CSCM77
filtering & edges

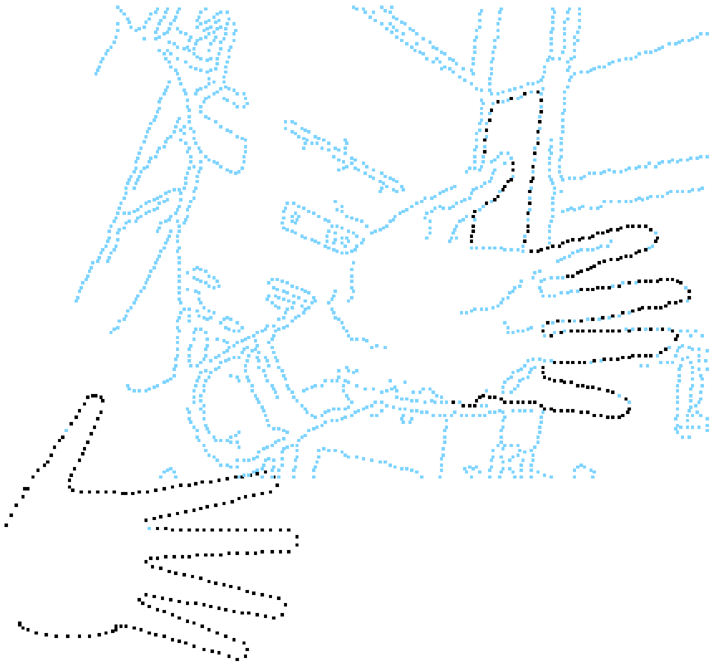
Prof. Xianghua Xie

x.xie@swansea.ac.uk

http://csvision.swan.ac.uk

Image Filtering & Edge Detection

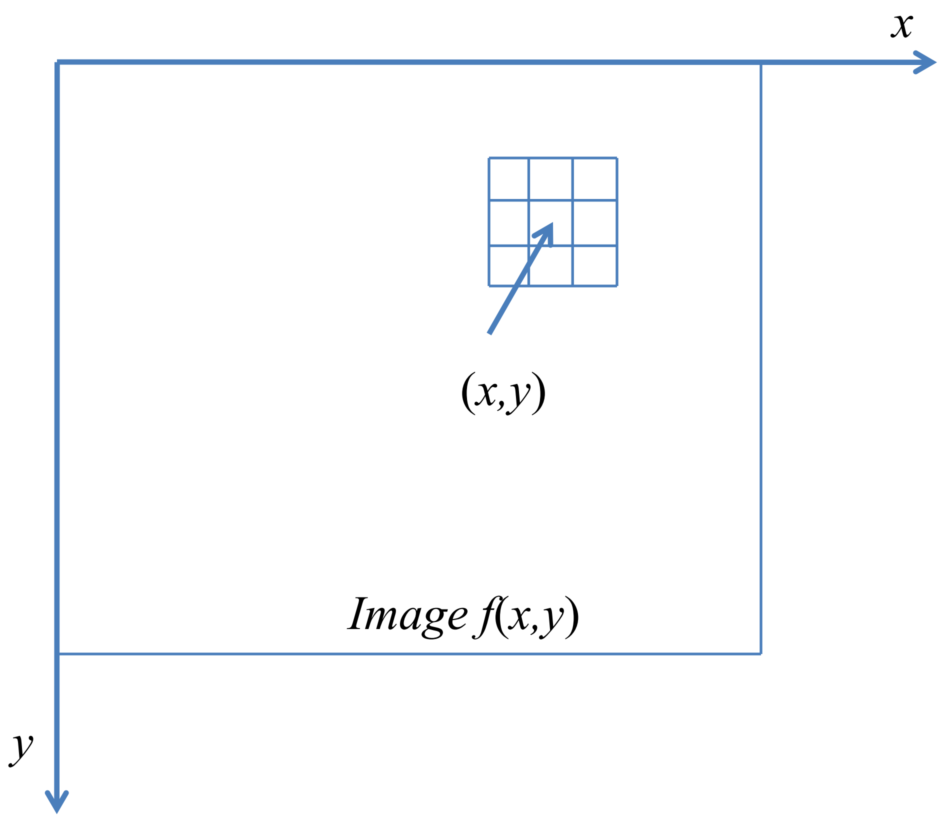
- Motivation



Stenger et al. ICCV 2003

Spatial domain filtering

- Spatial domain filtering
  - Linear filtering: convolution
  - Nonlinear filtering

