Band Pass Digital Transmission

→ a PCM s/g can not be directly Transmitted over the space because PCM is a type of baseband Modulation so it can only be used for wire comm^n.

→ in digital band pass transmission, firstly modulating s/g m(t) is converted into a digitally encoded s/g using PCM and then these ~~one~~ 1's & 0's are used to vary the important characteristics of high freq carrier s/g to generate ASK, PSK & FSK s/g

→ Orthogonality - two vectors are said to be orthogonal when their dot product is zero $\vec{A}, \vec{B}$

$$\vec{A} \cdot \vec{B} = AB\cos\theta = 0$$
$$\text{at } \theta = 90^\circ$$

So two s/g's are said to be orthogonal when they are in phase quadrature. ie the phase difference b/w them is $\pi/2$ and integration of product of two ~~orthogo~~ orthogonal s/g over a period is always equal to zero.

$\phi_1(t)$ & $\phi_2(t)$

$$\int_T \phi_1(t)\,\phi_2(t)\,dt = 0$$

Orthonormality

Orthonormal s/g = Orthogonal s/g + Normalization

when a orthogonal function has energy over a period equal to 1 then function is said to be a orthonormal function

$$\int_T |\phi_1(t)|^2 dt = 1$$

$\phi_1(t) = A\cos\omega t$

$$\int_0^T A^2\cos^2\omega t\, dt = 1$$

$$A^2 \int_0^T \frac{1+\cos 2\omega t}{2}\, dt = 1$$

$$\frac{A^2}{2}\int_0^T dt = 1$$

$$\frac{A^2 T}{2} = 1$$

$$\boxed{A = \sqrt{\frac{2}{T}}}$$

Gram - Schi midt procedure for Orthogonality

It is the procedure of constructing and orthogonal complete set consisting of only finite no. of functions.

The functions $s_1(t), s_2(t), \ldots\ldots s_n(t)$ are expressed as

$$s_1(t) = s_{11} \cdot u_1(t) + s_{12} u_2(t) + \ldots\ldots$$
$$\ldots\ldots s_{1N} \cdot u_N(t) \quad \text{——— (i)}$$

$$s_2(t) = s_{21} u_1(t) + s_{22} u_2(t) + \ldots\ldots$$
$$\ldots\ldots s_{2N} \cdot u_N(t) \quad \text{——— (ii)}$$

$$s_N(t) = s_{n1} u_1(t) + s_{n2} u_2(t) + \ldots\ldots$$
$$+ \ldots\ldots s_{nN} u_N(t) \ldots\ldots n$$

step 1

in eq (1) set all coefficient to zero except s_{11}

$$\boxed{s_1(t) = s_{11} u_1(t)}$$

Squaring and integrating over period T

$$\int_0^T s_1^2(t)\,dt = s_{11}^2 \int_0^T u_1^2(t)\,dt$$

$$\boxed{s_{11} = \sqrt{\int_0^T s_1^2(t)\,dt}}$$

$$u_1(t) = \frac{s_1(t)}{s_{11}}$$

Step 2

in eqn 2 set all coefficients to zero except S_{21} & S_{22}

$$S_2(t) = S_{21} U_1(t) + S_{22} U_2(t)$$

multiply both sides by $u_1(t)$ & integrate over period T

$$\int_0^T S_2(t) U_1(t)\, dt = S_{21} \overbrace{\int_0^T (U_1 t)^2 dt}^{1} + S_{22} \underbrace{\int_0^T (U_1(t))(U_2(t))\, dt}_{0}$$

$$S_{21} = \int_0^T S_2(t) U_1(t)\, dt$$

$$S_{22} U_2(t) = S_2(t) - S_{21} U_1(t)$$

squaring and integrating

$$S_{22}^2 = \int_0^T (S_2(t) - S_{21} U_1(t))^2 dt$$

$$S_{22} = \sqrt{\int_0^T (S_2(t) - S_{21} U_1(t))^2 dt}$$

$$U_2(t) = \frac{S_2(t) - S_{21} U_1(t)}{S_{22}}$$

Step 3

in eq 3 set all coefficient to zero except S_{31}; S_{32}, S_{33}

$$S_3(t) = S_{31} U_1(t) + S_{32} U_2(t) + S_{33} U_3(t)$$

multiply by $U_1(t)$ both side

$$\boxed{S_{31} = \int_0^T S_3(t) U_1(t)\, dt}$$

multiply by $U_2(t)$ bothsides

$$\boxed{S_{32} = \int_0^T S_3(t) U_2(t)\, dt}$$

$$S_{33} U_3(t) = S_3(t) - S_{31} U_1(t) - S_{32} U_2(t)$$

Squaring & integrating

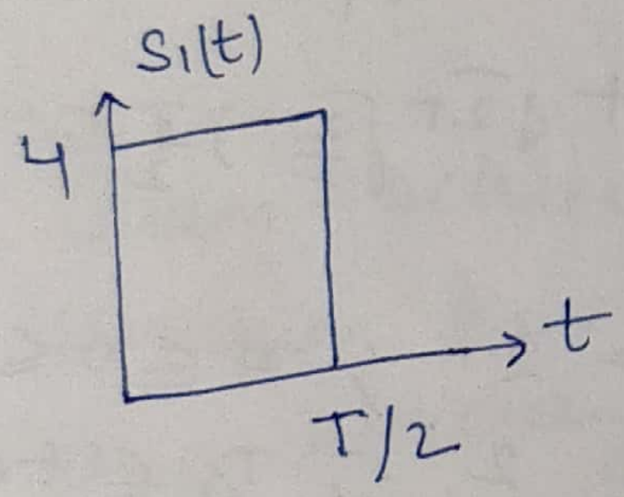
$$S_{33} = \sqrt{\int_0^T \left(S_3(t) - S_{31} U_1(t) - S_{32} U_2(t) \right)^2 dt}$$

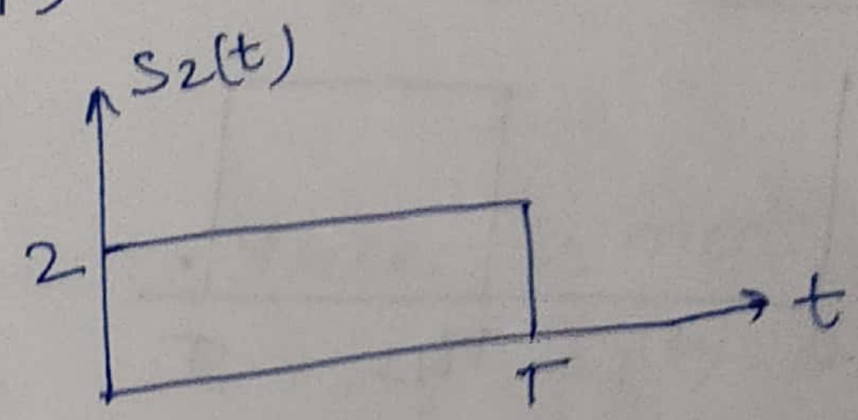
$$\boxed{U_3(t) = \frac{S_3(t) - S_{31} U_1(t) - S_{32} U_2(t)}{S_{33}}}$$

Step 4 the procedure is continued till all the N equ. are used & N orthonormal functions $U_1(t), U_2(t), \ldots U_N(t)$ are calculated

If there are N functions i.e. $S_1(t), S_2(t) \ldots S_N(t)$ but only M of them are linearly independent then the above procedure generates $M (\leq N)$ orthonormal function in terms of which we shall be able to express all N functions.

Q Express the following fn in terms of orthonormal fn's





$$S_{11} = \left[\int_0^T s_1^2(t)\,dt\right]^{1/2}$$

$$= \left[\int_0^{T/2} 16\,dt\right]^{1/2}$$

$$= \sqrt{8T}$$

$$U_1(t) = \frac{S_1(t)}{S_{11}}$$

$$U_1(t) = \frac{4}{\sqrt{8T}} \qquad 0 \leq t \leq T/2$$

$$U_1(t) = \sqrt{\frac{2}{T}}$$

$$S_{21} = \int_0^T S_2(t)\,U_1(t)\,dt$$

$$= \int_0^{T/2} 2 \times \sqrt{\frac{2}{T}}\,dt$$

$$= 2\sqrt{\frac{2}{T}}\,\frac{T}{2} = \sqrt{2T} \qquad 0 \leq t \leq T/2$$

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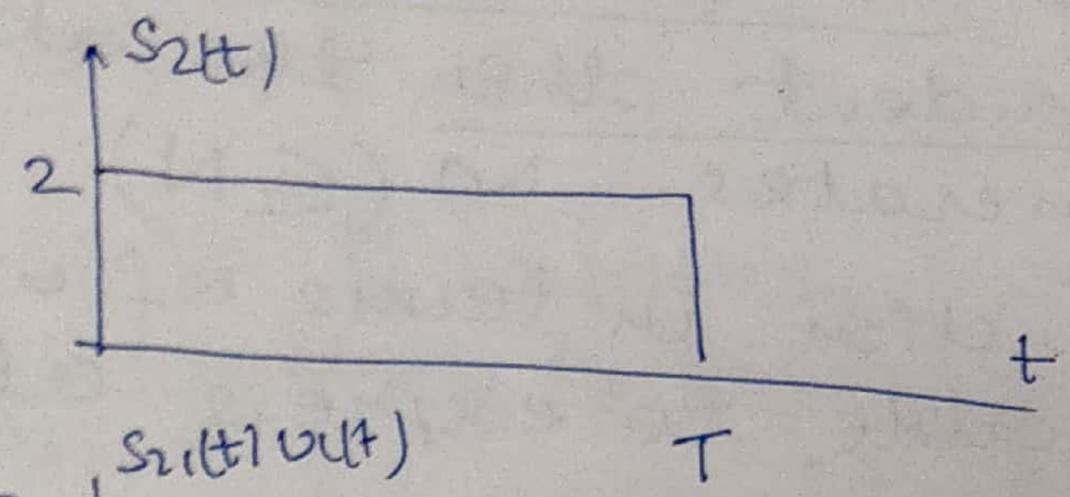
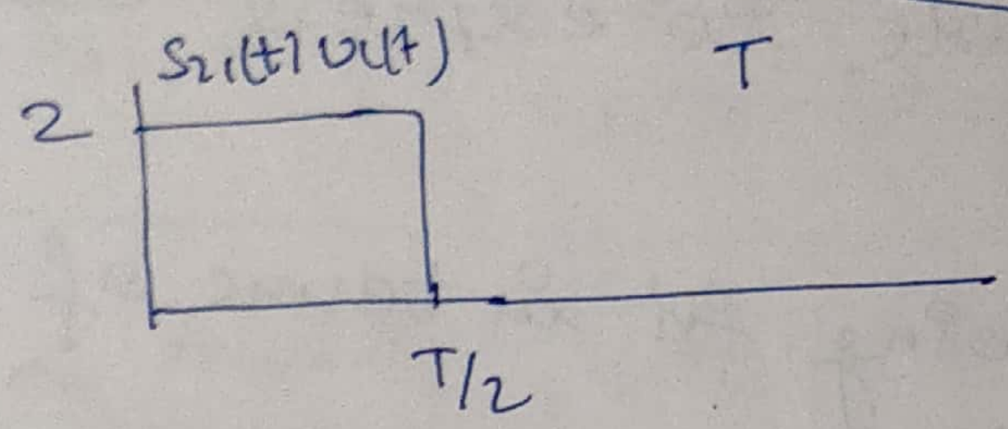
$$S_{22} = \left[\int_0^T \left[s_2(t) - s_{21} u_1(t) \right]^2 dt \right]^{1/2}$$





$s_1(t) = s_{11} u_1(t)$

$= \sqrt{8T} \sqrt{\frac{2}{T}} \quad 0 \le t \le T/2$

$= 4 \quad 0 \le t \le T/2$

$s_2(t) = s_{21} u_1(t) + s_{22} u_2(t)$

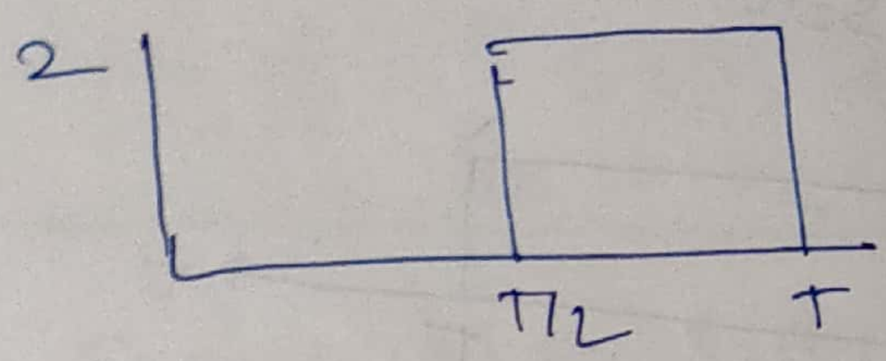
$= \sqrt{2T} \sqrt{\frac{2}{T}} \; ; 0 \le t \le \frac{T}{2}$

$+ \sqrt{2T} \sqrt{\frac{2}{T}} \; ; \frac{T}{2} \le t < T$

$= 2 \; ; 0 \le t \le T/2$

$2 \; ; T/2 \le t \le T$

subtract



$$S_{22} = \left[\int_{T/2}^{T} 4 \, dt \right]^{1/2}$$

$$= \sqrt{\frac{4T}{2}} = \sqrt{2T}$$

$$\boxed{S_{22} = \sqrt{2T}}$$

$$u_2(t) = \frac{s_2(t) - s_{21} u_1(t)}{S_{22}}$$

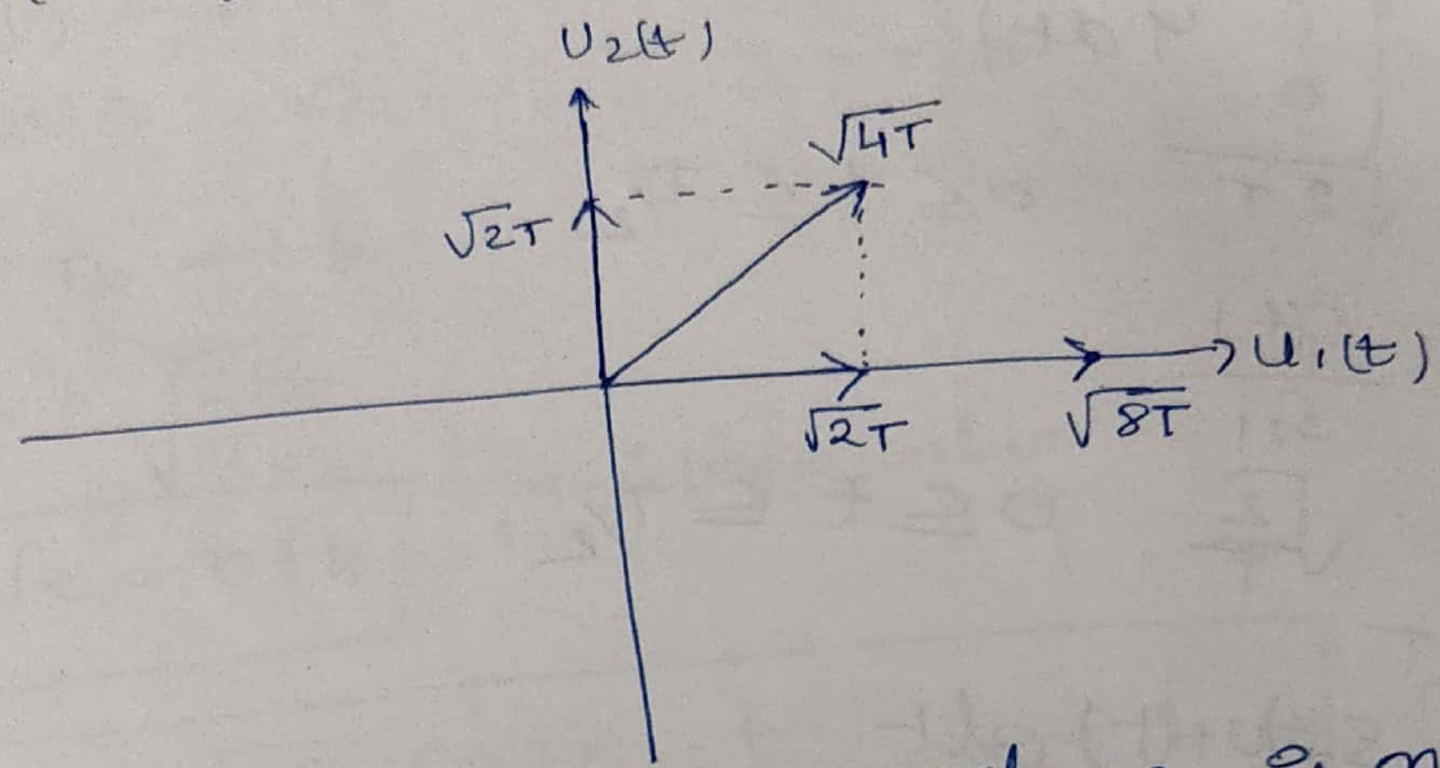
$$= \frac{2}{\sqrt{2T}} \; ; T/2 \le t \le T$$

$$\boxed{u_2(t) = \sqrt{\frac{2}{T}} \; ; T/2 \le t \le T}$$

Constellation diagram

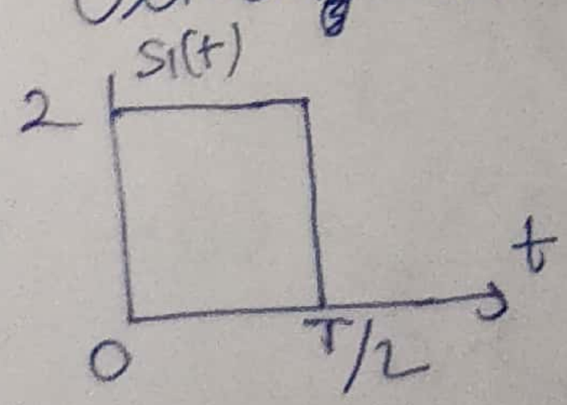
$S_1(t) = \sqrt{8T}\ U_1(t)$

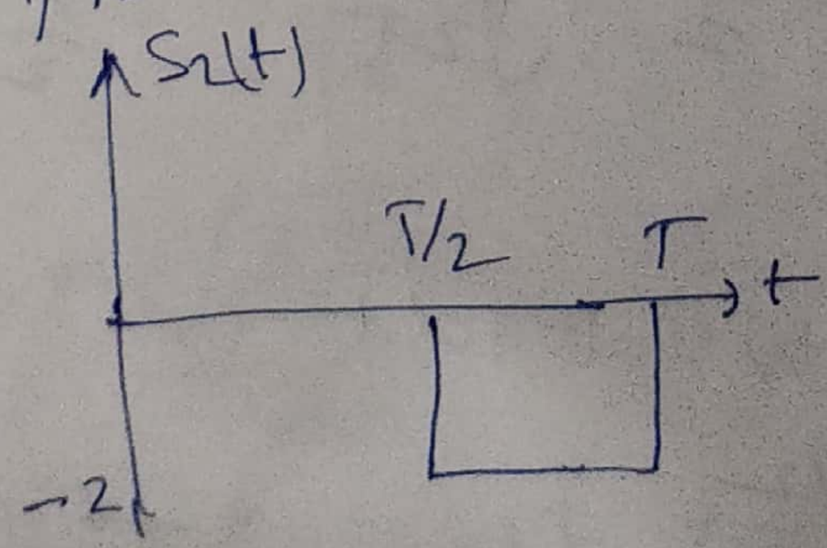
$S_2(t) = \sqrt{2T}\ U_1(t) + \sqrt{2T}\ U_2(t)$

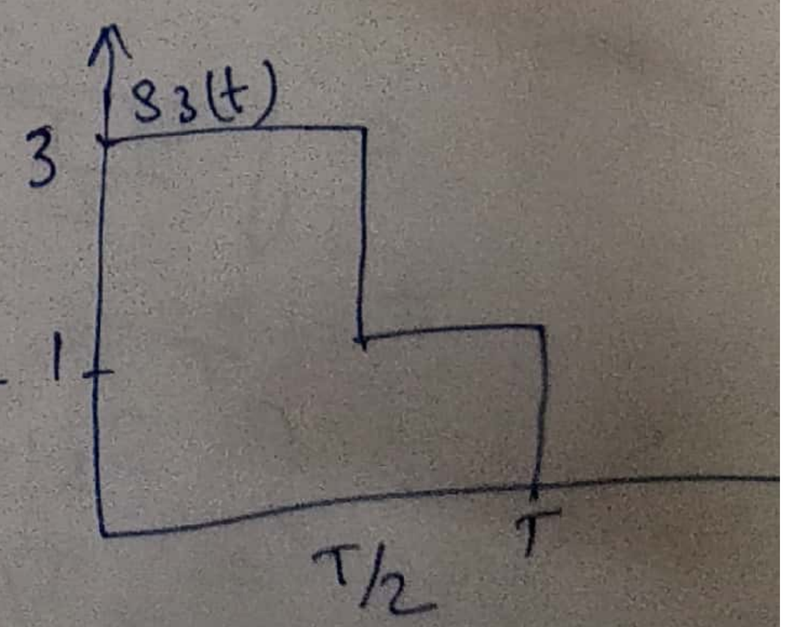


Gram-schimidt procedure is not a unique procedure as if $S_1(t)$ & $S_2(t)$ are interchanged, value of $U_1(t)$ & $U_2(t)$ will be different but the thing which does not change is the length of s/gs and it is only length of s/g which is detected in digital modulation schemes

