Proof 1

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \dots + \frac{1}{\sqrt{n-1}+\sqrt{n}} = \sqrt{n} - 1$$

Let: $F(n) = \frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \dots + \frac{1}{\sqrt{n-1}+\sqrt{n}}$

$P(n)$: $F(n) = \sqrt{n} - 1$

We will prove $P(n)$ for $n > 1$

Base case:

$P(2)$: $\frac{1}{\sqrt{1}+\sqrt{2}} = \sqrt{2} - 1$

I'm not sure how to show this but it seems to work

Inductive step: show $P(k) \Rightarrow P(k+1)$ for $k > 1$

Inductive hypothesis

assume $P(k)$: $F(k) = \sqrt{k} - 1$

show $P(k+1)$: $F(k+1) = \sqrt{k+1} - 1$

$$F(k+1) = \frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \dots + \frac{1}{\sqrt{k-1}+\sqrt{k}} + \frac{1}{\sqrt{k}+\sqrt{k+1}}$$

$$F(k+1) = F(k) + \frac{1}{\sqrt{k}+\sqrt{k+1}}$$

use inductive hypothesis

$$F(k+1) = \sqrt{k} - 1 + \frac{1}{\sqrt{k}+\sqrt{k+1}}$$

$$F(k+1) = \left(\sqrt{k} + \frac{1}{\sqrt{k}+\sqrt{k+1}}\right) - 1$$

This should equal $\sqrt{k+1}$, but I don't know how to show it

$$F(k+1) = \sqrt{k+1} - 1$$

We showed that $P(2)$ and $P(k) \Rightarrow P(k+1)$ for $k > 1$

$\therefore$ $P(n)$ is true for all $n > 1$

Proof 2

$2¢ + 5¢$ for all change 6 cents or higher

$2 \cdot 3 = 6$ $2 + 5 = 7$ $2 \cdot 4 + 5 \cdot 0 = 8$ $2 \cdot 2 + 5 \cdot 1 = 9$

Base Case $k = 6$ $2 \cdot (3) + 5 \cdot 0 = 6$

$P(k)$: $2a + 5b = k$ inductive Hypothesis

$P(k+1)$: $2c + 5d = k+1$ inductive Proof

my two cases are either I have a 5 or I don't when I go from k to k+1. if I don't have a 5 I trade in 2 "2"s for a 5. If I do have a 5 I swap 1 '5' for 3 "2"s

$2(a+3) + 5(b-1) = k+1$

$2a + 6 + 5b - 5 = k+1$

$2a + 5b + 1 = k+1$

using our inductive hypothesis of $2a + 5b = k$

$k+1 = k+1$ for all $k > 6$

Proof 3

$$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\cdots\left(1-\frac{1}{n}\right) = \frac{1}{n}$$

$P(n) = \frac{1}{n}$

Base $1-\frac{1}{2} = \frac{1}{2} = P(2)$

Induction

$P(n+1) = \frac{1}{n+1}$ *L

$P(n+1) = (1-\frac{1}{2})\cdots(1-\frac{1}{n+1})$ *R

$R = P(n) * (1-\frac{1}{n+1})$

$\uparrow$ hypothesis

$R = \frac{1}{n} - \frac{1}{n^2+n} = \frac{1}{n+1}$ *L

$* \, n^2+n$

$n+1-1 = \frac{n^2+n}{n+1} \cdot (n+1)$

$n^2+n = n^2+n$

$R = L$

$\therefore P(n) = \frac{1}{n}$ for $n > 1$

Proof 4

$$1 + 3 + 5 + \dots + (2n-1) = n^2$$

Base $1 = 1$!!

step by 1

$$1 + 3 + 5 + \dots + (2n-1) + (2(n+1)-1) = (n+1)^2$$

substituting

$$n^2 + (2n+1) = (n+1)^2$$

$$n^2 + 2n + 1 = (n+1)^2$$

$$(n+1)^2 = (n+1)^2$$

$P(k) \Rightarrow P(k+1) \; \forall n > 0$

qed

Proof 5

$P(k)$ $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + + \ldots + \frac{1}{(n-1)n} = \frac{n-1}{n}$ $\forall n > 1$

Goal show that $P(k) \Rightarrow P(k+1)$

Base case $n=1$ $\frac{1}{(1-1)1} = \frac{1-1}{1} \Rightarrow \cancel{\frac{1}{0} = \frac{0}{1}}$

Base case $n=2$ $\frac{1}{(2-1)2} = \frac{2-1}{2} \Rightarrow \frac{1}{2} = \frac{1}{2}$

~~Base case $n=3$ $\frac{1}{(3-1)3} = \frac{3-1}{3} \Rightarrow \frac{2}{6} = \frac{2}{3}$~~

