Digital Image Processing (CSE 478)
Lecture 24: Image Reconstruction from Projections

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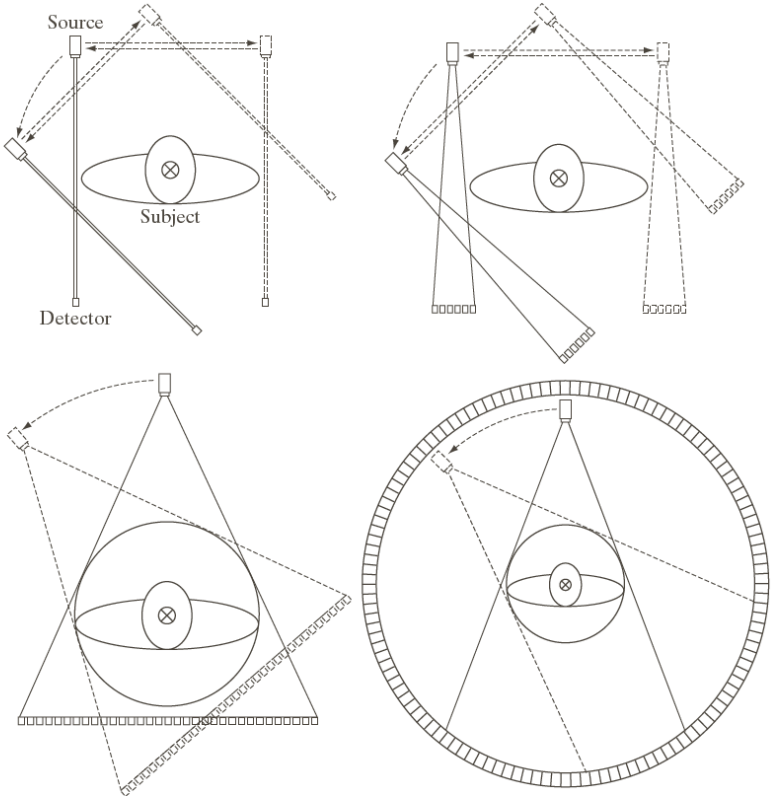
CT Scan