Q express following fn in terms of orthogonal fn's







$\therefore$

$$S_3(t) = \frac{3}{2} S_1(t) - \frac{1}{2} S_2(t)$$

$$S_{11} = \left(\int_0^T s_1^2(t)\,dt\right)^{1/2}$$

$$= \left(\int_0^{T/2} 4\,dt\right)^{1/2}$$

$$S_{11} = \sqrt{2T} \qquad 0 \le t \le T/2$$

$$u_1(t) = \frac{s_1(t)}{S_{11}}$$

$$= \sqrt{\frac{2}{T}} \qquad 0 \le t \le T/2$$

$$S_{21} = \int_0^T s_2(t)\,u_1(t)\,dt$$

$$= \underline{\underline{0}}$$

$$u_2(t) = -\sqrt{\frac{2}{T}} \quad ; T/2$$

$$S_{22} = \left[\int_0^T s_2^2(t)\,dt\right]^{1/2}$$

$$= \left[\int_{T/2}^{T} 4\,dt\right]^{1/2}$$

$$= \sqrt{\frac{4T}{2}} = \sqrt{2T} \qquad \frac{T}{2} \le t \le T$$

$$S_{31} = \int_0^T s_3(t)\,u_1(t)\,dt$$

$$= \int_0^{T/2} 3\sqrt{\frac{2}{T}}\,dt = 3\frac{T}{2}\sqrt{\frac{2}{T}} \qquad 0 \le t < T/2$$

$$= 3\sqrt{\frac{T}{2}}$$

$$S_{32} = \int_0^T s_3(t)\,u_2(t)\,dt = \int_{T/2}^{T} \cancel{+2\,dt} + \sqrt{\frac{2}{T}}$$

$$= -\frac{1}{2}\sqrt{2T}$$

$$= -2\frac{T}{2}$$

$$= -T \qquad \frac{T}{2} \le t < T$$

$S_{33} = 0$

ASK Amplitude shift keying

$S_1(t) = A \cos 2\pi f_c t \rightarrow '1'$

$S_2(t) = 0 \rightarrow '0'$

$\frac{A^2}{2} T_b = E_b$

$A = \sqrt{\frac{2E_b}{T_b}}$

$\boxed{f_c = n f_b}$ n-integer value

$$\boxed{\begin{aligned} S_1(t) &= \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t \rightarrow 1 \\ S_2(t) &= 0 \rightarrow '0' \end{aligned}}$$

$$S_{11} = \left[\int_0^{T_b} S_1^2(t)\, dt\right]^{1/2}$$

$$= \left[\frac{2E_b}{T_b} \int_0^{T_b} \frac{1 + \cos 4\pi f_c t}{2}\, dt\right]^{1/2}$$

$$= \left[\frac{E_b T_b}{T_b}\right]^{1/2} = \sqrt{\frac{E_b \cancel{T_b}}{\cancel{T_b}}} = \sqrt{E_b}$$

$$u_1(t) = \frac{S_1(t)}{S_{11}} = \frac{\sqrt{\frac{2E_b}{T_b}}}{\sqrt{E_b}} = \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$$

$$u_1(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$$

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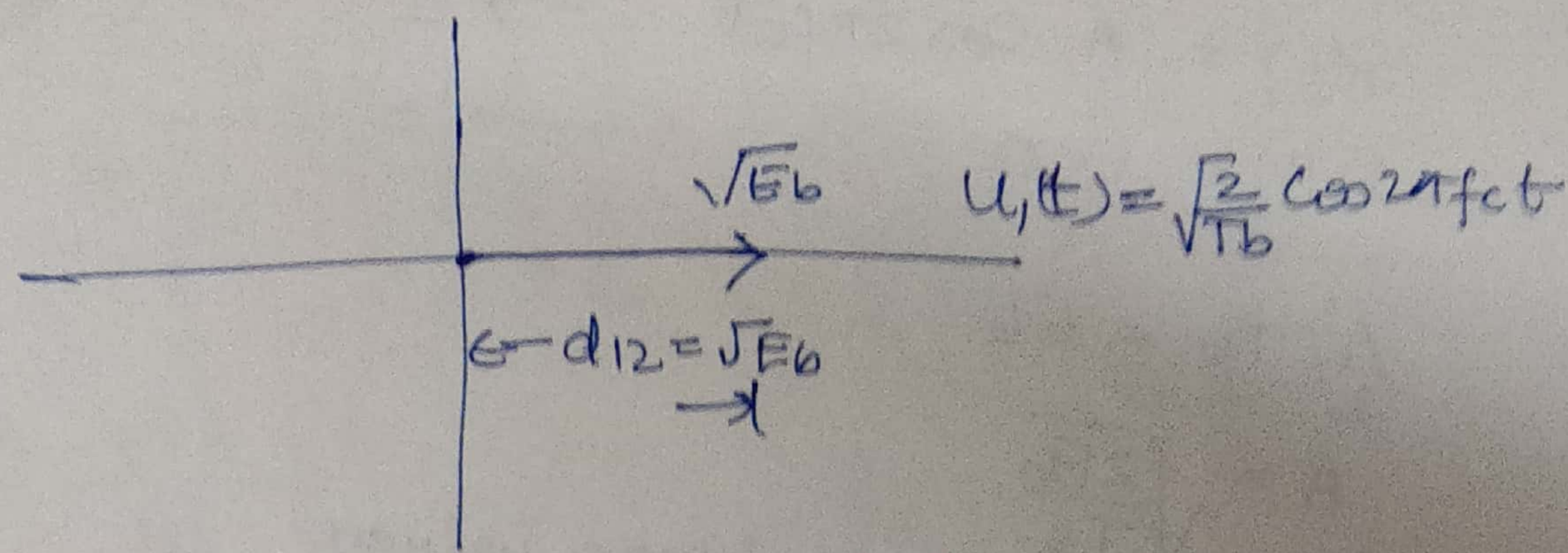
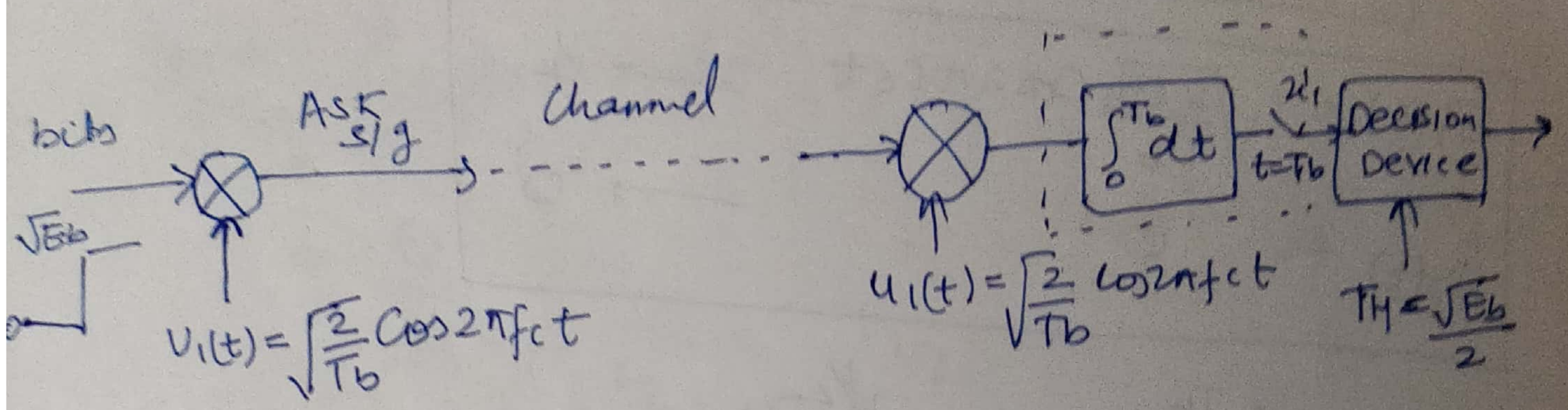
Constellation diagram

$S_1(t) = \sqrt{E_b} \times \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$

$S_2(t) = 0$



Generation & Detection of ASK s/g



Choose $1;\ x_1 > \frac{\sqrt{E_b}}{2}$

$0;\ x_1 < \frac{\sqrt{E_b}}{2}$

random; $x_1 = \frac{\sqrt{E_b}}{2}$

Synchronous ~~Transmission~~
Synchronous detection

$\sqrt{E_b} \cdot \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t \cdot \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$

at Rx

$\int_0^{T_b} \sqrt{E_b} \frac{2}{T_b} \cos^2 2\pi f_c t \, dt$

$= \sqrt{E_b} \frac{2}{T_b} \frac{T_b}{2}$

$\boxed{x_1 = \sqrt{E_b}}$

Working

At the TXing end, Bit stream bt which is a unipolar NRZ waveform having value $\sqrt{E_b}$ for bit '1' and 0 for bit '0'

→ the bit stream bt is multiplied by orthonormal function $u_1(t)$ to generate ASK S/g

→ The RXed ASK S/g is multiplied by locally generated orthonormal function to fascilitate coherent detection

→ The product is passed through integrator and dump switch RX which maximises SNR. at $t = T_b$ the sample of S/g plus noise is taken and is fed to the decision device.

→ The decision device decides b/w the 'zero & '1' according to its threshold value

~~Product~~ Probability of error in ASK

Pe in ASK

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{d_{12}^2}{4\eta}}\right) \qquad d_{12} = \sqrt{E_b}$$

$$= \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{4\eta}}\right)$$

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Binary Phase Shift Keying — BPSK

$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t \longrightarrow '1'$$

$$S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) \longrightarrow '0'$$

$$= -\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$$

$$S_2(t) = -S_1(t)$$

$$S_{11} = \left[\int_0^{T_b} \left(\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t \right)^2 dt \right]^{1/2}$$

$$S_{11} = \sqrt{E_b}$$

$$u_1(t) = \frac{S_1(t)}{S_{11}} = \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$$

$$S_{21} = \int_0^{T_b} S_2(t)\, u_1(t)\, dt$$

$$= \int_0^{T_b} -\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t \, dt$$

$$= \sqrt{\frac{2E_b}{T_b}} \sqrt{\frac{2}{T_b}} \frac{T_b}{2}$$

$$= -\sqrt{E_b}$$

$$S_{22} = \left[\int_0^{T_b} \left(S_2(t) - S_{21} u_1(t) \right)^2 dt \right]^{1/2}$$

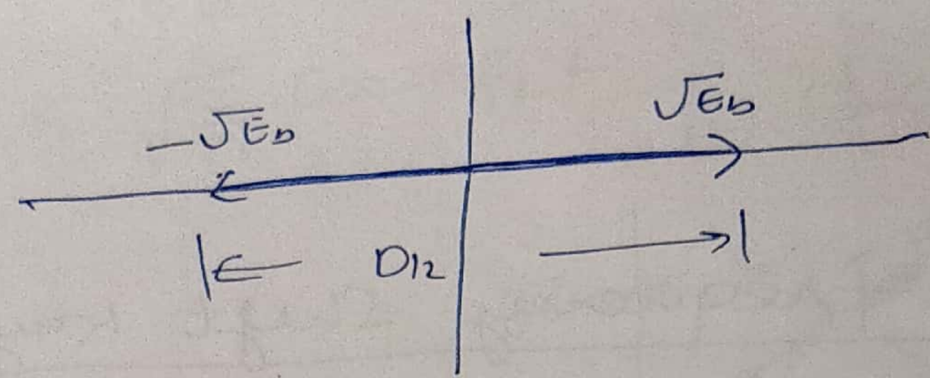
$$= \left[\int_0^{T_b} -\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t + \sqrt{E_b} \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t \right]^{1/2}$$

$$= 0$$

Constellation diagram

$s_1(t) = \sqrt{E_b} \cdot \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$

$s_2(t) = -\sqrt{E_b} \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$

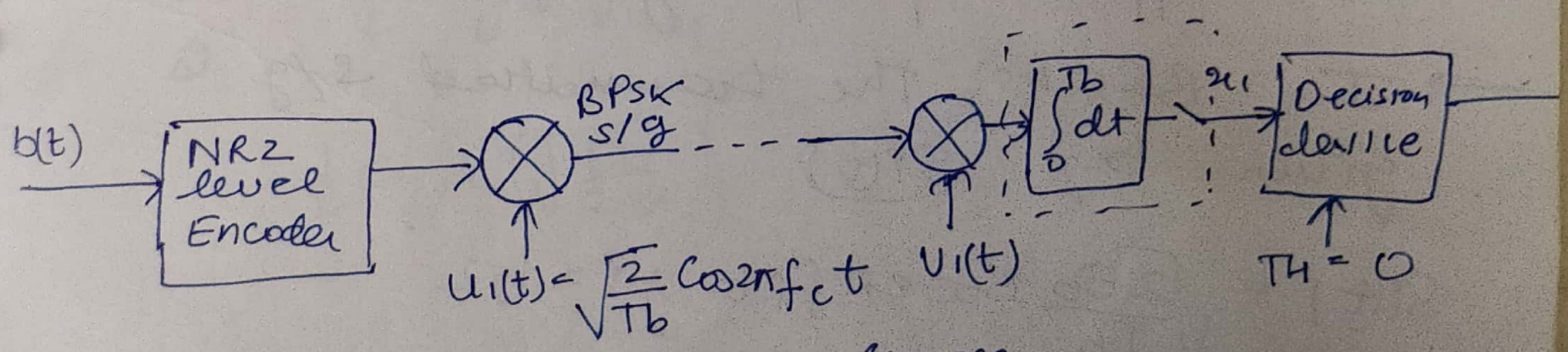


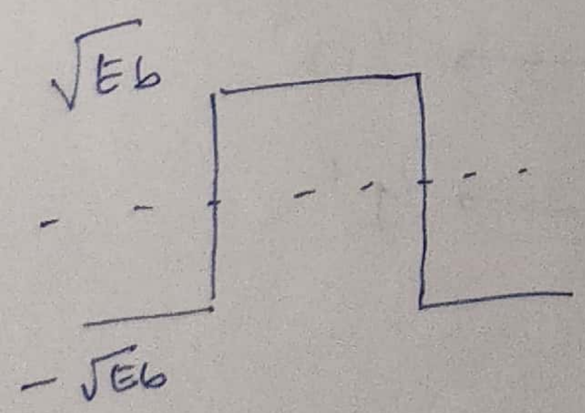
$u_1(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$

$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{D_{12}^2}{4N}}\right)$ $\quad D_{12} = 2\sqrt{E_b}$

$= \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{\eta}}$

generation and detection of BPSK s/g





choose

1 ; if $x_1 > 0$

0 ; if $x_1 < 0$

random ; if $x_1 = 0$

for imperfect phase synchronisation the probability of error in BPSK is given by $P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b \cos\theta}{\eta}}\right)$

θ = phase difference b/w transmitted carrier and locally generated carrier

$-\sqrt{E_b}\sqrt{\frac{2}{T_b}}\cos 2\pi f_c t \times \sqrt{\frac{2}{T_b}}\cos 2\pi f_c t$ at KX multiplier

at matched filter

$$= -\sqrt{E_b}\,\frac{2}{T_b}\int_0^{T_b}\frac{t}{2}\,dt$$

$$= -\sqrt{E_b}$$

BFSK - Binary frequency shift keying

this form of digital Modulation is an example of continuous phase FSK

in BFSK system, Bits '1's & '0's are distinguish from each other by transmitting one of the two sinusoidal waves that differ in freq. by a fixed amount

→ In BFSK, The transmitted s/g is given by $s_i(t)$

$$s_i(t) = \sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_i t \; ; i = 1,2$$

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_1 t \longrightarrow \text{'0'}$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_2 t \longrightarrow \text{'1'}$$

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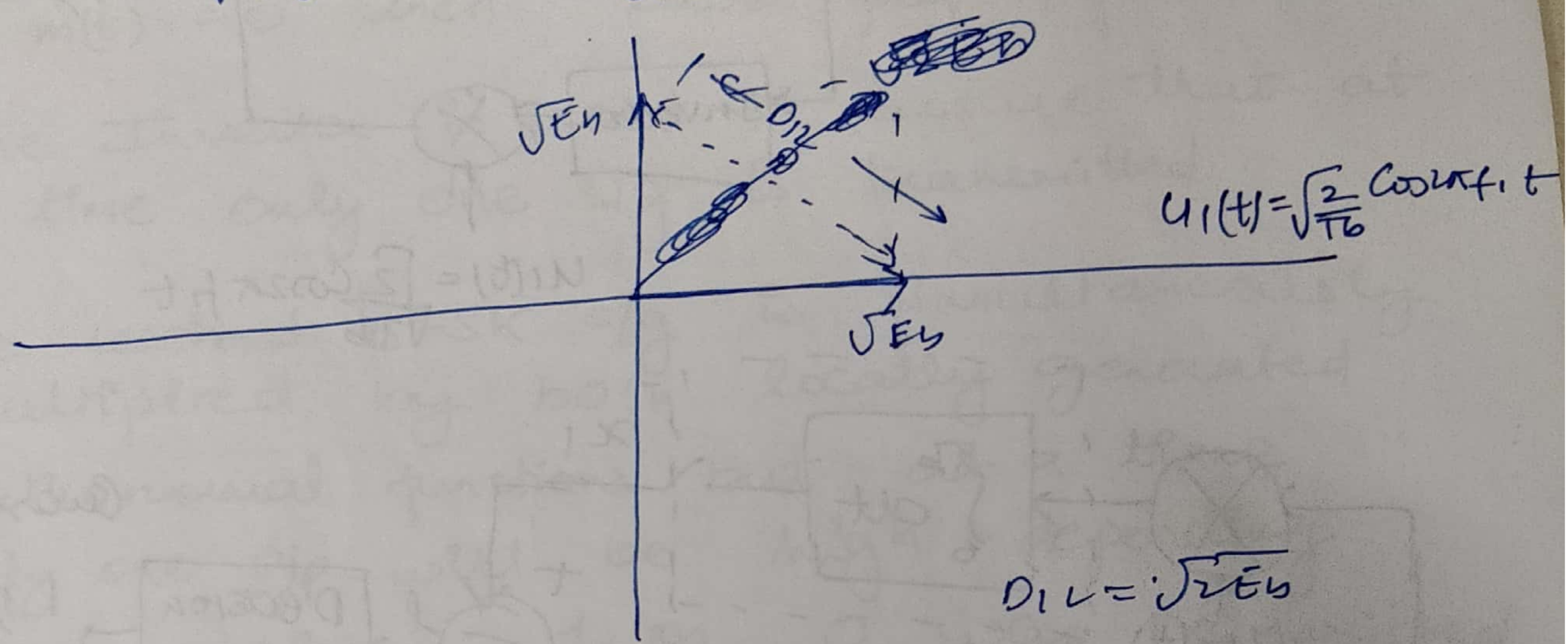
$S_{11} = \sqrt{E_b}$ $\qquad$ $S_{21} = 0$

