Mathematical Modelling of Chassis Dynamics of Electric Narrow Tilting Three Wheeled Vehicle

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***Abstract*—**

In recent years, substantial research efforts have been invested by industry and academia into the development of narrow tilting three wheeled vehicle technology as a viable alternative to four and two wheel vehicles for personal mobility. But there are challenges in order to make such vehicles safe, comfortable and acceptable to consumers. In this paper, a mathematical model of an electric narrow three wheel tilting vehicle, which has two front wheels driven by two in-wheel hub motors and one rear wheel, is proposed. The model is aimed at analysis of vehicle chassis dynamics along longitudinal, lateral and tilt (of body) motions using Simulink as implementation tool.

***Index Terms*—Narrow tilting three wheeled vehicle, in-wheel motor, vehicle dynamics, mathematical modelling, simulation**

I. Introduction

In recent years, industry and academia have put in substantial research efforts into development of narrow tilting three wheeled (NTTW) vehicle technology as a viable alternative to four and two wheelers for personal mobility as it combines positive aspects of a car (comfort, safety) and a motorcycle (low weight, dynamic driving performance), thus outperforming conventional cars on the aspects of consumption of less space in traffic and in parking [1]. But realization of this new technology offers stiff challenges in order to make it safe, comfortable and acceptable to the consumers. The vehicle dynamics in general consists of three translational motions (i.e. longitudinal, lateral and vertical) and three rotational motions (i.e. yaw, pitch and roll angles). While designing the narrow tilting three wheeled vehicle, one more dimension in the form of tilting stability is added and hence design of an automatic vehicle balance control system under various driving conditions like rapid maneuver, slippery road surfaces, banking for all speeds and acceleration becomes a challenge. As a result, lot of research has been directed towards design and development of tilt control strategies. In this paper, a detailed nonlinear mathematical model of a narrow three wheel tilting vehicle shall be proposed. The model is aimed at analysis of vehicle chassis dynamics along longitudinal, lateral and tilt (of body) motions using Simulink as implementation tool. Further, it is assumed that two in-wheel hub motors are used to drive the vehicle and achieve improved torque control in order to take advantage of high dynamic responsiveness of electric motors.

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In most of the literature [2, 3], the emphasis has been given to the combination of the lateral and tilting dynamics while cornering. It is mentioned in [4] that there is a scope to understand the effect of different driving conditions such as steering angle and speeds on the active tilting mechanism of a three wheel vehicle. Authors in [4] proposed a method to find optimal tilt angle for various speeds.

This research work considers the effect of longitudinal dynamics (e.g., speed, slip ratio) during the stability control design of such a narrow tilting vehicle. The usage of two in-wheel motors to facilitate the torque vectoring in the individual wheels during maneuvers adds to the complexity in the overall control of the platform.

II. Mathematical Model

In order to design chassis stability control system for a vehicle, it is important to know the dynamic forces experienced by the vehicle due to driver maneuvers and also due to the road conditions. Thus, it is important to approximate such parameters as close as possible to real values through simulation.

The mathematical model is built from first principles and consists of six major blocks. Each block represents the vehicle motion in certain degree of freedom. These blocks (the degrees of freedom of the vehicle) are coupled and interact during the vehicle motions.

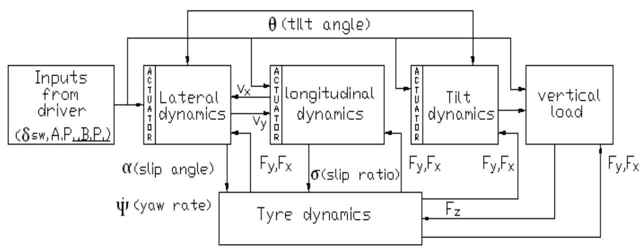

Fig. 1: Complete block diagram of vehicle

The model must include all components of vehicle dynamics that affect stability. Certain assumptions are made based on its applicability. The block schematic of the model is shown in Fig. 1. The inputs from the drivers are the steering wheel angle, accelerator pedal (A.P) and brake pedal (B.P). In this study, a non-linear mathematical model of vehicle chassis dynamics has been formulated and is implemented using

Simulink. The various forces on the vehicle are shown in Fig.2.

