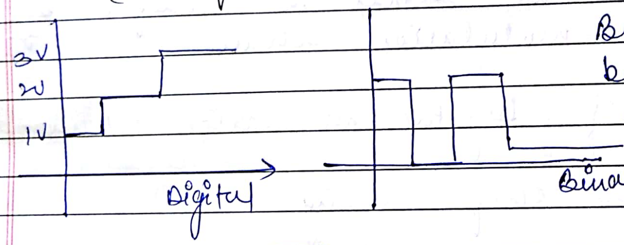
Digital communication

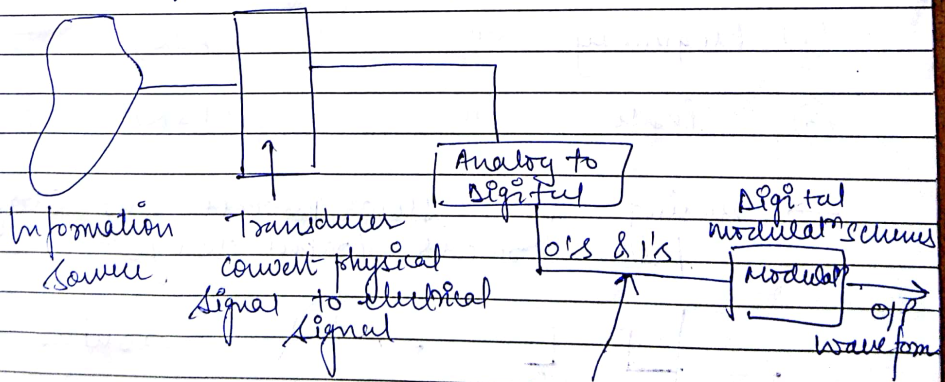
Digital includes binary.
Digital communication deals with physical layer.

Advantages. Programmable, reliable, less noise, hardwares are less as they are embodied inside IC, high S/N, Reconfigurable. It is general purpose.

Block diagram



Binary W/F can be digital but digital W/F may not be Bin.
↑
waveform



The input is analog but follows the command of digital system

Analog signals are sended to the channel.

Digital modulatn scheme converts the binary signal in a form suitable to be transmitted.

$T = 10^{-9}$ sec, $f = 10^{4}$ GHz and at-top freq is 0 ∴ lot of variation is there.

∴ continuous signal is modifies by Digital modulation scheme.

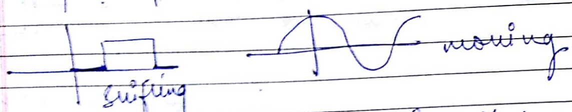
Types of Digital modulation scheme are -

① Amplitude shift keying (ASK)

② Frequency " " (FSK)

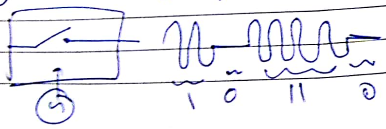
③ Phase " " (PSK)

→ Moving is slow process and shift is a fast process (rapid change)

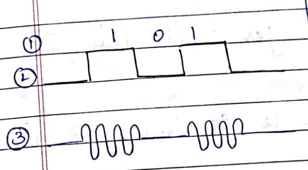


keying → security, security of lock

ASK 10110 →



Line Coding



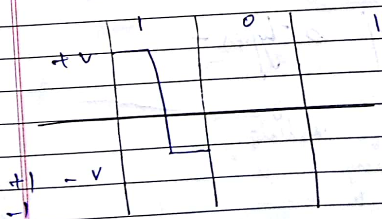
unipolar NRZ (non return to zero)

① Power requirement.
② Bandwidth
③ Detection
④ Probability of error
⑤ signal to noise ratio must be better (S/N).
⑥ Complexity (system must be complex at min level / signal).

There are different rules of reproducing byte code 0,1 for NRZ.

Split phase Manchester encoding

Mechanism of line coding should be so as to free more AC component and decrease the dc component.



1 → will change from high to low.

0 → will change from low to high.

Sampling - It is a process of converting continuous analog signal into a discrete signal.

No. of samples obtain per unit time is sample rate.

$f_s \rightarrow$ sampling frequency - It is measure in hertz. (Hz)
Frequency is the rate.

If 1MHz frequency then we are taking 1M samples per unit time (in one second)

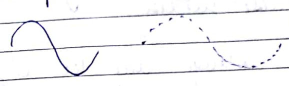
Sampling theorem says that if the max. frequency in the given signal is f_m then to reproduce the originality of the signal at the receiver the sampling rate (f_s) must follow the following condition -

$$\boxed{f_s \geq 2 f_m}$$

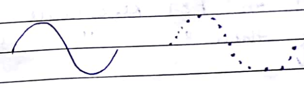
Sampling is of 3 types -

(1) Natural Sampling
(2) Instantaneous Sampling
(3) Flat Top Sampling

→ If sample is matching with original sample then it is natural sampling.



