Digital Image Processing (CSE/ECE 478)
Lecture8: Discrete Fourier Transform

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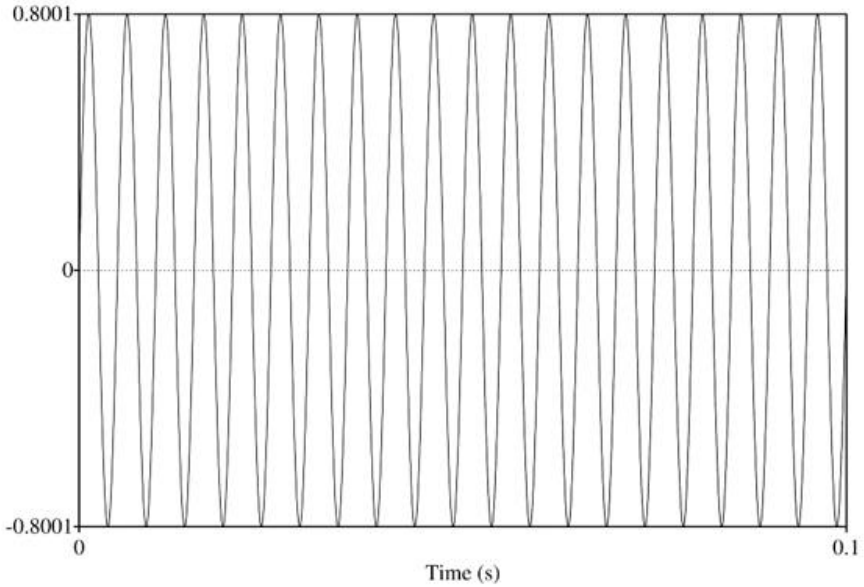
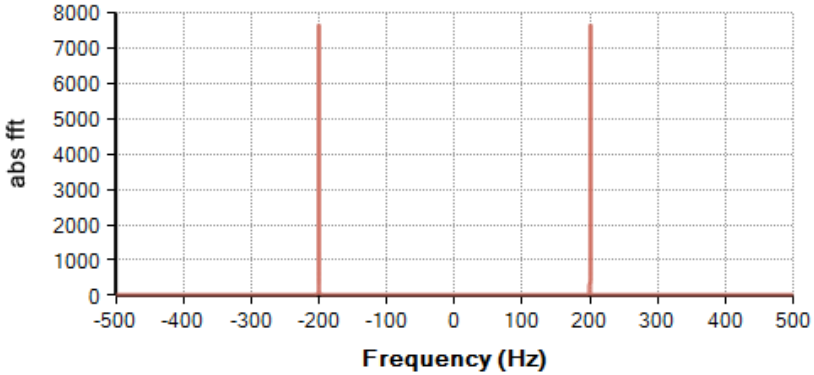
Transform based approach

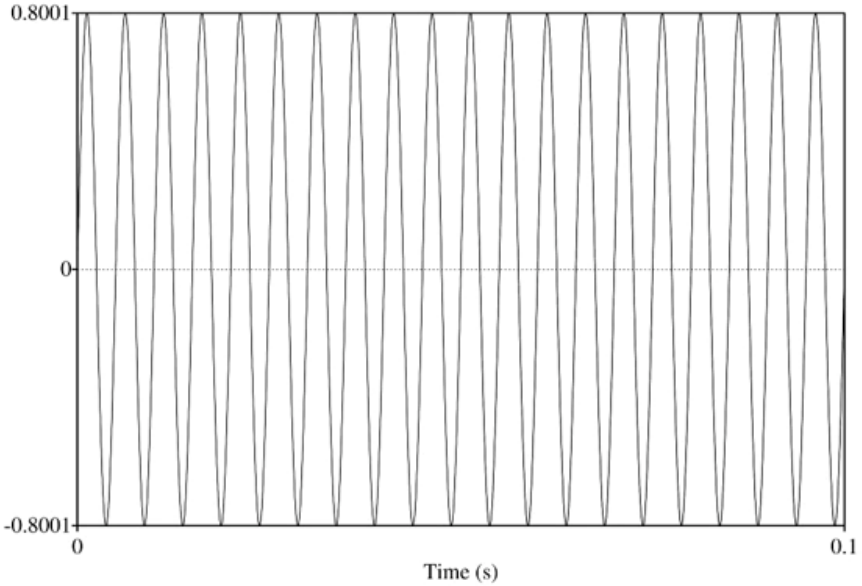
- A problem is defined in one setting
  - E.g. denoising a given rectangular image
- Transform the problem to a new domain (to a different basis)
  - Where it is more easily solvable
- Solve the problem in transformed setting
- Transform the solution back to original domain

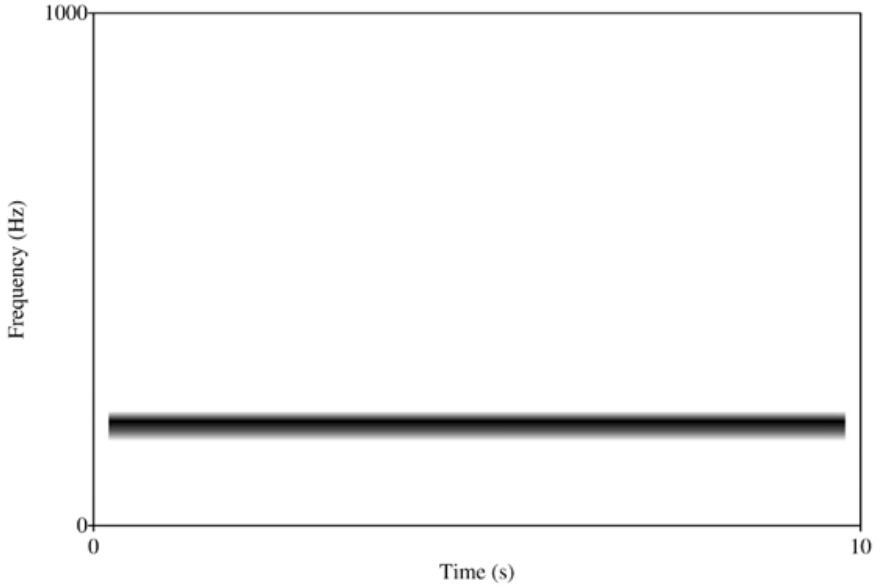
Motivation


0.8001
0
-0.8001
0
0.1
Time (s)


