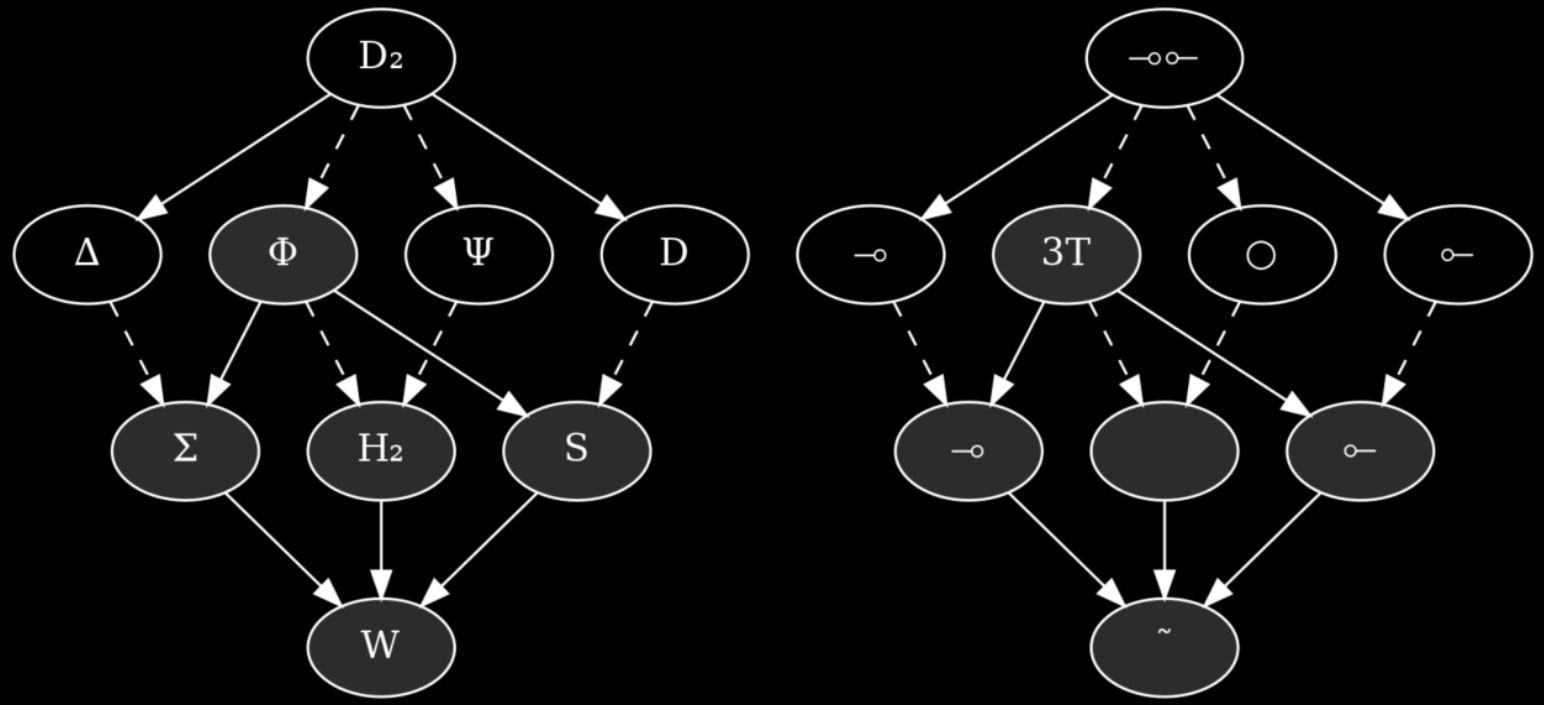
Composition Intuition



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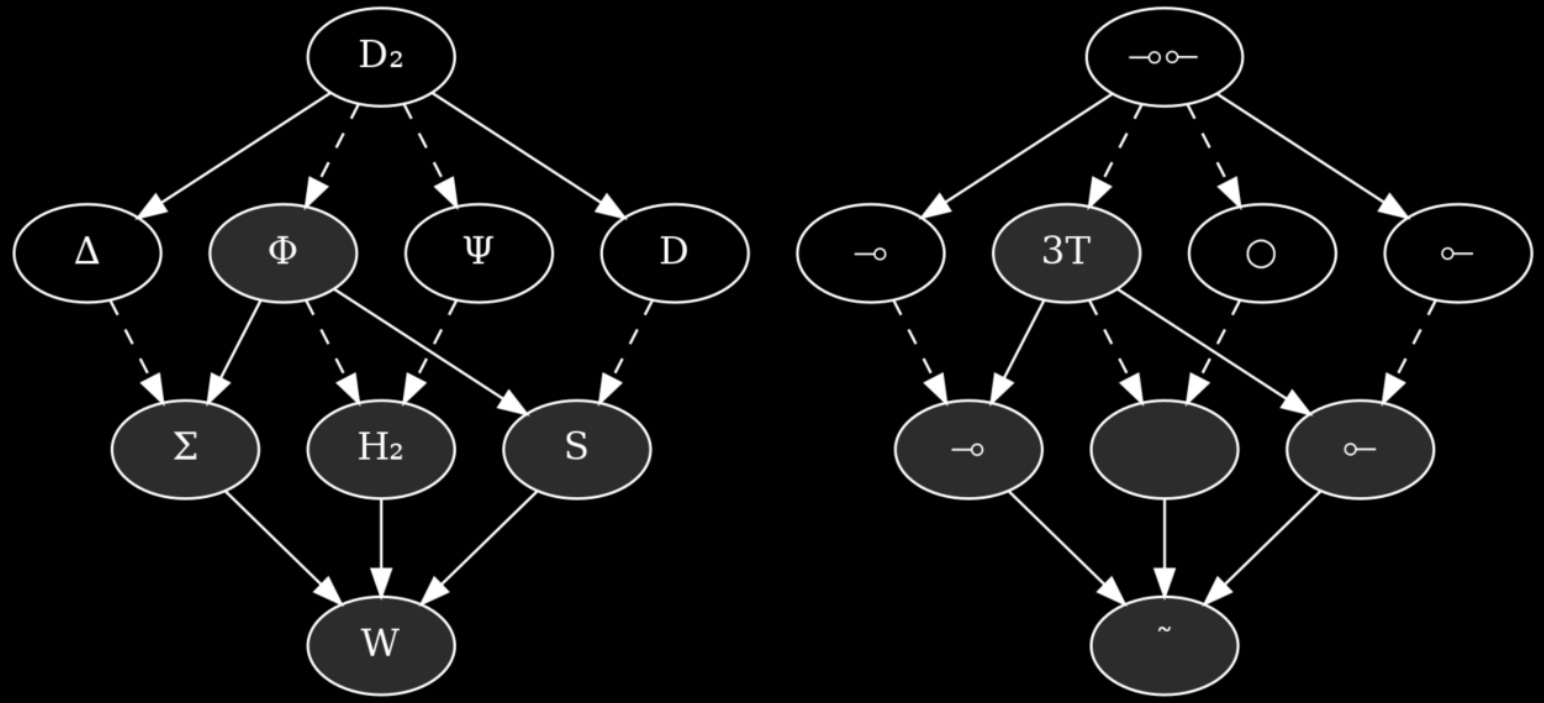
Composition Intuition

Combinators & Tacit Programming



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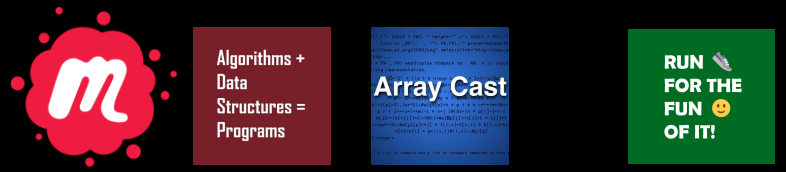

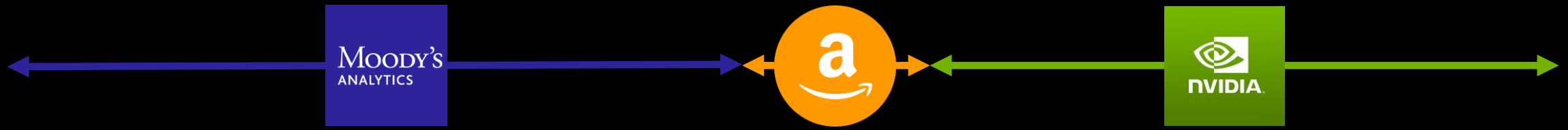

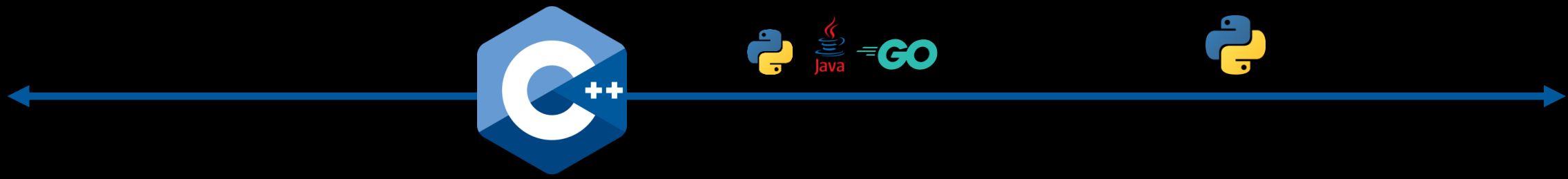

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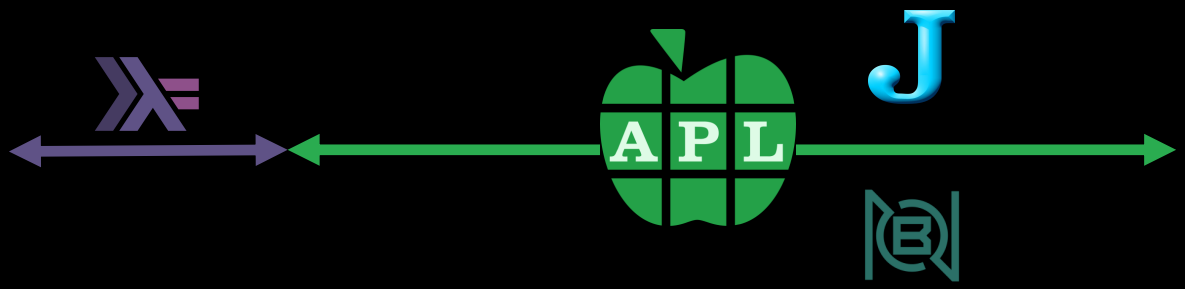
Conor Hoekstra / @code_report



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32 (20) Talks

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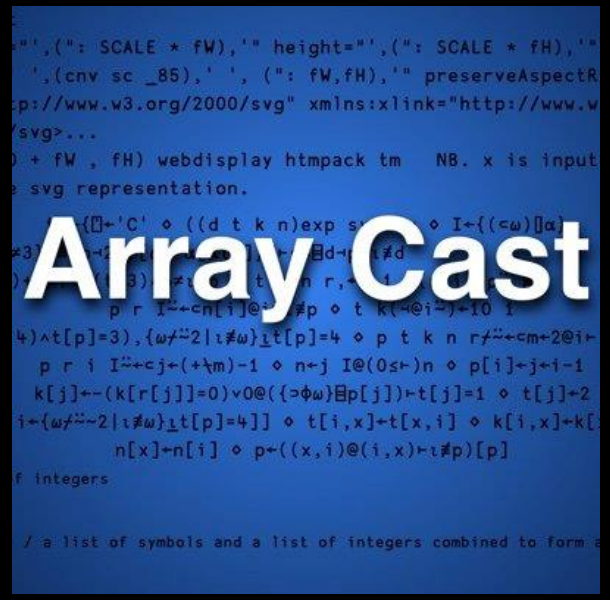
@adspthepodcast



54 Episodes
@arraycast

RUN FOR THE FUN OF IT!

9 Episodes
@conorhoekstra

www.plrank.com

Programming Language Rankings (2023 June)

by code_report

☑ StackOverflow ☑ Octoverse ☑ RedMonk ☑ Languish
☐ PYPL ☐ IEEE Spectrum ☐ TIOBE (rankings overview)

☑ Exclude "Edge Languages" | Number of Languages: 20 | Months for Delta (Δ): 3

	Language	Avg	StDev	n¹	Δ
1	JavaScript	1.25	0.5	4	-
2	Python	2.25	1.25	4	-
3	Java	4	1.41	4	-
4	TypeScript	5	2.16	4	-
5	C#	6	1.41	4	-
6	C++	6.75	1.7	4	-
7	PHP	8.25	3.5	4	-
8	C	10	0.81	4	-
9	Shell	10.5	3.51	4	-
10	Go	12	4.58	3	-
11	Rust	14.66	4.04	3	🟢 (1)
12	Ruby	14.75	6.55	4	🔴 (1)
13	Kotlin	16	1.73	3	-
14	R	16	4	3	-
15	Swift	16.33	4.61	3	-
16	Dart	18	2	3	-
17	PowerShell	18	6.55	3	-
18	Scala	24	8.18	3	🟢 (1)
19	Lua	24.5	2.12	2	🔴 (1)
20	VBA	25	5.65	2	🟢 (1)

1 - The number of (selected) ranking websites this language shows up in.

If you have suggestions or find a bug, you can open an issue here.

https://github.com/codereport/Content

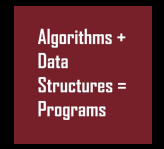
What is the best combinator programming language?

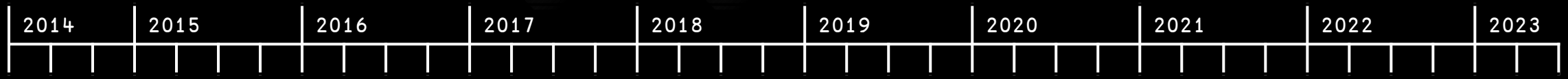
How did I discover array languages (and fall in love with them)?

https://www.youtube.com/watch?v=NiferfBvN3s

Dec 2019

How did I discover combinators?





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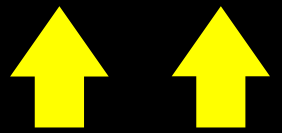
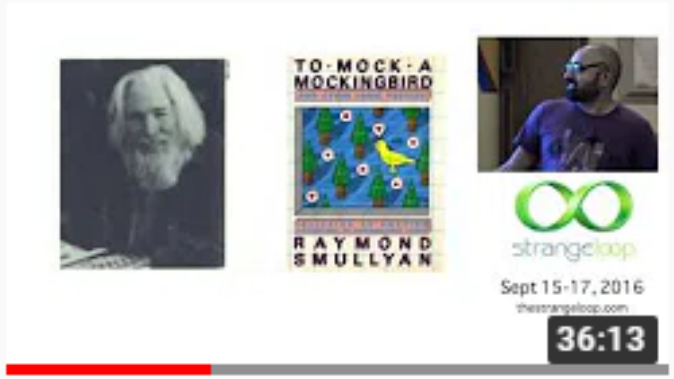


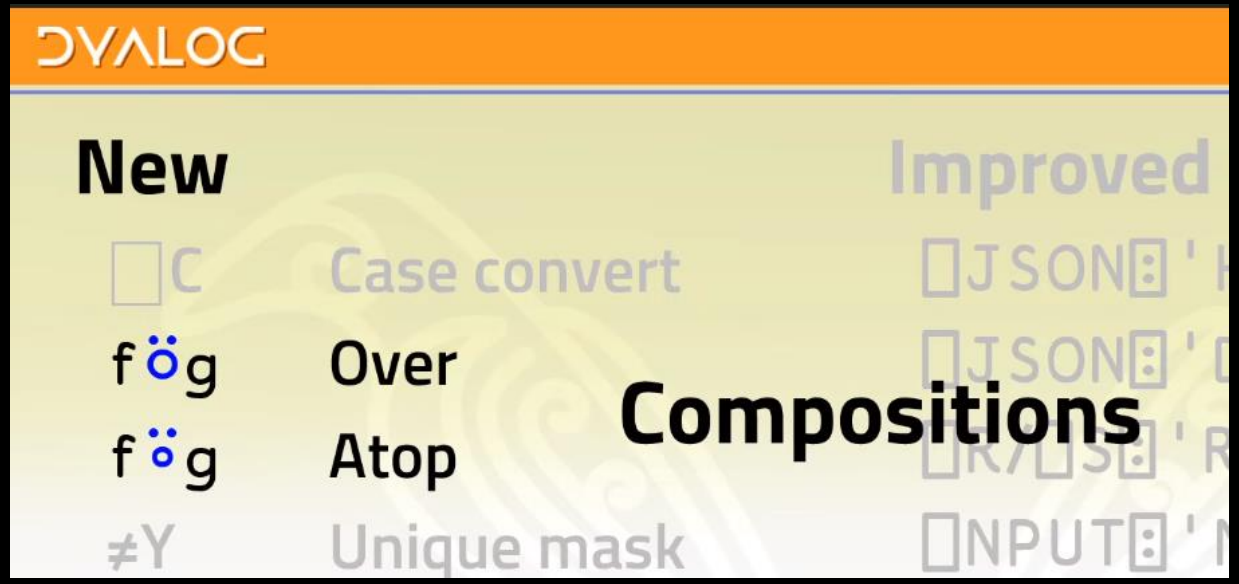


"Point-Free or Die: Tacit Programming in Haskell and Beyond" by Amar Shah

Strange Loop • 14K views • 3 years ago

Tacit programming, or programming in the "point-free" style, allows you to define a function without reference to one or more of its ...