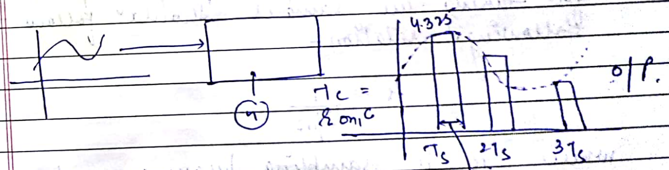
→ When sample are in the dotted form then that is a instantaneous sampling

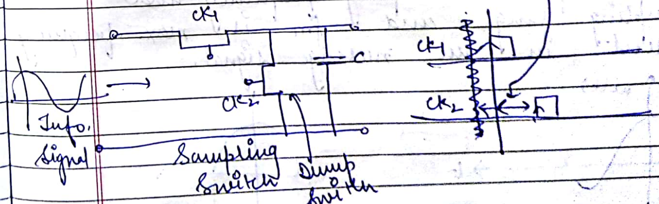


→ Sampling is a process and a sampler is a device which consists of switch.

Flat top sampling

T_s - It tells us above the next interval means at what interval it will take next sample.
As soon as it comes switch will turn on.





→ first ka leading edge aya tou charge ho gaya and second par discharge ho gaya. It is a pulse width.

→ Next sample ki voltage lene ke liye first should be zero that's we are discharging.

→ when switch is on FET Resistance is zero and when off then " " " infinite

$f_s \geq 2f_m$

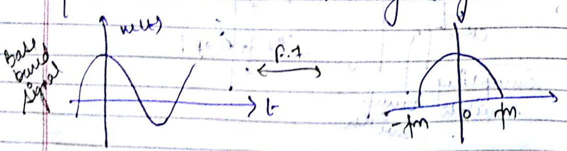
$T_s \geq \frac{1}{2f_m}$

$2f_m \geq \frac{1}{T_s}$

Sampling Theorem - This theorem states that the sampling frequency for taking the samples should follow the following relation -

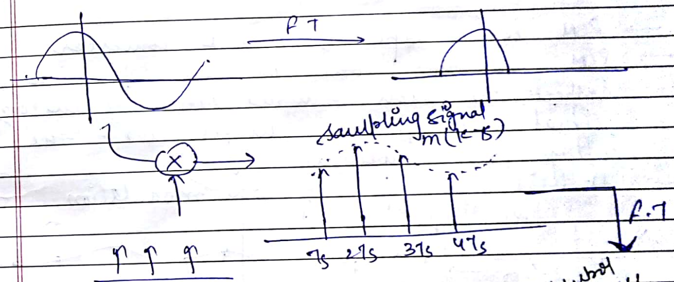
$f_s \geq \frac{1}{2f_m}$

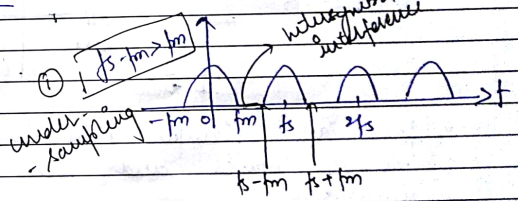
where f_s is sampling frequency and sampling range and f_m is max. frequency present in the message signal.



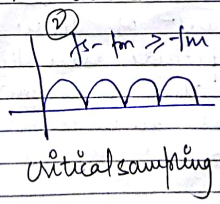
→ for calculating knowing the bandwidth of the signal we have to take the fourier transfer as to transfer the signal.

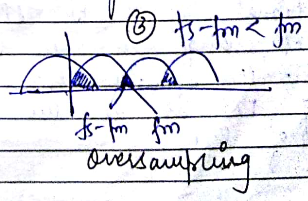
→ Base Band signal lend across zero frequency that's why known as base band.





To avoid this inter symbol interference -





Nyquist criteria

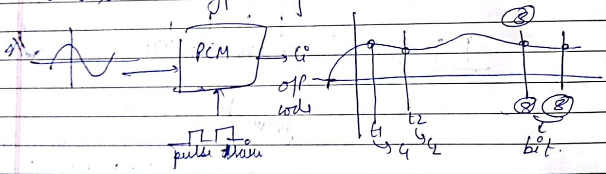
Mathematical Representatⁿ –

$$f_s - f_m \geq f_m$$

$$f_s \geq 2f_m$$

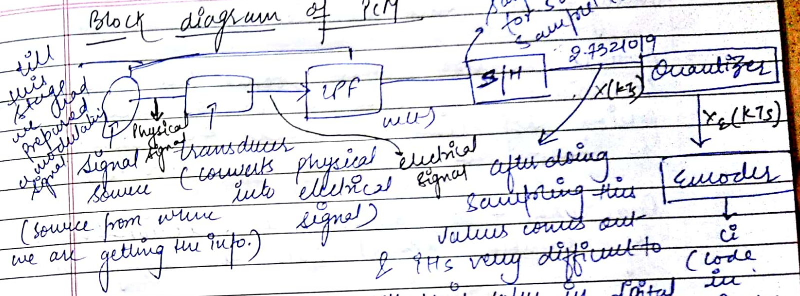
→ PCM stands for pulse code modulation

→ PCM is a type of A-to D converter
→ PCM gives a code in the O/P with respect to each sampled value of analog signal, the O/P code changes w.r.t the change in amplitude
→ It is a type of code modulation.



→ for each sample PCM transmits a fixed no. of waves and that is the code length.
→ even a voltage variation in a signal is not significant or there is no change in the amplitude even then the PCM will transmits the same no. of bits it is the biggest disadvantage of PCM.

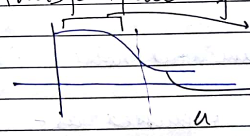
Block diagram of PCM



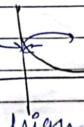
after doing sampling this values comes out & it's very difficult to convert this value in digital and to avoid this problem we are using quantizer block to round off it.