8000
7000
6000
5000
4000
3000
2000
1000
0
abs fft
-500
-400
-300
-200
-100
0
100
200
300
400
500
Frequency (Hz)

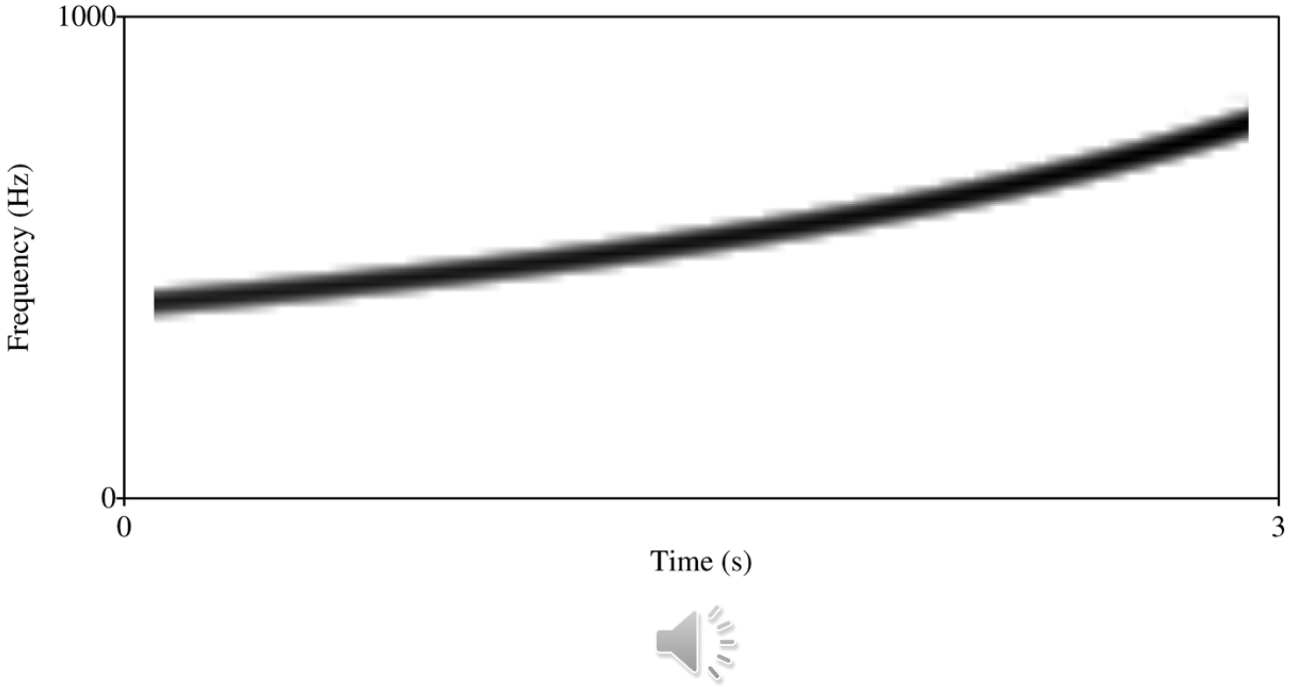
Motivation





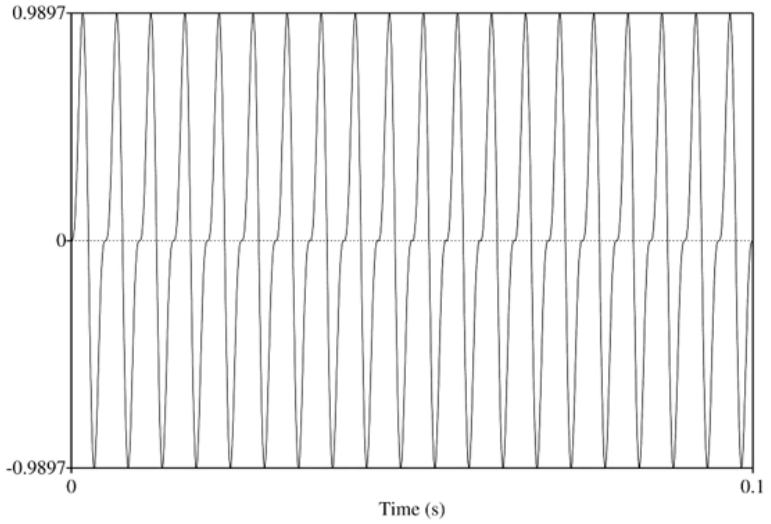
Courtesy: floridalinguistics.com