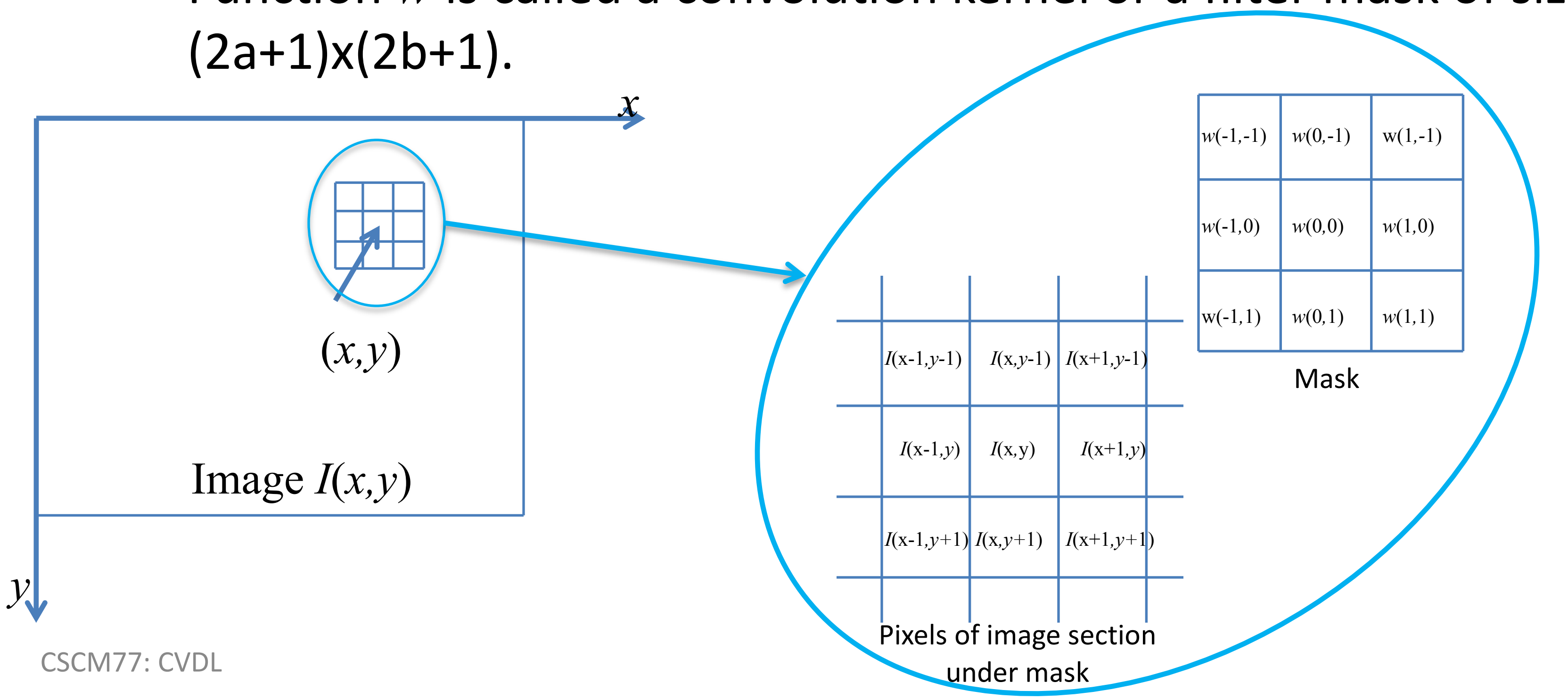


Linear Filtering

- Convolution is a form of linear filtering
  - A linear combination of brightness in a local neighborhood.
  - Discrete convolution:

$$g(x,y) = w * I = \sum_{m=-a}^{a} \sum_{n=-b}^{b} w(m,n) I(x-m, y-n)$$

  - Function w is called a convolution kernel or a filter mask of size (2a+1)x(2b+1).



Linear Filtering

- Convolution applies flipped kernel mask
  - Kernels (or masks) pre-rotated 180 degrees
  - Then apply direct pixel-to-pixel multiplication and compute the summation
- If kernel is not rotated
  - It is known as Correlation

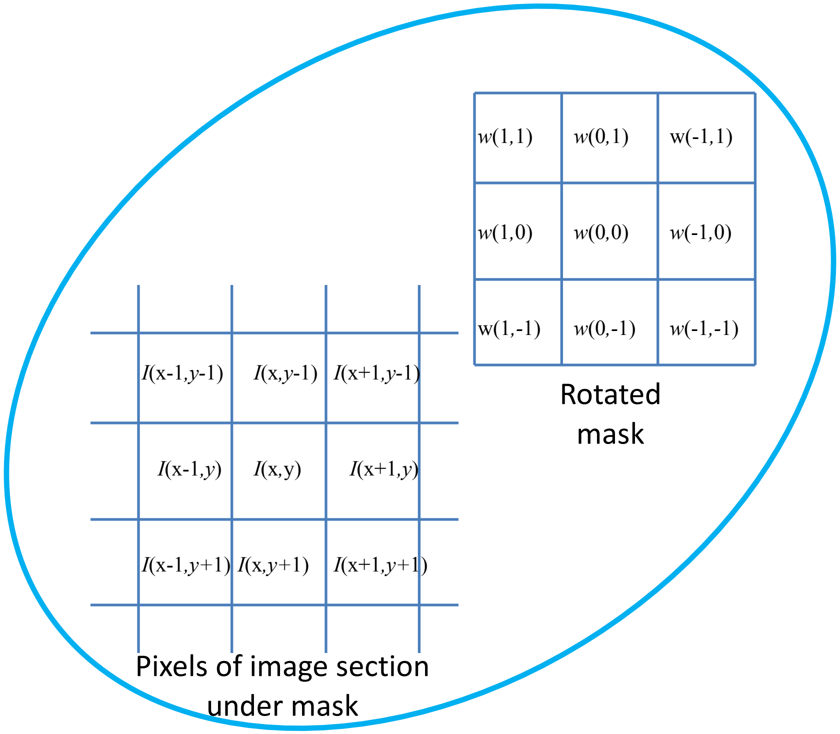
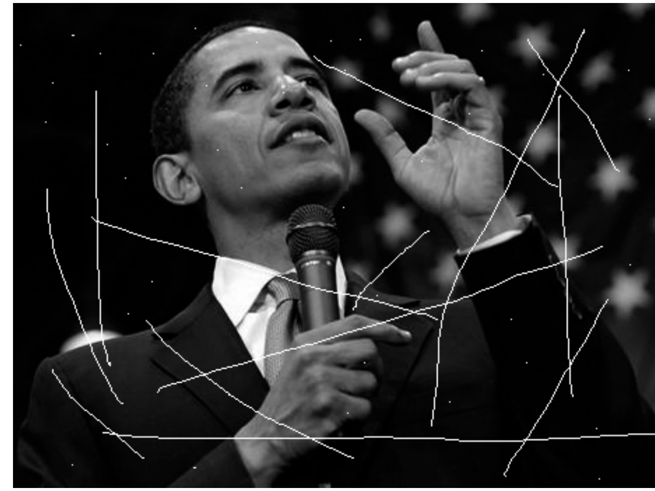
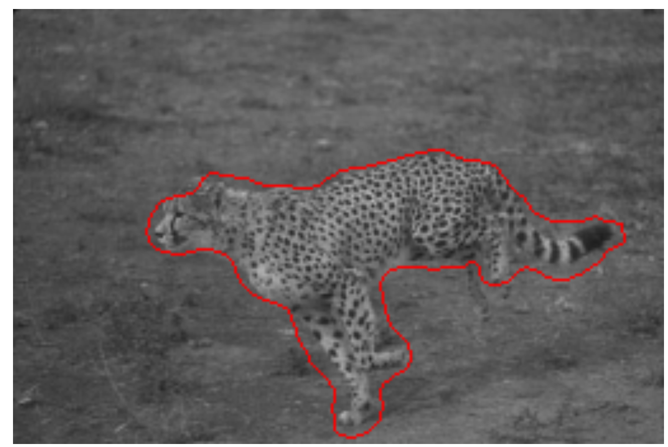


