Worksheet 14

To accompany Chapter 5.3 Fourier Transforms for Circuit and LTI Systems Analysis

We will step through this worksheet in class.

You are expected to have at least watched the video presentation of [Chapter 5.3 (https://cpjobling.github.io/eg-247-textbook/fourier_transform/3/ft3)](https://cpjobling.github.io/eg-247-textbook/fourier_transform/3/ft3) of the [notes (https://cpjobling.github.io/eg-247-textbook)](https://cpjobling.github.io/eg-247-textbook) before coming to class. If you haven't watch it afterwards!

The System Function

System response from system impulse response

Recall that the convolution integral of a system with impulse response $h(t)$ and input $u(t)$ is

$$h(t) * u(t) = \int_{-\infty}^{\infty} h(t-\tau)u(\tau)\, d\tau.$$

We let

$$g(t) = h(t) * u(t)$$

Then by the time convolution property

$$h(t) * u(t) = g(t) \Leftrightarrow G(\omega) = H(\omega).U(\omega)$$

The System Function

We call $H(\omega)$ the *system function*.

We note that the system function $H(\omega)$ and the impulse response $h(t)$ form the Fourier transform pair

$$h(t) \Leftrightarrow H(\omega)$$

Obtaining system response

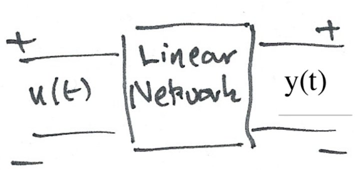
If we know the impulse resonse $h(t)$, we can compute the system response $g(t)$ of any input $u(t)$ by multiplying the Fourier transforms of $H(\omega)$ and $U(\omega)$ to obtain $G(\omega)$. Then we take the inverse Fourier transform of $G(\omega)$ to obtain the response $g(t)$.

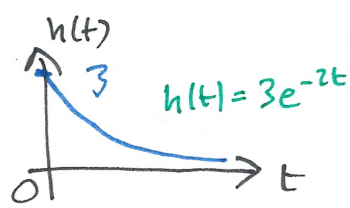
1. Transform $h(t) \rightarrow H(\omega)$
2. Transform $u(t) \rightarrow U(\omega)$
3. Compute $G(\omega) = H(\omega). U(\omega)$
4. Find $\mathcal{F}^{-1}\left\{G(\omega)\right\} \rightarrow g(t)$

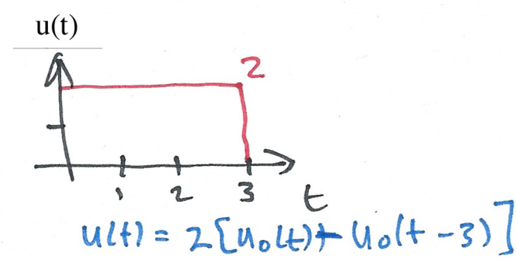
Examples

Example 1

Karris example 8.8: for the linear network shown below, the impulse response is $h(t) = 3e^{-2t}$. Use the Fourier transform to compute the response $y(t)$ when the input $u(t) = 2[u_0(t) - u_0(t-3)]$. Verify the result with MATLAB.







Solution

Matlab verification

In [ ]:

```
syms t w
U1 = fourier(2*heaviside(t),t,w)
```

In [ ]:

```
H = fourier(3*exp(-2*t)*heaviside(t),t,w)
```

In [ ]:

```
Y1=simplify(H*U1)
```

In [ ]:

```
y1 = simplify(ifourier(Y1,w,t))
```

Get y2

Substitute $t - 3$ into t.

In [ ]:

```
y2 = subs(y1,t,t-3)
```

In [ ]:

```
y = y1 - y2
```

Plot result

In [ ]:

```
ezplot(y)
title('Solution to Example 1')
ylabel('y(t)')
xlabel('t [s]')
grid
```

See [ft3_ex1.m (https://cpjobling.github.io/eg-247-textbook/fourier_transform/3/ft3_ex1.m)](https://cpjobling.github.io/eg-247-textbook/fourier_transform/3/ft3_ex1.m)

Result is equivalent to:

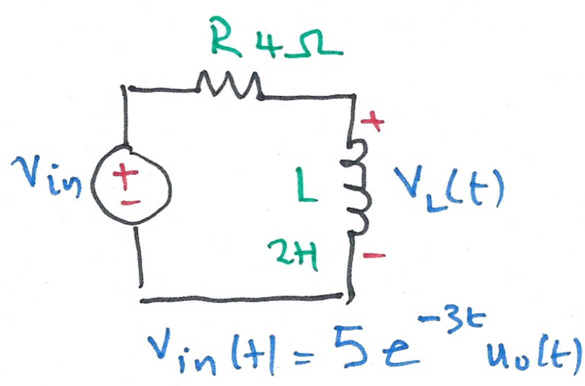
```
y = 3*heaviside(t) - 3*heaviside(t - 3) + 3*heaviside(t
- 3)*exp(6 - 2*t) - 3*exp(-2*t)*heaviside(t)
```

Which after gathering terms gives

$$y(t) = 3(1 - 3e^{-2t})u_0(t) - 3(1 - 3e^{-2(t-3)})u_0(t - 3)$$

Example 2

Karris example 8.9: for the circuit shown below, use the Fourier transfrom method, and the system function $H(\omega)$ to compute $V_L(t)$. Assume $i_L(0^-) = 0$. Verify the result with Matlab.



