Announcements

- To receive course-related announcements, sign up to the course list: cse478@lists.iiit.ac.in
- All assignments will be managed via the course portal
- For off-campus access to the course portal, contact the server room staff

TAs:

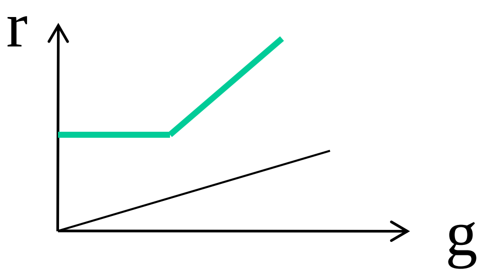
Aditi Gupta (aditi.gupta@students.iiit.ac.in)

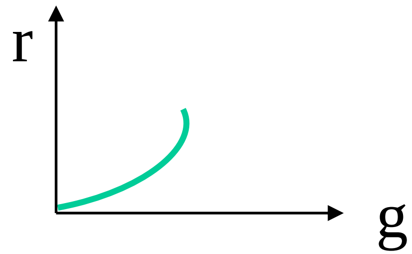
Shantanu Patil (shantanu.patil@students.iiit.ac.in)

Point processing

Point (local) processing

$g[m,n] \rightarrow r[m,n]$ where $r = f(g)$; $g, r \in [0, L\text{-}1]$





- *f* can be a one to one/many to one; linear/nonlinear mapping function
- This is <u>zero–memory</u> filtering
- Implementable using a look up table (LUT)

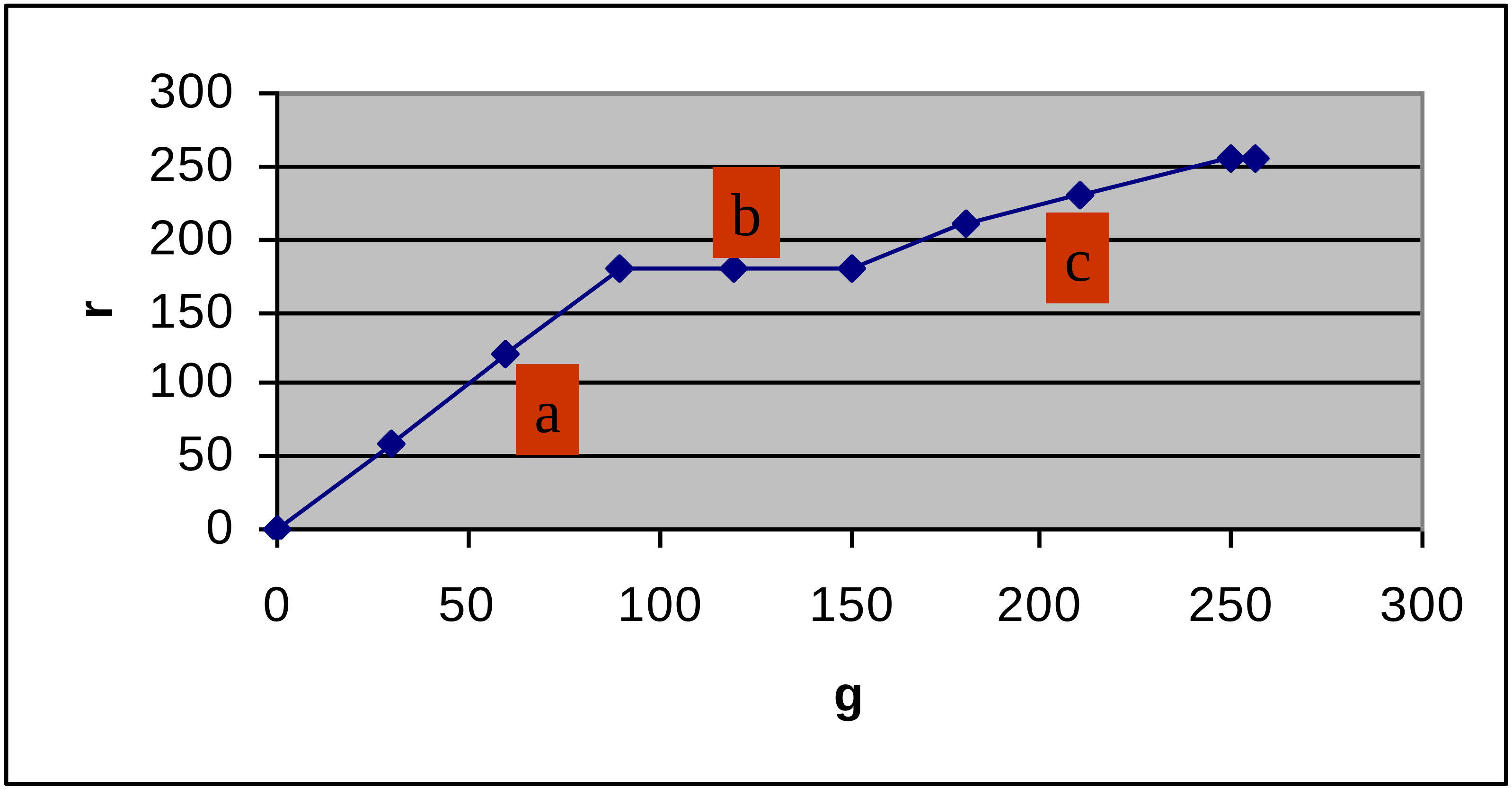
Point processing ..application

- $r = f(g)$ is *Contrast* manipulation

Used to

- compensate for differences in the dynamic range of the image vs. display devices
  - Ex. display a 16 bit image on 8 bit display
  - Ex. $\log(1+|F|)$ to display Fourier amplitude spectrum which has a huge dynamic range
- improve contrast of a given image
  - Ex. Image editing in Photoshop