CT Scan

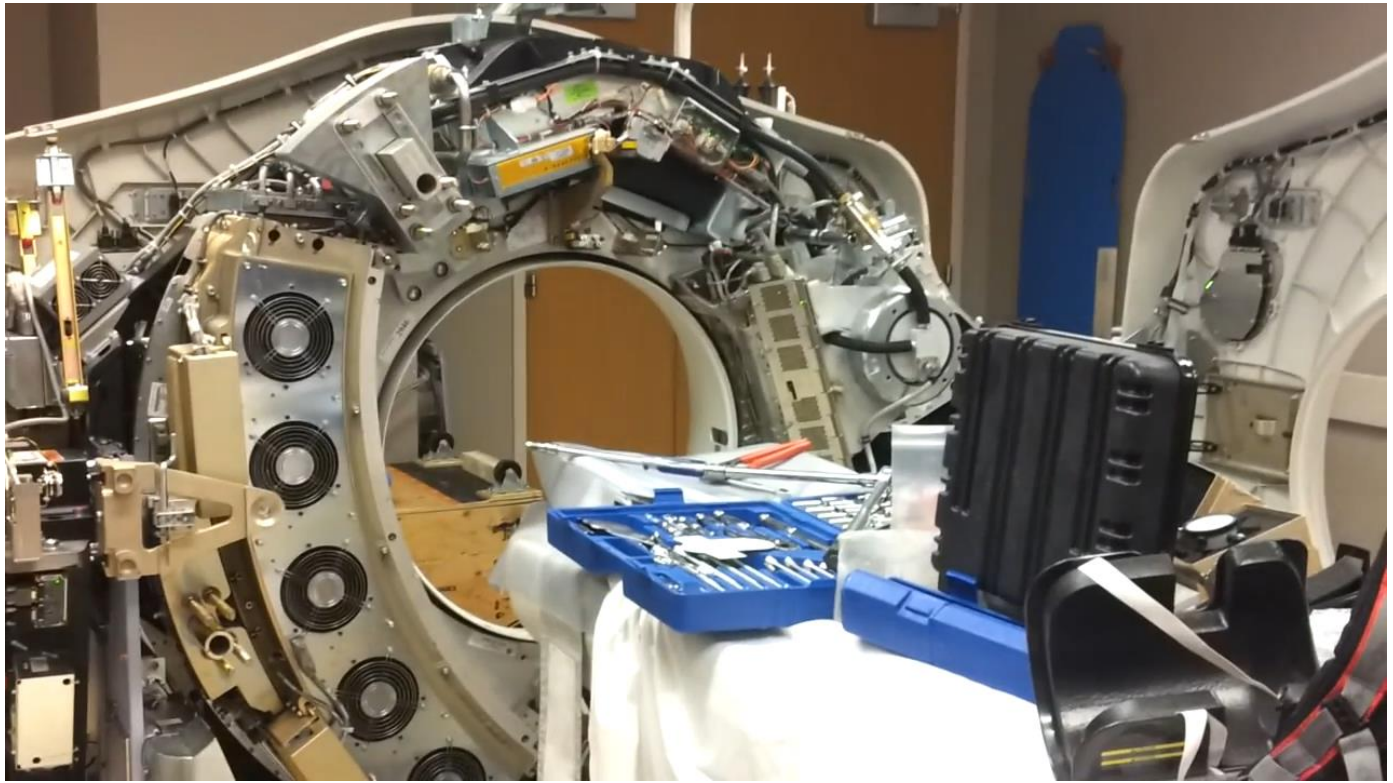
a b
c d

FIGURE 5.35 Four generations of CT scanners. The dotted arrow lines indicate incremental linear motion. The dotted arrow arcs indicate incremental rotation. The cross-mark on the subject's head indicates linear motion perpendicular to the plane of the paper. The double arrows in (a) and (b) indicate that the source/detector unit is translated and then brought back into its original position.