Solution

Matlab verification

In [ ]:

```
syms t w
H = j*w/(j*w + 2)
```

In [ ]:

```
Vin = fourier(5*exp(-3*t)*heaviside(t),t,w)
```

In [ ]:

```
Vout=simplify(H*Vin)
```

In [ ]:

```
vout = simplify(ifourier(Vout,w,t))
```

Plot result

In [ ]:

```
ezplot(vout)
title('Solution to Example 2')
ylabel('v_{out}(t) [V]')
xlabel('t [s]')
grid
```

See [ft3_ex2.m (https://cpjobling.github.io/eg-247-textbook/fourier_transform/3/ft3_ex2.m)](https://cpjobling.github.io/eg-247-textbook/fourier_transform/3/ft3_ex2.m)

Result is equivalent to:

```
vout = -5*exp(-3*t)*heaviside(t)*(2*exp(t) - 3)
```

Which after gathering terms gives

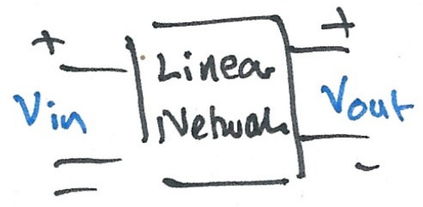
$$v_{\mathrm{out}} = 5\left(3e^{-3t} - 2e^{-2t}\right)u_0(t)$$

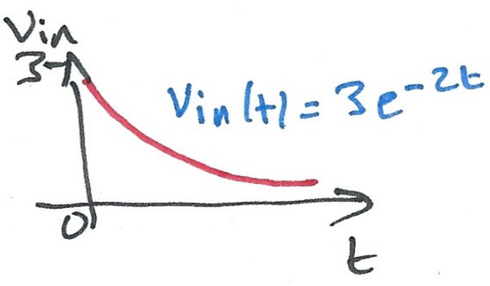
Example 3

Karris example 8.10: for the linear network shown below, the input-output relationship is:

$$\frac{d}{dt}v_{\text{out}} + 4v_{\text{out}} = 10v_{\text{in}}$$

where $v_{\text{in}} = 3e^{-2t}$. Use the Fourier transform method, and the system function $H(\omega)$ to compute the output v_{out}. Verify the result with Matlab.





Solution

Matlab verification

In [ ]:

```
syms t w
H = 10/(j*w + 4)
```

In [ ]:

```
Vin = fourier(3*exp(-2*t)*heaviside(t),t,w)
```

In [ ]:

```
Vout=simplify(H*Vin)
```

In [ ]:

```
vout = simplify(ifourier(Vout,w,t))
```

Plot result

In [ ]:

```
ezplot(vout)
title('Solution to Example 2')
ylabel('v_{out}(t) [V]')
xlabel('t [s]')
grid
```

See [ft3_ex3.m (https://cpjobling.github.io/eg-247-textbook/fourier_transform/3/ft3/ft3_ex3.m)](https://cpjobling.github.io/eg-247-textbook/fourier_transform/3/ft3/ft3_ex3.m)

Result is equiavlent to:

```
15*exp(-4*t)*heaviside(t)*(exp(2*t) - 1)
```

Which after gathering terms gives

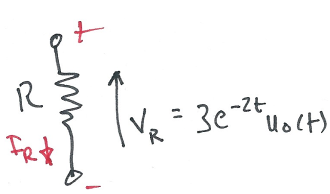
$$v_{\text{out}}(t) = 15\left(e^{-2t}\right) - e^{-4t}\right)u_0(t)$$

Example 4

Karris example 8.11: the voltage across a 1 Ω resistor is known to be $V_R(t) = 3e^{-2t}u_0(t)$. Compute the energy dissipated in the resistor for $0 < t < \infty$, and verify the result using Parseval's theorem. Verify the result with Matlab.

Note from [tables of integrals (http://en.wikipedia.org/wiki/Lists_of_integrals)](http://en.wikipedia.org/wiki/Lists_of_integrals)

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + C.$$



Solution

Matlab verification

In [ ]:

```
syms t w
```

Calcuate energy from time function

In [ ]:

```
Vr = 3*exp(-2*t)*heaviside(t);
R = 1;
Pr = Vr^2/R
Wr = int(Pr,t,0,inf)
```

Calculate using Parseval's theorem

In [ ]:

```
Fw = fourier(Vr,t,w)
```

In [ ]:

```
Fw2 = simplify(abs(Fw)^2)
```

In [ ]:

```
Wr=2/(2*pi)*int(Fw2,w,0,inf)
```

See [ft3_ex4.m (https://cpjobling.github.io/eg-247-textbook/fourier_transform/3/ft3/ft3_ex4.m)](https://cpjobling.github.io/eg-247-textbook/fourier_transform/3/ft3/ft3_ex4.m)

Solutions

See Worked Solutions in the [Week 7 Section (https://swanseauniversity-my.sharepoint.com/personal/c_p_jobling_swansea_ac_uk/_layouts/OneNote.aspx?id=%2Fpersonal%2Fc_p_jobling_swansea_ac_uk%2FDocuments%2FClass%20Not 247-EG-3068%20Signals%20and%20Systems%20%282019-2020%29&wd=target%28_Content%20Library%2FWorksheets.one%7C6DB507FF-A52A-442F-8ED4-3BCF0DC3D570%2FWorked%20Solutions%20to%20Examples%7C0827F501-1A74-104B-8FAA-7C96C30F1282%2F%29)](https://swanseauniversity-my.sharepoint.com/personal/c_p_jobling_swansea_ac_uk/_layouts/OneNote.aspx?id=%2Fpersonal%2Fc_p_jobling_swansea_ac_uk%2FDocuments%2FClass%20Not) of the OneNote Class Notebook