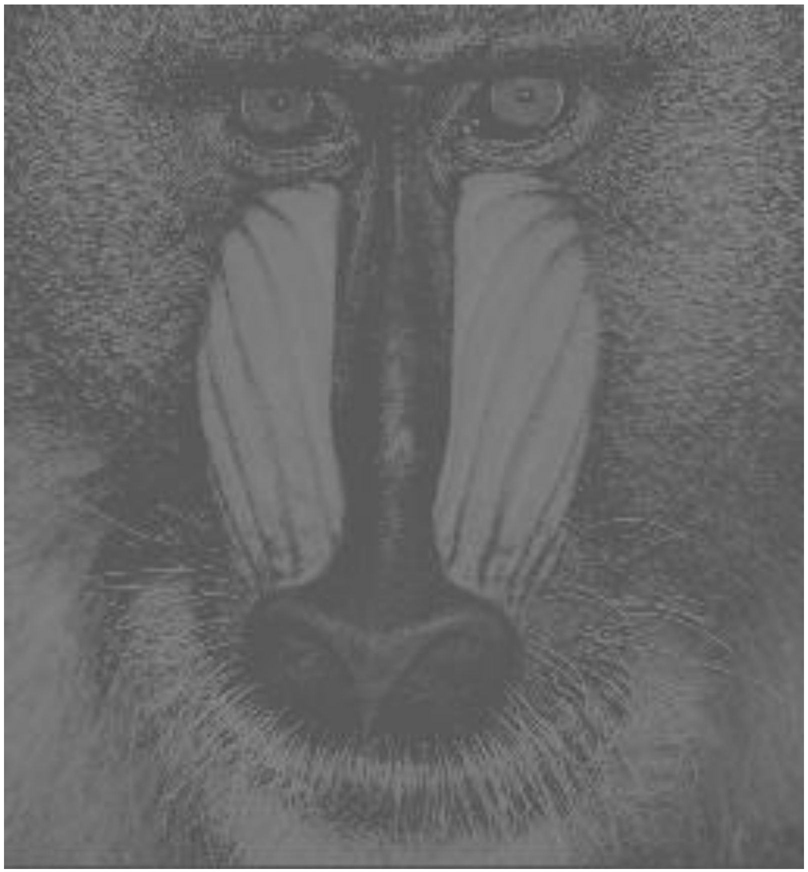
Contrast Stretching - example



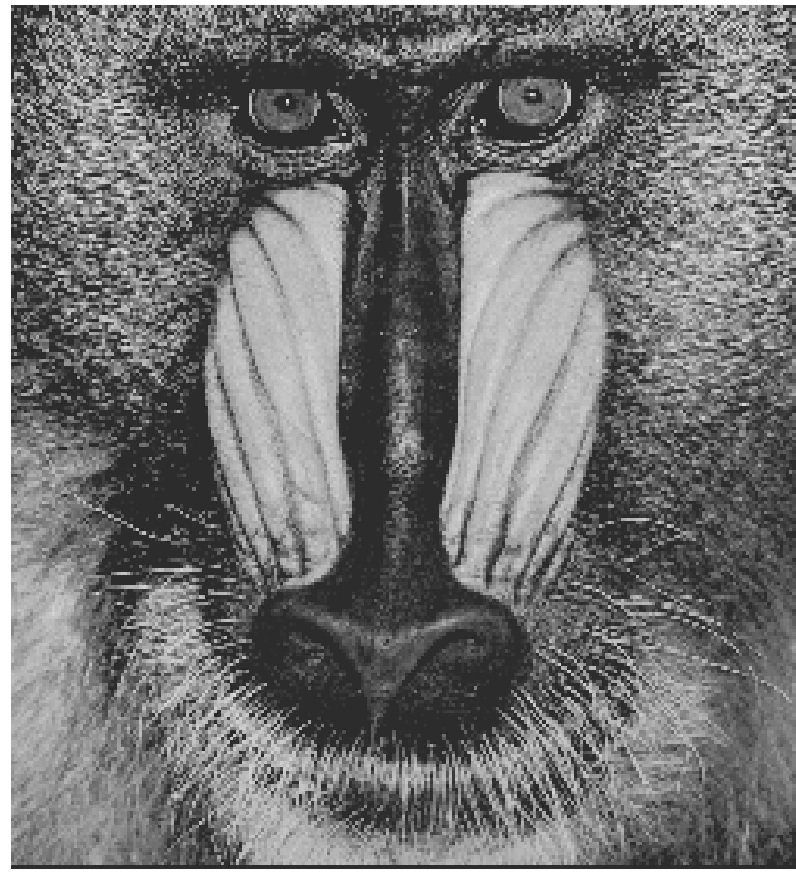
Piecewise linear function

Improving a poor contrast image

Before



After



What is the mapping function?

Examples of point processing

- **Negative image**: $r = L - g;\ g, r \in [0,L]$
  - Negation of a part of the range is *solarisation*

- **Binarisation**
  - With single threshold or dual thresholds

- **Clipping**

- **Intensity slicing**

Intensity slicing - example

Before

After

What is the mapping function?

Global operations

$$I_{0ut}[m_0,n_0] = f(I_{in}[m,n])$$

$f(\mathrm{I}_{in})$

- f needs to represent some global information about the input image
- Global info: grey level statistics or occurrence of greyvalues in the image
  - Ideally, all greyvalues should occur in equal measure

Probability density function - review

- The probability density function (pdf) of a (continuous) random variable ***x*** satisfies the following axioms.
  - Probability of ***x*** is a non-zero value for all ***x*** $p(x) \geq 0; \forall x$
  - The area under the pdf is 1 $\int_{-\infty}^{\infty} p(x)dx = 1$
  - The running integral of the pdf is called the cumulative distribution function or cdf $\int_{-\infty}^{x} p(a)da = F(x)$
  - The probability ***x*** takes on values in an interval is given by the area under the pdf in this interval

$$\int_{x1}^{x2} p(x)dx = P(x1 < x \leq x2)$$

Cumulative distribution function

- Cumulative distribution function (cdf)
  – describes the probability that a random variable `*x*' with a given probability distribution will be found at a value less than or equal to *x*.

$$F(a) = P(x \leq a)$$

  – it is the "*area so far*" function of the pdf

1. $F(-\infty) = 0$
2. $F(\infty) = 1$
3. $0 \leq F(x) \leq 1$
4. $F(x_1) \leq F(x_2)$ if $x_1 < x_2$
5. $P(x_1 < x \leq x_2) = F(x_2) - F(x_1)$

$$p(x) = \frac{dF(x)}{dx}$$

Grey level statistics

- Image histogram $h_I(g)$; $g \in [0,L\text{-}1]$
  - the distribution of the grey values in an image
    - frequency plot giving count of grey value occurrence in the image

- For a MxN image $h_I \in [0,MN]$

- After normalisation ($h_I \in [0,1]$), $h_I \sim$ pdf of the random variable g

- Key statistical metrics
  - $g_{mean,}$ mean grey value
  - σ_g, standard deviation / variance

Ex: narrow histogram $\Leftrightarrow$ low contrast image

(small std. devn.)

Caution

- Grey level statistics for an image is different from statistics across images

- Natural images have some regularity in their statistics
  - The power spectrum(2^{nd} order statistics) falls as $1/f^2$

Global processing

Image processing in the histogram space:

- Histogram manipulation
  - Transforming the pdf of the input image to a desired one
- Histogram equalisation
  - Special case of manipulation where the desired pdf is ‘uniform’

Histogram manipulation (HM)

- Find $r = f(g)$ such that $p_g(g)$ is modified to $p_r(r)$
  - p_g and p_r are probability density functions

Desirable properties of f

1. It is single-valued and monotonically increasing in [0, L-1]
   - To preserve the brightness order
2. Mapped values also lie in [0,L-1]
   - To satisfy the BIBO condition

HM for continuous case

- Let *r* and *g* be random variables with probability <u>density</u> functions $p_g(g)$ and $p_r(r)$

- HM requires the mapping:

$[g+dg] \rightarrow [r+dr] \;\; => \;\; p_g(g)\, dg = p_r(r)\, dr$

where *r* and *g* are $\in [0,L-1]$

$$\int_0^r p_r(\lambda)d\lambda = \int_0^g p_g(\alpha)d\alpha$$

i.e. the two grey level <u>cumulative</u> <u>distribution</u> functions are equal

Special case: $p_r(r)$ is a *uniform* density function

Histogram Equalisation (HE)

- When $p_r(r)$ is a uniform density function, we have histogram equalisation
- This process aims to uniformly distribute all the grey values in the final processed image
  - i.e. $p_r(r) = \text{constant}$ for $r \in [0,L]$

The required mapping is

$$\int_0^r d\lambda = r = \int_0^g p_g(\alpha)\,d\alpha$$

- Useful to *optimally* improve global contrast

HE mapping - discrete case

- g and r are discrete random variables taking values g_i and $r_i \in [0,1]; \; 0 \le i \le L$

- $p_g(g_i) = n_i/N$;

  n_i is the no. of pixels with grey value g_i

  N is the total no. of pixels in the image

$$r = \int_0^g p_g(\alpha)\, d\alpha \longrightarrow r_k = \sum_{i=0}^{k} \frac{n_i}{N}$$

Real-valued

Required mapping

HE..contd

- Finally, rescale the range of r_k to *[0,L]*

$$r_k = \max\{0, round(L\sum_{i=0}^{k}\frac{n_i}{N})\}$$

Example: 8x8, 3-bit image

g (given grey value)	**n_i**	**Cumulative value**	**r = f(g) r∈R**	**r = f(g) r∈Z+**
0	8	8	(8/64) x 7 = 0.875	1
1	10	18	(18/64) x 7 = 1.968	2
2	10	28	3.062	3
3	2	30	3.281	3
4	12	42	4.593	5
5	16	58	6.343	6
6	4	62	6.781	7
7	2	64	7	7