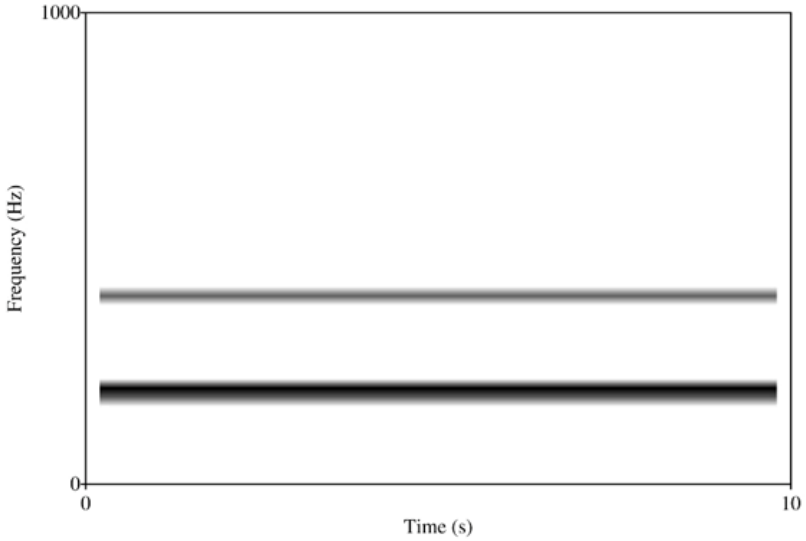
Motivation



Courtesy: floridalinguistics.com

Motivation





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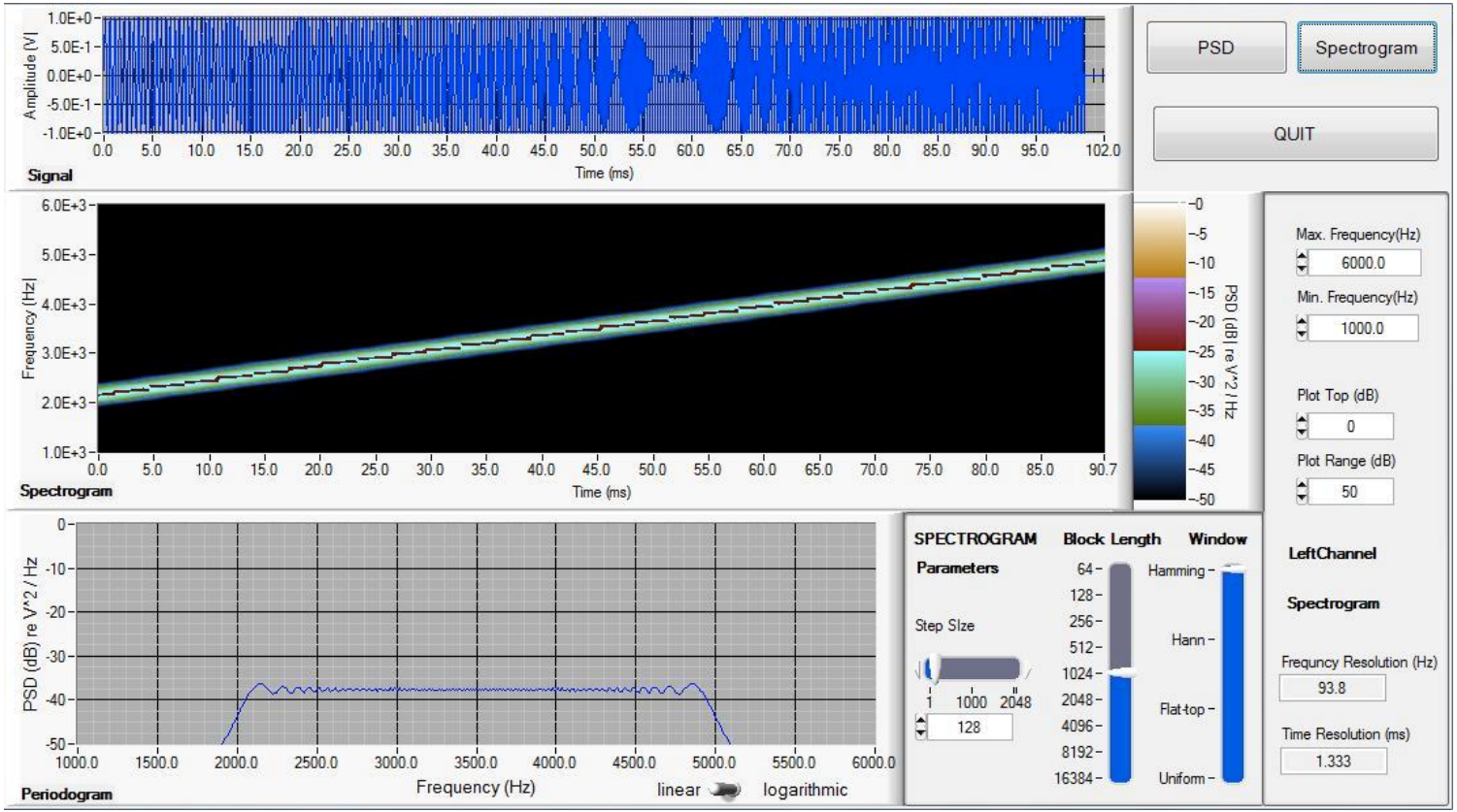
Motivation