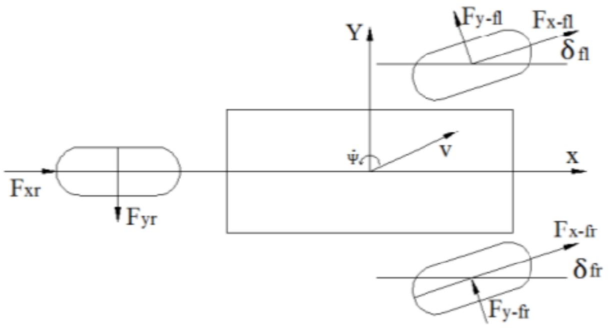

Fig. 2: Schematic of vehicle forces

Here, Fx_fl, Fx_fr and Fx_r are the longitudinal tyre forces at front left, right and rear wheels respectively. Similarly, Fy_fl, Fy_fr and Fy_r correspond to the lateral tyre forces.

A. Longitudinal dynamics modeling

This block serves as the power source of the vehicle and it drives the motion of the vehicle in forward direction. The states involved in this block are the velocity, V_x and angular speeds of front left, front right and rear wheels, ω_i where subscript i = fl (front left), fr (front right) and r (rear) wheels.

The slip ratio is calculated for each wheel and acts as input to the tyre model to obtain the tractive forces for the respective tyre. (1) to (5) gives the mathematical equations used to model the longitudinal block [6, 7]. '$m_{C.G}$' denotes the mass of the vehicle.

$$m_{C.G}\dot{V}_x = F_{x_{fl}}\cos\delta_{fl} + F_{x_{fr}}\cos\delta_{fr} + F_{x_r} - F_{y_{fl}}\sin\delta_{fl} - Fyfr\sin\delta fr + mC.G\varphi\, Vy \quad (1)$$

Wheel dynamics equations are given below:

$$I_{w_{fl}}\dot{\omega}_{fl} = T_{d_{fl}} - T_{b_{fl}} - rF_{x_{fl}} - rF_{roll_{fl}} \quad (2)$$

$$I_{w_{fr}}\dot{\omega}_{fr} = T_{d_{fr}} - T_{b_{fr}} - rF_{x_{fr}} - rF_{roll_{fr}} \quad (3)$$

$$I_{w_r}\dot{\omega}_r = T_{d_r} - T_{b_r} - rF_{x_r} - rF_{roll_r} \quad (4)$$

Wheel slip, $\sigma = \frac{\omega r - V_x}{\omega r}$ (5)

T_{d_i}, T_{b_i}, F_{x_i} and F_{roll_i} are the motor drive torque, brake torque, tyre force, rolling resistance force respectively. The overall block diagram of the longitudinal dynamics including the two in-wheel hub motors is given in Fig. 3. A simple DC motor model [7] is taken as in-wheel hub motors, which provide the necessary torque to achieve a velocity.

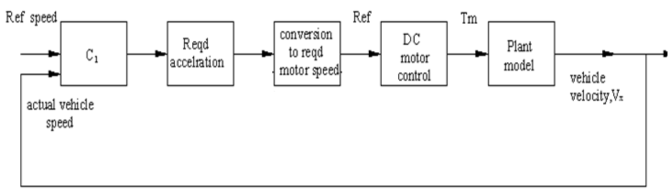

Fig. 3: Complete block diagram of longitudinal dynamics model

The driver provides the reference speed by pressing the throttle pedal, the actual speed from the plant model (longitudinal dynamics model) is fed back to the controller, C_1. Based on the error generated a simple PI controller generates a corrective action to generate the required acceleration to achieve the reference speed. The required acceleration block maps the command from 0 to 1 throttle ratio and this is again mapped into desired motor speed. In this paper, the desired motor speeds for front left and right wheels are assumed to be same. The DC motor speed is controlled using PI controller and the load on each motor is the resistance force experienced by the vehicle, which includes rolling resistance force and tyre forces. The motor needs to overcome this load and provide the required torque to move the vehicle in the forward direction. The torques from both the motors is provided to the plant model, through which the vehicle model consequently achieves the required longitudinal velocity.

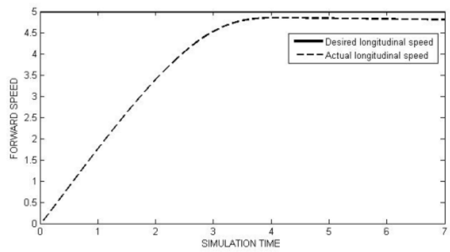

Fig. 4: Vehicle speed control plot

Figure 4 shows the longitudinal speed control plot, where the desired speed is 5 kmph.