July 5, 2020



Dyalog 18.0 Webinar
May 14, 2020

July 5, 2020

Sometime later in July

August 12

Combinatory logic

From Wikipedia, the free encyclopedia

Not to be confused with combinational logic, a topic in digital electronics.

Combinatory logic is a notation to eliminate the need for quantified variables in mathematical logic. It was introduced by Moses Schönfinkel[1] and Haskell Curry,[2] and has more recently been used in computer science as a theoretical model of computation and also as a basis for the design of functional programming languages. It is based on **combinators**, which were introduced by Schönfinkel in 1920 with the idea of providing an analogous way to build up functions—and to remove any mention of variables—particularly in predicate logic. A combinator is a higher-order function that uses only function application and earlier defined combinators to define a result from its arguments.

In mathematics [edit]

Combinatory logic was originally intended as a 'pre-logic' that would clarify the role of quantified variables in logic, essentially by eliminating them. Another way of eliminating quantified variables is Quine's predicate functor logic. While the expressive power of combinatory logic typically exceeds that of first-order logic, the expressive power of predicate functor logic is identical to that of first order logic (Quine 1960, 1966, 1976).

The original inventor of combinatory logic, Moses Schönfinkel, published nothing on combinatory logic after his original 1924 paper. Haskell Curry rediscovered the combinators while working as an instructor at Princeton University in late 1927.[3] In the late 1930s, Alonzo Church and his students at Princeton invented a rival formalism for functional abstraction, the lambda calculus, which proved more popular than combinatory logic. The upshot of these historical contingencies was that until theoretical computer science began taking an interest in combinatory logic in the 1960s and 1970s, nearly all work on the subject was by Haskell Curry and his students, or by Robert Feys in Belgium. Curry and Feys (1958), and Curry *et al.* (1972) survey the early history of combinatory logic. For a more modern treatment of combinatory logic and the lambda calculus together, see the book by Barendregt,[4] which reviews the models Dana Scott devised for combinatory logic in the 1960s and 1970s.

In computing [edit]

In computer science, combinatory logic is used as a simplified model of computation, used in computability theory and proof theory. Despite its simplicity, combinatory logic captures many essential features of computation.

Combinatory logic can be viewed as a variant of the lambda calculus, in which lambda expressions (representing functional abstraction) are replaced by a limited set of *combinators*, primitive functions without free variables. It is easy to transform lambda expressions into combinator expressions, and combinator reduction is much simpler than lambda reduction. Hence combinatory logic has been used to model some non-strict functional programming languages and hardware. The purest form of this view is the programming language Unlambda, whose sole primitives are the S and K combinators augmented with character input/output. Although not a practical programming language, Unlambda is of some theoretical interest.

Applications [edit]

Compilation of functional languages [edit]

David Turner used his combinators to implement the SASL programming language.

Kenneth E. Iverson used primitives based on Curry's combinators in his J programming language, a successor to APL. This enabled what Iverson called tacit programming, that is, programming in functional expressions containing no variables, along with powerful tools for working with such programs. It turns out that tacit programming is possible in any APL-like language with user-defined operators.[9]

Logic [edit]

The Curry–Howard isomorphism implies a connection between logic and programming: every proof of a theorem of intuitionistic logic corresponds to a reduction of a typed lambda term, and conversely. Moreover, theorems can be identified with function type signatures. Specifically, a typed combinatory logic corresponds to a Hilbert system in proof theory.

The **K** and **S** combinators correspond to the axioms

AK: $A \to (B \to A)$,
AS: $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$,

and function application corresponds to the detachment (modus ponens) rule

MP: from A and $A \to B$ infer B.

The calculus consisting of **AK**, **AS**, and **MP** is complete for the implicational fragment of the intuitionistic logic, which can be seen as follows. Consider the set W of all deductively closed sets of formulas, ordered by inclusion. Then $\langle W, \subseteq \rangle$ is an intuitionistic Kripke frame, and we define a model $\Vdash$ in this frame by

$$X \Vdash A \iff A \in X.$$

This definition obeys the conditions on satisfaction of $\to$: on one hand, if $X \Vdash A \to B$, and $Y \in W$ is such that $Y \supseteq X$ and $Y \Vdash A$, then $Y \Vdash B$ by modus ponens. On the other hand, if $X \nVdash A \to B$, then $X, A \nvdash B$ by the deduction theorem, thus the deductive closure of $X \cup \{A\}$ is an element $Y \in W$ such that $Y \supseteq X$, $Y \Vdash A$, and $Y \nVdash B$.

Let A be any formula which is not provable in the calculus. Then A does not belong to the deductive closure X of the empty set, thus $X \nVdash A$, and A is not intuitionistically valid.

See also [edit]

- Applicative computing systems
- B, C, K, W system
- Categorical abstract machine
- Combinatory categorial grammar
- Explicit substitution

Applications [edit]

Compilation of functional languages [edit]

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curry combinators and the "j programming language"

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Combinatory logic - Wikipedia

Combinatory logic is a notation to eliminate the need for quantified variables in mathematical ... Haskell **Curry** rediscovered the **combinators** while working as an instructor at Princeton University in late ... Kenneth E. Iverson used primitives based on **Curry's combinators** in his **J programming language**, a successor to APL.

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link.springer.com › chapter

Reduction languages and reduction systems | SpringerLink

May 28, 2005 - Backus, **J.: 'Programming Language** Semantics and Closed Applicative Languages'; San ... **Curry**, H.B.; Feys, R.: 'Combinatory Logic'; Vol. ... Hindley, J.R.; Seldin, J.P.: 'Introduction to **Combinators** and λ-Calculus'; London ...

by W Kluge - 1987 - Cited by 1 - Related articles

wiki.tcl-lang.org › page › Tacit+programming

Tacit programming - the Tcler's Wiki! - Tcl/Tk

Mar 25, 2014 - ... article [L2] about the **J programming language** (the "blessed successor" to APL, ... Only implicitly present is a powerful function **combinator** called "fork". ... to Schönfinkel/**Curry's** S **combinator** (see Hot **Curry** and **Combinator** ...

if 0 {As KBK pointed out in the Tcl chatroom, the "hook" pattern corresponds to Schönfinkel/Curry's S combinator (see Hot Curry and Combinator Engine), while "fork" is called S' there.

Google Scholar "combinatory logic" "combinator" "APL"

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include patents
include citations

Create alert

Pure functions in APL and J

[PDF] acm.org

E Cherlin - Proceedings of the international conference on APL'91, 1991 - dl.acm.org

... Any expression in **combinatory logic** made up of combinators and variables can be abstracted into apure **combinator** expres- sion applied to asequence of variables. Because there are great similarities between combinators and certain **APL** operators, a simihm result obtains in ...

☆ 99 Cited by 3 Related articles All 4 versions

[PDF] History of lambda-calculus and combinatory logic

[PDF] simons-rock.edu

F Cardone, JR Hindley - Handbook of the History of Logic, 2006 - hope.simons-rock.edu

... **Combinatory logic** was invented by Moses Ilyich Schönfinkel ... 2In today's notation (following [Curry and Feys, 1958, Ch.5]) their axiom-schemes are BXY Z = X(YZ), CXY Z = XZY , IX = X, KXY = X, SXY Z = XZ(YZ), and a **combinator** is any applicative combination of basic ...

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APL trivia

[PDF] acm.org

E Cherlin - ACM SIGAPL **APL** Quote Quad, 1990 - dl.acm.org

... The branch of mathematics that makes the most of trivia is Curry's **combinatory logic** [Cu58]. A **combinator** is, approximately, a trivial operator ... Each of these combinators corresponds somewhat to a Diction- ary **APL** function or operator, as also shown in Table 2, although ...

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[PDF] Phrasal forms

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EE McDonnell, KE Iverson - Conference proceedings on **APL** as a tool of ..., 1989 - dl.acm.org

... In **combinatory logic** one of the most useful primitive combinators is designated by S [Sch24] ... (The constancy **combinator** K is defined in infur notation so that cKz has the value c for all x.) Users of **APL** will appreciate the hook for the same reasons ...

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Phrasal Forms

K. E. Iverson
Toronto

E. E. McDonnell
I. P. Sharp Associates
Palo Alto

NOTE: In this paper we use the linguistic terms *verb* and *pronoun* interchangeably with the mathematical terms *function* and *variable.*

INTRODUCTION

In standard APL [ISO88] certain forms are ungrammatical, and new definitions could be adopted for them without conflict. Such definitions we shall call *phrasal forms* [AHD76]. For example, if b and c are pronouns, the phrase $b\ c$ is meaningless, and in APL2 [IBM85] the definition $(\subset b), (\subset c)$, where $\subset$ is the APL2 *enclose* function, is adopted for it.

VERB RANK

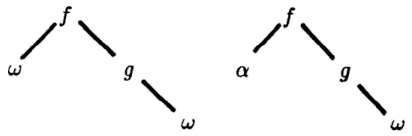
The notion of *verb rank*, first introduced by Iverson [Iv78], later elaborated by Keenan [Ke79], and further evolved by Whitney [Wh84], has been adopted by Iverson in his Dictionary [Iv87]. It refers to the rank of the subarrays of an argument which are the cells to which the verb aplies. For example, the cells that negate applies to are items, and items are ranlk zero objects, and thus we say the rank of negate is zero. Similarly, the cells to which reverse applies are lists, or rank ome objects, and thus we say the rank of reverse is one. Not only primitive verbs, but also derived and defined verbs have rank. The idea is a powerful one, producing great simplifications, and so we define the ranks of the new constructions we describe herein.

DEFINITIONS

In the following definitions, f, g, and h denote verbs and α and ω denote pronouns.

HOOK. A *hook* is denoted by $f\ g$ and is defined formally by identities and informally by hook-shaped diagrams as follows:

$$(fg)\omega \leftrightarrow \omega f g\omega; \qquad \alpha(fg)\omega \leftrightarrow \alpha f g\omega$$



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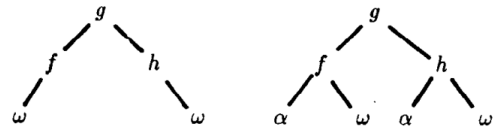
Copyright 1989 ACM 0-87971-327-2/0008/0197 $1.50

The rank of the hook $f\ g$ is the maximum of the ranks of f and g. Note that the verb is used monadically.

FORK. A *fork* is denoted by $f\ g\ h$ and is defined formally by identities and informally by forking diagrams as follows:

$$(fgh)\omega \leftrightarrow (f\omega)g(h\omega); \qquad \alpha(fgh)\omega \leftrightarrow (\alpha f\omega)g(\alpha h\omega)$$



The rank of the fork $f\ g\ h$ is the maximum of the ranks of f and g. Note that the central verb is used dyadically or monadically according to whether the fork is applied to two arguments or one. Parenthesis are required around hook and fork forms only to avoid ambiguity.

DISCUSSION OF HOOK AND FORK

HOOK. In combinatory logic one of the most useful primitive combinators is designated by **S** [Sch24]. Curry defines $\mathbf{S}fgx$ in prefix notation to be $fx(gx)$[CuFeCr74]. In common mathematical infix notation this would be given by $(x)f(g(x))$, which one can write in APL as $xfgx$, and this is the hook form $(fg)x$. The combinatory logician appreciates this form because of its great expressiveness: it can be shown that **S**, along with **K**, the *constancy* combinator, suffice to define all other combinators of interest [Ro50]. (The constancy combinator **K** is defined in infix notation so that $c\mathbf{K}x$ has the value c for all x.) Users of APL will appreciate the hook for the same reasons.

For example, **+÷** adds the reciprocal of the right argument to the left argument, a form used in describing continued fractions. Thus $(+\div)\backslash 3\ 7\ 16\ {}^{-}294$ gives the first four convergents to pi, which, to nine decimals, are 3, 3.1412857143, 3.14159292, and 3.141592654. Further, $=\lfloor$ is a proposition that tests whether its argument is an integer, the number of primes less than positive integer ω is approximately $(\div\circledast)\omega$; and to decompose a number ω into numerator and denominator, one can write $(\div\vee/)\omega, 1$ (where $\vee$ is the *greatest common divisor).*

FORK. The forks $f + h$ and $f \times h$ and $f \div h$ provide formal treatment of the identical but informal phrases used in mathematics [e.g. Ef89] for the sum and product and quotient,

er evolved by
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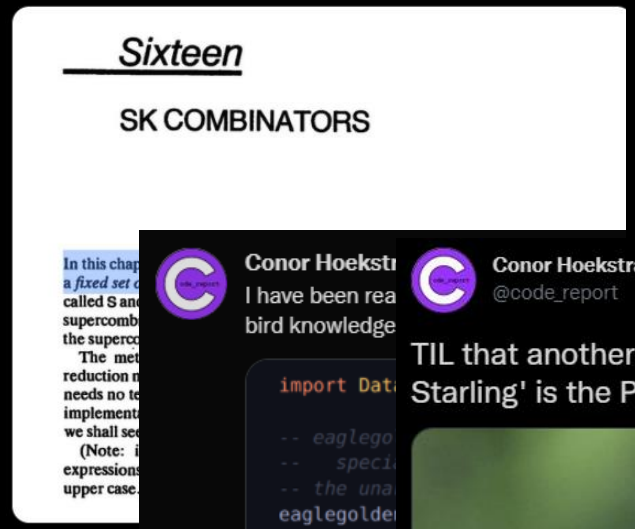
For example, +÷ adds the reciprocal of the right argument to the left argument, a form used in describing continued fractions. Thus (+÷)\3 7 16 ¯294 gives the first four con-

Curry [Cu31] defines a *formalizing combinator*, Φ, in prefix notation, such that $\Phi fghx$ means $f(gx)(hx)$. In common mathematical infix notation this would be designated by $(g(x))f(h(x))$. An example of this form is $\Phi+sin^2cos^2\theta$, meaning $sin^2\theta + cos^2\theta$. The fork $(f\ g\ h)\omega$ has the same meaning, namely $(f\omega)g(h\omega)$. Curry named this the *formalizing combinator* because of its role in defining formal implication in terms of ordinary implication.

