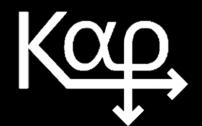


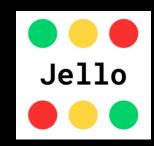
# Composition Intuition II







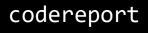




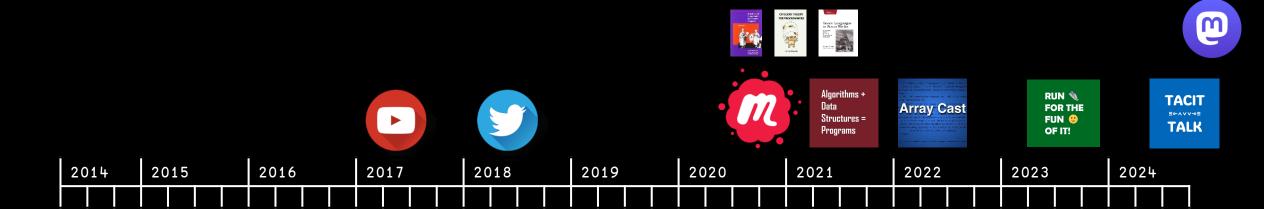
Conor Hoekstra

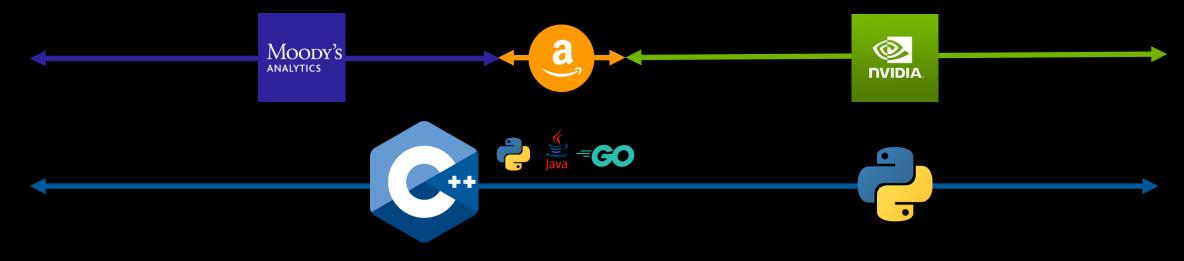


code\_report

















348 Videos

leos 40 (28) Talks

Algorithms + Data Structures = Programs







192 Episodes @adspthepodcast



84 Episodes @arraycast



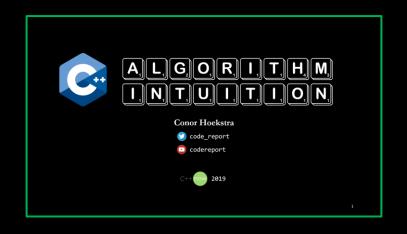
3 Episodes @codereport



19 Episodes @conorhoekstra

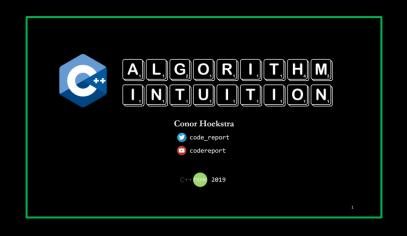


https://github.com/codereport/Content



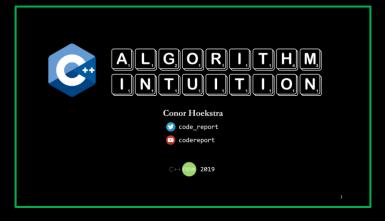


























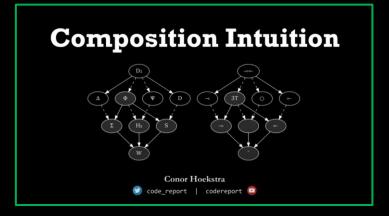




Meeting C++ 2019



#### The Composition Intuition Trilogy

















# Composition Intuition II







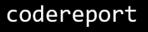




Conor Hoekstra



code\_report



## Agenda

Warm Up 

Combinator Recap

Example #1 

Example #2 

Example #2

## Warm Up

# Warm Up Sort Letters

#### Task 1: Sort Letters

You are given two arrays, @letters and @weights.

Write a script to sort the given array @letters based on the @weights.

#### Example 1

```
Input: @letters = ('R', 'E', 'P', 'L')
     @weights = (3, 2, 1, 4)
Output: PERL
```

#### Example 2

```
Input: @letters = ('A', 'U', 'R', 'K')
     @weights = (2, 4, 1, 3)
Output: RAKU
```

#### Example 3

```
Input: @letters = ('0', 'H', 'Y', 'N', 'P', 'T')
    @weights = (5, 4, 2, 6, 1, 3)
Output: PYTHON
```

3 2 1 4 R E P L 1 2 3 4 P E R L

# PERL

4 5 6 7 8 3 2 1 N O R T H P P C

## 1 2 3 4 5 6 7 8 C P P N O R T H

## CPPNORTH



```
auto sort_string(std::span<int> nums, std::string_view chars) -> std::string {
   // ...
}
```



```
auto sort_string(std::span<int> nums, std::string_view chars) -> std::string {
  auto pairs = std::vector<std::pair<int, char>>(nums.size());
  auto str = std::string(nums.size(), ' ');
  // ...
}
```













```
namespace rv = std::ranges::views; // std::views
namespace v3 = ranges;

auto sort_string(std::span<int> nums, std::string_view chars) -> std::string {
   auto pairs = rv::zip(nums, chars) | v3::to<std::vector>;
   std::ranges::sort(pairs);
   return rv::transform(pairs, [](auto p) { return std::get<1>(p); }) |
        v3::to<std::string>;
}
```