Image Filtering

- Often not for artistic reasons

- Rather:
  - Detect edges
  - Noise suppression
  - Find edges in busy patterns
  - Search objects





Gaussian Smoothing

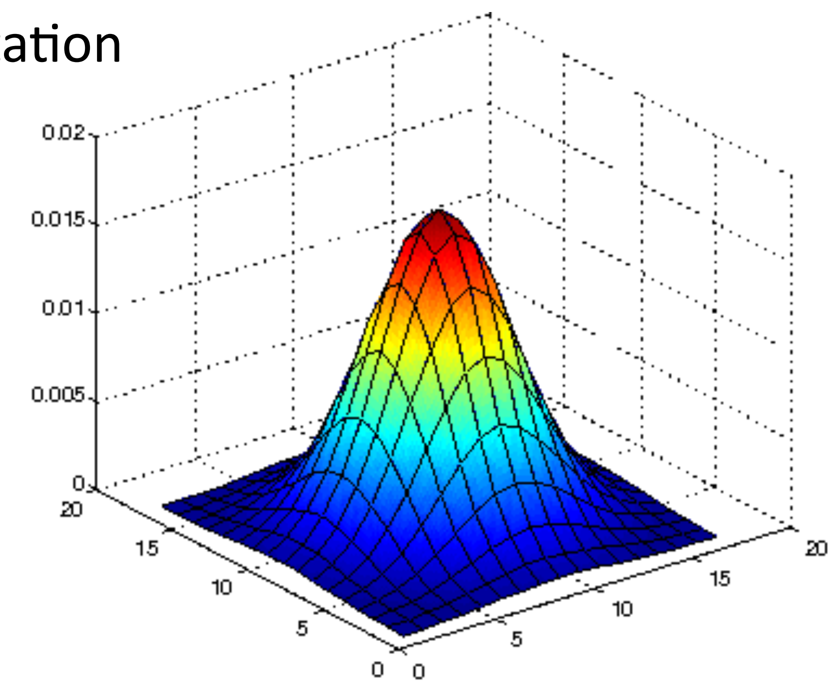
- Linear filtering

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

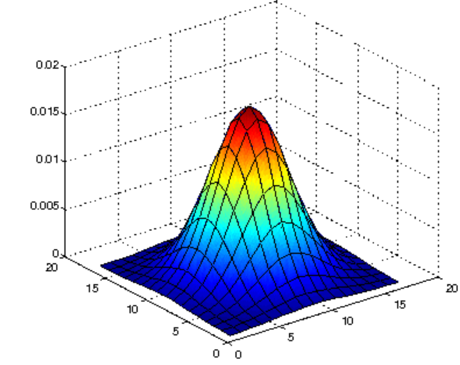
- σ controls spread of the Gaussian function
- Larger σ, larger smoothing
- Larger σ, larger kernel size
- Kernel separable: faster implementation

$$\begin{bmatrix} 0.0113 & 0.0838 & 0.0113 \\ 0.0838 & 0.6193 & 0.0838 \\ 0.0113 & 0.0838 & 0.0113 \end{bmatrix}$$

3x3, smallest kernel



Gaussian Smoothing



- Weighted average
- Isotropic
- Smoothing along the edges, as well as across the edges
- Not good at preserving edges
- New pixel values created across the edges

Median Filtering

- Nonlinear filtering
- Instead of using weighted mean, median value of local neighbourhood is used

80	89	78	80	81
82	88	86	90	85
81	83	52	85	83
78	80	78	82	80
76	77	77	75	73

[88, 86, 90, 83, 52, 85, 80, 78, 82]

sorting

[52, 78, 80, 82, 83, 85, 86, 88, 90]