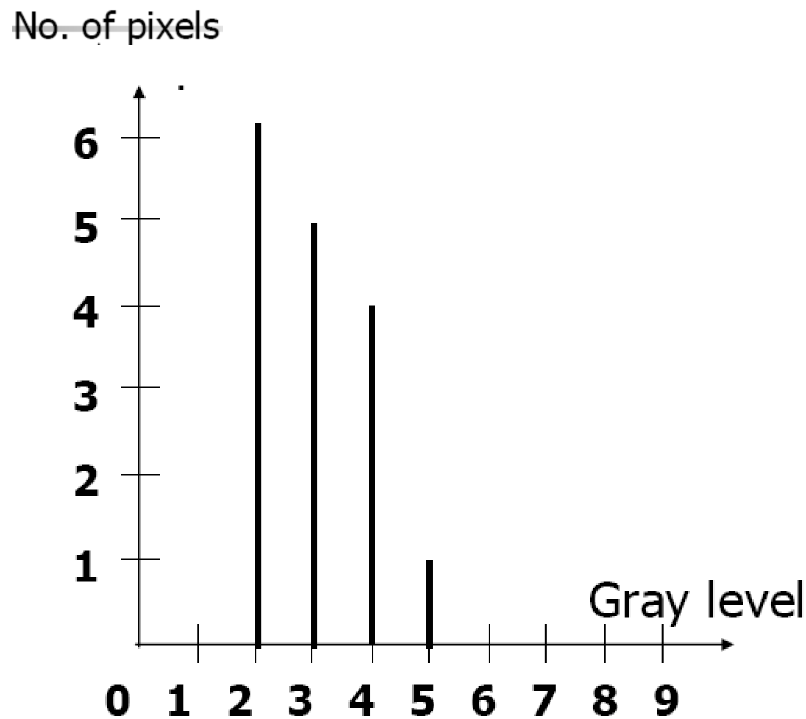
r	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
count	0	8	10	12	0	12	16	6

Example: 4x4 image

2	3	3	2
4	2	4	3
3	2	3	5
2	4	2	4

4x4 image

Gray scale = [0,9]



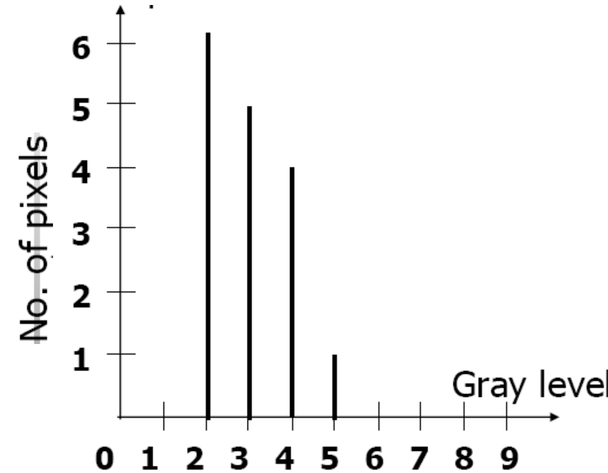
histogram

Gray Level(j)	0	1	2	3	4	5	6	7	8	9
No. of pixels	0	0	6	5	4	1	0	0	0	0
$\sum_{j=0}^{k} n_j$	0	0	6	11	15	16	16	16	16	16
$s = \sum_{j=0}^{k} \frac{n_j}{n}$	0	0	6 / 16	11 / 16	15 / 16	16 / 16	16 / 16	16 / 16	16 / 16	16 / 16
s x 9	0	0	3.3 ≈3	6.1 ≈6	8.4 ≈8	9	9	9	9	9

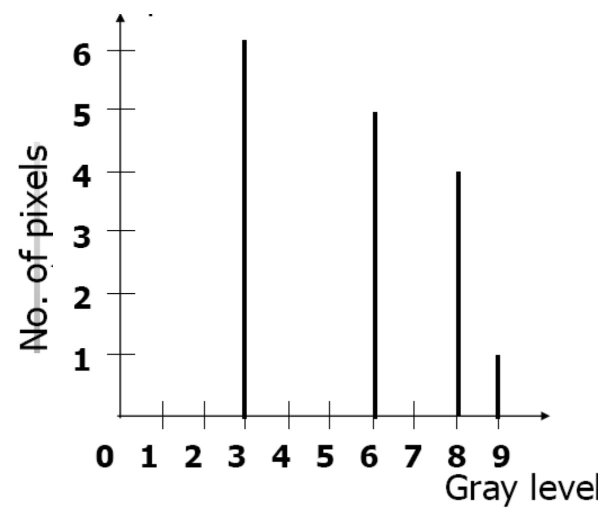
3	6	6	3
8	3	8	6
6	3	6	9
3	8	3	8

Output image

Gray scale = [0,9]



given histogram



After Histogram equalization

Effects of HE

- Brightness order of pixels is retained
- Minor variations in pixel values are amplified - added discrimination
- Pixels of different grey value can be assigned same value – loss of information
- Quantization prevents obtaining a flat histogram in practice