Conor Hoekstra
@code_report

For a second, I thought @simonpj0 had already implemented CEAF (Combinator-Enabled Algorithm Fusion) but alas the graph reduction he refers to is not that (it is reducing a program down to the S, K & I #combinators)



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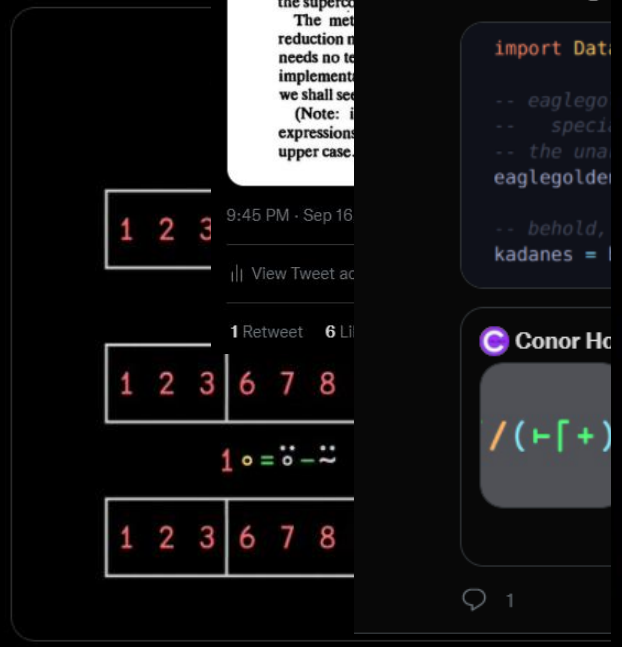
Conor Ho
@code_rep

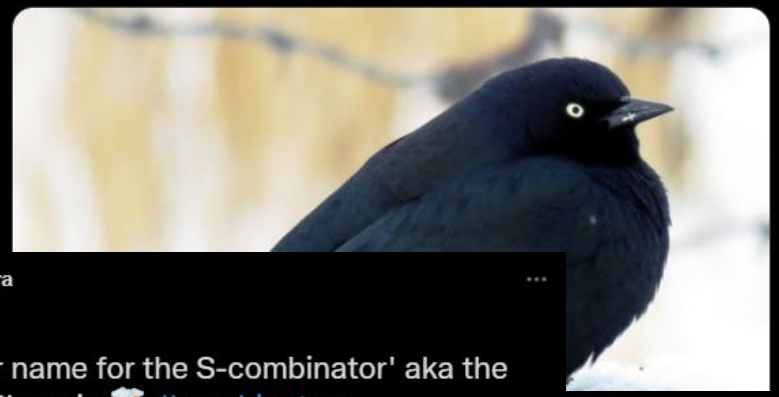
I am not sure
Combinatory
C, E, Ê, K an
Logic so muc



12:38 AM · Sep 24, 2021 · Twitter Web App

Conor Hoekstra
@code_report

And for maybe my most amazing combinator discovery to date, std::inner_product is the Blackbird 🤯 (which is what I was looking for when I stumbled across the Phoenix) #combinators #cpp #algorithms



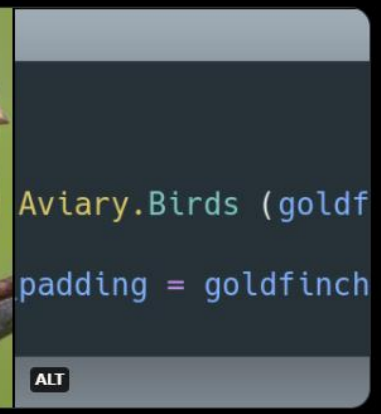
Conor Hoekstra
@code_report

TIL that another name for the S-combinator' aka the Starling' is the Phoenix 🤯 #combinators

9:06 PM · Oct 21, 2020 · Twitter Web App

Conor Hoekstra
@code_report

The Combinator Quest continues and I managed to use a new combinator today, the G-combinator, aka the Goldfinch 🙂



8:55 PM · Sep 10, 2020 · Twitter Web App

Conor Hoekstra
@code_report

In #APL, trains are ALL of the following:

- Bluebird (B combinator)
- Dove (D combinator)
- Phoenix (S' combinator)
- Blackbird (B1 combinator)
- Eagle (E combinator)
- Golden Eagle (E^ combinator specialization)

APL is a Combinatory Logic programming language.

2:56 AM · Sep 21, 2021 · Twitter Web App

A Brief History of Combinatory Logic (CL)
A Brief History of Array Languages
A Brief History of CL in Array Languages

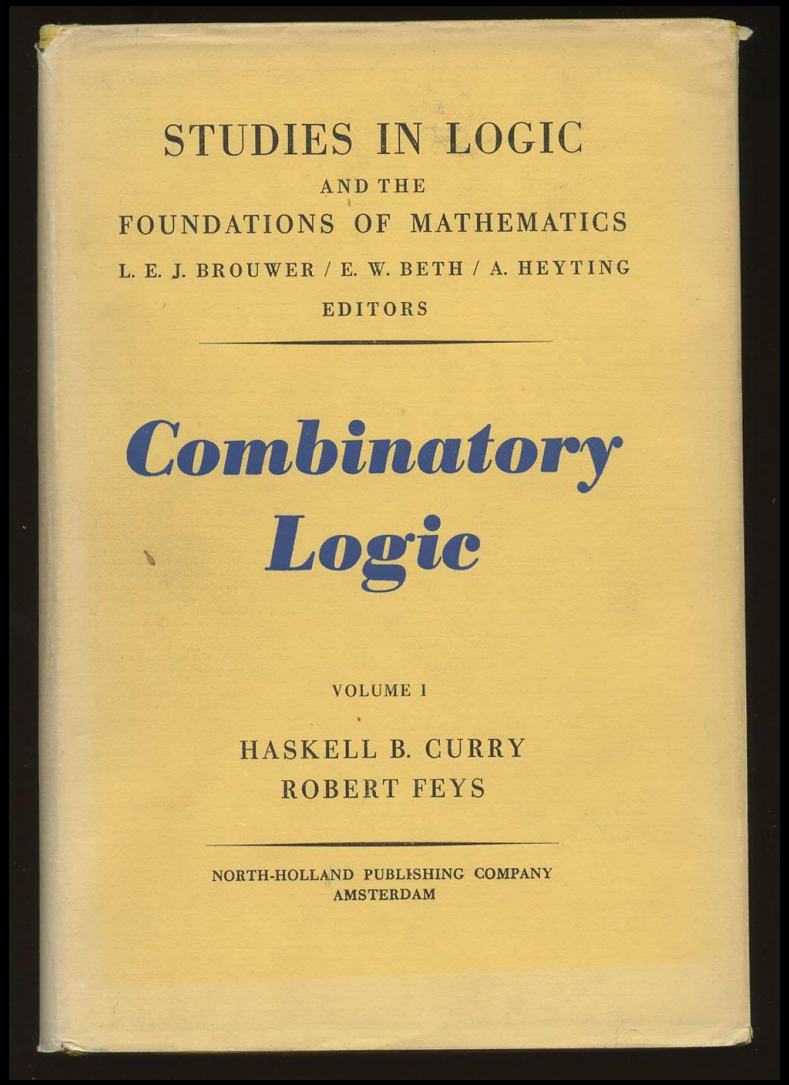
A Brief History of Combinatory Logic

Moses Schönfinkel
1988 - 1942

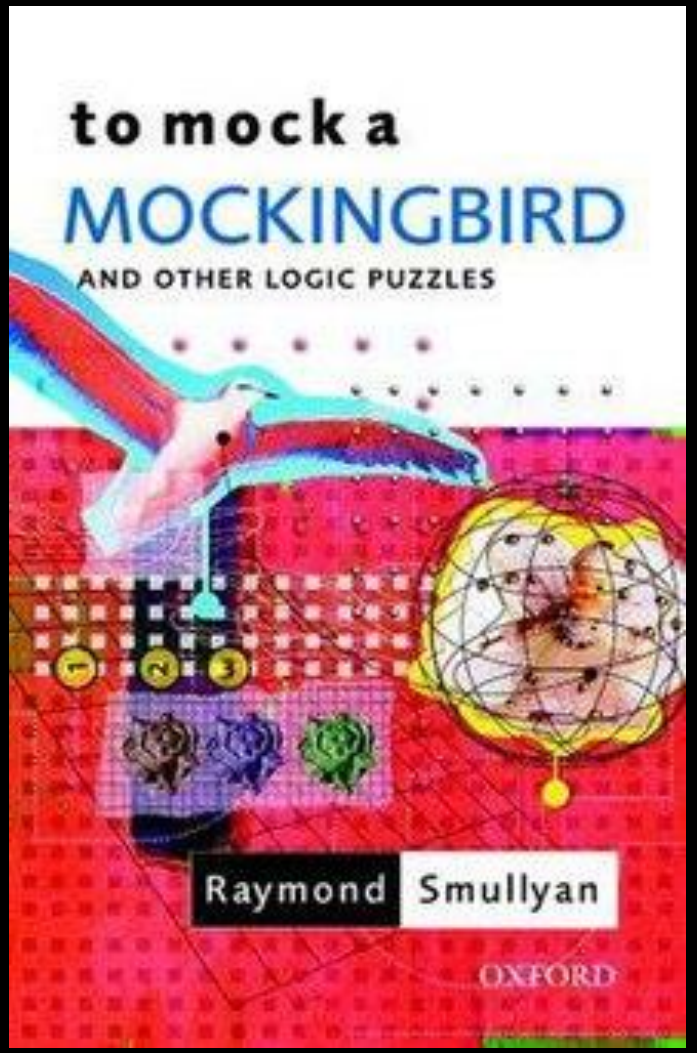
Moses Schönfinkel
1988 - 1942

Haskell Curry
1900-1982

Year	Author(s)	Title	Introduced
1924	Schönfinkel	On the building blocks of mathematical logic	S K I B C
1929	Curry	An Analysis of Logical Substitution	W
1930	Curry	The Foundations of Combinatory Logic	B_n (B_1, B_2, …)
1931	Curry	The Universal Quantifier in Combinatory Logic	Ψ, Φ_n (Φ, Φ_1, …)
1958	Curry and Feys	Combinatory Logic: Volume I	


STUDIES IN LOGIC
AND THE
FOUNDATIONS OF MATHEMATICS
L. E. J. BROUWER / E. W. BETH / A. HEYTING
EDITORS
Combinatory Logic
VOLUME I
HASKELL B. CURRY
ROBERT FEYS
NORTH-HOLLAND PUBLISHING COMPANY
AMSTERDAM

1958

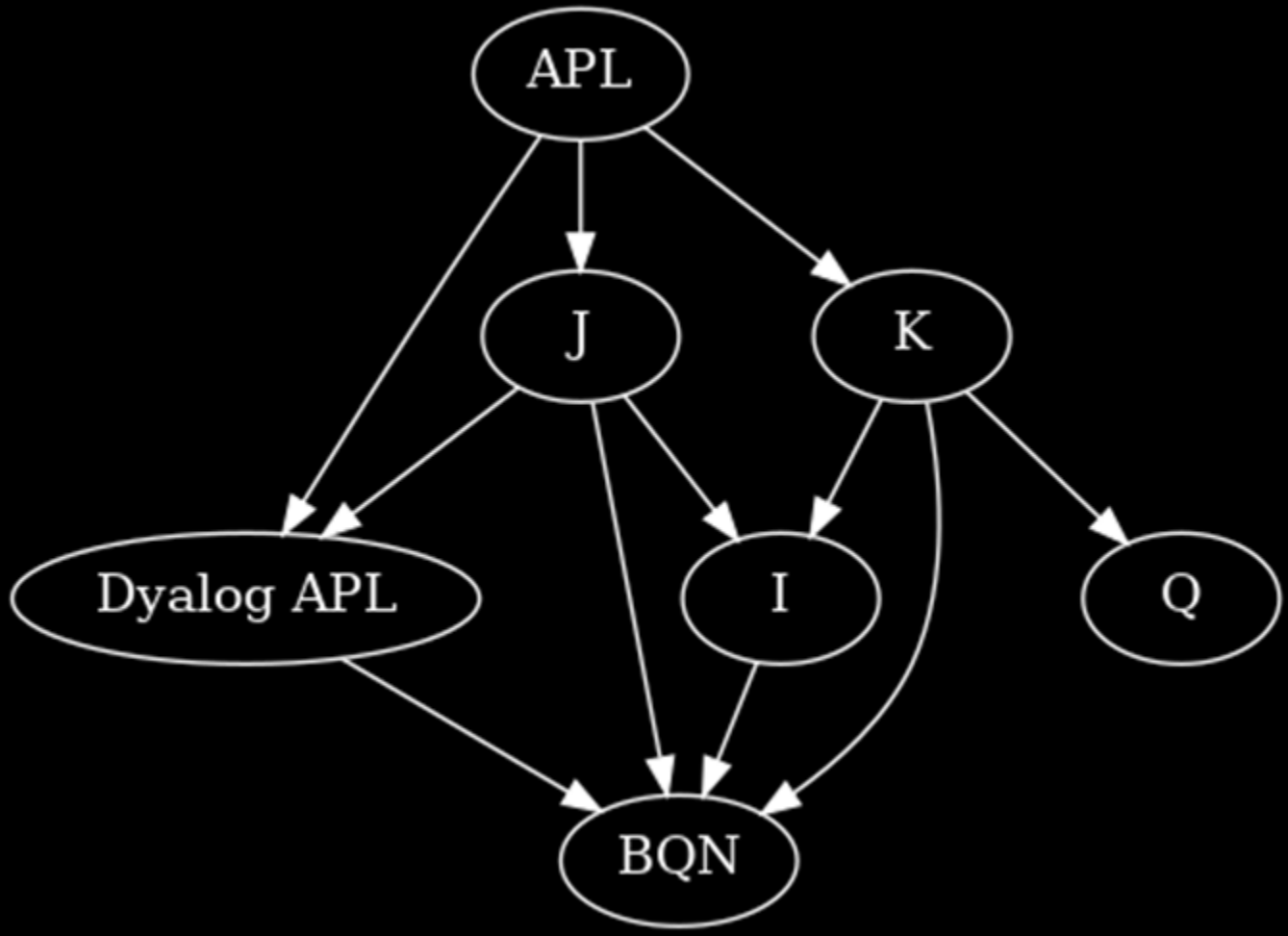

to mock a
MOCKINGBIRD
AND OTHER LOGIC PUZZLES
Raymond Smullyan
OXFORD

1985

www.combinatorylogic.com/table.html

	Author	Year	Paper
Sch24	Moses Schönfinkel	1924	On the building blocks of mathematical logic
Cur29	Haskell Curry	1929	An Analysis of Logical Substitution
Cur30	Haskell Curry	1931	Grundlagen der Kombinatorischen Logik (The Foundations of Combinatory Logic)
Cur31	Haskell Curry	1931	The universal quantifier in combinatory logic
Cur48	Haskell Curry	1948	A Simplification of the Theory of Combinators
Cur58	H. Curry & R. Feys	1958	Combinatory Logic: Volume I
Tur78	David Turner	1979	Another algorithm for bracket abstraction
Smu85	Raymond Smullyan	1985	To Mock a Mockingbird
Iv89	K. Iverson & E. McDonnell	1989	Phrasal Forms
Loc12	Marshall Lochbaum	2012	Added hook, backhook

A Brief History of Array Languages


APL
J
K
Dyalog APL
I
Q
BQN

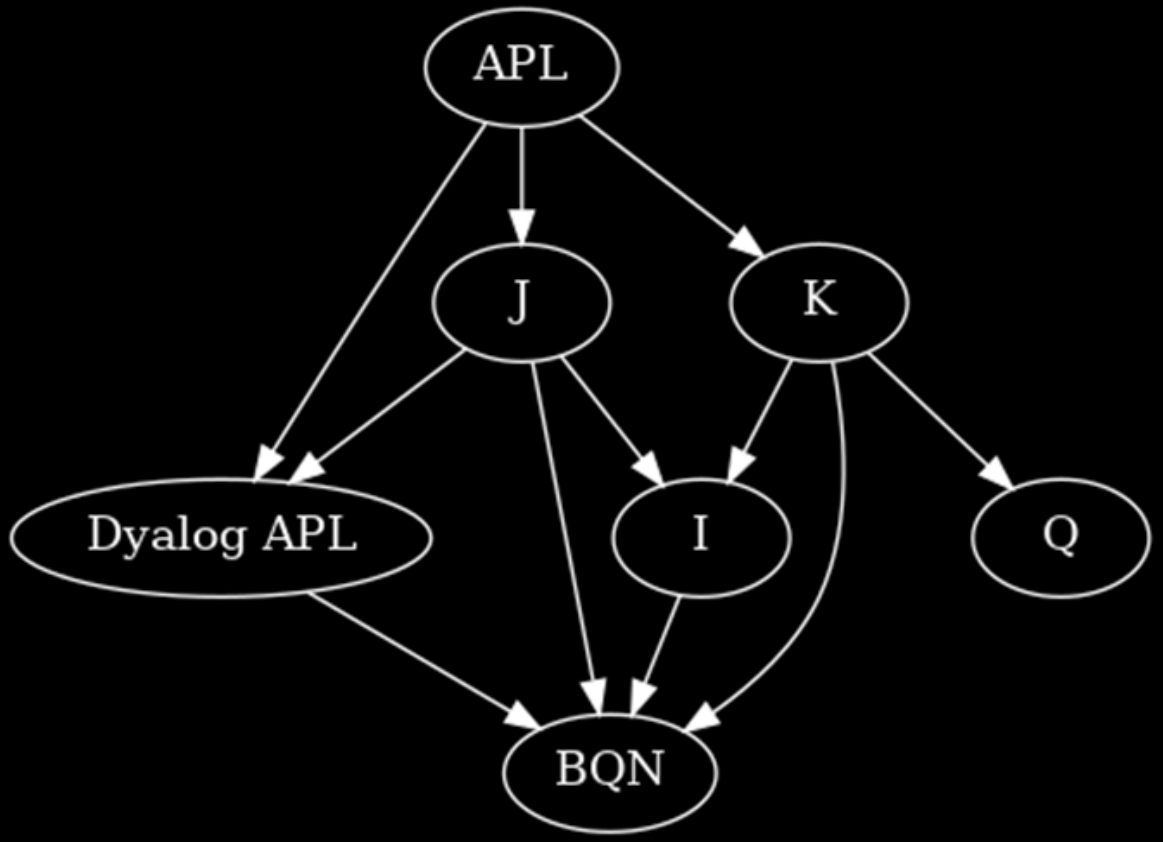


Table 1. Array languages timeline.