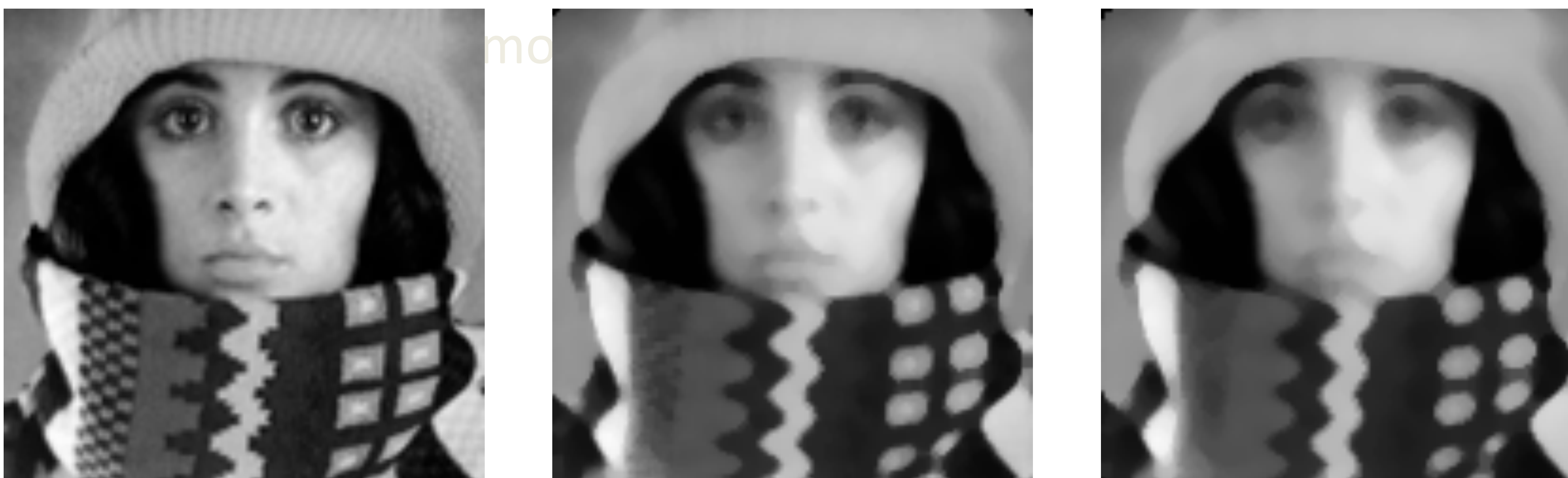
80	89	78	80	81
82	88	86	90	85
81	83	83	85	83
78	80	78	82	80
76	77	77	75	73

64, if using a 3X3 Gaussian kernel:

$$\begin{bmatrix} 0.0113 & 0.0838 & 0.0113 \\ 0.0838 & 0.6193 & 0.0838 \\ 0.0113 & 0.0838 & 0.0113 \end{bmatrix}$$

Median Filtering

- Median is a more robust average than the mean
- Less affected by outlier
- Good at removing lines and isolated dots
- Does not create new pixel values (reducing number of gray levels)
- Preserve edges better
- Smooth sharp corners

