

# Local Path Planning for Unmanned Surface Vehicle based on the Improved DWA Algorithm

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**Abstract:** The dynamic windows approach (DWA) is widely used in local path planning, however the path planned by the traditional dynamic window approach will bypass the periphery of the dense obstacle area, which makes the distance longer and close to the dynamic obstacle. In view of the problem, this paper proposes a local path planning algorithm for usv based on the improved dynamic windows approach algorithm. Firstly, the commonly used path planning algorithm and DWA algorithm are briefly introduced. According to the problem of traditional dynamic windows approach, the concept of obstacle search angle is proposed to better deal with the impact of obstacles on USV navigation. Numerical simulation results show that the improved method is superior to the traditional algorithm in path length, navigation time and average speed, and has stronger obstacle avoidance adjustment ability.

**Key Words:** unmanned surface vehicle, local path planning, dynamic windows approach, collision avoidance, optimal path

## 1 Introduction

With the development of information and communication technology and artificial intelligence technology, USVs with the advantages of unmanned operation, intelligence, low cost and high efficiency will occupy an important position in the future marine development and intelligent equipment application. Path planning is one of the key research directions of unmanned surface vehicle(USV). It is the basis and premise for USV to complete all navigation tasks. In the marine environment with obstacles, finding an optimal path from the initial state to the target state according to the performance indicators such as the shortest path, the lowest work cost and the highest security is an important factor to measure its intelligence level.

Path planning can be divided into global path planning which focuses on optimal path and local path planning focusing on safe obstacle avoidance. Global path planning is also called planning based on map. It avoids known static obstacles and looks for the optimal path according to the prior knowledge of the environment [1]. Common global path planning algorithms include planning algorithms based on research, such as Dijkstra algorithm [2], best first search algorithm [3], A\* algorithm [4], planning algorithms based on sampling, such as PRM algorithm [5], RRT algorithm [6], intelligent planning algorithms, such as ant colony algorithm [7], genetic algorithm [8], and improved algorithms based on these algorithms. However, the premise of these global path planning algorithms is ideal. When there are unknown or dynamic obstacles in the working environment of USV, local path planning algorithm needs to be used to realize real-time obstacle avoidance.

Local path planning, also known as dynamic re planning, recalculates the path, generates reasonable path points, and avoids unknown dynamic obstacles by controlling the navigation speed and direction of USV in real time. The main local path planning algorithms include artificial potential field [9], velocity obstacle algorithm [10], dynamic window approach (DWA algorithm) [11-16]. The principle

of artificial potential field is to regard the target point as the gravitational point and the obstacle as the repulsive point. The USV advances under the joint action of gravity and repulsion, but it is easy to fall into the local optimal solution. Sang et al. [17] divided the global path generated by A\* algorithm into sub target points and formed a sequence of sub target points. By converting the target points, the probability of falling into the local optimal solution of the artificial potential field method can be reduced. Dynamic window method is a local path planning algorithm that has been studied more recently. On the premise of ensuring no collision with obstacles, it obtains all feasible velocity angular velocity combinations of USV, predicts the motion trajectory generated after a certain time, and selects the best state combination through the evaluation function, so as to obtain the velocity angular velocity that can be adopted at the moment, Until the USV reaches the end. Han et al. [11] adaptively adjusted the weight of each item of the evaluation function according to the real-time distance between the USV and the obstacle, so that the operation strategy of the USV can be improved with the change of the environment. Li et al. [13] Based on the evaluation function of traditional DWA algorithm, divided the obstacle distance evaluation function into two items, representing the shortest distance between known obstacles and unknown obstacles respectively. Zhang et al. [16] further subdivided the evaluation function into seven items and introduced the distance between the tail point of the track and the target point, local target point and global planning path. Although it is considered in detail, it is too complicated and difficult to deal with the coefficient weight between the seven sub functions.