→ Encoder will give us the digital output.
→ LPF (low pass filter) – It will reduce the background noise and will pass the required signal lying in low frequency region.

Transfer functⁿ of LPF.



→ low pass frequency region. so it will pass.
→ but here will produce a lot of attenuation.



In this case time is very less i.e Δt in nano sec so therefore frequency __ ∴ known as high frequency region $\boxed{f = \frac{1}{\Delta t}}$

Sample & Hold – It samples the signal at sampling instance and hold that varying for some duration of the sample.

→ A-to-D converter takes some time to converting the sample so for that period of time it holds the sample so that no error can be produced.

Quantizer – It converts a fractional value of sample signal into the integer value.

→ The quantizer introduces an error which is called quantization error.

→ The true value is known as major value.
In case of 2.73 i.e the major value and error will be –ve as
2.73 – 3.00 = ~~-0~~.27
and in case of 3.01 error is +ve.
3.01 – 3.00 = 0.01

→ Quantizer is the fundamental noise of PCM.

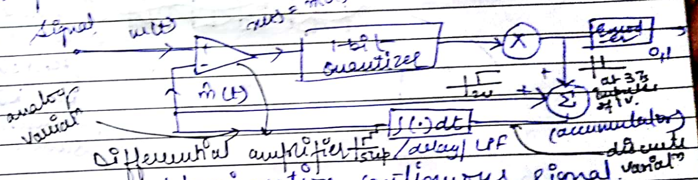
Quantization error is expressed as –

$$e_q = \underbrace{X(KT_s)}_{\text{sampled value (it may be a fractional value)}} - \underbrace{X_q(KT_s)}_{\text{quantized value (it is always in integers)}}$$

+ → non-inverting terminal
– → inverting terminal.

Delta Modulation (Δ-modulation)

1) It is a type of differential PCM.
2) It transmits smaller no. of bits as compare to PCM.



m̂(t) – approximation continuous signal

→ all the 3 i.e. integrator, delay and LPF can be used.

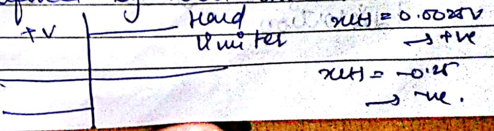
→ 1-bit quantizer – assign point values of x(t) into integer.

Integrator – It is an LPF. When a signal is pass through integrator it converts from analog to digital variation.

→ Delay provide by integrator which is T_s.
→ It also feedback.

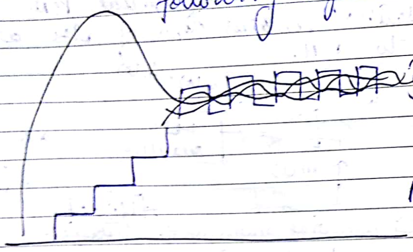
T_s → time at which sample should be taken.

1 bit quantizer have 2 values +ve & –ve. These are defined by hard limiter will assign value



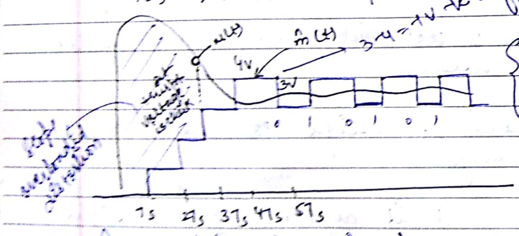
→ Impulse → step → ramp → parabolic after integration.

Δ-modulation behaviour is denoted by following signal -



}→ here so many signal gets changed but binary pattern remains same.

$\hat{m}(t)$ - staircase waveform



{ Binary pattern doesn't gets change due to signal variation }

Δ-modulation give 1 bit for every sample. So, Δ-modulation is much better than PCM as this Δ-modulation require less bandwidth.

If x>0 then reject it.

→ Signal tries to catch the staircase waveform but didn't able to catch it as there is a lot of difference and this problem is called slope overloaded distortion.

→ we can reduce slope overloaded distortion ~~by reducing~~ problem by ↑se in step size.

→ Encoder in Δ-modulation block diagram will give 0 or 1 in o/p and waveform will be like

→ Encoder convert +ve signal into 1 and negative signal into 0.

→ binary output of staircase waveform is 1111010101

Limitation of Δ-modulation

1) Slope overload distortion. - when the signal, slope rises sharply then the staircase approximation is not able to track the analog signal, hence information loss comes into this picture, this is called slope overload distortion.

2) Granular ~~Rise~~ noise - When signal variation is very much small then a noise will come which is called Granular noise.

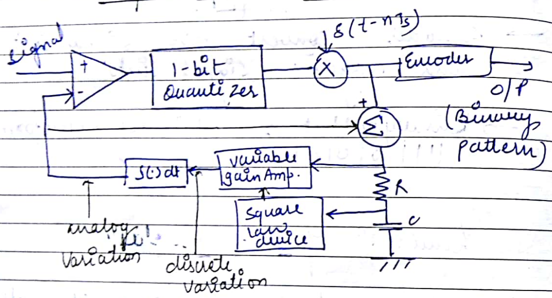
→ at 3Ts instant the value of staircase value is 20

→ The characteristic feature of Δ modulation is that it takes only 1 bit for each step of comparison.

→ If the difference is zero it will neglect.

→ Slope overloaded distortion - as the slope of $m(t)$ and $\hat{m}(t)$ differs a lot.

To avoid the limitation of Δ-modulation we use adaptive Delta modulator circuit.