sortString = map snd .: sortOn fst .: zip

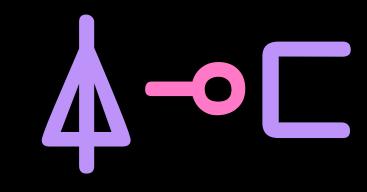






$$\{(Ax)Cx\}$$







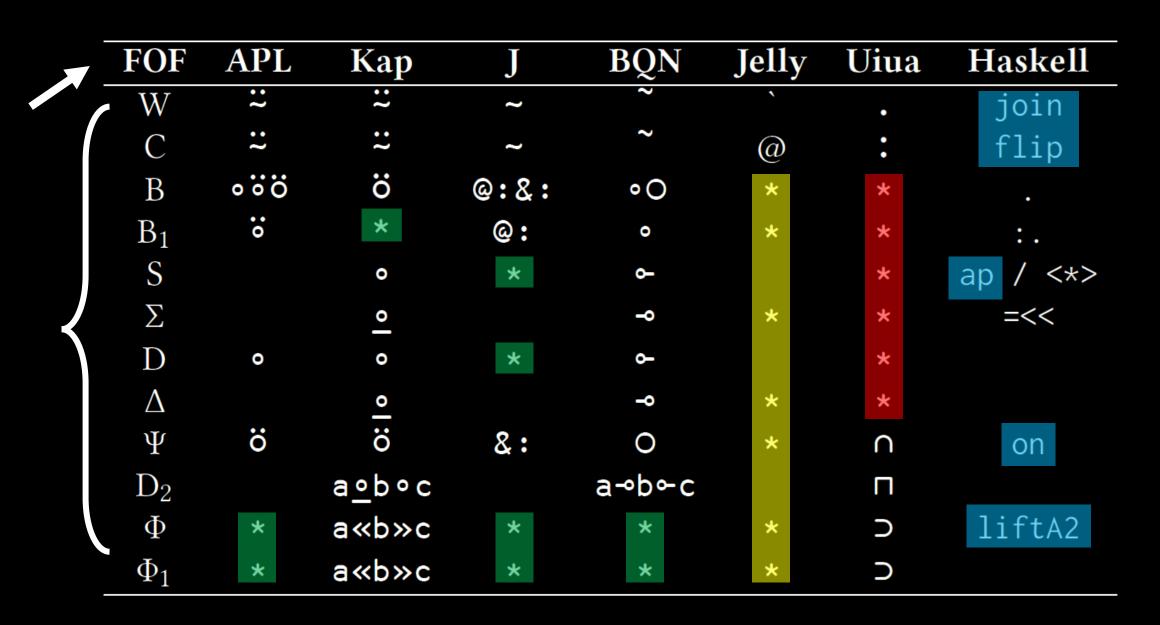




## **Function Composition**

### **Function Composition**

- 1. Operators
- 2. Functions
- 3. Trains
- 4. Chains
- 5. Stacks\*



Operators Functions Trains Chains Stacks\*

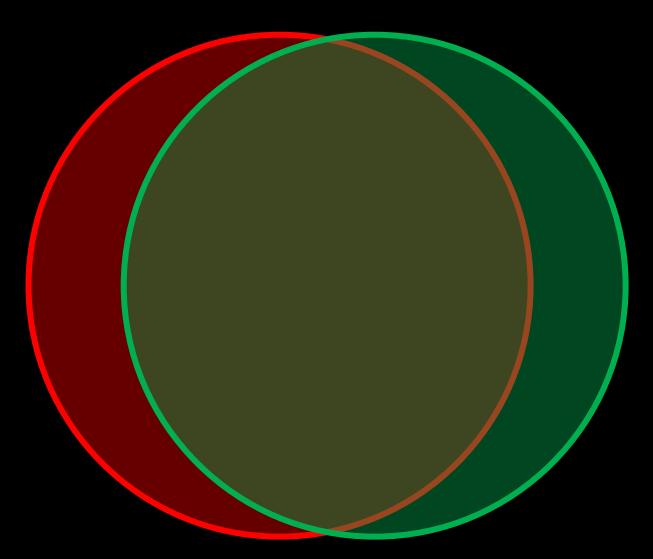
FOF: a combinator that <u>only</u> consumes <u>AND</u> produces functions