$u_1(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_1 t$

$S_{22} = \sqrt{E_b}$

$u_2(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_2 t$



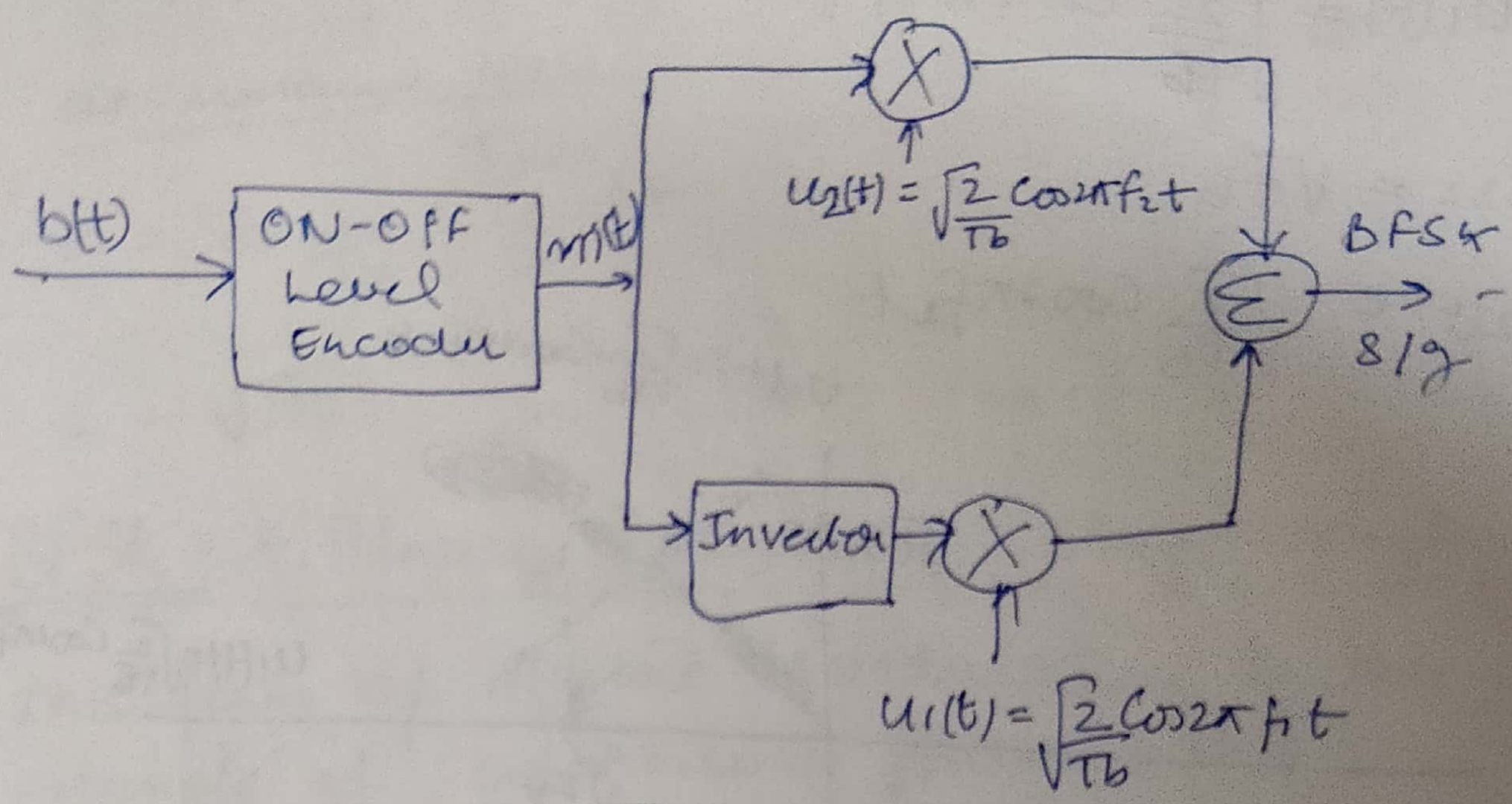
$D_{12} = \sqrt{2E_b}$

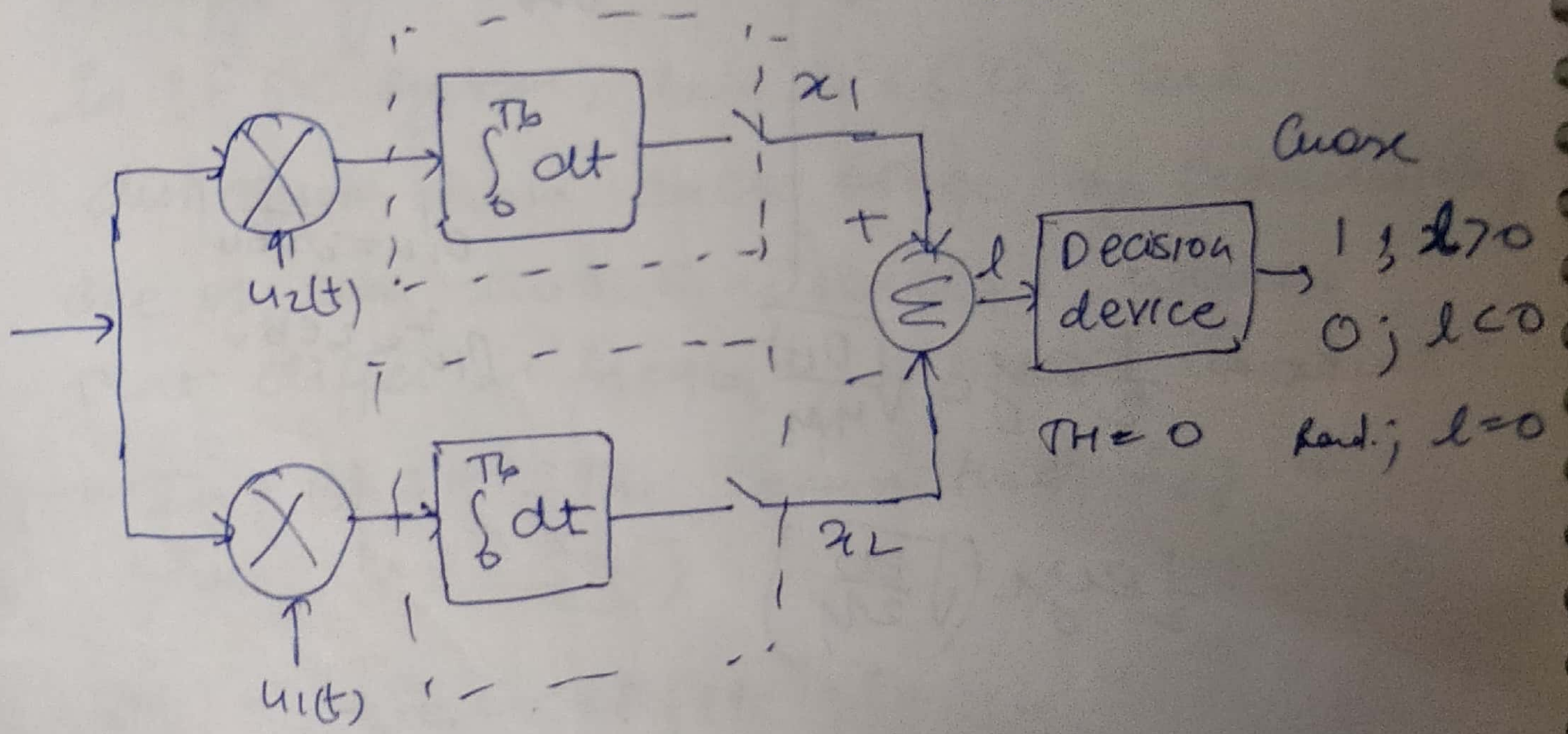
$D_{12}^2 = 2E_b$

$P_c = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{D_{12}^2}{4N}} =$

$\frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N}}\right)$

Generation & Detection of BFSK sig





for 1

$$\int_0^{T_b} \sqrt{E_b} \sqrt{\frac{2}{T_b}} \cos 2\pi f_2 t \times \underbrace{\sqrt{\frac{2}{T_b}} \cos 2\pi f_1 t}_{\text{orthonormal}} \, dt$$

so = 0

$$+ \int_0^{T_b} \sqrt{E_b} \sqrt{\frac{2}{T_b}} \cos 2\pi f_2 t \times \sqrt{\frac{2}{T_b}} \cos 2\pi f_2 t \, dt$$

$$= \sqrt{E_b}$$

Working

the binary data sequence b(t) is first applied to an (optional) ON-OFF level encoder to ensure that m(t) is a unipolar NRZ waveform

if m(t) = 1 then higher freq s/g is fixed
m(t) = 0 then lower freq s/g is fixed

the Inverter is used to ensure that at a time only one s/g is transmitted

→ The received BFSK s/g is simultaneously multiplied by both locally generated orthonomal functions but at a time only one o/p will be high depending upon whether 1 or 0 was transmitted

→ the product is passed through Integrator & Dump switch receiver which maximises s/g to Noise Ratio

→ At $t = T_b$ two samples x_1 & x_2 are taken the samples are substracted to generate resultant sample l and based upon the value of l, decision device decides b/w '0' & '1'

$$u_1(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_1 t$$

$$u_2(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_2 t$$

$$\int_0^{T_b} \cos 2\pi f_2 t \cdot \cos 2\pi f_1 t \, dt = 0$$

$$\int_0^{T_b} \cos(\omega_2 + \omega_1) t \, dt + \int_0^{T_b} \cos(\omega_2 - \omega_1) t \, dt = 0$$

$\omega_2 + \omega_1 = n\omega_b$ $\qquad$ $\omega_2 - \omega_1 = m\omega_b$

$$\boxed{f_2 + f_1 = n f_b \qquad f_2 - f_1 = m f_b}$$

→ condition for orthogonality

$$P_e(ASK) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{4\eta}}\right)$$

$$\frac{E_b}{\eta} = \frac{\text{watt} \times \text{sec}}{\frac{\text{watt}}{\text{Hz}}} = \frac{\text{watt}}{\text{watt}}$$

↓

Sig to Noise Ratio.

$$P_e(BFSK) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2\eta}}\right)$$

$$P_e(BPSK) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{\eta}}\right)$$

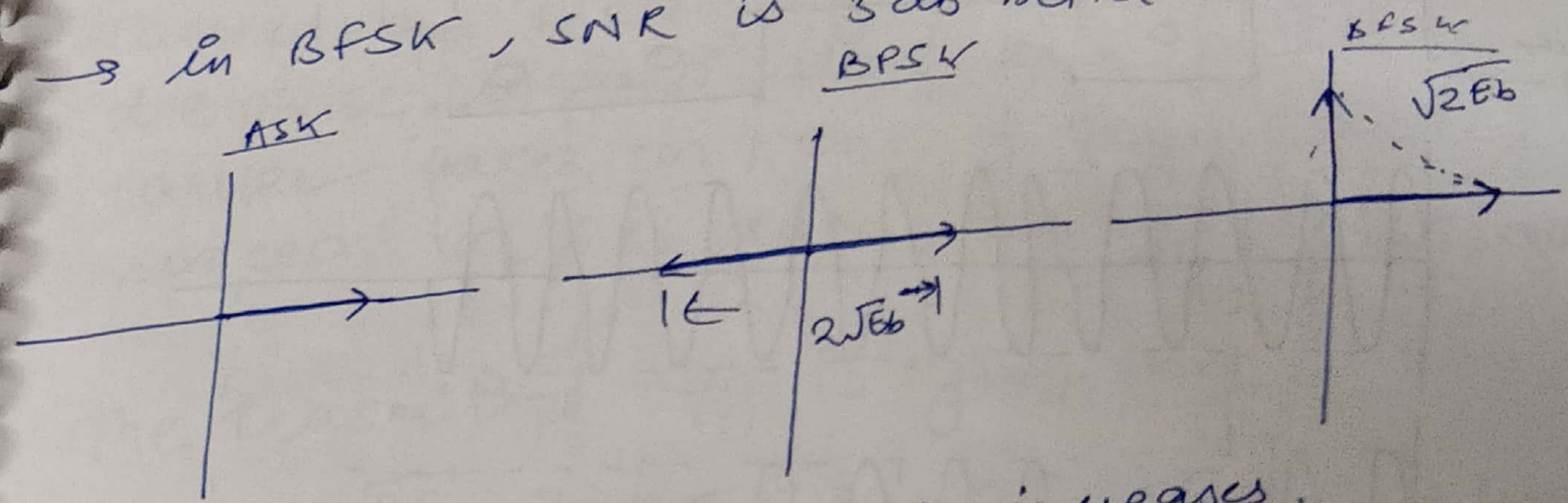
$\frac{E_b}{\eta} = 0\,dB$ $\qquad$ as reference

$\frac{E_b}{2\eta} = -3\,dB$

$\frac{E_b}{4\eta} = -6\,dB$

→ In BPSK, SNR is 3 dB better than BFSK and 6 dB better than ASK

→ In BFSK, SNR is 3 dB better than ASK



→ "As the distance b/w S/g increases, SNR increases and probability of error decreases"

Q

$E_b = 20 \times 10^{-20}$

$\frac{\eta}{2} = 10^{-20}$

$\eta = 2 \times 10^{-20}$

$\frac{E_b}{\eta} = 10$

$P_{ASK} = \frac{1}{2} erfc\sqrt{\frac{E_b}{\eta}} = \sqrt{\frac{10}{4}}$

$\frac{1}{2}\frac{e^{-2.5}}{1.58 \times \sqrt{\pi}}$

$P_{ASK} = 0.0146$

$P_{FSK} = \frac{1}{2} erfc\sqrt{\frac{E_b}{\eta/2}} = \frac{1}{2}\frac{e^{-5}}{\sqrt{5\pi}}$

$= 8.5 \times 10^{-4}$

$P_{PSK} = \frac{1}{2}\frac{e^{-10}}{\sqrt{10\pi}}$

$= 4.04 \times 10^{-6}$

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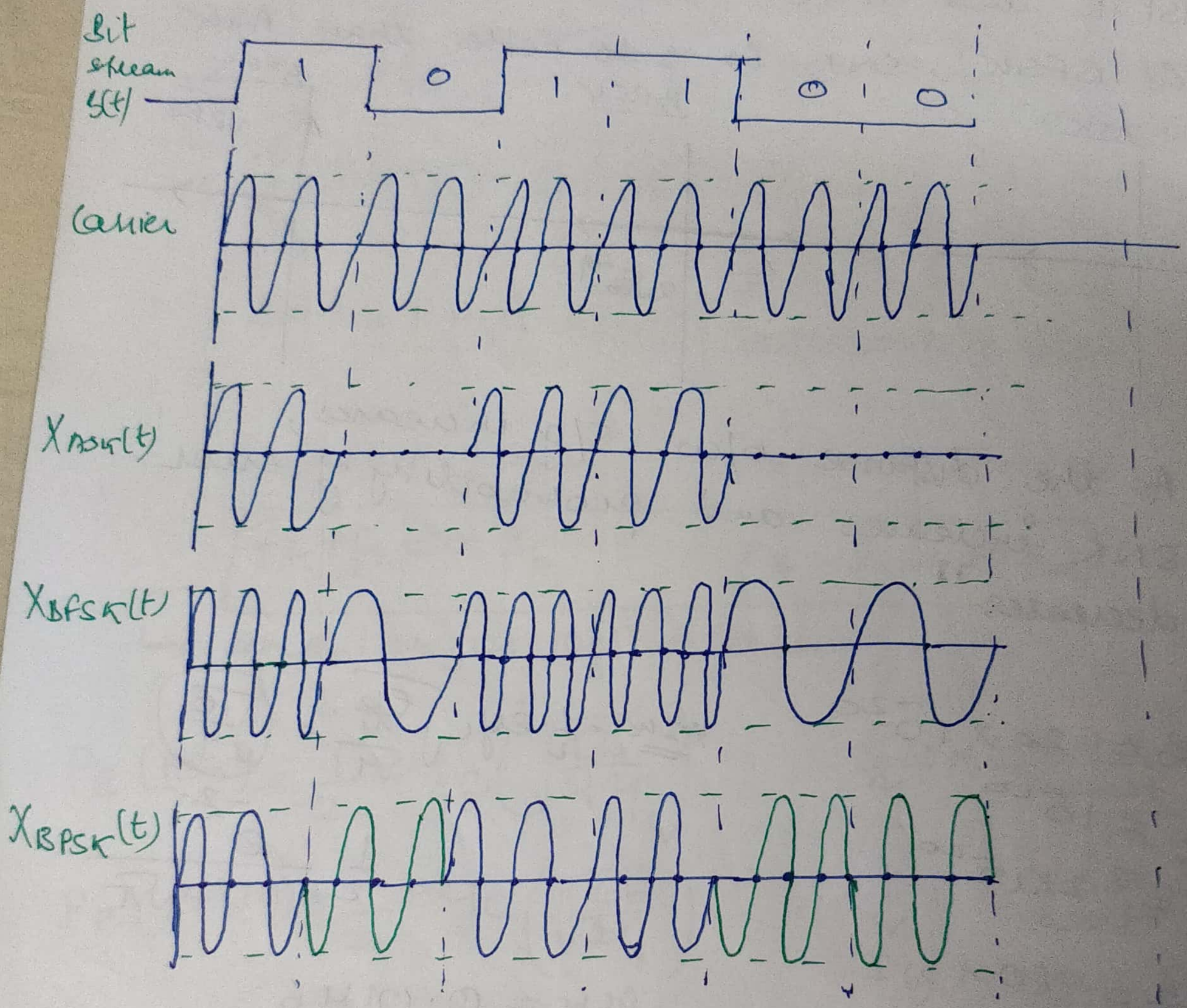
time domain Representation of ASK, BPSK & BFSK s/g's



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Quadrature phase shift keying - QPSK

In QPSK, as with binary PSK, information carried by Txed s/g is contained in the phase. In particular the phase of carrier takes on one of four equally spaced values such as $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

The trasmitted s/g is given by

$$S_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right] \; ; \; i = 1, 2, 3, 4$$

$E_s = 2E_b$

$T_s = 2T_b$

$$S_1(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t + \frac{\pi}{4}\right)$$

$$S_2(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t + \frac{3\pi}{4}\right)$$

$$S_3(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t + \frac{5\pi}{4}\right)$$

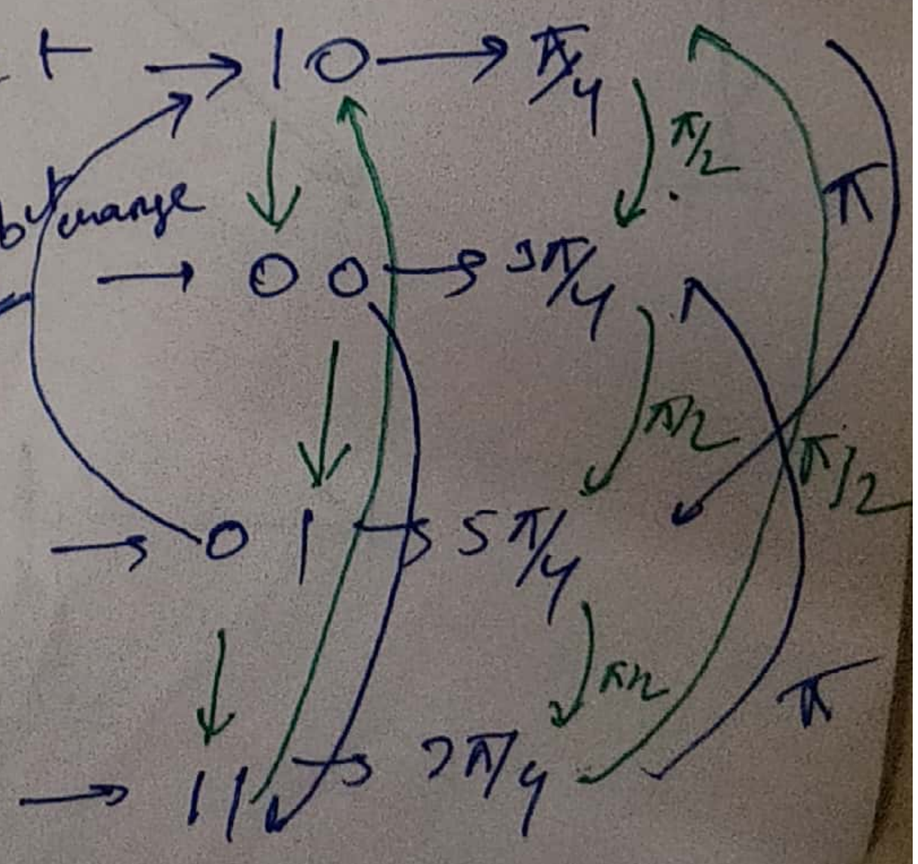
$$S_4(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t + \frac{7\pi}{4}\right)$$

$$S_1(t) = +\sqrt{\frac{E_s}{T_s}} \cos 2\pi f_c t - \sqrt{\frac{E_s}{T_s}} \sin 2\pi f_c t \rightarrow 10 \rightarrow \pi/4$$

$$S_2(t) = -\sqrt{\frac{E_s}{T_s}} \cos 2\pi f_c t - \sqrt{\frac{E_s}{T_s}} \sin 2\pi f_c t \rightarrow 00 \rightarrow 3\pi/4$$

$$S_3(t) = -\sqrt{\frac{E_s}{T_s}} \cos 2\pi f_c t + \sqrt{\frac{E_s}{T_s}} \sin 2\pi f_c t \rightarrow 01 \rightarrow 5\pi/4$$

$$S_4(t) = +\sqrt{\frac{E_s}{T_s}} \cos 2\pi f_c t + \sqrt{\frac{E_s}{T_s}} \sin 2\pi f_c t \rightarrow 11 \rightarrow 7\pi/4$$



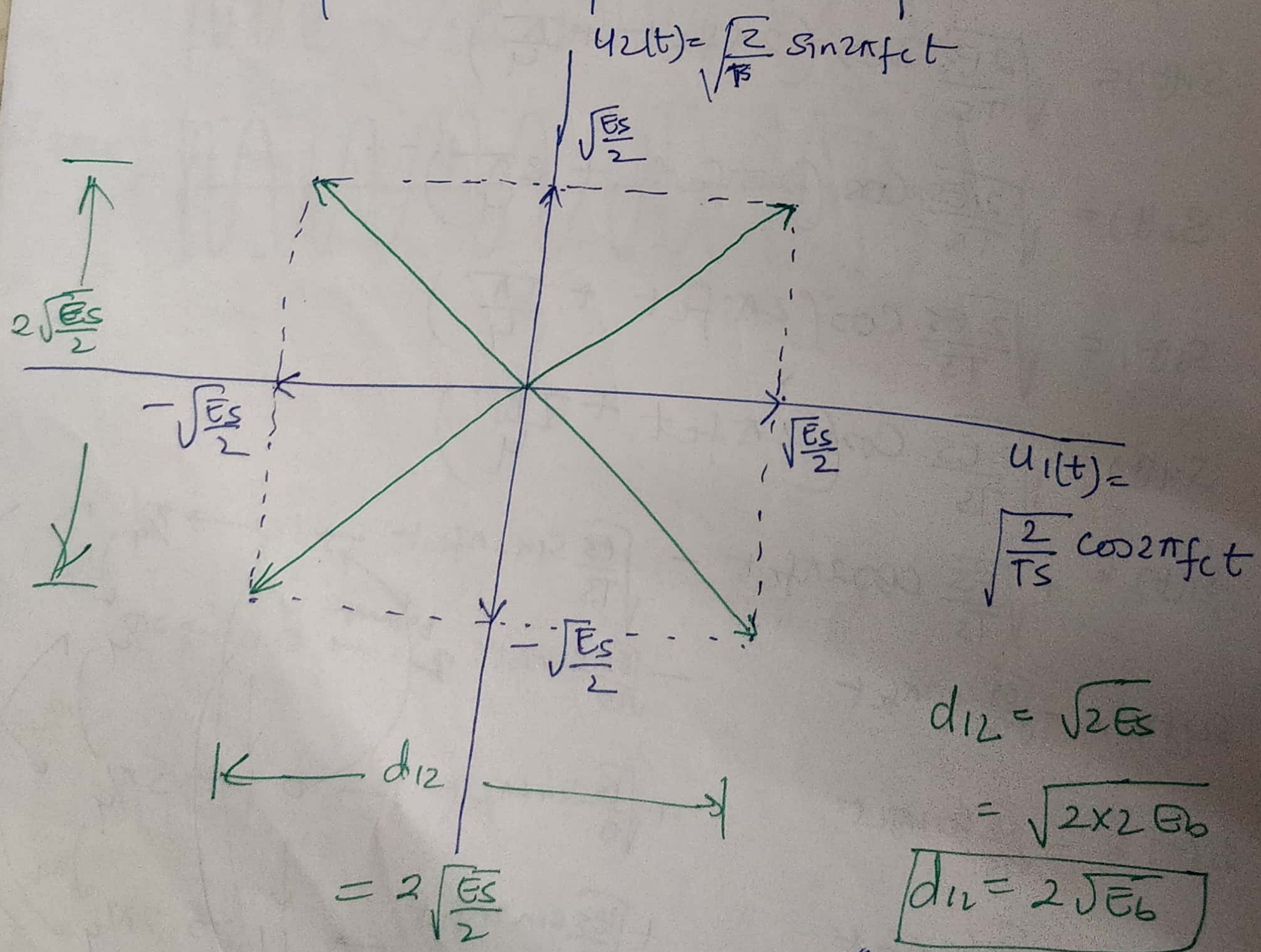
→ in QPSK for single bit change there is a phase shift of $\frac{\pi}{2}$ in resultant s/g

