



TRANSFORMATIONS AND OSGM15

User Guide

Transformations and OSGM15

User guide

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Preface

This user guide is designed to provide an overview of the Ordnance Survey grid transformations in Great Britain (OSTN15™), Northern Ireland and Ireland, and the Ordnance Survey Geoid model (OSGM15™). It gives guidelines and advice to help users understand the information contained in the data, as well as providing detailed technical information and the data format specification. It assumes that users have an understanding of coordinate systems and datums. If you find an error or omission in this user guide, or otherwise wish to make a comment or suggestion as to how we can improve the user guide, please contact us at the address shown below under contact details.

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Use of the transformation models

Within Great Britain coordinates are transformed using the Ordnance Survey National Grid Transformation model (OSTN15). Within the Republic of Ireland and Northern Ireland, the OSi/LPS Polynomial Transformation is used. OSGM15 is used to transform heights throughout the UK and Ireland.

The OSTN15, OSGM15 and OSi/LPS Polynomial transformation models have been created by a consortium comprising Ordnance Survey Limited Great Britain, Ordnance Survey Ireland (OSi), and Land & Property Services (LPS). These organisations are responsible for the official, definitive topographic mapping of their respective countries.

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The documentation is supplied in portable document format (PDF) only. Free Adobe® Reader® software, which displays the specification, incorporates search and zoom facilities and allows you to navigate within. Hyperlinks are used to navigate between associated parts of the specification and to relevant internet resources by clicking on the blue hyperlinks and the table of contents.

Chapter 1 Introduction

Coordinate transformations and the Geoid model

All Ordnance Survey mapping relates to a coordinate reference system. In Great Britain Ordnance Survey coordinates relate to OSGB36™ (the National Grid); within Northern Ireland and the Republic of Ireland the coordinate reference system is either the Irish Grid or the Irish Transverse Mercator (ITM). These reference systems were traditionally realised on the Earth's surface by monumented triangulation stations. The users of mapping products, in both the public and private sectors, have invested in geographical information systems (GIS) and asset management systems based on these grid systems, which have been accepted as de facto national standards.

The National Grid and the Irish Grid are capable of supporting surveying and mapping in UK and Ireland to meet all the requirements of users both now and in the future; however, an increasing number of spatial datasets are available in GNSS (Global Navigation Satellite System, e.g. GPS) compatible coordinate systems, such as ITM. When two or more coordinate datasets are to be integrated, it is essential that each relates to the same coordinate reference system, irrespective of accuracy issues.

In order to relate GNSS-derived positions to Ordnance Survey's mapping, GNSS coordinates need to be converted to Irish Grid or to National Grid, which requires a specialised datum transformation. For this reason Land & Property Services and Ordnance Survey Ireland have developed a polynomial transformation, which is the standard datum transformation for use with the Irish Grid throughout Ireland. Ordnance Survey of Great Britain has developed OSTN15, the standard datum transformation for Great Britain. OSTN15 replaces the previous model OSTN02™.

Ordnance Survey mapping also includes height information that relates to a regional vertical datum. Height information in Great Britain refers to Ordnance Datum Newlyn (ODN), which is established from mean sea level. Although ODN is the national height datum used across mainland Great Britain there are a number of additional datums that are used on the surrounding islands, for example: Lerwick on the Shetland Islands; Stornoway15 on the Outer Hebrides; Douglas02 on the Isle of Man and St Marys on the Scilly Isles. Land & Property Services relates heights within Northern Ireland to Belfast Lough datum, and Ordnance Survey Ireland relates heights within the Republic of Ireland to the Malin Head datum.

Orthometric heights in these systems have in the past been realised via a network of bench marks (BMs). These traditional levelling networks cover the whole of Great Britain, Northern Ireland and the Republic of Ireland. However, heights from precise GNSS surveying are relative to a reference ellipsoid that approximates to the shape of the Earth, but does not coincide with mean sea level. To enable GNSS to be used to determine orthometric heights, the Ordnance Surveys and LPS have jointly developed a model to establish the precise relationship between the two vertical reference surfaces. The resulting geoid model OSGM15 incorporates all the above vertical datums.

OSTN15

Ordnance Survey of Great Britain has developed the horizontal transformation OSTN15. This transformation consists of a 700km by 1,250km grid of translation vectors at 1km resolution. This provides a fit between the GNSS coordinate system European Terrestrial Reference System 1989 (ETRS89) and the OSGB36 National Grid. OSTN15 is in agreement with major triangulation stations at the level of 0.1m root mean square error (RMSE).

OSTN15 has been developed from the national primary, secondary and tertiary triangulation station network. It contains over 3,200 points directly observed by GNSS and more than 1,000 from the original retriangulation observations adjusted on the ETRS89 datum.

Within Great Britain OSTN15, in conjunction with the ETRS89 positions of the OS Net® permanent GNSS stations, is now the official definition of OSGB36 National Grid coordinate system. This means that using OSTN15 with the OS Net Network, surveyors using GNSS have no need to occupy triangulation stations in order to relate GNSS coordinates to National Grid coordinates.

OSi/LPS polynomial transformation

Ordnance Survey Ireland and Land & Property Services recommend the OSi/LPS polynomial transformation for all horizontal transformations in the Republic of Ireland and Northern Ireland. This transformation has been developed in association with the Institute of Engineering Surveying and Space Geodesy, University of Nottingham.

The transformation is based on 183 points evenly distributed throughout Ireland and Northern Ireland. The precise ETRS89 and Irish Grid coordinates of these points are determined by GNSS and terrestrial survey methods, and a one dimensional 3rd order polynomial individually fitted to the latitude and the longitude. The resulting polynomial allows calculation of the coordinate differences at additional points. The polynomial transformation has an accuracy of 0.4 m (95% data).

Ordnance Survey Geoid model: OSGM15

To provide the third dimension of the transformation, the Ordnance Surveys and LPS have, with others, developed the Geoid model OSGM15. The model is derived from precise gravity surveys across UK, Ireland, and surrounding waters; additionally, the model includes data from the global geopotential model (EGM96) and the GRACE gravity mission (GGM02). Alignment to each regional vertical datum is based on precise GNSS observations at bench marks. Within Great Britain these include the Ordnance Survey fundamental bench mark (FBM) network.

The OSGM15 model can be used with GNSS determined positions to establish height above mean sea level, as defined by the respective vertical datums, to the accuracies shown in the table below. The Ordnance Surveys and LPS recommend the use of the Geoid model OSGM15 and the national CORS networks (OS Net in GB) to produce orthometric height compatible with Ordnance Survey mapping. The standard error of the main datums are:

| OSGM15 region | Standard error (m) |
|-------------------------|---------------------------|
| Great Britain | 0.01 |
| Republic of Ireland | 0.02 |
| Northern Ireland | 0.01 |
| Orkney | 0.02 |
| Shetland | 0.02 |
| Outer Hebrides | 0.01 |
| Isle of Man | 0.03 |
| St Marys (Scilly Isles) | 0.01 |

Statistics (rms, m) of the changes in the various datums (OSGM15-OSGM02):

| | |
|-------------------------|-------------|
| Great Britain | 0.026 |
| Republic of Ireland | 0.093 |
| Northern Ireland | 0.018 |
| Orkney | 0.021 |
| Shetland | 0.013 |
| Outer Hebrides | 0.175 |
| Isle of Man | (no change) |
| St Marys (Scilly Isles) | 0.365 |

Ordnance Survey Great Britain intend that OSGM15 is the official definition of the relationship between GNSS ellipsoid heights and orthometric height in Great Britain. In the way that GNSS and the transformation model OSTN15 define the horizontal coordinate system, precise GNSS surveying using the Ordnance Survey Great Britain OS Net Network in conjunction with the Geoid model will become the standard method of determining orthometric height.