Language	Year
APL	1966
Dyalog APL 1.0	1983
J	1990
K	1994
Q	2003
I	2012
BQN	2020
Dyalog APL 18.0	2020

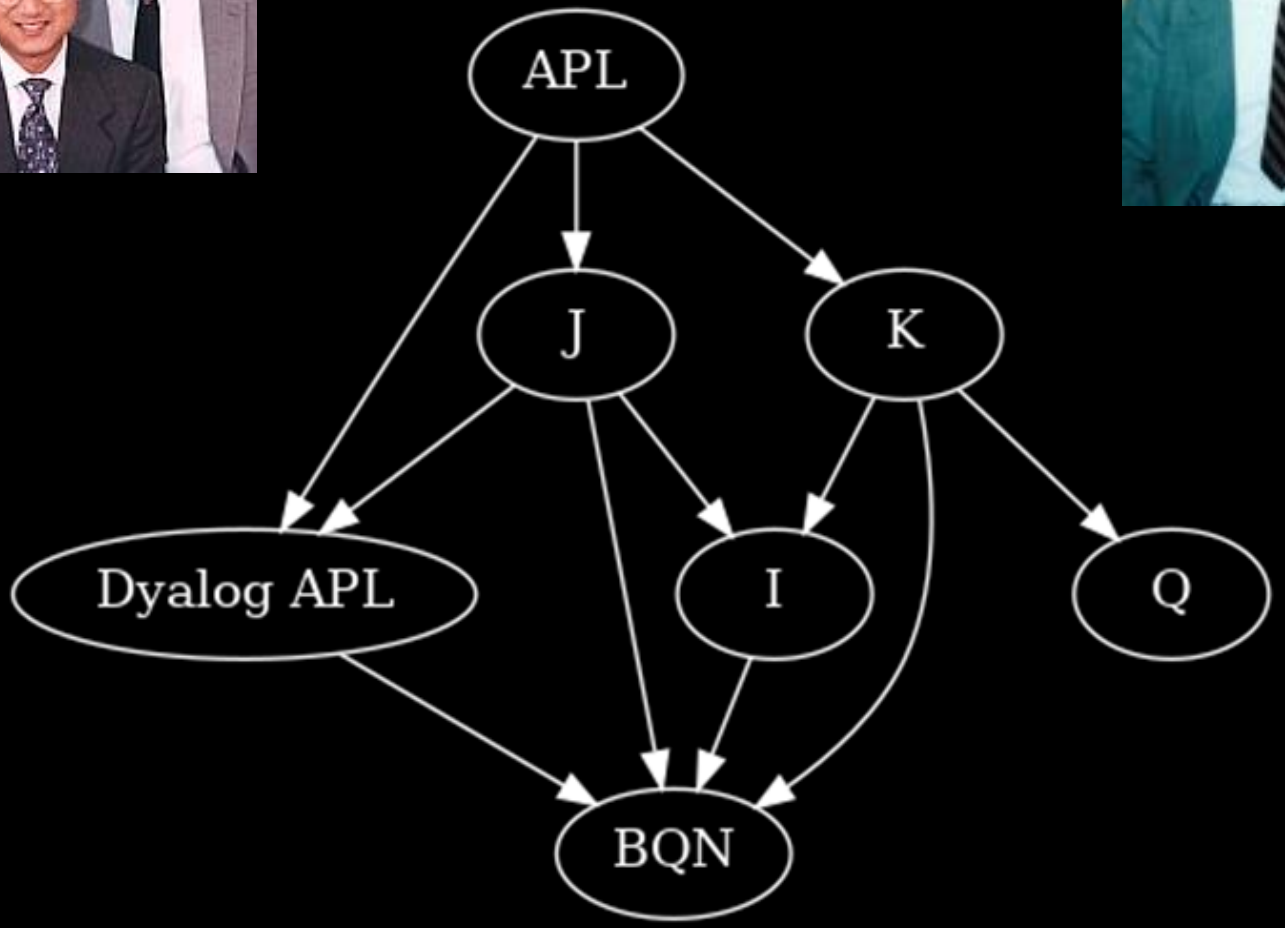


Table 1. Array languages timeline.

Language	Year
APL	1966
Dyalog APL 1.0	1983
J	1990
K	1994
Q	2003
I	2012
BQN	2020
Dyalog APL 18.0	2020

A Brief History of CL in Array Languages

Table 4. 2 and 3-trains in APL, BQN and J.

Year	Language	2-Train	3-Train
1990	J	S and D	Φ and Φ_1
2014	Dyalog APL	B and B_1	Φ and Φ_1
2020	BQN	B and B_1	Φ and Φ_1

Table 5. History of combinators in Dyalog APL.

Year	Version	Combinator	Spelling
1983	1.0	B, D, C	∘⍤
2003	10.0	W	⍨
2013	13.0	K	⊢
2014	14.0	B, B_1, Φ, $Φ_1$	trains
2020	18.0	B, B_1, Ψ, K	⍥⍤⍨

Combinator Table: Combinators, Birds, Spellings & More

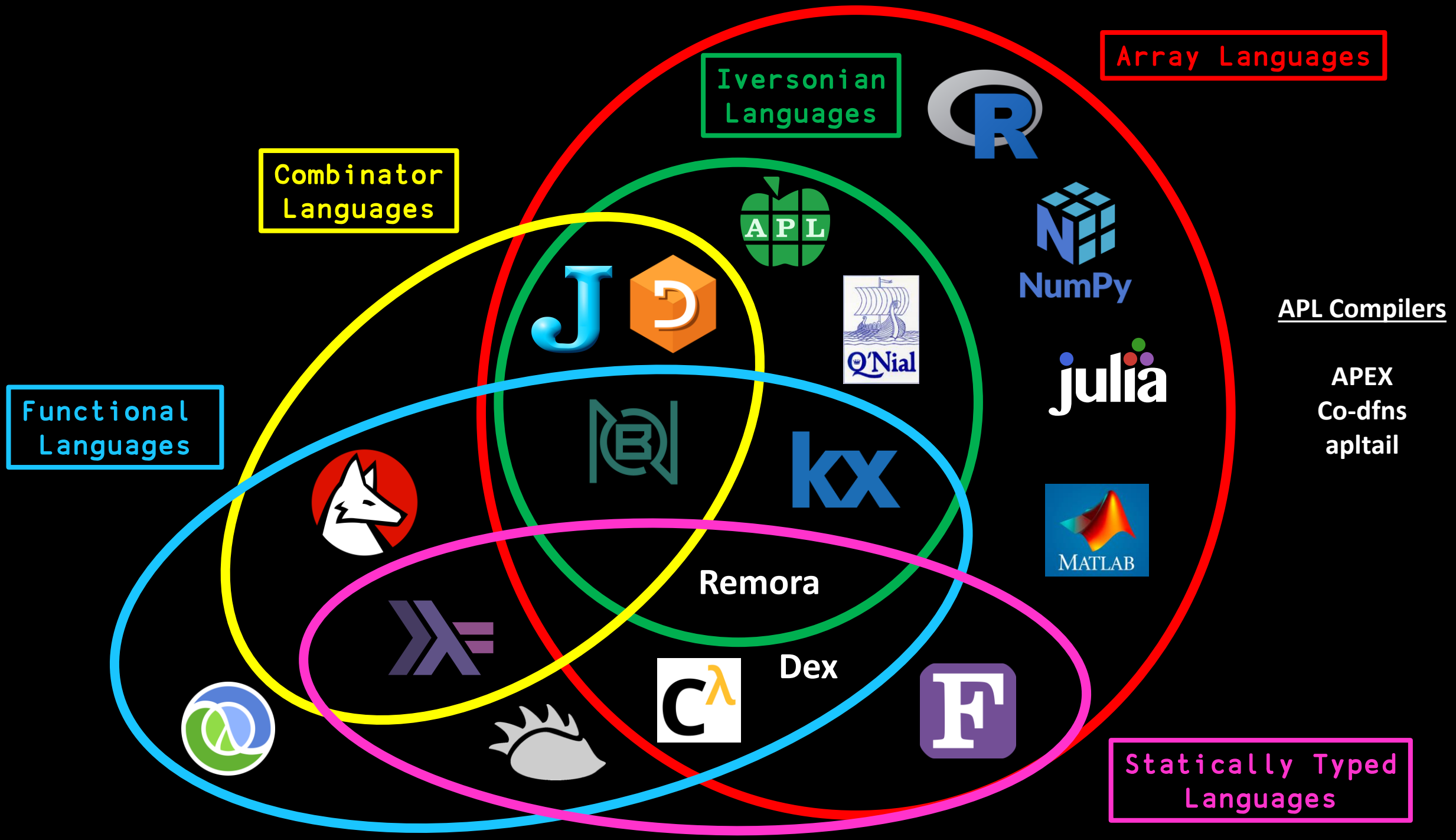
Function Abstraction	Symbol	Bird	CR[1]	CH[2]	Elem[3]	Haskell	APL	BQN	Intro
λabc.ac(bc)	S	Starling	S	S		<*> / ap		∘⊢	Sch24
λab.a	K	Kestrel	K	K	✅	const	⊢	⊢	Sch24
λa.a	I	Identity	SKK	SKK	✅	id	⊣⊢	⊣⊢	Sch24
λab.b	κ	Kite	KI	KI			⊣	⊣	
λab.abb	W	Warbler	C(BMR)	CSK	✅	join	⍨	˜	Cur29
λabc.acb	C	Cardinal	S(BBS)(KK)	B(ΦBS)KK	✅	flip	⍨	˜	Sch24
λabc.a(bc)	B	Bluebird	S(KS)K	S(KS)K	✅	.	∘⍤2T	∘○ 2T	Sch24
λabcd.a(bcd)	B_1	Blackbird	BBB	DB		.:	⍤2T	∘ 2T	Cur30
λabcde.a(bcde)	B_2	Bunting	B(BBB)B	DB_1					Cur30
λabcd.a(b(cd))	B_3	Becard	B(BB)B	BDB					Cur30
λabcd.ab(cd)	D	Dove	BB	BB			∘	∘⊢	Smu85
λabcd.a(bd)(cd)	Φ	Phoenix	-	B_1SB		liftA2	3T	3T	Cur31
λabcd.a(bc)(bd)	Ψ	Psi	-	B(SΦCB)B		on	⍤	○	Cur31
λabcde.abc(de)	D_1	Dickcissel	B(BB)	BD					
λabcde.a(bc)(de)	D_2	Dovekies	BB(BB)	DD					Smu85
λabcde.ab(cde)	E	Eagle	B(BBB)	BB_1					Smu85
λabcde.a(bde)(cde)	$Φ_1$	Pheasant	-	BΦΦ			3T	3T	Cur31
λabcdefg.a(bcd)(efg)	Ê	Bald Eagle	B(BBB)(B(BBB))	D_2D_2D					Smu85

	APL	BQN	J
I	⊣⊢	⊣⊢	][
K	⊢	⊢	]
KI	⊣	⊣	[
S		⍥	2 Train
B	∘⍤2	∘○ 2	@: &:
C	⍨	˜	~
W	⍨	˜	~
B_1	⍤2	∘ 2	@:
D	∘	⍥	2 Train
Ψ	⍤	○	&:
Φ	3 Train	3 Train	3 Train
$Φ_1$	3 Train	3 Train	3 Train
D_2		⍥⍥	

	APL	BQN	J
I	⊣⊢	⊣⊢	][
K	⊢	⊢	]
KI	⊣	⊣	[
S		⍥	2 Train
B	∘⍤⍤ 2T	∘○ 2T	@: &:
C	⍨	˜	~
W	⍨	˜	~
B_1	⍤ 2T	∘ 2T	@:
D	∘	⍥	2 Train
Ψ	⍤	○	&:
Φ	3 Train	3 Train	3 Train
$Φ_1$	3 Train	3 Train	3 Train
D_2		⍥⍥	

	APL	BQN	J
I	⊣⊢	⊣⊢	][
K	⊢	⊢	]
KI	⊣	⊣	[
S		⍥	2 Train
B	∘⍤⍤ 2T	∘○ 2T	@: &:
C	⍨	˜	~
W	⍨	˜	~
B_1	⍤ 2T	∘ 2T	@:
D	∘	⍥	2 Train
Ψ	⍤	○	&:
Φ	3 Train	3 Train	3 Train
$Φ_1$	3 Train	3 Train	3 Train
D_2		⍥⍥	

	APL	BQN	J
I	⊣⊢	⊣⊢	][
K	⊢	⊢	]
KI	⊣	⊣	[
S		⍥	2 Train
B	∘⍤⍤ 2T	∘○ 2T	@: &:
C	⍨	˜	~
W	⍨	˜	~
B_1	⍤ 2T	∘ 2T	@:
D	∘	⍥	2 Train
Ψ	⍤	○	&:
Φ	3 Train	3 Train	3 Train
$Φ_1$	3 Train	3 Train	3 Train
D_2		⍥⍥	
Σ		⍥	
Δ		⍥	


Array Languages
Iversonian Languages
Combinator Languages
Functional Languages
Statically Typed Languages
APL
NumPy
Q'Nial
julia
kx
MATLAB
Remora
Dex
APL Compilers
APEX
Co-dfns
apltail

What is a combinator?