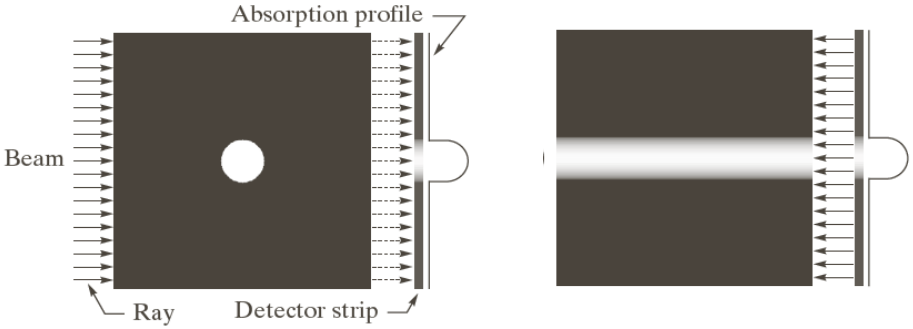


Source: Gonzalez and Woods

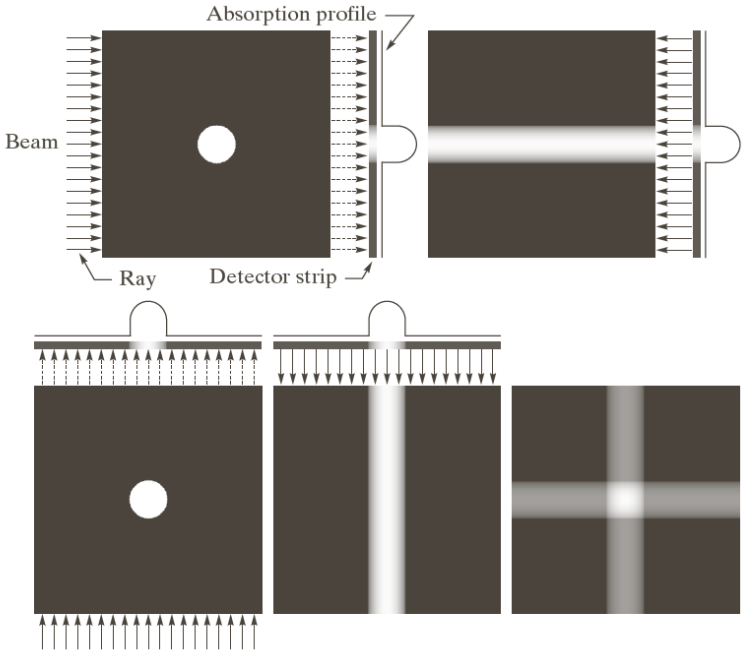
CT Scan



Projection with simple objects



Projection with simple objects



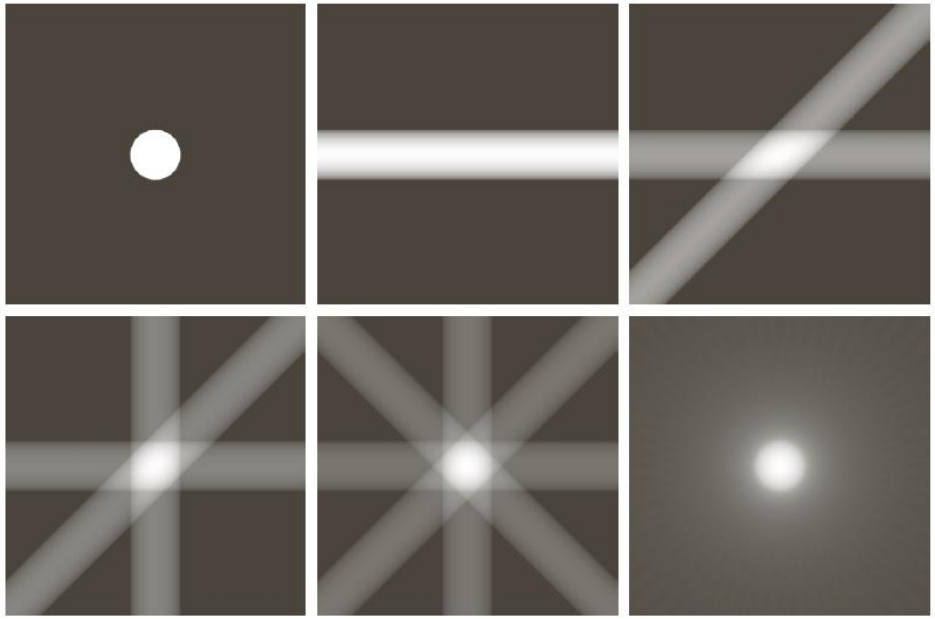
a b
c d e

FIGURE 5.32
(a) Flat region showing a simple object, an input parallel beam, and a detector strip. (b) Result of back-projecting the sensed strip data (i.e., the 1-D absorption profile). (c) The beam and detectors rotated by 90°. (d) Back-projection. (e) The sum of (b) and (d). The intensity where the back-projections intersect is twice the intensity of the individual back-projections.

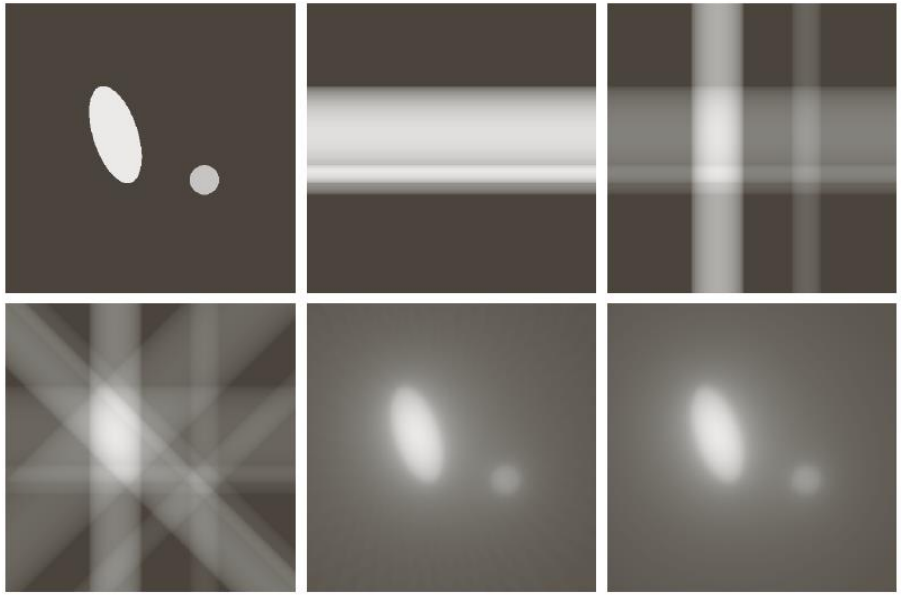
Projection with simple objects

a b c
d e f

FIGURE 5.33
(a) Same as Fig. 5.32(a).
(b)–(e) Reconstruction using 1, 2, 3, and 4 backprojections 45° apart.
(f) Reconstruction with 32 backprojections 5.625° apart (note the blurring).