ETRS89 explained

The Ordnance Survey transformations and OSGM15 link the Ordnance Survey coordinate reference systems and vertical datums to the GNSS-compatible coordinate system ETRS89. In Europe, ETRS89 is a precise version of the better known WGS84 reference system optimised for use in Europe; however, for most purposes it can be considered equivalent to WGS84.

Specifically, the motion of the European continental plate is not apparent in ETRS89, which allows a fixed relationship to be established between this system and Ordnance Survey mapping coordinate systems.

Additional precise versions of WGS84 are currently in use, notably ITRS (International Terrestrial Reference System); these are not equivalent to ETRS89. The difference between ITRS and ETRS89 is in the order of 0.25 m (in 1999), and growing by 0.025 m per year in UK and Ireland. This effect is only relevant in international scientific applications. For all navigation, mapping, GIS, and engineering applications within the tectonically stable parts of Europe (including UK and Ireland), the term ETRS89 should be taken as synonymous with WGS84.

Benefits

Together, the Ordnance Survey transformations and OSGM15 provide the complete solution to relating GNSS (WGS84) datasets to Ordnance Survey mapping in three dimensions. Used with the OS Net GNSS network, they allow GNSS surveying within the National Grid or the Irish Grid, and to the appropriate vertical datum, without the need to visit any Ordnance Survey traditional control points. OSGM15 additionally brings improvements over the previous model (OSGM02) in particular conformity with the latest coordinate realisation of the National GNSS networks, and local improvements in the Outer Hebrides, Scilly Isles and the west coast of Scotland.

The Outer Hebrides are now represented with a single homogenous datum (Stornoway15) aligned to the Stornoway Tide Gauge Bench Mark. This means that archive orthometric heights of benchmarks on North Uist, Benbecula, South Uist and Barra will no longer be realised by OSGM15 and should not be used unless there is a need to directly compare with previous surveys.

The Scilly Isles datum, St Marys, is now better realised by OSGM15, however, results from the previous model (OSGM02) will show a large discrepancy when compared to OSGM15.

Applications

The Ordnance Survey transformations and OSGM15 are of interest to:

- GNSS surveyors who need to relate their survey to the National Grid or the Irish Grid and/or OD orthometric heights – used with the national CORS networks (OS Net in GB), these products remove the need to visit traditional Ordnance Survey horizontal and vertical control points; and
- GIS, GPS, CAD and navigation system developers who need to integrate GNSS (WGS84) datasets with Ordnance Survey mapping – these products provide the complete solution to these users at all Ordnance Survey mapping scales.

Software for OSTN15, OSGM15 and OSi/LPS polynomial transformation

All the transformations have been coded into a software application – “Grid InQuest II”. The software allows for individual coordinate input and output via a GUI and also batch input/output via text files. A command line interface and dll, along with examples of their use in a variety of programming languages, are also included. Users wishing to incorporate the pre-prepared .dll into other applications should refer to the Grid InQuest II user guide.

Grid InQuest II download packages for Windows® (32 bit and 64 bit), Linux® (32 bit and 64 bit) and OSX® are available from <https://bitbucket.org/PaulFMichell/gridinquestii>

Chapter 2 Data Overview

Basic principles

Specifications of OSTN15 horizontal transformation

| | |
|----------------------|---|
| Transformation: | horizontal datum transformation between ETRS89 and OSGB36 |
| Transformation type: | interpolated square grids of easting and northing shifts |
| Estimation method: | Delaunay triangulation |
| Grid resolution: | 1km |
| Grid interpolation: | bilinear |
| Accuracy: | 0.1m (RMS) with respect to OSGB36 primary, secondary and tertiary triangulation monuments |
| Extent: | 700km east by 1,250km north |

Specifications of OSi/LPS polynomial transformation

| | |
|----------------------|--|
| Transformation: | datum transformation between Irish Grid and ETRS89 |
| Transformation type: | 3rd order polynomial |
| Accuracy: | 0.4 m (95% of data) |
| Extent: | Republic of Ireland and Northern Ireland |

Specifications of OSGM15 Geoid model in Great Britain

| | | | |
|----------------------|--|----------------|---------|
| Transformation: | vertical, ETRS89 ellipsoid to orthometric height | | |
| Transformation type: | interpolated square grid of geoid heights above ETRS89 ellipsoid | | |
| Estimation method: | Spherical Fast Fourier transformation with modified Stokes kernels | | |
| Grid resolution: | 1km (same grid as OSTN15) | | |
| Grid interpolation: | bilinear | | |
| Accuracy: | Area specific: | Mainland GB | 1cm rms |
| | | Orkney | 2cm rms |
| | | Shetland | 2cm rms |
| | | Outer Hebrides | 1cm rms |
| | | Isle of Man | 3cm rms |
| | | Scilly Isles | 1cm rms |

Specifications of OSGM15 Geoid model in Ireland / Northern Ireland

| | | |
|----------------------|---|-----------------------|
| Transformation: | vertical, ETRS89 ellipsoid to orthometric height | |
| Transformation type: | interpolated latitude/longitude graticule of geoid heights above ETRS89 ellipsoid | |
| Estimation method: | Spherical Fast Fourier transformation with modified Stokes kernels | |
| Grid resolution: | 0.013333° Lat by 0.02° Long | |
| Grid interpolation: | bilinear | |
| Accuracy: | Republic of Ireland | 2.3cm standard error, |
| | Northern Ireland | 1.4cm standard error |

Data structure

OSTN15/OSGM15 within Great Britain – format and layout of the data

Within Great Britain OSTN15 and OSGM15 are released as a combined data file using the same 1km grid. This grid covers an area 700km east–west and 1,250km north–south, the origin being the origin of the projected ETRS89 coordinates (see annexe B).

In Great Britain the entire OSTN15 transformation grid is fully populated so as to avoid a transformation “cliff” at the 10km boundary that was part of the previous transformation (OSTN02). **HOWEVER – great caution should be exercised to avoid using OSTN15 in areas where OSGB36 National Grid is not practical or required.**

Each record occupies a separate line with the south-west corner of the grid being the first record in the file. The format of each record is indicated by the following table:

| Record no ¹ | ETRS89 easting ² (m) | ETRS89 northing ³ (m) | OSTN15 east shift ⁴ (m) | OSTN15 north shift ⁵ (m) | OSGM15 Geoid Ht ⁶ (m) | Geoid datum flag ⁷ |
|------------------------|---------------------------------|----------------------------------|------------------------------------|-------------------------------------|----------------------------------|-------------------------------|
| 1 | 0 | 0 | | | | |
| 2 | 1,000 | 0 | | | | |
| 3 | 2,000 | 0 | | | | |
| and so on | and so on | and so on | | | | |
| 701 | 700,000 | 0 | | | | |
| 702 | 0 | 1,000 | | | | |
| 703 | 1,000 | 1,000 | | | | |
| and so on | and so on | and so on | | | | |
| 876 948 | 697,000 | 1,250,000 | | | | |
| 876 949 | 698,000 | 1,250,000 | | | | |
| 876 950 | 699,000 | 1,250,000 | | | | |
| 876 951 | 700,000 | 1,250,000 | | | | |