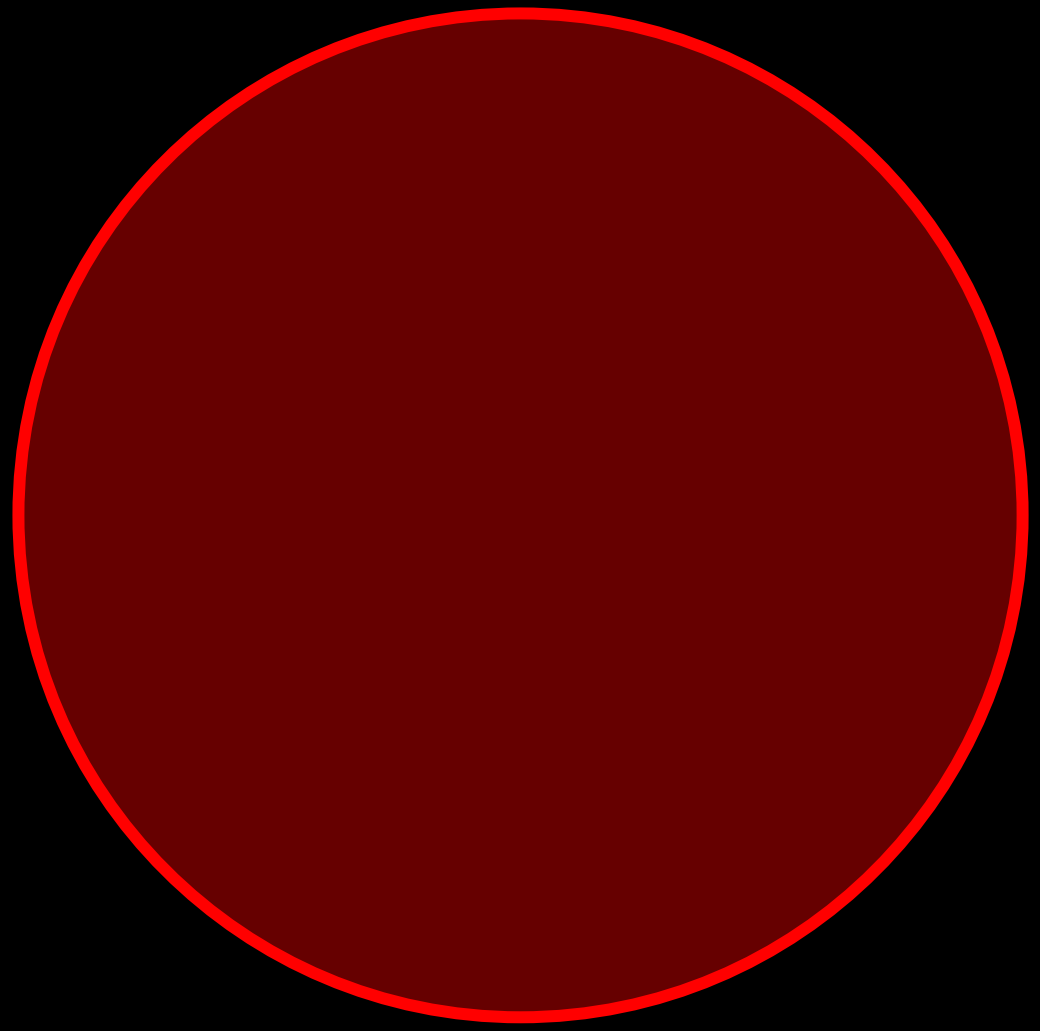
combinator: a lambda expression
containing no free variables

combinator: a function that deals only in its arguments

combinator = pure function?