Courtesy: benthowave.com

Motivation

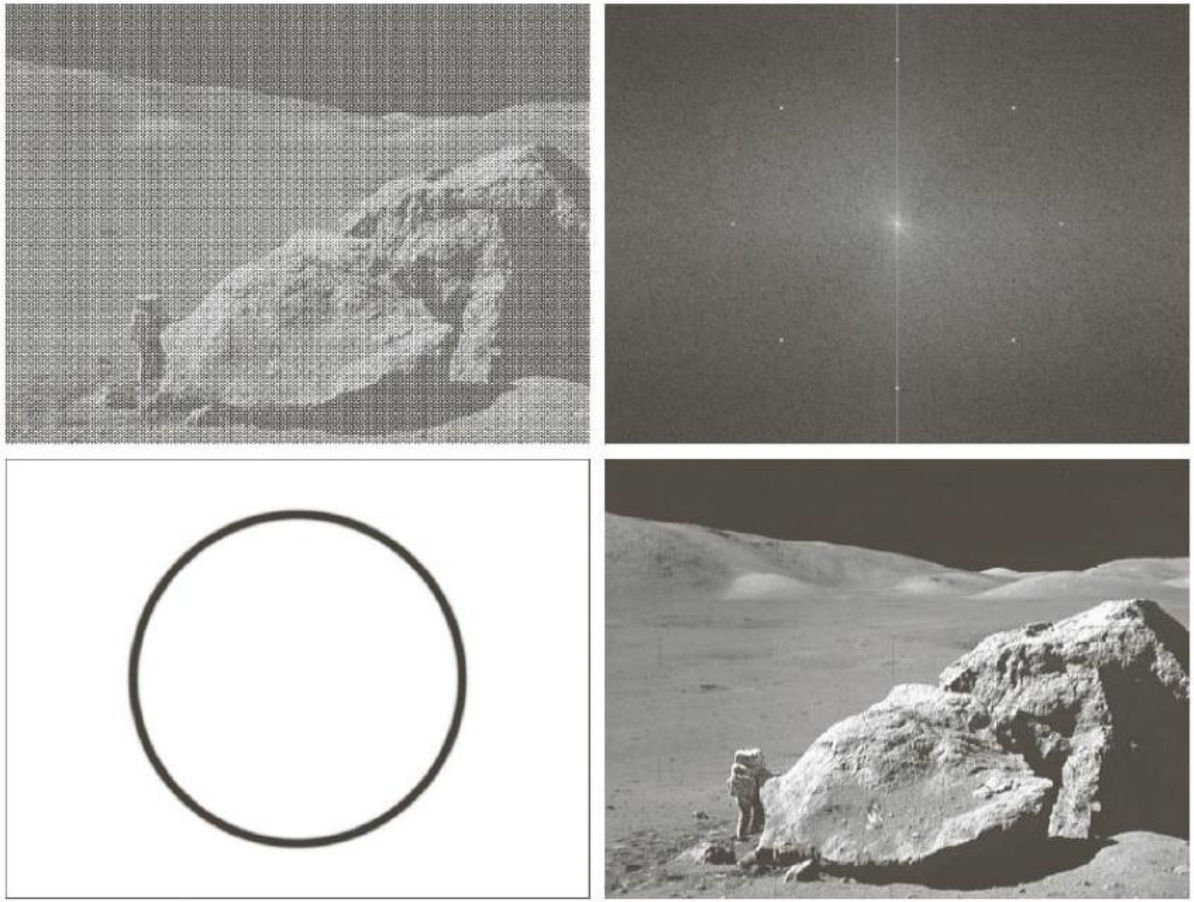


Image courtesy: Gonzalez and Woods

Todays lecture

- Understand the intuition behind DFT
- Discuss with some illustrative examples

Fourier Transform – 1D

DFT

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi\mu x} dx$$

Inverse DFT

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du$$

Pair

Discrete Fourier Transform (DFT) – 1D

DFT

$$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) e^{-j2\pi\mu x/M}$$

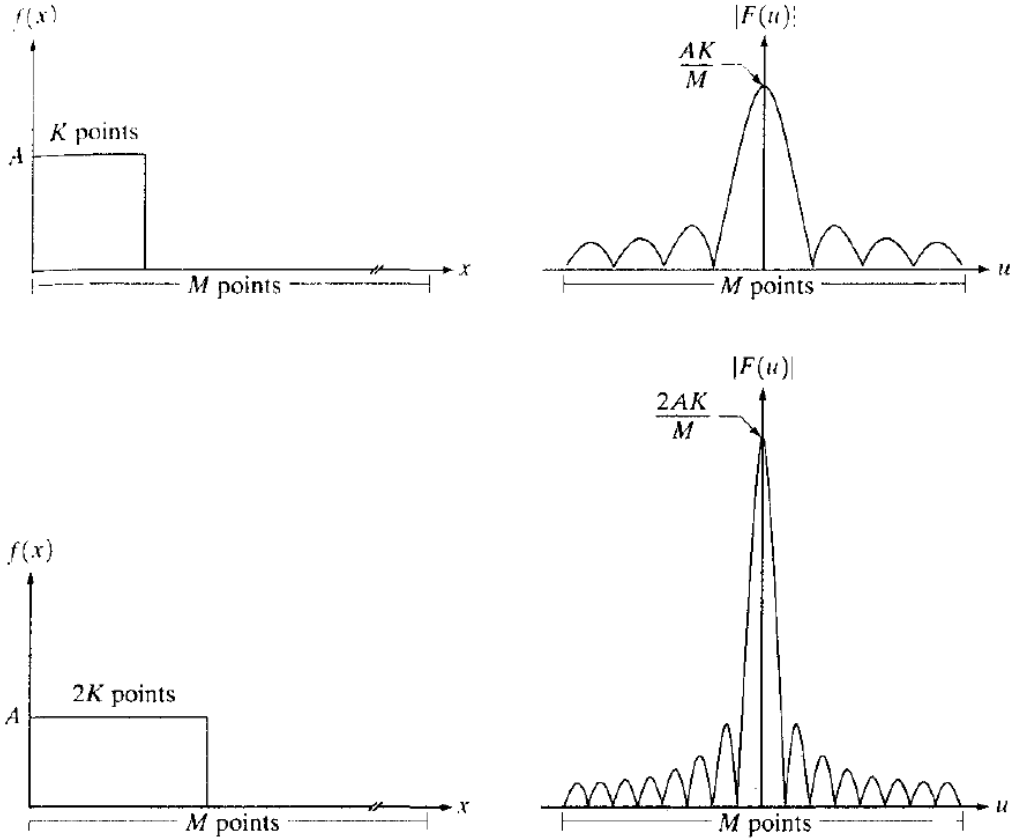
Inverse DFT

$$f(x) = \sum_{x=0}^{M-1} F(u) e^{j2\pi\mu x/M}$$

Important Terms

- Magnitude spectrum
- Phase Spectrum
- Power Spectrum

Relationship between u and x



a b
c d

FIGURE 4.2 (a) A discrete function of M points, and (b) its Fourier spectrum. (c) A discrete function with twice the number of nonzero points, and (d) its Fourier spectrum.

$$\Delta u = \frac{1}{M\Delta x}$$

Fourier Transform – 2D

DFT

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(ux+vy)}dxdy$$

Inverse DFT

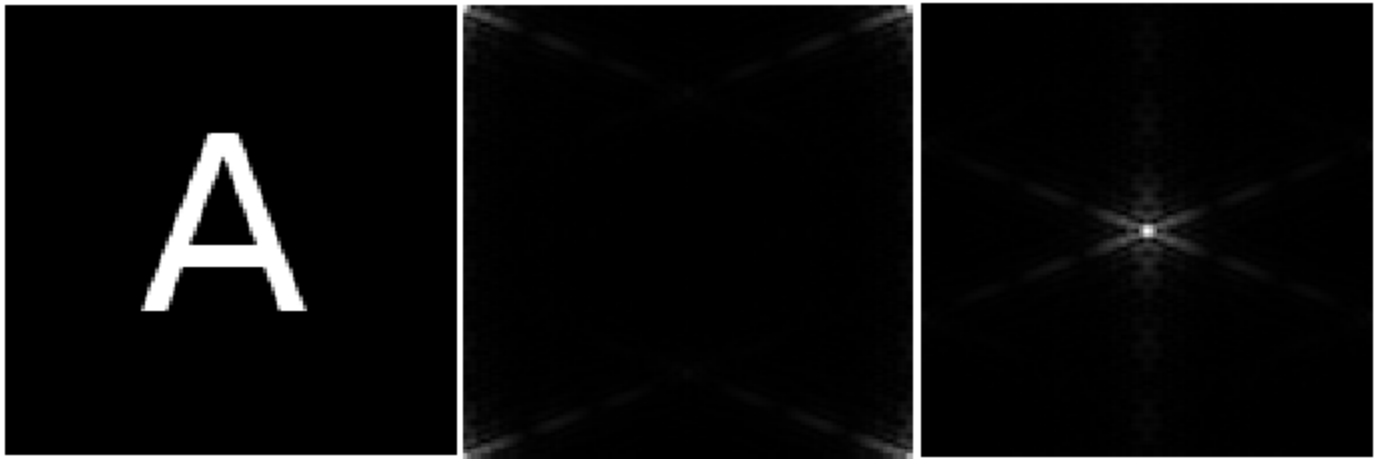
$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)e^{j2\pi(ux+vy)}dudv$$