Where:

- 1 The record number is a sequential number starting at 1 for the origin point (0,0) and finishing at 876 951 for the north-east corner (700 000, 1 250 000).
- 2 ETRS89, National Grid projection, grid intersection easting coordinate in metres.
- 3 ETRS89, National Grid projection, grid intersection northing coordinate in metres.
- 4 The shift in eastings, at the intersection, between ETRS89 and OSGB36 National Grid, that is:
ETRS89 east + OSTN15 east shift = OSGB36 National Grid easting.
- 5 The shift in northings, at the intersection, between ETRS89 and OSGB36 National Grid, that is:
ETRS89 north + OSTN15 north shift = OSGB36 National Grid northing.
- 6 The height of the Geoid above the ETRS89 ellipsoid, in metres, at the intersection, that is:
ETRS89 height – OSGM15 Geoid height = orthometric height above mean sea level.
- 7 The Geoid datum flag is a number representing the local height datum or area of applicability of the transformation. See the table below for details of the datum flag references.

Table 1

| Geoid datum flag | Datum name | Region |
|------------------|-----------------------------|--|
| 1 | Newlyn | UK mainland |
| 2 | St Marys | Scilly Isles |
| 3 | Douglas02 | Isle of Man |
| 4 | Stornoway15 | Outer Hebrides |
| 6 | Lerwick | Shetland Isles |
| 7 | Newlyn (Orkney) | Orkney Isles |
| 15 | Newlyn Offshore | Offshore (from 2km offshore up to transformation boundary) |
| 16 | Outside transformation area | Outside transformation area |

OSGM15 within Ireland and Northern Ireland – format and layout of the data

Within Ireland and Northern Ireland OSGM15 is released as two data files – one for Ireland relating to the Malin Head datum and another for Northern Ireland relating to the Belfast datum.

The corrector surfaces are supplied in a standard ASCII ‘grid file’ format.

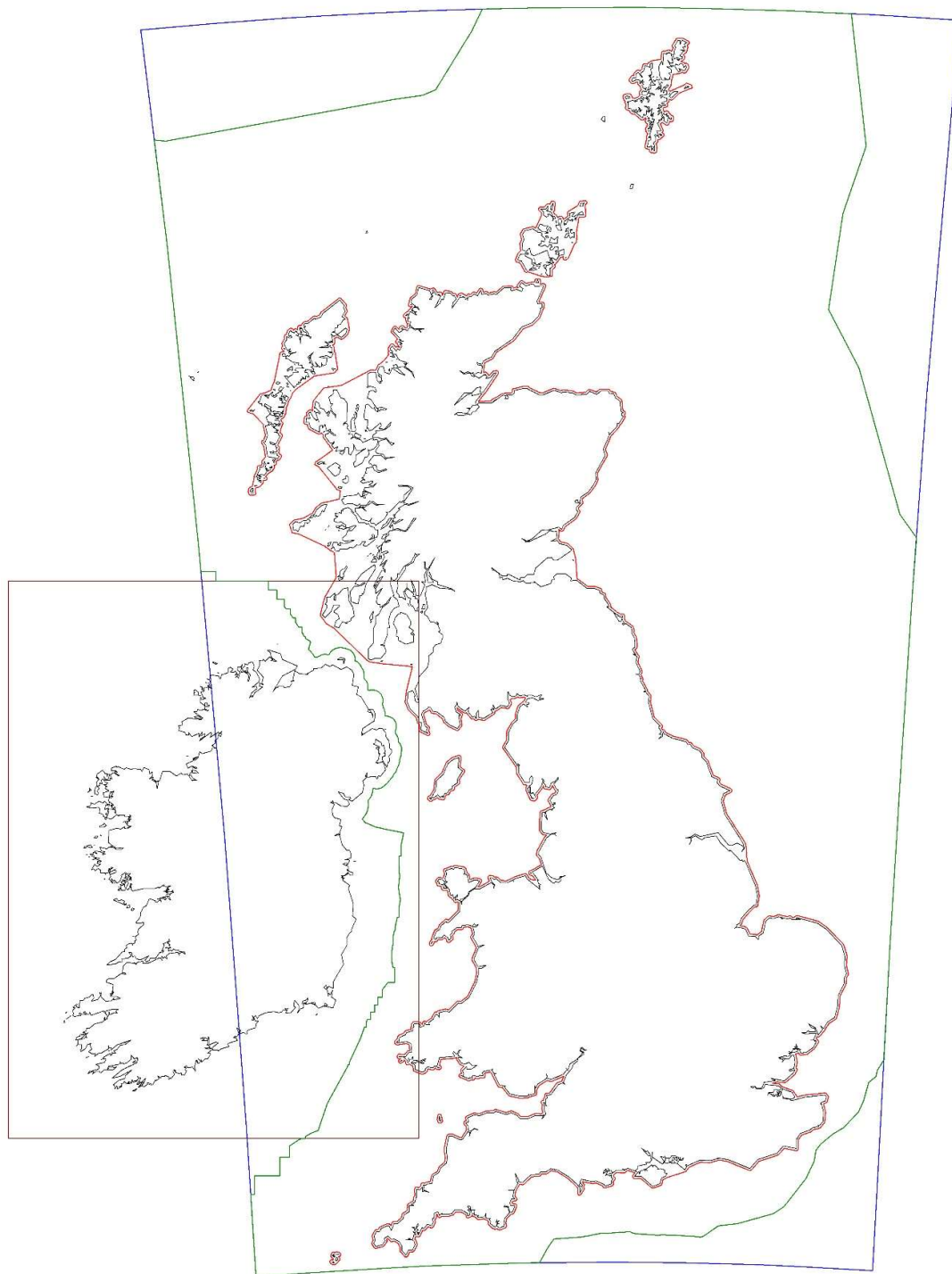
Each file begins with a header in the format $\Phi_{\min}, \Phi_{\max}, \lambda_{\min}, \lambda_{\max}, \sigma\Phi, \sigma\lambda$ (where Φ represents latitude and λ represents longitude).

The data is arranged in paired blocks; a block of 240 data points (30 rows, 8 columns) followed by a block of 86 data points (10 rows, 8 columns + 1 row, 6 columns). Each pair of blocks (240+86 points) gives a line of 326 points along a particular parallel of latitude, at a longitude spacing of 0.02° ($\sigma\lambda$), starting from the west (λ_{\min}) and running eastwards (with the last point at λ_{\max}).

Each line of points is separated by a latitude spacing of 0.013333° ($\sigma\Phi$). The first line is at latitude Φ_{\max} and the last line in the file at latitude Φ_{\min} .

All heights are expressed in metres above the GRS80 ellipsoid. The post spacing in the grid file is of the order of a data point every 1.5 km in both north-south and east-west directions.

Map of transformation extents



- = Extent of OSGM15 Irish grids
- = Extent of OSTN15/OSGM15 grid
- = Extent of OSGM15 “Newlyn Offshore” datum (flag 15)
- = Extents of Great Britain OSGM15 land based datums: Newlyn, St Marys, Douglas02, Stornoway15, Lerwick, Newlyn (Orkney)

Chapter 3 Ordnance Survey transformations and OSGM15 explained

This chapter explains the algorithms that must be coded to implement the Ordnance Survey transformations (OSTN15 and the OSi/LPS polynomial) and OSGM15.