B. Lateral dynamics

The driver commands desired steering input and based on Ackerman principle, the inner and outer wheel angles are calculated and fed in the lateral dynamics block. The states are the lateral velocity, V_y and yaw rate, $\dot{\varphi}$. Additionally the tilting effect, term'f(θ)', is part of the equation as given in [8]. The slip angle of the wheels is calculated based on the state values to feed into tyre model to obtain the resultant tyre forces. The lateral dynamic equations (6) to (11) are given below [6, 7 and 8].

$$m_{C.G}\dot{V}_y = F_{x_{fl}}\sin\delta_{fl} + F_{x_{fr}}\sin\delta_{fr} + F_{y_r} + F_{y_{fl}}\cos\delta_{fl} + Fyfr\cos\delta fr - mC.G\varphi\, Vx + \mathrm{f}\theta \quad (6)$$

$$\mathrm{f}(\theta) = m_{C.G}h_{C.G}\,\ddot{\theta}\cos\theta - m_{C.G}h_{C.G}\,\dot{\theta}^2\sin\theta \quad (7)$$

$$I_{vehicle_{zz}}\ddot{\varphi} = (F_{x_{fl}}\sin\delta_{fl} + F_{x_{fr}}\sin\delta_{fr})L_f + (F_{y_{fl}}\cos\delta_{fl} + F_{y_{fr}}\cos\delta_{fr})L_f - F_{y_r}L_r + \frac{T_w}{2}\left(F_{x_{fr}}\cos\delta_{fr} - F_{x_{fl}}\cos\delta_{fl}\right) + \frac{T_w}{2}\left(F_{y_{fl}}\sin\delta_{fl} - F_{y_{fr}}\cos\delta_{fr}\right)) \quad (8)$$

Wheel slip angles are given by (9), (10) and (11):

$$\alpha_{fl} = \delta_{fl} - \frac{V_y + L_f\dot{\varphi}}{V_x - \frac{T_w}{2}\dot{\varphi}} \quad (9)$$

$$\alpha_{fr} = \delta_{fr} - \frac{V_y + L_f\dot{\varphi}}{V_x + \frac{T_w}{2}\dot{\varphi}} \quad (10)$$

$$\alpha_r = -\frac{V_y - L_r\dot{\varphi}}{V_x} \quad (11)$$

Here, δ_{fl} and δ_{fr} represent the front left and front right wheel angles. T_w symbolizes the track-width of the vehicle and α represents the slip angles of respective wheels.

C. Vertical dynamics Equations

The vertical force represented by F_{z_i} applied on each tyre dynamically is approximated by neglecting suspension dynamics and coupling of vehicle pitch and roll. Generally, the load transfer due to acceleration, deceleration and cornering are taken into account [9]. In this paper, load transfer during cornering also accounts for the tilting moment of the narrow three wheel vehicle as well as due to its geometric construction, ε. Any effect from in-wheel motors on load transfer is neglected and will be included in further refinements to the vehicle model. The vertical forces at each tyre are given by (12), (13) and (14). Fig. 5 depicts the forces considered to calculate load transfer during various maneuvers.

