# Digital Image Processing (CSE 478)
# Lecture12: Morphological operations

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# Today’s Lecture

- Morphology is a branch of biology dealing with the study of the form and structure of organisms and their specific structural features (wikipedia)

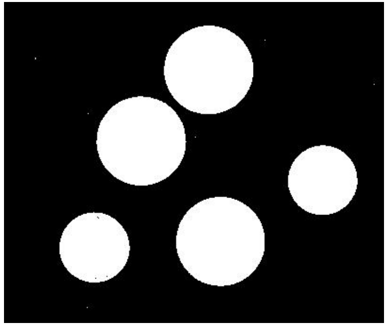
- Shape, form, structure
  - Useful for extracting and describing image component regions
  - Usually applied to binary images
  - Based on set theory

# Today's Lecture

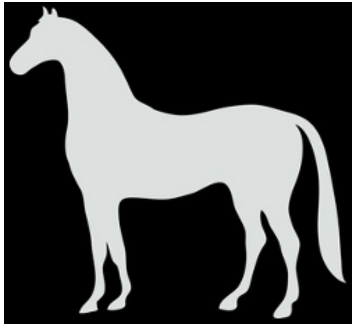
- Set theory
- Binary operations: dilation, erosion, opening, closing
- Connected components
- Morphological algorithms

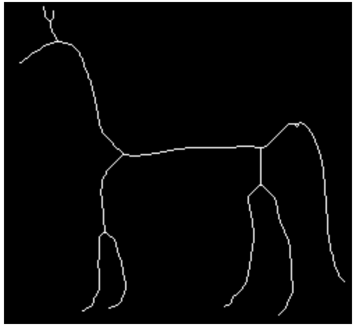
# Binary Images

- Thresholding (im2bw matlab)



- Binary regions
  - Count number of binary regions
  - Measure shape
- Typical applications
  - Character and texts
  - Maps
  - Fingerprints





# Set Theory Concepts

- 2D binary image as a set of points
  - A is unorderd set of pairs (x,y) such that the image value at (x,y) is equal to 1

$$A = \{(x, y) \mid I(x, y) = 1\}$$

- Union of two sets $(A \cup B)$
  - The set belonging to A,B, or both

- Intersection of two sets $(A \cap B)$
  - The set belonging to both A and B

- w “is an element of” set A
  - $w \in A$

# Set Theory Concepts

- Complement $(A^c)$
  - The set elements that are not in A

$$A^c = \{\ w \mid w \notin A\}$$

- Difference of two sets $(A - B)$
  - The set belonging to A, but not to B

$$A - B \ = \{\ w \mid w \in A, w \notin B\}$$

- Subset $(A \subseteq B)$
  - A is subset of B if every element of A is also in B

- Empty set {}
  - $\phi$

# Set Operations

$$A = \{(x,y) \mid I_A(x,y) = 1\}, \quad B = \{(x,y) \mid I_B(x,y) = 1\}$$

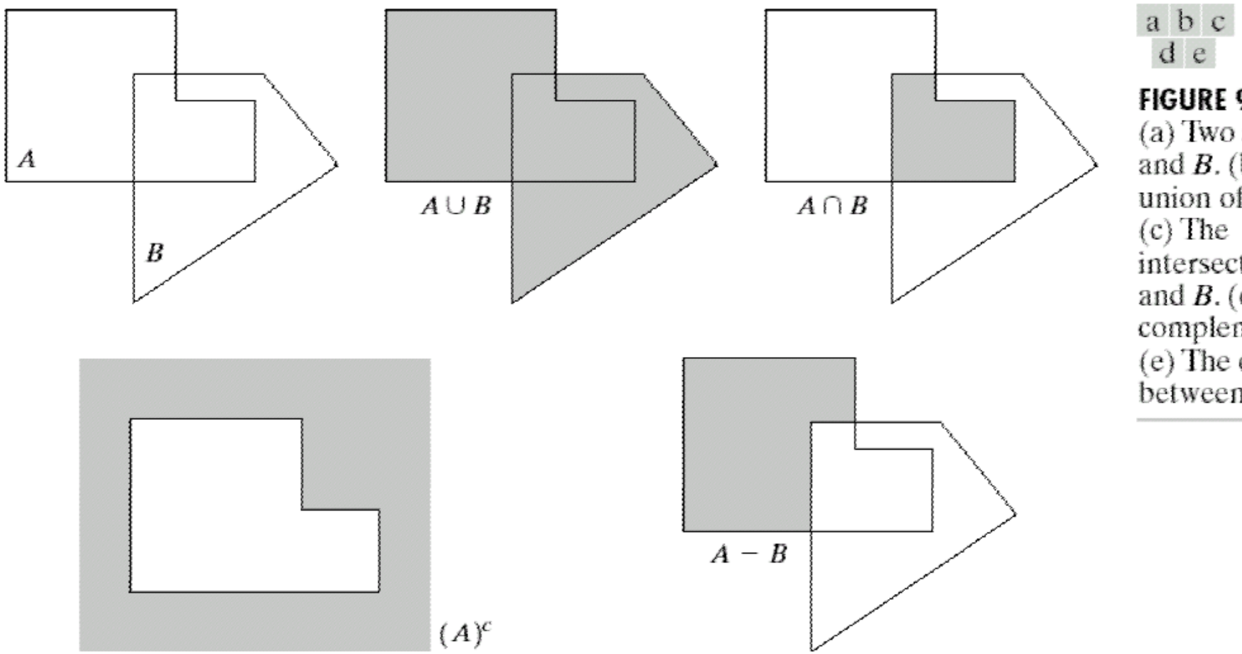
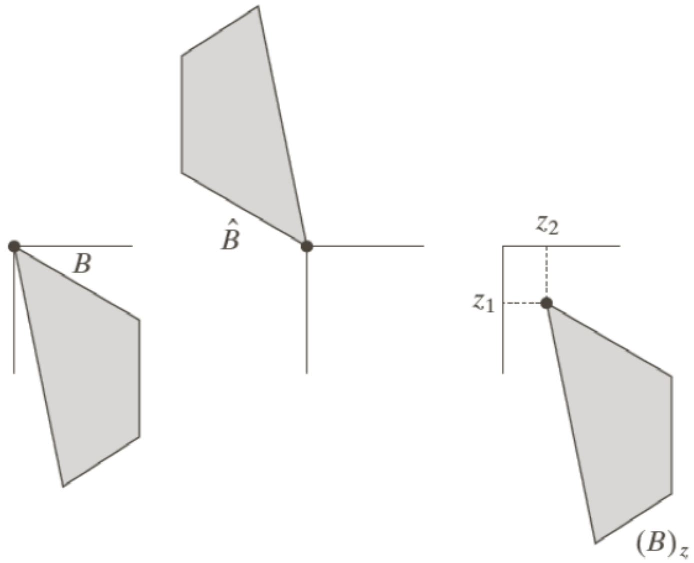



**FIGURE 9.1** (a) Two sets $A$ and $B$. (b) The union of $A$ and $B$. (c) The intersection of $A$ and $B$. (d) The complement of $A$. (e) The difference between $A$ and $B$.

# Reflection and Translation

$$\hat{B} = \left\{ \vec{w} \,\middle|\, \vec{w} = -\vec{b}, \quad for\ \vec{b} \in B \right\}$$

$$(B)_z = \left\{ \vec{c} \,\middle|\, \vec{c} = \vec{b} + \vec{z}, \quad for\ \vec{b} \in B \right\}$$




a b c

**FIGURE 9.1**
(a) A set, (b) its reflection, and (c) its translation by $z$.

# Structuring Elements (SEs)

- Morphological operations are defined based on "structuring elements (SEs)"
- SEs → small sets or sub images used to probe an image under study

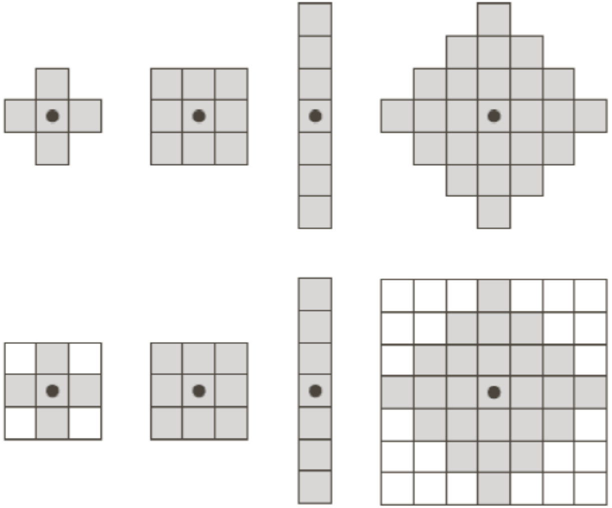
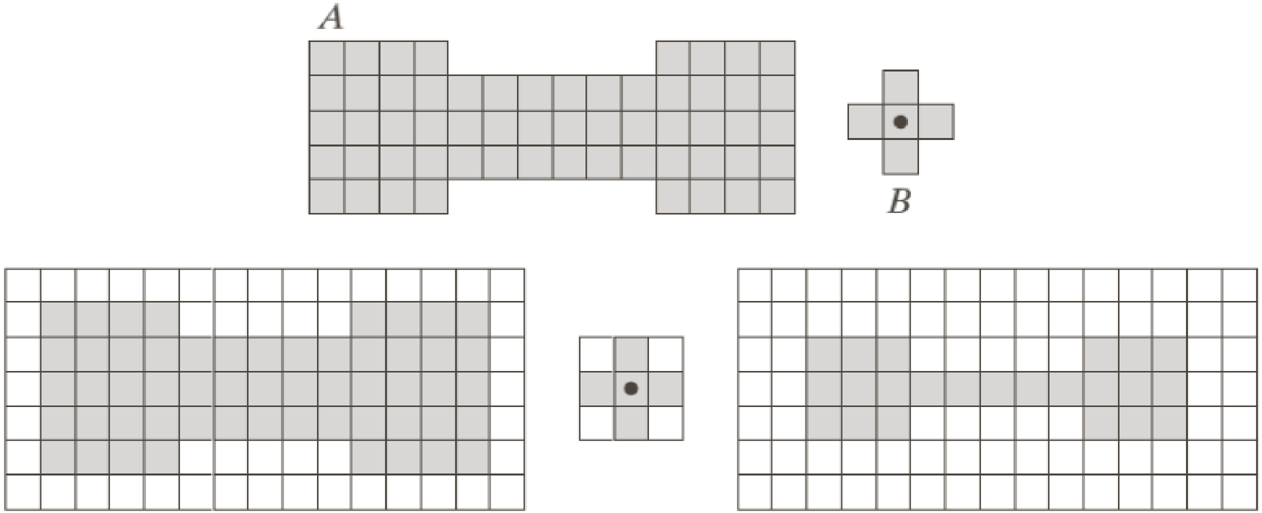