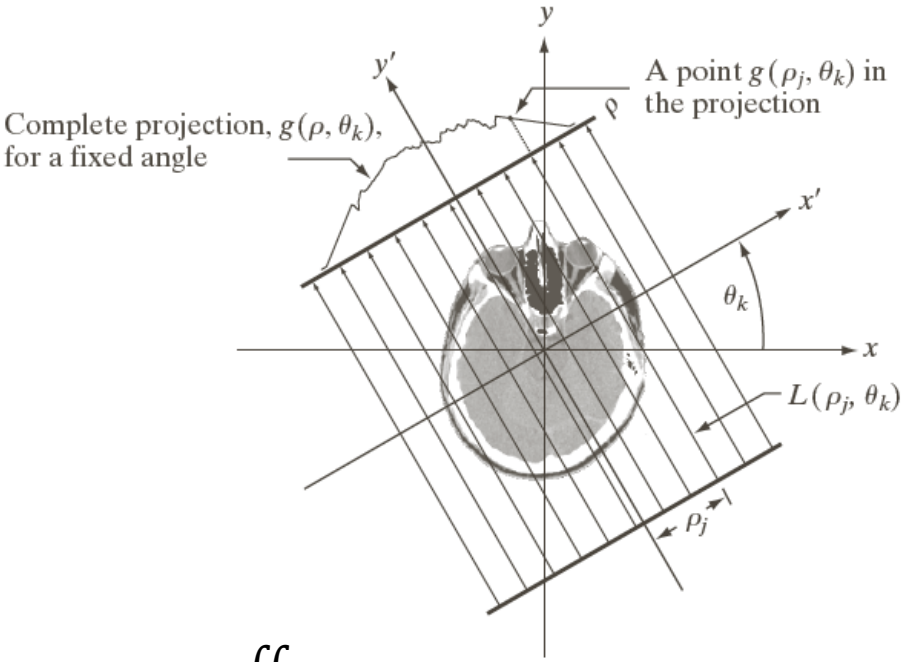
Projection with simple objects



a b c
d e f

FIGURE 5.34 (a) A region with two objects. (b)–(d) Reconstruction using 1, 2, and 4 backprojections 45° apart. (e) Reconstruction with 32 backprojections 5.625° apart. (f) Reconstruction with 64 backprojections 2.8125° apart.