→ and when both bits changes simultaneously then there is a corresponding phase shift of π

$$S_3(t) = -S_1(t)$$
$$S_4(t) = -S_2(t)$$

$$\begin{cases} u_1(t) = \sqrt{\frac{2}{T_s}} \cos 2\pi f_c t \\ u_2(t) = \sqrt{\frac{2}{T_s}} \sin 2\pi f_c t \end{cases}$$

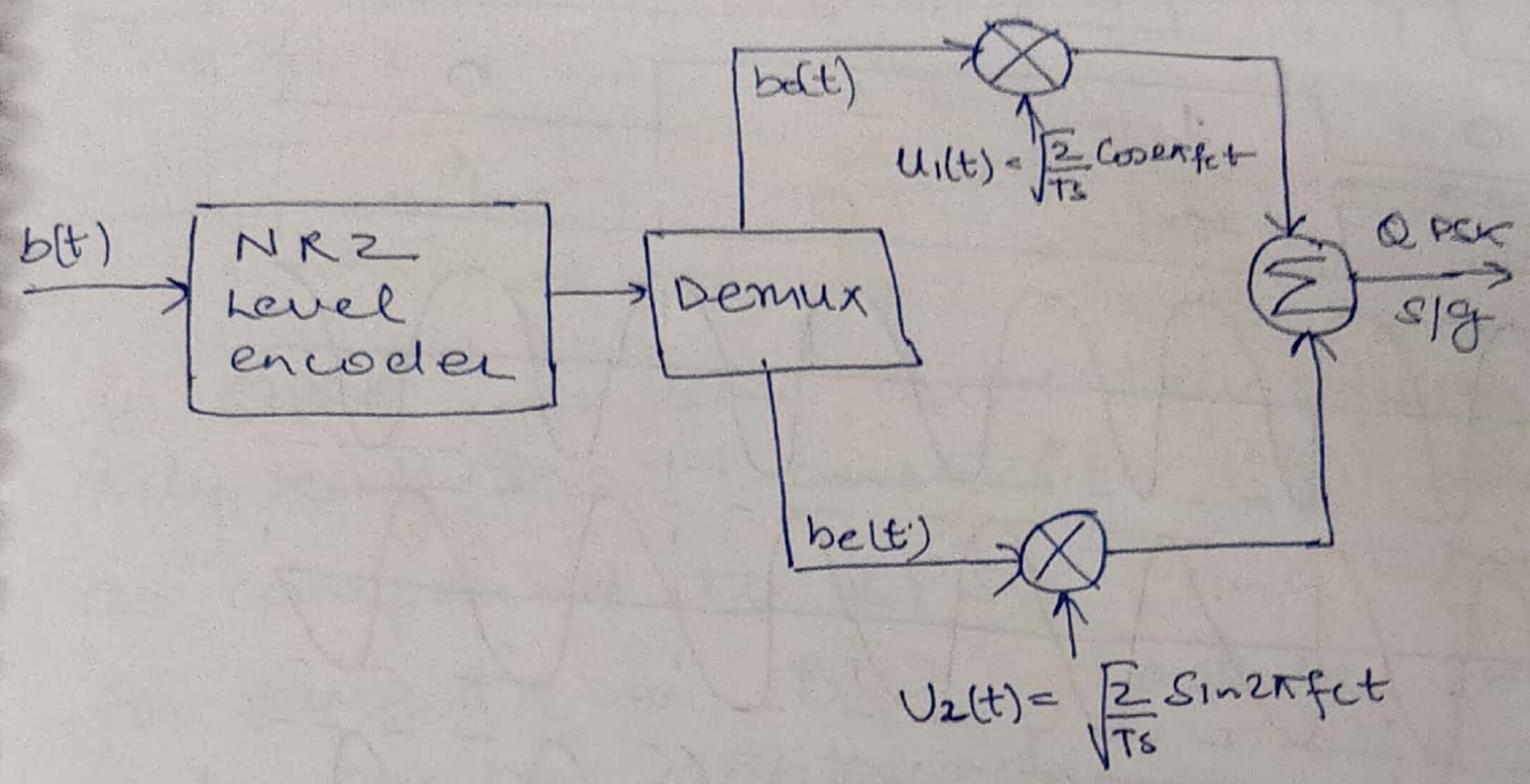
$S_{11} = \sqrt{\frac{E_s}{2}}$	$S_{21} = -\sqrt{\frac{E_s}{2}}$	$S_{31} = -\sqrt{\frac{E_s}{2}}$	$S_{41} = \sqrt{\frac{E_s}{2}}$
$S_{12} = -\sqrt{\frac{E_s}{2}}$	$S_{22} = -\sqrt{\frac{E_s}{2}}$	$S_{32} = +\sqrt{\frac{E_s}{2}}$	$S_{42} = \sqrt{\frac{E_s}{2}}$

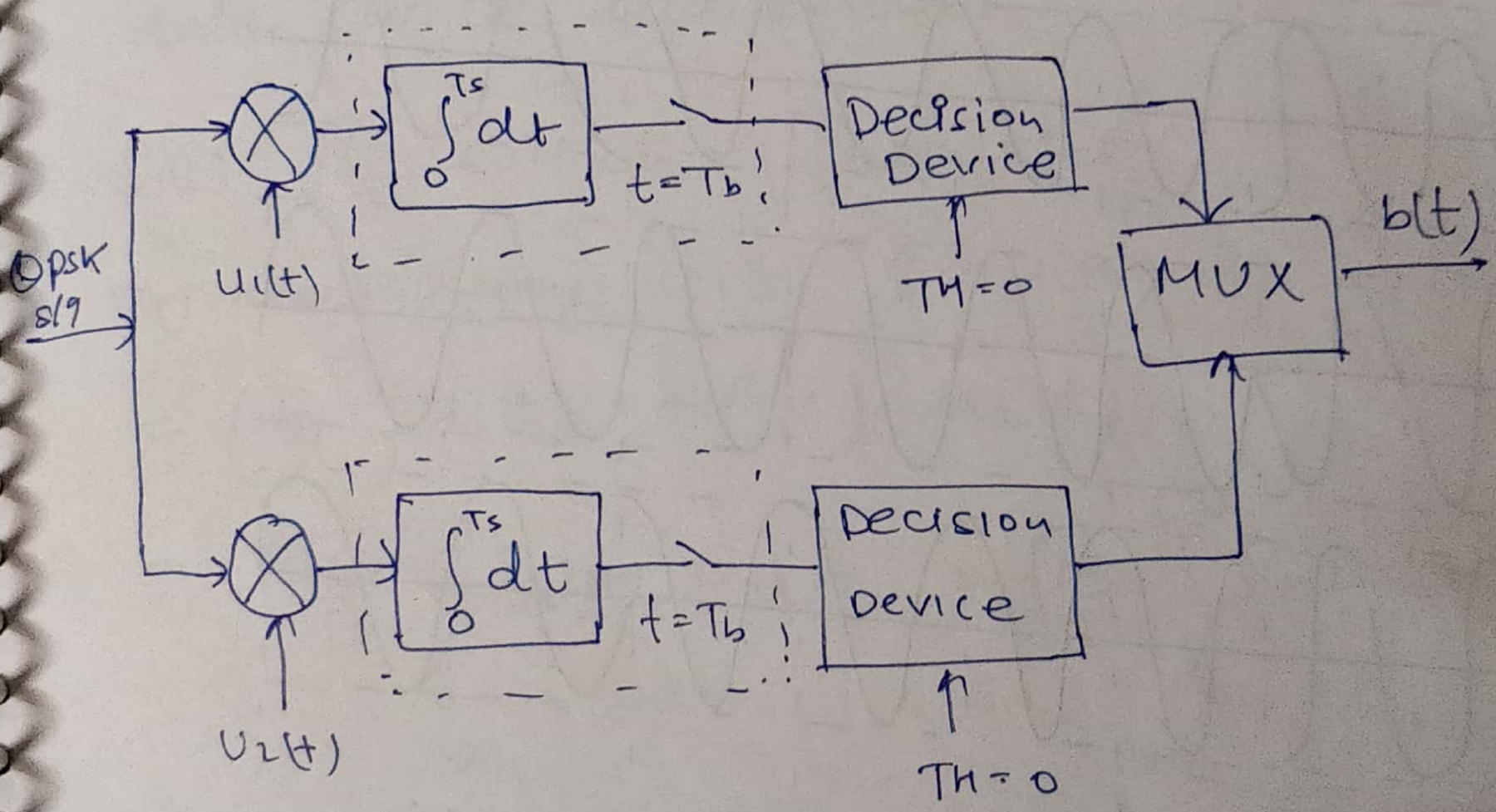


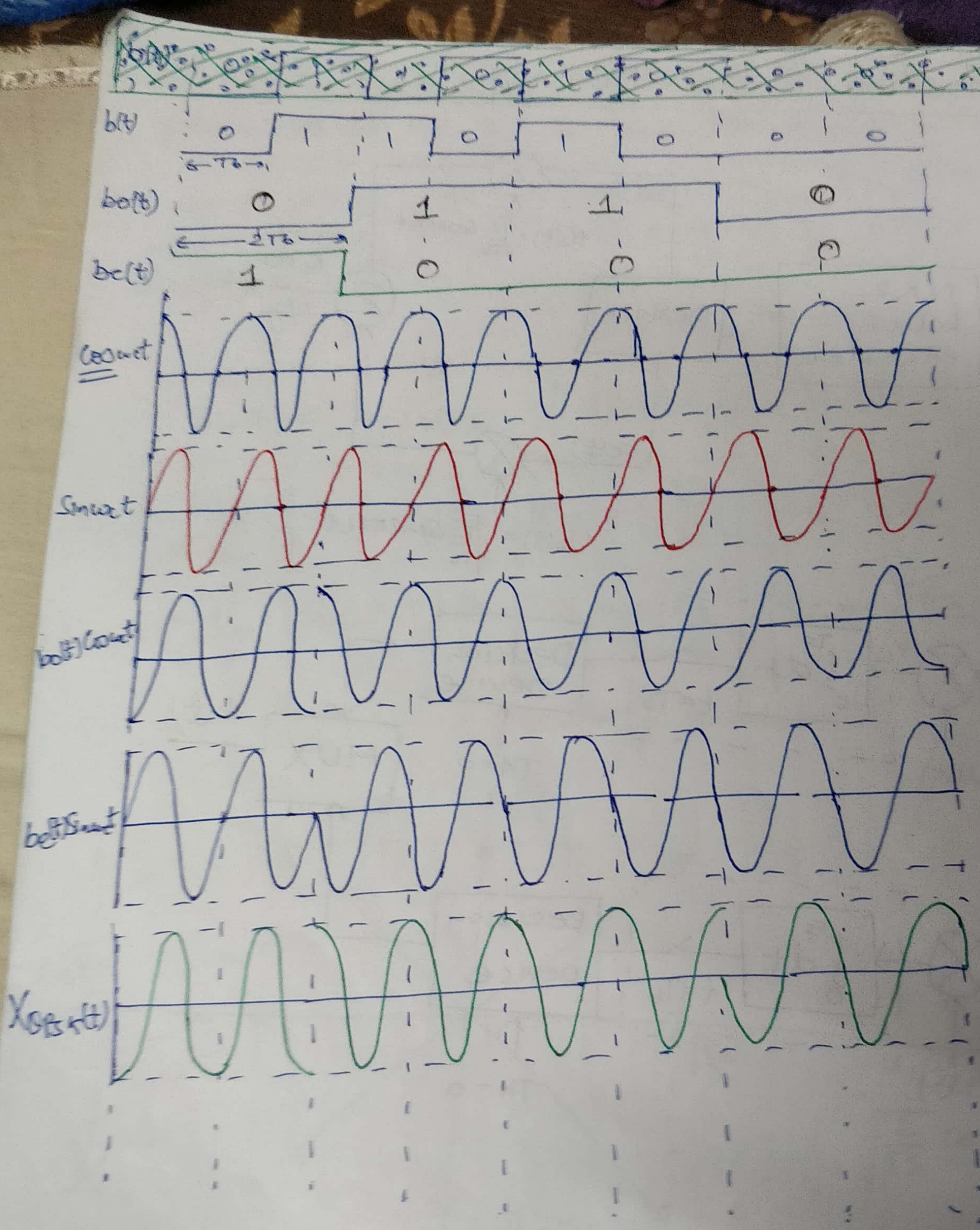
$$d_{12} = \sqrt{2E_s}$$
$$= \sqrt{2 \times 2 E_b}$$
$$\boxed{d_{12} = 2\sqrt{E_b}}$$

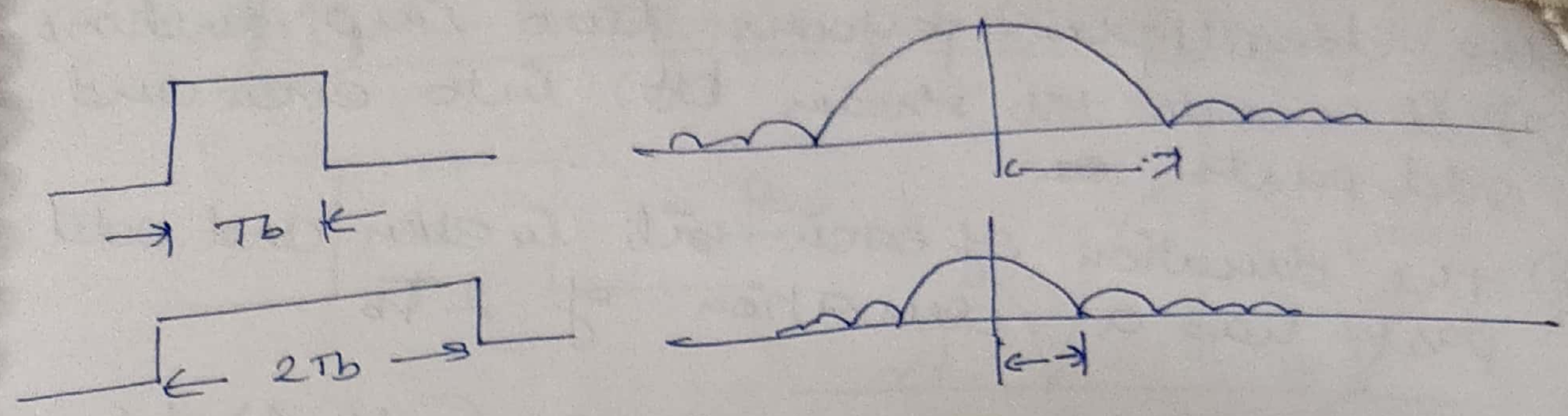
for worst case

Generation & detection of QPSK s/g:






b(t)
0 1 1 0 1 0 0 0
Tb
bo(t)
0 1 1 0
2Tb
bc(t)
1 0 0 0
cos ωct
sin ωct
bo(t) cos ωct
bc(t) sin ωct
XBPSK(t)



in QPSK, for same data transmission Rate only half the Transmission BW is required as compared to BPSK and alternately for same tx'ssion BW double the data rate can be achieved.

$$S_1(t) = \sqrt{\frac{E_s}{T_s}} \cos 2\pi f_c t - \sqrt{\frac{E_s}{T_s}} \sin 2\pi f_c t$$

by multiplying $u_1(t)$ & $u_2(t)$

$$= (\,''\,)\, u_1(t) + (\,''\,)\, u_2(t)$$

$$= \underbrace{\sqrt{\frac{2E_s}{T_s}} \int_0^{T_s} \cos^2 2\pi f_c t \, dt}_{x_1} - \underbrace{\sqrt{\frac{2E_s}{T_s}} \int_0^{T_s} \sin^2 2\pi f_c t \, dt}_{x_2}$$

$$= \quad x_1 = \sqrt{\frac{E_s}{2}} \qquad x_2 = -\sqrt{\frac{E_s}{2}}$$

Working

the answer sequence data b(t) which is a unipolar NRZ waveform is first converted into bipolar NRZ waveform by passing it through a NRZ level encoder.

the demultiplex performs two imp functions
(a) it converts bit stream b(t) into even and odd parts
(b) the duration of each bit in even and odd part has a duration of $2T_b$

③ → the Bit stream b(t) is multiplied by orthonormal function $U_1(t)$ and b(t) is multiplied to $U_2(t)$. the addition of these two waveform is transmitted as QPSK S/g

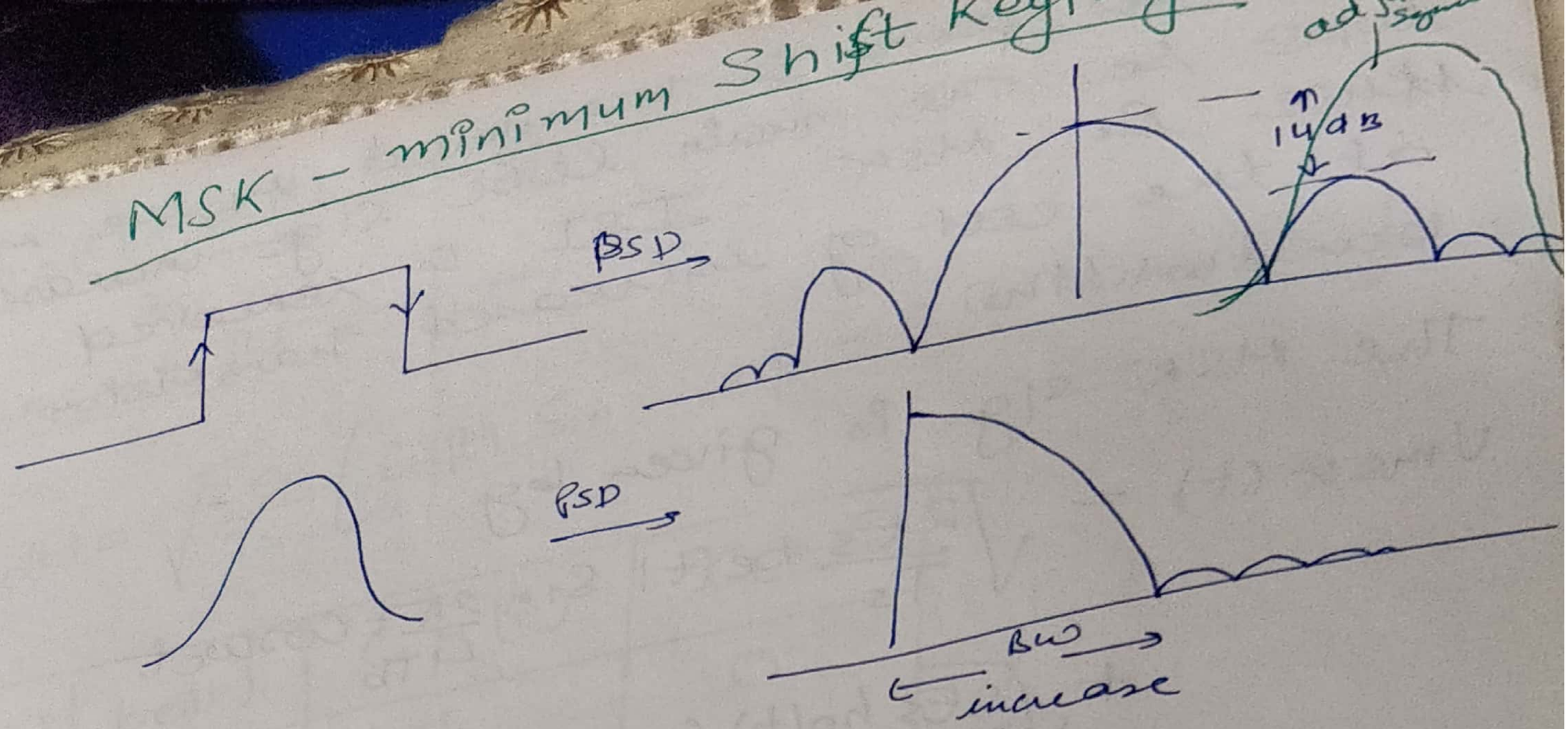
Rx

the Received QPSK S/g is simultaneously multiplied with locally generated orthonormal functions $U_1(t)$ & $U_2(t)$.
the Integrator and dump switch Rx. maximises SNR and samples are taken at $t = T_s$