**FIGURE 9.2** First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

The dots denote the centers of the SEs

# SE operation example

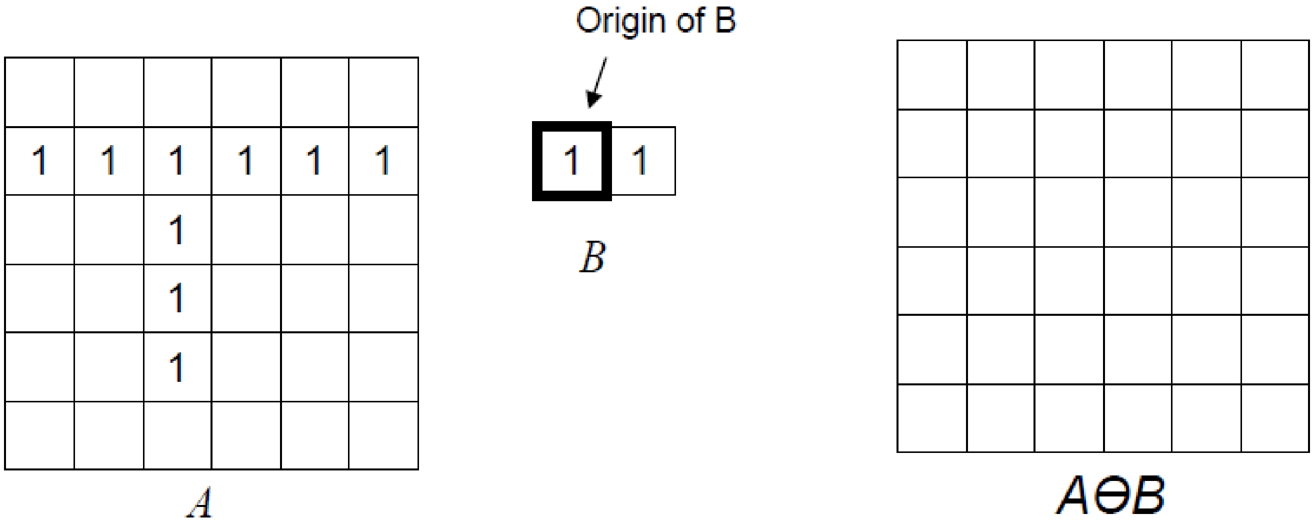
- Run B over A (origin of B visits every element of A)
- Mark if B is completely contained in A



# EROSION

- The erosion of set A by set (structuring element) B is

$$A \ominus B = \{z | (B)_z \subseteq A\}$$



# EROSION

- The erosion of set A by set (structuring element) B is

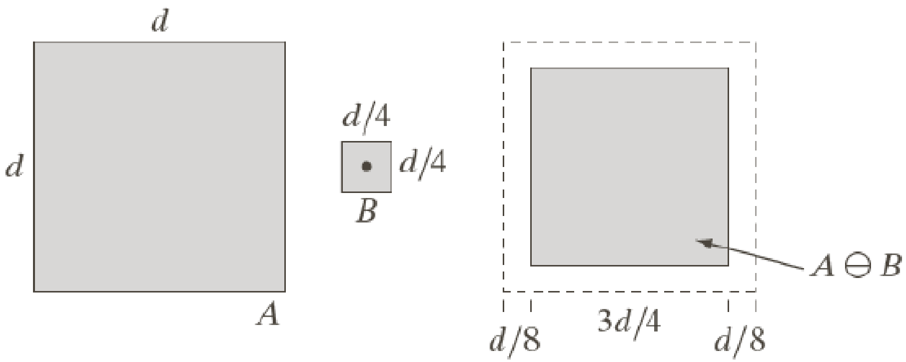
$$A \ominus B = \{z | (B)_z \subseteq A\}$$

Completely contained is equivalent to not sharing any common elements with the background

$$A \ominus B = \{z | (B)_z \cap A^c = \emptyset\}$$

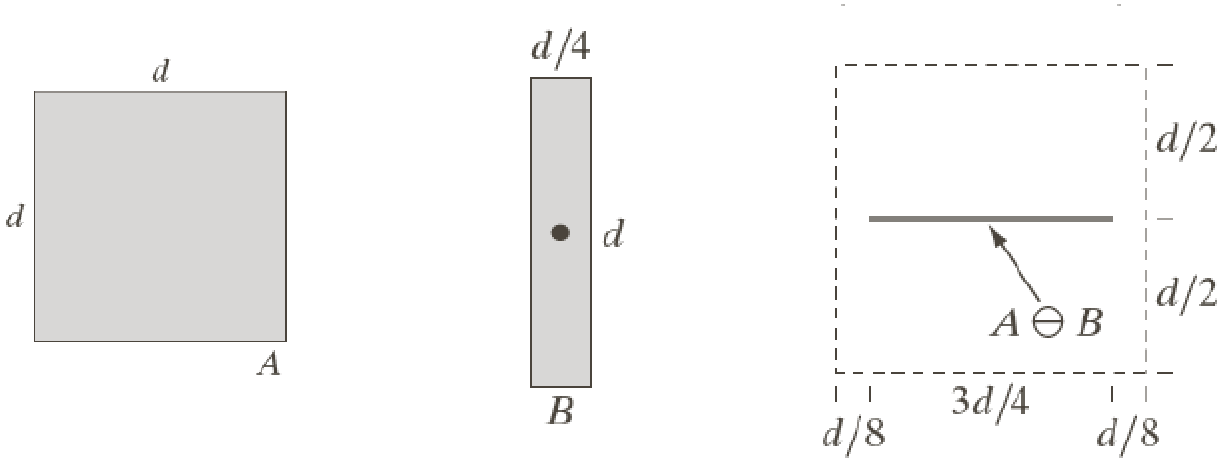
# EROSION

- More examples



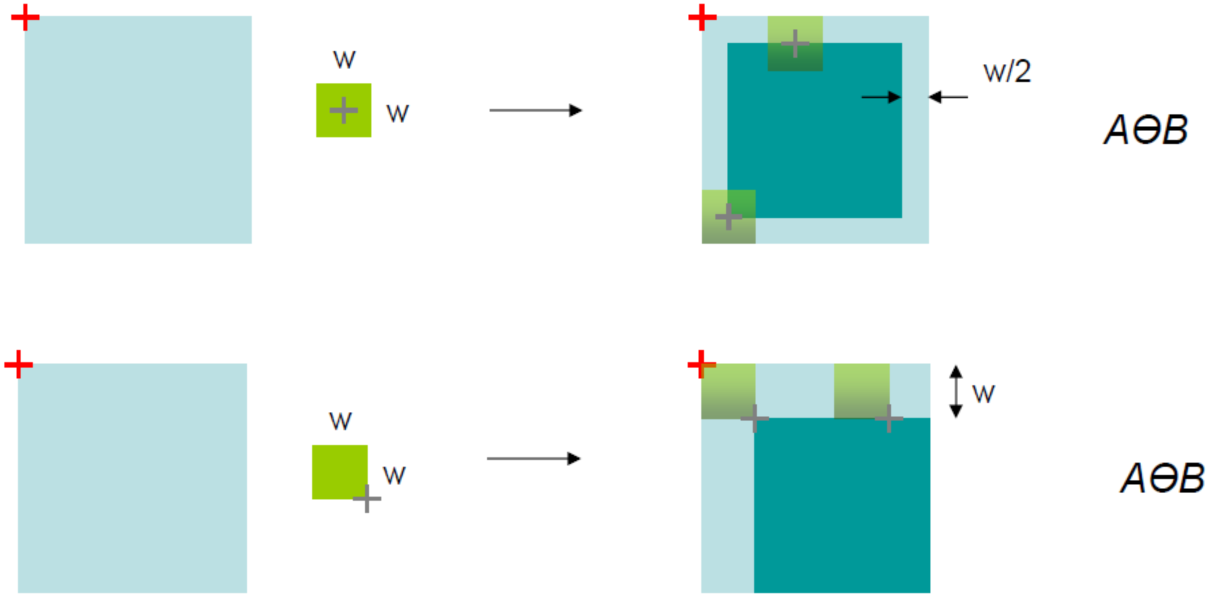
# EROSION

- More examples



# EROSION

- Changing the center

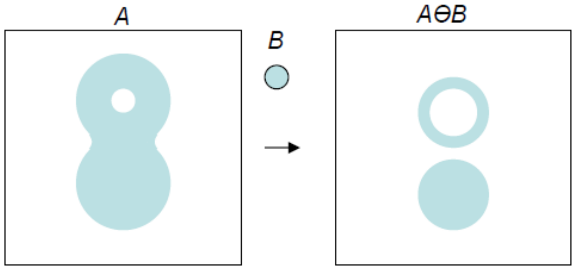


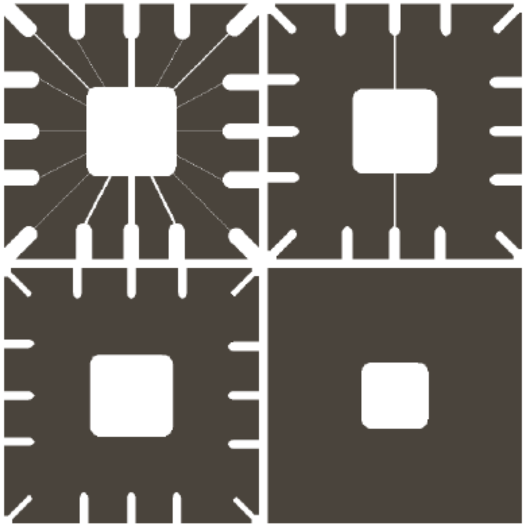

Source: William Hoff

# EROSION

- Enlarges holes, breaks thin parts, shrinks object
- Not commutative

$$A \Theta B \neq B \Theta A$$



a b
c d

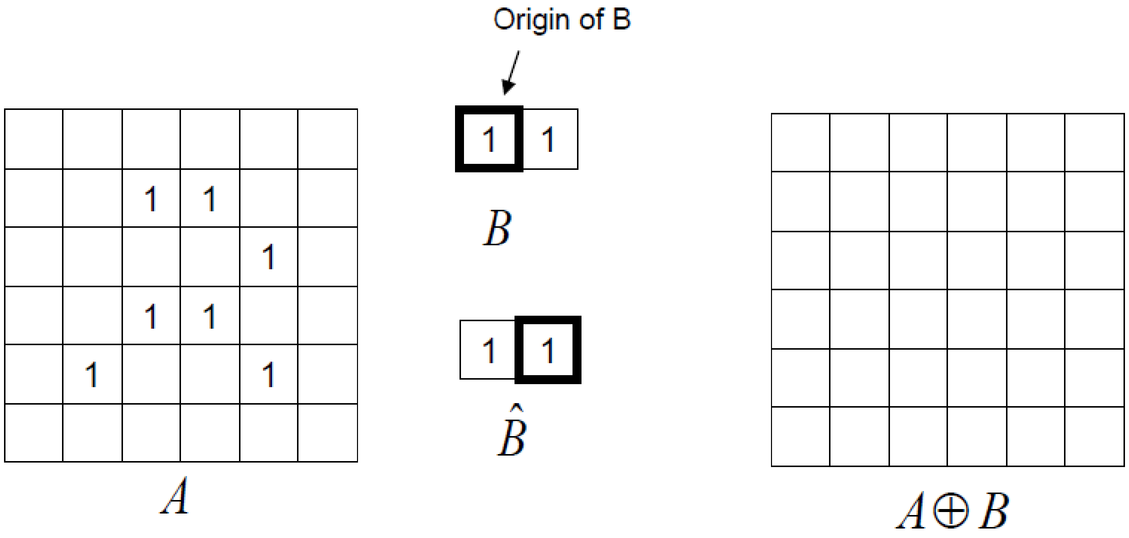
**FIGURE 9.5** Using erosion to remove image components. (a) A $486 \times 486$ binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes $11 \times 11$, $15 \times 15$, and $45 \times 45$, respectively. The elements of the SEs were all 1s.