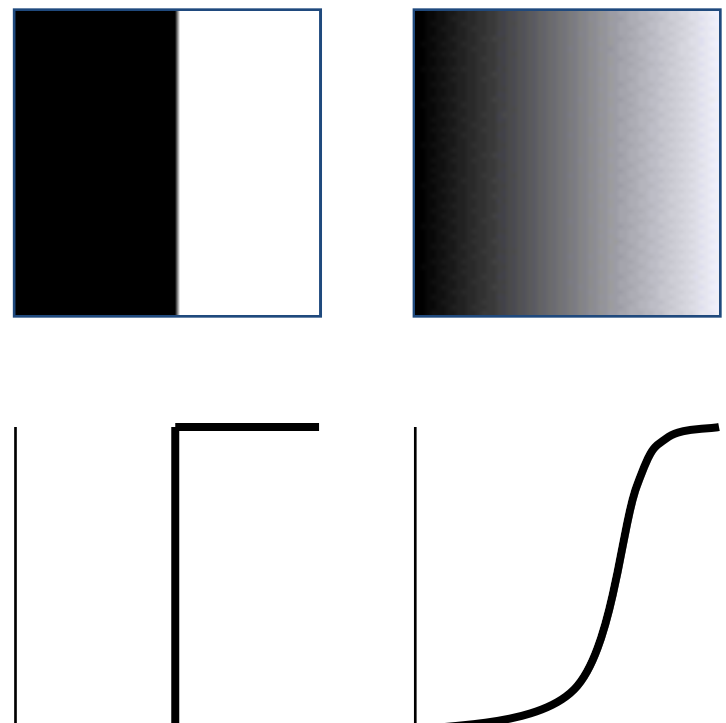


Edge Detection

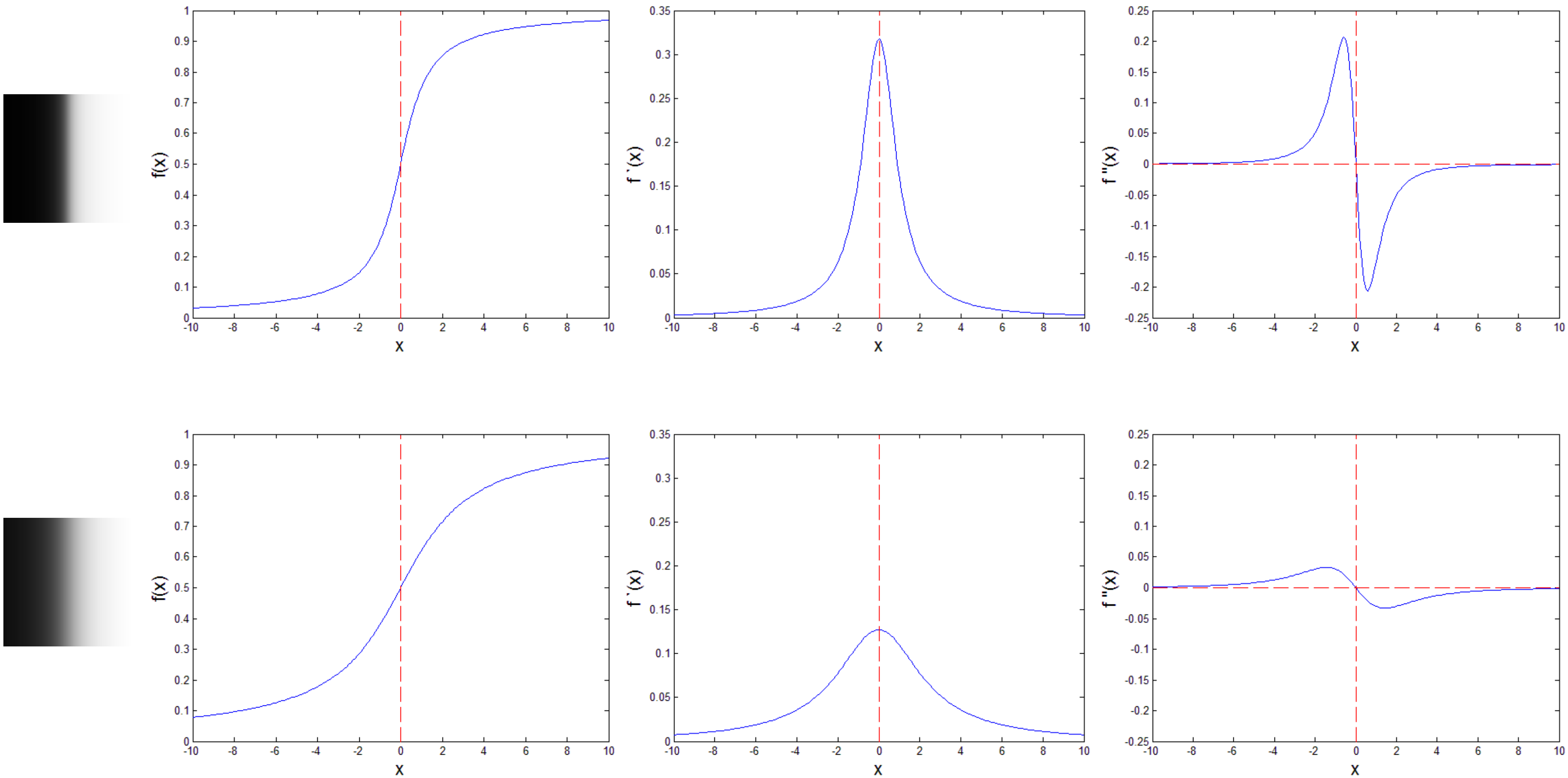
- Roughly speaking, edges are abrupt changes in intensity.
- A property attached to individual pixels
- Calculated by looking at the relationship a pixel has with its neighborhood pixels.

- If a pixel's gray-level value is similar to those around it, there is probably not an edge.

- If a pixel's has neighbours with widely varying gray levels, it may present an edge point.



Edge Detection



Edge profile | 1st order derivative | 2nd order derivative

Edge Detection

- Edges can be found by computing derivatives of image function
- Many are implemented with convolution masks
- Those masks are discrete approximations to differential operations.
- Measure the rate of change in the image brightness function.

- We will look at:
  - Prewitt operator (1st order)
  - Sobel operator (1st order)
  - Laplacian of Gaussian operator (2nd order)

- Some return both edge strength and edge direction

Prewitt

- Gradient of image denoted as: ∇I
- Two components: horizontal and vertical

- Prewitt operator is one of the simplest forms
- First order central difference (remember to rotate the mask when applying convolution!)

$$M_x = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \qquad M_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

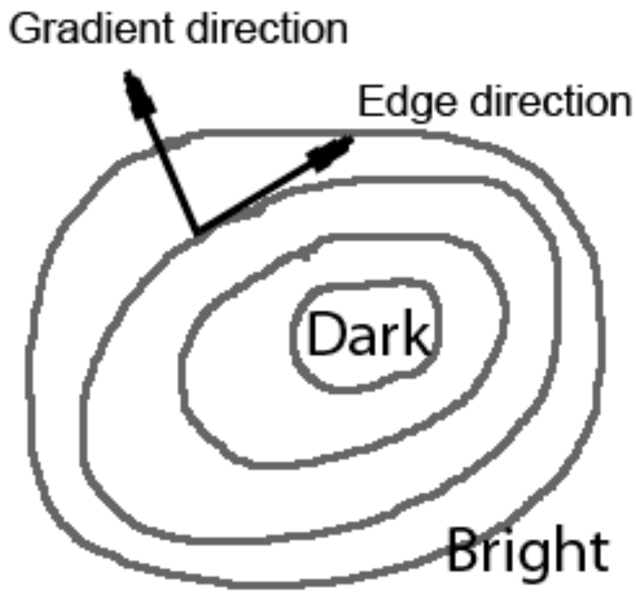
$$\nabla I = \begin{pmatrix} I_x \\ I_y \end{pmatrix} \cong \begin{pmatrix} M_x * I \\ M_y * I \end{pmatrix}$$