Discrete Fourier Transform (DFT) – 2D

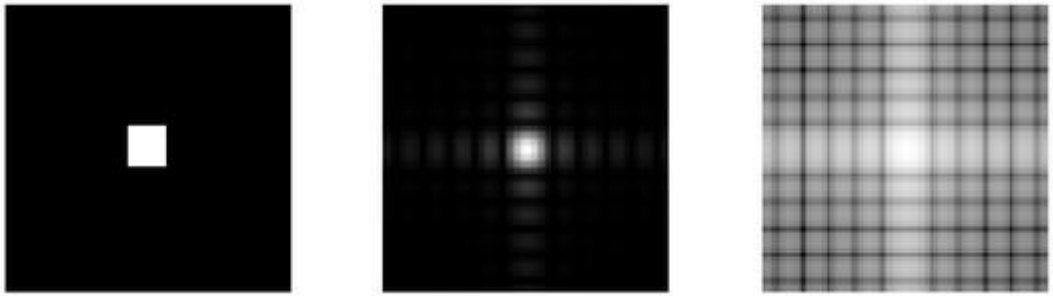
DFT $$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M+vy/N)}$$

Inverse DFT $$f(x,y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi(ux/M+vy/N)}$$

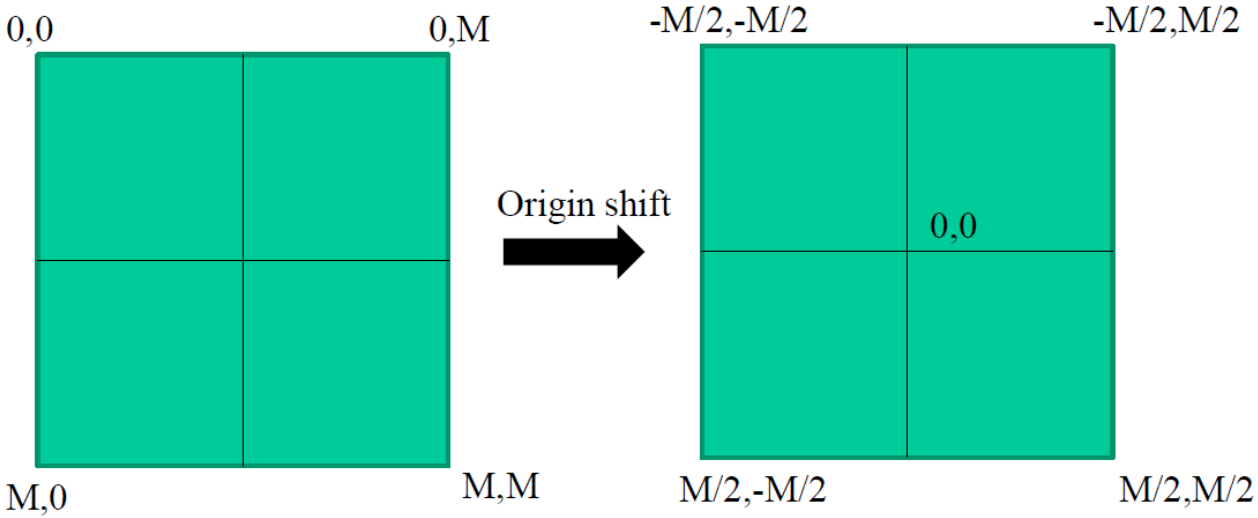
Discrete Fourier Transform (DFT) – 2D



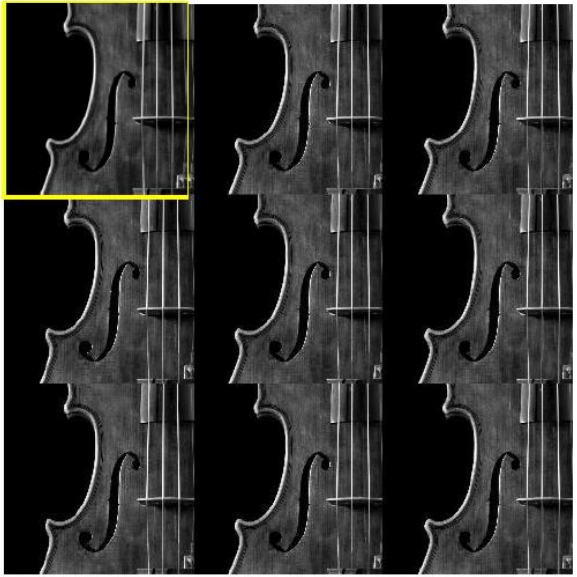
Discrete Fourier Transform (DFT) – 2D

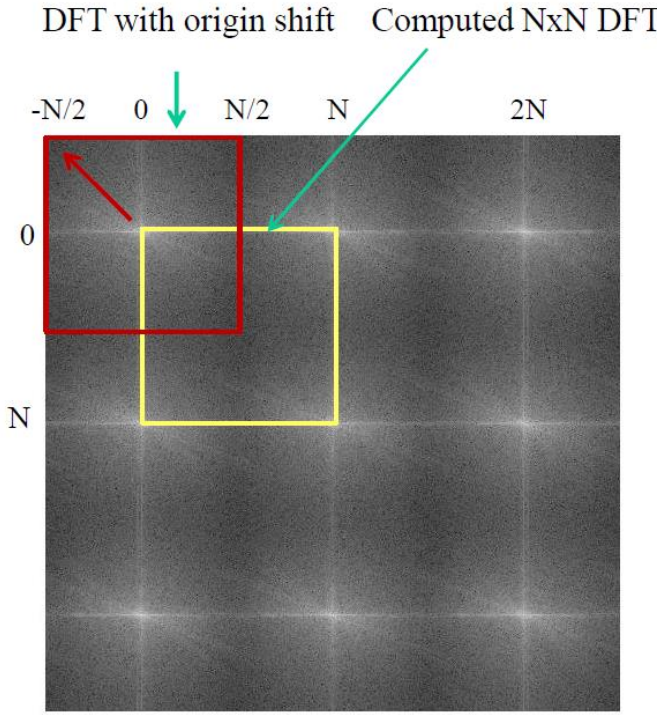


Shifting origin

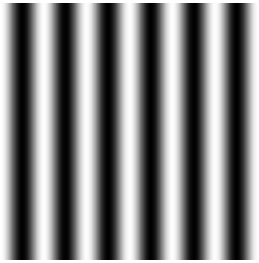