# DILATION

- The dilation of set A by set (structuring element) B is

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \emptyset\}$$

- Interpretation: reflect B, shift by z, if it overlaps with even one element, output 1



# DILATION

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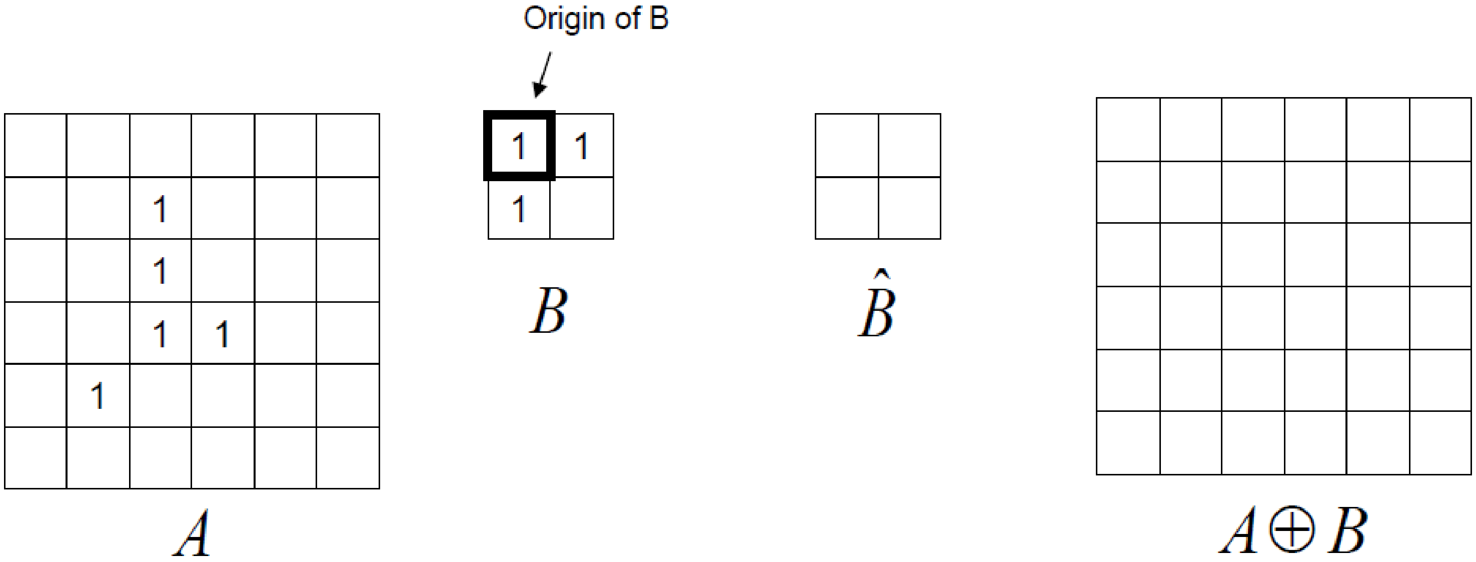


# DILATION

- The dilation of set A by set (structuring element) B is

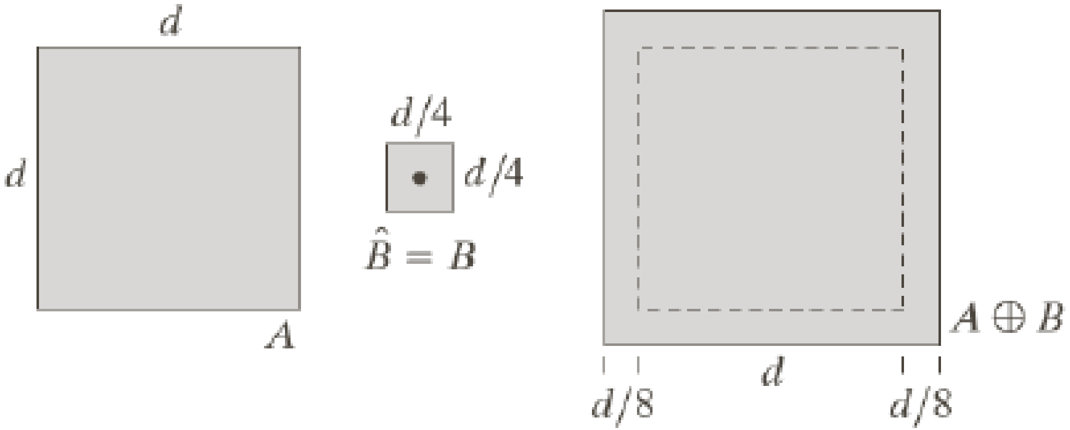
$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \emptyset\}$$

Can be equivalently written as:

$$A \oplus B = \{z \mid [(\hat{B})_z \cap A] \subseteq A\}$$

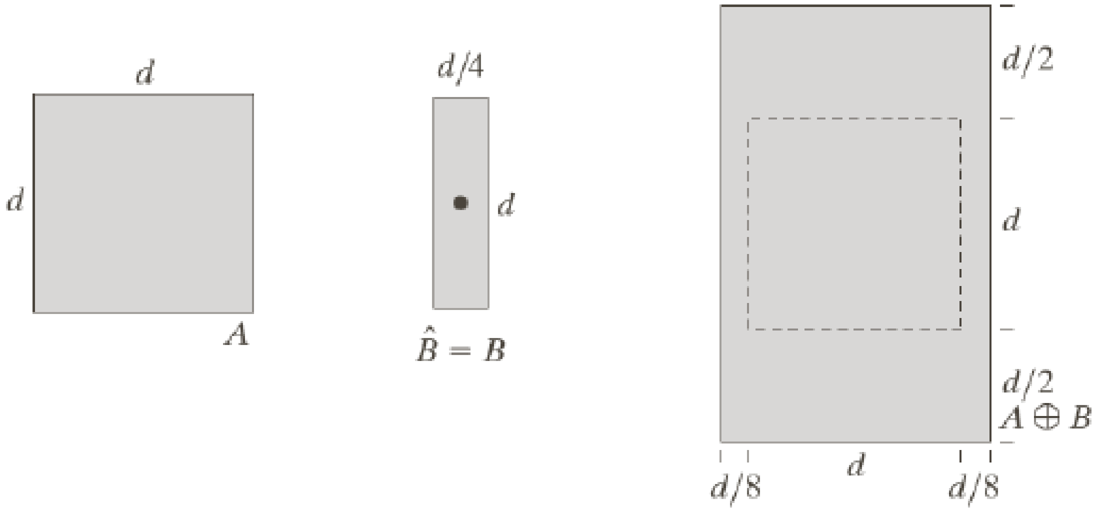
# DILATION

- More examples



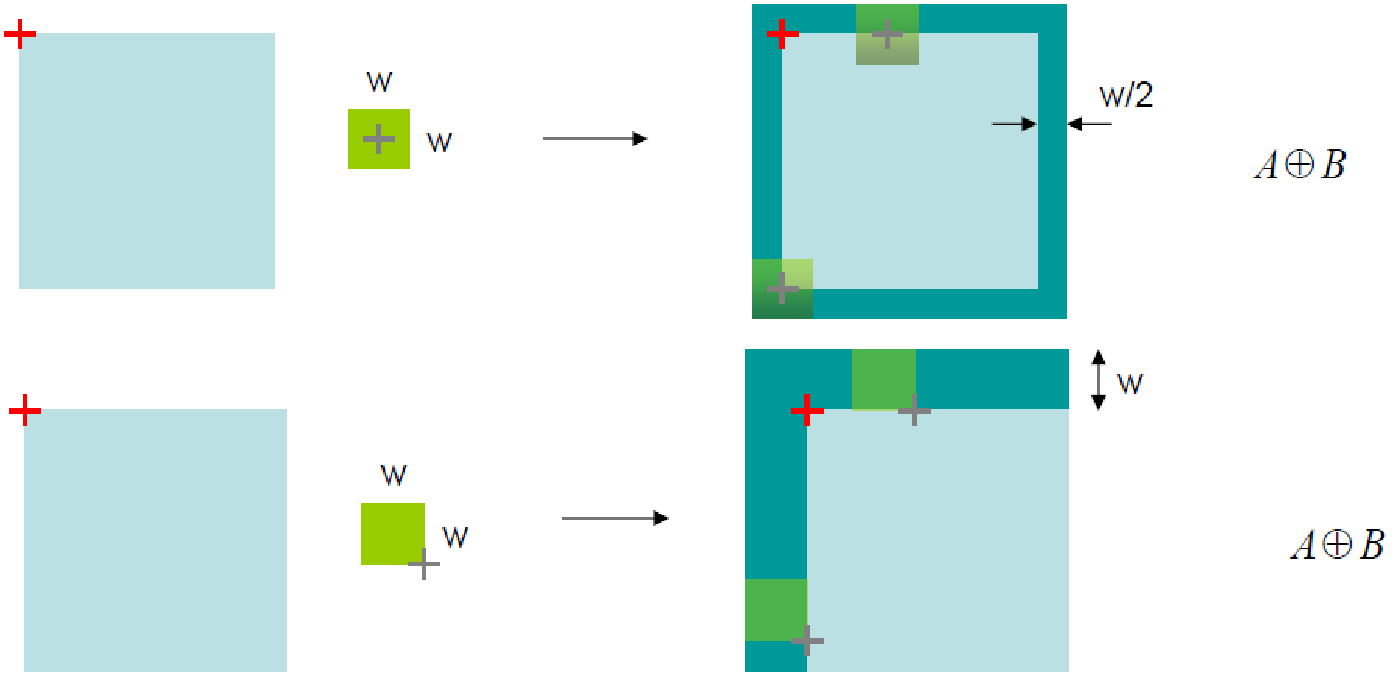
# DILATION

- More examples



# DILATION

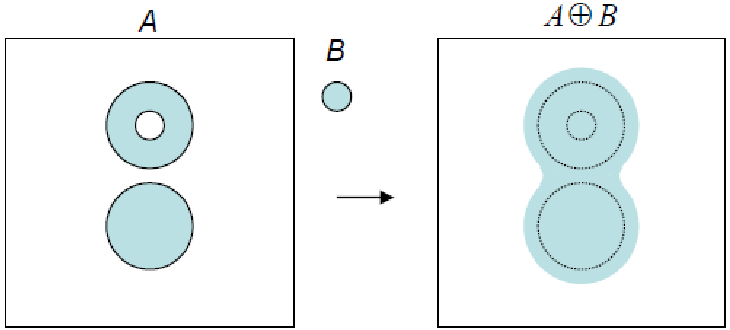
- More examples

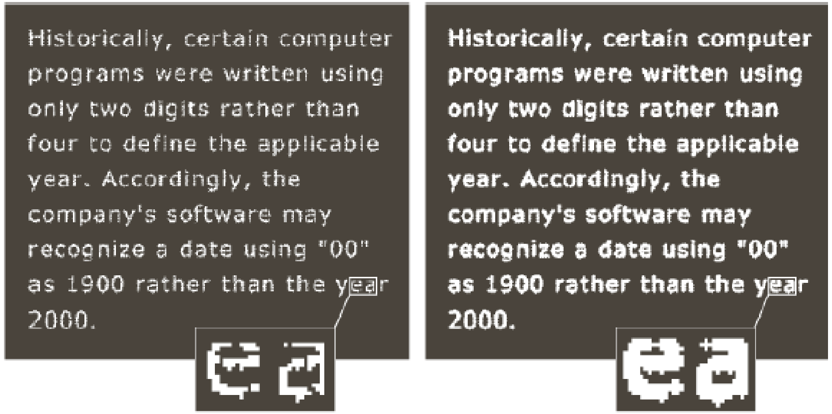



Source: William Hoff

# DILATION

- Fills in holes, thickens thin parts, grows object




| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

Source: William Hoff

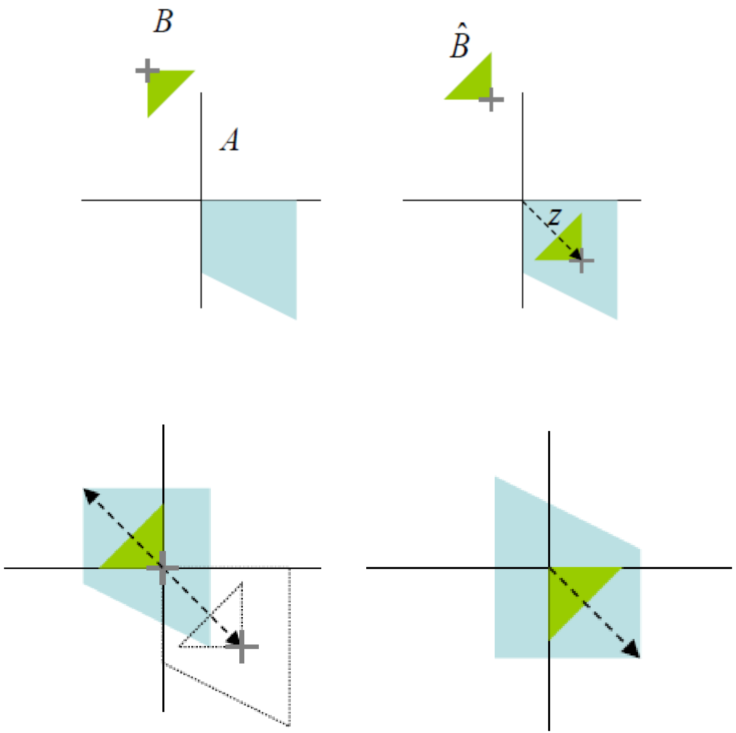
# DILATION Properties

- Commutative $A \oplus B = B \oplus A$
- Associative $A \oplus (B \oplus C) = (A \oplus B) \oplus C$
- Proof

$$A \oplus B = \left\{ z \mid \left(\hat{B}\right)_z \cap A \neq \varnothing \right\}$$

$$= \left\{ z \mid \hat{B} \cap \left(A\right)_{-z} \neq \varnothing \right\}$$

$$= \left\{ z \mid B \cap \left(\hat{A}\right)_z \neq \varnothing \right\} = B \oplus A$$




Source: William Hoff

# DUALITY

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

- Proof

$$A \ominus B = \{ z \mid (B)_z \subseteq A \} = \{ z \mid (B)_z \cap A^c = \varnothing \}$$

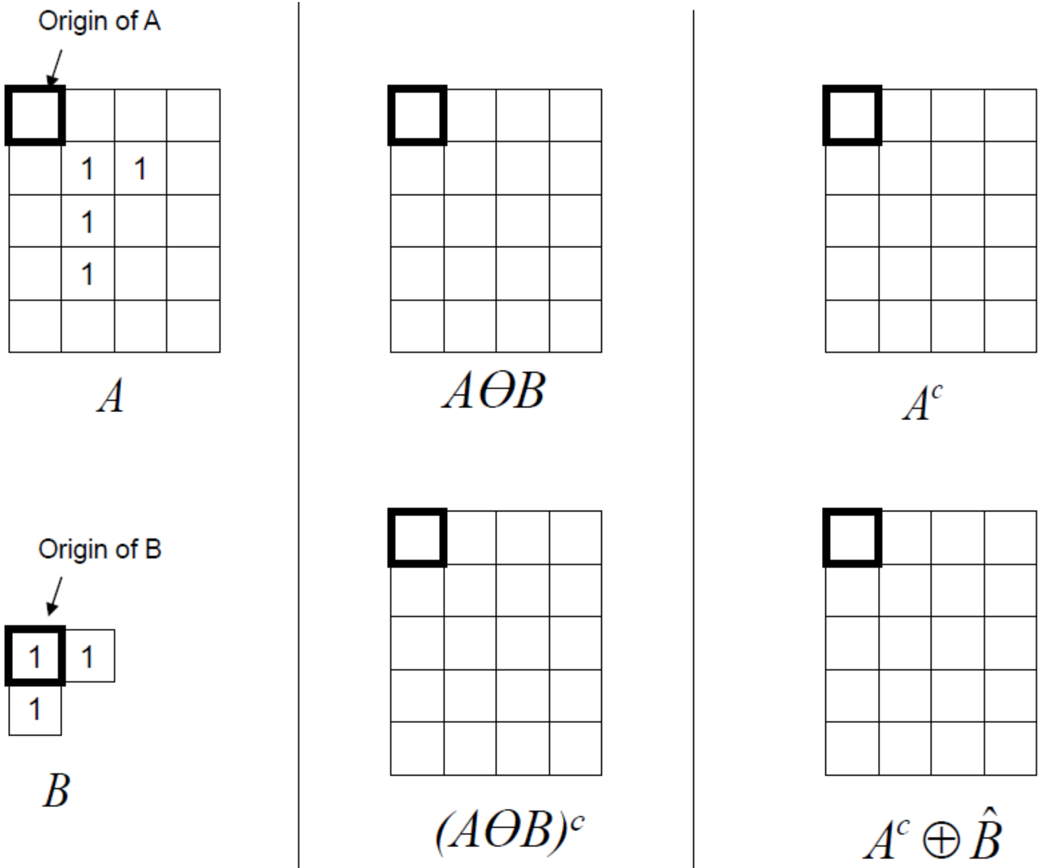
If set $(B)_z$ is contained in $A$, then the intersection of $(B)_z$ with the complement of $A$ is empty