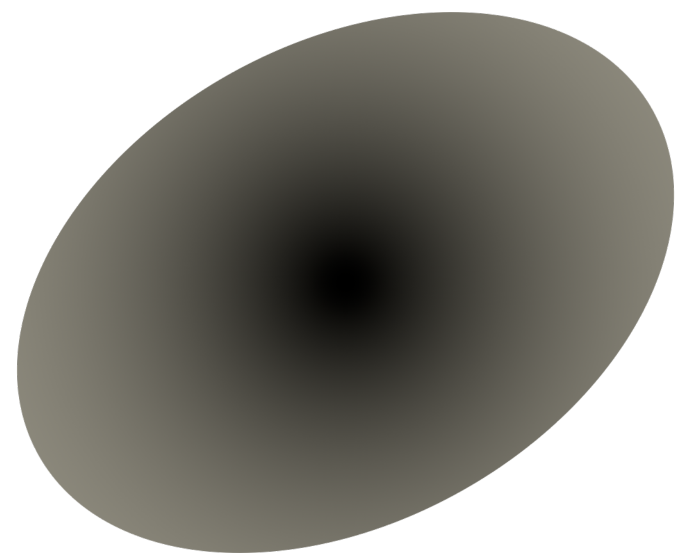
- Provides edge strength and edge direction

$$\text{magnitude} = \sqrt{{I_x}^2 + {I_y}^2} \qquad \text{gradient direction} = \arctan\left(\frac{I_y}{I_x}\right)$$

Prewitt

- The gradient direction gives the direction of maximal growth of the function
- Edge direction is a rotation of gradient direction





- Sensitive to noise

Sobel

- Similar to Prewitt operator
  - However, less sensitive to noise
- Combining Low Pass filtering, i.e. smoothing (G - Gaussian):

$$\nabla(G * I) = (\nabla G) * I$$

- Convolution kernels:

$$M_x = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} \qquad M_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} \qquad \nabla I = \begin{pmatrix} I_x \\ I_y \end{pmatrix} \cong \begin{pmatrix} M_x * I \\ M_y * I \end{pmatrix}$$

- Approximations of Gaussian derivatives
- Provides edge strength and edge direction

$$\text{magnitude} = \sqrt{I_x^{\ 2} + I_y^{\ 2}} \qquad \text{gradient direction} = \arctan\left(\frac{I_y}{I_x}\right)$$

Sobel

- Kernel decomposition (faster implementation)

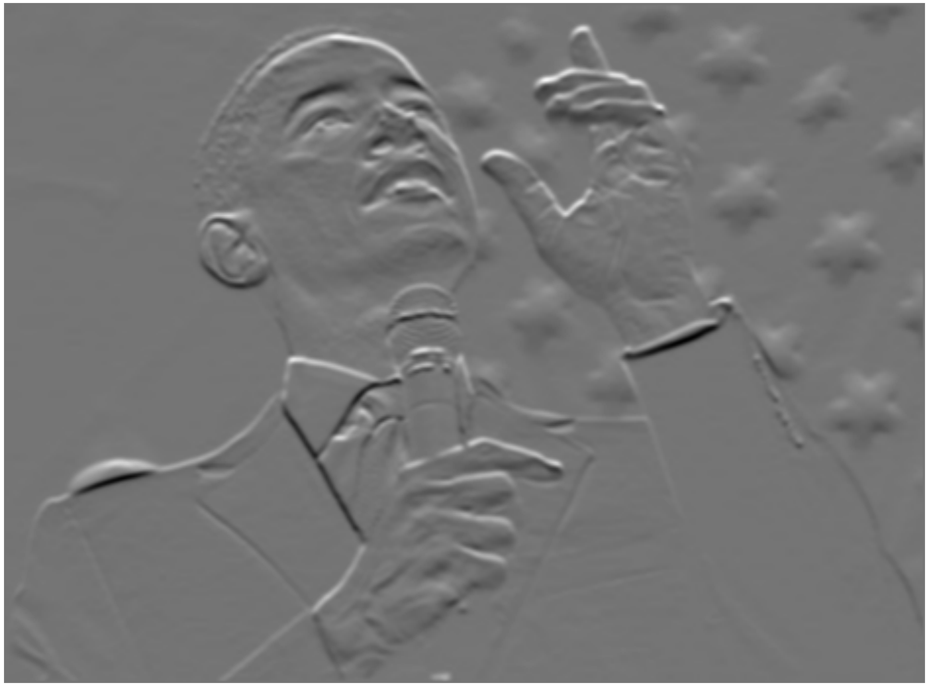
$$\begin{bmatrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} +1 & 0 & -1 \end{bmatrix} \qquad \begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} = \begin{bmatrix} +1 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

- Example:

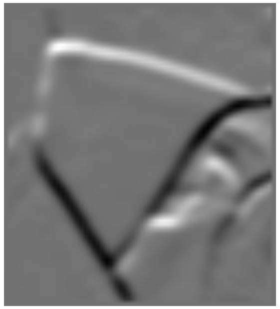
Sobel

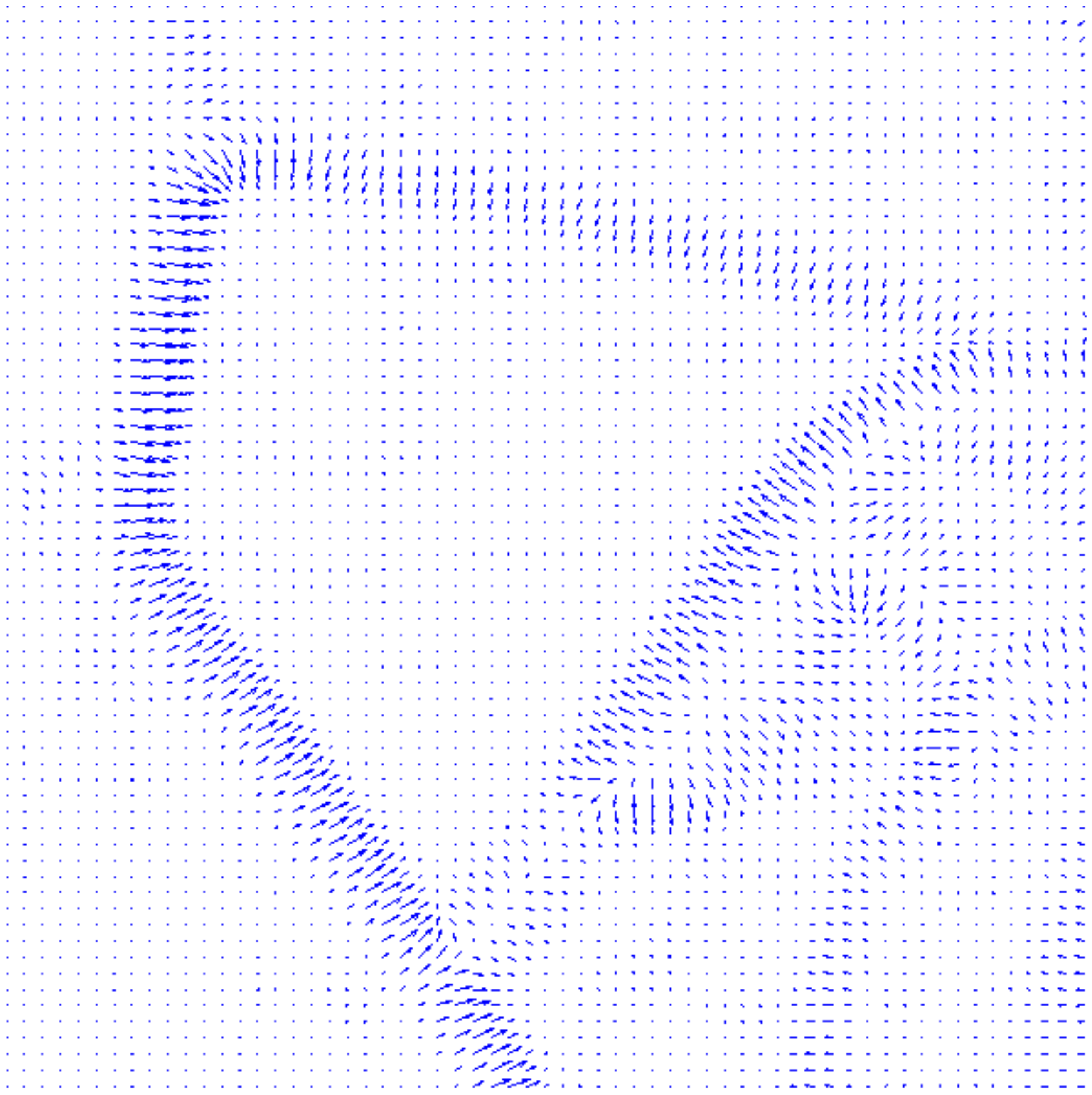
- Example (gradient direction):