pure function: same input = same output / no side effects

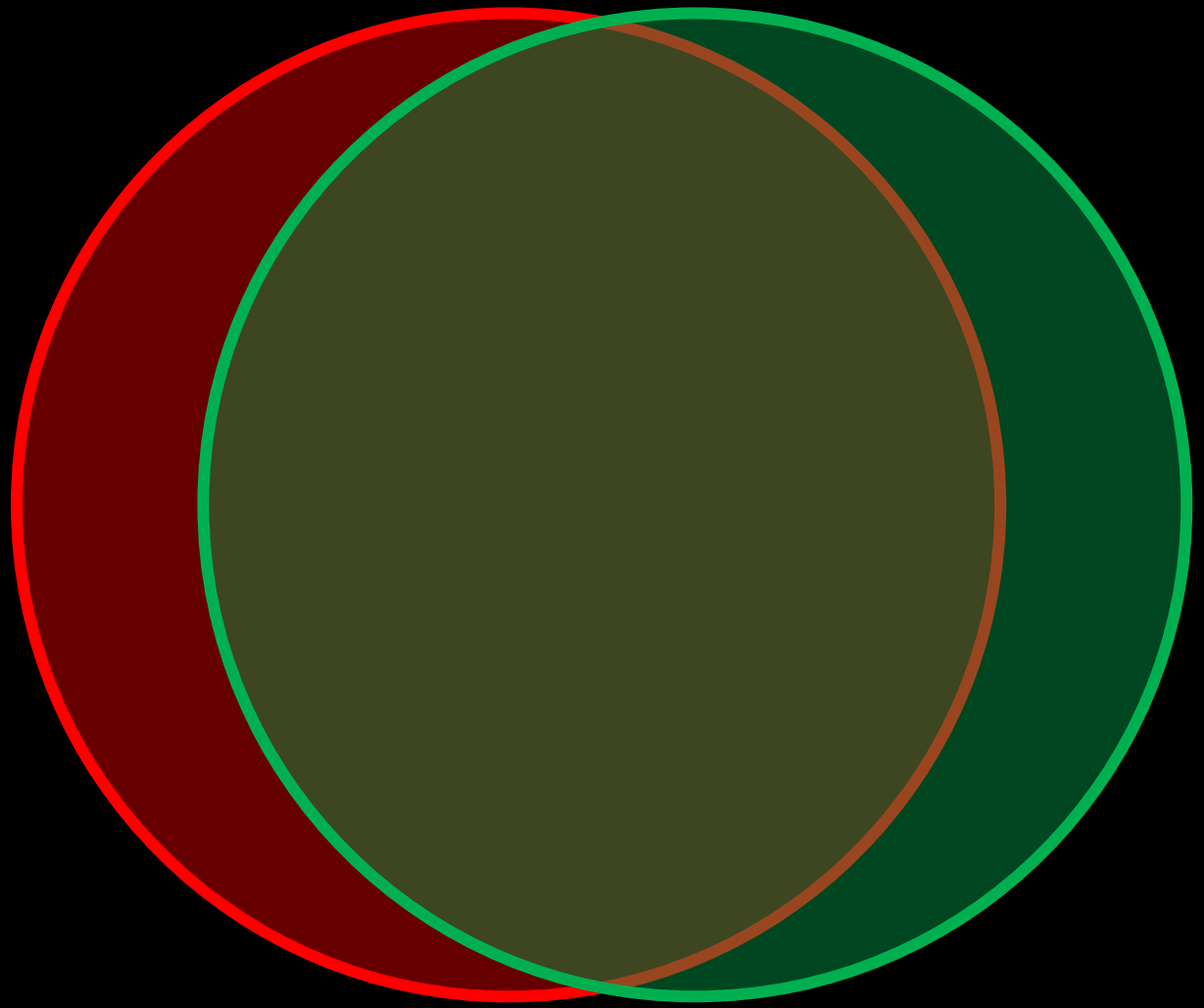
pure function



combinator

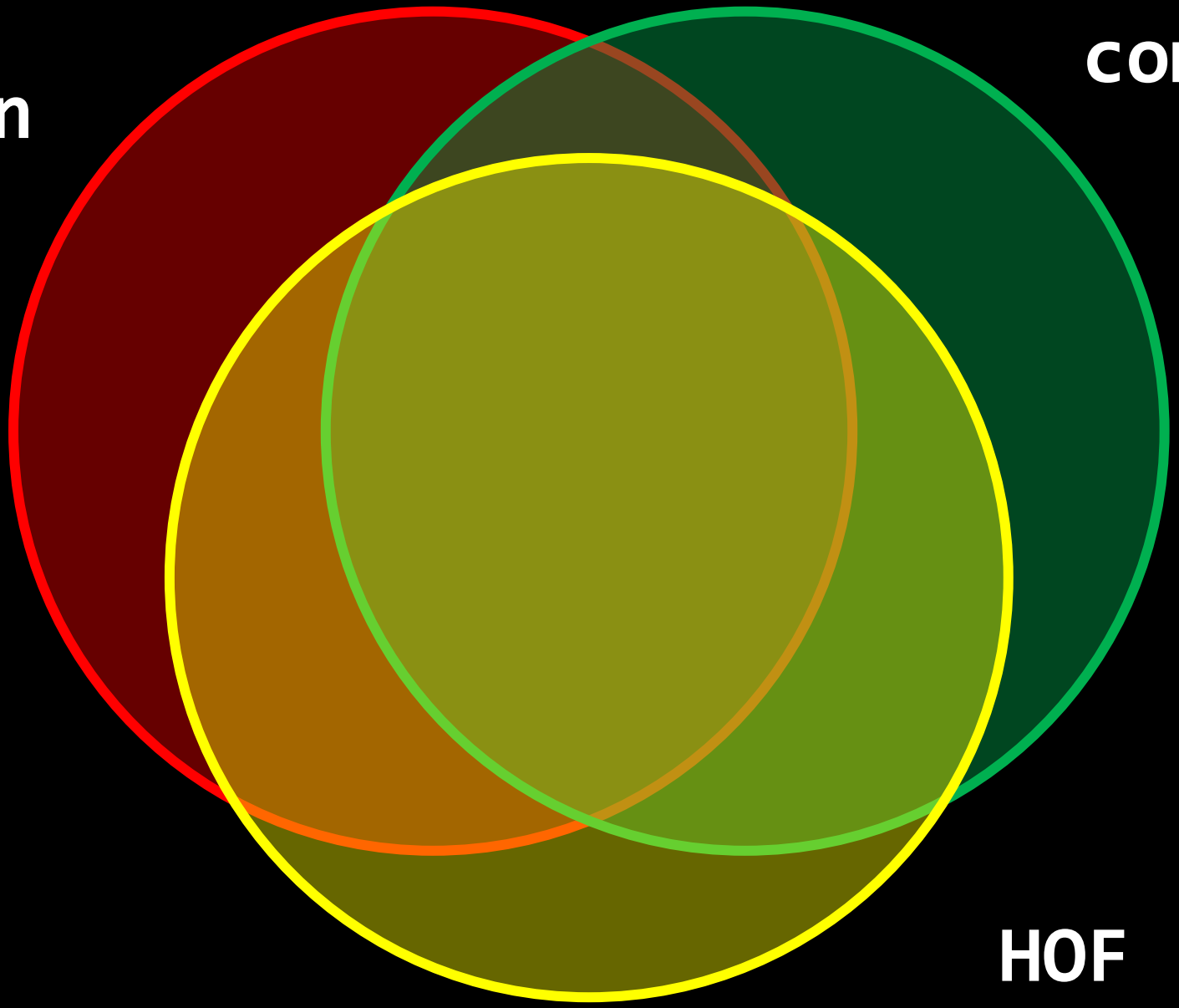
pure function

combinator



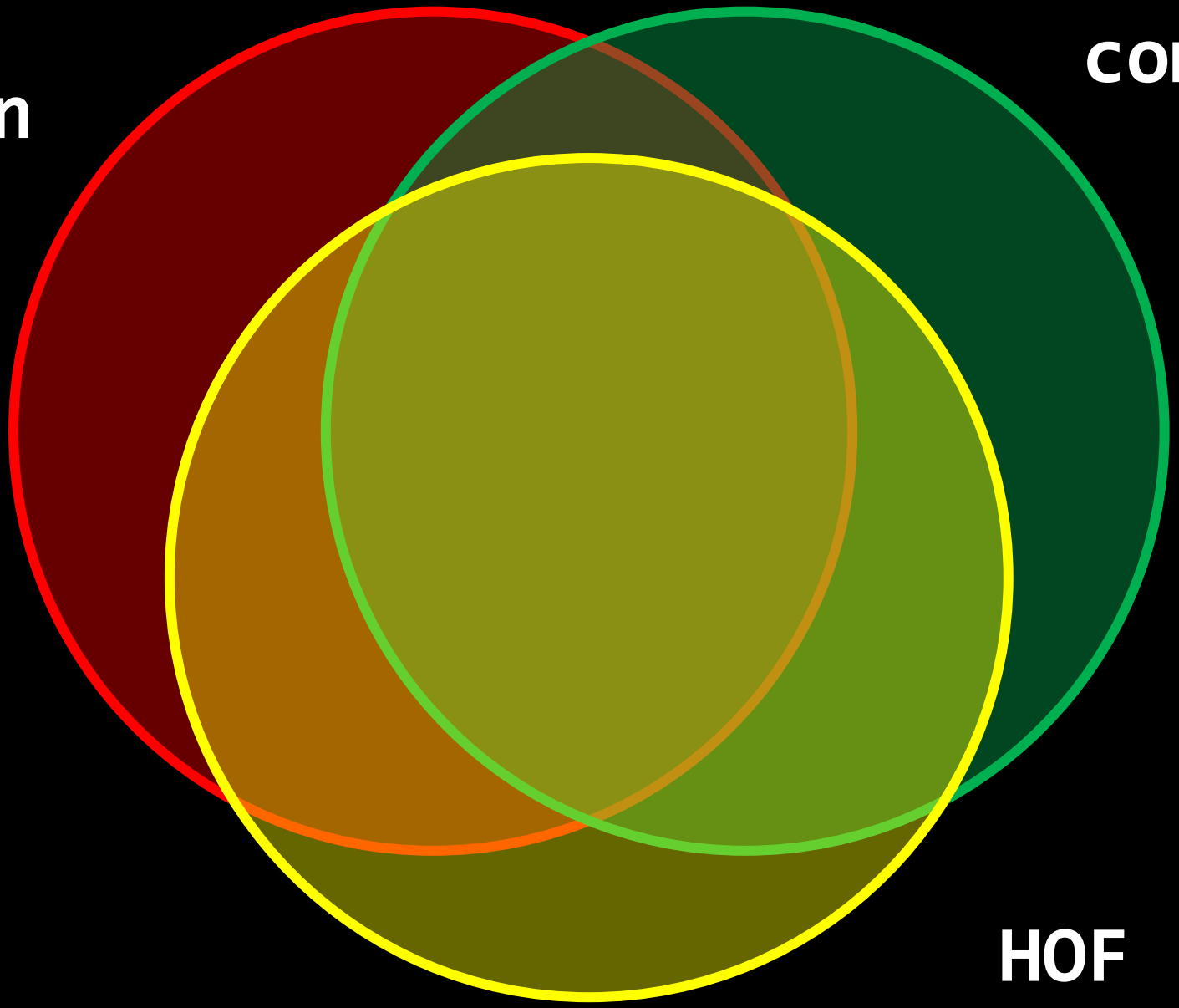
pure
function

combinator



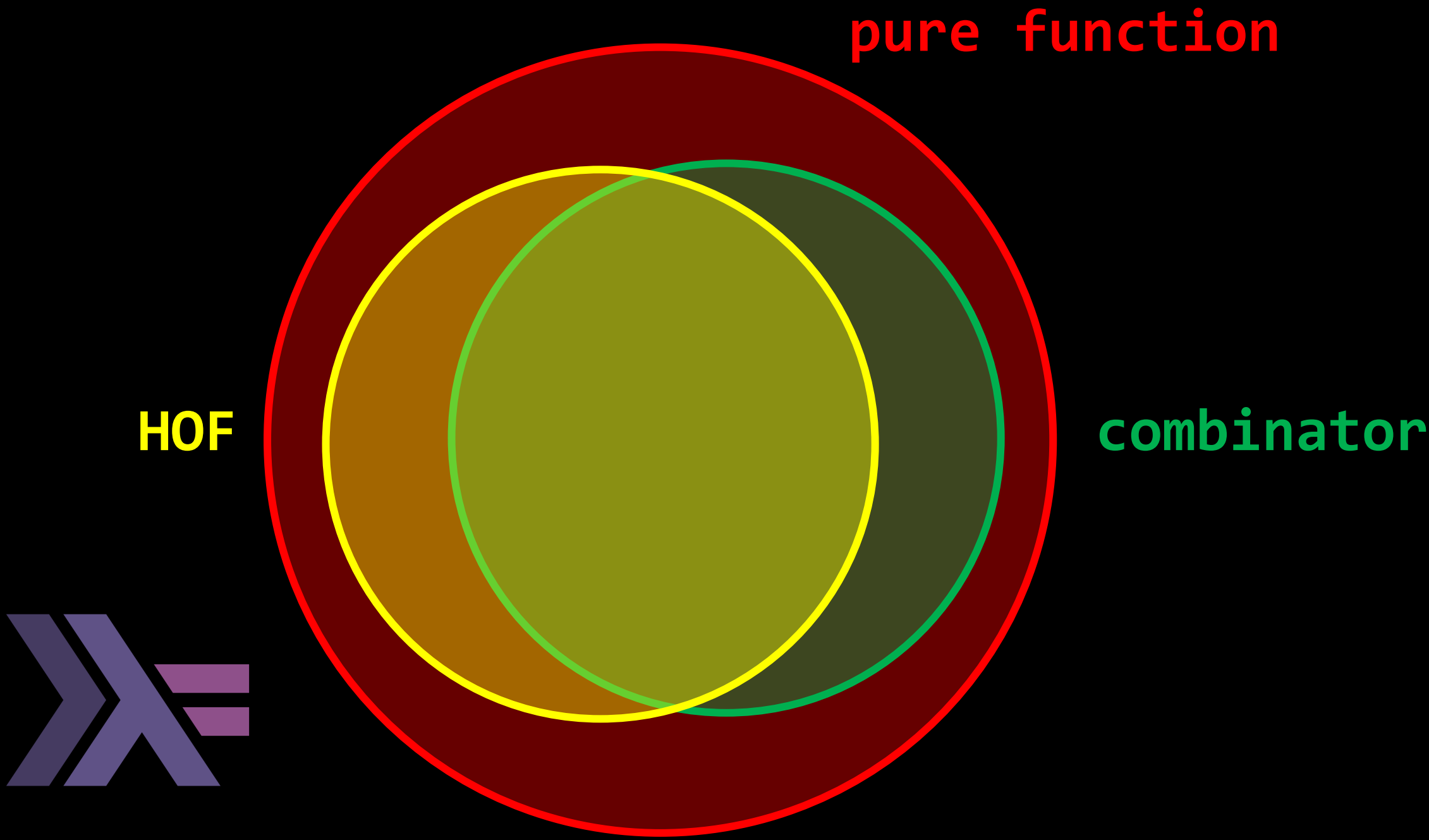
HOF

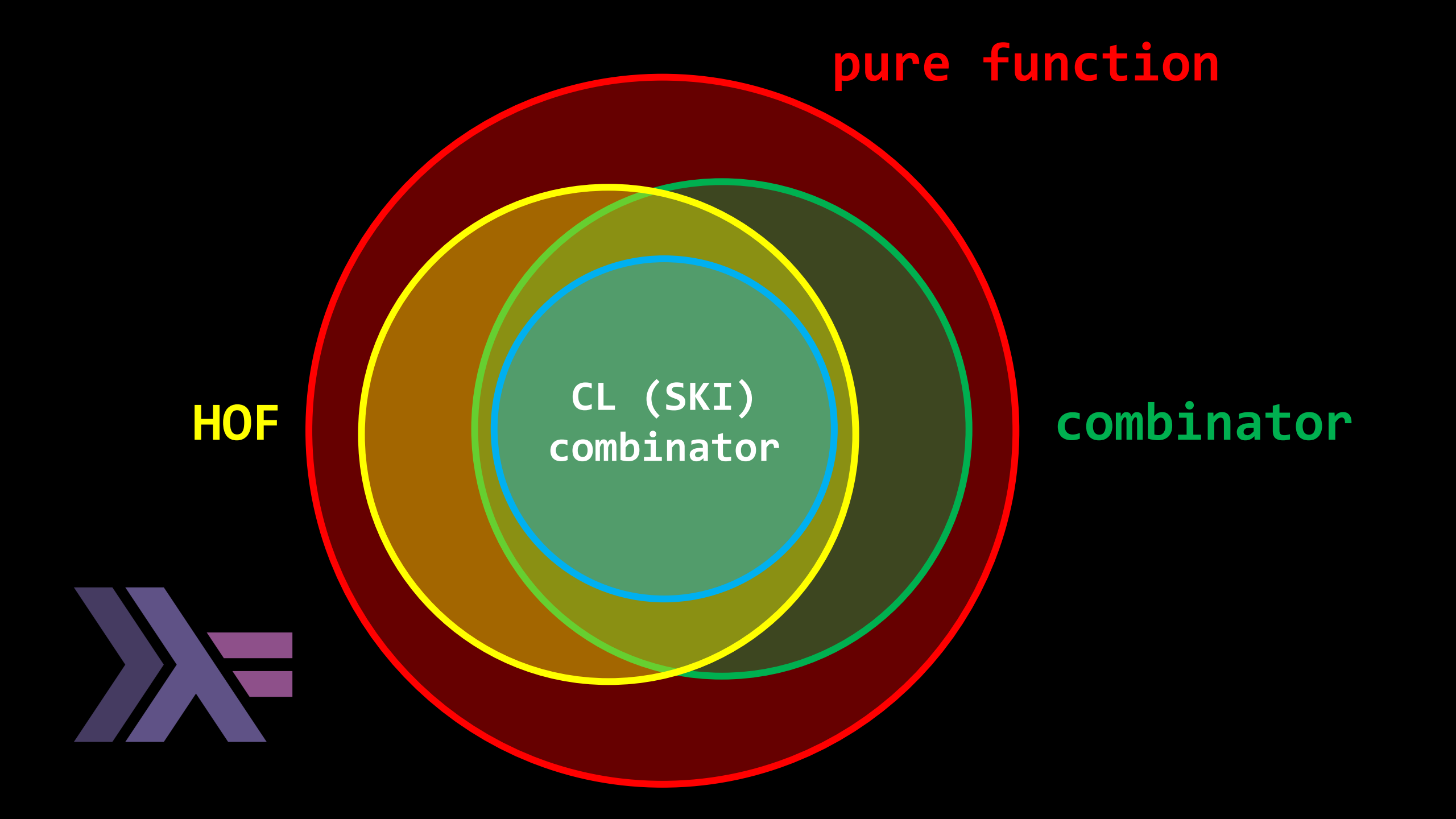
pure
function

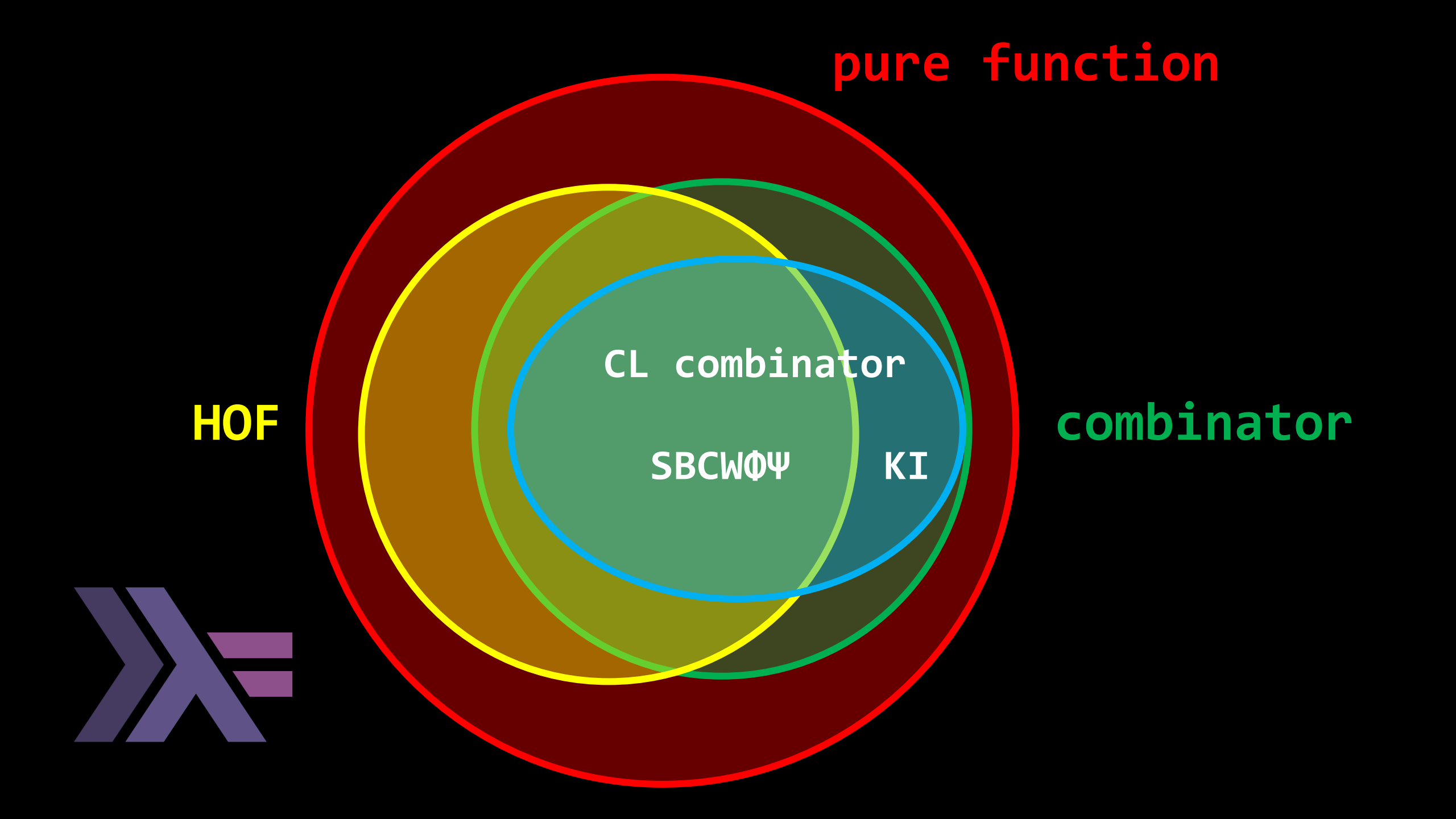


combinator

HOF


pure function
HOF
combinator


pure function
HOF
combinator
CL (SKI)
combinator


pure function
HOF
combinator
CL combinator
SBCWΦΨ
KI

combinator: a function that deals only in its arguments

CL combinator: a combinator that only consumes AND produces functions*

pure function: same input = same output / no side effects

HOF: consumes OR produces a function

Combinator Table: Combinators, Birds, Spellings & More

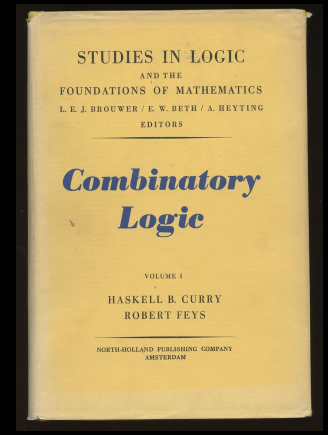
Function Abstraction	Symbol	Bird	CR[1]	CH[2]	Elem[3]	Haskell	APL	BQN	Intro
λabc.ac(bc)	S	Starling	S	S		<*> / ap		∘⊢	Sch24
λab.a	K	Kestrel	K	K	✅	const	⊢	⊢	Sch24
λa.a	I	Identity	SKK	SKK	✅	id	⊣⊢	⊣⊢	Sch24
λab.b	κ	Kite	KI	KI			⊣	⊣	
λab.abb	W	Warbler	C(BMR)	CSK	✅	join	⍨	˜	Cur29
λabc.acb	C	Cardinal	S(BBS)(KK)	B(ΦBS)KK	✅	flip	⍨	˜	Sch24
λabc.a(bc)	B	Bluebird	S(KS)K	S(KS)K	✅	.	∘⍤2T	∘○ 2T	Sch24
λabcd.a(bcd)	B_1	Blackbird	BBB	DB		.:	⍤2T	∘ 2T	Cur30
λabcde.a(bcde)	B_2	Bunting	B(BBB)B	DB_1					Cur30
λabcd.a(b(cd))	B_3	Becard	B(BB)B	BDB					Cur30
λabcd.ab(cd)	D	Dove	BB	BB			∘	∘⊢	Smu85
λabcd.a(bd)(cd)	Φ	Phoenix	-	B_1SB		liftA2	3T	3T	Cur31
λabcd.a(bc)(bd)	Ψ	Psi	-	B(SΦCB)B		on	⍤	○	Cur31
λabcde.abc(de)	D_1	Dickcissel	B(BB)	BD					
λabcde.a(bc)(de)	D_2	Dovekies	BB(BB)	DD					Smu85
λabcde.ab(cde)	E	Eagle	B(BBB)	BB_1					Smu85
λabcde.a(bde)(cde)	$Φ_1$	Pheasant	-	BΦΦ			3T	3T	Cur31
λabcdefg.a(bcd)(efg)	Ê	Bald Eagle	B(BBB)(B(BBB))	D_2D_2D					Smu85

Show Us Some Combinators!

THE ELEMENTARY COMBINATORS

Table 2. The elementary combinators.

Combinator	Elementary Name
I	Elementary Identificator
C	Elementary Permutator
W	Elementary Duplicator
B	Elementary Compositor
K	Elementary Cancellator



```
def i(x):
    return x
```

```
def k(x, y):
    return x
```

```
def w(f):
    return lambda x: f(x, x)
```

```
def b(f, g):
    return lambda x: f(g(x))
```

```
def s(f, g):
    return lambda x: f(x, g(x))
```

```
def i   (x):          return x
def k   (x, y):       return x
def ki  (x, y):       return y
def s   (f, g):       return lambda x:       f(x, g(x))
def b   (f, g):       return lambda x:       f(g(x))
def c   (f):          return lambda x, y:    f(y, x)
def w   (f):          return lambda x:       f(x, x)
def d   (f, g):       return lambda x, y:    f(x, g(y))
def b1  (f, g):       return lambda x, y:    f(g(x, y))
def psi (f, g):       return lambda x, y:    f(g(x), g(y))
def phi (f, g, h):    return lambda x:       g(f(x), h(x))
```

```
def b  (f, g):    return lambda x:    f(g(x))

def b1 (f, g):    return lambda x, y: f(g(x, y))
```

```
def b  (f, g):    return lambda x:    f(g(x))
def b1 (f, g):    return lambda x, y: f(g(x, y))
```

[[digression]]

Conor Hoekstra @code_report · Jan 8, 2022

Also, I apologize for my above average number of tweets 🐦 today, but this table of Greek/Latin words for describing function **arity** will be necessary for a future talk.