OSTN15 and OSGM15 in Great Britain

Transforming ETRS89 coordinates to OSGB36 National Grid and orthometric heights in GB overview

To transform a 3D ETRS89 coordinate to OSGB36 plane coordinates and an orthometric height, the ETRS89 easting and northing is first obtained using the algorithm, GRS80 ellipsoid parameters and National Grid projection parameters in annexe B. Within the kilometre square where the point falls, a bilinear interpolation is used to obtain the exact transformation value for the point from the values at the four corners of the kilometre square. These values are added to the ETRS89 easting and northing to obtain the OSGB36 values and subtracted from the ETRS89 height to obtain an orthometric height. The inverse transformation (OSGB36 to ETRS89) is accomplished by an iterative procedure.

Calculating which data record to use

To find the record number corresponding to a given ETRS89 easting and northing, use the following algorithm:

```
east_index      = integer_part_of (easting/1,000)
north_index     = integer_part_of (northing/1,000)
record_number   = east_index + (north_index x 701) + 1
```

For example, to find the record for (2,000E, 1,000N):

```
east_index      = integer_part_of (2,000/1,000)
                = 2
north_index     = integer_part_of (1,000/1,000)
                = 1
record_number   = east_index + (north_index x 701) + 1
                = 2 + 1 x 701 + 1
                = 704
```

Procedure for transforming ETRS89 to OSGB36 coordinates and orthometric height

To convert an ETRS89 easting and northing (x, y) obtained using annexe B to a National Grid easting and northing (e, n), the easting and northing shifts from the data file should be added to the x and y coordinates, respectively. The ETRS89 height is transformed to orthometric by subtracting the geoid shift.

The point to be transformed is unlikely to lie exactly on one of the nodes of the grid, so to calculate the shifts at any other points an interpolation is required.

The first stage in the transformation is to identify in which grid cell the ETRS89 point lies. This simply requires an integer division of the (x, y) coordinates, where x and y are in metres:

```
east_index    =    integer_part_of (x/1,000)
north_index   =    integer_part_of (y/1,000)
```

Having located the correct cell, find the values of the shifts and datum flags at the four corners of the cell:

```
se0 se1, se2, se3 for the shifts in eastings,
sn0, sn1, sn2, sn3 for the shifts in northings,
sg0, sg1, sg2, sg3 for the shifts in height
sf0, sf1, sf2, sf3 for the datum flags
```

and the offsets of the point x, y from the bottom left corner of the cell (x₀, y₀) – shown in figure 1 below.

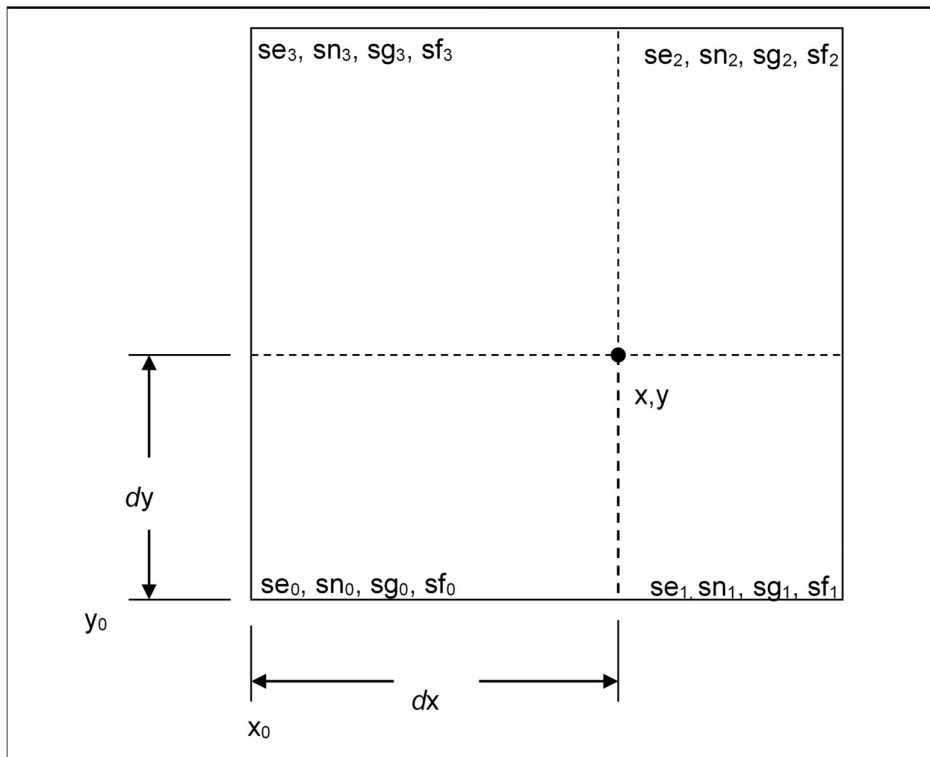


Figure 1 Calculating the OSTN15 se and sn horizontal shifts and the sg vertical shifts for OSGM15.

Shifts for x, y are:

$se_0 = \text{east_shift}(\text{east_index}, \text{north_index})$

NOTE: recall that the record number in the data file will be $(\text{east_index} + (\text{north_index} \times 701) + 1)$

$se_1 = \text{east_shift}(\text{east_index} + 1, \text{north_index})$

$se_2 = \text{east_shift}(\text{east_index} + 1, \text{north_index} + 1)$

$se_3 = \text{east_shift}(\text{east_index}, \text{north_index} + 1)$

$sn_0 = \text{north_shift}(\text{east_index}, \text{north_index})$

$sn_1 = \text{north_shift}(\text{east_index} + 1, \text{north_index})$

$sn_2 = \text{north_shift}(\text{east_index} + 1, \text{north_index} + 1)$

$sn_3 = \text{north_shift}(\text{east_index}, \text{north_index} + 1)$

$sg_0 = \text{height_shift}(\text{east_index}, \text{north_index})$

$sg_1 = \text{height_shift}(\text{east_index} + 1, \text{north_index})$

$sg_2 = \text{height_shift}(\text{east_index} + 1, \text{north_index} + 1)$

$sg_3 = \text{height_shift}(\text{east_index}, \text{north_index} + 1)$

Offsets are:

$dx = x - x_0$

$dy = y - y_0$

The value of the east shift (se), north shift (sn) and height shift at the point x, y is given by the following formulae:

$t = dx / 1\,000$

$u = dy / 1\,000$

$se = (1 - t)(1 - u) se_0 + (t)(1 - u) se_1 + (t)(u) se_2 + (1 - t)(u) se_3$

$sn = (1 - t)(1 - u) sn_0 + (t)(1 - u) sn_1 + (t)(u) sn_2 + (1 - t)(u) sn_3$

$sg = (1 - t)(1 - u) sg_0 + (t)(1 - u) sg_1 + (t)(u) sg_2 + (1 - t)(u) sg_3$

These shifts must then be added to the point x, y to give the National Grid position (e, n):

$e = x + se$

$n = y + sn$

The orthometric height is calculated by subtracting the height shift from the ETRS89 height:

$h = H - sg$

To determine the appropriate datum flag to apply to the point use the following algorithm:

```
if (sf0 = sf1) and (sf1 = sf2) and (sf2 = sf3) # all flags are equal
    then DatumFlag = sf0
else if (t <= 0.5) and (u <= 0.5) # point is in SW quadrant (or dead centre)
    then DatumFlag = sf0
else if (t > 0.5) and (u <= 0.5) # point is in SE quadrant
    then DatumFlag = sf1
else if (t > 0.5) and (u > 0.5) # point is in NE quadrant
    then DatumFlag = sf2
else # if none of the above are true point must be in NW quadrant
    then DatumFlag = sf3
```

Inverse transformation (OSGB36 to ETRS89)

To compute ETRS89 eastings and northings from OSGB36 coordinates, an iterative procedure is required:

Step 1

To start the iteration, compute the ETRS89 to OSGB36 easting, northing and height shifts at the OSGB36 point, using the OSGB36 easting and northing and the method described above.