Based on the traditional DWA algorithm, this paper introduces the concept of obstacle search angle, that is, when the static obstacle is not in a certain angle range of the forward direction of the USV, its influence on the subsequent movement of the USV is no longer considered. Moreover, the improved algorithm improves the adaptability

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of obstacle avoidance in different scenes. Finally, the improved method is verified by numerical simulation.

The rest of this paper is organized as follows. The traditional DWA algorithm is introduced in next section. Section 3 briefly introduces improvement strategy for it. In section 4, the simulation will show the results of improved DWA compared with traditional DWA's from various obstacle maps. Conclusions and some later research planning are summarized in final section.

## 2 Traditional DWA algorithm

The dynamic window method transforms the position constraint problem of the USV into the speed constraint problem, and transforms the obstacle avoidance problem of the USV into the optimal speed execution problem. The trajectory of the ship to be sampled and the speed limit of the environment to be sampled are fully considered for the evaluation of the algorithm, and the maximum speed of the ship to be sampled in the simulation window is fully considered for the evaluation of the trajectory to be sampled according to the time constraint of the obstacle. The path obtained by this algorithm not only realizes safe obstacle avoidance, but also fully considers the motion characteristics of USV. It has high reliability. It is a common algorithm in local path planning.

When using velocity mode to predict the trajectory of USV, it is necessary to analyze the motion model of the ship.  $v(t)$  and  $w(t)$  represent the linear velocity and angular velocity of the USV in the geodetic coordinate system respectively,  $\psi(t)$  is the heading angle of USV. In each sampling period, the moving track of the ship is approximated, and the moving path in each sampling period is regarded as a straight line, so the position  $(x(t+1), y(t+1))$  at time  $t+1$  and the heading angle of the unmanned surface vehicle are as follows:

$$\begin{aligned} x(t+1) &= x(t) + v(t)\Delta t \cos(\psi(t)) \\ y(t+1) &= y(t) + v(t)\Delta t \sin(\psi(t)) \\ \psi(t+1) &= \psi(t) + w(t)\Delta t \end{aligned} \quad (1)$$

That is, the USV moves  $v(t)\Delta t$  linearly along the heading angle direction, and its heading angle rotates  $w(t)\Delta t$  relative to the geodetic coordinate system.

According to the motion model of the USV, the trajectory can be calculated on the basis of obtaining the speed. Therefore, the two cores of the dynamic window algorithm are: ① Form speed constraints according to the obstacle environment and the motion characteristics of the USV, and generate a dynamic window for speed sampling. ② According to the evaluation function, the predicted trajectory corresponding to the sampled speed is scored, so as to obtain the optimal path and execute it. During speed sampling, the speed of USV is mainly subject to the following constraints:

(1) The unmanned surface vehicle is limited by its maximum and minimum speed, which is also the maximum range of the speed of DWA algorithm:

$$V_i = \{(v, w) \mid v \in [v_{\min}, v_{\max}], w \in [w_{\min}, w_{\max}]\} \quad (2)$$

(2) Due to the influence of the USV's own motor, the torque provided by its speed increase and deceleration is limited. Therefore, in the cycle of simulating the forward movement of the USV, there is a dynamic window, that is, the speed in the window is the actual speed  $V_j$  that the USV can achieve under the influence of its own dynamic characteristics:

$$V_j = \left\{ (v, w) \mid \begin{array}{l} v \in [v_c - \dot{v}_b \Delta t, v_c + \dot{v}_a \Delta t] \\ w \in [w_c - \dot{w}_b \Delta t, w_c + \dot{w}_a \Delta t] \end{array} \right\} \quad (3)$$

Where:  $v_c$  is the current linear speed of the USV;  $\dot{v}_a$  ( $\dot{v}_b$ ) is the maximum acceleration (deceleration) linear speed of USV;  $w_c$  is the current angular speed of the ship;  $\dot{w}_a$  ( $\dot{w}_b$ ) is the maximum acceleration (deceleration) angular velocity of the ship.