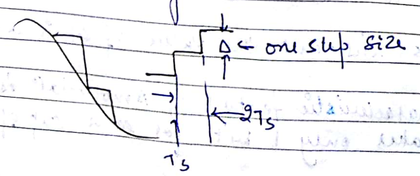


→ Square law device - check how much amplitude is varying.

→ Slope will be sensed by RC combination.

→ variable gain amplifier - ↑es the step size to avoid slope overload distortion.

→ Step stair case waveform - It is used to trace the signal.



Slope of stair case waveform -

$$\boxed{\text{slope} = \frac{\Delta y}{\Delta x} = \frac{\Delta}{T_s}}$$

Slope of the signal is obtained by differentiating the analog signal $m(t)$ -

$m(t) = A_m \sin \omega_m t$

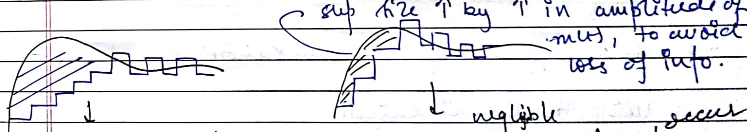
$$\left|\frac{dm(t)}{dt}\right| = A_m(\cos \omega_m t)\omega_m$$

$= \omega_m A_m \cos \omega_m t$

$= A_m \omega_m \cos \omega_m t$

so, magnitude of slope $= A_m \omega_m$

step size ↑ by ↑ in amplitude of $m(t)$, to avoid loss of info.



lot of loss of info as slope overloaded is too much and step size didn't get increased here.

~~no~~ negligible loss of info. as slope overloaded is less and step size has been ↑ed here.

→ 3 condⁿ are there -

① $(\text{slope})_{\text{signal}} = (\text{slope})_{\text{staircase}}$ less slope overloaded occurs

~~X~~ $(\text{slope})_{\text{signal}} > (\text{slope})_{\text{staircase}}$ (slope overloading too much)

3) $(\text{slope})_{\text{signal}} < (\text{slope})_{\text{staircase}}$ ✓

So, condⁿ will be –

$(\text{slope})_{\text{signal}} \leq (\text{slope})_{\text{staircase}}$ as

(slope overloaded will be less)

→ So →

$A_m \omega_m \leq \dfrac{\Delta}{T_s}$

$A_m \leq \dfrac{\Delta f_s}{2\pi f_m}$

} to avoid slope overloaded distortion

Quantization Noise for uniform Quantization

→ Two types of Quantization are there –

1) Uniform Quantization
2) Non-uniform Quantization

1) uniform – when step size remains fixed and sampling is done at uniform rate then it is called as uniform Quantization.

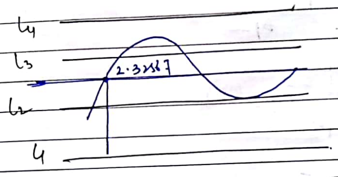
→ Difference in the levels is step size.

$\Delta = (L_2 - L_1)$

step size diff. in 2 levels.

More no. of levels, less error will occur.

$$\boxed{\text{step size } (\Delta) = \frac{\text{signal range}}{L}}$$

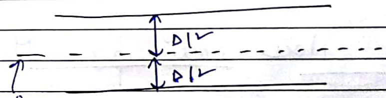


$L \rightarrow$ no. of levels assign

n – no. of bits assign in o/p wave.

$$\boxed{2^n = L}, \quad \boxed{n = \log_2 L}$$

While finding out the quantization error, the decision is taken at the mid wave value of step size, then quantization error can change from $-\Delta/2$ to $\Delta/2$.

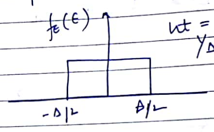


decision boundary

The error is the effective one and is always taken to be as effective value which is the mean square value.

$$\boxed{E[\epsilon^2] = \int_{-\Delta/2}^{\Delta/2} \epsilon^2 f_\epsilon(\epsilon)\, d\epsilon}$$

The given function is uniformly distributed b/w $-\Delta/2$ and $\Delta/2$.



$$E[\epsilon^2] = \int_{-\Delta/2}^{\Delta/2} \epsilon^2 f_\epsilon(\epsilon)\, d\epsilon$$

$$= \int_{-\Delta/2}^{\Delta/2} \epsilon^2 \frac{1}{\Delta}\, d\epsilon$$

$$= \frac{1}{\Delta}\left(\frac{\epsilon^3}{3}\right)_{-\Delta/2}^{\Delta/2}$$

$$= \frac{1}{3\Delta}\left(\frac{\Delta^3}{8} + \frac{\Delta^3}{8}\right)$$

$$= \frac{\cancel{\Delta}^1}{3\cancel{\Delta}}\left(\frac{\cancel{2}\Delta^2}{\cancel{8}4}\right) = \frac{\Delta^2}{12}$$

Signal to Noise Ratio for PCM

Companding - These word comprised of 2 words that are compressing and Expanding

Compounding: $\underbrace{\text{Com}}_{\text{taken from compressing}}\underbrace{\text{panding}}_{\text{taken from expanding}}$

Sampling Rate (r) - Rate to which some ~~sampling~~ signals are transmitted.