Subtract these shifts from the OSGB36 coordinates to obtain the first estimate of the ETRS89 easting and northing and height.

Step 2

Use this estimate of the ETRS89 easting and northing to obtain improved values for the easting and northing and height shifts, and subtract these from the OSGB36 coordinates to obtain improved values of the ETRS89 easting and northing and height.

Step 3

If the difference between the first shift value and second shift value is more than 0.0001 metres in either easting or northing, repeat step 2 until this is not the case.

Step 4

If ETRS89 latitude and longitude coordinates are required, obtain these from the ETRS89 easting and northing by the procedure described in annexe C.

The OSi/LPS polynomial transformation

To some extent distortions within traditional triangulation networks are inevitable. Within the triangulation network of the Republic of Ireland and Northern Ireland these distortions are not generally significant; however, regional distortions do occur. A third order polynomial transformation has been developed to model these distortions.

A polynomial expression was fitted to the coordinate differences of a number of points in the different coordinate reference systems. This is a one-dimensional fitting method that is applied to the geographical coordinate, requiring independent parameters to be computed for both latitude and longitude.

In general, the polynomial model can be expressed as:

$$\Delta\phi = \sum \sum A_{ij} (\phi - \phi_m)^i (\lambda - \lambda_m)^j$$

$$\Delta\lambda = \sum \sum B_{ij} (\phi - \phi_m)^i (\lambda - \lambda_m)^j$$

The fully expanded forms of the 3rd order polynomial are as follows:

$$\Delta\phi = [A_{00} + A_{10}U + A_{01}V + A_{11}UV + A_{20}U^2 + A_{02}V^2 + A_{21}U^2V + A_{12}UV^2 + A_{22}U^2V^2 + A_{30}U^3 + A_{03}V^3 + A_{31}U^3V + A_{13}UV^3 + A_{32}U^3V^2 + A_{23}U^2V^3 + A_{33}U^3V^3] / 3600$$

$$\Delta\lambda = [B_{00} + B_{10}U + B_{01}V + B_{11}UV + B_{20}U^2 + B_{02}V^2 + B_{21}U^2V + B_{12}UV^2 + B_{22}U^2V^2 + A_{30}U^3 + B_{03}V^3 + B_{31}U^3V + B_{13}UV^3 + B_{32}U^3V^2 + B_{23}U^2V^3 + B_{33}U^3V^3] / 3600$$

Where A_{ij} and B_{ij} are the computed parameters, and U and V are the normalised coordinates calculated as follows:

$$U = k_0 (\phi - \phi_m) \quad \text{and} \quad V = k_0 (\lambda - \lambda_m)$$

Where ϕ_m and λ_m are the coordinates of the approximate centre of the region. The parameters A_{ij} , B_{ij} , K_0 , ϕ_m and λ_m are given in table 2 below. The transformed geographical coordinates are then obtained as follows:

$$\lambda_{\text{ETRS}} = \lambda_{\text{IG}} + \Delta\lambda \quad \text{and} \quad \phi_{\text{ETRS}} = \phi_{\text{IG}} + \Delta\phi$$

The reverse transformation from ETRS89 to Irish Grid cannot be calculated directly and requires iteration.

Conversions between geographical and grid coordinates are computed using standard Transverse Mercator projection formulae in association with the published Irish Grid parameters.

Table of coefficients for OSi/LPS polynomial transformation

| Coefficient i, j | Latitude (Ai, j) | Longitude (Bi, j) |
|------------------|------------------|-------------------|
| 0, 0 | 0.763 | -2.810 |
| 0, 1 | 0.123 | -4.680 |
| 0, 2 | 0.183 | 0.170 |
| 0, 3 | -0.374 | 2.163 |
| 1, 0 | -4.487 | -0.341 |
| 1, 1 | -0.515 | -0.119 |
| 1, 2 | 0.414 | 3.913 |
| 1, 3 | 13.110 | 18.867 |
| 2, 0 | 0.215 | 1.196 |
| 2, 1 | -0.570 | 4.877 |
| 2, 2 | 5.703 | -27.795 |
| 2, 3 | 113.743 | -284.294 |
| 3, 0 | -0.265 | -0.887 |
| 3, 1 | 2.852 | -46.666 |
| 3, 2 | -61.678 | -95.377 |
| 3, 3 | -265.898 | -853.950 |

Other parameters

$\phi_m = 53.5$

$\lambda_m = -7.7$

$K_0 = 0.1$

OSGM15 in Ireland and Northern Ireland

Orthometric height (h) in the the British Isles can be found by the formula:

$$h = H - N$$

Where H is the GRS80 ellipsoidal height, and N is the geoid undulation (geoid-ellipsoid separation).

Please note: some publications use the notations of h and H the other way round.

The Ireland and Northern Ireland grid files are in geodetic format and when projected to ITM they have a post spacing of the order of a data point every 1.5km in both north-south and east-west directions. (See the 'OSGM15 within Ireland and Northern Ireland' section of Chapter 2).

A 1km ITM grid needs to be produced via interpolation from the original 1.5km grid before following the steps below. Similar to the Ordnance Survey (Great Britain) OSTN15 transformation, the first stage in calculating the geoid undulation is to identify in which grid cell the ETRS89 point lies.

To identify the appropriate grid cell in the Northern Ireland dataset, use the following formulae:

$$east_index = \left(integer_part_of \left(\frac{x}{1000} \right) \right) - 550$$

$$north_index = \left(integer_part_of \left(\frac{y}{1000} \right) \right) - 800$$

$$record_number = east_index + (north_index \times 251) + 1$$

To identify the appropriate grid cell and record numbers in the Republic of Ireland dataset, use:

$$east_index = \left(integer_part_of \left(\frac{x}{1000} \right) \right) - 400$$

$$north_index = \left(integer_part_of \left(\frac{y}{1000} \right) \right) - 500$$

$$record_number = east_index + (north_index \times 351) + 1$$

For both Irish datasets the ETRS89 eastings (x) and ETRS89 northings (y) must be computed using the GRS80 ellipsoid and ITM projection (see Annexe B).

Having located the correct cell, find the values of the geoid undulations at the four corners of the cell (sg_0, sg_1, sg_2, sg_3) and the offsets of the point (dx, dy) from the bottom left corner of the cell (x_0, y_0)

The value of the geoid undulation at point x, y is given as follows:

$$N = (1 - t)(1 - u)sg_0 + (t)(1 - u)sg_1 + (t)(u)sg_2 + (1 - t)(u)sg_3$$

Where: $t = \frac{dx}{1000}$ and $u = \frac{dy}{1000}$

The resulting geoid undulation is subtracted from the ellipsoidal height (H) to give orthometric height (h).