In order to achieve safe obstacle avoidance and avoid collision with obstacles occupying a certain space, the range  $V_k$  can be obtained under the condition of deceleration and maximum acceleration to further reduce the dynamic window range:

$$V_k = \left\{ (v, w) \mid \begin{array}{l} v \leq \sqrt{2 \cdot \text{dist}(v, w) \cdot \dot{v}_b} \\ w \leq \sqrt{2 \cdot \text{dist}(v, w) \cdot \dot{w}_b} \end{array} \right\} \quad (4)$$

where  $\text{dist}(v, w)$  is the minimum obstacle distance on the predicted trajectory of the corresponding speed.

To sum up, according to the mechanical characteristics of the USV and the obstacle environment, the dynamic window can be defined as

$$V_r = V_i \cap V_j \cap V_k \quad (5)$$

The motion trajectory is mainly generated according to the sampling points of each linear velocity and angular velocity of the USV and the forward simulation time  $t_{\text{sim}}$ .

After obtaining the motion trajectory of the USV, the evaluation function is required to score each path, and the one with the highest score is selected as the comprehensive optimal path and executed:

$$G(v, w) = \alpha \cdot \text{heading}(v, w) + \beta \cdot \text{distance}(v, w) + \gamma \cdot \text{velocity}(v, w) \quad (6)$$

where:  $\text{heading}(v, w)$  is the deflection angle evaluation sub function, which is used to evaluate the angle difference between the end direction of the track and the target point at the simulated track speed, and its formula is  $180^\circ - \theta$  (The smaller the  $\theta$ , the higher the score, among which,  $\theta$  is the angle between the end point of the sampled trajectory and the connecting line between the USV and the target point). The main function of this sub function is to promote the USV to keep its azimuth towards the target point during its movement;  $\text{distance}(v, w)$  is the safety factor evaluation sub function, which is used to eliminate the sampling paths that may collide or contact with obstacles, so as to realize the safe obstacle avoidance of USV s. In order to avoid the excessive proportion of the evaluation function, when scoring the sampling paths without obstacles, the safety factor evaluation sub function is set as a constant;  $\text{velocity}(v, w)$  is the velocity evaluation sub function, which is used to select the fastest path in the sampling trajectory that can achieve

safe obstacle avoidance, so as to reach the target point as soon as possible.

### 3 Improved DWA algorithm

As shown in Figure 1,  $P$  is the predicted position of the USV after the forward simulation time  $t_{sim}$ ,  $v$  is the predicted motion direction of the USV,  $a$ ,  $b$  and  $c$  are the static obstacles around it,  $d_1$ ,  $d_2$ ,  $d_3$  is the distance between the USV and the obstacle respectively,  $d_3 < d_1 < d_2$ . According to the conventional DWA algorithm, the shortest distance between the USV and the obstacle is  $dist = d_3$ . However, according to the moving direction of the ship, the USV has bypassed the static obstacle  $c$  at this time, and will not collide with the obstacle  $c$  in subsequent navigation operations. However, when  $dist$  takes the smaller value  $d_3$ , according to equation (4), the maximum speed of the USV will be affected in terms of braking distance. According to equation (6), the evaluation function value of the predicted state will also be low due to the small  $dist$  value, so that the ship will abandon this predicted state and choose other obstacle avoidance trajectories, so as to increase the navigation time and distance of the USV. Therefore, at this time, the existence of obstacle  $c$  should not be considered when calculating the minimum obstacle distance of USV.