→ Sample x_1 & x_2 are applied to the decision device which decides accordingly b/w 0 & 1

→ The odd & even bit stream are fed to a multiplexer which converts the parallel data in serial o/p binary sequence b(t)

MSK – minimum Shift Keying



$(BW)_{MSK} = 1.5\,(BW)_{QPSK}$

in QPSK due to abrupt phase change of sig's, the level of side lobe power increases. Practically it has been determined that in QPSK difference b/w highest point of main lobe and side lobe level is only 14 dB. So there is considerable amount of power present in side lobe level which interferes the transmission of adjacent symbol. This gives rise to intersymbol Interference (ISI)

→ to remove ISI abrupt phase change sig's are multiplied by smoothening carriers which ensure smooth phase change. This reduces the power present in side lobe

and this power is transferred in the main lobe so. the main lobe sig increases hence in MSK ISI is removed at the cost of increased transmission bandwidth

The MSK sig is given by

$$V_{msk}(t) = \sqrt{\frac{2E_s}{T_s}}\, b_e(t) \sin\frac{2\pi t}{4T_b} \cos\omega_c t$$

$$+ \sqrt{\frac{2E_s}{T_s}}\, b_o(t) \cos\frac{2\pi t}{4T_b} \sin\omega_c t$$

Let $\frac{2\pi}{4T_b} = \omega_1$

$$V_{msk}(t) = \sqrt{\frac{2E_s}{T_s}}\, b_e(t) \sin\omega_1 t \cos\omega_c t + \sqrt{\frac{2E_s}{T_s}}\, b_o t \cos\omega_1 t \sin\omega_c t$$

$$= \sqrt{\frac{2E_s}{T_s}}\left[\frac{b_e(t)}{2}\left(\sin(\omega_c+\omega_1)t - \sin(\omega_c-\omega_1)t\right) + \frac{b_o(t)}{2}\left(\sin(\omega_c+\omega_1)t + \sin(\omega_c-\omega_1)t\right)\right]$$

$$= \sqrt{\frac{2E_s}{T_s}}\left[\frac{b_e(t)+b_o(t)}{2} \sin(\omega_c+\omega_1)t \; \frac{b_o(t)-b_e(t)}{2} \sin(\omega_c-\omega_1)t\right]$$

$\omega_c+\omega_1 = \omega_H \quad \omega_c-\omega_1 = \omega_L$

$$\frac{b_o(t) + b_e(t)}{2} = C_H(t)$$

$$\frac{b_o(t) - b_e(t)}{2} = C_L(t)$$

$$V_{msk}(t) = \sqrt{\frac{2E_s}{T_s}}\left[C_H(t)\sin\omega_H t + C_L(t)\sin\omega_L t\right]$$

$b_o(t)$	$b_e(t)$	$C_H(t)$	$C_L(t)$
-1	-1	-1	0
-1	$+1$	0	-1
$+1$	-1	0	$+1$
$+1$	$+1$	$+1$	0

$$S_1(t) = -\sqrt{\frac{2E_s}{T_s}}\sin\omega_H t \rightarrow 00$$

$$S_2(t) = -\sqrt{\frac{2E_s}{T_s}}\sin\omega_L t \rightarrow 01$$

$$S_3(t) = \sqrt{\frac{2E_s}{T_s}}\sin\omega_L t \rightarrow 10$$

$$S_4(t) = \sqrt{\frac{2E_s}{T_s}}\sin\omega_H t \rightarrow 11$$

$$u_1(t) = \sqrt{\frac{2}{T_s}}\sin\omega_H t$$

$$u_2(t) = \sqrt{\frac{2}{T_s}}\sin\omega_L t$$

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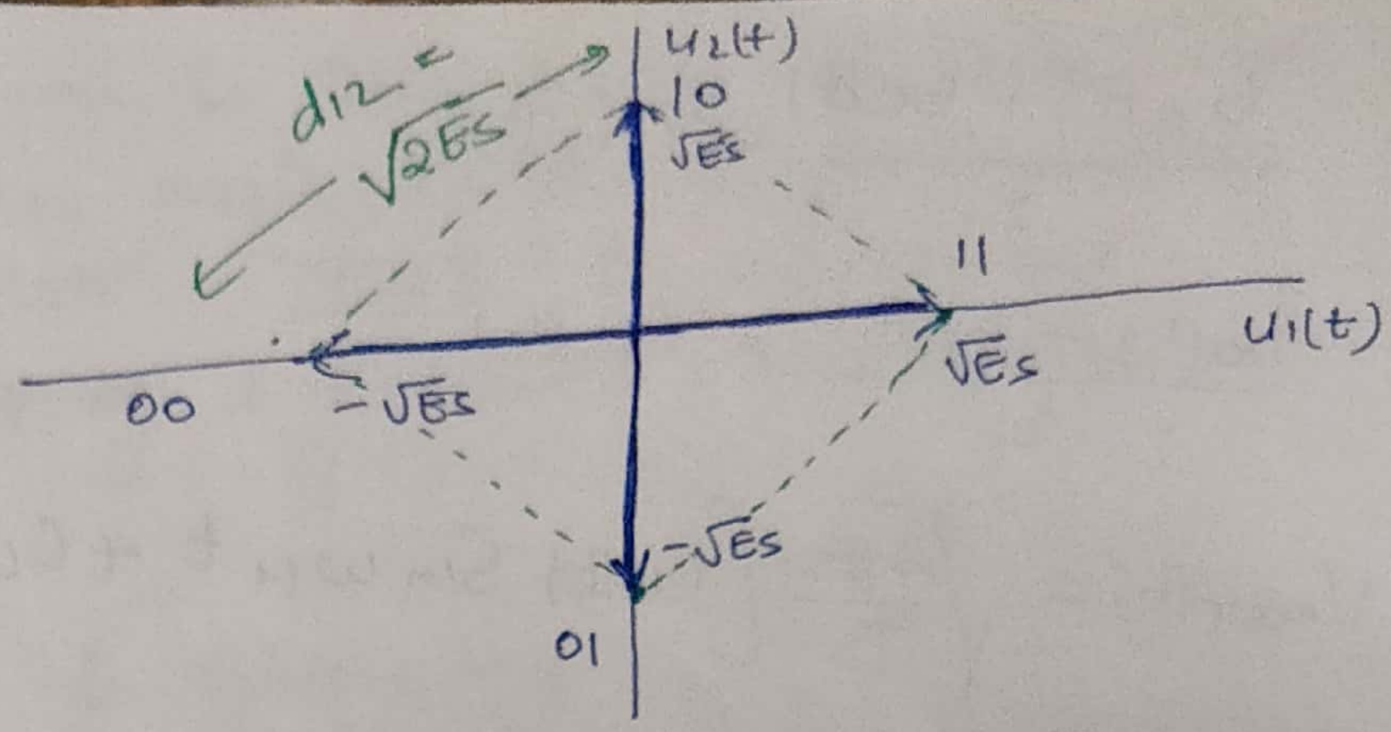
$S_{11} = -\sqrt{E_s}$

$S_{22} = -\sqrt{E_s}$

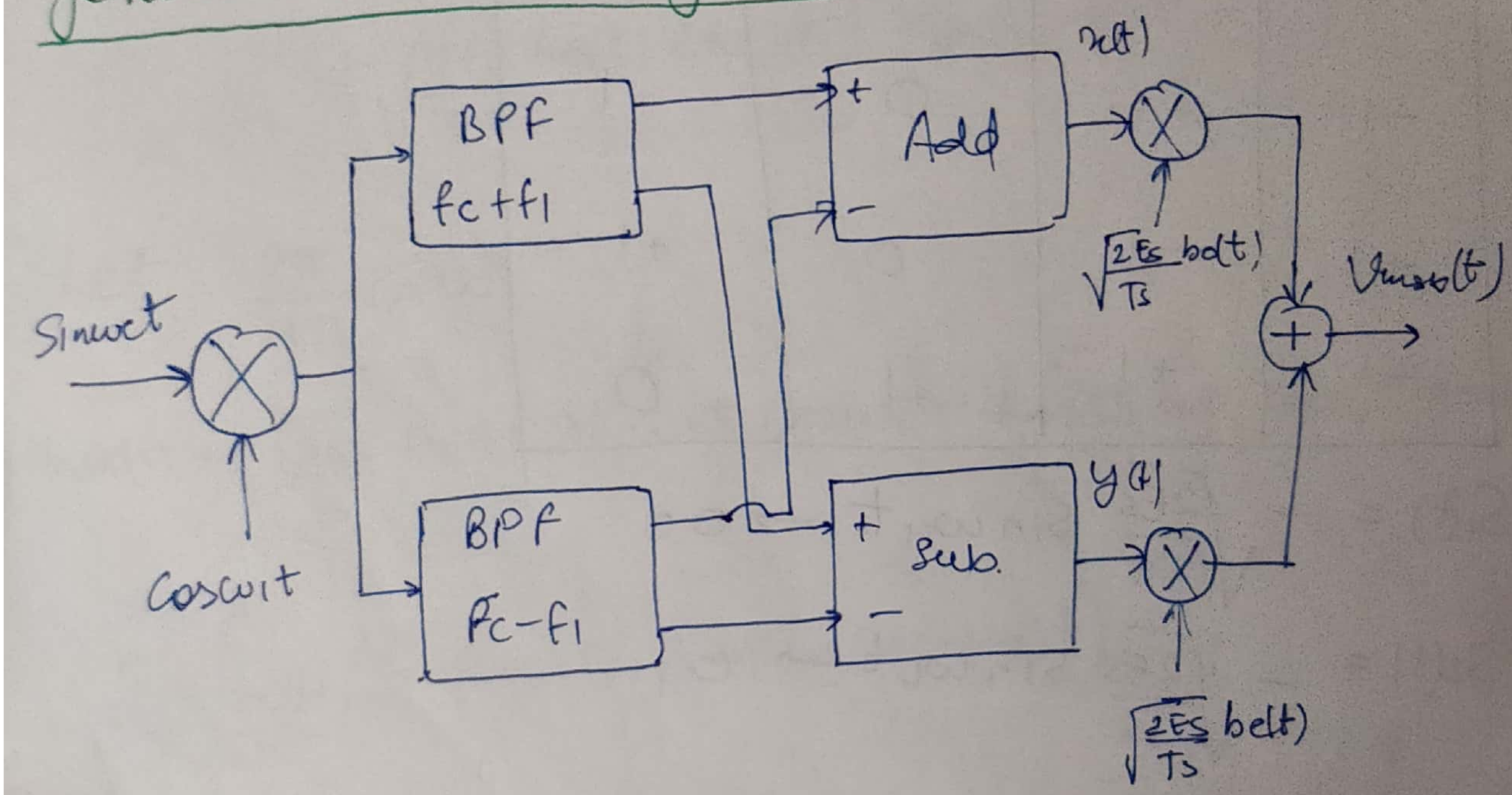
$S_{32} = \sqrt{E_s}$

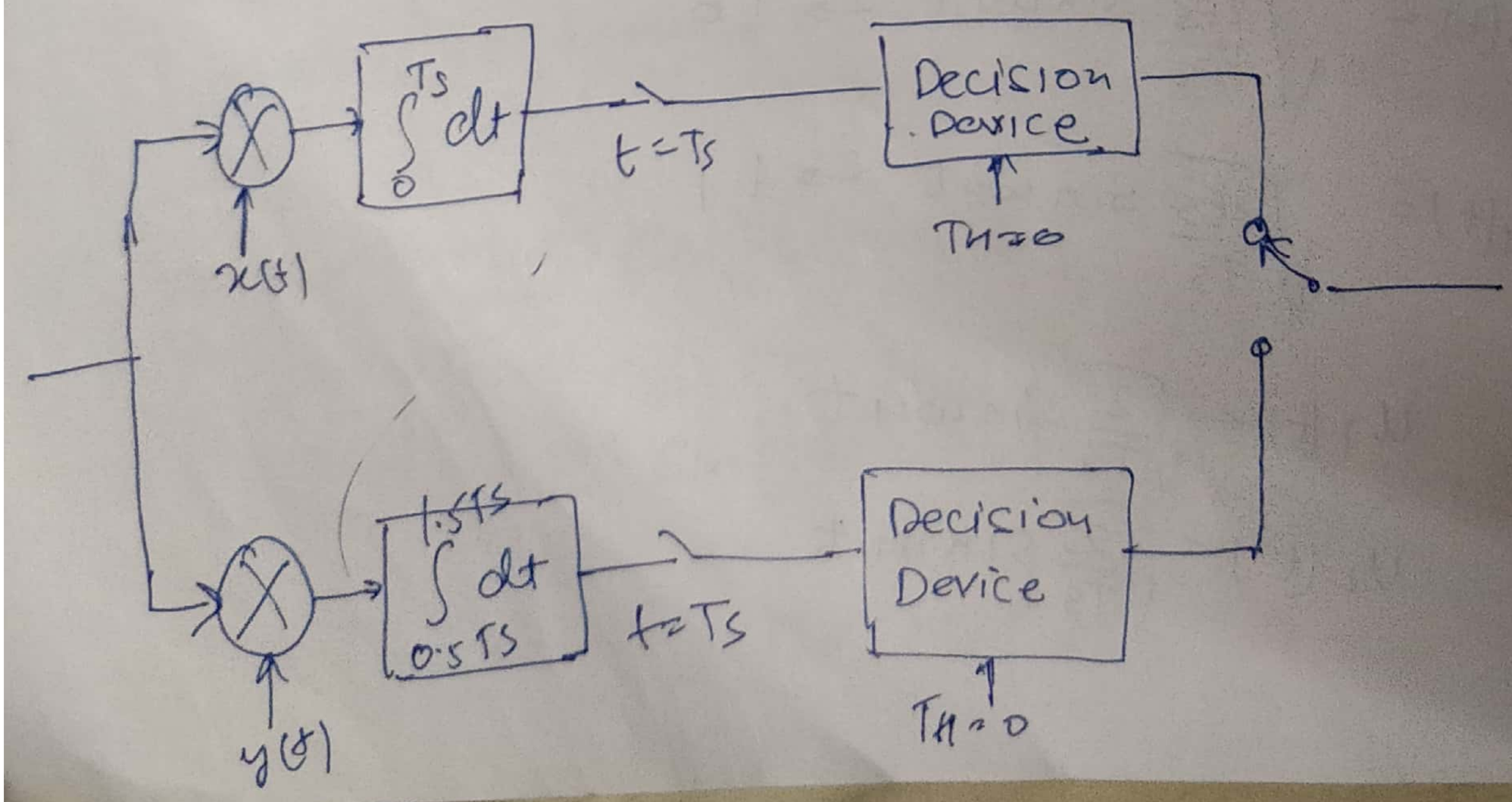
$S_{41} = \sqrt{E_s}$

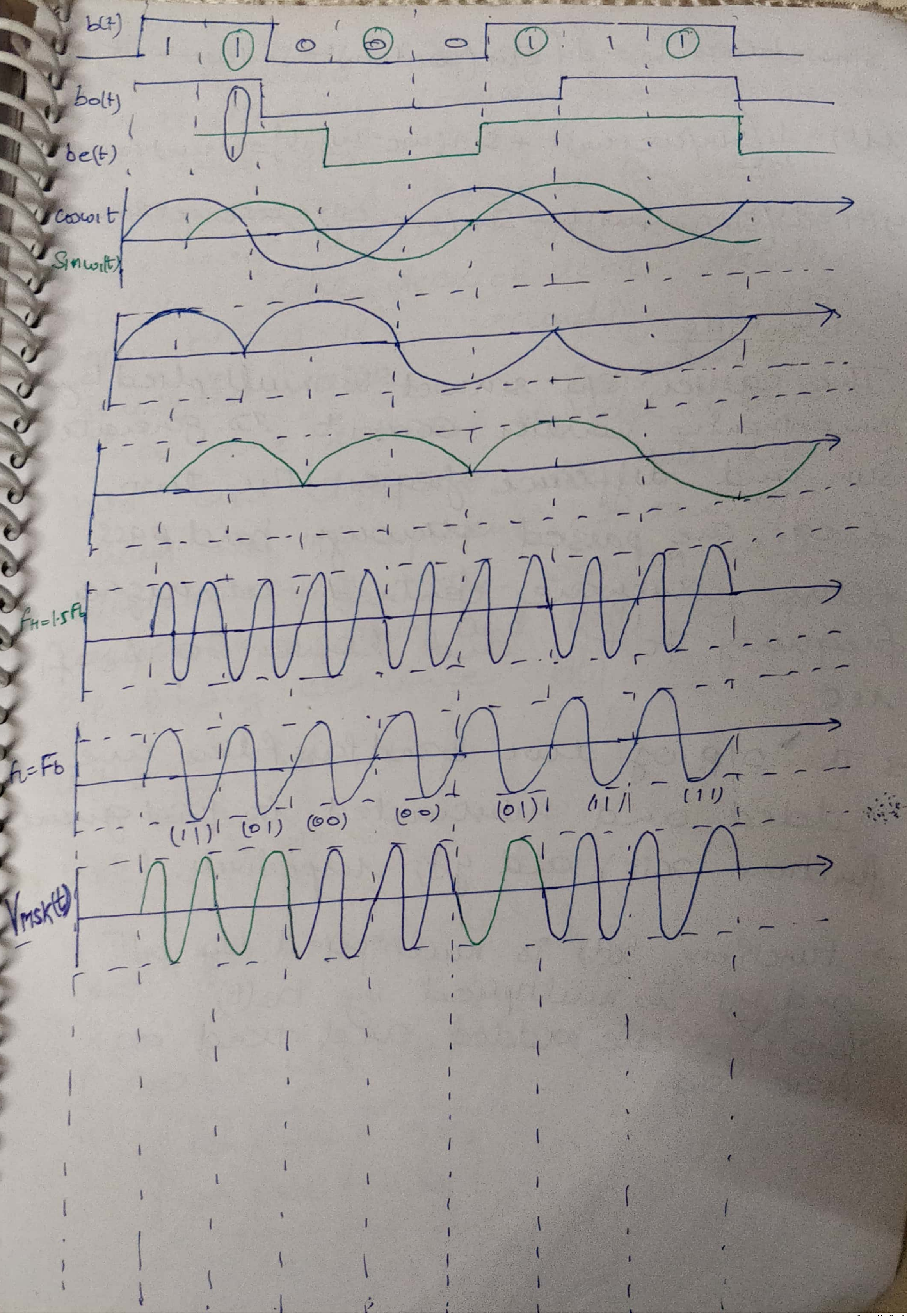


$$d_{12} = \sqrt{2 \times 2E_b} = 2\sqrt{E_b}$$

Generation & detection of MSK S/g






b(t)
1 1 0 0 0 1 1 1
bo(t)
be(t)
cosω1t
sinω1(t)
fH=1.5fb
fL=Fb
(11) (01) (00) (00) (01) (11) (11)
VMSK(t)

$\sin\omega_c t \cos\omega_1 t = \frac{1}{2}\left[\sin(\omega_c+\omega_1)t + \sin(\omega_c-\omega_1)t\right]$

$x(t) = \frac{1}{2}\left[\sin(\omega_c+\omega_1)t + \sin(\omega_c-\omega_1)t\right] = \sin\omega_c t \cos\omega_1 t$

$y(t) = \frac{1}{2}\left(\sin(\omega_c+\omega_1)t - \sin(\omega_c-\omega_1)t\right) = \cos\omega_c t \sin\omega_1 t$

Working

The carrier s/g $\sin\omega_c t$ is multiplied by smoothening carrier $\cos\omega_1 t$ to generate sum and difference freqies the two freq. are passed through band pass filters which are centred at higher frequency $f_c + f_1$ and lower freq $f_c - f_1$ resp.

→ The o/p of two Band Pass filters are added and substracted to ~~form~~ generate functions $x(t)$ and $y(t)$ respectively.

→ Function $x(t)$ is multiplied by $b_d(t)$ and $y(t)$ is multiplied by $b_e(t)$. The two s/gs are added and used as MSK s/g

Ans / RX

the Received MSK s/g is simultaneously multiplied by locally generated function $x(t)$ and $y(t)$.

Integrator and dump switch receiver maximises SNR and generate samples x_1 & x_2. The decision device desides o/w '0' and '1' according to its threshold value.