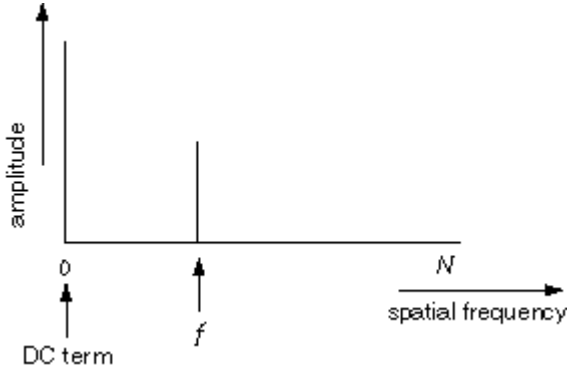
Shifting origin





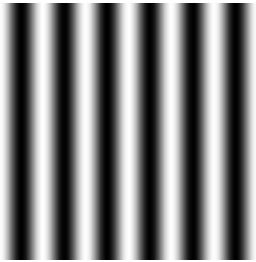
DFT Example

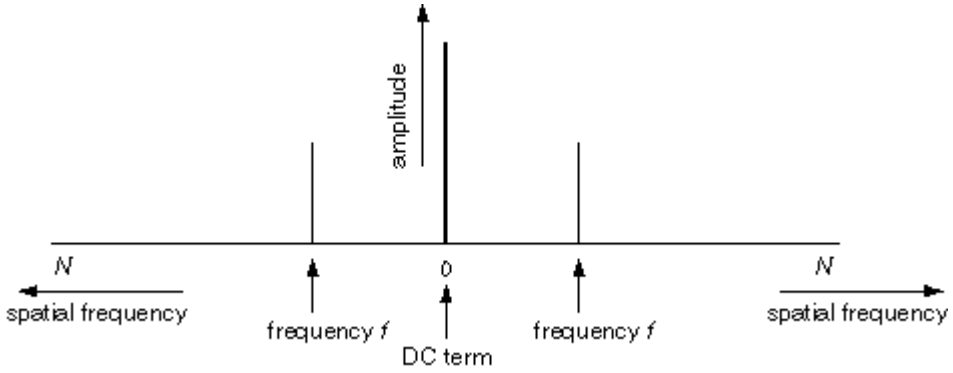




Courtesy: http://cns-alumni.bu.edu/~slehar/fourier/fourier.html

DFT Example





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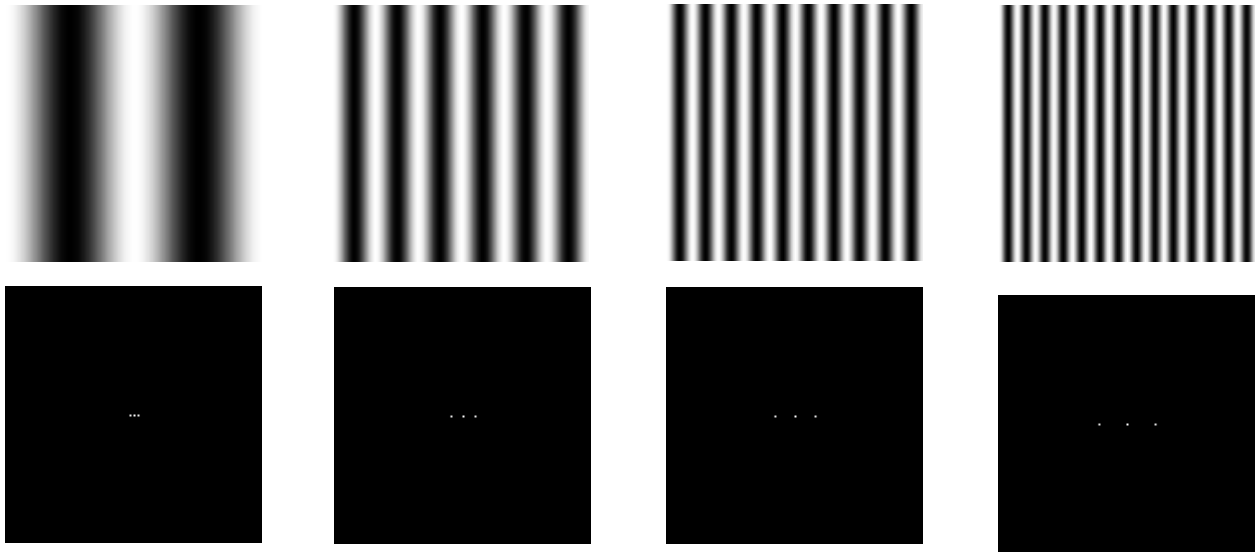
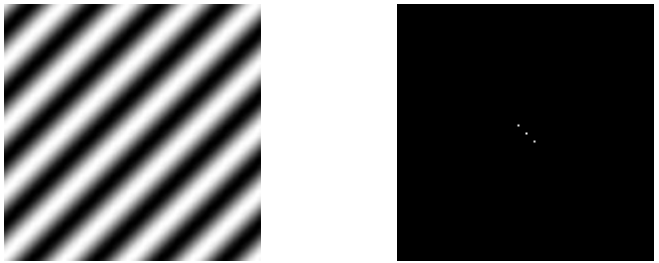
DFT Example



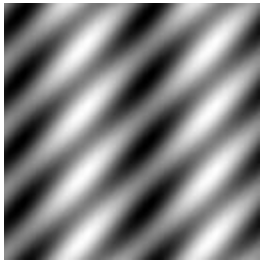
Courtesy: http://cns-alumni.bu.edu/~slehar/fourier/fourier.html