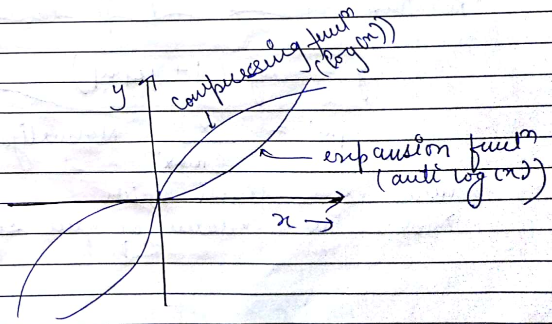
1000 Samples $\rightarrow$ $\boxed{\text{A to D Converter}}$ $\rightarrow$ $(n \times 1000)$ bits/sec
8000 bits/sec

$f_s = 1000$ Hz

$$\text{Bandwidth} \geq r = \frac{n f_s}{2} = \frac{n(2 f_m)}{2} = n f_m$$

$\boxed{\text{compressing function } \log x}$

compressing function $\log x$ = Expansion funct^n anti $\log(x)$



Decibel (DB) - It is a unit on logarithmic scale.

Companding has two laws -
1) A law (value of A is 87.56)
2) μ law (Value of μ → 255)

These are the values of A and μ are used commercially for compressing and expanding the files.

Linearity - Signal at transmitter and receiver end should almost be same but amplitude may change but waveform should be same.
e.g. → Mobile communication
→ Voice tone at transmitter and receiver end should be same. to achieved the linearity.

1) μ law

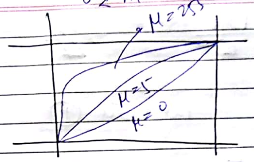
~~Relation b/w A and μ law~~

$$y = \log(1 + \mu|m|)$$

$$|m| = \frac{x}{x_{max}} \quad \text{Normalization} < 1.$$

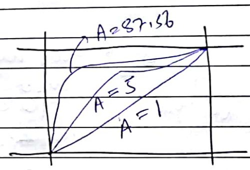
where x - signal value
x_{max} = max. value of the signal

$0 < m < 1$



(2) A law

$$y = \begin{cases} \dfrac{A|m|}{1+\log A} & 0 \le m < \dfrac{1}{A} \\[2ex] \dfrac{1+\log A|m|}{1+\log A} & \dfrac{1}{A} \le |m| \le 1 \end{cases}$$



Signal to Noise ratio for PCM -

$\left(\frac{S}{N}\right)_{dB}$ → Signal power / → Noise power } → measured in dB.

Capacity ⇒ $C = B \log_2(1 + S/N)$
measure in dB.

N → in digital comm. is always Quantisation noise.
let S → P and
N → Q/N → $\Delta^2/12$

$$= \frac{P}{\Delta^2/12} = \frac{12P}{\Delta^2} = \cancel{2\Delta m} \quad \text{— (1)}$$

$$\Delta = \frac{2V_m}{L} \quad \text{Put in (1)}$$

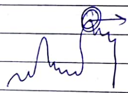
$$= \frac{12PL^2}{4V_m^2} = \frac{3PL^2}{V_m^2}$$

and $L = 2^n$

$$= \frac{3P(2^n)^2}{V_m^2} = \frac{3P(2^{2n})}{V_m^2}$$

$$(S/N) = \frac{2^{2n}\cdot 3P}{V_m^2} = \frac{3P(2^{2n})}{V_m^2}$$

→ If $P = V_m^2$

Spectrum of signal  → they will check this peak value and square it and we'll get the power. this is for random signals.

for random signals -

~~S/N~~ $P \to (\text{Peak value})^2 = V_m^2$

$$(S/N) = \frac{3\cancel{V_m^2}(2^{2n})}{\cancel{V_m^2}}$$

$$(S/N) = 3\cdot 2^{2n}$$

In decibels — $10\log_{10}(3\times 2^{2n})$

$$= 10\log_{10}3 + 10\cdot 2n\log_{10}2$$

$$= 10\log_{10}3 + 20n\log_{10}2$$

$$= 10\times 0.477 + 20\times n\times 0.3010$$

$$= 4.77 + n\times 6.02$$

$$= 4.77 + 6.02n$$

$$= 4.8 + 6n \quad (\text{by approx.})$$

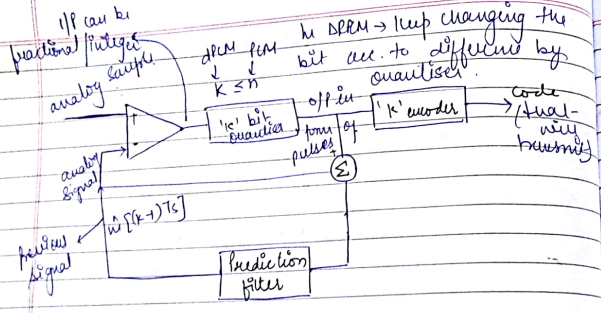
→ this is called 6dB rule, i.e if one bit is increased in code then S/N changes to 6 times of S/N.

DPCM - As the name suggest differential pulse code modulation sounds the limitation of PCM which transmit the same no. of bits irrespective of change in information. Hence PCM is not capable of utilising the given bandwidth properly.

The DPCM effectively utilises the given bandwidth by changing no. of bit to be transmitted bcz we know that required transmission bandwidth

In PCM, only n bits are provided.

In DPCM → keep changing the bit acc. to difference by quantiser.



→ DPCM means to use the given bandwidth efficiently means (min bandwidth ko max. way me use karna, min no. of bits se max gain lena).

→ Efficiency means to use the resources minimum and make o/p max. from those minimum resources.