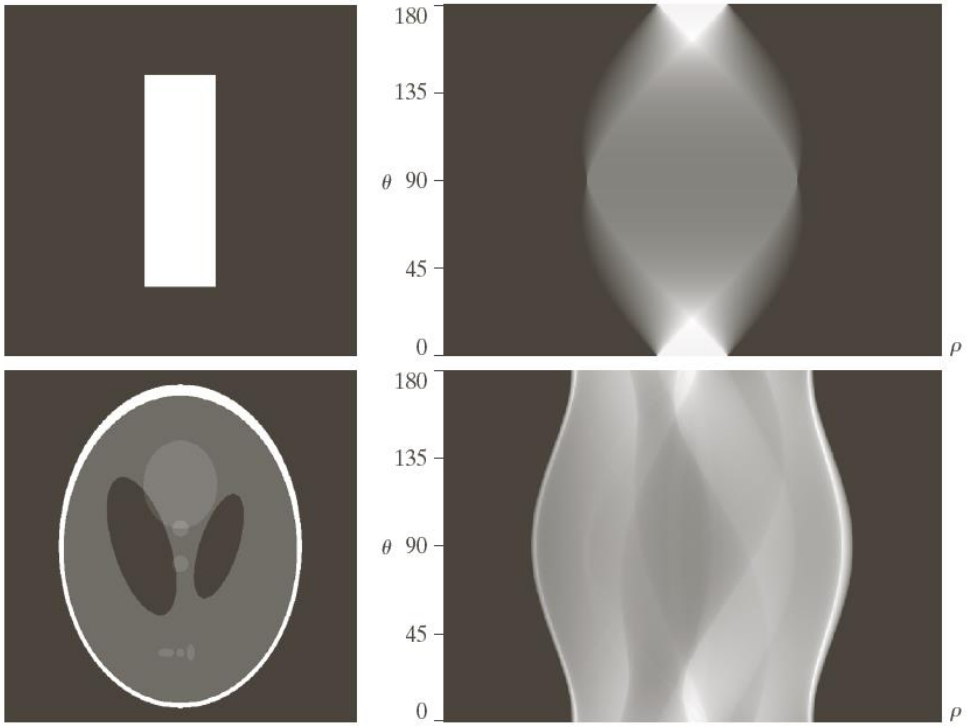
Radon Transform



$$g(\rho_j, \theta_k) = \iint f(x,y)\delta\left(x\cos\theta_k + \sin\theta_k - \rho_j\right)dxdy$$

$$g(\rho\ \ , \theta\ \ ) = \iint f(x,y)\delta(x\cos\theta\ \ + \sin\theta\ \ - \rho\ \ )dxdy$$

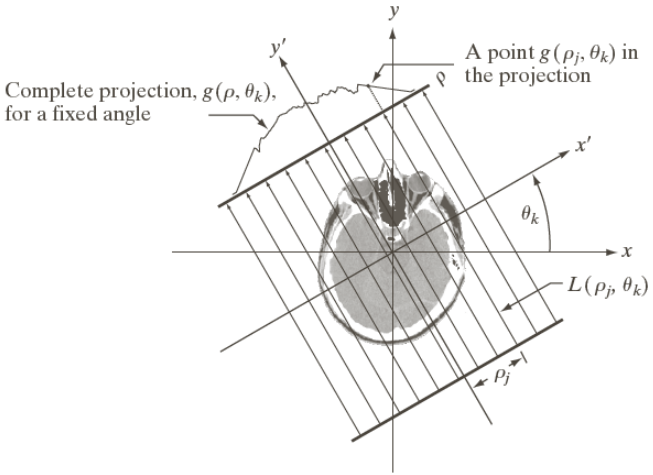
Sinograms



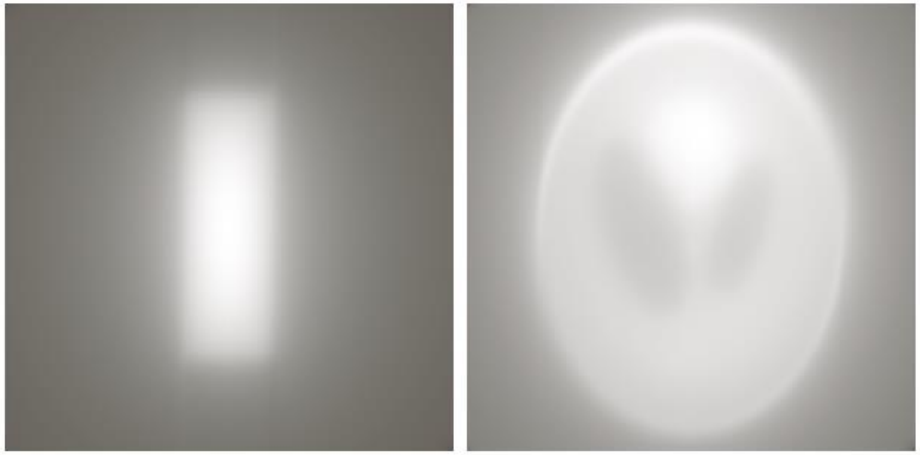
Reconstruction

$$f_\theta(x,y) = g(\rho,\theta_k) = g(x\,cos\theta_k + y\,sin\theta_k, \theta_k)$$

$$f(x,y) = \int_{\theta=0}^{\pi} f_\theta(x,y)$$



Reconstruction



a b

FIGURE 5.40
Backprojections of the sinograms in Fig. 5.39.