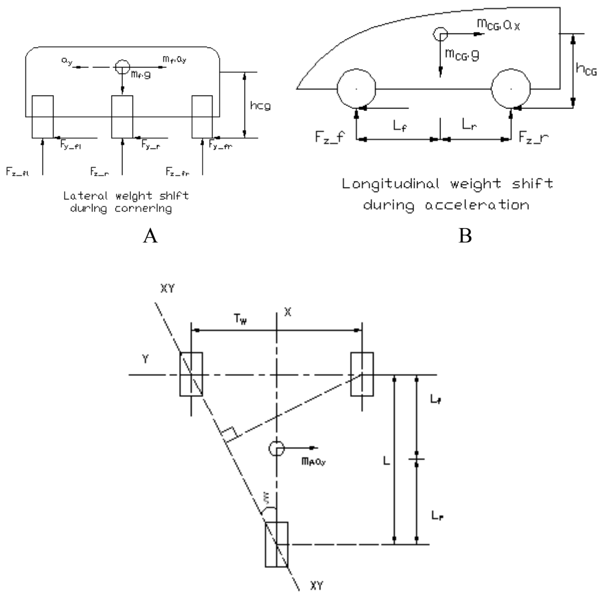

Fig. 5: Load transfer due to (A) Lateral, (B) Longitudinal and (C) Tilt dynamics

$$F_{z_{fl}} = m_{C.G}\left[\frac{L_r}{L}g - \frac{h_{C.G}}{L}a_x\right]\left[\frac{1}{2} - \frac{(a_y h_{C.G}\cos\varepsilon)}{(gT_W)}\right] - \frac{M_\theta}{T_W} \quad (12)$$

$$F_{z_{fr}} = m_{C.G}\left[\frac{L_r}{L}g - \frac{h_{C.G}}{L}a_x\right]\left[\frac{1}{2} + \frac{(a_y h_{C.G}\cos\varepsilon)}{(gT_W)}\right] + \frac{M_\theta}{T_W} \quad (13)$$

$$F_{z_r} = m_{C.G}\left[\frac{L_f}{L}g + \frac{h_{C.G}}{L}a_x\right] \quad (14)$$

Here, M_θ indicates the moment generated due to tilting action. Equal vertical forces on tyres are considered in all maneuvers [8], whereas [2] considers load transfers during cornering. In this study the load transfer effects due to dynamic motions of the vehicle in the longitudinal, lateral and tilt axis are taken into account.

D. Tyre modelling

The tyre interaction with the vehicle dynamics is one of the most important aspects of a vehicle chassis model. The tyre connects the chassis to the road and enables the vehicle to maneuver as per requirement by generating the relevant tyre forces.

In literature, Pacejka tyre model [2] is used for calculating the dynamic tyre forces. Pacejka tyre model is based on experimental data and the tyre model is generated by fitting this experimental data in a mathematical form. Acquiring such data of tyres becomes difficult as it requires a highly instrumented set up. In this paper, tyre is modelled using the analytical Dugoff tyre model equations [6, 8] and are given below (15) to (16). The Dugoff model provides the forces using friction circle concept (Fig. 6). The Fig. 6 shows the variation of friction circle (the unit of forces is N) with slip ratio.

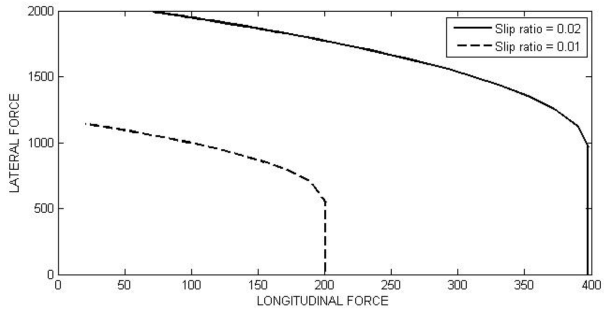

Fig. 6: Friction circle of the Dugoff tyre model for slip angle varied from 0 to 1 degree

The tyre model requires inputs from the lateral, longitudinal and tilt dynamics model, i.e. slip angle, slip ratio and the body tilt angle to calculate the tyre forces. The forces generated from the tyre model must be input to the longitudinal, lateral and tilt dynamics model in (1), (6) and (8). Also, a friction calculation block is included, which calculates the friction of the tyre based on the slip ratio of the wheels. The relationship between adhesion (μ) and slip ratio (σ) is plotted graphically in Fig. 7 below.

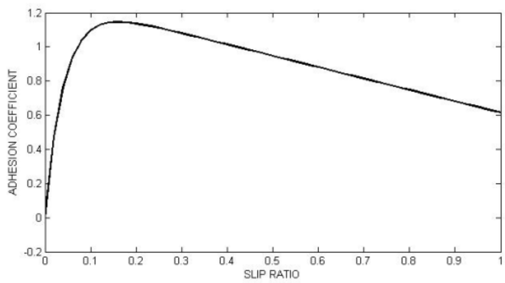

Fig. 7: Plot for $\boldsymbol{\sigma}$ **vs** $\boldsymbol{\mu}$

$$F_x = C_\sigma\left({\sigma}/{1+\sigma}\right)f(k) \quad (15)$$

$$F_y = C_\alpha\left({\tan\alpha}/{1+\sigma}\right)f(k) + C_\gamma\theta \quad (16)$$

Where,

$$k = \frac{\mu F_z(1+\sigma)}{2\sqrt{(C_\sigma\sigma)^2 + (C_\alpha\tan\alpha)^2}}$$

And

$$f(k) = (2-k)k \quad if\ k < 1$$
$$f(k) = 1 \quad if\ k \geq 1$$

E. Tilt dynamics

Tilting of body of vehicle during cornering is a unique feature of NTTW vehicles and the inclusion of this degree of freedom is both interesting and complex because of the coupling effects among tilting, lateral and longitudinal degree of freedoms.