Description

1) Differential Amplifier - It gives a +ve or -ve difference on the basis of the signal value in comparison with - $\hat{m}[(k-1)T_s]$

It is reconstructed approximatⁿ of analog sampled signal ($m(kT_s)$) and it also amplifies the difference. Hence it is called differential amplifier and because of it the differential word is there in DPCM.

2) K-bit Quantizer - This block converts a fractional value into the pre-defined integer value which is suitable to be encoded. It introduces some arbitrary error which is called Quantization error.
(e.g. 2.03 and it gets converts to 2 so there will be error of 0.03 and this is insignificant error.

3) Accumulator - This is accumulator which sums up the present result of Quantizer and previous result of prediction filter, and gives the o/p to the prediction filter.

imp. 4) This is important and internal part of DCPM called as Prediction filter. It consist of a dedicated processor and it's memory for maintaining database. The processor is programmed so as to so as to make efficient decision for predicting the next incoming analog signal so that the difference is minimizing.

→ Quantizer convert fractional value to integer value. o/p of quantiser as +ve / -ve.

Diff. b/w Δ & DPCM - predictⁿ filter is used in DPCM and in Δ, integer is used.

By utilising efficiently means transmit

max. no. of info by using small no. of bits.

DPCM working - works on principle that predict/keeping info about +ve next incoming signal by consider past record signal.

Errors in DPCM is non-significant.

23/8/18: S/N ratio for delta modulation -

$$S = \left(\frac{A_m}{\sqrt{2}}\right)^2 = \frac{A^2_m}{2}$$

To avoid slope overload, $A_m \le \frac{\Delta}{\omega_m T_s}$

$$S = \frac{\Delta^2}{2\omega_m^2 T_s^2}$$

Noise power

cut f_M

f_M

$f_M > f_m$ signal freq.

cut off frequency

$$N' = E(\epsilon)^2$$

$$= \int_{-\Delta}^{\Delta} \epsilon^2 f_\epsilon(\epsilon)\, d\epsilon$$

$$= \int_{-\Delta}^{\Delta} \epsilon^2 \left(\frac{1}{2\Delta}\right) d\epsilon$$

$$= \frac{1}{2\Delta}\left(\frac{\epsilon^3}{3}\right)_{-\Delta}^{\Delta}$$

$$= \frac{1}{2\Delta}\left(\frac{\Delta^3}{3} + \frac{\Delta^3}{3}\right)$$

$$= \frac{\Delta^2}{3}$$

$\Rightarrow \left(\frac{\Delta^2}{3}\right)\left(\frac{f_M}{f_m}\right)$

↳ Noise power

$$S = \frac{\Delta^2}{2\omega_m^2 T_s^2}$$

$$N = \frac{\Delta^2}{3}\left(\frac{f_M}{f_s}\right)$$

Q. An analog signal $5\cos(10^4 t)$ is Δ-modulated. Find out the step size to be kept to avoid the slope overloaded distortion if sampling rate is following the Nyquist rate.

Sol. $5\cos 10^4 t$

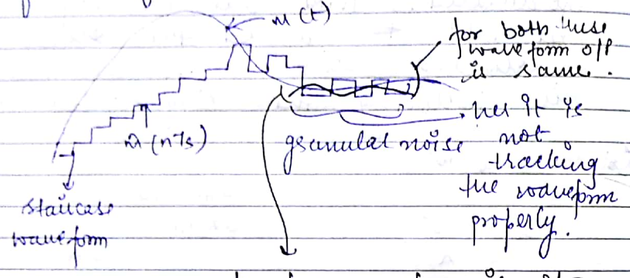
$\omega_m = 10^4$

$2\pi f_m = 10^4$, $f_m = \frac{10^4}{2\pi}$

$f_s = \frac{10^4}{\pi}$

Adaptive Δ- modulation

when the signal waveform is having small variation ranging in the dimensions of grain then staircase waveform will not be able to track the changes then it will cause a loss of information.



therefore waveform is not detected properly due to this granular noise arises and the distortion which occurs here is known as granular noise distortion, because for

both these binary o/p is 01010 for both the signal which is wrong as for 2 waveform o/p can't be same and noise arise too much.

→ The step size of staircase waveform is adaptive to the amplitude variation of input signal.

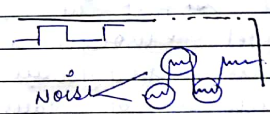
If signal amplitude is large then step size increase sharply.

Random variable

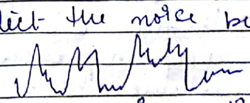
PCM signal Passing through gaussian Noise channel

1) The PCM o/p is in the form of binary data and when it is transmitted to a channel then noise is added into it. This type of noise is generally gaussian in nature whose pdf is given by

→ Most of the channel suffer from AWGN (Added white Gaussian Noise)
Here, Added means the noise will keep on adding up with the original signal as the channel length increases.



at a certain distance we are not able to detect the noise bcz signal becomes like



at a certain distance signal is lost inside the noise as noise flow keeps on increasing.

This situatⁿ can be solved out by using repeaters.

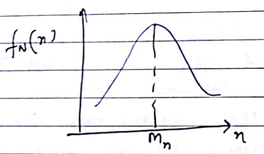
→ **White** means it will contain all the frequency because it contains all the colours so known as white Noise.