Chapter 4 Quality statement

Coverage

OSTN15 covers Great Britain and the Isle of Man. The OSi/LPS polynomial transformation covers the Republic of Ireland and Northern Ireland. It should be noted that the Irish Grid and the National Grid are two independent coordinate reference systems, and that Irish Grid coordinates are not directly compatible with OSGB36 coordinates.

OSGM15 covers all of Great Britain, Isle of Man, Republic of Ireland, and Northern Ireland. The OSGM15 model in GB contains datum flags in order to relate to mean sea level as defined by the specific vertical datum for each region. The datum flag that forms part of each data record specifies to which datum the geoid-ellipsoid separation value relates. For Ireland and Northern Ireland there are separate OSGM15 files for the Malin Head and Belfast datums.

Accuracy of Ordnance Survey transformations

Within Great Britain, OSTN15 is the definitive OSGB36/ETRS89 transformation. OSTN15 in combination with the ETRS89 coordinates of the OS Net GNSS network stations, rather than the fixed triangulation network, now define the National Grid. This means that, for example, the National Grid coordinates of an existing OSGB36 point, refixed using GNSS from OS Net and OSTN15, will be the correct ones. The original archived OSGB36 National Grid coordinates of the point (if different) will no longer be true OSGB36, by definition, but the two coordinates (new and archived) will agree on average to better than 0.1 m, (68% probability).

Within the Republic of Ireland and Northern Ireland the OSi/LPS polynomial transformation is recommended for coordinate transformations between Irish Grid and ETRS89. Transformed ETRS89 coordinates will agree with Irish Grid coordinates derived from traditional survey control to within 0.4 m (95% data).

Accuracy of OSGM15

The heights output by precise GPS positioning in the ETRS89 coordinate system are geometric distance above the WGS84 (GRS80) reference ellipsoid. Note that GNSS heights are typically two to three times less precise than horizontal positions. OSGM15 converts GNSS ellipsoid heights to orthometric heights above mean sea level.

In mainland Great Britain, the datum (origin point) representing mean sea level is Ordnance Datum Newlyn, defined at Newlyn in Cornwall. In the Republic of Ireland, Northern Ireland, and the islands surrounding Great Britain, mean sea level is defined by specific independent vertical datums that are all incorporated in OSGM15 and hence OSGM15 is compatible with the products from each of the Ordnance Surveys and LPS. Other geoid models may give mean sea level heights that are incompatible with the Ordnance Surveys' and LPS's products.

The estimated accuracies of OSGM15 for each regional vertical datum are included in the table below. The figures quoted assume precise ellipsoidal heights are used; for lower quality GNSS observations additional error budget must be included.

| Regional datum | Standard error (m) |
|---------------------|--------------------|
| Great Britain | 0.01 |
| Republic of Ireland | 0.02 |
| Northern Ireland | 0.01 |
| Orkney | 0.02 |
| Shetland | 0.02 |
| Outer Hebrides | 0.01 |
| Isle of Man | 0.03 |
| Scilly Isles | 0.01 |

Any discrepancy found between an Ordnance Survey levelled bench mark and a OSGM15 computed orthometric height is likely to be due to bench mark subsidence or uplift and, assuming precise GNSS survey has been carefully carried out, the orthometric height given by OSGM15 should be considered correct in preference to archive bench mark heights.

Annexe A Transforming ETRS89 GNSS coordinates to OSGB36 and orthometric height

Worked example

To convert the coordinates of Caister Water Tower, at position given by ETRS89 geographical coordinates 52° 39' 28.8282" N, 1° 42' 57.8663" E, 108.05 m, to OSGB36 and orthometric height.

Step 1: Compute ETRS89 eastings and northings – see [annexe B](#)

Latitude = 52 39' 28.8282" N
= 52.658007833°

Longitude = 1 42' 57.8663" E
= 1.716073972°

The parameters for the GRS80 ellipsoid are:

a = 6 378 137.0000
b = 6 356 752.3141

Following the procedure in [annexe B](#), the calculation steps yield the following:

e^2 = 6.69438004e-03
v = 6.38912542e+06
p = 6.37332179e+06
 η^2 = 2.47965409e-03
M = 4.06772557e+05
P = 6.48577261e-02
I = 3.06772557e+05
II = 1.54055171e+06
III = 1.56081387e+05
IIIA = -2.06739447e+04
IV = 3.87545974e+06
V = -1.70023086e+05
VI = -1.01356325e+05
eastings = 651 307.0030
northings = 313 255.6859

The ETRS89 eastings and northings (to the nearest mm) are therefore:

x = 651 307.003
y = 313 255.686

Step 2: Transform ETRS89 eastings and northings to OSGB36 and ETRS89 height to orthometric height

First calculate the grid cell in which the point lies:

east_index = integer_part_of (x/1,000)
= 651

north_index = integer_part_of (y/1,000)
= 313

The eastings and northings of the south-west corner of the cell are therefore:

x0, y0 = (651 000, 313 000)

The easting, northing and geoid shifts for the four corners of the cell are given by:

(se₀, sn₀, sg₀) = shifts (east_index, north_index)
= shifts (651, 313)
= record (651 + (313 x 701) + 1)
= record (220 065)
= (102.787, -78.242, 44.236)

$$\begin{aligned}
 (se_1, sn_1, sg_1) &= \text{shifts (652, 313)} \\
 &= \text{record (220 066)} \\
 &= (102.825, -78.244, 44.221)
 \end{aligned}$$

$$\begin{aligned}
 (se_2, sn_2, sg_2) &= \text{shifts (652, 314)} \\
 &= \text{record (220 767)} \\
 &= (102.834, -78.225, 44.210)
 \end{aligned}$$

$$\begin{aligned}
 (se_3, sn_3, sg_3) &= \text{shifts (651, 314)} \\
 &= \text{record (220 766)} \\
 &= (102.795, -78.213, 44.224)
 \end{aligned}$$

The offset values are given by:

$$\begin{aligned}
 dx &= x - x_0 \\
 &= 307.003 \\
 dy &= y - y_0 \\
 &= 255.686 \\
 t &= dx/1\,000 \\
 &= 0.3070032 \\
 u &= dy/1\,000 \\
 &= 0.2556860
 \end{aligned}$$

The shifts are therefore:

$$\begin{aligned}
 se &= (1 - t)(1 - u) se_0 + (t)(1 - u) se_1 + (t)(u) se_2 + (1 - t)(u) se_3 \\
 &= 102.801 \\
 sn &= (1 - t)(1 - u) sn_0 + (t)(1 - u) sn_1 + (t)(u) sn_2 + (1 - t)(u) sn_3 \\
 &= -78.236 \\
 sg &= (1 - t)(1 - u) sg_0 + (t)(1 - u) sg_1 + (t)(u) sg_2 + (1 - t)(u) sg_3 \\
 &= 44.228
 \end{aligned}$$

And finally, the National Grid (OSGB36) eastings and northings coordinates are given by:

$$\begin{aligned}
 e &= x + se \\
 &= 651\,409.804 \\
 n &= y + sn \\
 &= 313\,177.450
 \end{aligned}$$

The orthometric height h is given by

$$\begin{aligned}
 h &= 108.05 - sg \\
 &= 108.05 - 44.228 \\
 &= 63.822
 \end{aligned}$$

So Caister Water Tower has National Grid (OSGB36) coordinates (651 409.804, 313 177.450) and orthometric height 63.822 m relative to the vertical datum as indicated by the datum flag field – which in this case = 1, indicating Ordnance Survey Datum Newlyn.