Histogram **Specification**

- Here $p_r(r)$ is some specified function

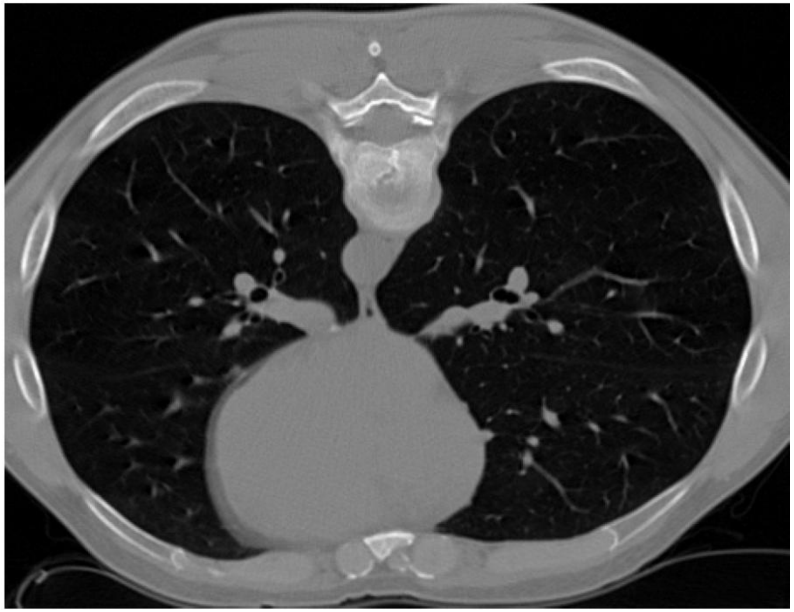
- These functions are

  sampled versions of continuous density functions

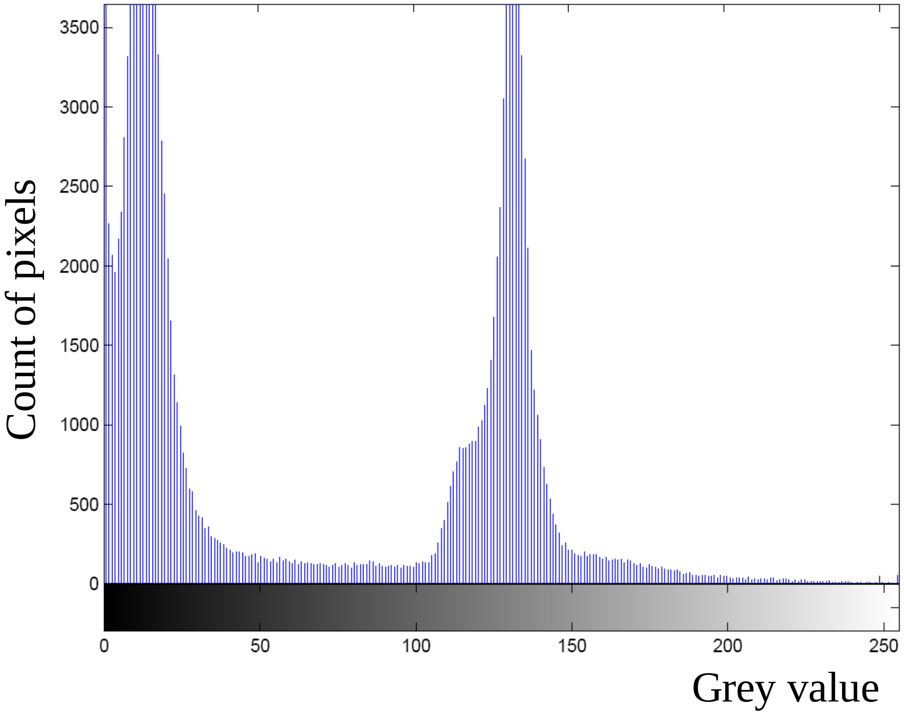
  OR

  Specified interactively

Global information source - grey level statistics

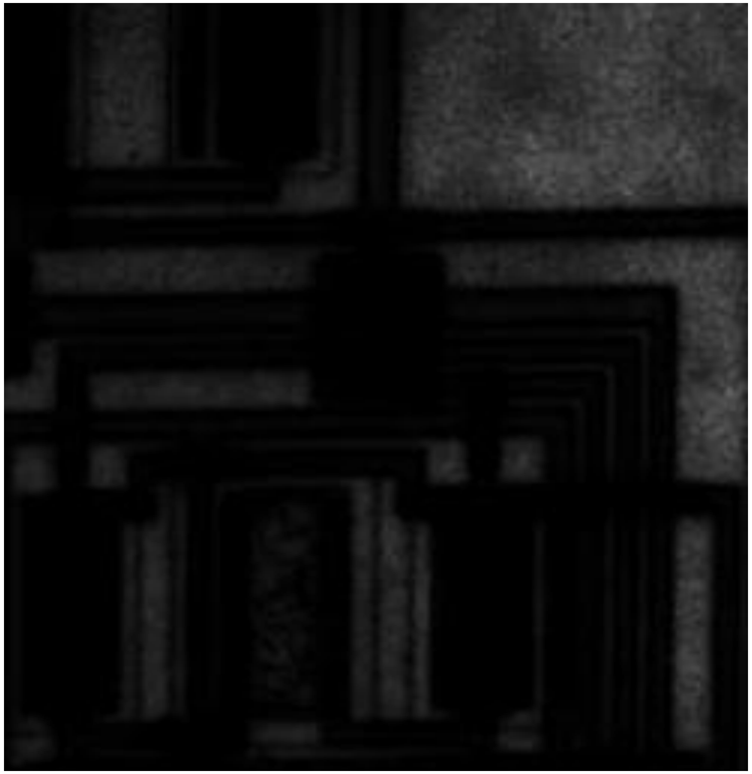


CT slice

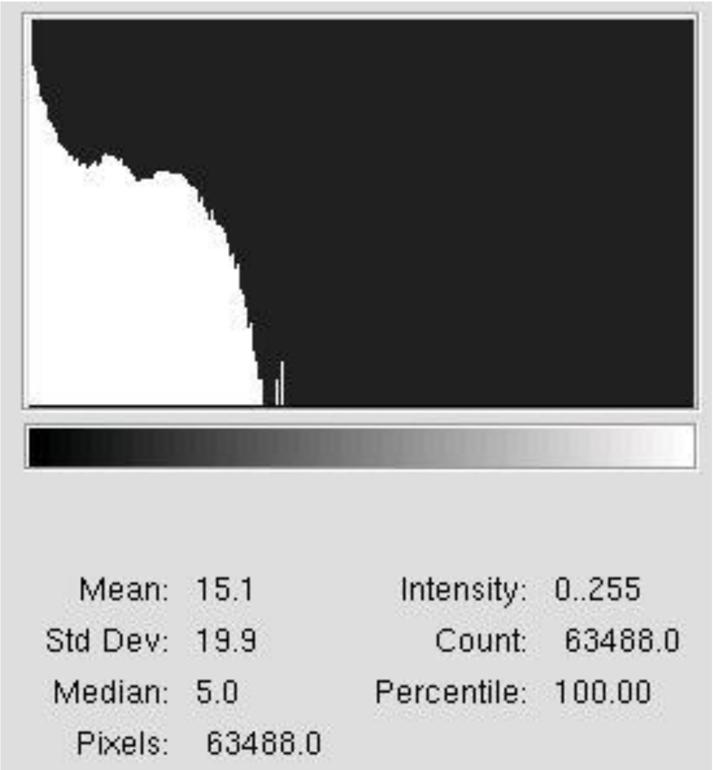


Histogram of the slice

HE – example 1