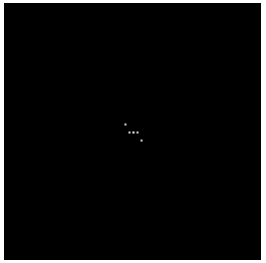
DFT Example



Courtesy: http://cns-alumni.bu.edu/~slehar/fourier/fourier.html

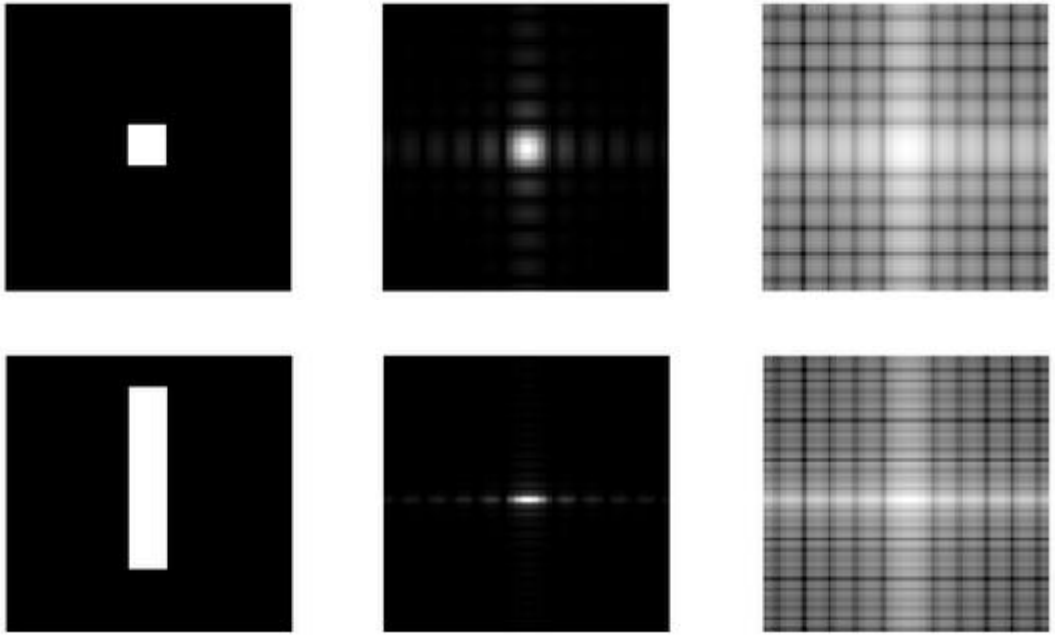
DFT Example



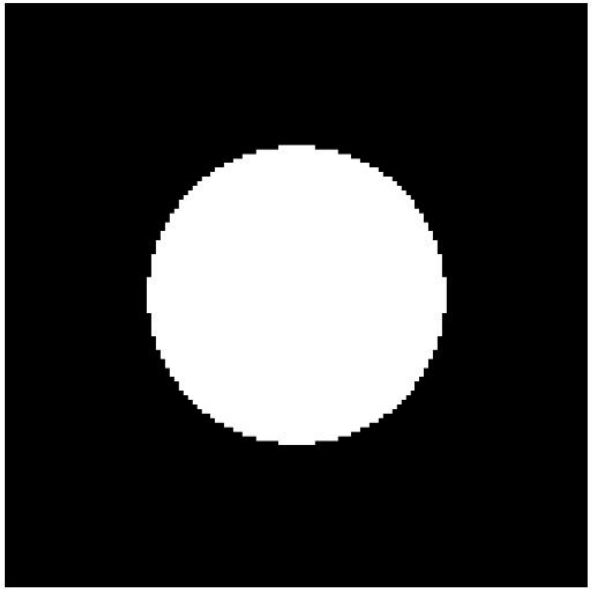


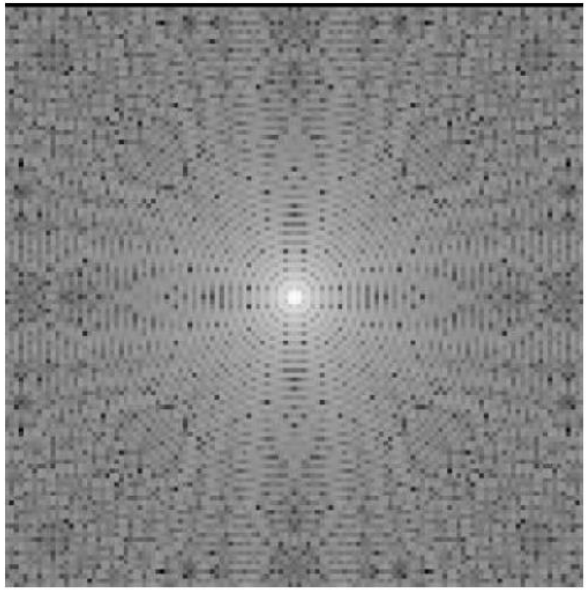
Courtesy: http://cns-alumni.bu.edu/~slehar/fourier/fourier.html

