Simultaneous Productions:
A Fully General Grammar Specification

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2021-06-10

Contents

1 Motivation for a New Grammar Specification

This paper is the first of several on a parsing method we will refer to as "Simultaneous Productions" (or "S.P." for short).zThis name was chosen to emphasize two goals of this method:

1. An S.P. *grammar* is composed of a set of *productions*, very similar to most existing concepts of formal grammars [1]. However, unlike many common parsing algorithms [2], an S.P. grammar can represent a recursively enumerable language [3].

2. When parsing a string, these productions can be independently evaluated over separate parts of the input string, and adjacent successful matches can then be merged to form a successful parse. Unlike many common parsing algorithms [4], this feature avoids any intrinsic serial dependencies that require parsing the beginning of the string first, hence allowing for *parallelism* during the parsing process.

1.1 Goals

We have noted that the above features are not shared by many commonly-used parsing algorithms [5]. In this paper, we will describe the S.P. *grammar-grammar*, i.e. the specification which defines any S.P. grammar.

The S.P. model was created to make grammar definitions significantly more composable by **separating the entry point for parsing/string matching from the rest of the grammar's definition**. The hope is that this can allow for **global agreement upon a safe reliable subset of "primitive" parsers (e.g. integer and float parsers) across codebases**, while end users are still able to add custom logic on top seamlessly.

We hope to show that:

1. The S.P. grammar-grammar should be proven equivalent to Chomsky's canonical specification of an formal language [6].

2. The S.P. grammar input format should be able to represent all rungs of the Chomsky hierarchy distinctly to the *grammar-writer*, who may then pick and choose the kind of complexity they want instead of getting surprised. For example:

    - regular expressions (DFAs) [7],

[1] cite: chomsky formal grammars

[2] cite: common parsing algorithms – frequency and power? maybe cite history of parsing page?

[3] cite: what is RecEnum?

[4] cite: might need to expand on this more? need to think about what "common" means

[5] cite: the history of parsing webpage

[6] cite: chomsky formal grammars again, maybe just turing machine?

[7] cite: what are DFAs/regex

- EBNF syntax commonly used for CFGs [8],
- regex with backrefs (recursively enumerable?) [9].

3. Furthermore, representing a grammar with S.P. **should follow strongly a "pay for what you use" policy**, i.e. having more complex features should not create needless complexity for users who only use the simpler features. **This requires creating a hierarchical-ish concept of feature complexity.**

4. Separate the output of a parse from the algorithm that produces it: see (section 1.2). We hope that **standardizing parser output** into the fully general [10] PoP format as well as **sharing high-quality grammars in a modular way** can reduce what we see as a frustrating amount of mental overhead and duplicated work for everyone who wants to write any sort of compiler or interpreter. We believe this situation can be improved.

1.2 No Parsing Allowed

We will not rely on the details of any specific parsing algorithm. Indeed, we will avoid introducing any iterative algorithms at all in this paper. Further paper(s) will describe an efficient parsing algorithm to evaluate an S.P. grammar over a specific input string.

The intention of this separation is to allow the S.P. grammar-grammar to be reviewed and criticized separately from the evaluation method. This is done because the author believes that the S.P. grammar-grammar has merit in itself, as a "lingua franca" for *executable formal grammars*, that is, grammars which can be efficiently parsed by computer.

1.3 Notation

- Named concepts in the rest of this paper will be represented in *italics* when first defined.
- Capital letters generally refer to sets, while lowercase letters generally refer to elements of some set. This may not be true in all cases.
- As abbreviations:
  - $[n] = [1, n] \, \forall \, n \in \mathbb{N}$.
  - $|$ should be translated as "for some" or "for which".

[8] cite: use of EBNF for CFGs
[9] cite: backrefs are RecEnum?
[10] is this true?

2 Definition

We first define the *S.P. grammar-grammar*, as a kind of meta-grammar which specifies all concrete S.P. grammars. We use the term grammar-grammar to emphasize the regular structure of an S.P. grammar. We believe this formulation is relatively simple to analyze, and in subsequent work we will demonstrate that it admits a relatively performant *parsing algorithm*, or *evaluation method*. It is possible this representation can be further improved.