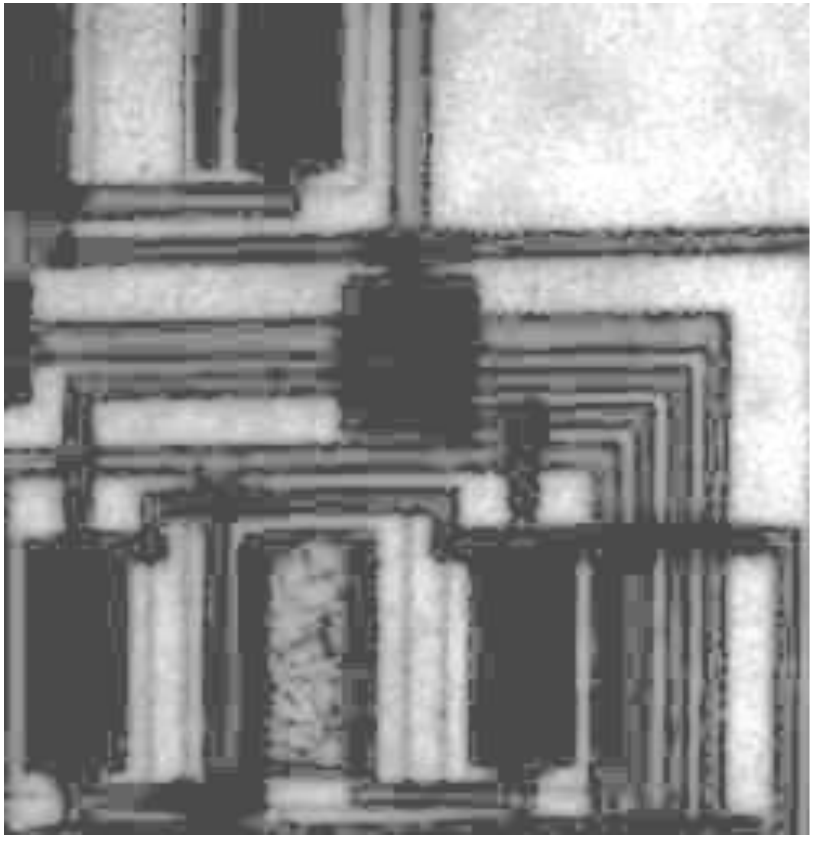


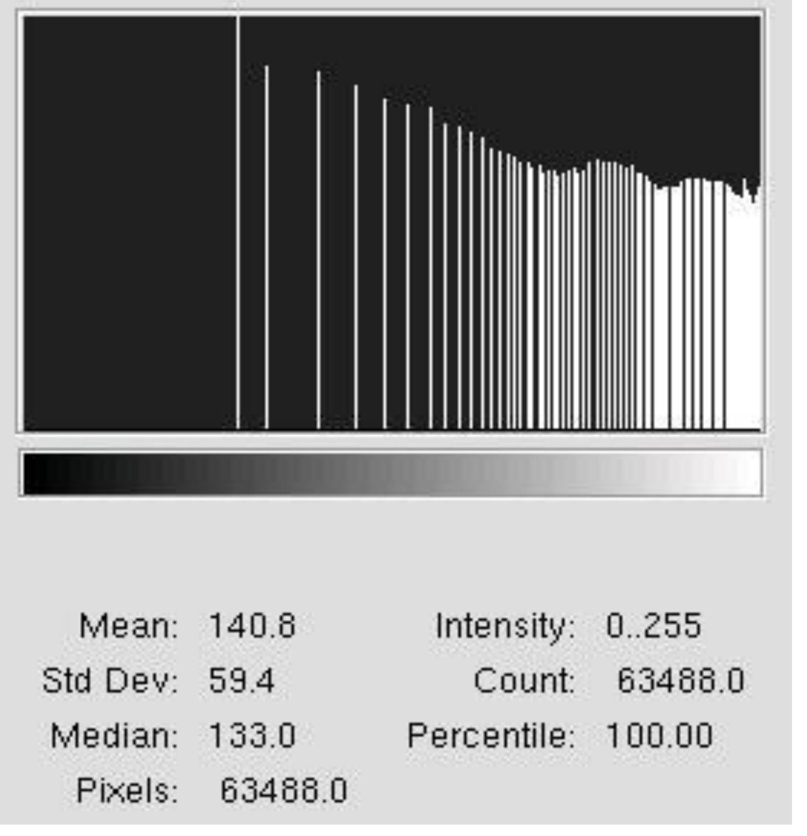
A dark image



Its histogram

HE- example 1

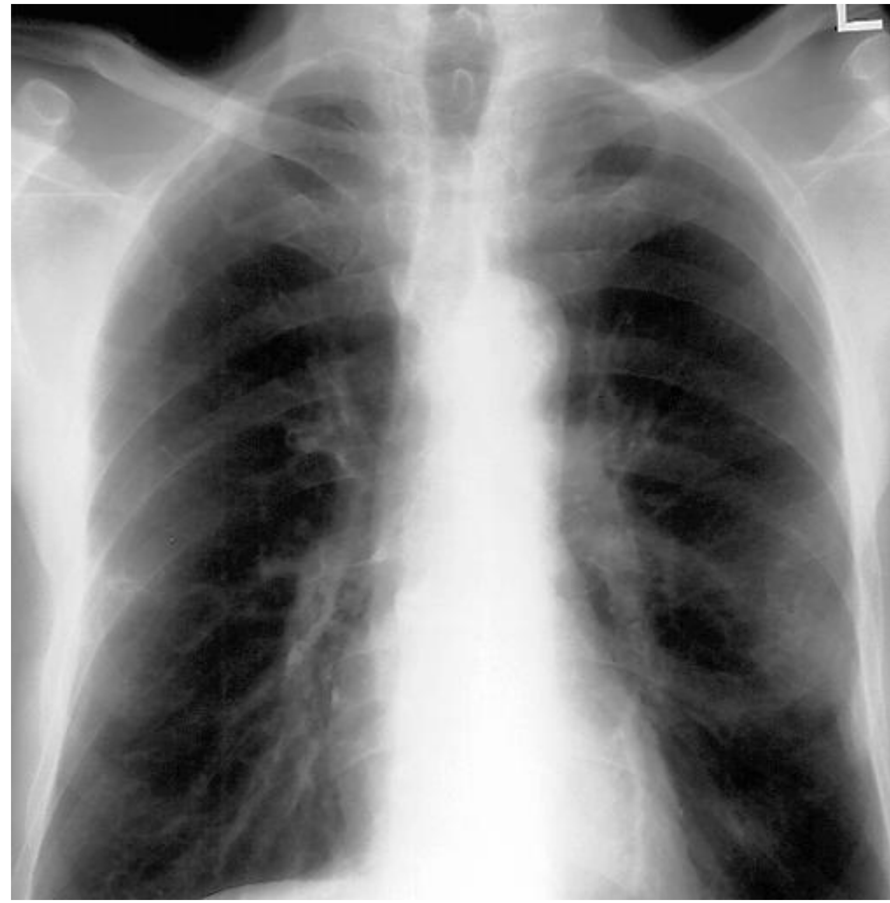




After HE

HE - example 2

Before



After HE

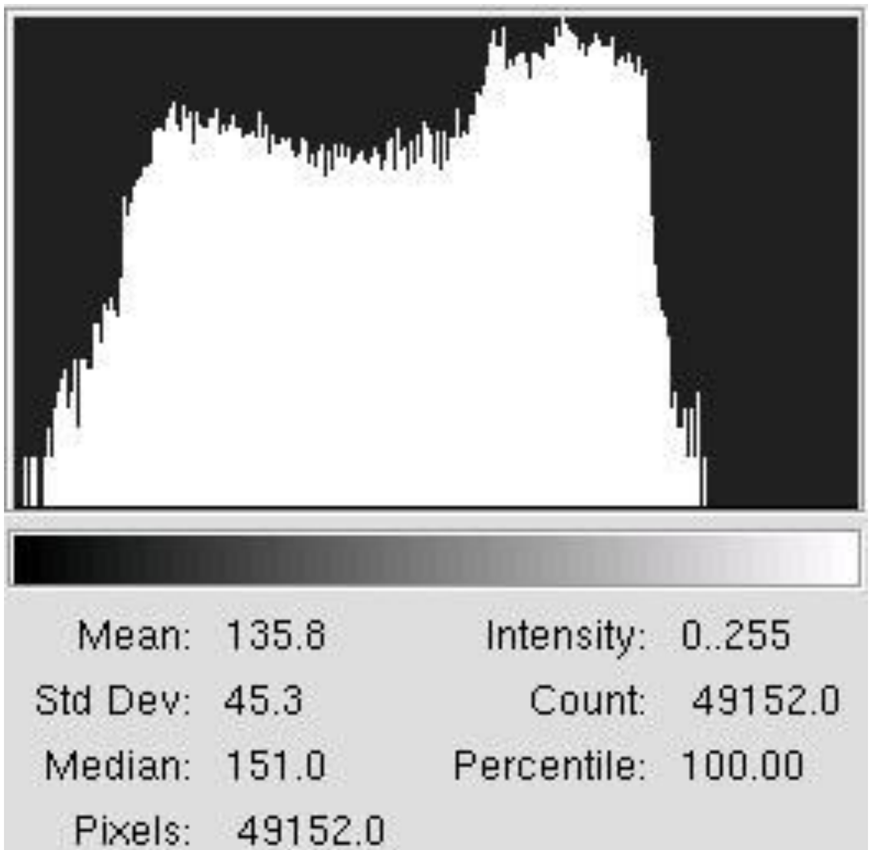
HE - example 3

Before

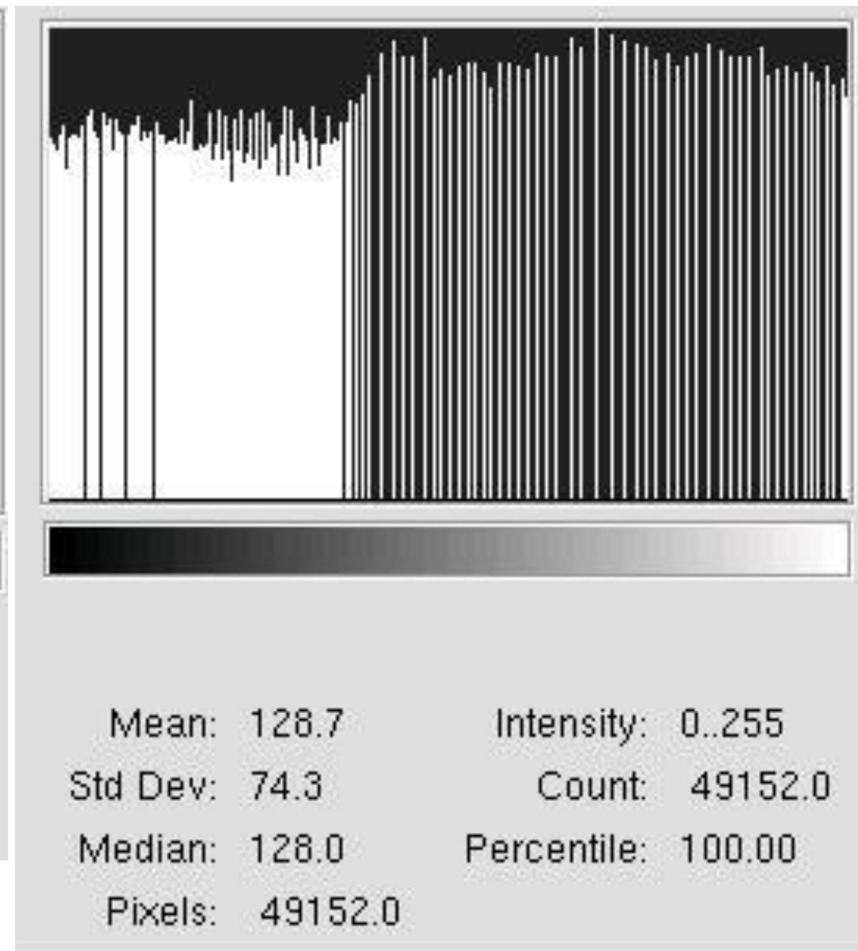
After HE

HE – example 3

Before



After



Adaptive HE

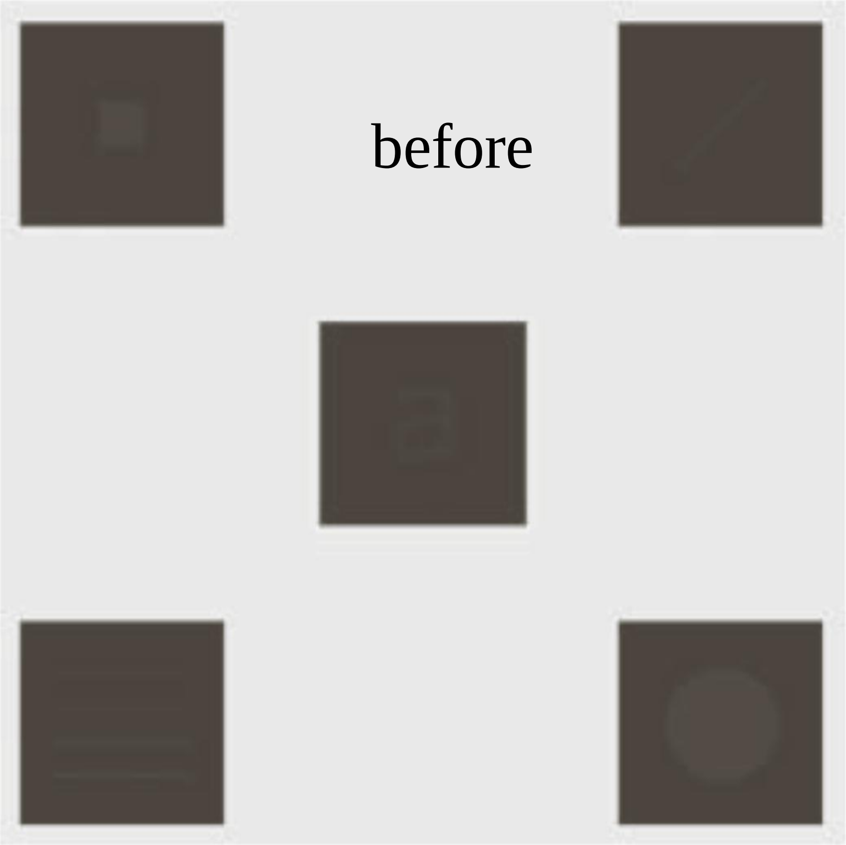
- HE works well for images with homogeneous contrast (poor)