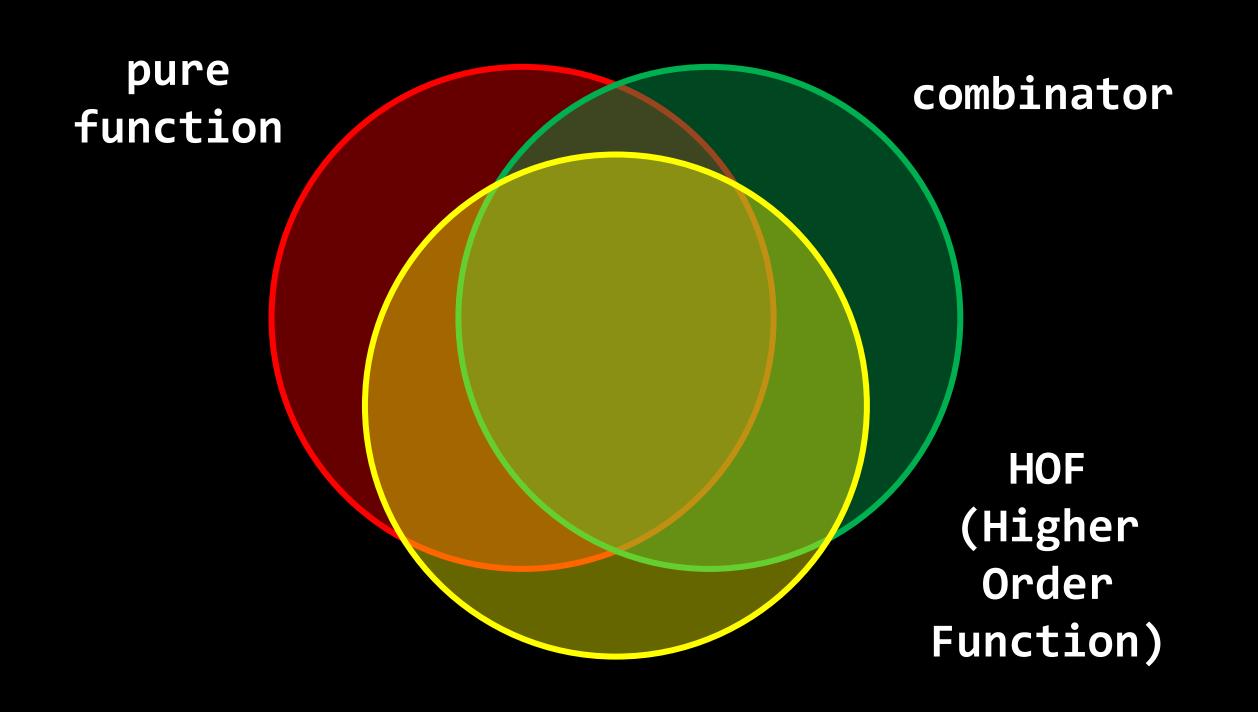
HOF: consumes OR produces a function

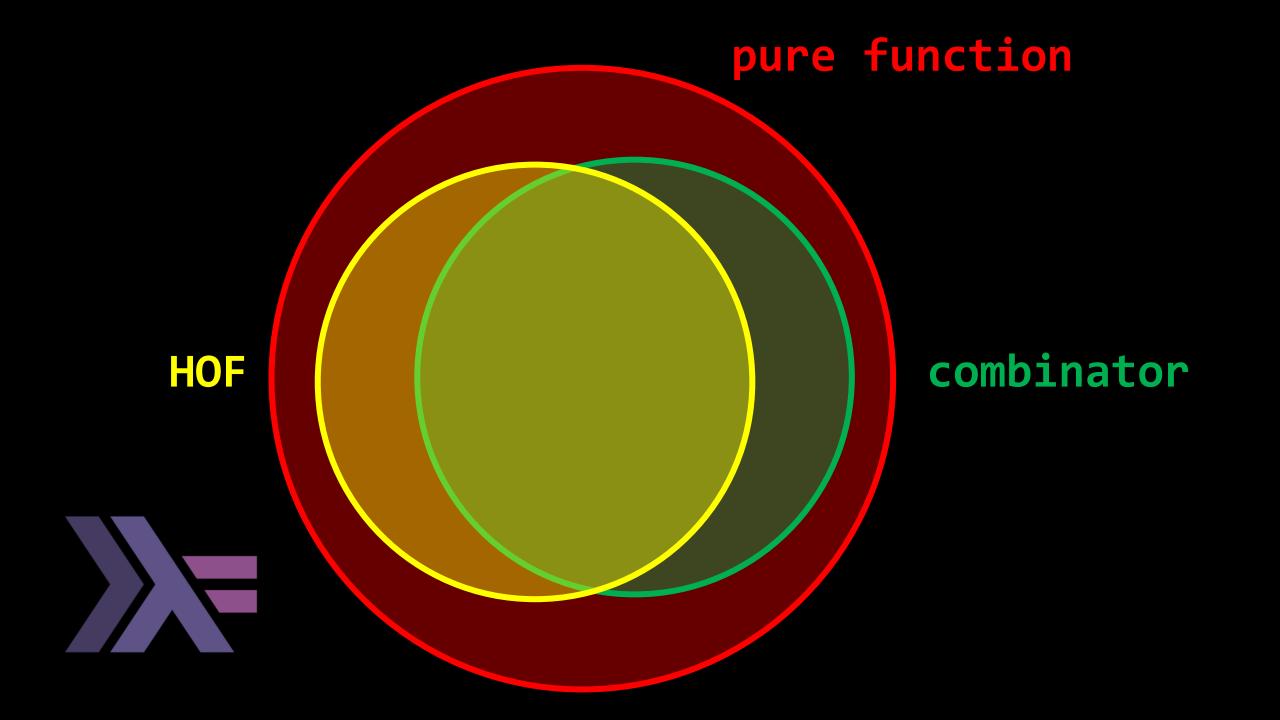
# pure function combinator

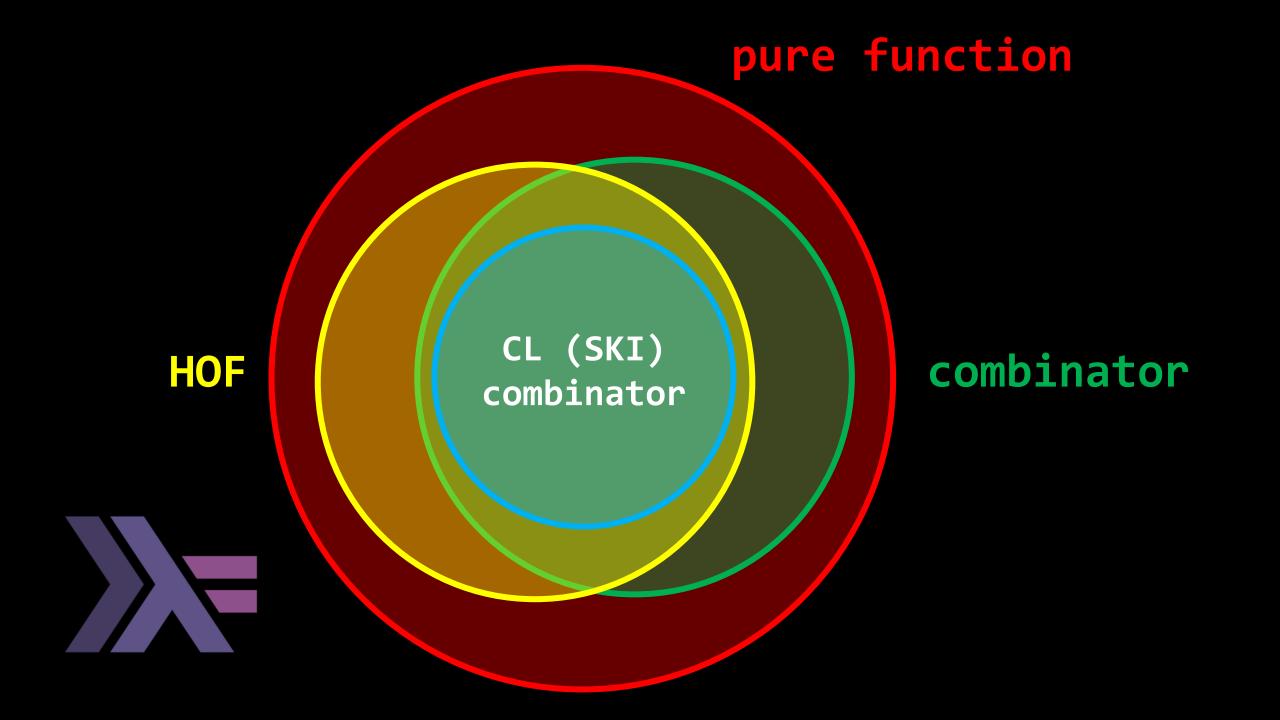
#### pure function

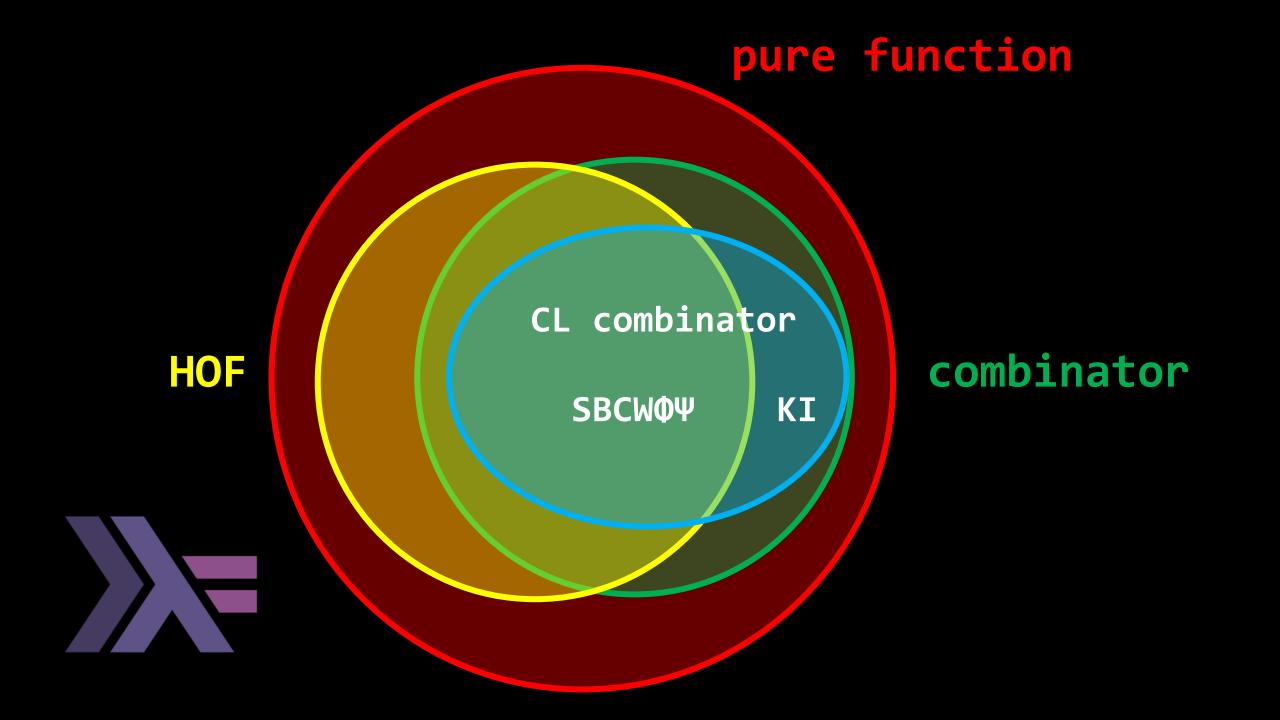
#### combinator

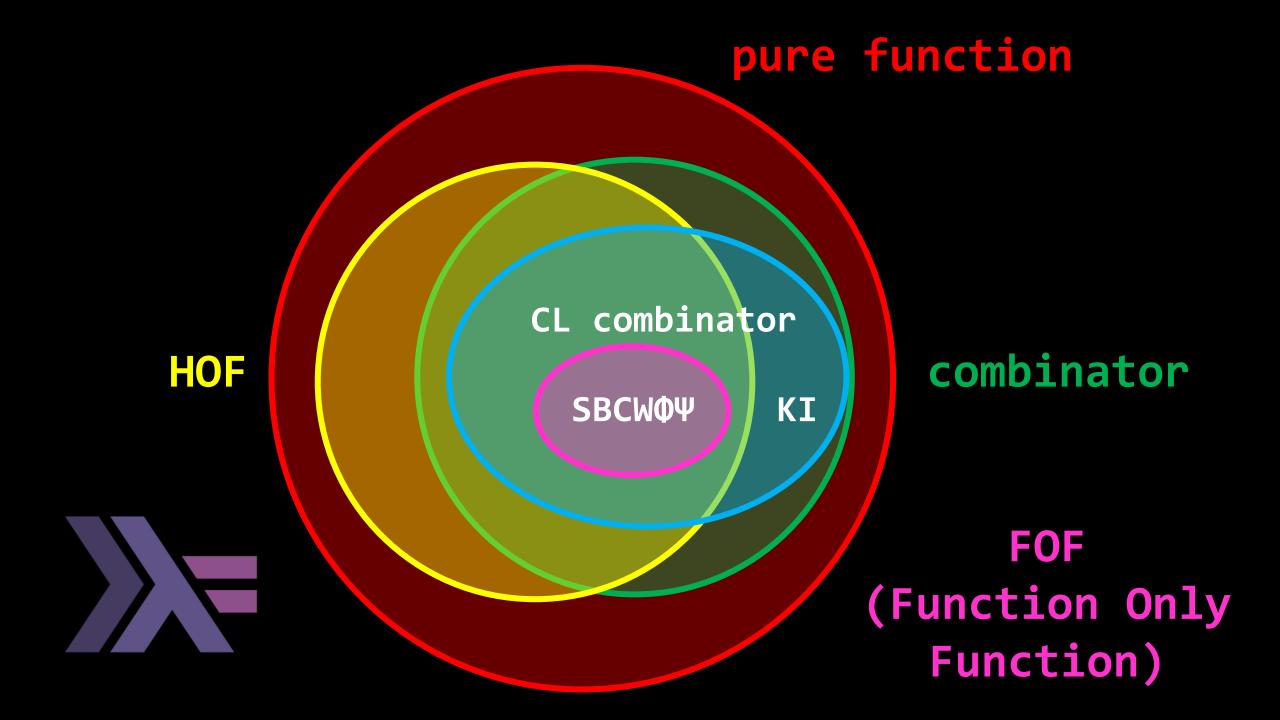


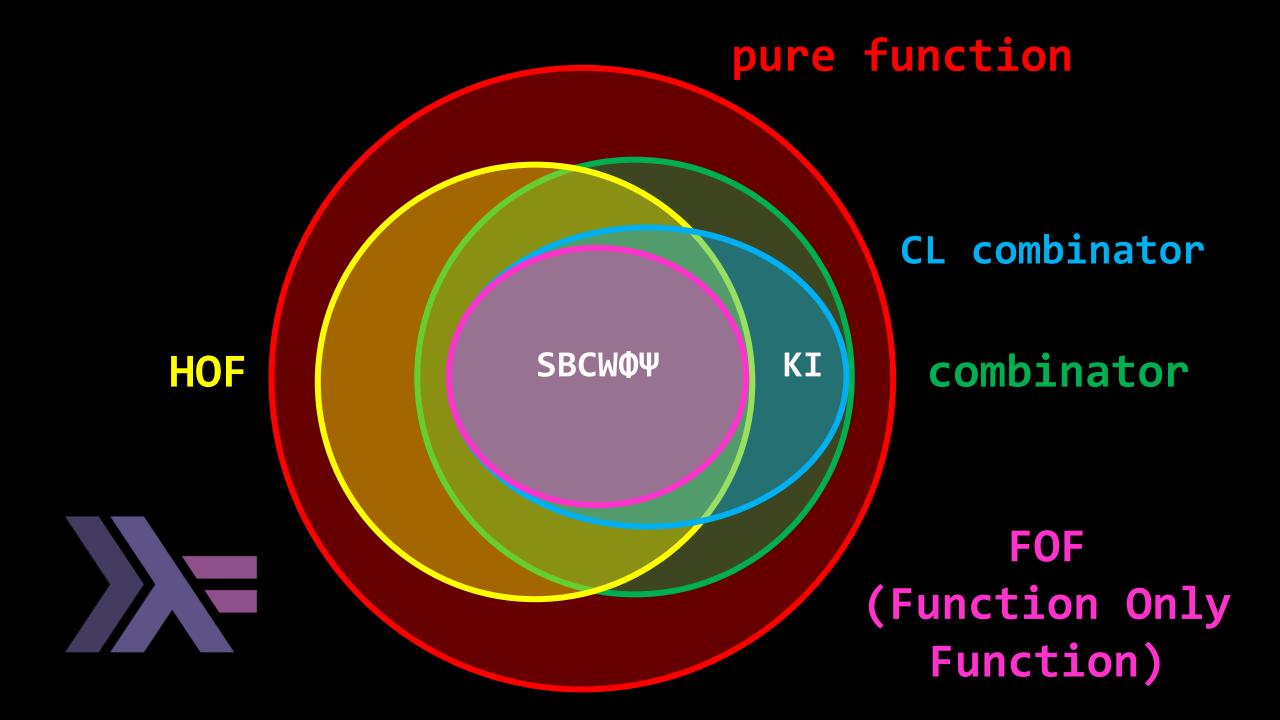












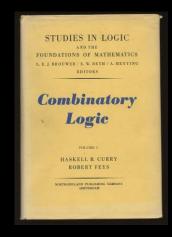
FOF: a combinator that <u>only</u> consumes <u>AND</u> produces functions

HOF: consumes OR produces a function

### Combinators

## THE ELEMENTARY COMBINATORS

Combinator	Elementary Name
I	Elementary Identificator
C	<b>Elementary Permutator</b>
W	<b>Elementary Duplicator</b>
В	<b>Elementary Compositor</b>
K	Elementary Cancellator





## def i(x): return x



## def k(x, y): return x



```
def w(f):
return lambda x: f(x, x)
```

### [[digression]]

#### Conor Hoekstra @code\_report · Jan 8, 2022

Also, I apologize for my above average number of tweets  $\P$  today, but this table of Greek/Latin words for describing function **arity** will be necessary for a future talk.