The Ê combinator is "tetradic"

Unary/Monadic
Binary/Dyadic
Ternary/Triadic
Quaternary/Tetradic

Terminology [edit]

Latinate names are commonly used for specific arities, primarily bas
cardinal numbers or ordinal numbers. For example, 1-ary is based o

x-ary	Arity (Latin based)	Adicity (Greek based)
0-ary	*Nullary* (from *nūllus*)	*Niladic*
1-ary	*Unary*	*Monadic*
2-ary	*Binary*	*Dyadic*
3-ary	*Ternary*	*Triadic*
4-ary	*Quaternary*	*Tetradic*

6 2 46

[[end of digression]]

"Point-Free or Die: Tacit Programming in Haskell and Beyond" by Amar Shah

2.00

```
exactMatches = length . filter id ... zipWith (==)
```

strangeloop

Sept 15-17, 2016

thestrangeloop.com

23:08 / 36:12

"Point-Free or Die: Tacit Programm

2.00

exactMatches =

trangeloop

ot 15-17, 2016

nestrangeloop.com

23:08 / 36:12

"Point-Free or Die: Tacit Programming in Haskell and Beyond" by Amar Shah

2.00

```
exactMatches = length . filter id ... zipWith (==)
```

strangeloop

Sept 15-17, 2016

thestrangeloop.com

23:08 / 36:12

```
exactMatches = length . filter id .: zipWith (==)
```

```
exactMatches     = length . filter id .: zipWith (==)
exactMatches c g = length (filter id (zipWith (==) c g))
```

```
exactMatches     = length . filter id .: zipWith (==)
exactMatches c g = length (filter id (zipWith (==) c g))

= exactMathes "RRGG" "RYGB"
= length (filter id (zipWith (==) "RRGG" "RYGB"))
= length (filter id [True, False, True, False])
= length [True, True]
= 2
```

```
length . filter id .: zipWith (==)
```

```
sum .: zipWith (==)
```

```
sum . map fromEnum .: zipWith (==)
```

```
sum .: zipWith (fromEnum .: (==))
```

```
countElem True .: zipWith (==)
```

```
length . filter id .: zipWith (==)
sum . map fromEnum .: zipWith (==)
 sum .: zipWith (fromEnum .: (==))
   countElem True .: zipWith (==)
```

```
length . filter id .: zipWith (==)
sum . map fromEnum .: zipWith (==)
 sum .: zipWith (fromEnum .: (==))
   countElem True .: zipWith (==)
```

```
length . filter id .: zipWith (==)
sum . map fromEnum .: zipWith (==)
 sum .: zipWith (fromEnum .: (==))
   countElem True .: zipWith (==)
```

=

```
length . filter id .: zipWith (==)
sum . map fromEnum .: zipWith (==)
 sum .: zipWith (fromEnum .: (==))
   countElem True .: zipWith (==)
```


```
+/=
```

```
length . filter id .: zipWith (==)
sum . map fromEnum .: zipWith (==)
 sum .: zipWith (fromEnum .: (==))
   countElem True .: zipWith (==)
```


```
{+/α=ω}
```

```
length . filter id .: zipWith (==)
sum . map fromEnum .: zipWith (==)
 sum .: zipWith (fromEnum .: (==))
   countElem True .: zipWith (==)
```


```
+/=
```

```
length . filter id .: zipWith (==)
sum . map fromEnum .: zipWith (==)
 sum .: zipWith (fromEnum .: (==))
   countElem True .: zipWith (==)
```


```
+ . =
```

```
length . filter id .: zipWith (==)
sum . map fromEnum .: zipWith (==)
 sum .: zipWith (fromEnum .: (==))
   countElem True .: zipWith (==)
```


```
+/=
```

```cpp
auto exact_matches(std::string_view code,
                   std::string_view guess) -> int {
    return std::transform_reduce(
        code.begin(), code.end(), guess.begin(), 0,
        std::plus{},
        std::equal_to{});
}
```

```haskell
exactMatches =  length
             .  filter id
             .: zipWith (==)
```


```apl
ExactMatches ← +/=
```

Conor Hoekstra @code_report · Oct 21, 2020

And for maybe my most amazing combinator discovery to date, std::inner_product is the **Blackbird** 🤯 (which is what I was looking for when I stumbled across the Phoenix) #combinators #cpp #algorithms

2 7

https://www.youtube.com/watch?v=U6I-Kwj-AvY


```
{(+/0>⍵),(+/0<⍵)}
```


```
(+/0>⊢),(+/0<⊢)
```

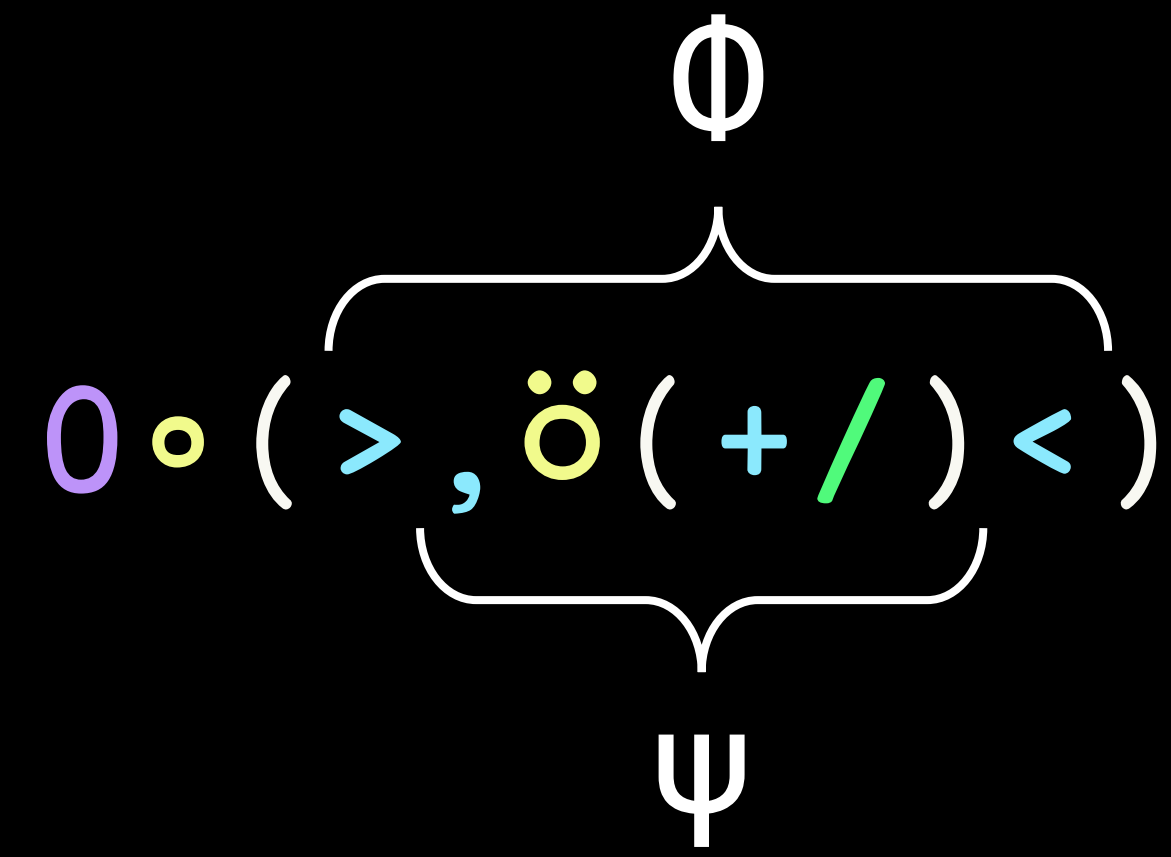

```
(   0>⊢),(   0<⊢)
      +/
```


```
(0>⊢),⍤(+/)(0<⊢)
```


```
0∘(>,⍤(+/)<)
```


```
0∘(>,⍤(+/)<)
```

ψ


⌽
0∘(>,⍤(+/)<)
⍦


```
def psi(f, g):    return lambda x, y: f(g(x), g(y))
def phi(f, g, h): return lambda x:    g(f(x), h(x))
```

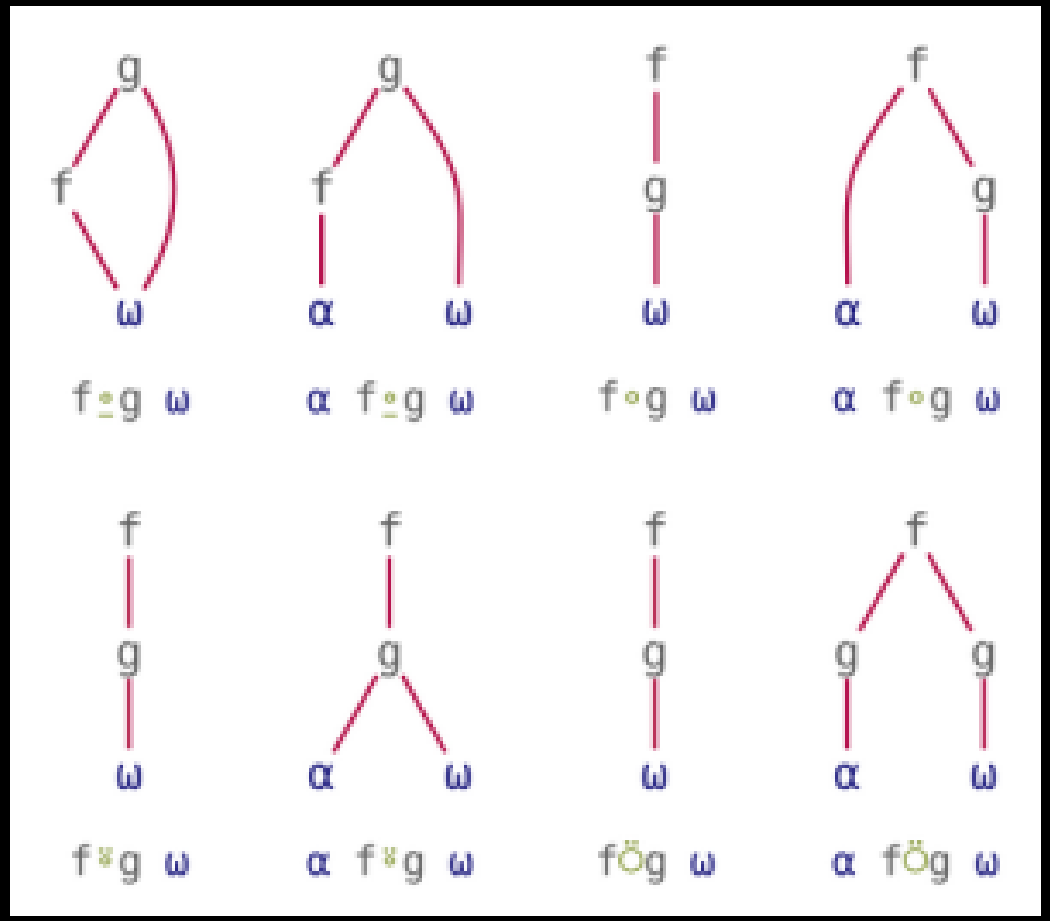
```
      ϕ
  ┌─────────┐
0∘(>,⍤(+/)<)
  └─────────┘
      ψ
```

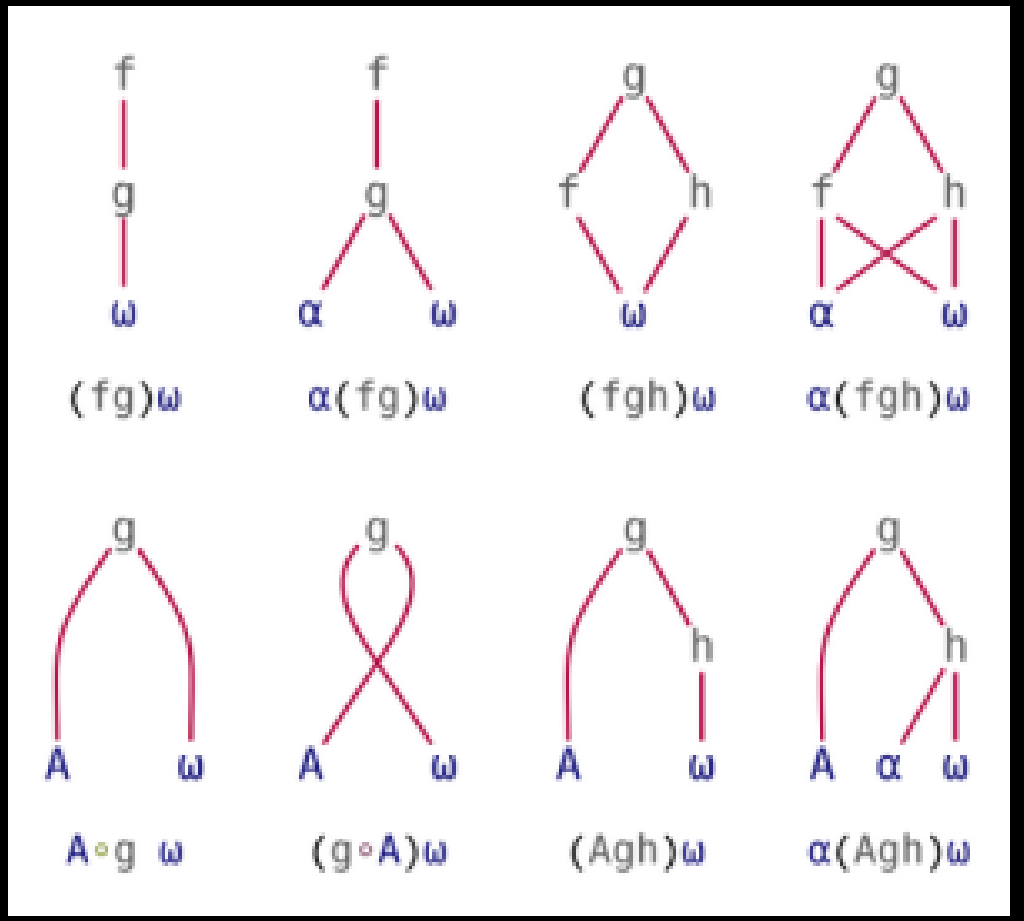

```
0∘(>,⍤(+/)<)
```

```
liftA2 (on (,)
           (length . filter id))
       (map (>0))
       (map (<0))
```

Visualizing Combinators

https://aplwiki.com/wiki/Tacit_programming



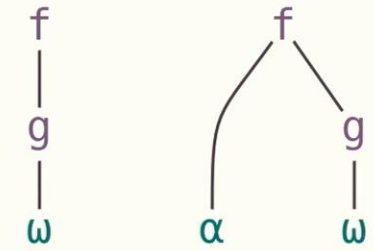


Tacit Techniques in Dyalog 18.0

Composition operators

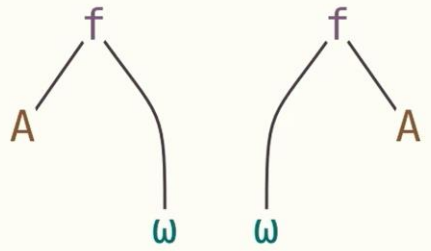
~~Compose~~

Beside



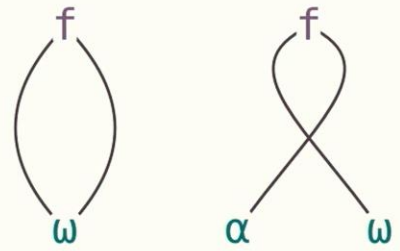
f∘g ω α f∘g ω

Bind



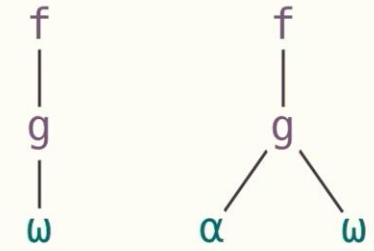
A∘f ω f∘A ω

Commute



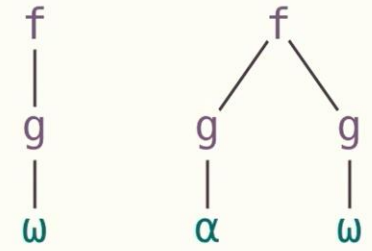
f⍨ ω α f⍨ ω

Atop



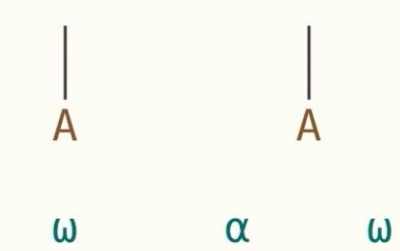
f⍤g ω α f⍤g ω
(f g)ω α(f g)ω

Over



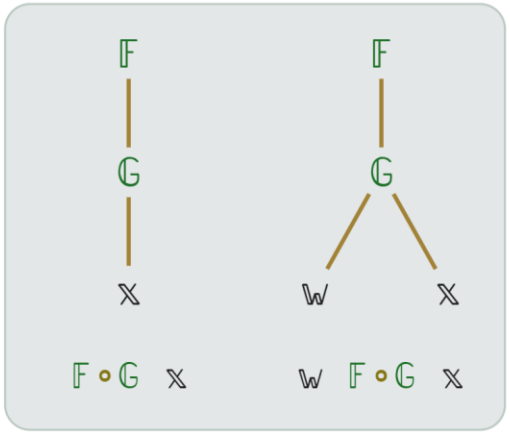
f⍥g ω α f⍥g ω

Constant

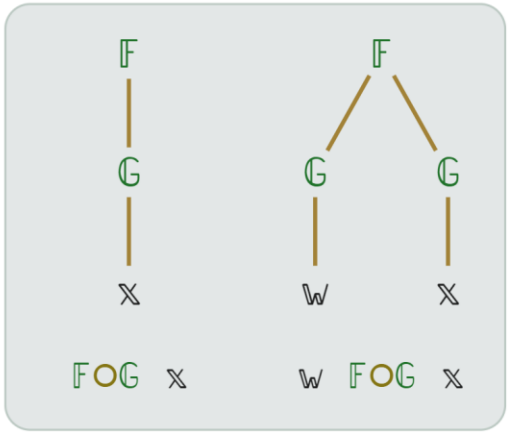


(A⍨) ω α (A⍨) ω

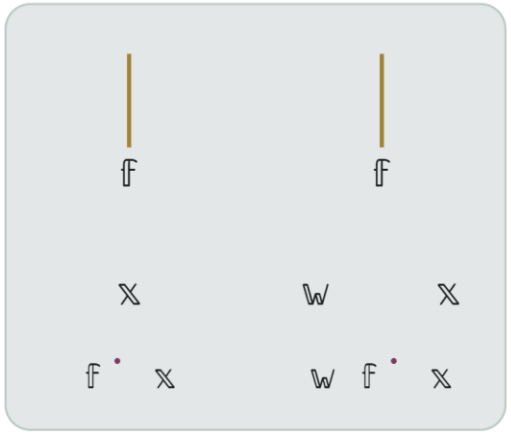
https://mlochbaum.github.io/BQN/doc/tacit.html#combinators