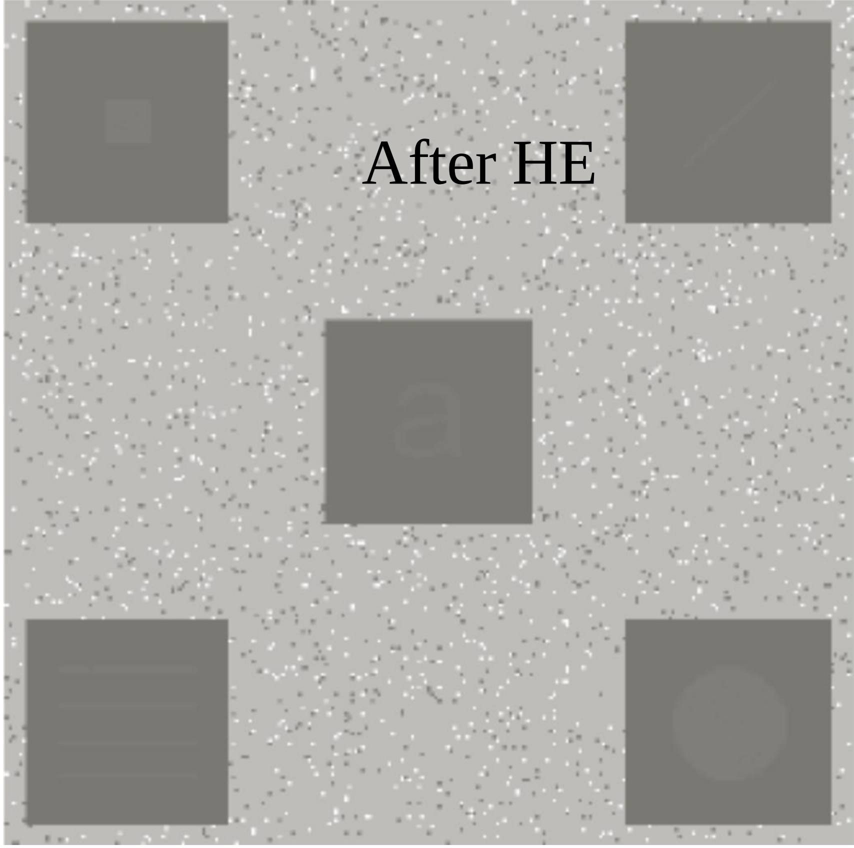
- Many images have inhomogeneous contrast
  - ex. Images with partial shadows

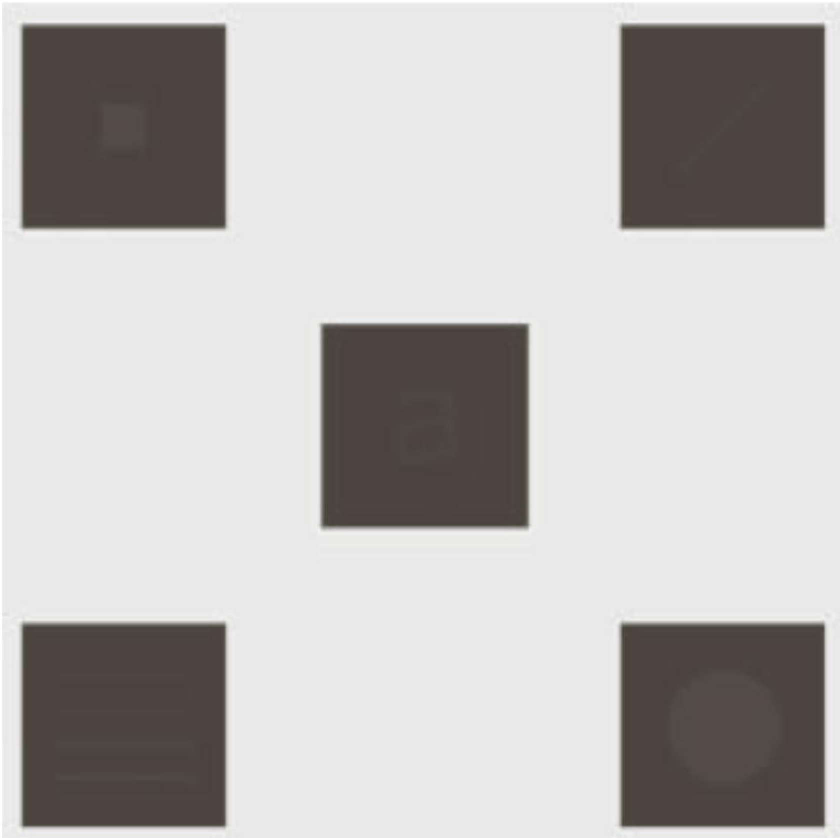
Solution: **Adaptive HE**

Approach: use local statistics to do HE

1. divide the image into blocks/regions
2. apply HE to each region


before


After HE



Original

Adaptive HE with 3 X 3

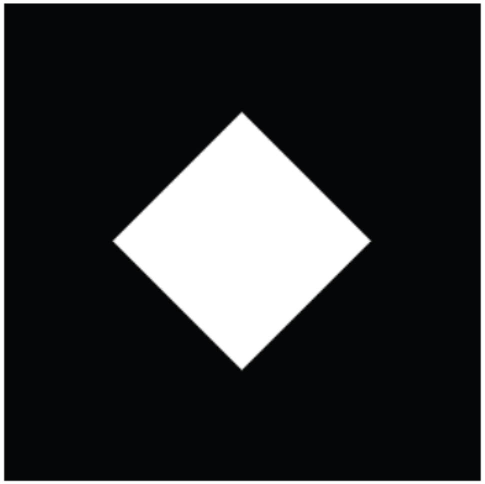
What if we increase the block size?

