

Z - Algorithm

$$\begin{array}{l} \rightarrow \text{Naive} \rightarrow (P*T) \\ \rightarrow \text{Rabin-Karp} \rightarrow (\underline{P} \times \underline{T}) \rightarrow (\underline{T}) \end{array}$$

→ Core of the algorithm is the function

$Z(k)$  = longest substring starting at  $(k)$   
which is also prefix of the string.

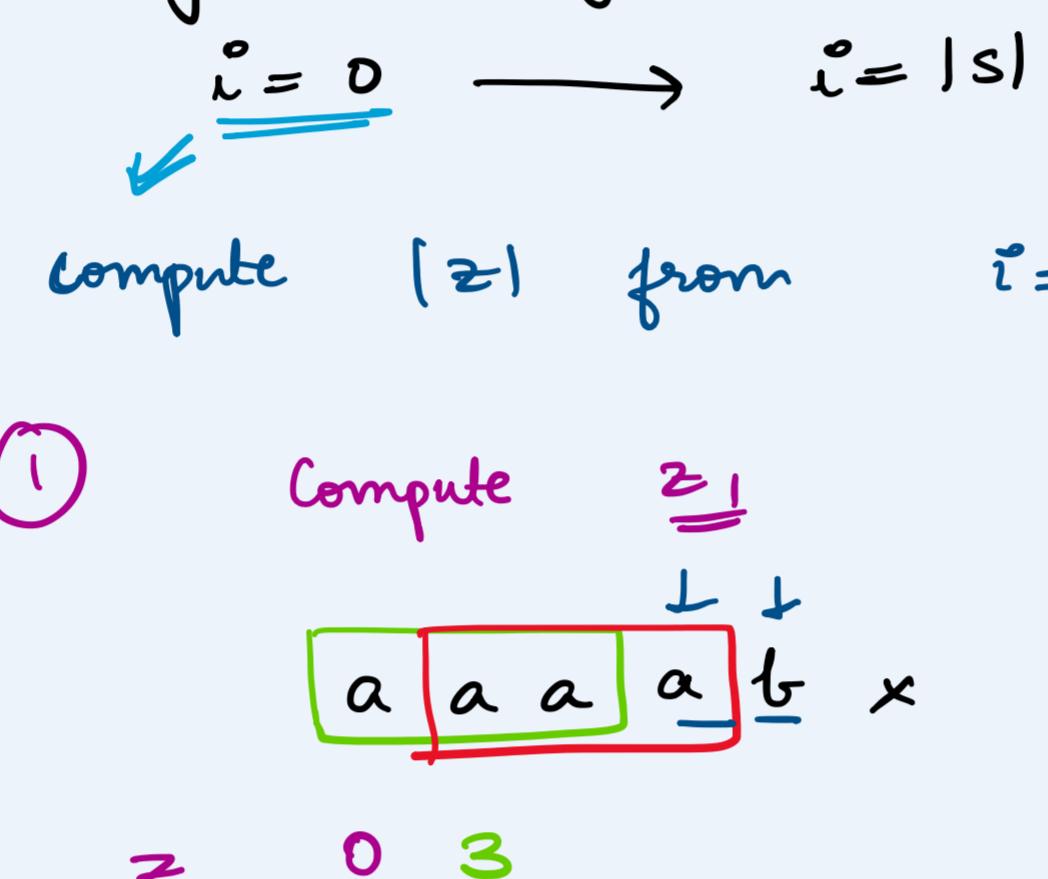
String → a a b x a a y a a b  
 $Z \rightarrow 0 \ 1 \ 0 \ 0 \ 2 \ 1 \ 0 \ 3 \ 1 \ 0$   
 ↓  
 substring at index 1,  
 length 1 is a prefix of  
 this string.

1 To use this information for pattern matching

2 How to compute the  $z$ -array efficiently.

3  $\text{Text} \rightarrow x \boxed{abc} abz \boxed{abc} \checkmark$   
 $\text{pattern} \rightarrow abc$

① Concat pattern + text and separate them by a character which is not present in any of these strings.



② Calculate the  $z$ -array of this new concatenated string.

$$\begin{array}{l} \rightarrow abc \ $ xabcabzabc \ \text{len}(P) \\ z \rightarrow 00000 \boxed{3} 00200 \boxed{3} 00 \end{array}$$

③ look for positions where length of the pattern is equal to the  $z$ -array element.