The mathematical model of the tilt dynamics is derived using the first principle methods similar to other sub-blocks. The vehicle body tilt angle and the tyre tilt angle are assumed to be the same and the equations are formulated such that the vehicle is considered to be inverted pendulum [2, 8]. The schematic of the tilting body is shown in Fig. 8 below. The moment generated by the tilt angle shown by theta is effected by the vertical forces on each tyre, tyre lateral force, height of centre of gravity and the track-width of the vehicle. Tilt dynamics block is modelled using (17) as given below.

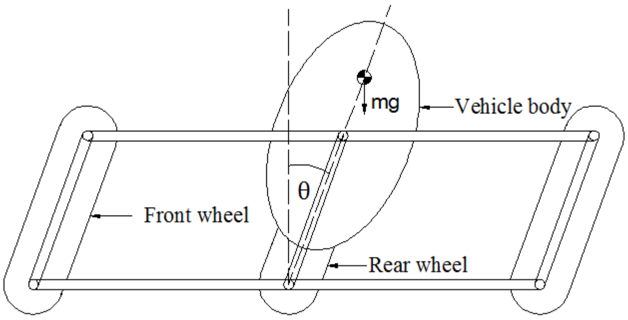

Fig. 8: Tilting body of the vehicle

$$I_z\ddot{\theta} = F_{z_{fr}}\left(\frac{T_w}{2} + h_{C.G}\sin\theta\right) + F_{z_r} * h_{C.G}\sin\theta - F_{z_{fl}}\left(\frac{T_w}{2} - h_{C.G}\sin\theta\right) + F_y h_{C.G}\cos\theta) + M_{tilt_{input}} \quad (17)$$

$$\theta_{des} = tan^{-1}\left(\frac{V_x^2}{Rg}\right) \quad (18)$$

Where, V_x is the longitudinal or forward velocity and R is calculated by $\frac{L}{\delta_f}$

Based on the reference/desired tilt angle, the tilting actuator provides a torque input to the system (tilt dynamics). In this paper, the desired tilt angle is calculated such that the components of gravitational forces and the lateral forces balance each other to keep the vehicle stable [8]. The equation is given by (18) above. The overall block diagram of the tilt dynamics block is shown in Fig. 9.

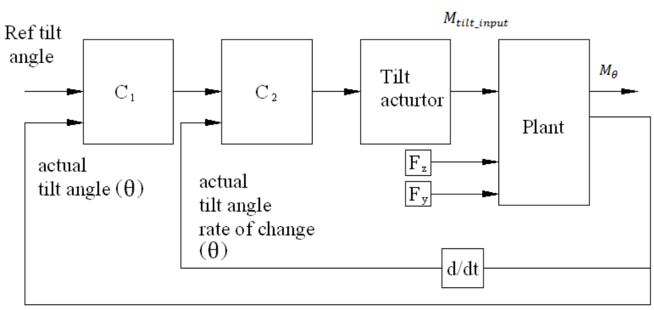

Fig. 9: The overall block diagram of the tilt dynamics model

The tilt actuator is considered as a first order model and controllers C_1 and C_2 are PI controllers. The M_{tilt_input} from the controller and feedbacks from the vertical and lateral tyre forces are provided to the plant model (tilt dynamics model) to finally get the resultant moment generated due to tilting motion. The generated resultant moment is input to the vertical dynamics because this moment will influence the load at the tyres. The tilt angle is also an input to the lateral dynamics block as shown in the lateral dynamics model section. Fig. 10 shows that the control is successfully able to track the given reference tilt angle.