The Ê combinator is "tetradic"

Unary/Monadic Binary/Dyadic Ternary/Triadic Quaternary/Tetradic

#### Terminology [edit]

Latinate names are commonly used for specific arities, primarily base cardinal numbers or ordinal numbers. For example, 1-ary is based or

x-ary	Arity (Latin based)	Adicity (Greek based)
0-ary	Nullary (from nūllus)	Niladic
1-ary	Unary	Monadic
2-ary	Binary	Dyadic
3-ary	Ternary	Triadic
4-ary	Quaternary	Tetradic











### [[ end of digression ]]



```
def w(f):
return lambda x: f(x, x)
```



```
def b(f, g):
return lambda x: f(g(x))
```



```
def c(f):
return lambda x, y: f(y, x)
```



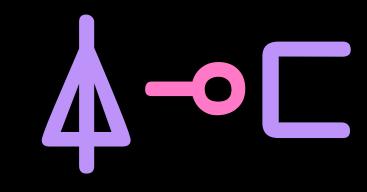
```
def s(f, g):
return lambda x: f(x, g(x))
```



```
defi(x):
                 return x
def k (x, y):
                 return x
def ki(x, y):
                 return y
def s (f, g):
                 return lambda x: f(x, g(x))
                 return lambda x: f(g(x))
def b (f, q):