To find the original index in text where pattern is present,

$$\begin{array}{l} \text{Index} = (\text{length of pattern} + \text{special char}) \\ 5 = (3 + 1) = \boxed{1} \rightarrow \text{original array indices.} \\ 11 = (3 + 1) = \boxed{7} \end{array}$$

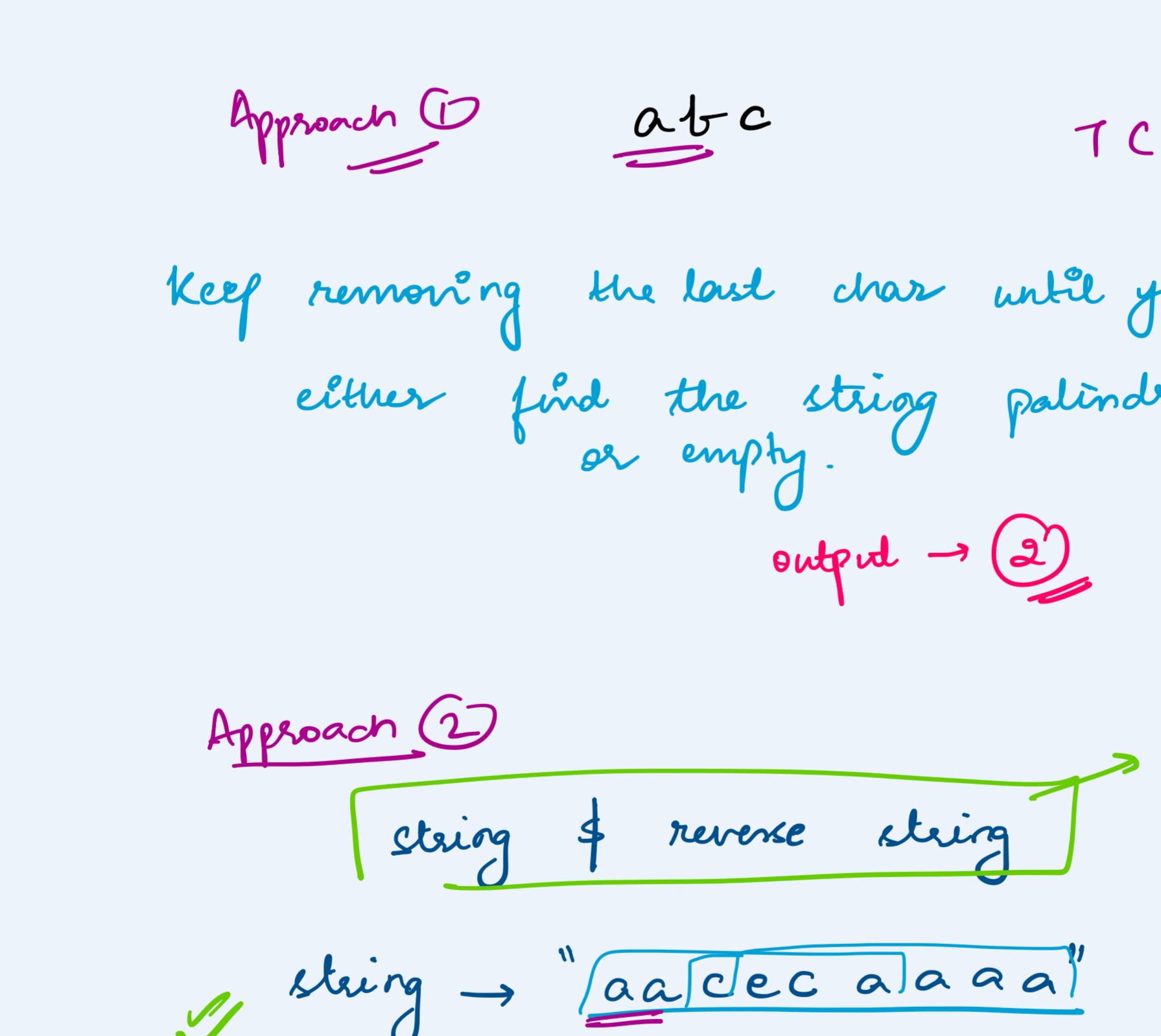
$$\text{Time Complexity} \rightarrow O(|P| + |T|) \quad \begin{array}{l} \text{length of text} \\ \text{length of pattern} \end{array}$$

Quick ques

$$S = P \neq T \rightarrow \boxed{S = T \neq P} ?? \quad \text{No}$$

This is not a symmetric property.