$$(A \ominus B)^c = \{ z \mid (B)_z \cap A^c = \varnothing \}^c$$

$$= \{ z \mid (B)_z \cap A^c \neq \varnothing \}$$

$$= A^c \oplus \hat{B}$$

# DUALITY



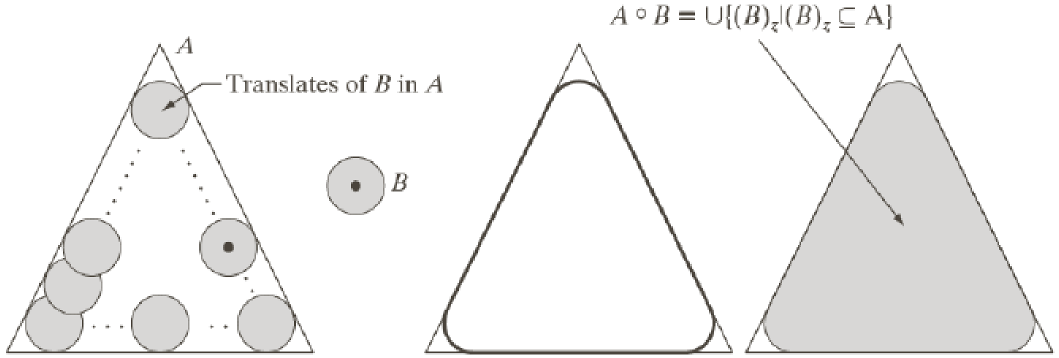

Source: William Hoff

# OPENING

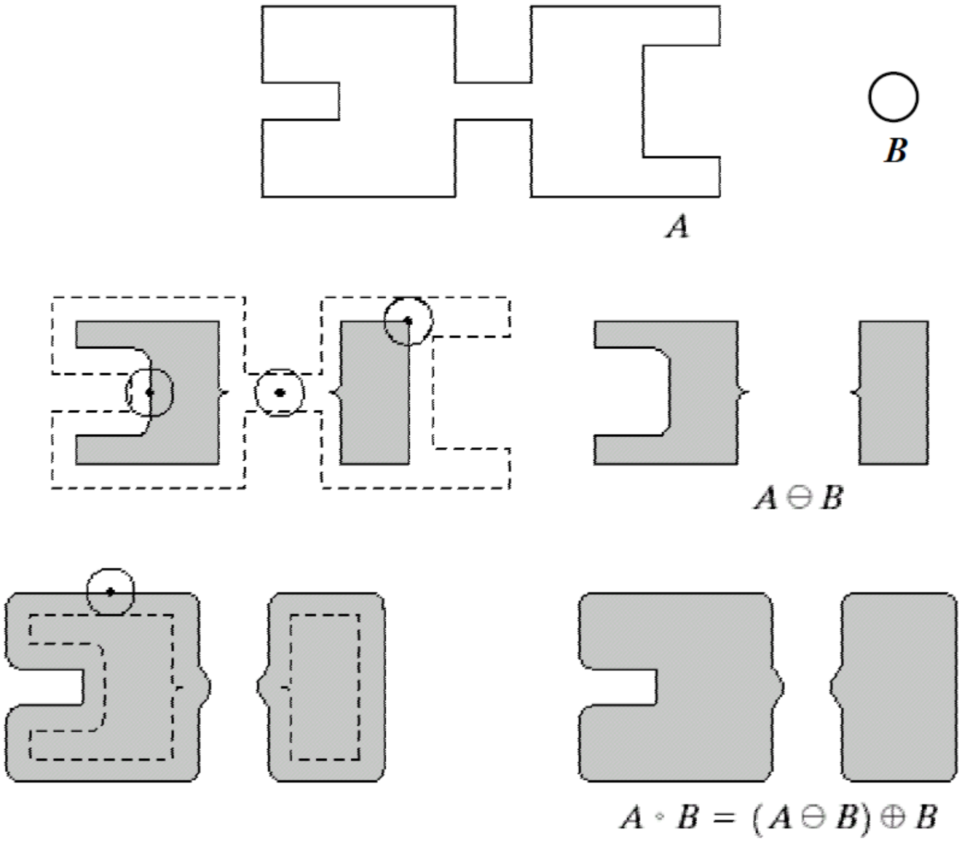
- Erosion followed by a dilation

$$A \circ B = (A \theta B) \oplus B$$

- Smoothes contour, breaks narrow isthmuses, and eliminate thin protrusions



# Example OPENING

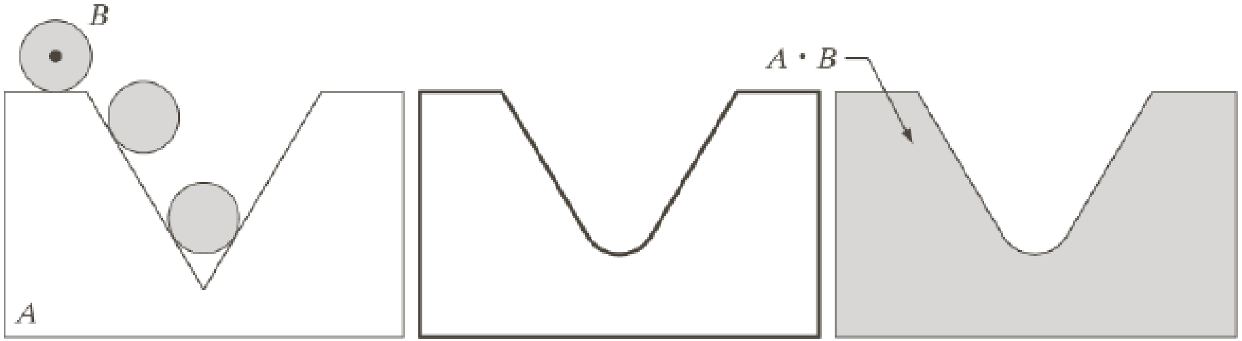


# CLOSING

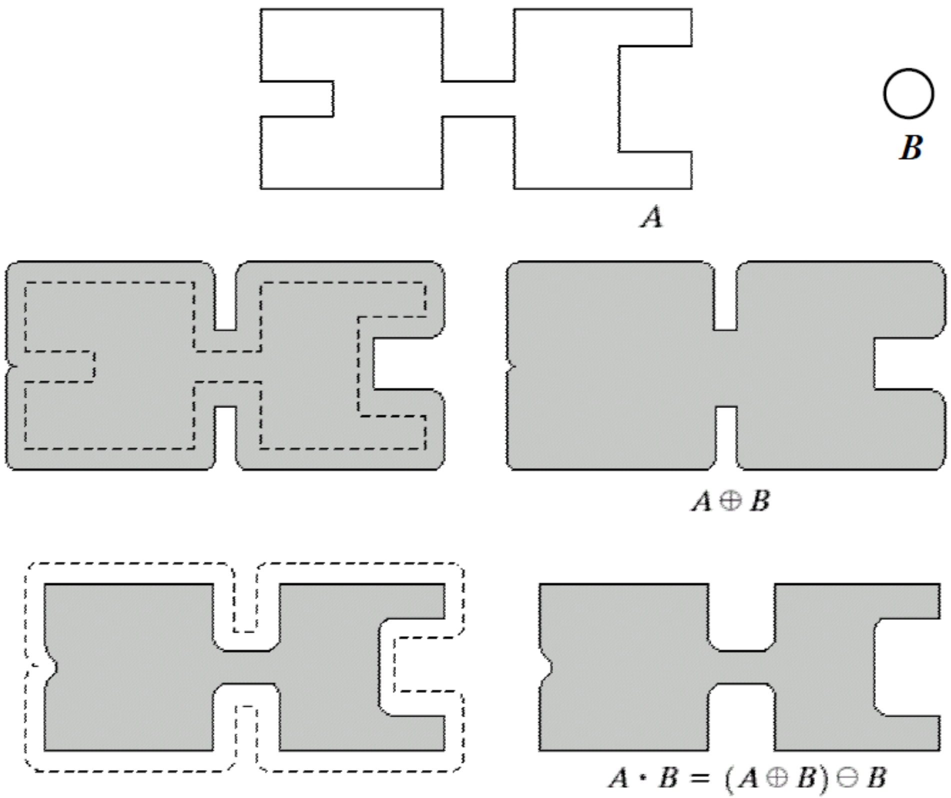
- Dilation followed by a erosion

$$A \bullet B = (A \oplus B) \theta B$$

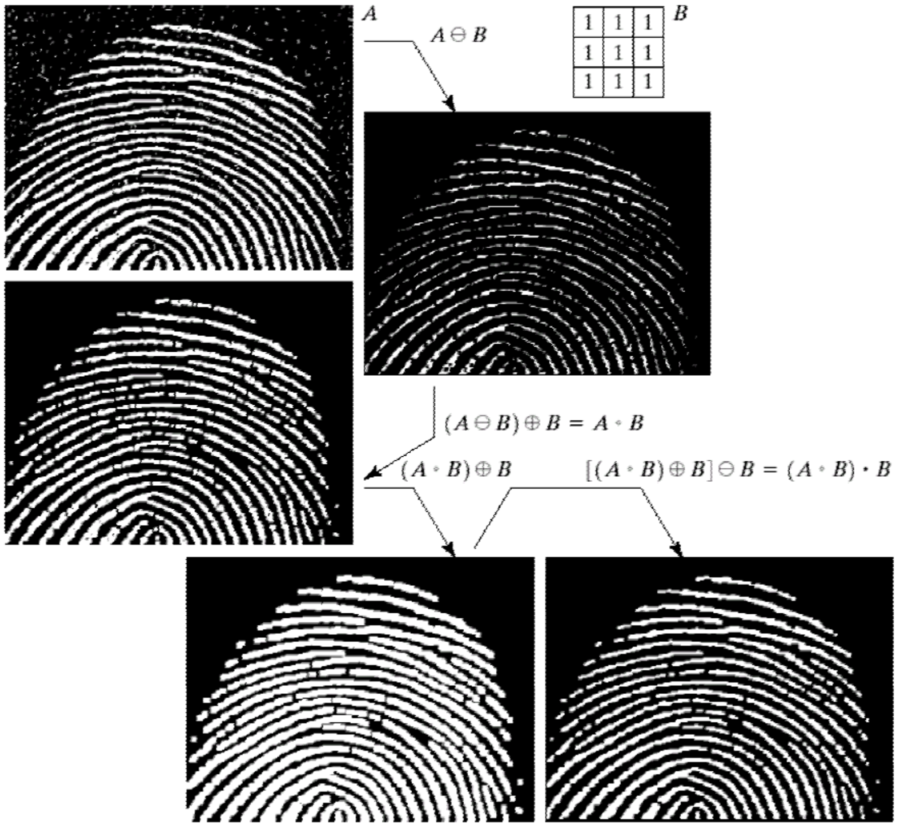
- Fuses narrow bleaks and long thin gulfs, eliminates small holes, and fills gap in contour



# Example CLOSING



# Application



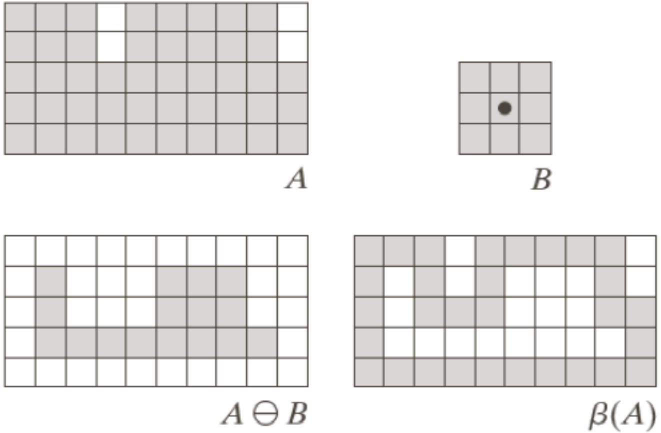
# Morphological Algorithms

- Boundary Extraction
- Hit and Miss transform
- Region Filling
- Convex Hull
- Skeletonization

# Boundary Extraction

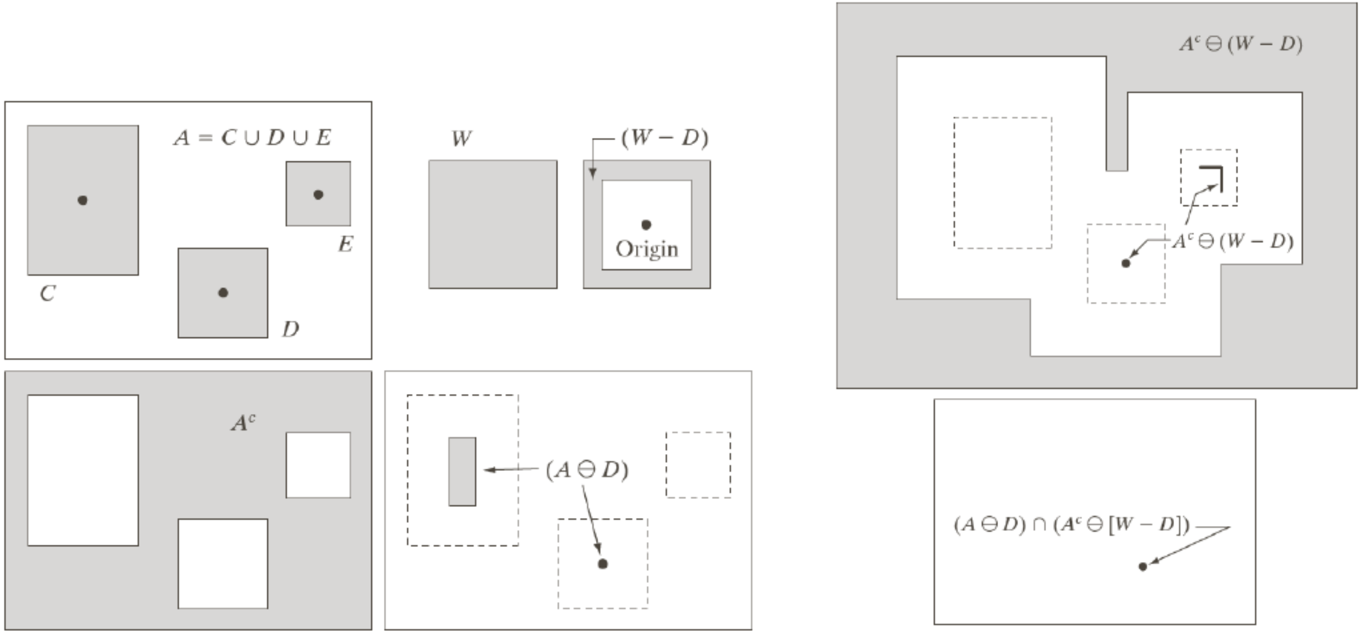
- Set difference between A and its erosion with B