def c (f):
                 return lambda x, y: f(y, x)
def w (f):
                 return lambda x: f(x, x)
                 return lambda x, y: f(x, g(y))
def d (f, g):
                 return lambda x, y: f(g(x, y))
def b1 (f, g):
def psi(f, g):
                 return lambda x, y: f(g(x), g(y))
def phi(f, g, h): return lambda x: g(f(x), h(x))
```

### Revisiting the Warmup







```
_Sigma_ ← {(Fw)Gx}

# Before (Monadic)

# Reverse Hook (I)
```

## Live Coding

```
_Sigma_ ← {(Fw)Gx} # ~

SortString ← A _Sigma_ □
```







0

```
_B1_ ← {Fwcx}
# Atop (Dyadic)
```

## Live Coding

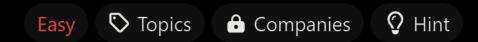
```
_B1_ ← {FwGx} # ∘ & 2 Train
Zip ← ⋈¨
Sort ← ∧
Snd ← ⊑Φ

SortString ← Snd¨ Sort _B1_ Zip
```

## Example #1

## Example #1 Special Array

#### 3151. Special Array I

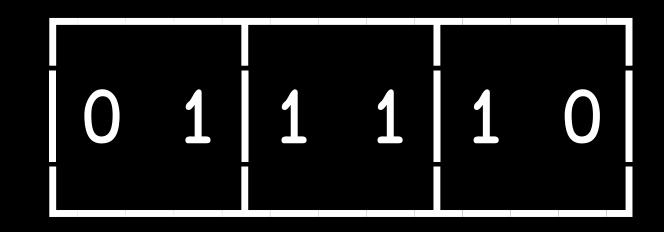


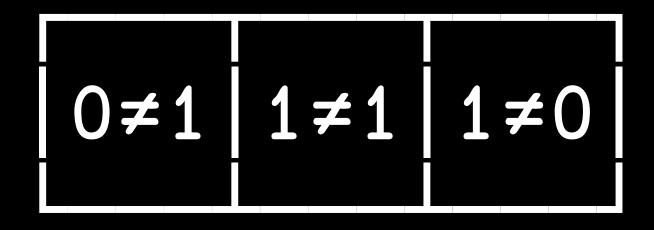
An array is considered **special** if every pair of its adjacent elements contains two numbers with different parity.