Now  $Z$ -algorithm,



Now, given at position  $i$ , where  $z_i > 0$ ,

→ starting at position  $i$ , there is some substring that matches the prefix of the string

$$\boxed{y \neq x}$$

string of length 1s  
 $i=0 \rightarrow i=1s$

compute  $|z|$  from  $i=1$  to  $i=1s$

① Compute  $\boxed{z_1}$

$$\begin{array}{c} a \boxed{a} a a b x \\ z = 0 \ 3 = \end{array}$$

for just computing

$$z_1 = 4 \text{ comparisons}$$

worst case ??

$$\rightarrow \begin{array}{c} a a a a a a a a \\ z_1 = 4 \\ \text{compute} \rightarrow z_1 \\ l = 5 \\ \text{comparisons} \rightarrow 5 \end{array}$$

② Naive approach

→ very ineffective.



$$z \rightarrow 0 \ 1 \ 0 \ 0 \ 4 \ 1 \ 0 \ 0 \ 0 \ 8 \ 1 \ 0 \ 0 \ 5 \ 1 \ 0 \ 0 \ 1 \ 0$$

$$19 \approx 28$$

$$\text{comparisons} \rightarrow \boxed{\cancel{11} \ \cancel{11} \ \cancel{11} \ \cancel{11} \ \cancel{11}}$$

when copying  $z$ -values, if the sum of  $z$ -value + current index  $\geq$  right boundary, if  $\neq$  box,

then we need to re-calculate the  $z$ -value.

Approach 1: XOR of a no with a same no is 0.

$$\begin{array}{|c|c|c|} \hline A & B & Z \\ \hline 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ \hline \end{array}$$

$$A \oplus A = 0$$

→ So, we have to count how many times substring of  $\text{B}$  matches with string  $\text{A}$ .

→ Concatenate  $\text{B}$  with  $\text{B}$ , so that it contains all cyclic permutations

$$10110$$

→ Now the problem is reduced to finding the occurrences of pattern  $A$  in string  $S = B \cdot B$ .

→  $Z$ -algo.

$$\boxed{101 \neq 10110}$$

$$\boxed{001 \ 0 \ \boxed{3} 0110}$$

$$\Rightarrow \text{ans} \rightarrow \boxed{1}$$

② Given a string  $A$  of size  $N$  consisting of uppercase alphabets. The operation allowed is to insert characters in the beginning of the string.

\* Find the return how many characters are needed to make the string a palindrome string.

Eg:  $(A) \rightarrow "ABC"$  output →  $\boxed{2}$

$$\begin{array}{l} \text{Insert } B \rightarrow \overset{\downarrow \uparrow \downarrow \uparrow}{\text{BABC}} \\ \text{Insert } C \rightarrow \overset{\downarrow \uparrow \downarrow \uparrow}{\text{CBABC}} \end{array}$$

Approach ①  $\boxed{abc}$   $\rightarrow$   $\boxed{abc}$   $\rightarrow$   $T C \rightarrow O(n^2)$

Keep removing the last char until you either find the string palindromic or empty.

$$\text{output} \rightarrow \boxed{2}$$

Approach ②  $\boxed{\text{string} \neq \text{reverse string}}$

string →  $"\text{aa}|\text{dec alaka a}"$

$$\boxed{a a c e c a a a a a a a c e c a a a}$$

$\rightarrow$  find the max val of the array.

$$9 - 7 = \boxed{2}$$

$\cancel{a a a a c e c a a a a a a c e c a a a}$   $\neq$   $\cancel{a a c e c a a a a a a a c e c a a a}$

$$\boxed{2} \rightarrow 0321000210210004321$$

③ Given a very large number  $n$ , count the total no. of ways such that if we divide the number into two parts  $A$  and  $B$ , then no.  $a$

can be obtained by integral division of number  $B$  by some power  $p$

$$7 \neq 0$$

Eg:  $220$  output →  $1$

Explanation

$$\rightarrow 220$$

$$A = 2 \quad B = 20 \quad p = 1$$

$$220 / 10^1 = 2$$

Eg:  $2202200$  output →  $2$

Explanation

$$\rightarrow 2202200$$

$$A = 220 \quad B = 20200 \quad p = 5$$

$$20200 / 10^5 = 2$$

Eg:  $2202200$  output →  $2$

Explanation

$$\rightarrow 2202200$$

$$A = 220 \quad B = 2200 \quad p = 3$$

$$2200 / 10^3 = 2$$

① Irregular division of no is not allowed

$$289 \rightarrow 29 \text{ and } 8 \neq 0$$

②  $a$  and  $b$  should not contain leading zeroes.

③ Integral division of no is allowed  $F(\frac{a}{b})$

④  $1 \leq$  no. of digits in  $N \leq 10^5$