2.1 S.P. Grammar-Grammar

$$SP = (\Sigma, \mathcal{P}). \tag{1}$$

An *S.P. grammar* SP is a 2-tuple with an arbitrary finite set Σ and a finite set of productions $\mathcal{P}$ defined in (equation 2). We refer to Σ as the *alphabet*.

$$p = \mathcal{C}_p \, \forall \, p \in \mathcal{P}. \tag{2}$$

Each *production* p is a finite set of cases $\mathcal{C}_p$, defined in (equation 3).

$$c_p = \{e_j\}_{j=1}^{m} \, \forall \, c_p \in \mathcal{C}_p. \tag{3}$$

Each *case* c_p is a finite sequence of case elements $\{e_j\}_{j=1}^{m}$, where m is the number of elements in the sequence c_p, with e_j defined in (equation 4).

$$e_j = \left\{ \begin{array}{c} t \in \Sigma, \\ p \in \mathcal{P}. \end{array} \right\} \forall \, j \in [m]. \tag{4}$$

Each *case element* e_j is either a *terminal* $t \in \Sigma$ or *nonterminal* $p \in \mathcal{P}$.

3 PoP: Proof of Parsing

We describe a tree-like data structure "proof of parsing" (PoP) which retains sufficient information to validate that a string belongs to a recursively enumerable language, as well as the validation function "validate" applied to the data structure.

3.1 Grammar Specialization

$$p^* \in \mathcal{P}. \tag{5}$$

An S.P. grammar SP alone is not sufficient information to unambiguously parse a string – a specific production must also be provided (we have defined it to be this way). Therefore to get an *executable grammar*, we select a single "top" production $p^* \in \mathcal{P}$. This is vaguely reminiscent of the *start symbol* found in Chomsky grammars (section 5).

TODO: this mechanism is the same way we should use for specifying precedence of non-top productions when a parse is completely ambiguous in a subtree!!!

$$SP^* = (\Sigma, \mathcal{P}, p^*). \tag{6}$$

The tuple SP^* formed from the selection of p^* is referred to as a *specialized grammar*. For this reason, we may also refer to a grammar SP (without having chosen any p^* yet) as an *unspecialized grammar*.

3.2 Input Specification

$$I = \{t_i\}_{i=1}^{n} \,|\, t_i \in \Sigma \,\forall\, i \in [n]. \tag{7}$$

An *input string* I is a finite sequence of *tokens* $\{t_i\}_{i=1}^{n}$ from the alphabet Σ, where $|I| = n$ is the number of elements in the sequence I.

TODO: we should not distinguish the act of parsing a finite vs infinite stream!

3.3 Matching a String

We represent the conditions necessary to match a production $p \in \mathcal{P}$ recursively, by defining the "matches" function over multiple separate domains:

1. $\text{matches}_{(\mathcal{P})}$: the top-level function that matches against a single production p.
2. $\text{matches}_{(\mathcal{C}_p)}$: matches against a single case c_p from a production p.
3. $\text{matches}_{(e_j)}$: matches against a single case element e_j from the case c_p.

3.3.1 Matching a Production $p \in \mathcal{P}$

$$\begin{aligned} \text{matches}_{(\mathcal{P})} &: \mathcal{P} \times \{I\} \to \{\text{true}, \text{false}\}. \\ \text{matches}_{(\mathcal{P})}(p, I) &= \{\, \exists\, c_p \in \mathcal{C}_p \,|\, \text{matches}_{(\mathcal{C}_p)}(c_p, I) \Leftrightarrow \text{true}\}. \end{aligned} \tag{8}$$

A production $p \in \mathcal{P}$ *matches* an input string I when **any** of its cases $c_p \in \mathcal{C}_p$ match I via $\text{matches}_{(\mathcal{C}_p)}$ as defined in (equation 9).