DFT Example

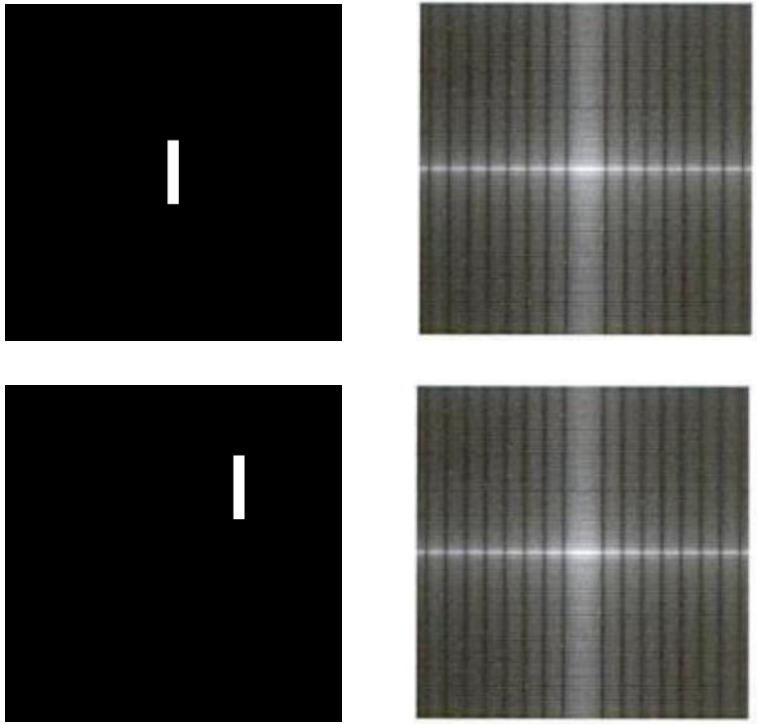


DFT Example

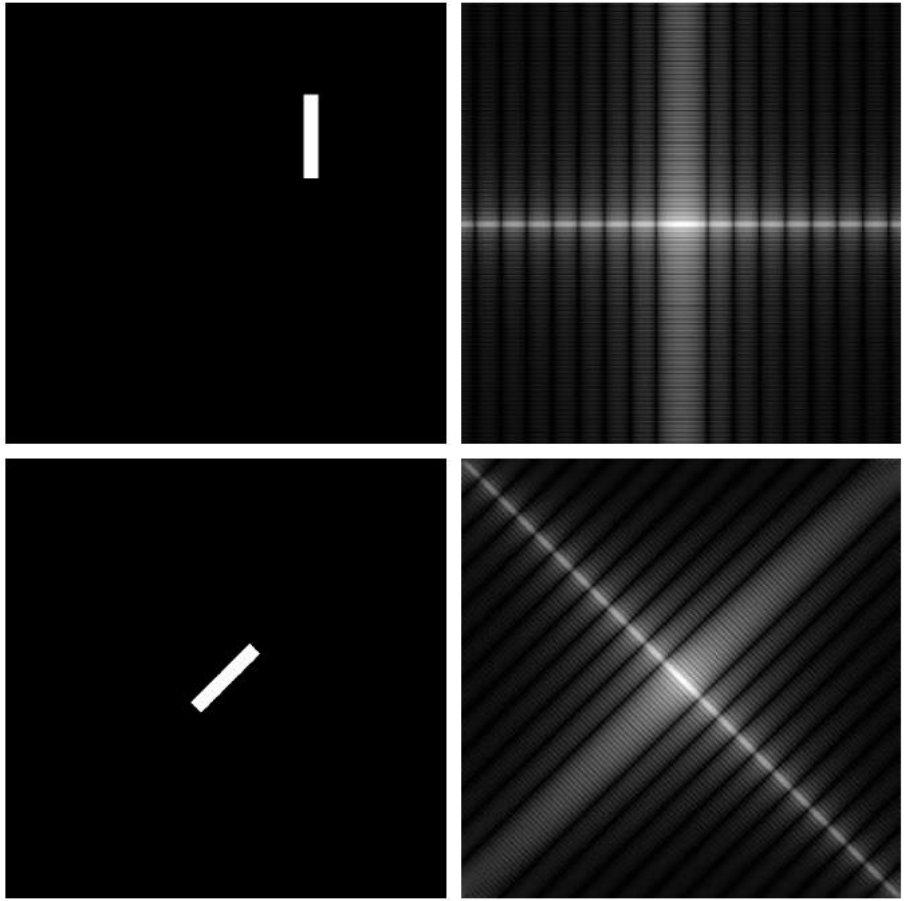




DFT Example

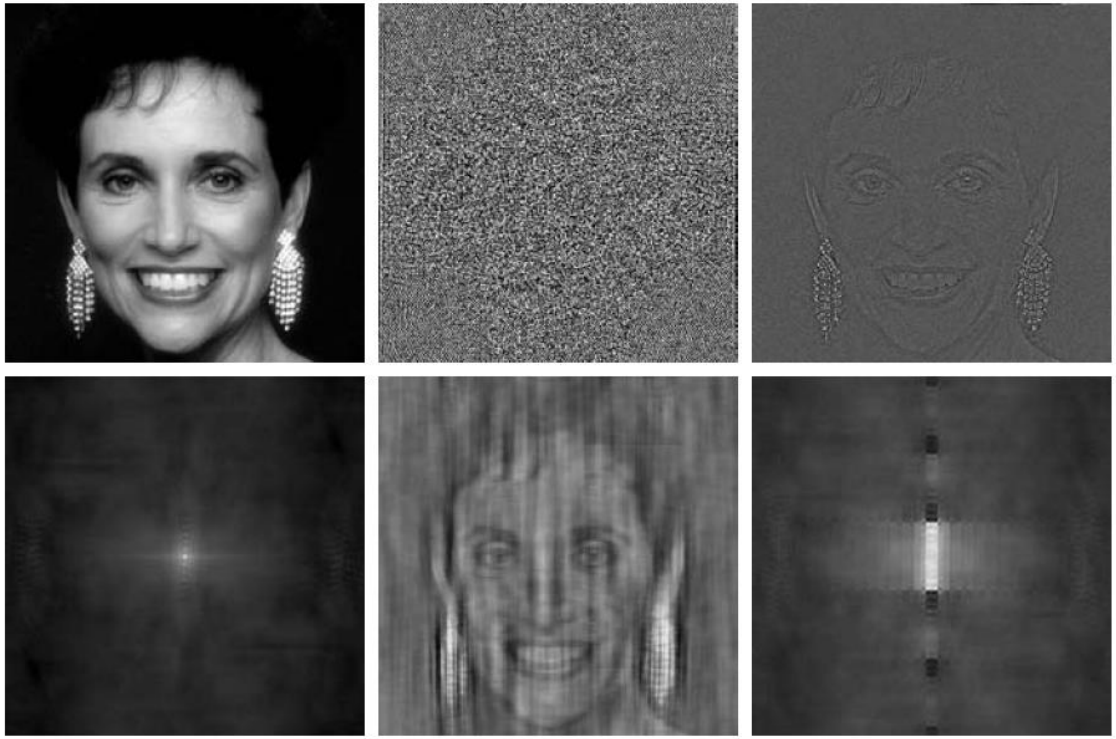


DFT Example



a b
c d

FIGURE 4.25
(a) The rectangle in Fig. 4.24(a) translated, and (b) the corresponding spectrum. (c) Rotated rectangle, and (d) the corresponding spectrum. The spectrum corresponding to the translated rectangle is identical to the spectrum corresponding to the original image in Fig. 4.24(a).



a b c
d e f

FIGURE 4.27 (a) Woman. (b) Phase angle. (c) Woman reconstructed using only the phase angle. (d) Woman reconstructed using only the spectrum. (e) Reconstruction using the phase angle corresponding to the woman and the spectrum corresponding to the rectangle in Fig. 4.24(a). (f) Reconstruction using the phase of the rectangle and the spectrum of the woman.