You are given an array of integers nums. Return true if nums is a **special** array, otherwise, return false.

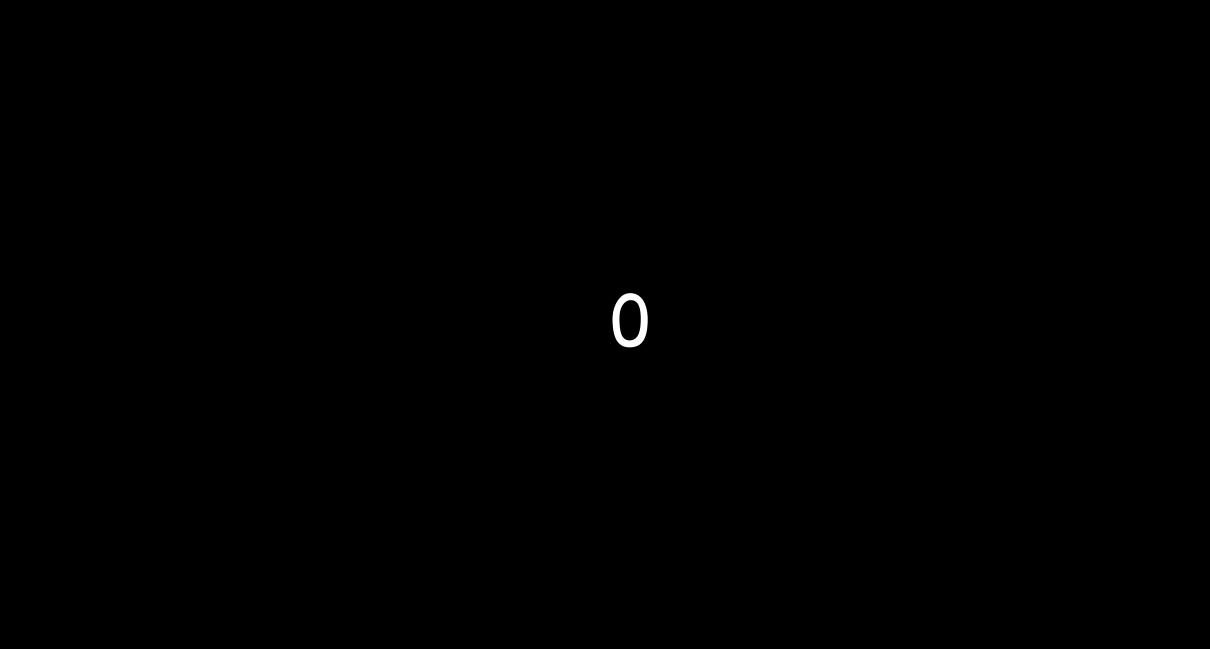
## 4 3 1 6

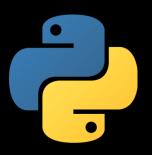
4 3 3 1 1 6





# 1 0 1

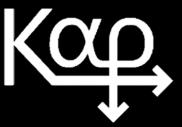
























## Python & C++

**Function Composition** 

- Functions
- **X** Operators
- **X** Trains
- **X** Chains
- X Stacks



```
def isArraySpecial(nums):
    for i in range(len(nums) - 1):
        if nums[i] % 2 == nums[i + 1] % 2:
            return False
    return True
```



```
def isArraySpecial(nums):
    for x, y in zip(nums, nums[1:]):
        if x % 2 == y % 2:
            return False
    return True
```







```
namespace rv = std::ranges::views; // std::views

auto is_array_special(std::vector<int> nums) -> bool {
   return std::ranges::all_of(
      rv::zip_transform(
       [](auto a, auto b) { return a % 2 != b % 2; },
      nums, nums | rv::drop(1)),
   std::identity{});
}
```



```
namespace rv = std::ranges::views; // std::views

auto is_array_special(std::vector<int> nums) -> bool {
  return std::ranges::all_of(
    rv::adjacent_transform<2>(
        nums,
        [](auto a, auto b) { return a % 2 != b % 2; }),
    std::identity{});
}
```





```
def psi(f, g):
return lambda x, y: f(g(x), g(y))
```



```
_Psi_ ← {(Gx)FGx}

# Over

# on (Haskell)
```

```
Psi_ ← {(Gx)F(Gx)}

# Over

# on (Haskell)
```





```
from dovekie import psi, odd
from operator import ne

def isArraySpecial(nums):
    return all(map(psi(ne, odd), nums, nums[1:]))
```



```
namespace rv = std::ranges::views; // std::views

auto is_array_special(std::vector<int> nums) -> bool {
  return std::ranges::all_of(
    rv::adjacent_transform<2>(
        nums,
        [](auto a, auto b) { return a % 2 != b % 2; }),
    std::identity{});
}
```



```
namespace rv = std::ranges::views; // std::views
auto is_array_special(std::vector<int> nums) -> bool {
   return std::ranges::all_of(
     rv::adjacent_transform<2>(nums, _psi(_neq_, _odd)), _id);
}
```



```
namespace rv = std::ranges::views; // std::views
auto is_array_special(std::vector<int> nums) -> bool {
   return std::ranges::all_of(
     rv::pairwise_transform(nums, _psi(_neq_, _odd)), _id);
}
```



```
from dovekie import psi, odd
from operator import ne

def isArraySpecial(nums):
    return all(map(psi(ne, odd), nums, nums[1:]))
```



from dovekie import psi, odd from operator import ne

```
def isArraySpecial(nums):
    return all(map(psi(ne, odd), nums, nums[1:]))
```



```
from dovekie import psi, odd
from operator import ne
from itertools import pairwise

def adjacentMap(xs, op):
    return [op(a, b) for a, b in pairwise(xs)]

def isArraySpecial(nums):
    return all(map(psi(ne, odd), nums, nums[1:]))
```



```
from dovekie import psi, odd
from operator import ne
from itertools import pairwise

def adjacentMap(xs, op):
    return [op(a, b) for a, b in pairwise(xs)]

def isArraySpecial(nums):
    return all(adjacentMap(nums, psi(ne, odd)))
```



```
from dovekie import odd
from operator import ne
from itertools import pairwise

def adjacentMap(xs, op):
    return [op(a, b) for a, b in pairwise(xs)]

def isArraySpecial(nums):
    return all(adjacentMap(map(odd, nums), ne))
```