3.3.2 Matching a Case $c_p \in \mathcal{C}_p$

$$\begin{aligned} \text{matches}_{(\mathcal{C}_p)} &: \mathcal{C}_p \times \{I\} \to \{\text{true}, \text{false}\}. \\ \text{matches}_{(\mathcal{C}_p)}(c_p, I) &= \{\textbf{TODO: ???}\}. \end{aligned} \tag{9}$$

TODO: >= context-sensitive can be modelled as coroutines, compared to the immediate function call of DFA/CFGs?

3.3.3 Matching a Case Element $e_j \in c_p$

$$\left\{ \begin{array}{lllll} e_j = t \in \Sigma & \Rightarrow & \bar{I} = \bar{I}_{l_1,l_2}, l_1 = l_2, I_{l_1} = t & \Leftrightarrow & \text{true}, \\ e_j = p' \in \mathcal{P} & \Rightarrow & \text{matches}_{(\mathcal{P})}(SP, p', \bar{I}) & \Leftrightarrow & \text{true}. \end{array} \right\} \begin{array}{l} [m] \times \{\bar{I}\} \to \{\text{true}, \text{false}\} \quad : \text{matches}_{(e_j)}. \\ = \text{matches}_{(e_j)}(j, \bar{I}). \end{array} \tag{10}$$

A case element e_j matches an input subsequence $\bar{I}$ when e_j is a token $t \in \Sigma$, in which case $\bar{I}$ is a length-1 substring of I containing the single token t, or when e_j is a production p', in which case the subsequence $\bar{I}$ must match the production p' as defined in (equation 8).

3.4 Summary of Adjacency

At this stage, we note a few important points:

1. A production p may match a finite or countably infinite number of subsequences $\bar{I}$ of I, not just one. So, if we say p matches $\bar{I}$ for some case c_p, it may still match other subsequences $\bar{I}'$, either for the same case c_p, or other cases $c'_p \in \mathcal{C}_p$.

4 Proof of Turing-Equivalence

TODO: do reduction from lambda calculus!!

5 Chomsky Equivalence

We have described an S.P. grammar $SP = (\Sigma, \mathcal{P})$ (section 2.1), and we have *specialized* the grammar into $SP^* = (\Sigma, \mathcal{P}, p^*)$ by selecting a production $p \in \mathcal{P}$ (section 3.1). We have described the conditions under which p^* is said to successfully match an input string I consisting of tokens from Σ (section 3.3).

We attempt to directly reduce the canonical specification of a formal grammar (often attributed to Noam Chomsky) into specialized or executable form SP^* [11].

5.1 Chomsky Construct

The definition of a "formal grammar" we copy from **TODO: cite!!!** as follows [12]:

[11] cite: chomsky grammars
[12] cite: chomsky!!!

5.1.1 Formal Grammar Definition

$$\Sigma\text{: terminal symbols.} \tag{11}$$
$$N\text{: nonterminal symbols.} \tag{12}$$
$$S\text{: start symbol.} \in N \tag{13}$$
$$P\text{: productions.} \tag{14}$$
$$P = \{(\Sigma \cup N)^* N (\Sigma \cup N)^* \to (\Sigma \cup N)^*\}. \tag{15}$$
$$G = (N, \Sigma, P, S). \tag{16}$$

5.1.2 Parsing a Formal Grammar

asdf

5.2 Equivalence Proof

We will perform a Cook-Levin-style reduction from a Chomsky grammar $G = (N, \Sigma, P, S)$ (section 5.1) into a specialized S.P grammar $SP^* = (\Sigma, \mathcal{P}, p^*)$ (section 3.1) [13].

5.2.1 Construction of Σ_{SP^*}

$$\Sigma_{SP^*} = \Sigma_G. \tag{17}$$

The alphabet Σ is exactly the same in both S.P. and the Chomsky formulation.

5.2.2 Construction of $\mathcal{P}$

$$asdf \tag{18}$$

6 Relevant Prior Art / Notes

6.1 Overview

asdf

[13] cite: cook-levin!