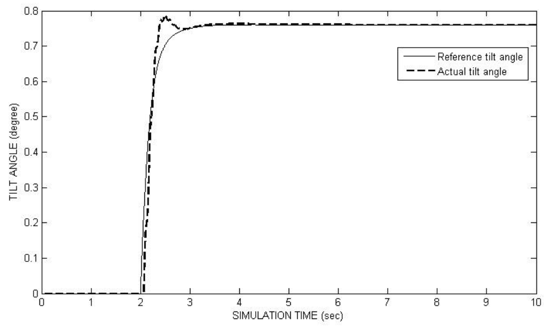

Fig. 10: Tilt angle control plot

Table I gives the vehicle parameters used for modeling the narrow tilting three wheeled vehicle.

TABLE I
PARAMETERS DEFINITION

Parameter	Symbol	Value	Unit
Mass	$m_{C.G}$	450	kg
Wheelbase	L	1.7	m
Distance of centre of gravity to front wheels	Lf	0.6	m
Width	Tw	0.85	m
Height of Centre of gravity	$h_{C.G}$	0.6	m
Tyre diameter	dia	0.55	m

III. RESULTS AND ANALYSIS

The mathematical model is implemented and simulated in Simulink. Each of the blocks are first individually tested and then integrated to analyze their interactions. The model is tested based on relationships between driver inputs and outputs. Plots of the states of the complete model are shown from Fig. 11-14 and showcase the coupling between the degrees of freedom of the vehicle. There are three factors, steering angle to tilt angle, steering angle to settling time of lateral states with and without provision of tilt and influence of longitudinal dynamics on the tilt and lateral dynamics.

A. Effect of tilt on yaw rate and lateral acceleration

The steering ratio, i.e. steering wheel to vehicle wheel angle is assumed as one. During the simulation, the steering angle is step input with final value of 10 degree and the forward speed is 20 kmph. When tilt is provided, the desired tilt angle is calculated as per (18) to be 18 degree.

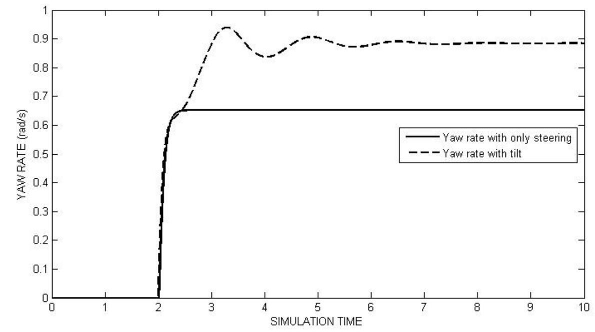

Fig. 11: Yaw rate plot

Fig. 11 shows yaw rate plots with only steering as input (solid line) and with added tilting of vehicle body (dotted line). It is observed that the rate of heading change increases as we tilt the vehicle body while cornering.

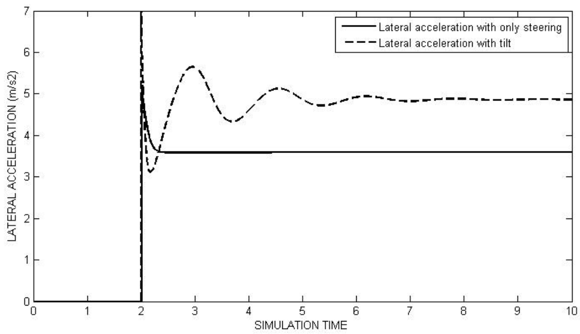

Fig. 12: Lateral acceleration plot

Similarly, as seen in Fig. 12, the value of lateral acceleration increases as tilt is introduced while cornering. These observations are in line with the understanding of mathematical equations and indicate that tilting aids the vehicle in taking sharper and faster maneuvers without losing stability. It can be seen from Fig. 11 and Fig. 12 that the tilting maneuver increases settling time affecting the transient state and hence these aspects must be taken into account by the control strategies.

B. Effect on the trajectory

The tilting of vehicle body while cornering helps in taking quicker and sharper turn as shown in Fig. 13.