### Haskell

**Function Composition** 

- **Functions**
- Operators
- **X** Trains
- **X** Chains
- X Stacks

### 0-

```
_S_ ← {xF(Gx)}
# After (Monadic)
# Hook (J/I)
```

FOF	APL	Kap	J	BQN	Jelly	Uiua	Haskell
W	∺	::	~	~	•		join
C	::	<b>::</b>	~	~	<u>@</u>	:	flip
В	• • • •	ö	@:&:	° O	*	*	•
$\mathrm{B}_1$	:0	*	<b>@:</b>	•	*	*	:.
S		•	*	<b>6</b> -		*	ap / <*>
$\sum$		0		9	*	*	=<<
D	0	0	*	<b>6-</b>		*	
Δ		<u> </u>		-0	*	*	
Ψ	ö	ö	&:	0	*	$\cap$	on
$D_2$		a <u>∘</u> b∘c		a⊸b⊶c		П	
Φ	*	a«b»c	*	*	*	$\supset$	liftA2
$\Phi_1$	*	a«b»c	*	*	*	$\supset$	

Operators Functions Trains Chains Stacks\*







# APL, BQN, J, Kap

**Function Composition** 

- **X** Functions
- Operators
- Trains
- **X** Chains
- X Stacks



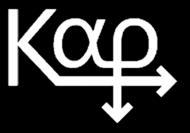


Table 4: 2 and 3-trains in APL, Kap, BQN and J.

Year	Language	2-Train	3-Train
1990	J	S and D	$\Phi$ and $\Phi_1$
2014	Dyalog APL	B and B <sub>1</sub>	$\Phi$ and $\Phi_1$
2020	Kap	B and B <sub>1</sub>	_
2020	BQN	B and B <sub>1</sub>	$\Phi$ and $\Phi_1$



```
defi(x):
                 return x
def k (x, y):
                 return x
def ki(x, y):
                 return y
def s (f, g):
                 return lambda x: f(x, g(x))
                 return lambda x: f(g(x))
def b (f, q):
def c (f):
                 return lambda x, y: f(y, x)
def w (f):
                 return lambda x: f(x, x)
                 return lambda x, y: f(x, g(y))
def d (f, g):
                 return lambda x, y: f(g(x, y))
def b1 (f, g):
def psi(f, g):
                 return lambda x, y: f(g(x), g(y))
def phi(f, g, h): return lambda x: g(f(x), h(x))
```



```
def s (f, g): return lambda x: f(x, g(x)) def b (f, g): return lambda x: f(g(x))

def d (f, g): return lambda x, y: f(x, g(y)) def b1 (f, g): return lambda x, y: f(g(x, y))

def phi(f, g, h): return lambda x: g(f(x), h(x))
```



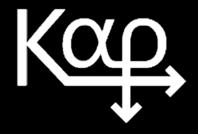
```
def b (f, g): return lambda x: f(g(x)) def b1 (f, g): return lambda x, y: f(g(x, y)) def phi (f, g, h): return lambda x: g(f(x), h(x)) def phi1(f, g, h): return lambda x, y: g(f(x, y), h(x, y))
```





## Live Coding





#### Jelly

**Function Composition** 

- **X** Functions
- Operators
- **X** Trains
- **Chains**
- **X** Stacks

> [4,1,6]

```
Jello
```

```
Jello
```

```
Jello
```

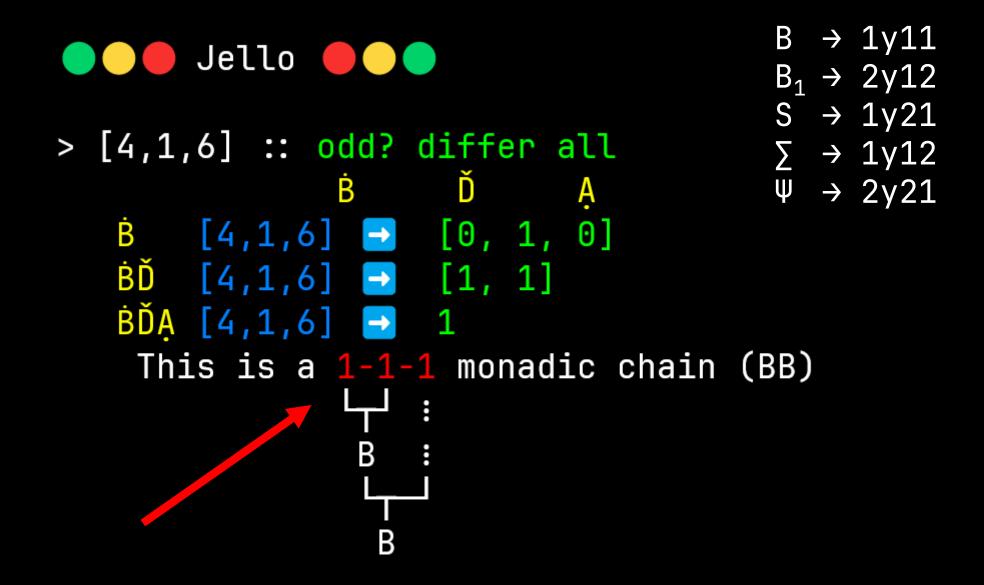
```
> [4,1,6] :: odd? differ all 

B Ď Å

B Ď [4,1,6] □ [0, 1, 0]

BĎ [4,1,6] □ [1, 1]

BĎĀ [4,1,6] □ 1
```



```
Jello
> [4,1,6] :: odd? differ all
    \dot{B} [4,1,6] \Box [0, 1, 0]
    \dot{B}\dot{D} [4,1,6] \longrightarrow [1, 1]
    \dot{B}DA [4,1,6] \rightarrow 1
     This is a 1-1-1 monadic chain (BB)
```

B  $\rightarrow$  1y11 B<sub>1</sub>  $\rightarrow$  2y12 S  $\rightarrow$  1y21  $\sum \rightarrow$  1y12  $\Psi \rightarrow$  2y21