→ Since the interval of integration of odd and even bits are different so the two bit stream are not present at the o/p simultaneously. The switch alternately connects b/w $b_o(t)$ and $b_e(t)$ to generate o/p binary sequence $b(t)$.

$$u_1(t) = \sqrt{\frac{2}{T_s}} \sin \omega_H t$$

$$u_2(t) = \sqrt{\frac{2}{T_s}} \sin \omega_L t$$

$$\int_0^{T_s} \sin \omega_H t \sin \omega_L t \, dt$$

$$\int_0^{T_s} \cos(\omega_H - \omega_L)t \, dt - \int_0^{T_s} \cos(\omega_H + \omega_L)t \, dt$$

$$\boxed{\begin{aligned} \omega_H - \omega_L &= n\omega_s \\ \omega_H + \omega_L &= m\omega_s \end{aligned}}$$

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$f_H - f_L = n f_s$

$T_s = 2T_b$

$f_s = \frac{f_b}{2}$

$\omega_1 = \frac{2\pi}{4T_b}$

$f_1 = \frac{f_b}{4}$

$\omega_H = \omega_c + \omega_1$

$\omega_L = \omega_c - \omega_1$

$f_c + f_1 - f_c + f_1 = n\frac{f_b}{2}$

$2f_1 = n\frac{f_b}{2}$

$\frac{2f_b}{4} = n\frac{f_b}{2}$

$\therefore n = 1$

$f_H + f_L = m f_s$

$f_c + f_1 + f_c - f_1 = m\frac{f_b}{2}$

$2f_c = \frac{m f_b}{2}$

$$\boxed{f_c = \frac{m f_b}{4}}$$

$f_H = f_c + f_1 = \frac{m f_b}{4} + \frac{f_b}{4}$

$f_H = (m+1)\frac{f_b}{4}$

$f_L = f_c - f_1 = \frac{m f_b}{4} - \frac{f_b}{4}$

$= (m-1)\frac{f_b}{4}$

at $m = 5$

$$\boxed{f_L = f_b}$$

→ $\underline{\underline{m = 1}}$

this is the "minimum" integer value for which the two functions $\sin \omega_H(t)$ and $\sin \omega_L(t)$ becomes orthogonal. that is why MSK is called so.

M-ary Sig

BPSK → 2ary PSK signal $M=2$

QPSK → 4-ary PSK signal $M=4$

$M=8$ → 8-ary PSK signal

$M=16$ → 16-ary PSK signal

$$\boxed{T_s = T_b \log_2 M}$$

$M=2$, $T_s = T_b$

$M=4$; $T_s = 2T_b$

$M=8$; $T_s = 3T_b$

⋮

$$R_b = \frac{1}{T_b}$$

$$R_s = \frac{1}{T_s}$$

$$\boxed{\text{Baud Rate} = \frac{R_b}{\log_2 M} \text{ samples / sec}}$$

Bandwidth efficiency ρ

M-ary PSK S/g

for m-ary PSK S/g the channel BW is given by $\boxed{BW = \frac{2}{T_S}}$

$$BW = \frac{2}{T_b \log_2 M} = \frac{2 r_b}{\log_2 M}$$

$$\boxed{\rho = \frac{\log_2 M}{2}}$$

M-ary FSK S/g

For M-ary FSK S/g, the channel BW is given by

$$\boxed{BW = \frac{M}{2T_s}}$$

$$BW = \frac{M}{2 T_b \log_2 M}$$

$$BW = \frac{M r_b}{2 \log_2 M}$$

$$\boxed{\rho = \frac{2 \log_2 M}{M}}$$

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M-ary PSK

① BPSK : M=2

$\boxed{\rho = 0.5}$

② QPSK : Q=4

$\boxed{\rho = 1.0}$

③ 8-ary PSK

$\boxed{\rho = 1.5}$

④ 16-ary PSK

$\boxed{\rho = 2.0}$

① BFSK M=2

$\boxed{\rho = 1.0}$

② 4 ary PSK = M=4

$\boxed{\rho = 1.0}$

③ 8 ary PSK = M=8

$\boxed{\rho = 0.75}$

M=16

④ 16-ary PSK

$\boxed{\rho = 0.5}$

In M-ary PSK s/g, as more no. of bits are combined, the distance b/w s/gs go on decreasing & decreasing.

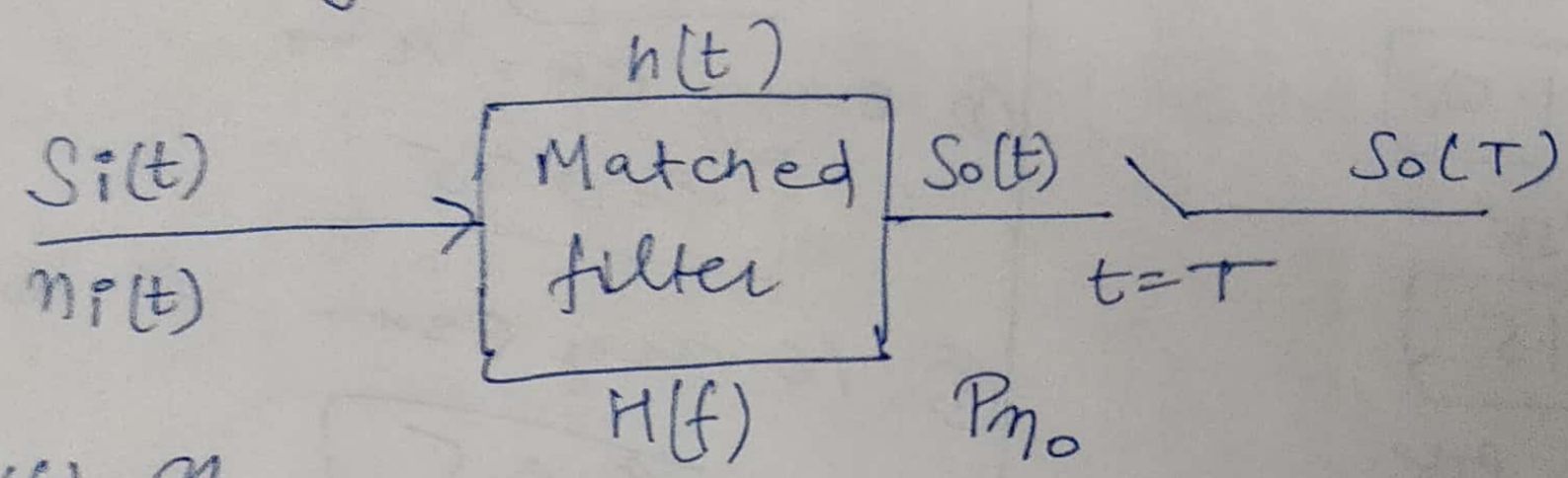
So Probability of error increases

Hence higher PSK s/g's are not used

→ BFSK provides same BW efficiency as QPSK but still it is not used because high prob of error.

Matched filter

A matched filter is called so. because its impulse responce is matched to the incoming s/g.



$$S_{ni}(f) = \frac{\eta}{2}$$

$$S_o(t) = S_i(t) * h(t)$$

$$S_o(f) = S_i(f)\, H(f)$$

$$S_o(t) = \int_{-\infty}^{\infty} S_o(f)\, e^{j2\pi f t}\, df$$

$$S_o(t) = \int_{-\infty}^{\infty} S_i(f)\, H(f)\, e^{j2\pi f t}\, df$$

$$S_{no}(f) = \frac{\eta}{2} |H(f)|^2$$

$$P_{no} = \frac{\eta}{2} \int_{-\infty}^{\infty} |H(f)|^2\, df$$

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At sampling time $t = T$ sec, filter should maximise o/p SNR

$$S_o(T) = \int_{-\infty}^{\infty} S_o(f) e^{j2\pi f T} df$$

$$|S_o(T)|^2 = \left| \left[\int_{-\infty}^{\infty} S_o(f) e^{j2\pi f T} df \right] \right|^2$$

$$SNR = \frac{|S_o(T)|^2}{P_{n_o}}$$

$$= \frac{\left| \int_{-\infty}^{\infty} H(f) S_i(f) e^{j2\pi f T} df \right|^2}{\frac{\eta}{2} \int_{-\infty}^{\infty} |H(f)|^2 df}$$

"Schwarz's inequality"

Schwarz's inequality states that "Square magnitude of total area under the product of two complex functions is always less than ~~of~~ or equal to the product of total area under the square magnitude of each of two functions"

and the equality is satisfied if and only if the first function is the complex conjugate of second."

$$\frac{\left|\int_{-\infty}^{\infty} H(f) S_i(f) e^{j2\pi f T} df\right|^2}{\frac{\eta}{2}\int_{-\infty}^{\infty} |H(f)|^2 df} \leq \frac{\int_{-\infty}^{\infty} |H(f)|^2 df \int_{-\infty}^{\infty} |S_i(f) e^{j2\pi f T}|^2 df}{\frac{\eta}{2}\int_{-\infty}^{\infty} |H(f)|^2 df}$$

$$SNR|_{max} = \frac{2}{\eta}\int_{-\infty}^{\infty} |S_i(f)|^2 df$$

$$H(f) = \left[S_i(f) e^{j2\pi f T}\right]^*$$

$$H(f) = S_i^*(f) e^{-j2\pi f T}$$

$$h(t) = \int_{-\infty}^{\infty} S_i^*(f) e^{-j2\pi f T} e^{j2\pi f t} df$$

$$h(t) = \int_{-\infty}^{\infty} S_i^*(f) e^{-j2\pi f (T-t)} df$$

$$h(t) = \int_{-\infty}^{\infty} S_i(-f) e^{-j2\pi f (T-t)} df$$

$$\boxed{h(t) = S_i(T-t)}$$

$$s_i(t) = \int_{-\infty}^{\infty} S_i(f)\, e^{j2\pi f t}\, df$$

replace t by $T-t$

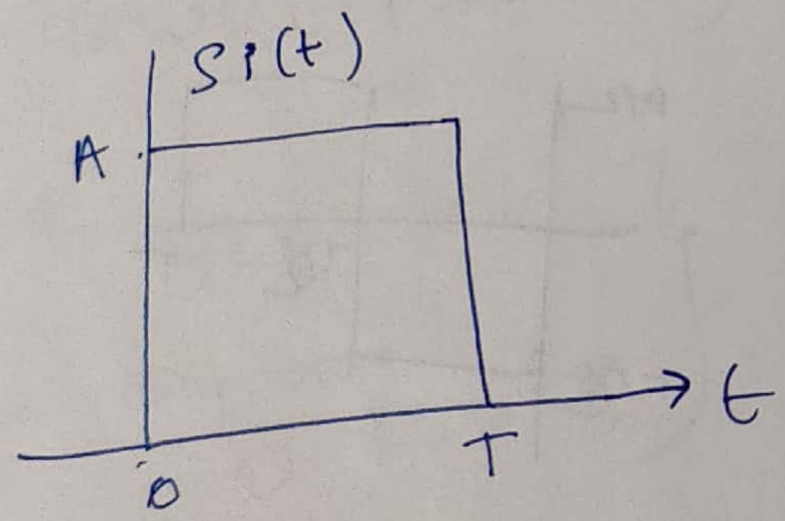
$$s_i(T-t) = \int_{-\infty}^{\infty} S_i(f)\, e^{j2\pi f (T-t)}\, df$$

$f = -f$

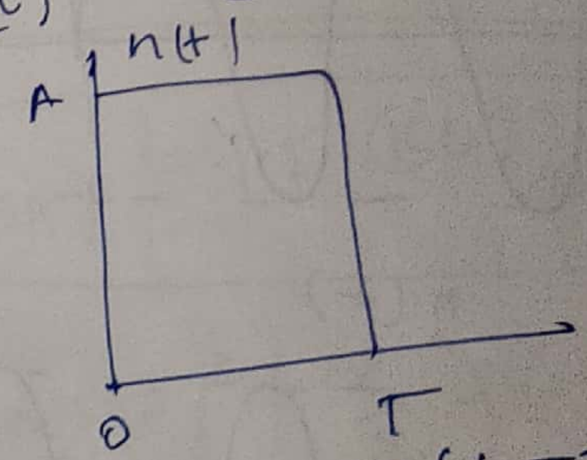
$df = -df$

$$\boxed{s_i(T-t) = \int_{-\infty}^{\infty} S_i(-f)\, e^{-j2\pi f (T-t)}\, df}$$

Q



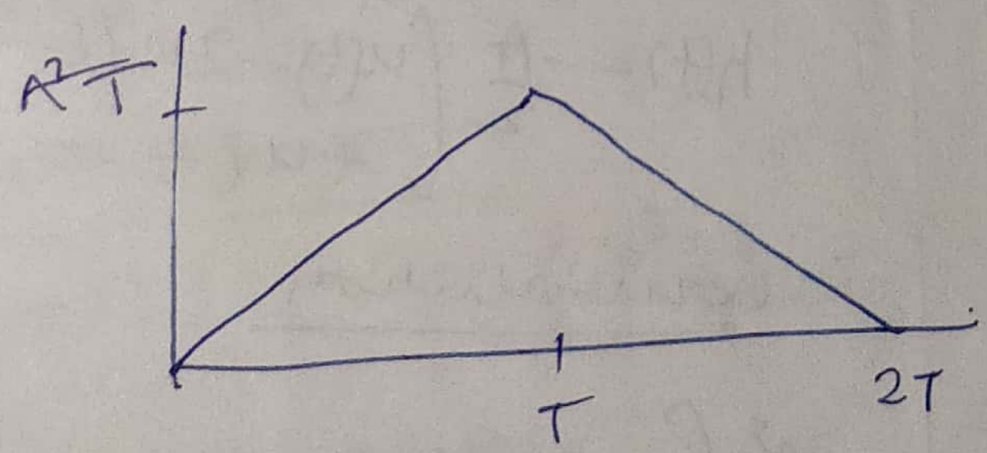
$h(t) = s(-t+T)$



$s_i(t) = A(u(t) - u(t-T))$

$s_i(t) * h(t) \longrightarrow = A^2 [u(t) - u(t-T)][u(t) - u(t-T)]$

$= A^2 (r(t) - 2r(t-T) + r(t-2T))$



(6)

$s(t) = 10\sin(2\pi \times 10^6)$

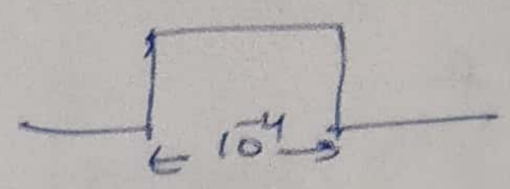
$; 0 < t < 10^{-4}$ sec

$A = 10$

$T = 10^{-4}$

$A^2T = 10^{-2}$

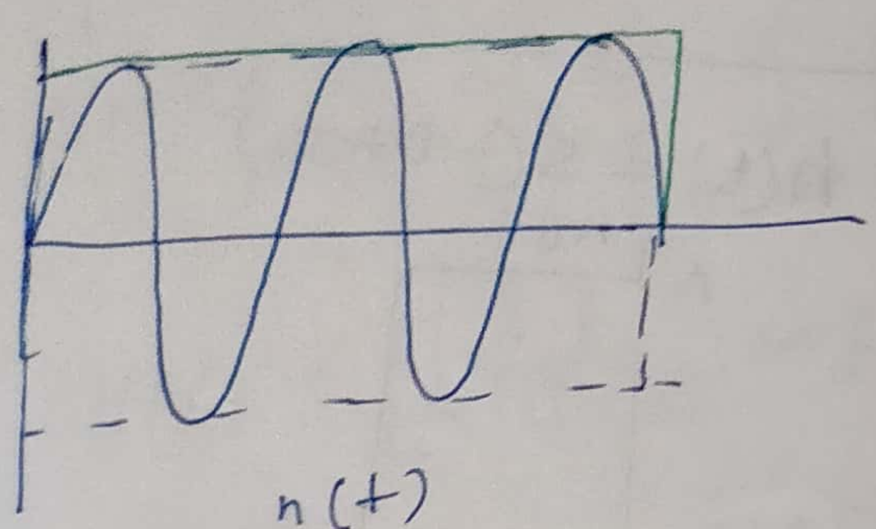
$= 10\,mV$



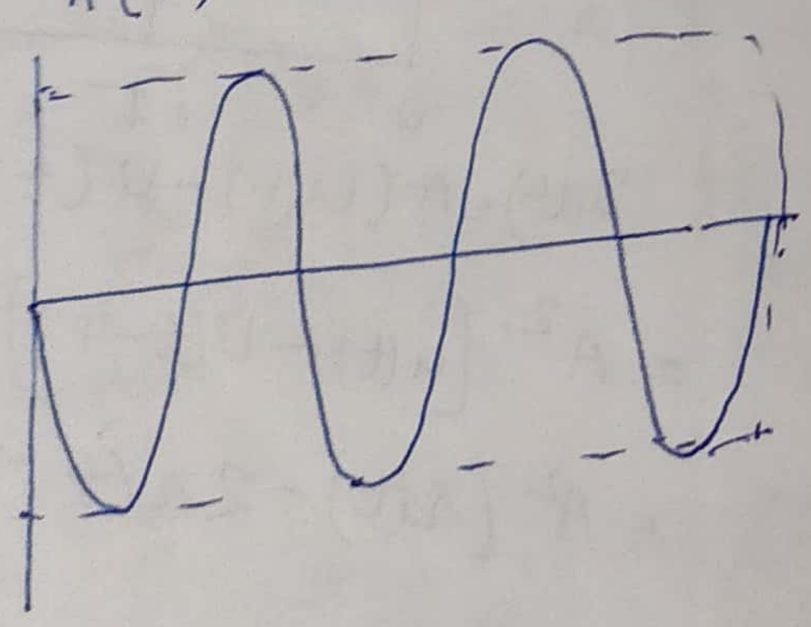
$f_c = 10^4\,Hz$

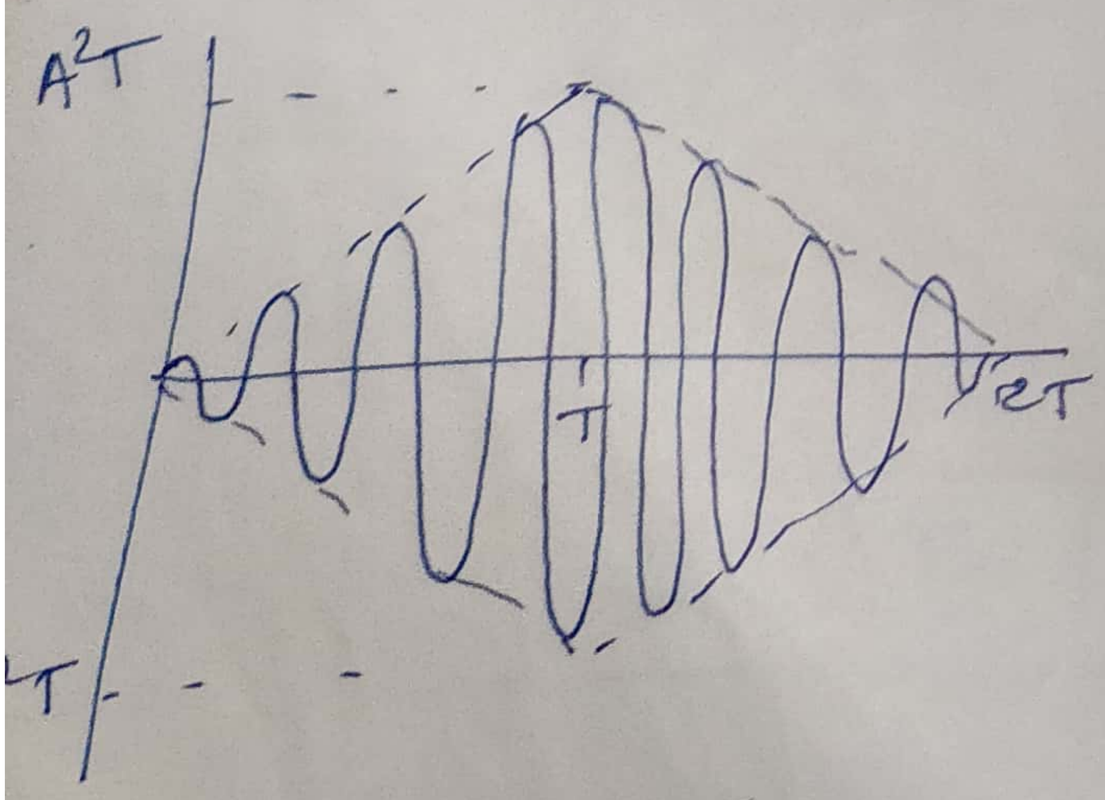
$f_c = 10\,kHz$

$f_c = 1\,MHz$

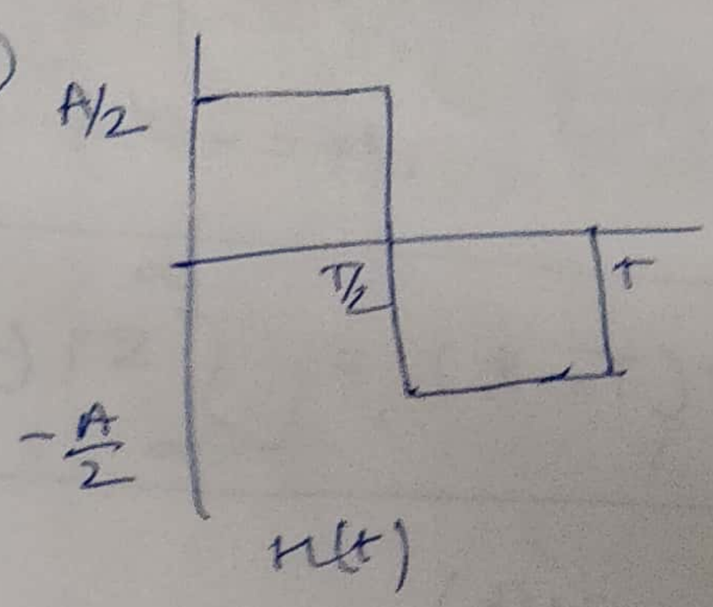


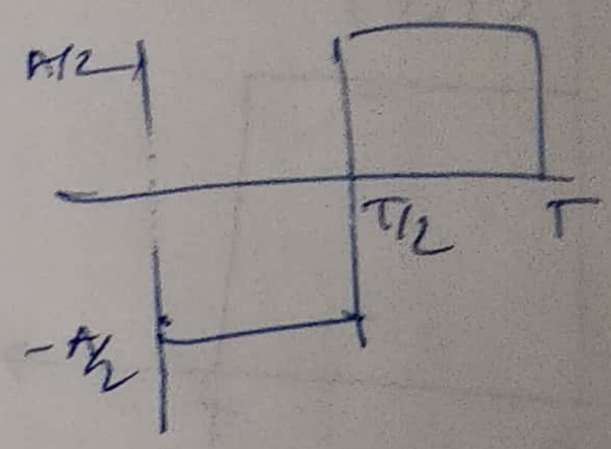
$n(t)$





(13)