$\frac{1}{(2-1)(2)}$

$\frac{1}{2} + \frac{1}{6}$

$\frac{4}{6} = \frac{1}{3}$

$\frac{1}{2} + \frac{1}{6}$

$\frac{3}{6} + \frac{1}{6} = \frac{4}{6}$

$\frac{2}{3}$

inductive hypothysis

$P(k)$: $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \ldots + \frac{1}{(k-1)k} = \frac{k-1}{k}$

inductive step

$P(k+1)$: $\underbrace{\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \ldots \frac{1}{(k-1)k}} + \frac{1}{(k+1-1)(k+1)} = \frac{k+1-1}{k+1}$

using the inductive hypothesis

$$\frac{k-1}{k} + \frac{1}{k(k+1)} = \frac{k}{k+1}$$

$$\frac{1}{k}\left(k-1 + \frac{1}{k+1}\right)$$

$$\frac{1}{k}\left(\frac{(k+1)(k-1)}{(k+1)} + \frac{1}{(k+1)}\right)$$

$$\frac{1}{k}\left(\frac{(k^2 \cancel{-k} \cancel{+k} \cancel{-1})}{(k+1)} + \frac{\cancel{1}}{(k+1)}\right)$$

$$\frac{1}{k}\left(\frac{k^{\cancel{2}}}{(k+1)}\right)$$

$$\frac{k}{k+1} = \frac{k}{k+1}$$

LHS = RHS $P(k) \Rightarrow P(k+1)$ $\forall k > 1$

Proof 6

$$\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

$$\frac{n+1}{2n+3} = \frac{n}{2n+1} + \frac{1}{2n+1}\cdot\frac{1}{2n+3} \quad *(2n+3) \quad \longleftarrow \frac{1}{3}$$ Base

$$n+1 = \frac{n(2n+3)+1}{2n+1} \quad (2n+1)$$

$$2n^2+3n+1 = 2n^2+3n+1$$ Induction

Proof 7

Prove $1\cdot 2 + 2\cdot 3 + 3\cdot 4 + \dots + n(n-1) = \frac{n(n-1)(n+1)}{3}$

Base case: $n=1$

$1\cdot 2 = \frac{1(\overset{0}{\cancel{1-1}})(1+1)}{3}$

$2 = \frac{0}{3}$

~~$2 = 0$~~

$n=2$

$1\cdot 2 = \frac{2(2-1)(2+1)}{3}$

$2 = \frac{2(1)(3)}{3}$

$2 = 6/3$

$2 \stackrel{\checkmark}{=} 2$

Sorry!

Inductive Step: Assume $1\cdot 2 + 2\cdot 3 + 3\cdot 4 + \dots + n(n-1) = \frac{n(n-1)(n+1)}{3}$,

Want to prove $1\cdot 2 + 2\cdot 3 + 3\cdot 4 + \dots + (n+1)((n+1)-1) \stackrel{?}{=} \frac{(n+1)((n+1)-1)((n+1)+1)}{3}$

$\underbrace{1\cdot 2 + 2\cdot 3 + 3\cdot 4 + \dots + n(n-1)}_{I.H.} + (n+1)(n) \stackrel{?}{=} \frac{(n+1)(n)(n+2)}{3}$

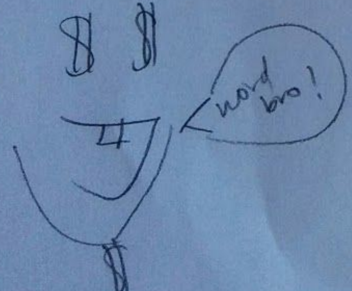
$\frac{n(n-1)(n+1)}{3} + \frac{(n+1)n}{1} \stackrel{?}{=} \frac{(n^2+n)(n+2)}{3}$

$\frac{(n^2-n)(n+1)}{3} + \frac{3(n^2+n)}{3} \stackrel{?}{=} \frac{n^3+2n^2+n^2+2n}{3}$

$\frac{n^3+\cancel{n^2}-\cancel{n^2}-n+3n^2+3n}{3} = \frac{n^3+3n^2+2n}{3}$

$\frac{n^3+3n^2+2n}{3}$

$\checkmark$ $\therefore$ QED



Cha-Ching!

Prove $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + (n-1)n = \frac{n(n-1)(n+1)}{3}$

$n(n^2-1)$

$\frac{n^3-n}{3}$

$1 \cdot 2 = \frac{n^3-n}{3}$

$1 \cdot 2 = \frac{(2)^3-2}{3}$

$1 \cdot 2 = \frac{8-2}{3}$

$2 = 2$ ✓ (cha-CHING!!)

$(n-1)n = \frac{n^3-n}{3}$

I tried!

$3(n-1)n = n^3-n$

$3n = n\cancel{(n-1)}(n+1)$

$3n = n(n+1)$

$3 = n+1$ ... I have no idea anymore...