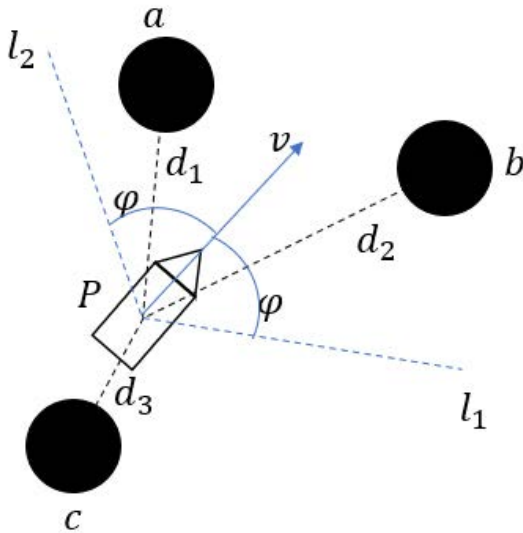


Fig.1: Obstacle avoidance of USV

Take  $P$  as the vertex and make two rays so that the included angles  $\angle l_1 P V$  and  $\angle V P l_2$  are  $\varphi$ , which is the obstacle search angle. When the obstacle expansion circle is not in the sector  $(l_2 P l_1)$  and there is no intersection with rays  $l_1$ ,  $l_2$ , the impact of the obstacle on the subsequent navigation of the ship is not considered. As shown in Figure 1, at this time, the nearest distance between the USV and the obstacle  $dist = d_1$ .

### 4 Simulation experiment

In the simulation experiment, we compare the running results of the conventional DWA algorithm with the improved algorithm. The static obstacles are expanded according to the same radius, and the USV is regarded as a

particle. The weight of USV state parameters and evaluation function is shown in the table below:

Table 1: Simulation experiment parameter

Parameter	Value	Parameter	Value
$v_{max}$	1.0 m/s	$v_{min}$	-1.0 m/s
$w_{max}$	0.5236 rad/s	$w_{min}$	-0.5236 rad/s
$\dot{v}_a$	0.2 m/s <sup>2</sup>	$\dot{v}_b$	-0.2 m/s <sup>2</sup>
$\dot{w}_a$	0.8727 rad/s <sup>2</sup>	$\dot{w}_b$	-0.8727 rad/s <sup>2</sup>
$\alpha$	0.05	$\beta$	0.2
$\gamma$	0.1	$t_{sim}$	3.0 s
$\varphi$	$\pi/3$		
$r$	Radius of obstacle expansion circle		

1) Simulation experiment of different obstacle radius on the same map

First, the obstacles are randomly generated. In the same map, the obstacles are expanded according to the expansion circle radius of 0.2m, 0.5m and 0.7m respectively. Considering that the ship can navigate effectively when the expansion radius is 0.7m, the obstacle rate is 0.2. The simulation results of traditional algorithm and improved algorithm under different expansion radius are shown in Figure 2, 3 and 4, and the results are shown in Table 2.

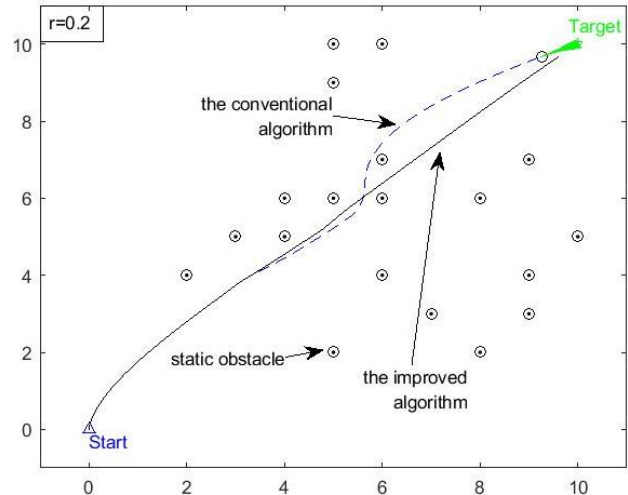


Fig.2: Comparison diagram of simulation path when the radius of obstacle expansion circle is 0.2m