Usefulness of HE

- So far we saw HE helps in image enhancement
- It can also be used to binarise a greyscale image

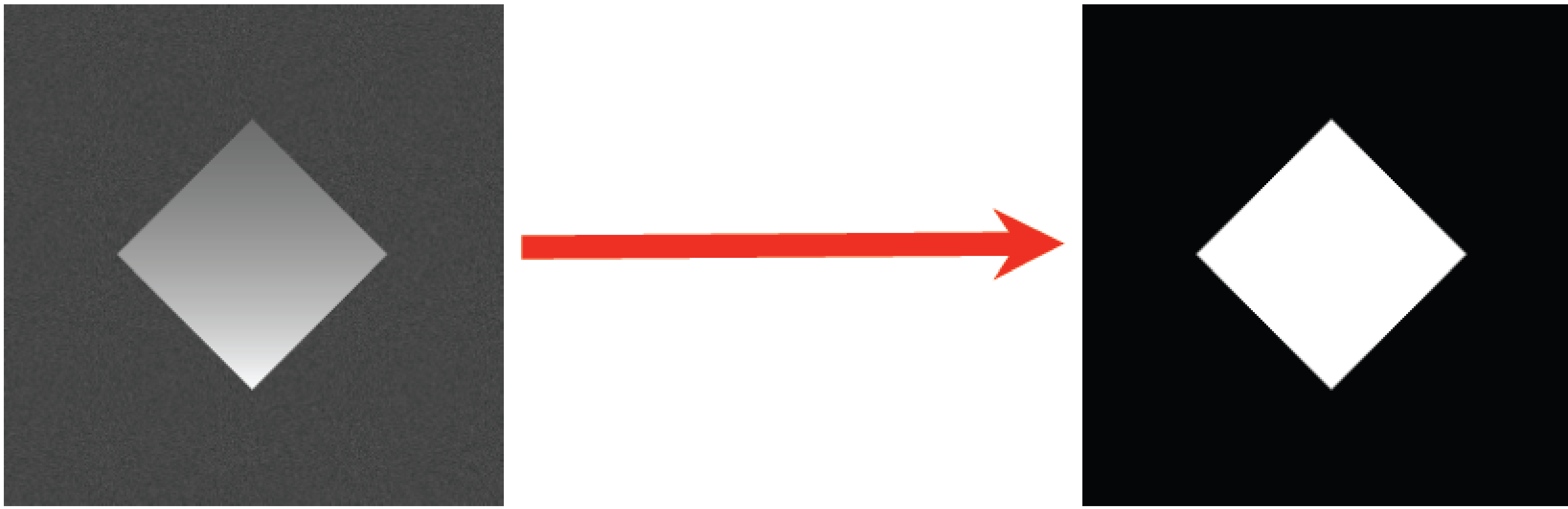
Example:

Input image
intensities 0-255



Segmentation output
0 (background)
1 (foreground)

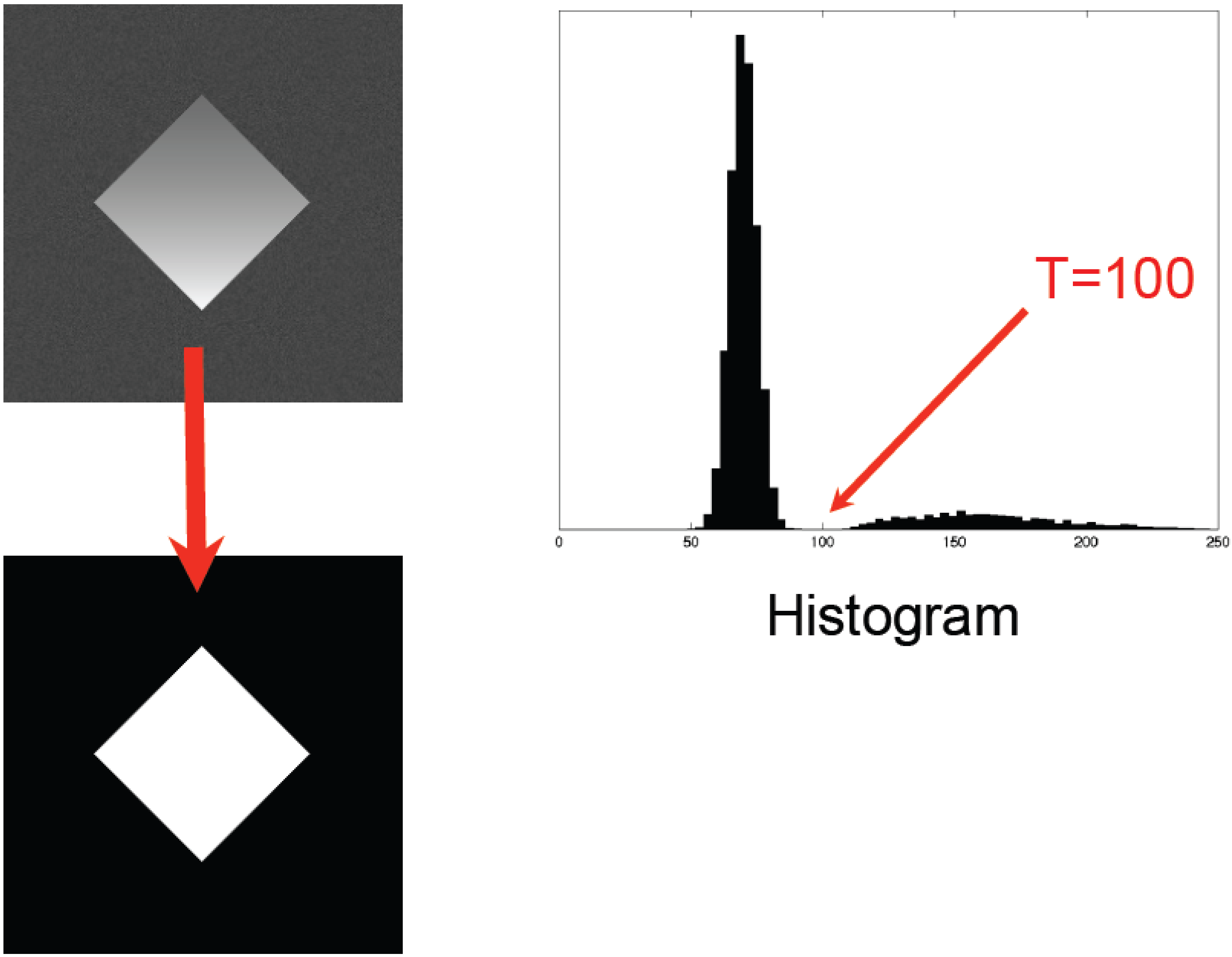
$$g(x,y) = \begin{cases} 1 & if \quad f(x,y) > T \\ 0 & if \quad f(x,y) \leq T \end{cases}$$