→ **Gaussian** here means

it has Gaussian pdf –

$$f_N(n) = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-(n-M_n)^2/2\sigma_n^2}$$

and follow a bell shape functⁿ.



when the PCM signal in the form of code travel through a channel then noise is added in the codded information which is of gaussian nature. This noise distorted the binary informatⁿ and as the channel length increases the effect of noise keep on increasing after a certain distance the noise overcome over signal and signal is unable to keep it's identity hence it is lost under the noise of flow. So, receiver is not able to detect the signal.

→ To solve this problem, regenerative repeaters are used after a certain distance in a channel.

Concept of Regenerative repeaters for PCM data transmission –

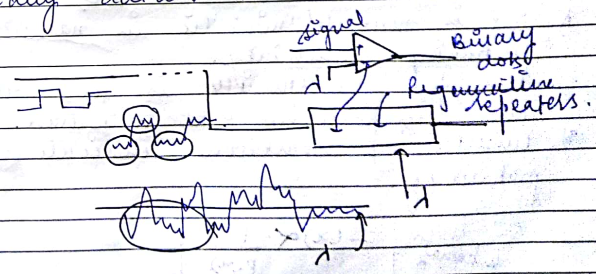
→ The main advantage of dcomm is that the binary data can be regenerated.

Process

→ Here for this purpose regenerative repeaters are used.

→ Here is a threshold value (λ) is pre-decided with which the noisy data is compared.

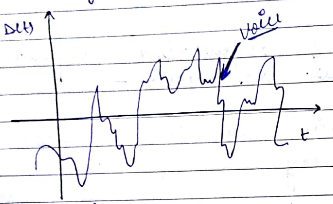
→ If the signal is more than λ then one (1) will be assigned in the o/p and if it is less then zero (0) will be assigned in the o/p then binary data will be recovered.



| diff. b/w data & voice. |

Random Variables

frequency (fm) is required to transmit the signal, otherwise signal gets distorted



→ waveform is totally random in nature.
→ it is random bcz we don't have any idea of future value.
→ waveform is of continuous random variable type.

R.V
- Discrete R.V
- Continuous R.V

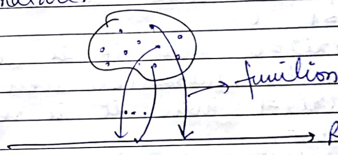
1) R.V is non-deterministic in nature. It is containing uncertainty.
2) More the uncertainty it is containing more information it will have.
3) uncertainty somehow indicate less probability.

information → $\boxed{I(x) \propto \frac{1}{P(x)}}$ ← Probability

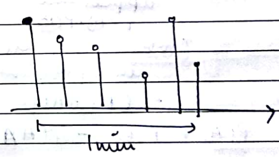
define pt. – whose radius tends to zero
pt. is physical in nature

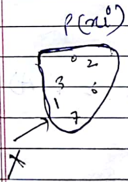
Probability –

Random variable
→ It is a rule by means of which we can map the physical outcome of certain non-deterministic event on to the real axis.
→ Random variable takes values which are partially deterministic or partially probabilistic in nature.



In pictorial form we can represent discrete random variable as –





X → event
10
0
1
3
6
7
1
2
1
6

Probability

If we perform an experiment so many times then data accuracy will be obtained.

$$P(x_i) = \lim_{N \to \infty} \frac{n(x_i)}{N}$$

experiment is conducted for ∞ times.

Probability lies b/w 0 and 1.

x is our observa^n^.

Properties

1. Prob. of any event is non-negative quantity
2. $0 \le P(x_i) \le 1$.
3. when $P(x_i) = 0$ then it is set to be uncertain i.e impossible even and when $P(x_i) = 1$ then it is very much sure event.
4. Joint probability $\{P(A, B)$ or $P(A|B)$ or $P(B|A)\}$
5. $\sum_{i=1}^{n} P(x_i) = 1$

Joint appearance $= P(A) \cdot P(B)$ (when they are independent)

$= P(A \cap B)$

$\rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Conditional Probability

$$P(A|B) = \frac{P(A, B)}{P(B)} = \frac{P(A) \cdot \cancel{P(B)}}{\cancel{P(B)}} = P(A)$$

$$P(B|A) = P(B)$$

$$P(A, B) = P(A|B) \cdot P(B)$$
$$= P(B|A) \cdot P(A)$$

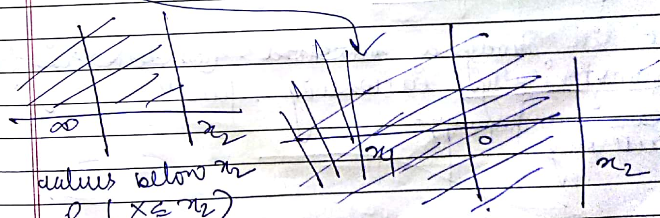
$$\frac{P(A)}{P(B)} = \frac{P(B|A)}{P(A|B)}$$

The probability $P(A)$ & $P(B)$, these probabilities are suitable for discrete random variable.

Cdf, Pdf

Cdf - it is the probability where the random variable X can take all the possible value with in the range x_1 & x_2.

$$P(x_1 \le X \le x_2) \text{ cdf} = \int_{x_1}^{x_2} f_X(x)\,dx = F_X(x_2) - F_X(x_1)$$
$$= P(X \le x_2) - P(X \le x_1)$$

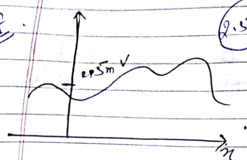


values below x_2

$P(X \le x_2)$

CDF and PDF

CDF.