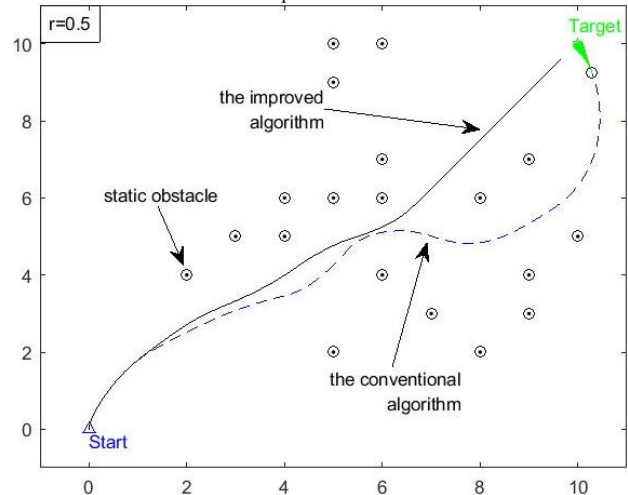


Fig.3: Comparison diagram of simulation path when the radius of obstacle expansion circle is 0.5m



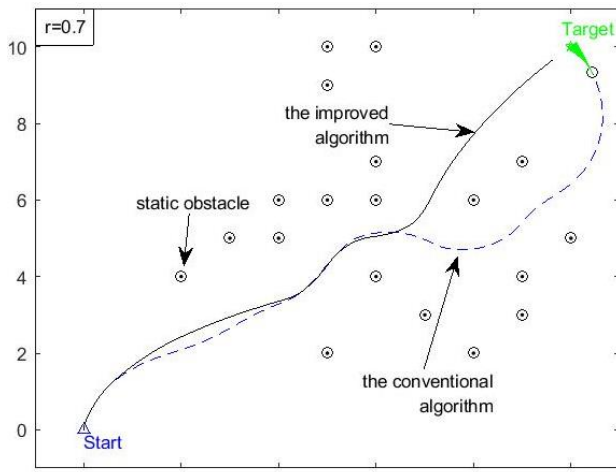


Fig.4: Comparison diagram of simulation path when the radius of obstacle expansion circle is 0.7m

Table 2: Table of obstacle avoidance simulation results of static obstacles with the same obstacle and different expansion circle radius

Radius of obstacle expansion circle(m)	0.2	0.5	0.7
Navigation time of traditional algorithm(s)	52.62	47.46	61.62
Navigation time of improved algorithm(s)	24.47	22.66	36.24
Path length of traditional algorithm(m)	14.68	16.60	16.86
Path length of improved algorithm(m)	14.26	14.42	14.69
Average speed of traditional algorithm(m/s)	0.28	0.35	0.27
Average speed of improved algorithm(m/s)	0.58	0.64	0.41

It can be observed from the above figures 2, 3 and 4 that when the static obstacles are distributed in the same way, on the premise of keeping a safe distance from the obstacles, compared with the traditional algorithm, the path generated by the improved algorithm can pass between the obstacles better, and the path turns less, which is conducive to the steering and navigation of USVs. When the radius of the obstacle expansion circle is too large or too small, the paths choose to bypass or pass through, so the path improvement effect is not obvious. When the radius of the obstacle expansion circle is 0.5, the path improvement effect is more obvious. It can be seen from table 2 that when the radius of the obstacle expansion circle is 0.2, the path generated by the improved algorithm is 2.9% shorter than that of the traditional algorithm, 13.1% shorter when the radius of the obstacle is 0.5, and 12.9% shorter when the radius of the obstacle is 0.7. In terms of time, the improved algorithm can save half of the time. When the obstacle radius is 0.5, the time can be saved by 52.9% at most. In terms of speed improvement, when the obstacle radius is 0.2, the average

speed can be increased by 1.07 times, when the obstacle radius is 0.5, the average speed can be increased by 0.83 times, and when the obstacle radius is 0.7, the average speed can be increased by 0.52 times. Therefore, compared with the traditional algorithm, the improved algorithm has great improvement in path length, navigation time and average speed.

## 2) Simulation experiment of different obstacle maps

Different obstacle maps are randomly generated according to different obstacle rates. When the radius  $r$  of obstacle expansion circle is 0.2m, 0.5m and 0.7m respectively, the obstacle rates are 0.3, 0.3 and 0.2 respectively. The simulation results of traditional algorithm and improved algorithm in different obstacle maps are shown in the figure below, and the results are shown in Table 3.