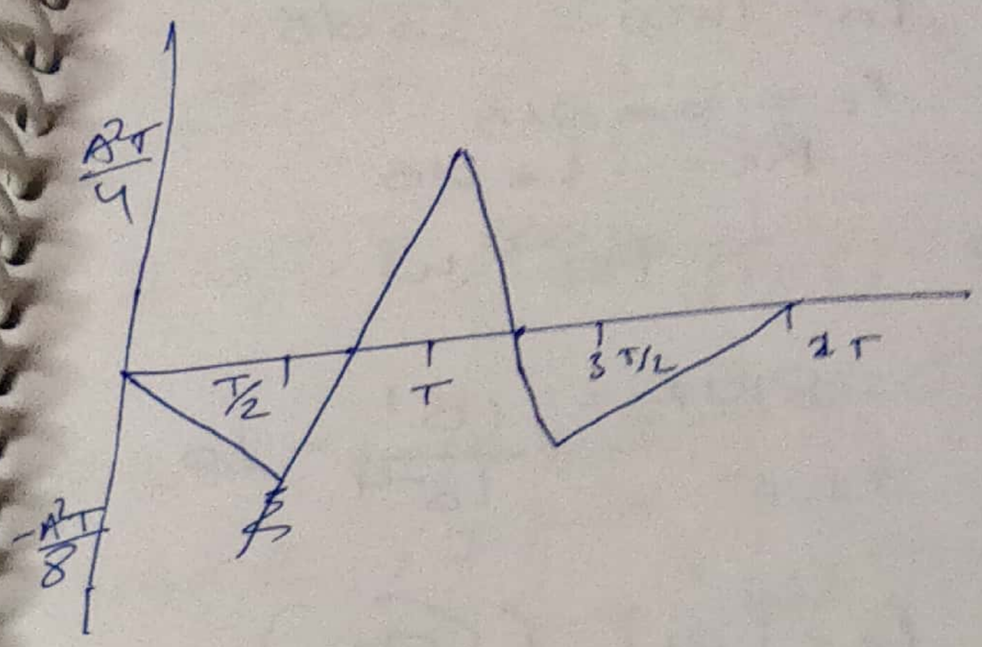


$$s(t) = \frac{A}{2}\left[u(t) - 2u(t - T/2) + u(t - T)\right]$$

$$h(t) = -\frac{A}{2}\left[u(t) - 2u(t - T/2) + u(t - T)\right]$$

<u>Convolution</u>

$$-\frac{A^2}{4}\left[r(t) - 4r(t - T/2) + 6r(t - T) - 4r(t - 3T/2) + r(t - 2T)\right]$$



(14) $P(t) = \dfrac{\sin 4\pi\omega t}{4\pi\omega t(1-16\omega^2t^2)}$

at $t = \frac{1}{4\omega}$

$= \dfrac{\sin\pi}{0} \quad \frac{0}{0}$

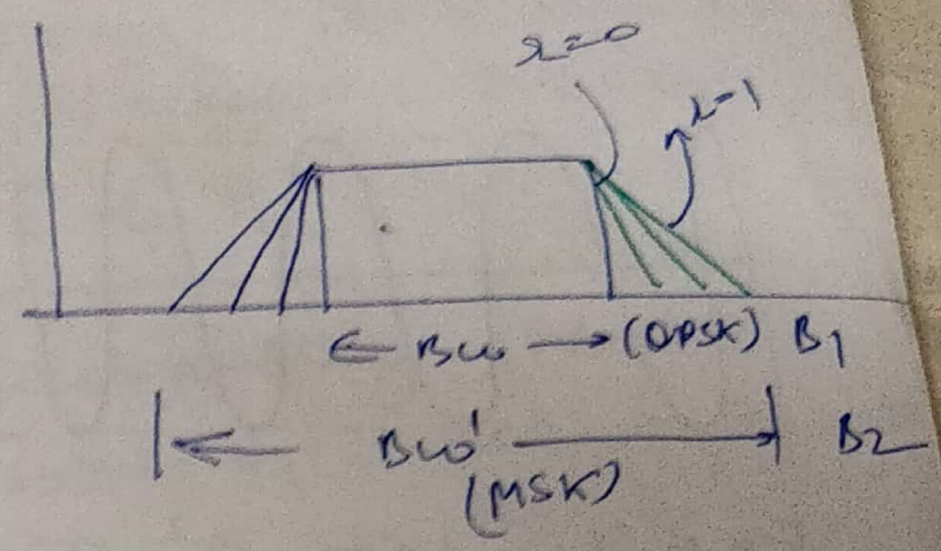
using LHR

$= \dfrac{4\pi\omega\cos 4\pi\omega t}{4\pi\omega - 3\times 64\pi\omega^3 t^2}$

$= \dfrac{4\pi\omega\cos\pi}{4\pi\omega\left(1 - \frac{3\times 64\pi\omega^3}{4\pi\omega}\times\frac{1}{16\omega^2}\right)}$

$= \dfrac{-1}{-2} = 0.5$

Unity Roll-off factor



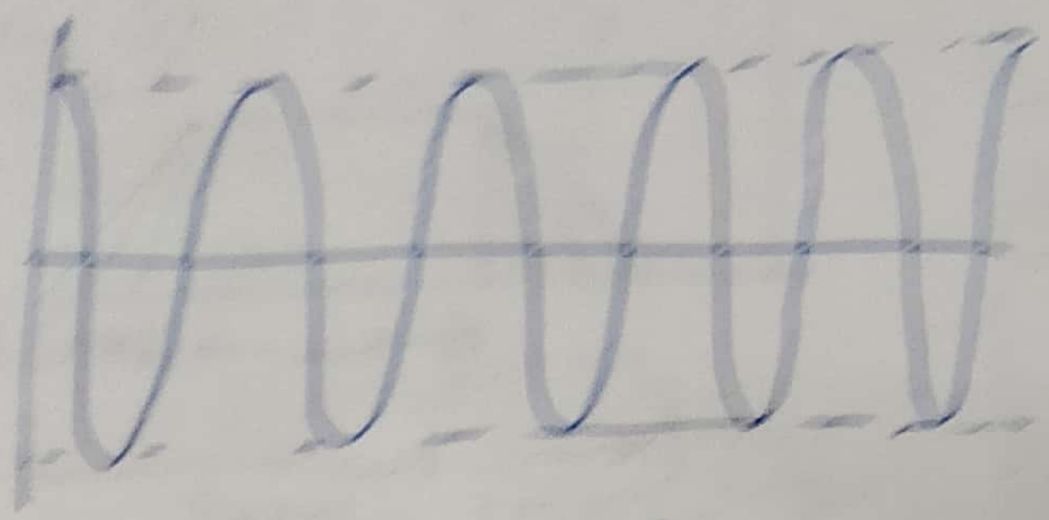
$$B_2 = \left(1 + \frac{r}{2}\right) B_1$$

$$BW_{(MSK)} = 1.5\,(BW)_{OPSK}$$

$$BW_{(MSK)} = \left(1 + \frac{r}{2}\right)(BW)_{OPSK}$$

(24) $v(t) = 100\sin 5\pi t$

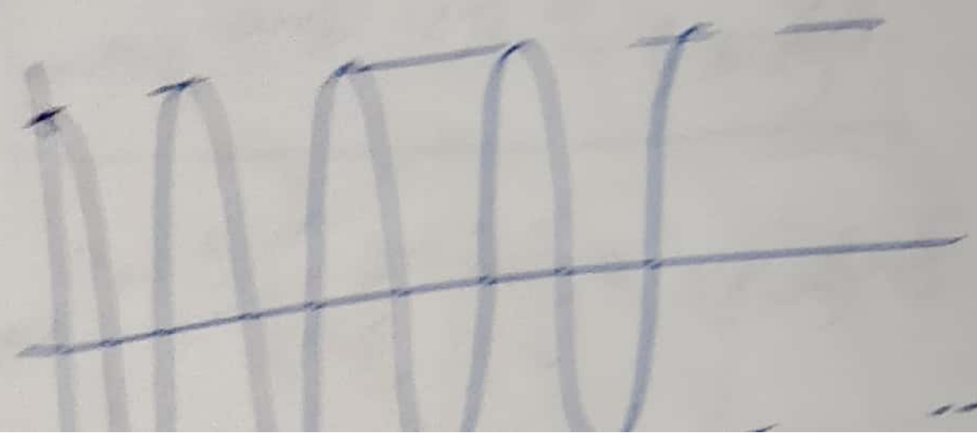
$f_c = \frac{5}{T}$



$t_0 = 10^{-3}\,s = \frac{\pi^2}{2} T$

$\frac{1}{2} T = 10^{-3}$

$T = 2\times10^{-3}$ sec



$P_{av} = 100 = 30\,dB$

$P_L = 60\,dB$

$P_{av} - 30\,dB$

$= 10^{-3}\,W$

$SNR = \frac{10^{-3}}{10^{-4}} = 10$

$P_e = \frac{1}{2} erfc\left(\sqrt{\frac{SNR}{2}}\right)$

$= \frac{1}{2} erfc\sqrt{10}$

$\frac{e^{-10}}{2\sqrt{10\pi}}$

$= 4.049\times10^{-6}$

Information theory & Coding

$$I(x_i) \propto \frac{1}{P(x_i)}$$

$$I(x_i) = \log_b \frac{1}{P(x_i)}$$

① when $b=10$, $I \rightarrow$ decit

$b=e$, $I \rightarrow$ "nat"

$b=2$, $I \rightarrow$ "bit"

$$\boxed{I_{x_i} = \log_2 \frac{1}{P(x_i)}} \text{ bits}$$

Properties of Information

① Information I is always non-negative

$$\boxed{I \geq 0}$$

② for a message with probability approaching its highest value 1, the amount of Information in it should approach its lowest value

③ for two different messages x_i & x_j with respective probabilities P_i & P_j, the one with lower probability should contain more information

i.e if $P_i < P_j$

then $I_i > I_j$

④ Total information conveyed by two messages will be sum of their respective information contents

i.e. $x_i \rightarrow I_i$

$x_j \rightarrow I_j$

$$\boxed{I = I_i + I_j}$$

$$= \log_2 \frac{1}{P_i} + \log_2 \frac{1}{P_j}$$

$$= \log_2 \left(\frac{1}{P_i P_j} \right)$$

Scanned by CamScanner

ENTROPY $H(X)$

In Any practical comm. system, we usually transmitt long sequence of messages from an information source. So we would be interested in average information that is source produces then information content of a single message. So Entropy is defined as Average information per message and is given by

$$H(X) = \sum_{i=1}^{M} P(x_i) \log_2 \frac{1}{P(x_i)}$$

— bits / message
— bits / symbol
— bits / sample

Consider only two messages with probability P & $1-P$

$$H(X) = P \log \frac{1}{P} + (1-P) \log \frac{1}{(1-P)}$$

$$= -P \log_2 P - (1-P) \log_2 (1-P)$$

$$\frac{dH(X)}{dP} = 0$$ for maximizing

$$= -\left[\cancel{P \frac{1}{P}} + 1 \log_2 P + \frac{\cancel{(1-P)}(-1)}{\cancel{(1-P)}} (-1) \log (1-P) \right] = 0$$

$$\log_2 P - \log (1-P) = 0$$

$$\log_2 \left(\frac{P}{1-P} \right) = 0$$

$$\frac{P}{1-P} = 1$$

$$P = 1-P$$

$$\boxed{P = \frac{1}{2}}$$

$$\boxed{1-P = \frac{1}{2}}$$

Scanned by CamScanner

"a msg source X generating M no. of messages will provide max. average information per message that is max. Entropy when all messages are equally probable or equally likely

$$H(X)|_{max} = \frac{1}{M}\log_2 M + \frac{1}{M}\log_2 M + \cdots + \frac{1}{M}\log_2 M$$

$$H(X)|_{max} = M \times \frac{1}{M}\log_2 M$$

$$\boxed{H(X)|_{max} = \log_2 M}$$

Information rate (R)

Let r = no of messages generated per sec

$H(X)$ = average information per message

$R = r\,H(X)$ bits/sec

$$\boxed{R_{max} = r\,H(X)|_{max}}$$

Q. An analog sig is band limited to BHz, sample at nyquist rate & samples are quantised into 4 levels. the quantisation levels q_1, q_2, q_3, q_4 are assumed $1/8$ $3/8$ $3/8$ $1/8$ independent and occur with probability $\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$

(1) Information rate

(2).

$$H(X) = \left(\frac{1}{8}\log_2 8\right)2 + \frac{3}{8}\left(\log \frac{8}{3}\right)\times 2$$

$H(X) = 1.8$ bits/sec

$r = 2B$ sample

$K = 3.68\,B$ bits/sec

$K_{max} = 4B$ bits/sec

Source coding

if the messages are not equally likely then it is not advisable to encode each msg. with same no. of bits. msges must be assigned bits according to their respective probabilities. That is high probability msgs should by assigned lesser no. of bits and vice versa this is performed by source coding. There are two source coding techniques

(1) Shanon fano coding

(2) Huffman coding

Shanon fano coding algo.

Consider five msges with their respective probabilities $\frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{16}, \frac{1}{2}$

		no of bits
$m_5 = 0.5$	0	1
$m_3 = 0.25$	1 0	2
$m_2 = 0.125$	1 1 0	3
$m_1 = 0.0625$	1 1 1 0	4
$m_4 = 0.0625$	1 1 1 1	4

Coding efficiency η

it is defined as the ratio of entropy to average code word length and is given by

$$\eta = \frac{H(X)}{\bar{L}} \times 100\%$$

$$\bar{L} = \sum_{i=1}^{n} p_i n_i$$

$$H(X) = \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4 + \frac{1}{8}\log_2 8 + \left(\frac{1}{16}\log_2 16\right) \times 2$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{1}{2}$$

$$= 1.875 + 0.375$$

$$= 1.875 \text{ bits/message}$$

$$\bar{L} = \frac{1}{2} + 2 \times 0.25 + 3 \times 0.125 + \left(4 \times \frac{1}{16}\right) 2$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{1}{2}$$

$$= 1.875 \text{ bit/message}$$

$$\eta = \frac{1.875}{1.875} \times 100\%$$

$$\boxed{\eta = 100\%}$$

Q Consider 5 messages with their resp. probabilities 0.4, 0.2, 0.2, 0.1, 0.1

0.4	0	0	2
0.2	0	1	2
0.2	1	0	2
0.1	1	1	3
0.1	1	1	3

0.2 ↑ 2.2 ↓ 0.8

12 bits

$$H(X) = 0.4 \log_2 \frac{1}{0.4} \rightarrow 0.528$$

$$+ \left(0.2 \log \frac{1}{0.2}\right) 2 \rightarrow 0.928$$

$$+ \left(0.1 \log_2 \frac{1}{0.1}\right) 2 \rightarrow 0.664$$

$$= 2.120 \text{ bits/message}$$

$$\bar{L} = 2 \times 0.4 + 2 \times 0.2 + 2 \times 0.2 + 3 \times 0.1 + 3 \times 0.1$$

$$= 2.2 \text{ bits/sec}$$

$$\boxed{\eta = 96.45\%}$$

age

Tuneis
,0.2
.1

0.2

--2.2

0.8

528

28

4

0.4	0
0.2	1 0
0.2	1 1 0
0.1	1 1 1 0
0.1	1 1 1 1

1
2 - - 2.2 ↑ 1.2
3
4 ↓ 1.8
4

spreading along mean

14 bits

Huffman Coding

0.4, 0.2, 0.2, 0.1, 0.1

less — more

0.4 0.4 0.4 0.6 0
0.2 0.2 0.4 0.4 1
0.2 0.2 0.2 1
0.1 0 1
0.1 1

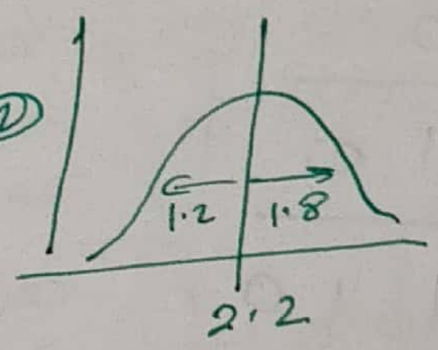
		no. of bits
0.4 →	00	2
0.2 →	10	2
0.2 →	11	2
0.1 →	010	3
0.1 →	011	3

$H(X) = 2.122$

$\bar{L} = 0.4 + 0.2\times2 + 0.2\times3 + 0.1\times4 \times 0.1\times4$

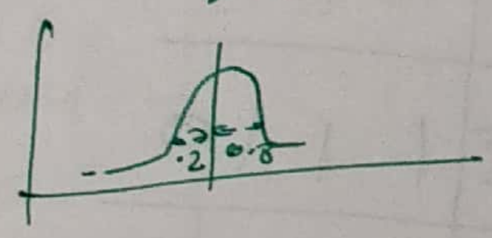
$= \underline{\underline{2.2}}$

②



0.2	00
0.2	01
0.1	110
0.1	111

①



$0.4 + 0.6 + 0.6 + .3 + .3$

$= \underline{\underline{2.2}}$

0.4 0.4 0.4 0.6 0
0.2 0.2 0.4 0.4 1
0.2 0.2 0.2 1
0.1 0 0.2 1
0.1 1

Variance of Average code word length is given by

$$\sigma^2 = \sum_{i=1}^{K} P_i (l_K - \bar{L})^2$$

method ① 0.016 + 0.016

$\sigma^2 = 0.4(2-2.2)^2 + 0.2(2-2.2)^2\times2 + 0.1(3-2.2)^2\times2$

$= 0.16$

		no. of bits
0.4 →	1	1
0.2 →	01	2
0.2 →	000	3
0.1 →	0010	4
0.1 →	0011	4

method ②

$\sigma^2 = 0.4(1-2.2)^2 + 0.2(2-2.2)^2 + 0.2(3-2.2)^2 + 0.1\times2(4-2.2)^2$

$= 1.36$

Q1) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \frac{1}{256}$

$$H(X) = \frac{1}{2} + \frac{1}{4}\times 2 + \frac{1}{8}\times 3 + \frac{1}{16}\times 4 + \frac{1}{32}\times 5 + \frac{6}{64} + \frac{7}{128} + \frac{8}{256}$$

$= 1.96$ bits/symbol

$000 \rightarrow \frac{1}{2}$

$001 \rightarrow \left(\frac{1}{12} + \frac{1}{12} + \frac{1}{12}\right)$ for $0 \Rightarrow \frac{1}{6}$

$010 \rightarrow$

Prob. occurr of occurr of 0

$\frac{1}{2} \rightarrow 000 \rightarrow \frac{1}{2}$ Pro 0 $\frac{1}{2}$

$\frac{1}{4} \rightarrow 001 \rightarrow \left(\frac{1}{12}, \frac{1}{12}, \frac{1}{12}\right) \frac{1}{6}$

$\frac{1}{8} \rightarrow 010 \rightarrow \left(\frac{1}{24}, \frac{1}{24}, \frac{1}{24}\right) \cdot \frac{1}{12}$

$\frac{1}{16} \rightarrow 011 \rightarrow \left(\frac{1}{48}, \frac{1}{48}, \frac{1}{48}\right) \frac{1}{48}$

$\frac{1}{32} \rightarrow 100 \rightarrow \left(\frac{1}{96}, \frac{1}{96}, \frac{1}{96}\right) \frac{1}{48}$

$\frac{1}{64} \rightarrow 101$

$\frac{1}{128} \rightarrow 110$

$\frac{1}{256} \rightarrow 110$

$\frac{1}{2}$	0	1
$\frac{1}{4}$	1 0	2
$\frac{1}{8}$	1 1 0	3
$\frac{1}{16}$	1 1 1 0	4
$\frac{1}{32}$	1 1 1 1 0	5
$\frac{1}{64}$	1 1 1 1 1 0	6
$\frac{1}{128}$	1 1 1 1 1 1 0	7
$\frac{1}{256}$	1 1 1 1 1 1 1	7

Pro 1 = 0

0.8

total = 0.2

$$\bar{L} = 3(0.996)$$
$$= 2.988$$
$$\simeq 2.99 \text{ bits/symbol}$$

$$\eta(X) = \frac{1.96}{2.99}$$
$$= 65.55\%$$

$$\bar{L} = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \frac{5}{32} + \frac{6}{64} + \frac{7}{128} + \frac{7}{256}$$

$= 1.957$

$$\eta = \frac{1.96}{1.957} \times 100\% = 100.15\%$$

Q2

0.5

0.4

0.1

Q2 0.5, 0.4, 0.1

0.5	0
0.4	10
0.1	11

$L = 0.5 + \underbrace{0.4}_{0.52} +$

$= 1.352$ bits

$\bar{L} = 1.5$ bits/sec

$\eta = 90.7\%$

Second order extension Code

for $m_1 m_2 m_3$

$m_1 m_1$ – 0.25
$m_1 m_2$ – 0.20
$m_1 m_3$ – 0.05 } → 0.5

$m_2 m_1$ – 0.20
$m_2 m_2$ – 0.16
$m_2 m_3$ – 0.04 } → 0.4

$m_3 m_1$ – 0.05
$m_3 m_2$ – 0.04
$m_3 m_3$ – 0.01 } → 0.1

Capacity of a gaussian channel

According to Shannon Hartley law, the capacity of a white band limited gaussian channel is given by

$$\boxed{C = B\log_2\left(1 + \frac{S}{N}\right)}$$

$\frac{S}{N}$ → SNR B – channel BW.