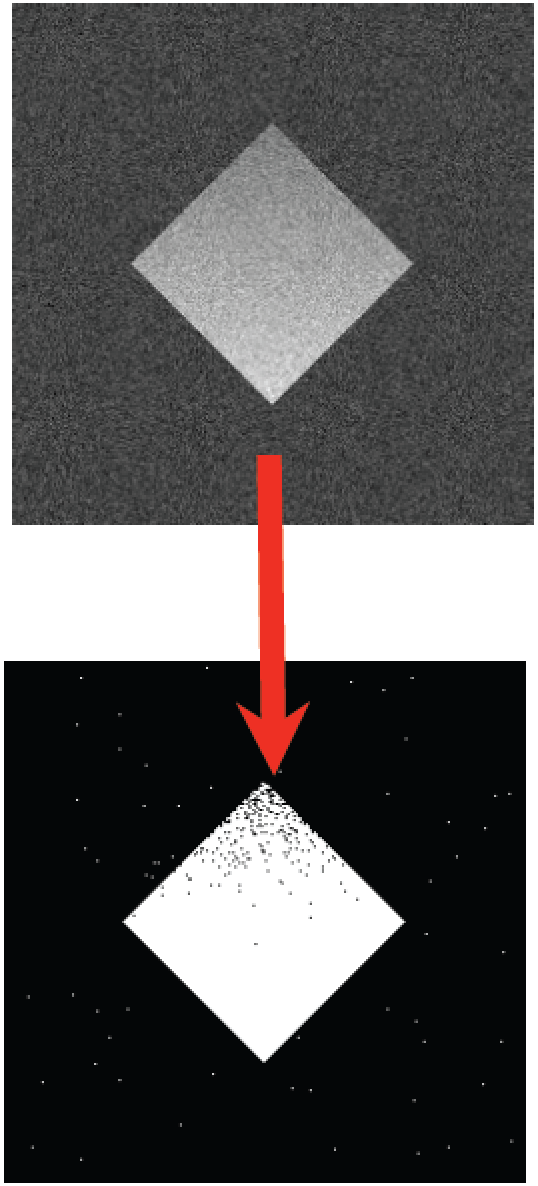


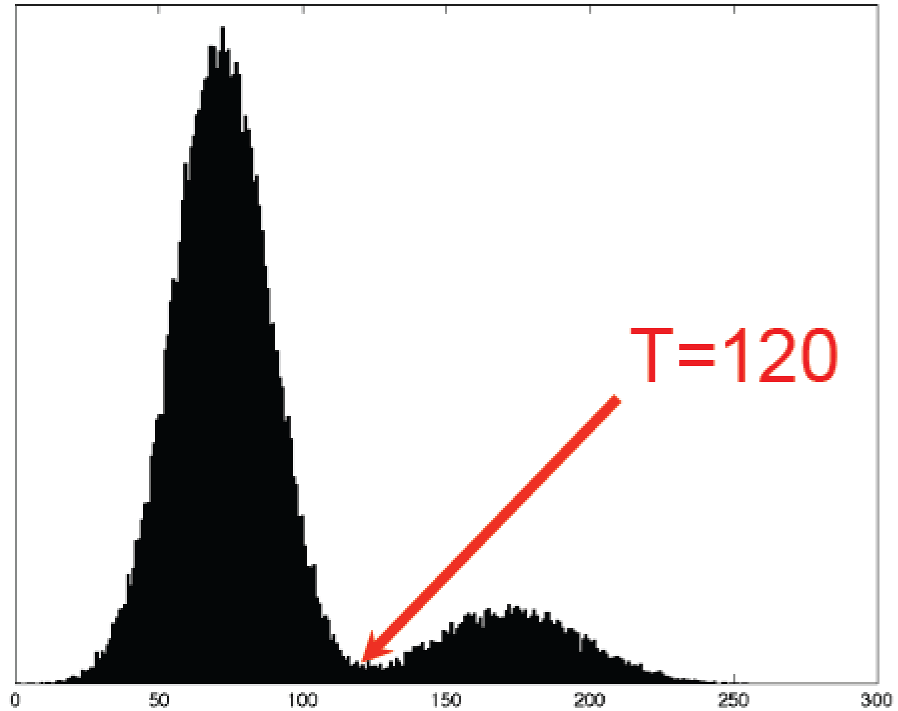
Input image f(x,y)
intensities 0-255

Segmentation output g(x,y)
0 (background)
1 (foreground)

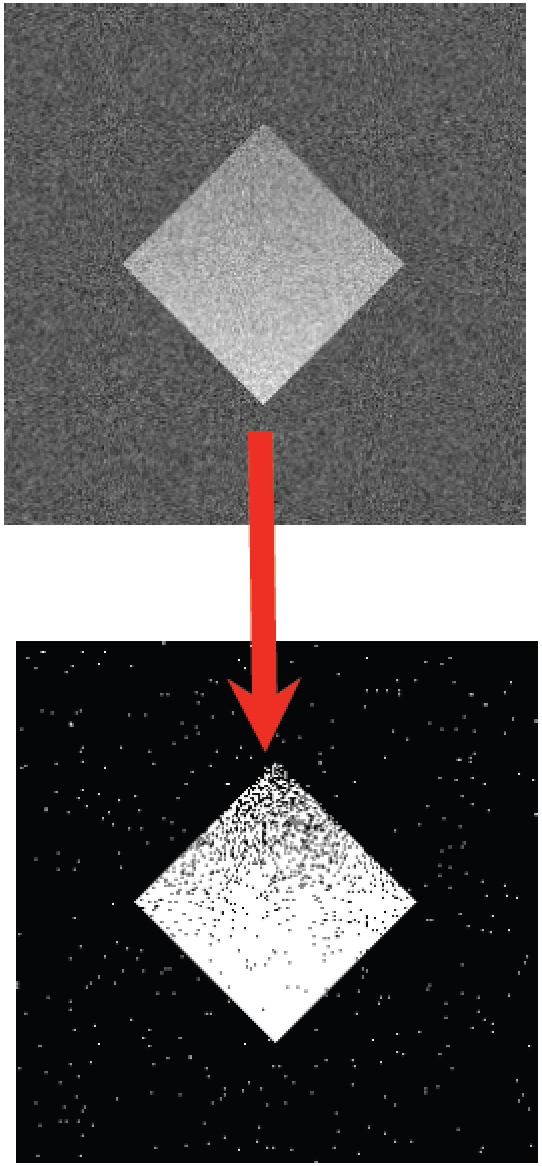
- How can we choose T?
  - –Trial and error
  - –Use the histogram of f(x,y)

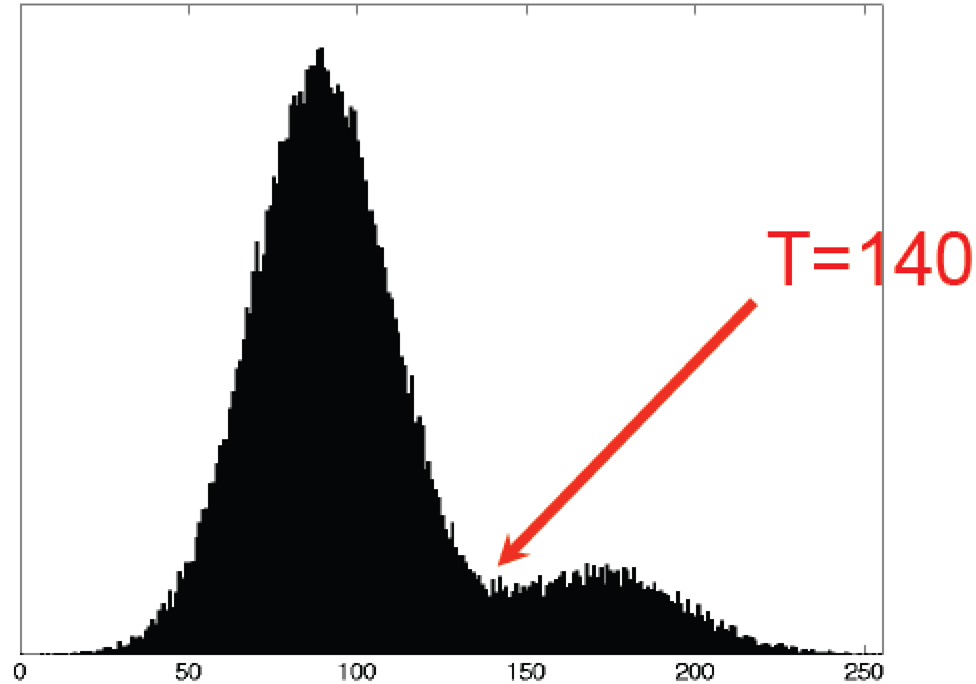

T=100
0
50
100
150
200
250
Histogram





Noise





Low SNR

Evaluating enhanced images

- Image enhancement seeks to alter the subjective appearance of an image
  - Improve contrast, remove shadows, correct for non-uniform illumination

How do you evaluate the results?

- Need a metric to quantify improvement

Global Contrast of an Image I

Contrast is a measure of the dynamic range of the grey (pixel) values

- Simple definition

  g is the grey value $$C_I = \frac{g_{\max}}{g_{\min}}$$

- Statistical definition $C_S = \sigma_I$

Global Contrast …contd.

- Weber's definition

$$C_{WI} = \frac{g_{\max} - g_{mean}}{g_{mean}}$$

- Michelson's definition (for sinusoidal gratings)

$$C_{MI} = \frac{g_{\max} - g_{\min}}{2g_{mean}} = \frac{g_{\max} - g_{\min}}{g_{\max} + g_{\min}}$$