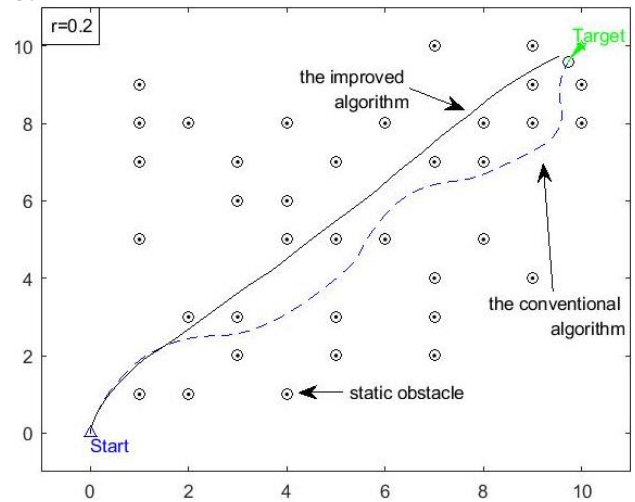


Fig.5: Comparison diagram of simulation path when the radius of obstacle expansion circle is = 0.2

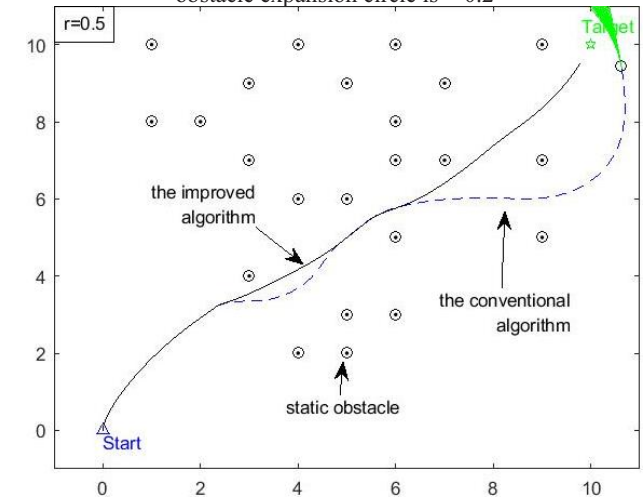


Fig.6: Comparison diagram of simulation path when the radius of obstacle expansion circle is = 0.5

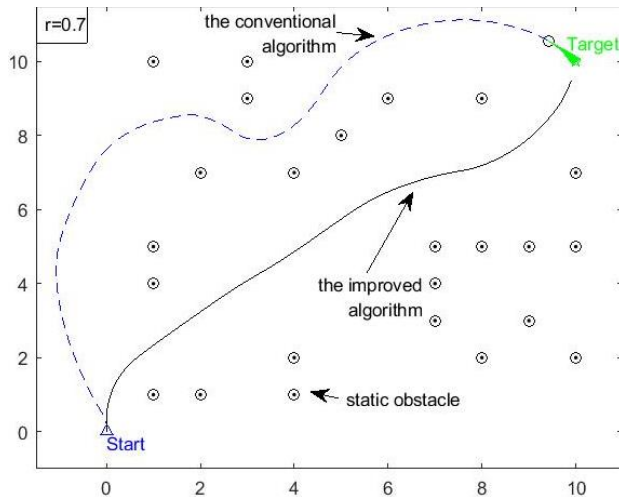


Fig.7: Comparison diagram of simulation path when the radius of obstacle expansion circle is = 0.7

Table 3: Comparison of obstacle avoidance simulation results of improved algorithm

Radius of obstacle expansion circle (m)	0.2	0.5	0.7
Navigation time of traditional algorithm(s)	62.62	77.50	60.60
Navigation time of improved algorithm(s)	26.63	34.33	21.13

Table 4: Comparison of static obstacle avoidance simulation results of improved algorithm