$$\beta(A) = A - (A \ominus B)$$




# Hit and Miss Transform

$$A \circledast B = (A \ominus D) \cap [A^c \ominus (W - D)]$$

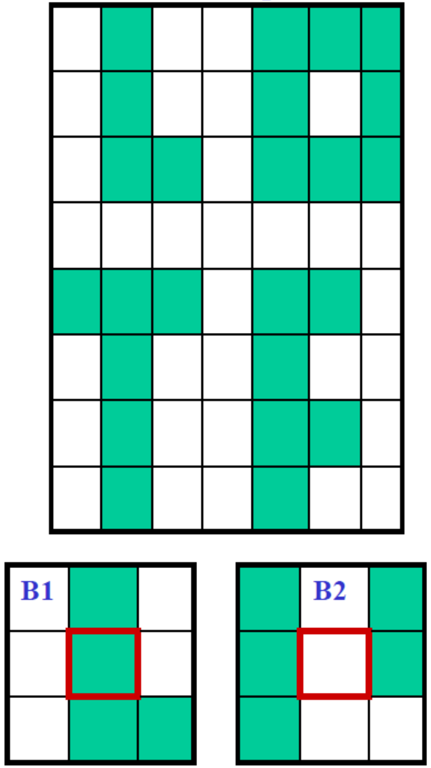


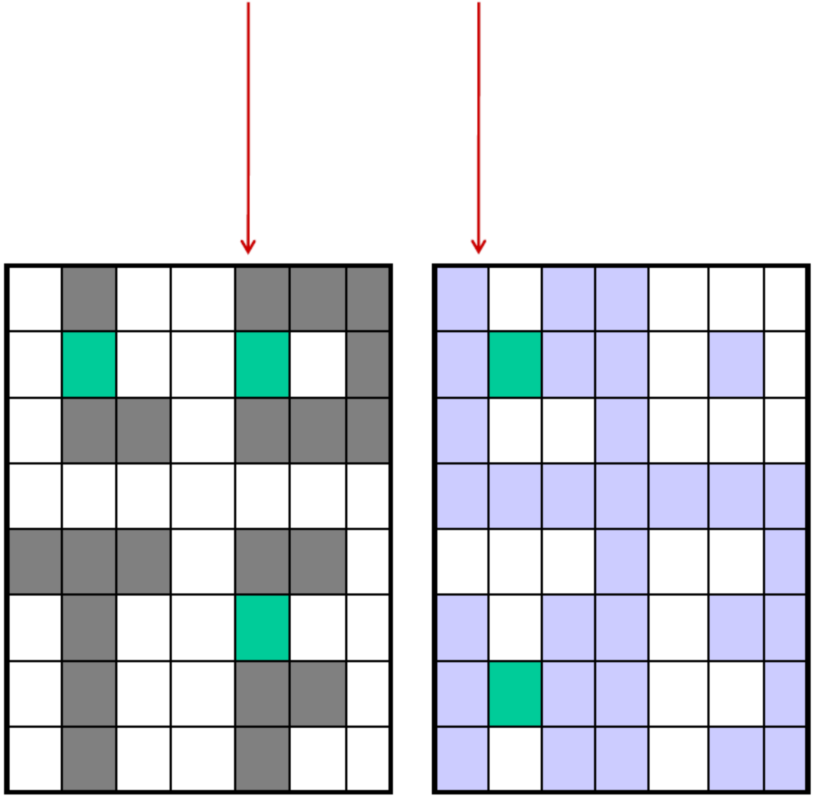
# Hit and Miss Transform

$A$ : all green pixels

$A^c$ : all white pixels

$$A \circledast B = (A \ominus B1) \cap (A^c \ominus B2)$$



Source: J. Sivaswami

# Hit and Miss Transform

| | | |
|---|---|---|
| | 1 | |
| 0 | 1 | 1 |
| 0 | 0 | |

| | | |
|---|---|---|
| | 1 | |
| 1 | 1 | 0 |
| | 0 | 0 |

| | | |
|---|---|---|
| | 0 | 0 |
| 1 | 1 | 0 |
| | 1 | |

| | | |
|---|---|---|
| 0 | 0 | |
| 0 | 1 | 1 |
| | 1 | |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

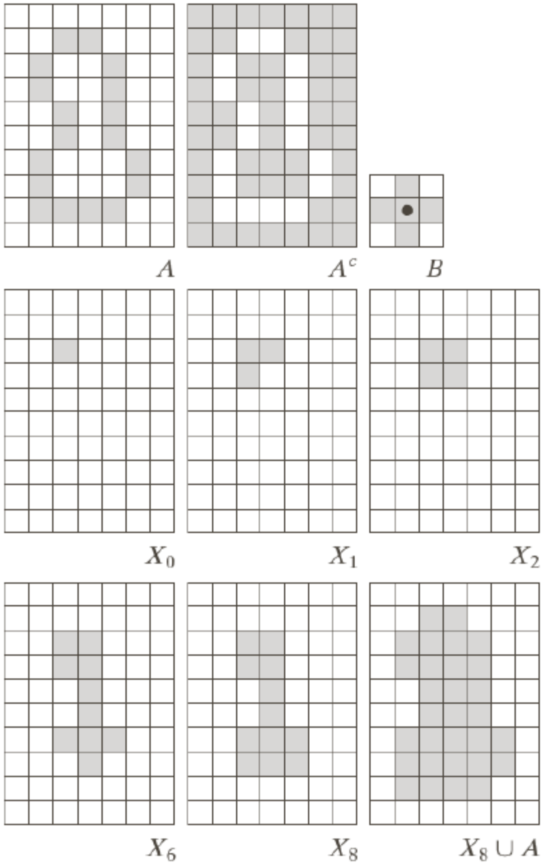
Source: http://homepages.inf.ed.ac.uk

# Region Filling

- Let A be a the set of 8-connected boundary points
- Start with a point inside the region
- Dilate
- Take intersection with compliment of A (repeat)