2.5mV

$$F_X(x) = P(X \le x)$$
$$= P(X \le 2.5\text{mV})$$
$$= \int_{-\infty}^{2.5\text{mV}} f_X(x)\,dx$$

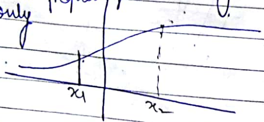
$$P(X > 2.5) = 1 - \int_{-\infty}^{2.5\text{mV}} f_X(x)\,dx$$

$$= \int_{-\infty}^{\infty} f_X(x)\,dx - \int_{-\infty}^{2.5\text{mV}} f_X(x)\,dx$$

$$F_X(\bar{x}) = \int_{2.5\text{mV}}^{\infty} f_X(x)\,dx$$
or
$P(X > 2.5)$

Properties of CDF

① It is always a ~~monotone~~ - ~~non-decreasing funct^n~~ monotonically increasing funct^n.
means one-frequency direct^n only



② It is a non-decreasing function.

③ $F_X(x_1) < F_X(x_2)$, if $x_2 > x_1$.

④ $F_X(\lambda) = P(X \le \lambda)$ — it means possible values.

⑤ $F_X(\infty) = P(X \le \infty) = 1 = \int_{-\infty}^{\infty} f_X(x)\,dx$

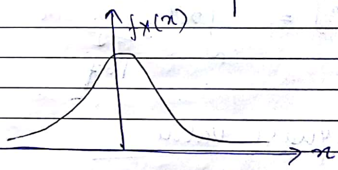
$F_X(-\infty) = 0 = P(X \le -\infty)$ — it means impossible values

⑥ $F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$

Properties of PDF

① PDF is a distribution of probability of a given random variable.

② It indicate the slope variation of CDF.



③ PDF is always a +ve quantity, it can never be a negative ".

④ total area coming under pdf unity.

Mean Value - It is also known as the average and expected value.

Representat^n - m_X, $\bar{X}$, $E[X]$

V[X] = ...

→ $v(t) = 5 \sin 100 t$ | deterministic signal

first cal. period of this & then –

$$\bar{v}(t) = \frac{1}{T} \int_{-T/2}^{T/2} v(t)\,dt$$

→ for continuous signal – (for random variable)

$$E[X] = \int_{-\infty}^{\infty} x\, f_X(x)\,dx$$

→ the nth moment of any given random variable x is given by –

$$E[x^n] = \int_{-\infty}^{\infty} x^n \underbrace{f_X(x)}_{\text{Pdf}}\,dx$$

Mean square value

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$$

Marginal PDF and CDF

If Joint CDF & PDF is given and we are finding the PDF & CDF for one variable then i.e known as Marginal PDF and CDF.

joint PDF & CDF

$$f_{XY}(x,y) = f_X(x) \cdot f_Y(y)$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy$$

lly

$$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx$$

$$= \int_{-\infty}^{\infty} f_X(x) \cdot f_Y(y)\,dy$$

$$= f_X(x) \underbrace{\int_{-\infty}^{\infty} f_Y(y)\,dy}_{\text{it becomes unity as (area under the PDF curve is unity)}} = \underbrace{f_X(x)}_{\text{Marginal pdf}}$$

→ If joint CDF is given and marginal pdf is to be obtain –

step ① $f_{XY}(x,y) = \frac{d^2}{dx\,dy} F_{XY}(x,y)$

② $f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy$

Marginal CDF

To obtain the marginal CDF from joint PDF

$F_X(x) = F_{XY}(x, \infty)$

$\quad \rightarrow P(-\infty \le X \le x, -\infty \le Y \le \infty)$

$$= \int_{-\infty}^{x} \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx\,dy$$

$F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx \rightarrow$ It is the marginal CDF.

$F_Y(y) = F_{XY}(y, \infty)$

$\quad \rightarrow P(-\infty \le X \le \infty, -\infty \le Y \le y)$

$$= \int_{-\infty}^{y} \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx\,dy$$

$F_Y(y) = \int_{-\infty}^{y} f_Y(y)\,dy \rightarrow$ It is the marginal CDF.

Mean Value

n^{th} central moment.

$$E[(X - m_X)^n] = \int_{-\infty}^{\infty} (x - m_x)^n f_X(x)\,dx$$

for $n = 2$

$$E[(X - m_x)^2] = \int_{-\infty}^{\infty} (x - m_x)^2 f_X(x)\,dx$$

$$\boxed{\sigma^2 = E\{(x - m_x)^2\}}$$

or

$$\boxed{\sigma^2 = E[X^2] - m^2_x}$$

$$\sigma^2 = E[x^2 + m_x^2 - 2x m_x]$$

$$= \cancel{E[x^2] + E\,m\,E[\mu_x^2 - 2x\mu_x]}$$

$$= \cancel{E[x^2] + E[\mu_x[\mu_x - 2x]]}$$

$$= \cancel{E[x^2] + E[\mu_x}$$

$$= E[x^2] \cancel{+ E[\mu_x^2 - 2x\mu_x + \mu_y^2 - \mu_y^2]}$$

$$= E[x^2] \cancel{+}$$

$$= E[x^2 + m_x^2 - 2x m_x]$$

$$= E[x^2] + m^2_x - 2m_x E[x]$$

$$= E[x^2] + m^2_x - 2m_x^2$$

$$= E[x^2] - m^2_x.$$

Scanned by CamScanner