Using the procedure in [annexe C](#), these coordinates can be converted to latitude and longitude. A worked example of this step is given in [annexe C](#).

Inverse transformation: OSGB36 to ETRS89

Worked example

Taking the OSGB36 coordinates and height from the example above, that is 651 409.804, 313 177.450, 63.822 the procedure for the inverse transformation (OSGB36 to ETRS89) gives the following iterative solution:

| Iteration No. | Se | Sn | Sg | ETRSEast | ETRSNorth | ETRSHeight |
|---------------|----------|----------|---------|-------------|-------------|------------|
| 1 | 102.8041 | -78.2384 | 44.2278 | 651306.9999 | 313255.6884 | 108.0498 |
| 2 | 102.8008 | -78.2360 | 44.2284 | 651307.0032 | 313255.6860 | 108.0504 |
| 3 | 102.8008 | -78.2360 | 44.2284 | 651307.0032 | 313255.6860 | 108.0504 |

Since the second and third iterations show convergence at the required level, the calculation is stopped.

Using the procedure in [annexe C](#), to convert the ETRS89East and ETRS89North coordinates to latitude and longitude gives:

52.658007833, 1.716073972

Annexe B Converting latitude and longitude to easting and northing

The formulae in this annexe and [annexe C](#) require ellipsoid constants and projection constants, given in the tables below.

Important note: When converting OSGB36 coordinates between (easting, northing) and (latitude, longitude) in either direction, use the Airy 1830 ellipsoid constants. When converting ETRS89 coordinates between (easting, northing) and (latitude, longitude) in either direction, use the GRS80 ellipsoid constants. Use the same National Grid projection constants for both ETRS89 and OSGB36 coordinates.

The ITM (Irish Transverse Mercator) projection is required to obtain ETRS89 eastings and ETRS89 northings for use with the OSGM15 Geoid model data files for Northern Ireland and the Republic of Ireland. The ITM projection should only be used with the GRS80 ellipsoid.

Ellipsoid constants

| Ellipsoid | Semi-major axis <i>a</i> (metres) | Semi-minor axis <i>b</i> (metres) | Used for the following coordinate system |
|-----------|--------------------------------------|--------------------------------------|---|
| Airy 1830 | 6 377 563.396 | 6 356 256.909 ¹ | OSGB36 National Grid |
| GRS80‡ | 6 378 137.000 | 6 356 752.3141 | ETRS89 (WGS84) |

‡ Also known as the WGS84 ellipsoid.

The ellipsoid squared eccentricity constant e^2 is computed from *a* and *b* by:

$$e^2 = \frac{a^2 - b^2}{a^2} \quad (\text{B1})$$

Projection constants

| Projection | Scale factor on central meridian (F_0) | True origin (ϕ_0 and λ_0) | Map coordinates of true origin (m) (E_0 and N_0) |
|---------------|---|---|--|
| National Grid | 0.9996012717 | lat 49° N long 2° W | E 400 000 N -100 000 |
| ITM | 0.99982 | lat 53° 30' N long 8° W | E 600 000 N 750 000 |

¹ For a long time, in previous versions of this publication and other Ordnance Survey publications, the Airy 1830 value for *b* was quoted as 6356256.910. Research (Empire Survey Review, Vol. XI, No.84, 1952) shows the correct rounding is actually .909. The original dimensions for the Airy 1830 ellipsoid are quoted as *a* = 20,923,713 feet and *b* = 20,853,810 feet. The conversion of these values to metres is derived from the length of a standard bar ('O1'). This bar was the length standard for the principal triangulation and the retriangulation. The defined conversion to metric is:

$$10^{(\log(\text{axis}) + 9.48401603)}$$

This results in a metric value for the axis given in tenths of a nanometre. An easier way to express the conversion to metres is to multiply the axis length in feet by:

$$\left(\frac{10^{0.48401603}}{10} \right)$$

Both methods result in the 3-decimal place values in the table above. The resulting difference in eastings and northings when using the .909 or .910 values for *b* is approximately 0.016mm and is therefore insignificant.

To convert a position from the graticule of latitude and longitude coordinates (λ, ϕ) to a grid of easting and northing coordinates (E, N) using a transverse mercator projection, for example OSGB36 National Grid, ITM or UTM (Universal Transverse Mercator), compute the following formulae. Remember to express all angles in radians. You will need the ellipsoid constants a, b and e^2 and the projection constants listed below:

N_0 – northing of true origin;

E_0 – easting of true origin;

F_0 – scale factor on central meridian;

ϕ_0 – latitude of true origin; and

λ_0 – longitude of true origin and central meridian.

$$n = \frac{a-b}{a+b} \quad (\text{B2})$$

$$v = aF_0(1 - e^2 \sin^2 \phi)^{-0.5} \quad (\text{B3})$$

$$\rho = aF_0(1 - e^2)(1 - e^2 \sin^2 \phi)^{-1.5} \quad (\text{B4})$$

$$\eta^2 = \frac{v}{\rho} - 1 \quad (\text{B5})$$

$$M = bF_0 \left(\begin{aligned} &\left(1 + n + \frac{5}{4}n^2 + \frac{5}{4}n^3\right)(\phi - \phi_0) - \left(3n + 3n^2 + \frac{21}{8}n^3\right)\sin(\phi - \phi_0)\cos(\phi + \phi_0) \\ &+ \left(\frac{15}{8}n^2 + \frac{15}{8}n^3\right)\sin(2(\phi - \phi_0))\cos(2(\phi + \phi_0)) - \frac{35}{24}n^3\sin(3(\phi - \phi_0))\cos(3(\phi + \phi_0)) \end{aligned} \right) \quad (\text{B6})$$

$$I = M + N_0$$

$$II = \frac{v}{2} \sin \phi \cos \phi$$

$$III = \frac{v}{24} \sin \phi \cos^3 \phi (5 - \tan^2 \phi + 9\eta^2)$$

$$IIIA = \frac{v}{720} \sin \phi \cos^5 \phi (61 - 58 \tan^2 \phi + \tan^4 \phi)$$

$$IV = v \cos \phi$$

$$V = \frac{v}{6} \cos^3 \phi \left(\frac{v}{\rho} - \tan^2 \phi \right)$$

$$VI = \frac{v}{120} \cos^5 \phi (5 - 18 \tan^2 \phi + \tan^4 \phi + 14\eta^2 - 58\eta^2 \tan^2 \phi)$$

$$N = I + II(\lambda - \lambda_0)^2 + III(\lambda - \lambda_0)^4 + IIIA(\lambda - \lambda_0)^6 \quad (\text{B7})$$

$$E = E_0 + IV(\lambda - \lambda_0) + V(\lambda - \lambda_0)^3 + VI(\lambda - \lambda_0)^5 \quad (\text{B8})$$

Worked example using the Airy 1830 ellipsoid and National Grid

Intermediate values are shown here to 10 decimal places. Compute all values using double-precision arithmetic.

ϕ 52° 39' 27.2531" N
 λ 1° 43' 4.5177" E

ν 6.3885023339E+06
 ρ 6.3727564398E+06
 η^2 2.4708137334E-03
 M 4.0668829595E+05
I 3.0668829595E+05
II 1.5404079094E+06
III 1.5606875430E+05
IIIA -2.0671123013E+04
IV 3.8751205752E+06
V -1.7000078207E+05
VI -1.0134470437E+05

E 651409.903 m
N 313177.270 m

Annexe C Converting easting and northing to latitude and longitude

Obtaining (λ, ϕ) from (E, N) is an iterative procedure. You need values for the ellipsoid and projection constants $a, b, e^2, N_0, E_0, F_0, \phi_0$ and λ_0 given in [annexe B](#). Remember to express all angles in radians.