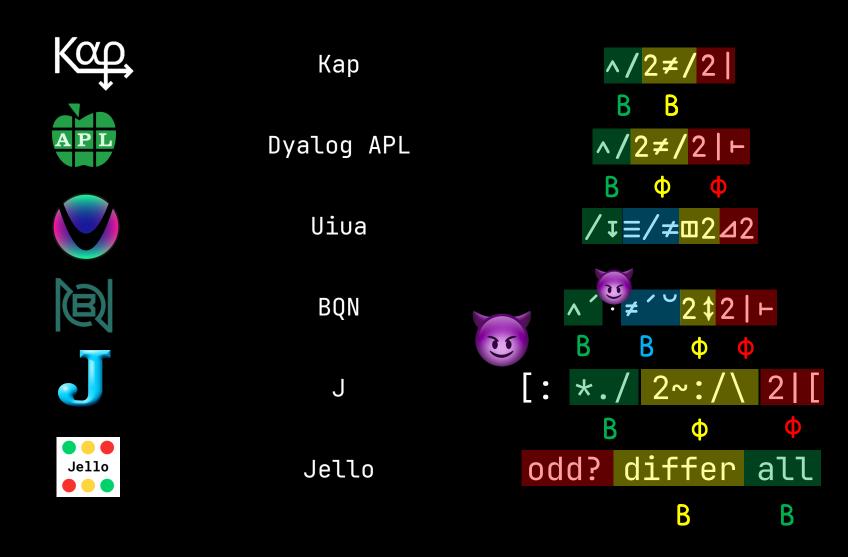
```
Jello 🛑 🛑
> [4,1,6] :: odd? # prior and fold
             Ė
                 n p a /
   and fold can be replaced with all
      🤳 🍪 algorithm advisor 😘 🤳
   ≠ prior can be replaced with differ
      🤳 🍪 algorithm advisor 🍪 🤳
  \dot{B} [4,1,6] \blacksquare [0, 1, 0]
   Bnp [4,1,6] ■ [1, 1]
   Bnpa/ [4,1,6] ■
   This is a 1-2-q-2-q monadic chain (BB)
              : h₁
```



#### Uiua

**Function Composition** 

- **X** Functions
- Operators
- **X** Trains
- **X** Chains
- Stacks



### Example #2

# Example #2 Minimum Average

#### 3194. Minimum Average of Smallest and Largest Elements



You have an array of floating point numbers averages which is initially empty. You are given an array nums of n integers where n is even.

You repeat the following procedure n / 2 times:

- Remove the **smallest** element, minElement, and the **largest** element maxElement, from nums.
- Add (minElement + maxElement) / 2 to averages.

Return the **minimum** element in averages.

7 8 3 4 15 13 4 1

1 3 4 4 7 8 13 15

 1
 3
 4
 4

 15
 13
 8
 7

#### 16 16 12 11

#### 5.5











## Live Coding



```
B \rightarrow 1y11

B<sub>1</sub> \rightarrow 2y12

S \rightarrow 1y21

\sum \rightarrow 1y12

\psi \rightarrow 2y21
```

```
→ 1y11
B
    → 2y12
B_1
    → 1y21
S
    → 1y12
\sum
    → 2y21
Ψ
    → 1y121
Φ
    → 2y222
\Phi_1
```

```
→ 1y2
W
    \rightarrow 2y2
    → 1y11
B
     → 2y12
B_1
     → 1y21
S
    → 1y12
\sum
    \rightarrow 2y21
Ψ
     → 1y121
Ф
     → 2y222
\Phi_1
```

```
\rightarrow 1y2
W
     \rightarrow 2y2
     → 1y11
B
                      2 Train
     \rightarrow 2y12
B_1
S
     → 1y21
     → 1y12
Ψ
     → 2y21
     → 1y121
Ф
                      3 Train
     → 2y222
\Phi_1
```

#### In Conclusion



Кар

^/2≠/2|



Dyalog APL

^/2≠/2|⊢



Uiua

/↓≡/≠ш2⊿2



BQN

^´·≠´<sup>2</sup>2\$2|⊢



J

[: \*./ 2~:/\ 2|[



Jello

odd? differ all

lucid, systematic, and penetrating treatment of basic and dynamic data structures, sorting, recursive algorithms, language structures, and compiling

NIKLAUS WIRTH

Algorithms +

Combinators =

Christian Beautiful Code

PRENTICE-HALL SERIES IN AUTOMATIC COMPUTATION

FOF	APL	Kap	J	BQN	Jelly	Uiua	Haskell
W	::	::	~	~	`	•	join
C	::	<b>::</b>	~	~	<u>@</u>	:	flip
В	• • • •	ö	@:&:	• O	*	*	
$\mathrm{B}_1$	· · ·	*	<b>@:</b>	•	*	*	:.
S		•	*	٥-		*	ap / <*>
$\sum$		<u>o</u>		•	*	*	=<<
D	0	•	*	٠		*	
Δ		<u> </u>		-•	*	*	
Ψ	ö	ö	&:	0	*	$\cap$	on
$D_2$		a <u>∘</u> b∘c		a⊸b⊶c		П	
Φ	*	a«b»c	*	*	*	$\supset$	liftA2
$\Phi_1$	*	a«b»c	*	*	*	$\supset$	

Operators Functions Trains Chains Stacks\*

		C.		APL	Καρ,	J		Jello	
Functions				X	X	X	X	X	X
Operators	X	X							
Trains	X	X	X					X	X
Chains	X	X	X	X	X	X	X		X
Stacks	X	X	X	X	X	X	X	X	



```
from dovekie import odd
from operator import ne

def isArraySpecial(nums):
    return all(adjacentMap(map(odd, nums), ne))
```







Кар

^/2≠/2|



Dyalog APL

^/2≠/2|⊢



Uiua

/I≡/≠m2⊿2



BQN

^´·≠´<sup>2</sup>2\$2|⊢



J

[: \*./ 2~:/\ 2|[



Jello

odd? differ all

### **Online REPLS**

Language	Link				
APL	https://tryapl.org/				
Кар	https://kapdemo.dhsdevelopments.com/clientweb2/				
BQN	https://bqnpad.mechanize.systems/				
J	https://jsoftware.github.io/j-playground/bin/html2/				
Uiua	https://www.uiua.dev/pad				





**Blackbird** 



### Dovekie

### dovekie 0.7.0

pip install dovekie 🕒



## Blackbird

```
# --- Fetch blackbird ---------
FetchContent_Declare(blackbird
 GIT_REPOSITORY https://github.com/codereport/blackbird
 GIT TAG main
FetchContent_GetProperties(blackbird)
if(NOT blackbird_POPULATED)
 FetchContent_Populate(blackbird)
 add_subdirectory(${blackbird_SOURCE_DIR} ${blackbird_BINARY_DIR} EXCLUDE_FROM_ALL)
endif()
```

# www.combinatorylogic.com



# Thank You

https://github.com/codereport/Content/Talks

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# Questions?

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