Radius of obstacle expansion circle (m)	Average sailing time			Average path length			Average sailing speed		
	Traditional algorithm(s)	Improved algorithm(s)	Average reduction rate	Traditional algorithm (m)	Improved algorithm (m)	Average reduction rate	Traditional algorithm (m/s)	Improved algorithm (m/s)	Average increase rate
0.2	53.17	25.92	50.62%	14.87	14.27	3.95%	0.28	0.55	97.02%
0.5	58.82	33.40	42.52%	16.60	14.62	11.07%	0.31	0.48	60.76%
0.7	57.84	27.03	53.91%	18.50	15.04	16.54%	0.36	0.66	84.32%

In order to verify the adaptability of the improved algorithm, 20 different obstacle maps are randomly generated according to different obstacle rates on the basis of simulation 2). Table 4 shows the experimental results. The analysis of the experimental results shows that when the radius of the expansion circle of the obstacle is  $r = 0.2$ , the traditional algorithm and the improved algorithm have less bends in selecting the path, and can better cross between the obstacle and the obstacle. Therefore, the path length is not improved, and the average distance shortening rate is 3.95%. Because the improved algorithm only considers the obstacles that will affect the subsequent navigation of the USV when calculating the obstacle distance, the minimum obstacle distance obtained is greater than that of the traditional algorithm, which makes the speed selection of the USV larger, and the average speed improvement rate is 97.02%. Therefore, the navigation time can be greatly shortened, and the average reduction rate is 50.62%. When the radius of the obstacle expansion circle is  $r = 0.5$  and  $0.7$ , compared with the traditional algorithm, the improved algorithm prefers to walk around the obstacles. When selecting the path, the improved algorithm can better walk between the obstacles, with fewer path turns and shorter distance. With the increase of the radius of the obstacle

Path length of traditional algorithm(m)	15.31	17.13	20.20
Path length of improved algorithm(m)	14.30	14.46	14.88
Average speed of traditional algorithm(m/s)	0.24	0.22	0.33
Average speed of improved algorithm(m/s)	0.57	0.42	0.70

Figures 5, 6 and 7 show the simulation path comparison effects of traditional algorithm and improved algorithm in different maps, and table 3 shows the corresponding experimental results. It can be seen from the figure that although the obstacle distribution is different, compared with the traditional algorithm, the path generated by the improved algorithm has less bends and shorter length. The same results as those in Table 2 can be obtained from table 3. The improved algorithm is superior to the traditional algorithm in path length, navigation time and average speed. 3) Simulation experiment of multiple groups of different obstacle maps

In order to verify the adaptability of the improved algorithm, on the basis of simulation ②, 20 different obstacle maps are randomly generated according to different obstacle rates. When the radius of obstacle expansion circle is 0.2m, 0.5m and 0.7m respectively, the obstacle rates are 0.3, 0.3 and 0.2 respectively. The average value of 20 simulation experiment results is shown in Table 4.

expansion circle, the effect of road shortening is more obvious. When  $r = 0.7$ , the average distance shortening rate can reach 16.54%. At the same time, the improved algorithm can also improve the navigation speed of USVs, so the average reduction rate of navigation time of the improved algorithm is maintained at 40% - 50%. Experimental results verify the adaptability of the improved algorithm.

## 5 Conclusion

Aiming at the static obstacles that may be faced by the USV in the navigation environment, this paper classifies the static obstacles according to the navigation direction of the ship. When the static obstacles are not within the obstacle search angle, the impact on the subsequent navigation operation of the USV is not considered. The simulation results show that the improved DWA algorithm is superior to the conventional algorithm in path length, navigation time and average speed, and has stronger adjustment ability of obstacle avoidance. However, this paper only carried out the comparative experiment between the improved algorithm and the conventional DWA algorithm, and did not compare with other improved algorithms, which is the deficiency of this paper and the direction of future work.

This paper presents an effective method to realize the local path planning of USV. The results show the feasibility and applicability of the improved algorithm. The following work can consider combining this method with the global path planning algorithm to further improve the efficiency and engineering practicability of path planning. In addition, COLREGs regulations will be combined to obtain better local obstacle avoidance effect.

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