First compute:

$$\phi' = \left(\frac{N - N_0}{aF_0} \right) + \phi_0 \quad (\text{C1})$$

and M from equation **(B6)** in [annexe B](#), substituting ϕ' for ϕ .

If the absolute value of $(N - N_0 - M) \geq 0.01$ mm, obtain a new value for ϕ' using:

$$\phi'_{new} = \left(\frac{N - N_0 - M}{aF_0} \right) + \phi' \quad (\text{C2})$$

and recompute M substituting ϕ' for ϕ .

Iterate until the absolute value of $(N - N_0 - M) < 0.01$ mm, then compute ρ , ν and η^2 using equations **(B3, B4 and B5)** in [annexe B](#) and compute:

$$\text{VII} = \frac{\tan \phi'}{2\rho\nu}$$

$$\text{VIII} = \frac{\tan \phi'}{24\rho\nu^3} (5 + 3 \tan^2 \phi' + \eta^2 - 9 \tan^2 \phi' \eta^2)$$

$$\text{IX} = \frac{\tan \phi'}{720\rho\nu^5} (61 + 90 \tan^2 \phi' + 45 \tan^4 \phi')$$

$$\text{X} = \frac{\sec \phi'}{\nu}$$

$$\text{XI} = \frac{\sec \phi'}{6\nu^3} \left(\frac{\nu}{\rho} + 2 \tan^2 \phi' \right)$$

$$\text{XII} = \frac{\sec \phi'}{120\nu^5} (5 + 28 \tan^2 \phi' + 24 \tan^4 \phi')$$

$$\text{XIIA} = \frac{\sec \phi'}{5040\nu^7} (61 + 662 \tan^2 \phi' + 1320 \tan^4 \phi' + 720 \tan^6 \phi')$$

$$\phi = \phi' - \text{VII}(E - E_0)^2 + \text{VIII}(E - E_0)^4 - \text{IX}(E - E_0)^6 \quad (\text{C3})$$

$$\lambda = \lambda_0 + \text{X}(E - E_0) - \text{XI}(E - E_0)^3 + \text{XII}(E - E_0)^5 - \text{XIIA}(E - E_0)^7 \quad (\text{C4})$$

Worked example using Airy 1830 ellipsoid and National Grid

Intermediate values are shown here to 10 decimal places. Compute all values using double precision arithmetic.

E 651 409.903 m

N 313 177.270 m

| | |
|---------------|----------------------|
| $\phi' \#1$ | 9.2002324604E-01 rad |
| $M \#1$ | 4.1290347143E+05 |
| $N-N_0-M\#1$ | 2.7379857228E+02 |
| $\phi' \#2$ | 9.2006619470E-01 rad |
| $M \#2$ | 4.1317717541E+05 |
| $N-N_0-M\#2$ | 9.4594338385E-02 |
| $\phi' \#3$ | 9.2006620954E-01 rad |
| $M \#3$ | 4.1317726997E+05 |
| $N-N_0-M\#3$ | 3.2661366276E-05 |
| $\phi' \#4$ | 9.2006620954E-01 rad |
| $M \#4$ | 4.1317727000E+05 |
| $N-N_0-M\#4$ | 1.1350493878E-08 |
| final ϕ' | 9.2006620954E-01 rad |
| ν | 6.3885233415E+06 |
| ρ | 6.3728193094E+06 |
| η^2 | 2.4642206357E-03 |
| VII | 1.6130562489E-14 |
| VIII | 3.3395547427E-28 |
| IX | 9.4198561675E-42 |
| X | 2.5840062507E-07 |
| XI | 4.6985969956E-21 |
| XII | 1.6124316614E-34 |
| XIIA | 6.6577316285E-48 |

ϕ 52° 39' 27.2531" N

λ 1° 43' 4.5177" E

Annexe D Glossary

The following is a list of technical terms used in this user guide, together with a fuller definition.

datum

A point, line, surface or set of these, with respect to which positions of objects can be stated as unique sets of coordinates.

de facto national standard

A national standard by adoption rather than legally enforced.

ellipsoid (biaxial)

The 3D geometric figure obtained by rotating an ellipse about its minor axis. Used in geodesy to approximate the shape of the earth.

ETRF89

European Terrestrial Reference Frame 1989 – the Europe-fixed realisation of WGS84. Governed by EUREF as a standard reference frame for Europe.

ETRS89

European Terrestrial Reference System 1989 – a coordinate system that is the Europe-fixed precise version of WGS84. Governed by EUREF as the standard fixed reference system for Europe. ETRS89 is related to the state-of-the-art WGS84-consistent system ITRS2000 by a six-parameter kinematic transformation published by IERS.

EUREF

EUREF (European Reference Frame): a sub-commission of the International Association of Geodesy, Commission X.

geocentric datum

A reference system that uses the centre of mass of the earth as its origin; the popularity of these systems today derives from their usefulness in describing satellite orbits.

Geoid model

A model of the level surface which is closest to mean sea level over the oceans. This surface is continued under land as the fundamental reference surface for height measurement.

GNSS

Global Navigation Satellite System – generic term for one or more satellite navigation systems including (but not limited to) GPS (USA), GLONASS (Russia), Galileo (Europe), Beidou (China).

GPS

Global Positioning System – an outdoor positioning technique using a constellation of US Department of Defense satellites and a portable receiver to dynamically determine coordinates. For high precision, several receivers are used and their relative positions are determined.

GRS80

A global reference ellipsoid used in the WGS84 coordinate system. Also known as the WGS84 ellipsoid.

IERS

International Earth Rotation Service.

ITRF

International Terrestrial Reference Frame – the state-of-the-art global realisation of the WGS84 reference system, using observations from worldwide networks of active geodetic stations of the VLBI, SLR, GPS and DORIS techniques.

ODN

Ordnance Datum Newlyn – the levelling-based vertical reference frame for most of the British Isles, with a single tide gauge constraint in Newlyn in Cornwall.

OSGB36

Ordnance Survey Great Britain 1936 – the British horizontal mapping datum, observed by triangulation from 1936 and traditionally realised on the ground by triangulation stations. With the release of the definitive transformation, OSTN02, OSGB36 is now realised by the ETRS89 coordinates of the National GPS Network in conjunction with the OSTN02 transformation.

OSGM02

Ordnance Survey National Geoid Model 2002 – now superseded, a gravimetric Geoid model that is aligned with the national height datums of Great Britain, Northern Ireland and Ireland. Replaced by OSGM15.

OSGM15

Ordnance Survey National Geoid Model 2015 – a gravimetric Geoid model that is aligned with the national height datums of Great Britain, Northern Ireland and Ireland.

OSTN15

Ordnance Survey National Grid Transformation 2015 – a grid shift type horizontal transformation between the ETRS89 datum and OSGB36 National Grid.

realisation

A spatial reference system made real on the ground by monumented points with estimated coordinates and errors.

transformation

A procedure to change from one coordinate system to another.

WGS84

World Geodetic System 1984 – the global geodetic reference system used to describe the position of GPS satellites and ground stations.