Fourier Slice theorem

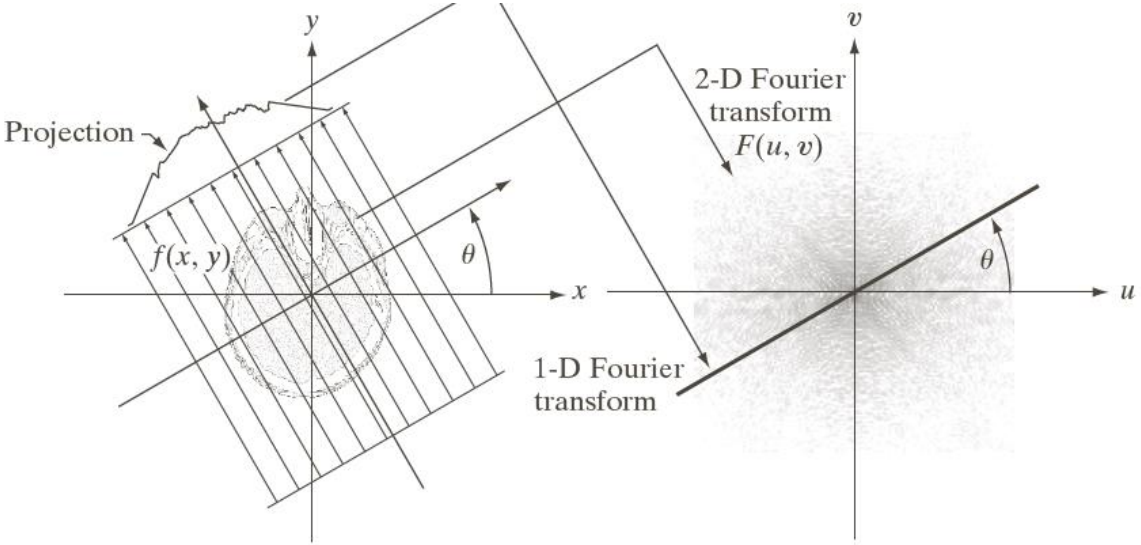


FIGURE 5.41 Illustration of the Fourier-slice theorem. The 1-D Fourier transform of a projection is a slice of the 2-D Fourier transform of the region from which the projection was obtained. Note the correspondence of the angle θ.

Filtered projection example

a b

FIGURE 5.44 Filtered backprojections of the head phantom using (a) a ramp filter, and (b) a Hamming-windowed ramp filter. Compare with Fig. 5.40(b).

CT Scan

a b
c d

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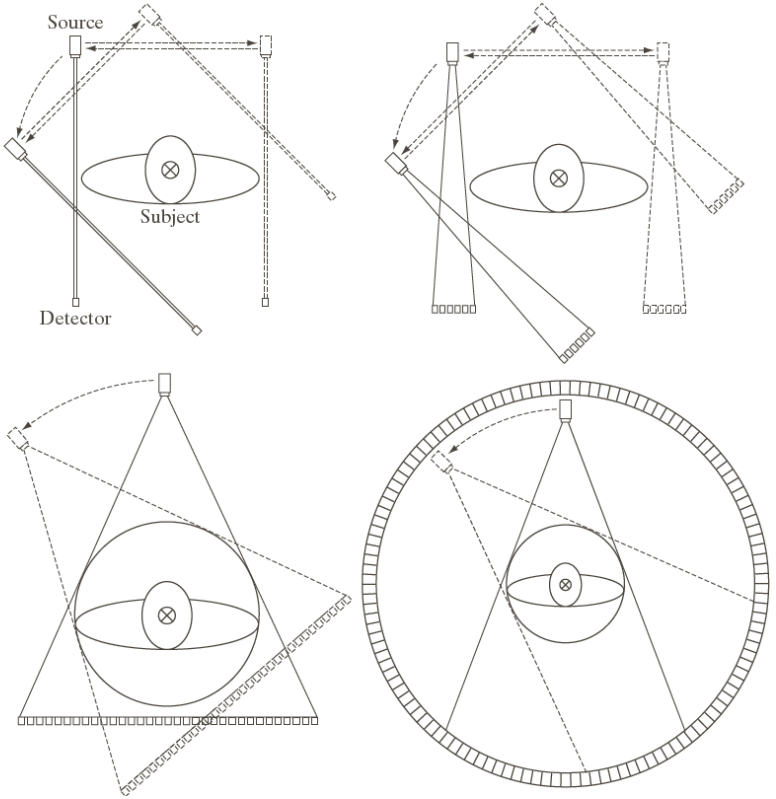

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