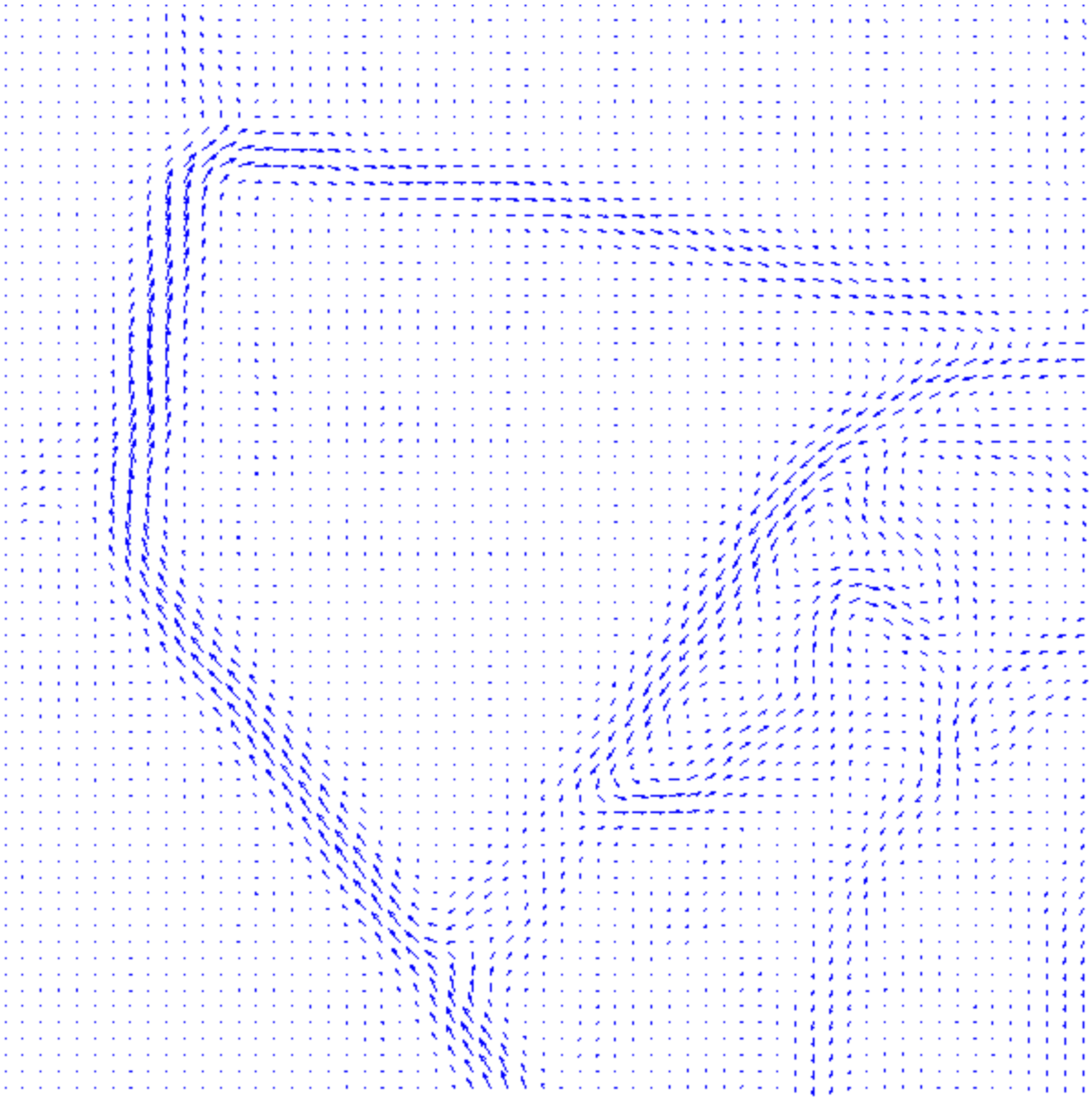
I_x



I_y







Sobel

- A simple visualisation of edge direction

Sobel

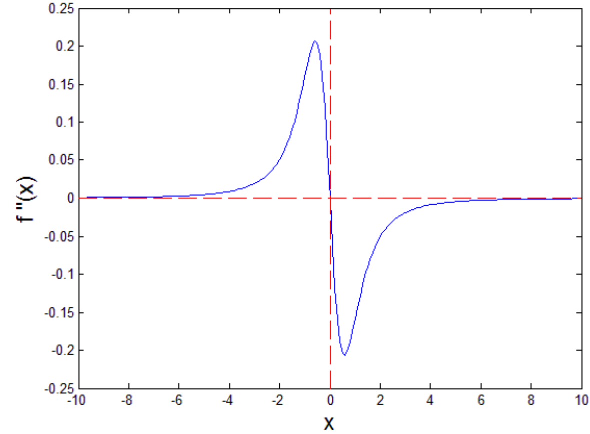
- Thresholded Sobel edge map has poor connectivity

Sobel

Thresholded

Laplacian of Gaussian

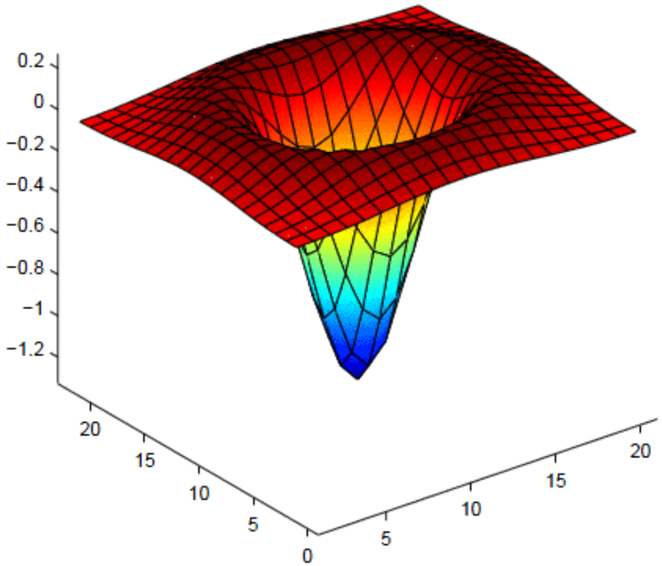
- Based on second order derivatives $\nabla^2 I$
- Divergence of image gradient
- Looking for zero crossing
- Sensitive to noise, smoothing is necessary



$$\nabla^2(G * I) = (\nabla^2 G) * I$$

- Discrete approximation, 5 by 5 case:
    - Symmetric kernel

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 1 & 2 & -16 & 2 & 1 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

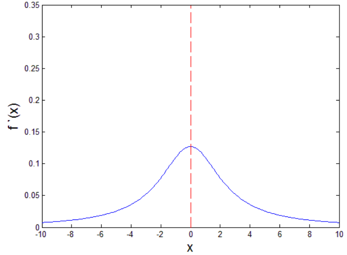


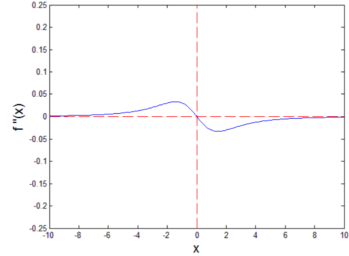
- No edge direction information

Laplacian of Gaussian

- Sensitive to noise
- No edge direction information
- More accurate in localising edge centre pixels (zero crossing)
    - We are looking for zero crossing, not just zeros (mid-gray)

Laplacian of Gaussian





- Sensitive to noise
- No edge direction information
- More accurate in localising edge centre pixels (zero crossing)

Summary – filtering and edge detection

- Filtering
- Convolution & Correlation
- Gaussian Smoothing
- Median Filtering
- Prewitt
  - 1^{st} order
  - Sensitive to noise
  - Edge magnitude and orientation
- Sobel
  - 1^{st} order
  - Edge magnitude and orientation
  - Thresholded edge has poor connectivity
- LoG
  - 2^{nd} order
  - Sensitive to noise
  - No edge direction information
  - Relatively more accurate in localising edge centre pixels