt-pants combinations can I make? 2 * 3 = 6

6. I will either wear a red shirt or gray pants. How many shirt-pants combinations can I make? 3 + 8 - 1 = 10

7. I will wear a gray article of clothing. How many shirt-pants combinations can I make? 3 + 8 - 1 = 10

In vs Out

1. How many ways can we choose a committee of 5 people from our class of 30? $\binom{30}{5} = \frac{30!}{5!*25!}$

2. How many ways can we choose 25 people not to be on the committee of 5 people from our class of 30? $\binom{30}{25} = \binom{30}{30-25} = \binom{30}{5}$

3. How many different strings of 2 digits are there? $10 * 10 = 100$

4. How many strings of 2 digits have a nine in them? (strings where first digit is a nine plus strings where second digit is a nine minus strings where first and second digits are nine) $10 + 10 - 1 = 19$

5. How many strings of 2 digits do not have a nine in them? $100 - 19 = 81$

6. License plates have 3 letters followed by 3 numbers. How many different license plates are there? $26^3 * 10^3$

7. How many different license plates are there that include a nine? First, how many three digit strings include a nine? Let's do it the easy way! All three digit strings = 10^3, three digit strings without any nines = 9^3. So three digit strings with at least 1 nine = $10^3 - 9^3 = 1000 - 729 = 271$!! Can we do it without this trick? Yes, but it sucks. Read on.

   We can have a nine and then a two digit string without any nines: 1 * 81 = 81. Or we can have any digit and then a two digit string with at least one nine: 10 * 19. Together this is 271. Another way: we can have 1 nine, 2 nines, or 3 nines. There is one way to have exactly 3 nine. If you have exactly 2 nines, the other number can be zero through eight. Then you can place this other number in one of three places in the string: 9 * 3 = 27. If you have exactly 1 nine, then you have two other numbers which are zero through eight. You can put the nine in front of these two, in between these two, or after these two: (9 * 9) * 3 = 243. How many strings have at least 1 nine = strings with 1 nine + strings with 2 nines + strings with 3 nines = 243 + 27 + 1 = 271.

   Finally!! How many plates include a nine? $26^3 * 271$

8. How many different license plates are there that don't include a nine? $26^3 * 10^3 - 26^3 * 271$

9. How many different license plates are there that don't have any letter appear more than once? ${}^{26}P_3 * 10^3 = 26 * 25 * 24 * 10^3$

10. How many different license plates are there that don't have any letter appear more than once and don't have any digit appear more than once? ${}^{26}P_3 * {}^{10}P_3 = 26 * 25 * 24 * 10 * 9 * 8$

11. How many different license plates are there that don't have any letter appear more than once or

don't have any digit appear more than once?
${}^{26}P_3 * 10^3 + 26^3 * {}^{10}P_3 - {}^{26}P_3 * {}^{10}P_3$