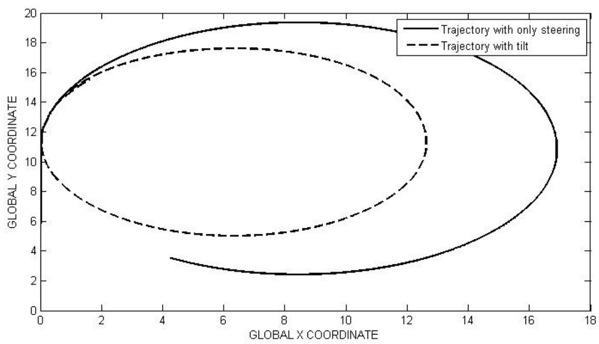

Fig. 13: Trajectory of the vehicle

C. Effect of slip-ratios

In the simulation, the slip ratio is an output from the longitudinal dynamics block which serves as an input to the tyre dynamics block. To study the effect of slip ratios on the lateral dynamics of the vehicle, the simulation is done with the step steering angle input of 20 degree and forward speed as 20 kmph. The desired tilt angle as per (18) is 30 degree. Two cases of slip ratios of 0.1 and 0.01 are considered.

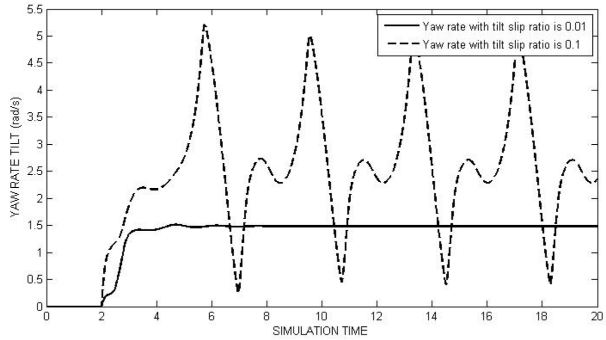

Fig. 24: Yaw rate plots for different slip ratios

Fig. 14 shows yaw rate plots for slip ratios of 0.1 (dotted line) and 0.01(solid line). It is observed that the vehicle becomes unstable when slip ratio is higher for a constant forward speed.

Thus, there is a need to determine an optimal tilt angle value as a function of vehicle wheel slip ratio as well. This shows that the influence of longitudinal dynamics is required to be accounted for finding out the optimal tilt angle while maneuvering.

D. Longitudinal dynamics

The Fig. 15 and 16 show some results from longitudinal dynamics block. The desired forward speed input is 5 kmph and the lateral state inputs are considered to be zero. Thus, the vehicle is moving in the forward direction driven by the in-wheel motors. The torques from the left and right in-wheel motors is identical as we have assumed equal tyre forces on the left and right wheels.

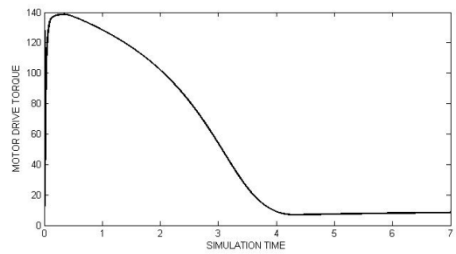

Fig. 15: In-wheel hub motor torque

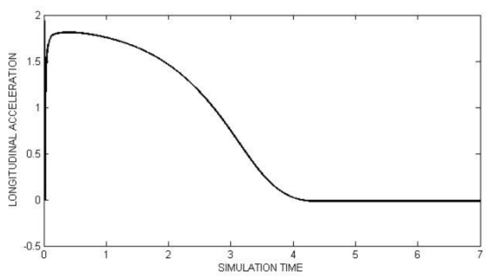

Figure 16: Vehicle longitudinal acceleration profile

Fig.15 shows the required torque input to vehicle dynamics against time of operation. The in-wheel motor torque increases to 140 Nm first and then decreases slowly up to 20 Nm steady state value. Based on the torque provided by the in-wheel motor, the vehicle is moved in forward direction and shows similar longitudinal acceleration profile of the vehicle as shown in Fig. 16. The acceleration profile is similar to the input torque profile of the in-wheel motor, which is expected as per the understanding of dynamics of the vehicle.

IV. CONCLUSION

In order to maintain stability of any vehicle, the control designer must be able to understand the complete behavior of the vehicle in all kinds of maneuvers, i.e. acceleration, braking, cornering, acceleration while cornering and braking while cornering. In this study, it is observed that the tilting of vehicle body improves the handling characteristics for NTTW vehicles. It is also understood that the optimum tilt angle during a maneuver depends on parameters like slip ratio, velocity and steering angle. The relationship among these parameters needs to be formulated and applied in the control strategy to obtain stable behavior of the narrow tilting three wheeled vehicle.

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