$$X_k = (X_{k-1} \oplus B) \cap A^c, \qquad k = 1,2,3 \ldots$$

- Stop when no more changes

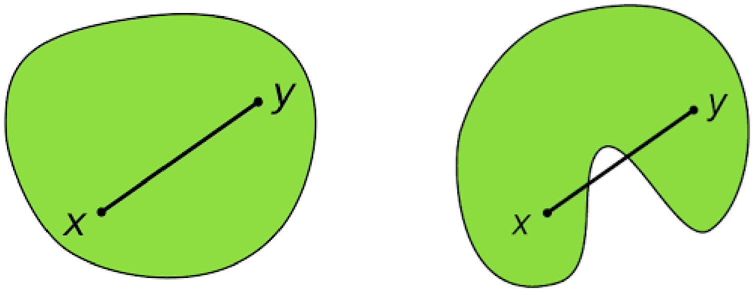


# Region Filling

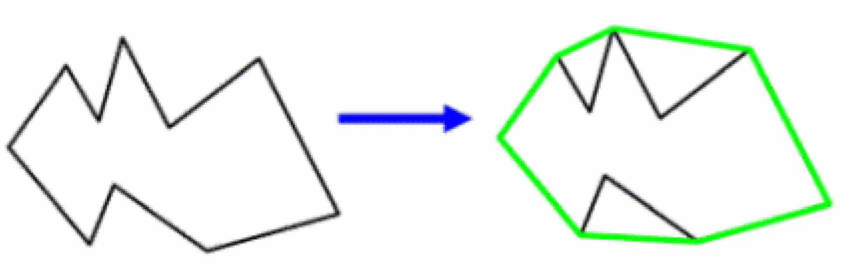
- Example

# Convex Hull

- Convex set



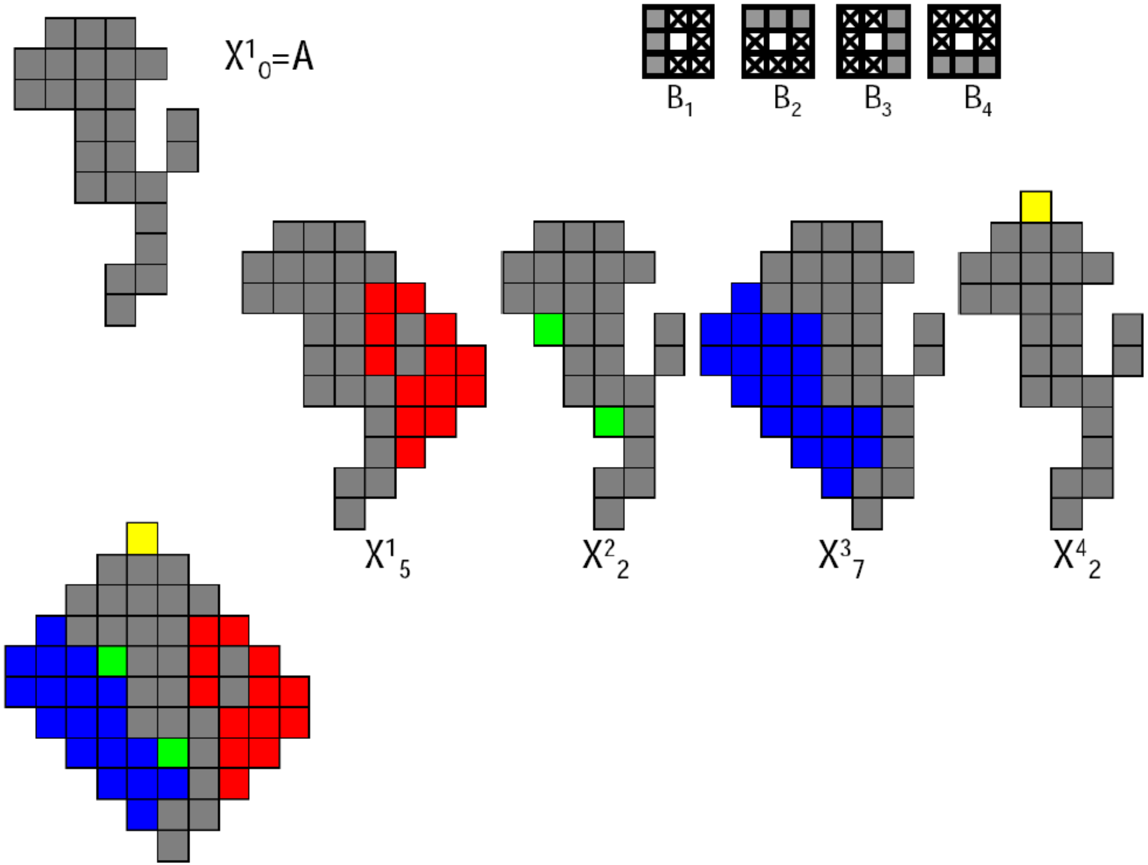

- Convex Hull : minimal envelope



# Convex Hull (Morphological Algorithm)

1. Iteratively perform Hit or Miss transform with each SE, until convergence

2. Take union of the four results

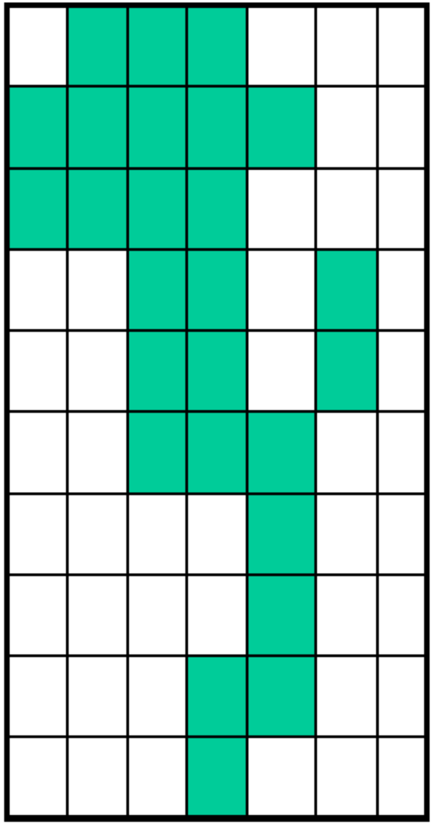


# Convex Hull (alternate approach)

1. For every pixel $i$ find the number $n_i$ of its neighbours which belong to the object

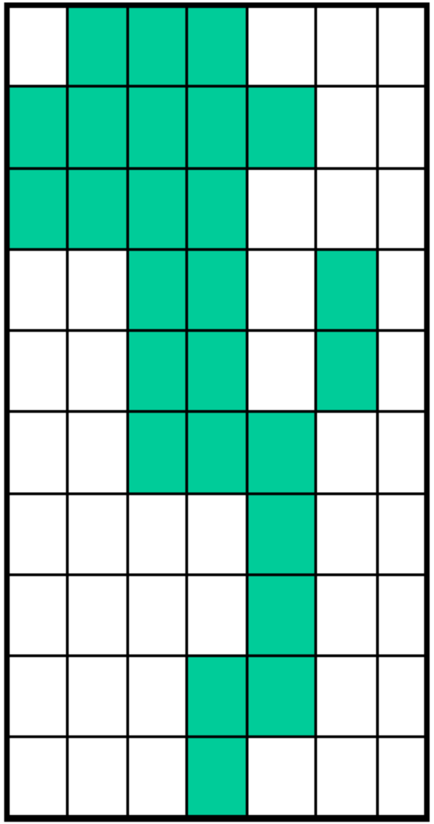
2. If $n_i > 3$ then mark the object pixel $I$

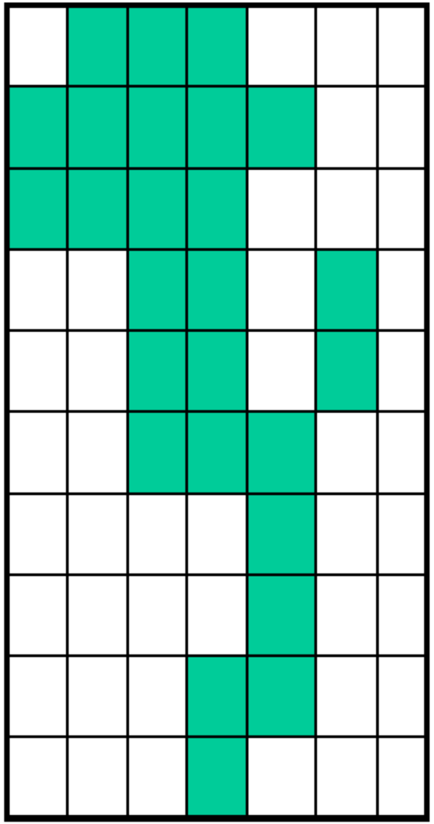
3. Repeat 1 and 2 until there are no pixels with more than 3 neighbours



Source: J. Sivaswami

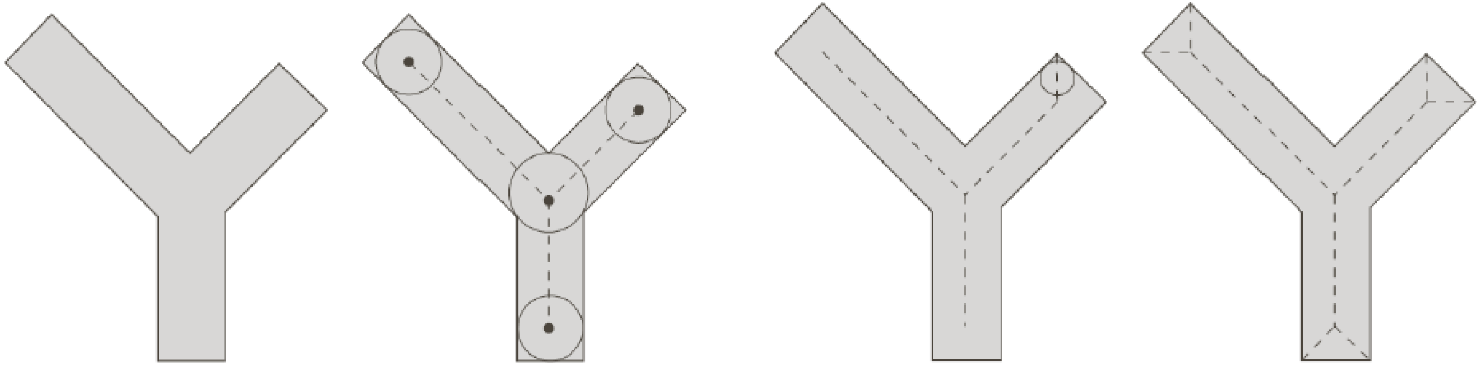
# Convex Hull





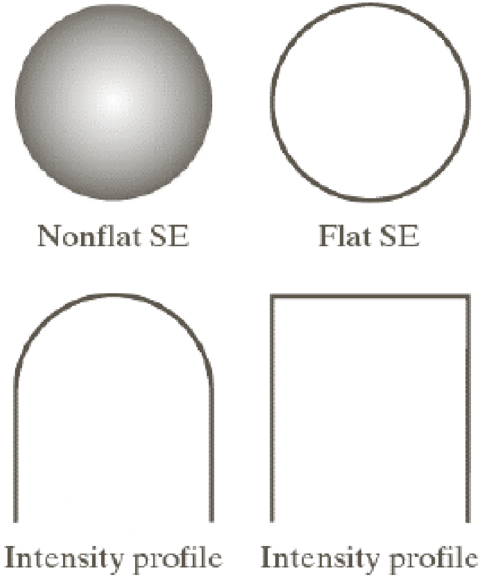
# Skeletons

- Set of all points that are equally distant from two closest points of the object boundary
- A concise representation of a shape



- Analogy
  - start a fire at the boundary, let it burn inward
  - Points where fire is quenched are the skeleton

# Gray scale morphology




| a | b |
|---|---|
| c | d |

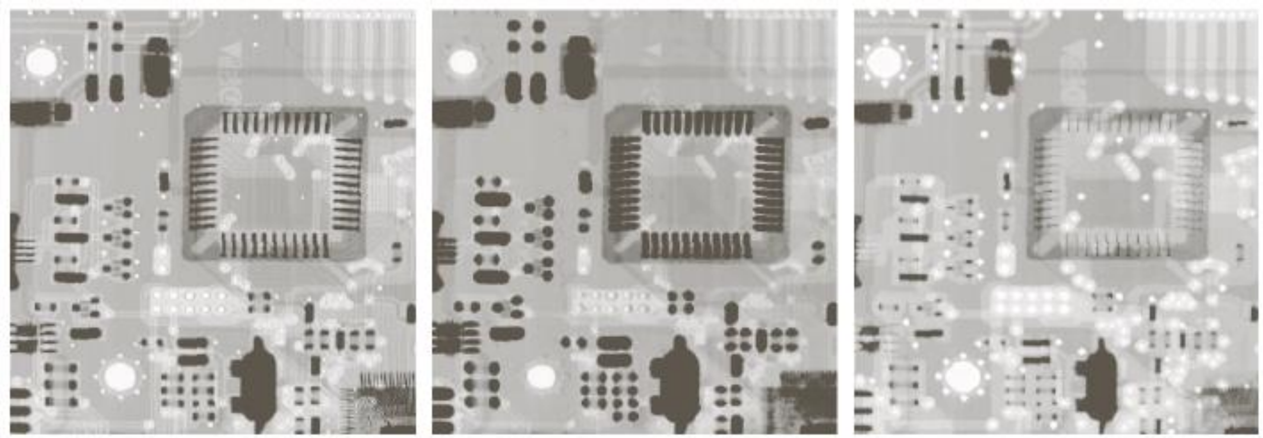
**FIGURE 9.34** Nonflat and flat structuring elements, and corresponding horizontal intensity profiles through their center. All examples in this section are based on flat SEs.

# Gray scale morphology

- Erosion and Dilation

$$[f \ominus b](x,y) = \min_{(s,t)\in b}\{f(x+s, y+t)\}$$

$$[f \oplus b](x,y) = \max_{(s,t)\in b}\{f(x-s, y-t)\}$$

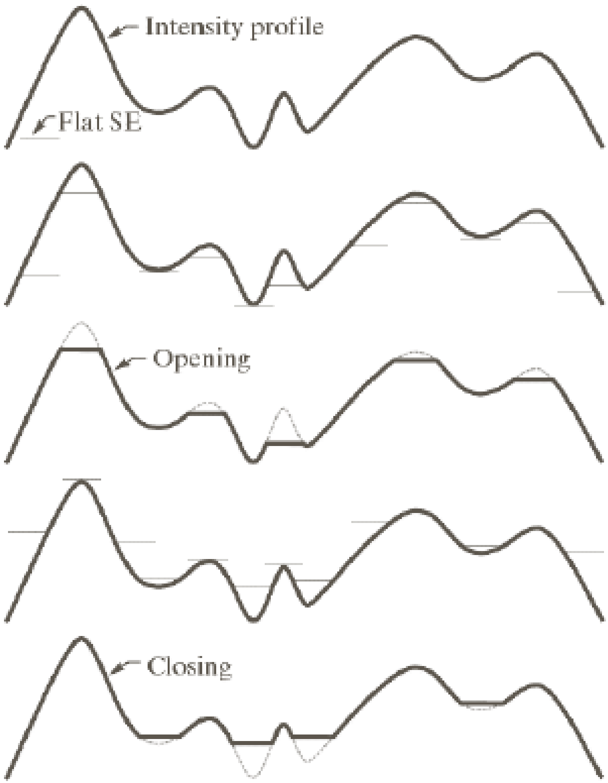


# Gray scale morphology

- Opening and Closing

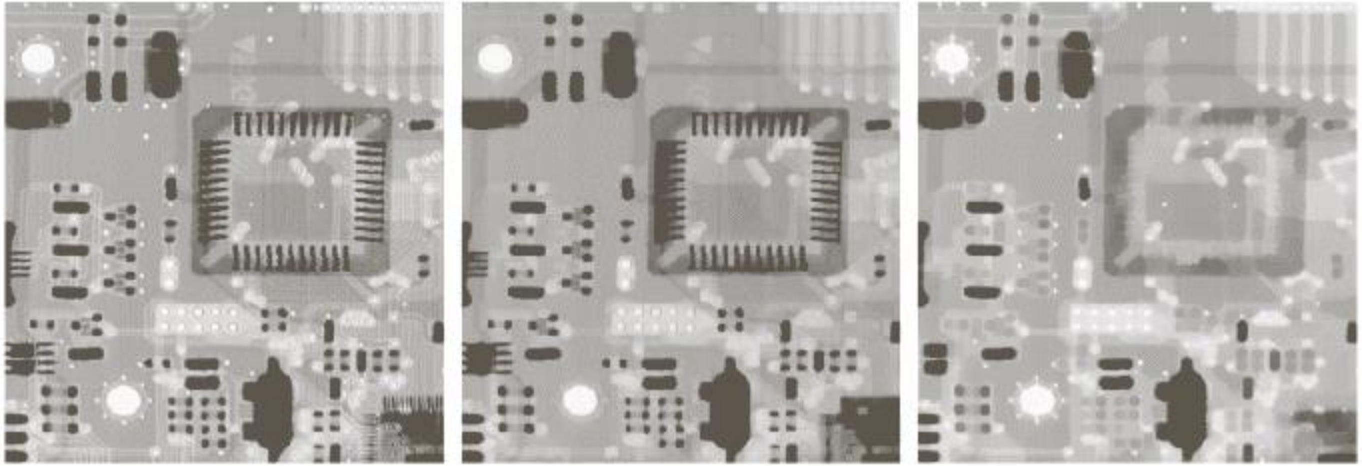
$$[f \circ b] = (f \ominus b) \oplus b$$