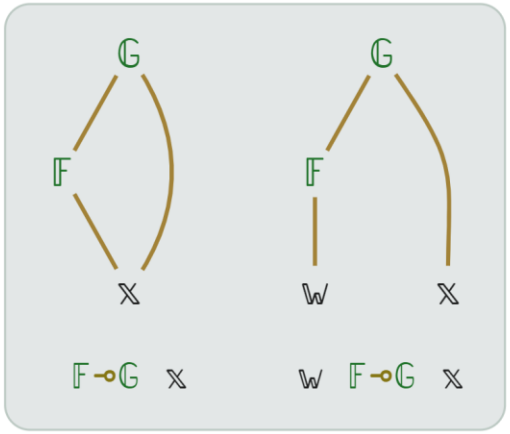
Atop



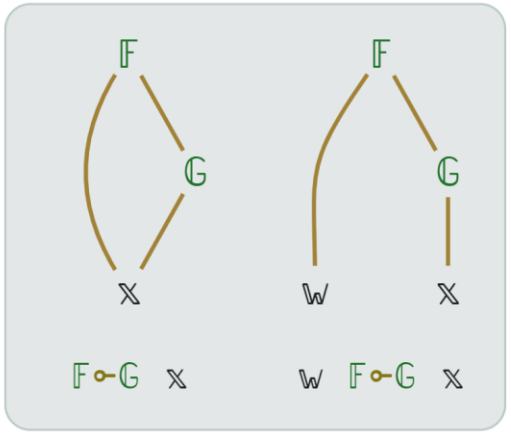
Over



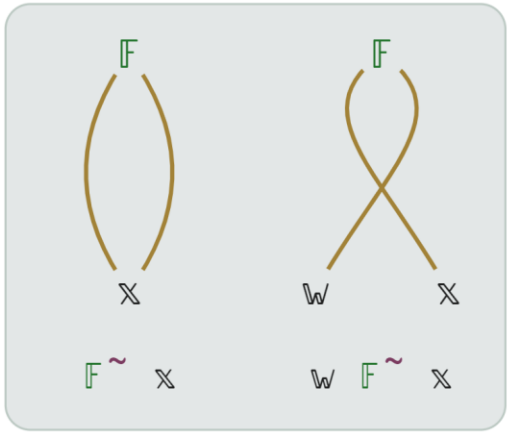
Constant



Before



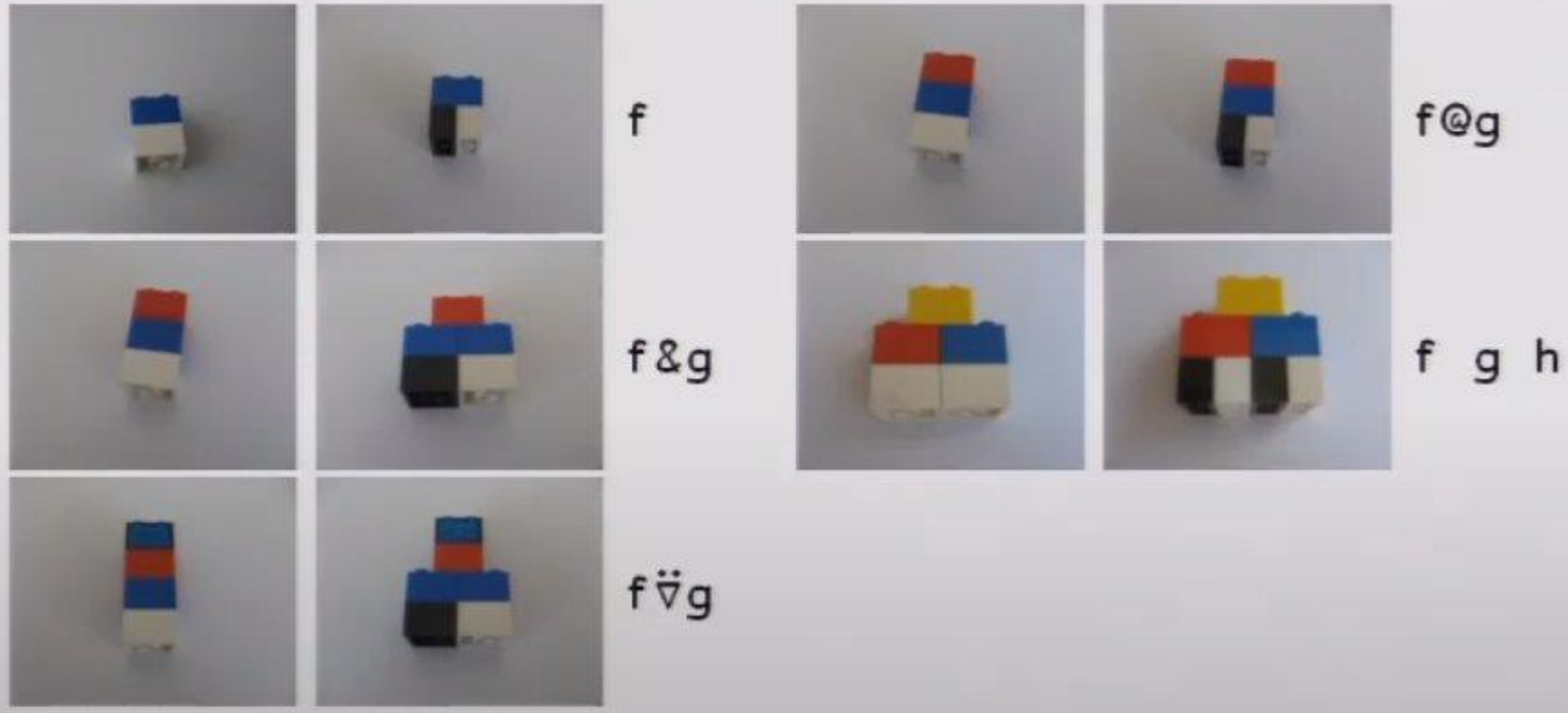
After



Self/Swap

Roger Hui – What is Functional Programming?

Combinators: System Summary



What was the goal of this talk?

What was the goal of this talk?

Composition is more than just $f \circ g$

What was the goal of this talk?

Composition is more than just f◦g

type state :

```
template <typename F1, typename F2>
auto compose(F1 f1, F2 f2)
{
  return [=](auto const& x) { return f1(f2(x)); };
}
```

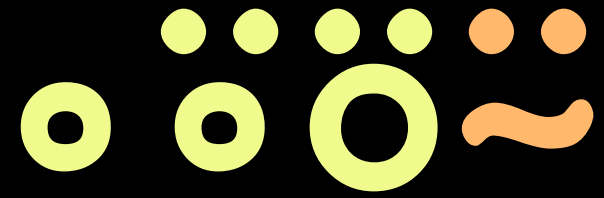
https://www.youtube.com/watch?v=qL6zUn7iiLg

What was the goal of this talk?

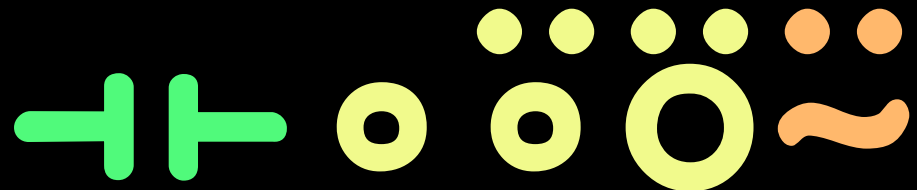
Composition is more than just $f \circ g$

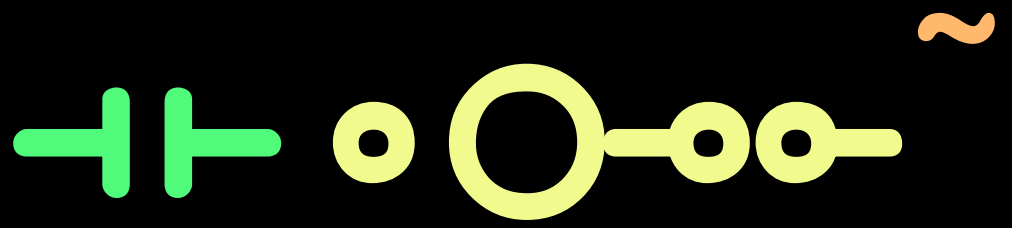
- [x] Sub goal 1: popularize array-combinator languages
- [x] Sub goal 2: story of how I discovered combinators
- [x] Sub goal 3: combinators are really important


APL



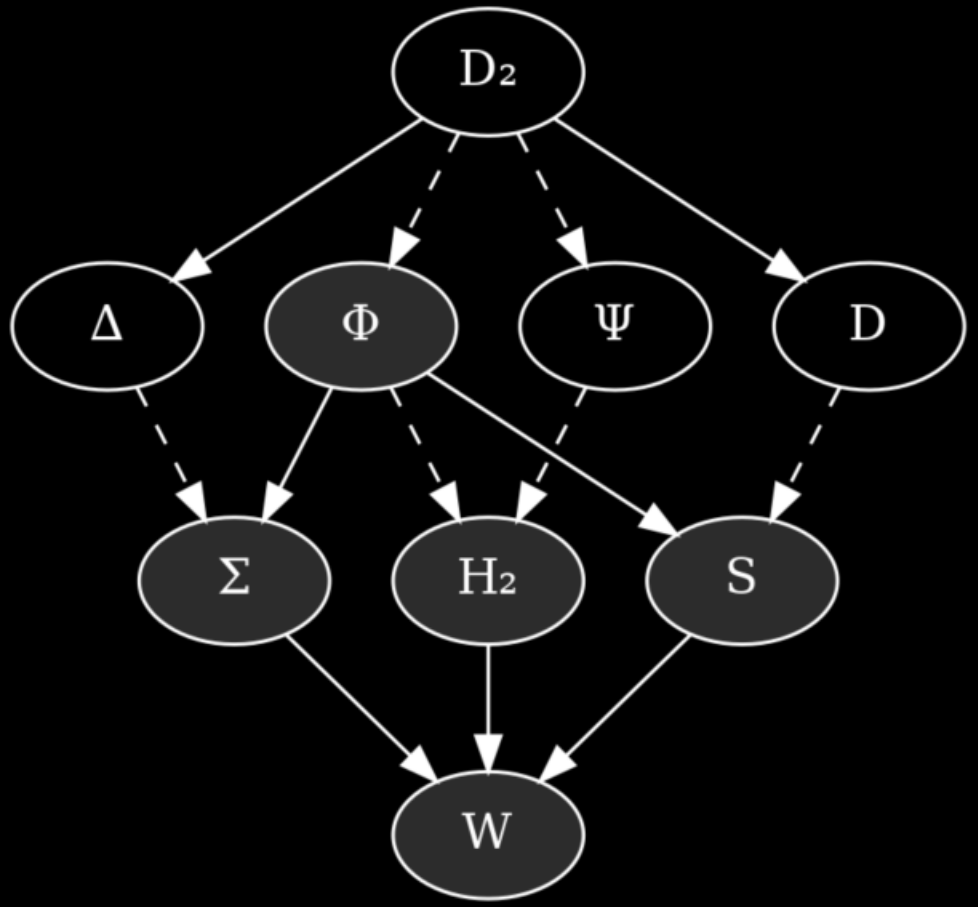

APL





Things I didn’t mention:

- Concatenative languages like FORTH, Joy
- FP & FL (John Backus & Alex Aiken)
- The implicit Φ combinator in The FL Project
- Function application & the curse of ()
- My discovery of the Ψ combinator
- The combinator hierarchy / specializations
- 4+-trains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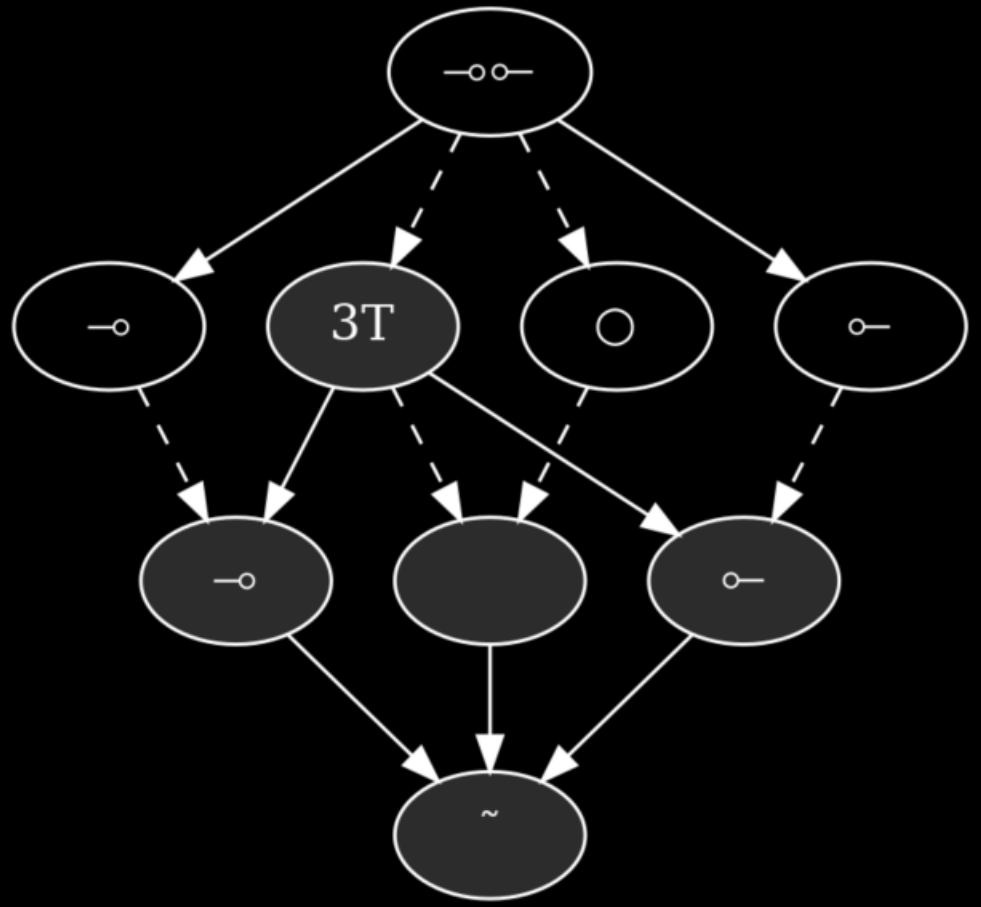
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~

www.combinatorylogic.com

Thank You

https://github.com/codereport/Content/Talks

Conor Hoekstra

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Questions?

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