$N = \underbrace{KT}_{\eta}B = \eta B$

C – max (error free) transmission rate

Bandwidth-SNR trade off

The Shannon Hartley theorem indicates that a noise less gaussian channel has an infinite capacity on the other hand, while the channel capacity does increase, it does not become infinite because with an increase in BW the Noise power also increases.

Thus for a fixed s/g power and in the presence of white gaussian noise the channel capacity approaches an upper limit with increasing BW.

$$C = B\log\left(1 + \frac{S}{\eta B}\right)$$

$$= \frac{S}{\eta}\left(\frac{\eta B}{S}\log\left(1 + \frac{S}{\eta B}\right)\right)$$

$$= \frac{S}{\eta}\log_2\left(1 + \frac{S}{\eta B}\right)^{\frac{\eta B}{S}}$$

$B \to \infty$

$$\left(1 + x\right)^{1/x} = e$$

$x \to 0$

$$C_{max} = \frac{S}{\eta} \log_2 e$$

$$\boxed{C_{max} = 1.44 \frac{S}{\eta}}$$

$$R \leq C$$

$$\boxed{R_{max} = C}$$

Channel coding

Hamming distance

total no. of successive bits by which two code words differ is known as hamming distance

$C_1 = 0101$
$C_2 = 1010$
$C_3 = 1111$

$d_{min} = 2$

Hamming weight

total no. of 1's present in the code word gives the hamming weight of the code word

Properties of Linear code

0000, 1010, 0101, 1111

① the sum of two code words belonging to the code is also a code word belonging to the code.

$1010 \oplus 0101 \rightarrow 1111$ (XOR)

② the all '0' word is always a code word

③ The minimum hamming distance is equal to minimum hamming weight

Linear Block codes

Data Bits	Parity Bits

$\leftarrow k \rightarrow \leftarrow r \rightarrow$

$\leftarrow n = k + r \rightarrow$

Parity check Matrix H

$$H = \left[P^T_{r \times k} \vdots I_{r \times r} \right]_{r \times n}$$

Generator Matrix G

$$G = \left[I_{k \times k} \vdots P_{k \times r} \right]_{k \times n}$$

Q generate linear block code of size 7,4 with parity check matrix given by

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$\boxed{C = DG}$$

$D = 1010$

$$[1010] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$[1\ 0\ 1\ 0\ 0\ 1\ 0]$$

$1 \oplus 1$

$1 \oplus 0$

$D = 1010$

Q $[1\ 1\ 0\ 1\ 0\ 1\ 0]$

$C = 1101010$

$Rec. = [1101011]$

$$[1\ 1\ 0\ 1\ 0\ 1\ 1] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= [0\ 0\ 1]$$

at $[1110100]$

$[1\ 0\ 1]$ at 3rd bit

Minimum Hamming distance	Error control Capability
1	error can not be detected
2	Single error detection
3	Single error correction
4	Single error correction + Double error detection
5	Double error correction
6	Double error correction + Triple error detection

In general if d is the minimum hamming distance of a block ~~code~~ then

$\frac{d-1}{2}$ error can be corrected if d = odd

$\frac{d-2}{2}$ error can be corrected

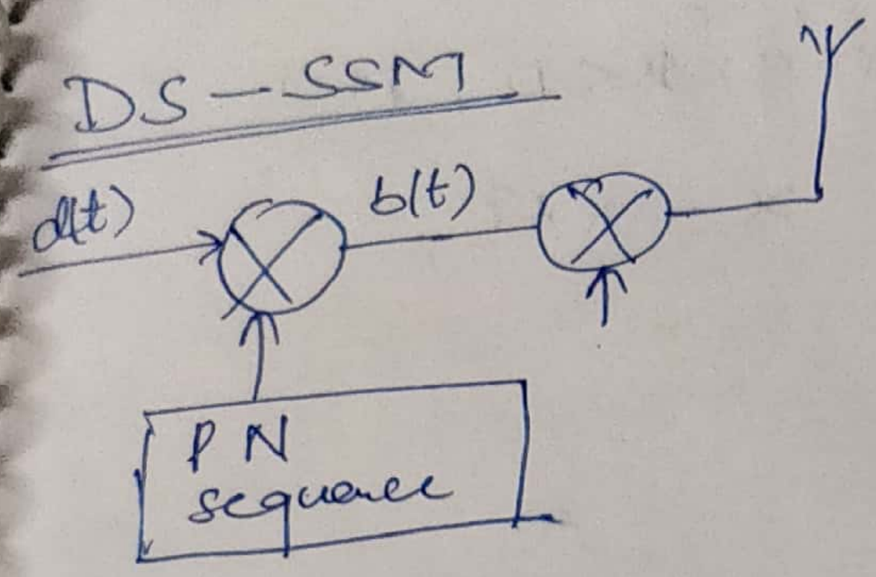
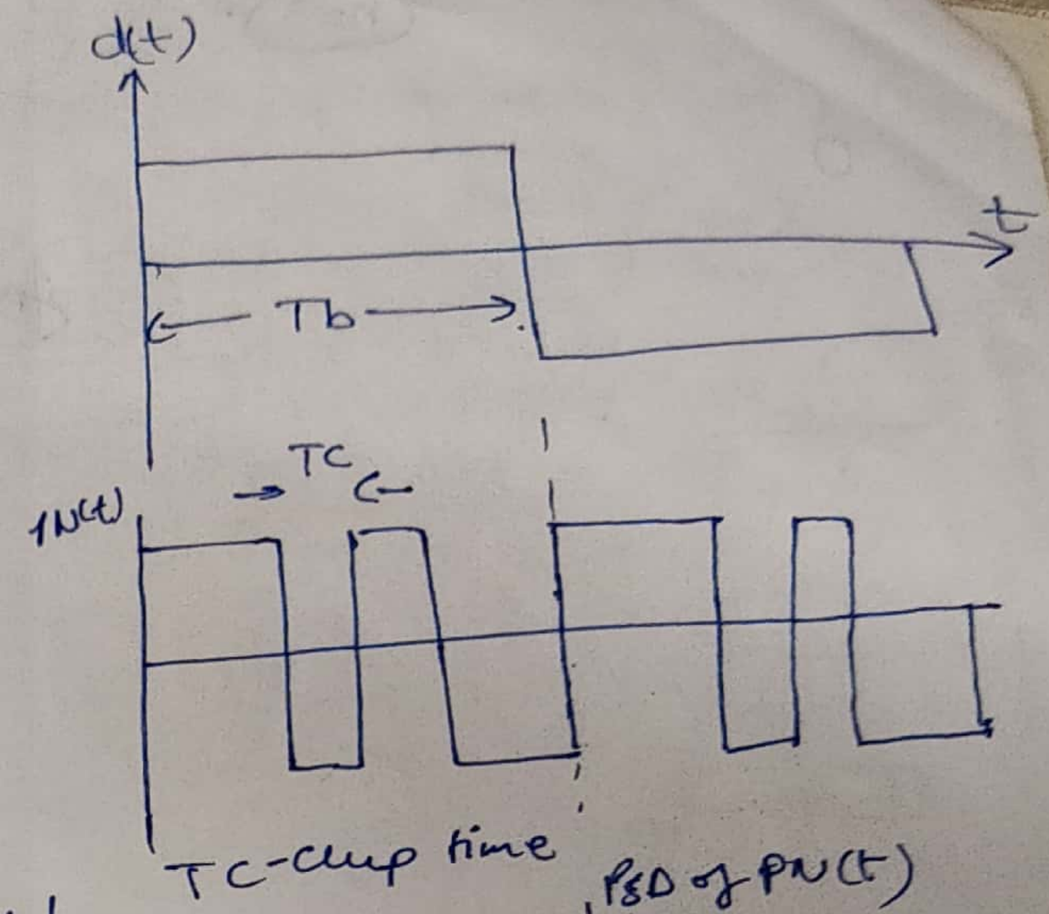
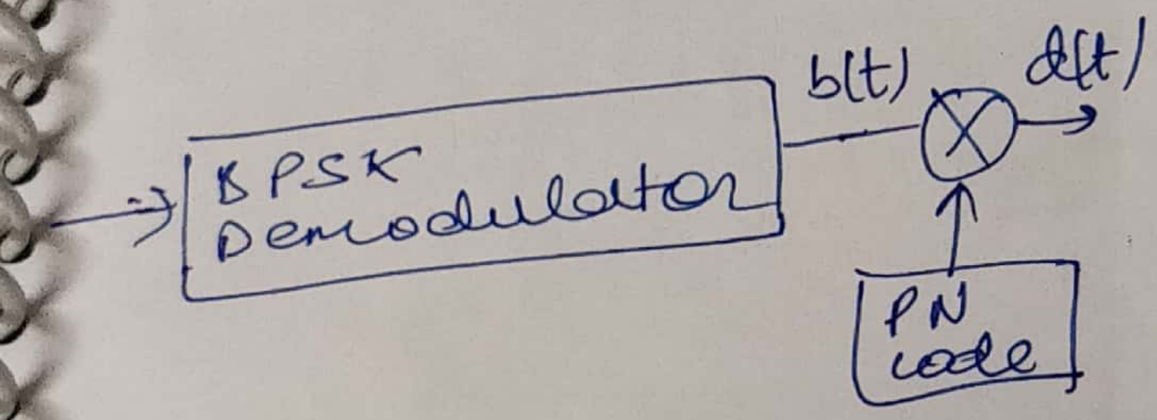
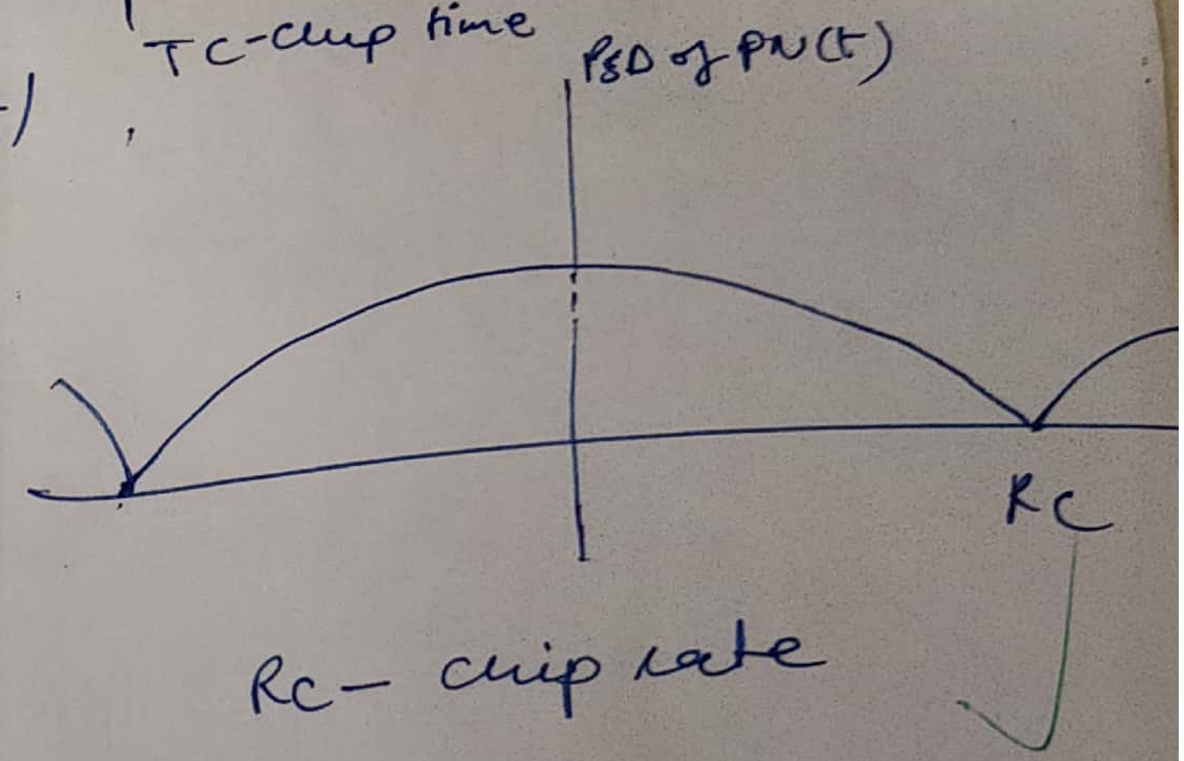
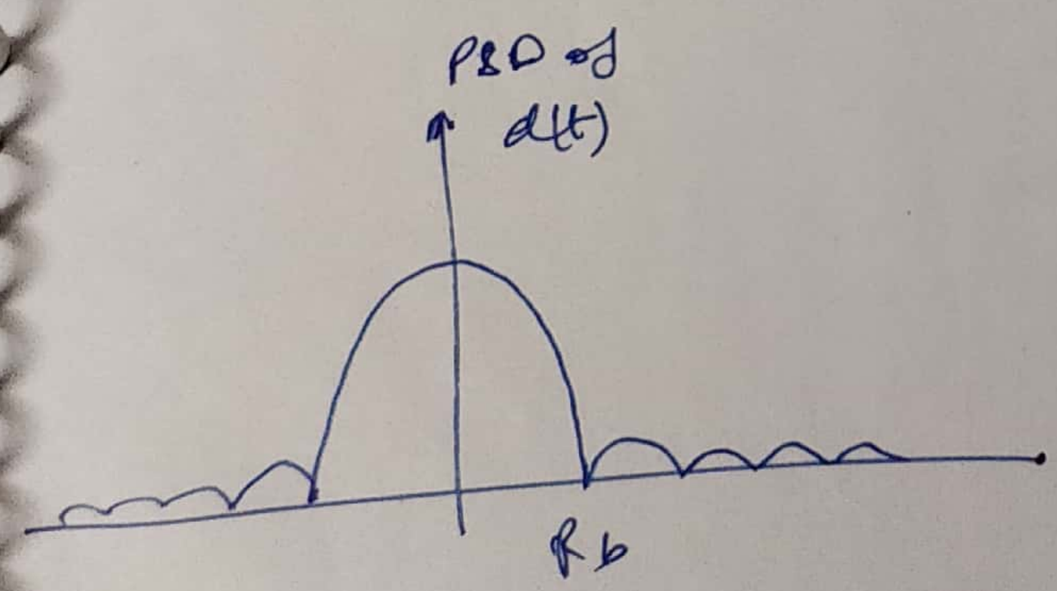
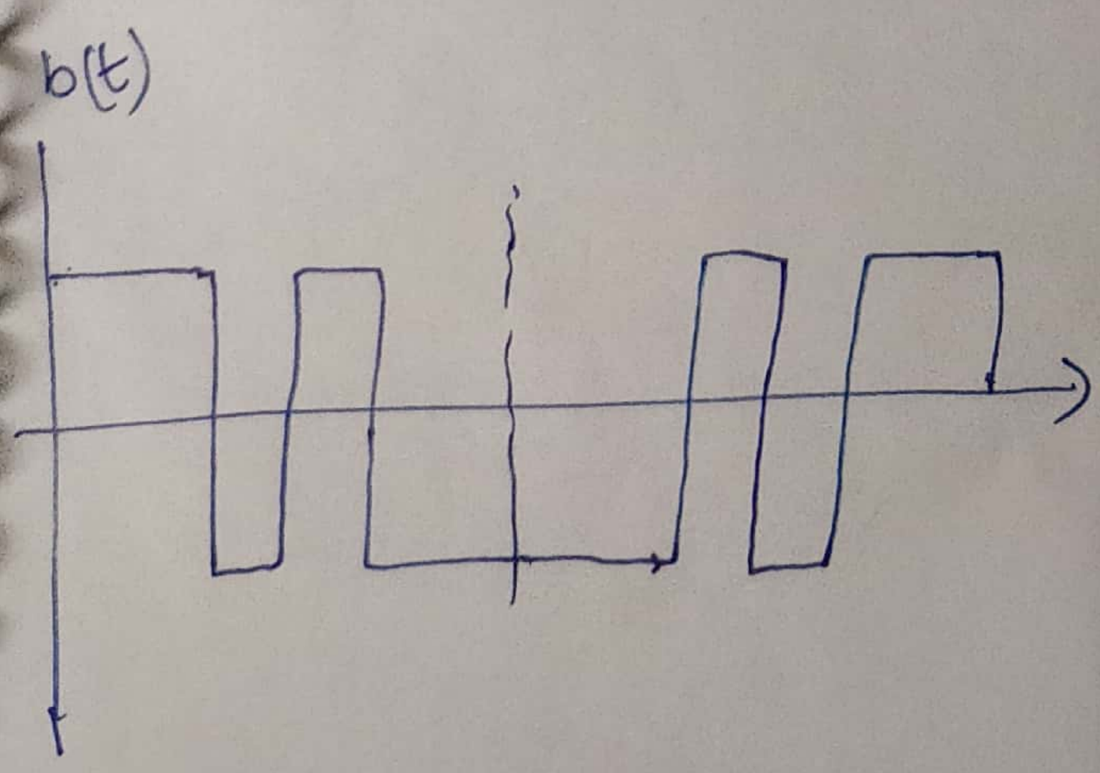
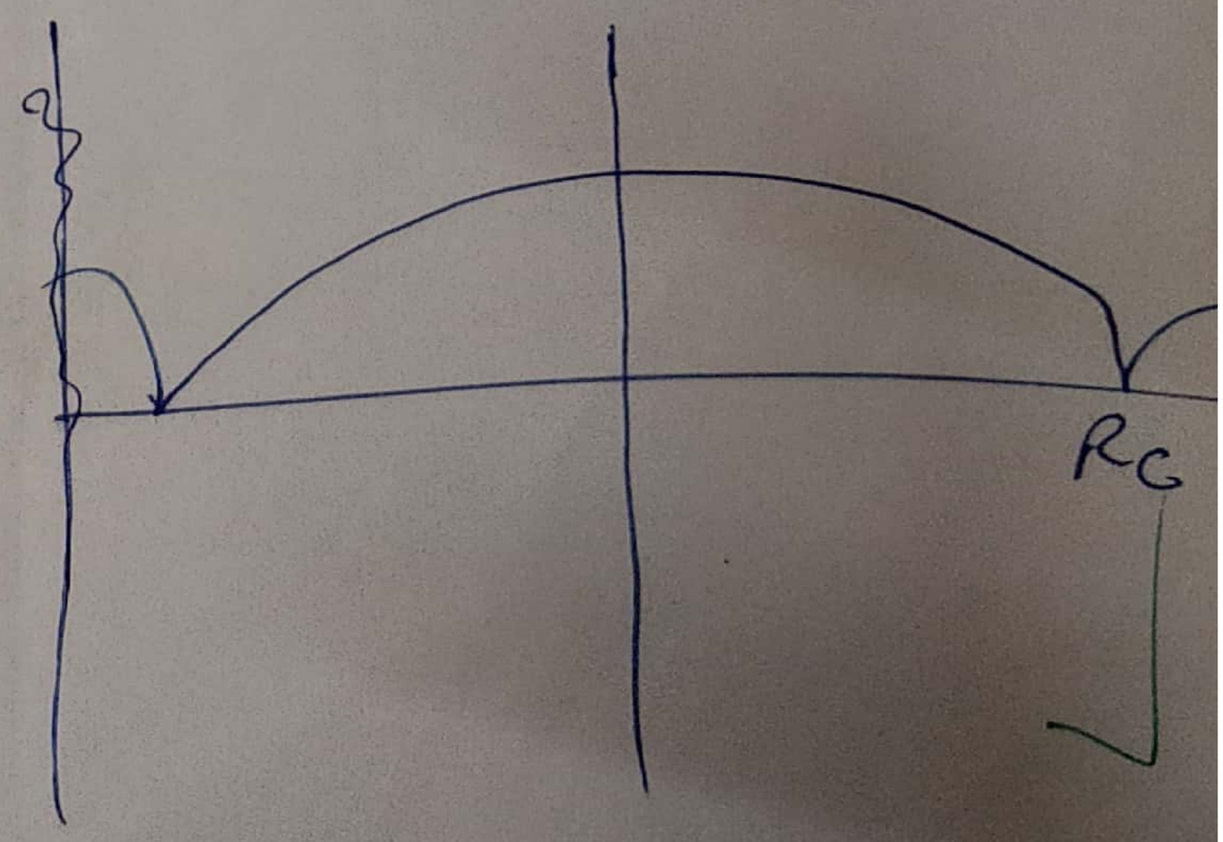
$\frac{d}{2}$ errors can be detected if d is even

Spread spectrum modulation

a Tx'sion technique in which Pseudo noise code (PN code) independent of information data is employed as a modulation waveform to spread the s/g energy over a BW much greater than the s/g information BW

At the RX, the s/g is despread using a ~~seq~~ synchronised Replica of PN code.

there are two types of SSM's

① Direct sequence SSM
② Frequency Hopped spread S.M.

DS – SSM