$$[f \cdot b] = (f \oplus b) \ominus b$$



# Gray scale morphology

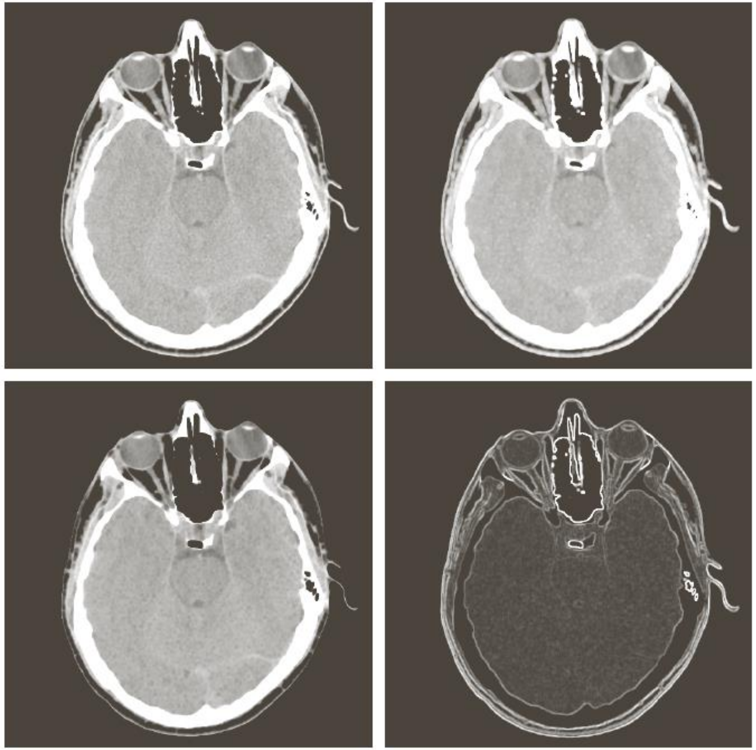
- Opening and Closing



a b c

**FIGURE 9.37** (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.

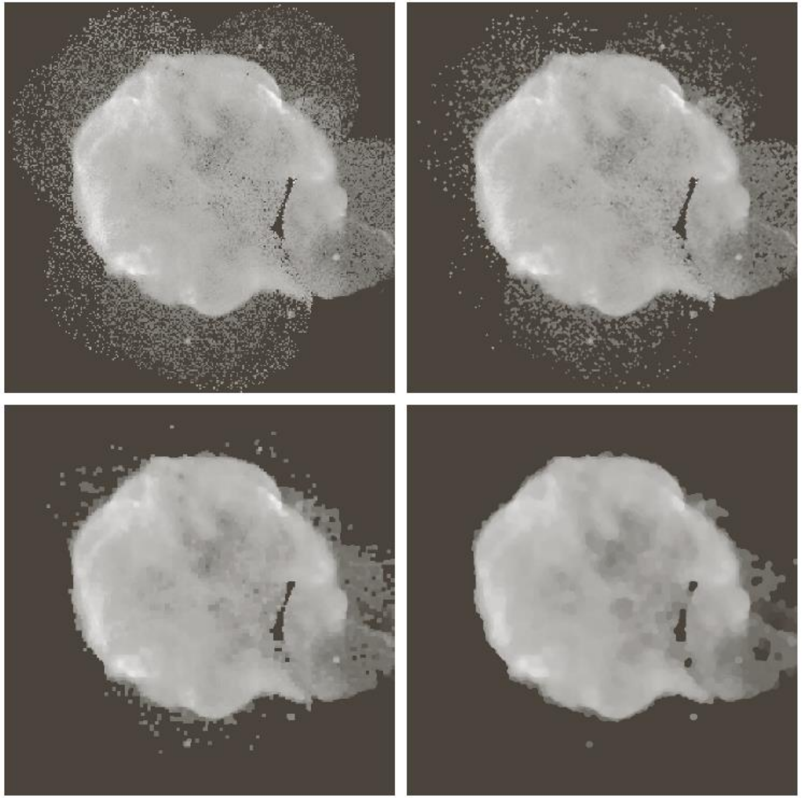
# Morphological Gradient



| a | b |
|---|---|
| c | d |

**FIGURE 9.39** (a) 512 × 512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

# Smoothing



a b
c d

**FIGURE 9.38** (a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

# Applications: Top hat transform

- Open the image with an structuring element, subtract it from original
- Leaves only details smaller than the structuring element

$$T_{hat}(f) = f - (f \circ b)$$

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# Top hat transform

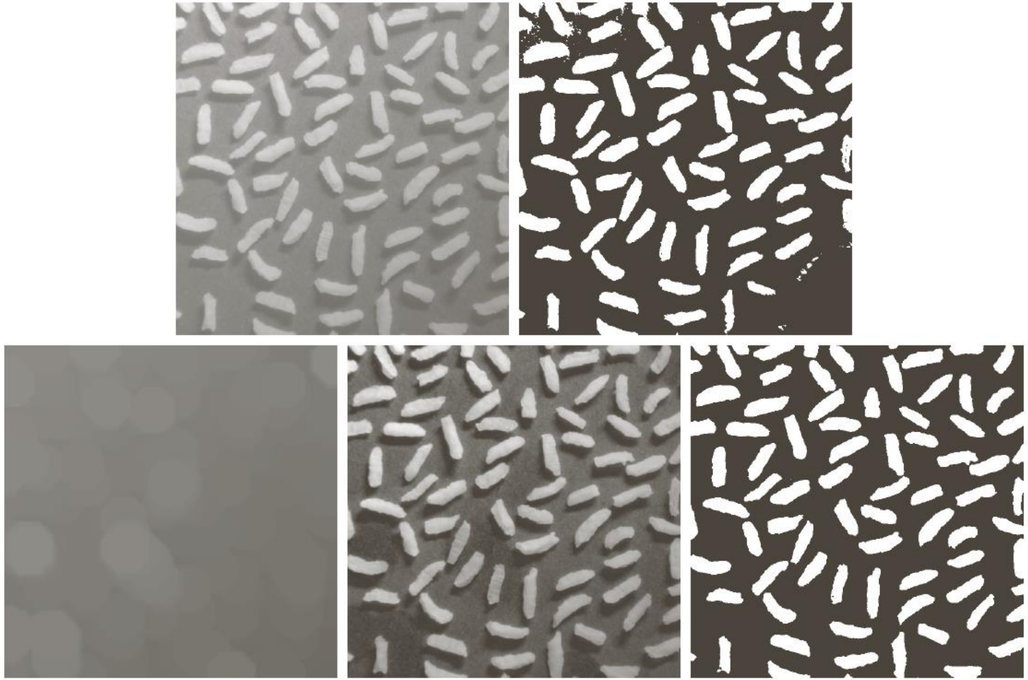
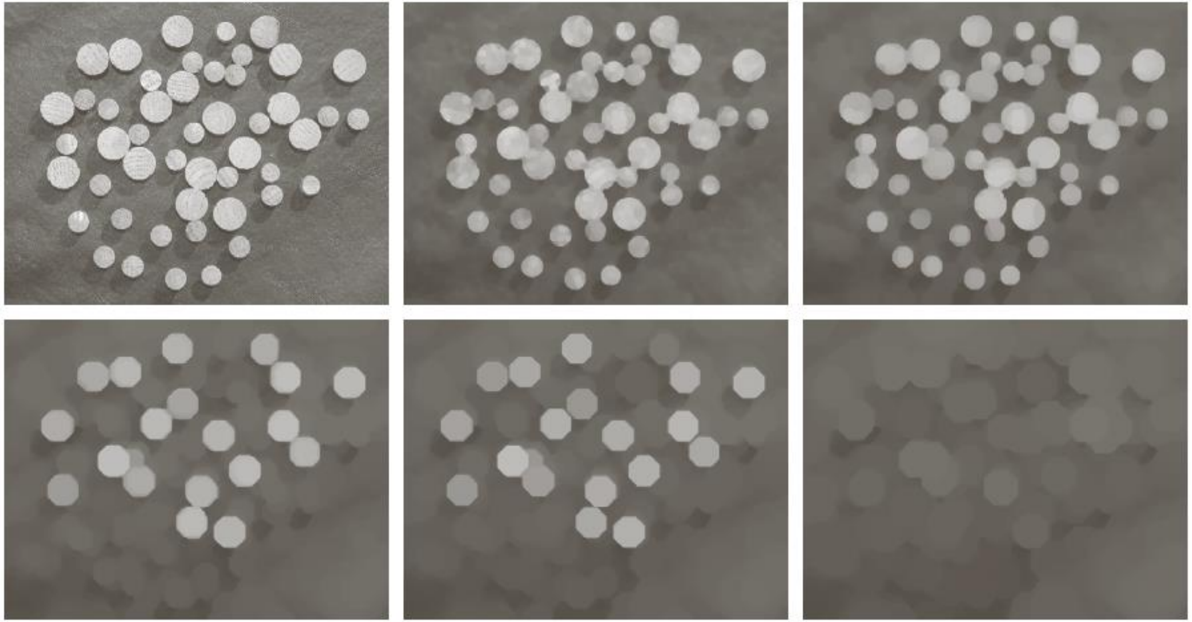



**FIGURE 9.40** Using the top-hat transformation for *shading correction*. (a) Original image of size $600 \times 600$ pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

# Granulometry



| | | |
|---|---|---|
| a | b | c |
| d | e | f |

**FIGURE 9.41** (a) 531 × 675 image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, The MathWorks, Inc.)