

# Unit 3.4: Transfer Functions

## Contents

- [Colophon](#)
- [Agenda](#)
- [Transfer Functions for Circuits](#)
- [Examples](#)
- [Using Transfer Functions in MATLAB for System Analysis](#)
- [Using Transfer Functions in Simulink for System Simulation](#)
- [Reference](#)

The preparatory reading for this section is [Chapter 4.4 \[Karris, 2012\]](#) which discusses transfer function models of electrical circuits.

## Colophon

An annotatable worksheet for this presentation is available as [Worksheet 7](#).

- The source code for this page is [laplace\\_transform/4/transfer\\_functions.ipynb](#).
- You can view the notes for this presentation as a webpage ([HTML](#)).
- This page is downloadable as a [PDF](#) file.

## Agenda

- Transfer Functions
- A Couple of Examples
- Circuit Analysis Using MATLAB LTI Transfer Function Block
- Circuit Simulation Using Simulink Transfer Function Block

```
% Matlab setup
clear all
cd ../matlab
pwd
format compact
```

```
ans =

'/Users/eechris/code/src/github.com/cpjobling/eg-247-
textbook/laplace_transform/matlab'
```

## Transfer Functions for Circuits

When doing circuit analysis with components defined in the complex frequency domain, the ratio of the output voltage  $V_{\text{out}}(s)$  to the input voltage  $V_{\text{in}}(s)$  *under zero initial conditions* is of great interest.

This ratio is known as the *voltage transfer function* denoted  $G_v(s)$ :

$$G_v(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)}$$

Similarly, the ratio of the output current  $I_{\text{out}}(s)$  to the input current  $I_{\text{in}}(s)$  under zero initial conditions, is called the *current transfer function* denoted  $G_i(s)$ :

$$G_i(s) = \frac{I_{\text{out}}(s)}{I_{\text{in}}(s)}$$

In practice, the current transfer function is rarely used, so we will use the voltage transfer function denoted:

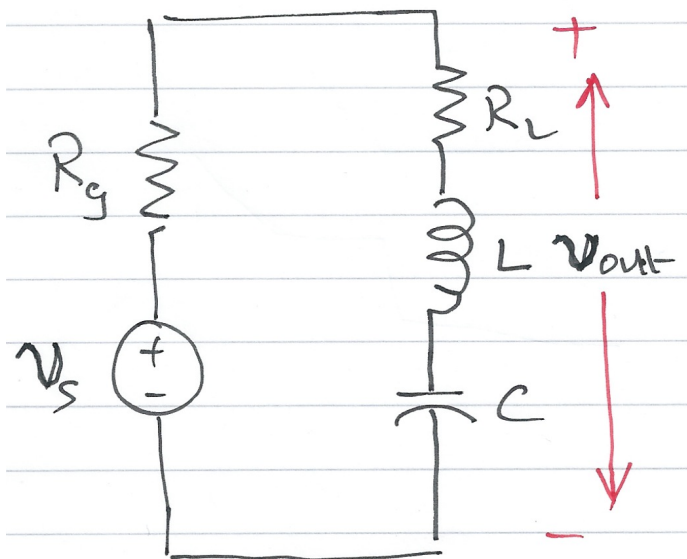
$$G(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)}$$

## Examples

See [Worksheet 7](#) for the worked solutions to the examples. We will work through these in class. Here I'll demonstrate the MATLAB solutions.

### Example 6

Derive an expression for the transfer function  $G(s)$  for the circuit below. In this circuit  $R_g$  represents the internal resistance of the applied (voltage) source  $v_s$ , and  $R_L$  represents the resistance of the load that consists of  $R_L$ ,  $L$  and  $C$ .



### Sketch of Solution

- Replace  $v_s(t)$ ,  $R_g$ ,  $R_L$ ,  $L$  and  $C$  by their transformed (*complex frequency*) equivalents:  $V_s(s)$ ,  $R_g$ ,  $R_L$ ,  $sL$  and  $1/(sC)$
- Use the *Voltage Divider Rule* to determine  $V_{\text{out}}(s)$  as a function of  $V_s(s)$
- Form  $G(s)$  by writing down the ratio  $V_{\text{out}}(s)/V_s(s)$

### Worked solution.

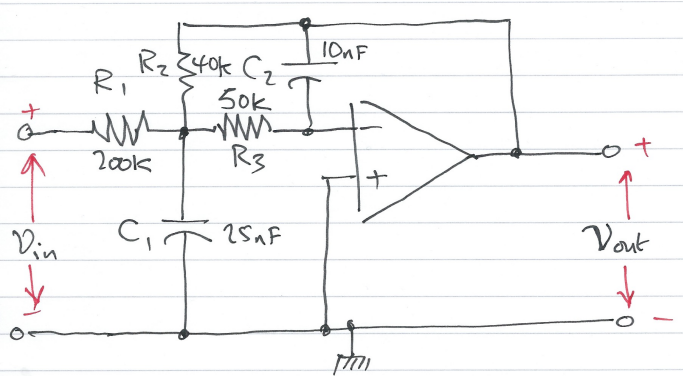
Pencast: [ex6.pdf](#) - open in Adobe Acrobat Reader.

### Answer

$$G(s) = \frac{V_{\text{out}}(s)}{V_s(s)} = \frac{R_L + sL + 1/sC}{R_g + R_L + sL + 1/sC}.$$

## Example 7

Compute the transfer function for the op-amp circuit shown below in terms of the circuit constants  $R_1, R_2, R_3, C_1$  and  $C_2$ .



Then replace the complex variable  $s$  with  $j\omega$ , and the circuit constants with their numerical values and plot the magnitude

$$|G(j\omega)| = \frac{|V_{\text{out}}(j\omega)|}{|V_{\text{in}}(j\omega)|}$$

versus radian frequency  $\omega$  rad/s.

## Sketch of Solution

- Replace the components and voltages in the circuit diagram with their complex frequency equivalents
- Use nodal analysis to determine the voltages at the nodes either side of the 50K resistor  $R_3$
- Note that the voltage at the input to the op-amp is a virtual ground
- Solve for  $V_{\text{out}}(s)$  as a function of  $V_{\text{in}}(s)$
- Form the reciprocal  $G(s) = V_{\text{out}}(s)/V_{\text{in}}(s)$
- Use MATLAB to calculate the component values, then replace  $s$  by  $j\omega$ .
- Plot on log-linear “paper”:

$$|G(j\omega)|$$

## Worked solution.

Pencast: [ex7.pdf](#) - open in Adobe Acrobat Reader.

## Answer

$$G(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{-1}{R_1 \left( (1/R_1 + 1/R_2 + 1/R_3 + sC_1) (sC_2R_3) + 1/R_2 \right)}.$$

## The Matlab Bit

See attached script: [solution7.m](#).

## Week 3: Solution 7

```
syms s;
```

```
R1 = 200*10^3;  
R2 = 40*10^3;  
R3 = 50*10^3;  
  
C1 = 25*10^(-9);  
C2 = 10*10^(-9);
```

```
den = R1*((1/R1+ 1/R2 + 1/R3 + s*C1)*(s*R3*C2) + 1/R2);  
simplify(den)
```

```
ans =
```

```
100*s*((7555786372591433*s)/302231454903657293676544 + 1/20000) + 5
```

Simplify coefficients of s in denominator

```
format long  
denG = sym2poly(ans)
```

```
denG =  
    0.000002500000000    0.005000000000000    5.000000000000000
```

```
numG = -1;
```

Plot

For convenience, define coefficients *a* and *b*:

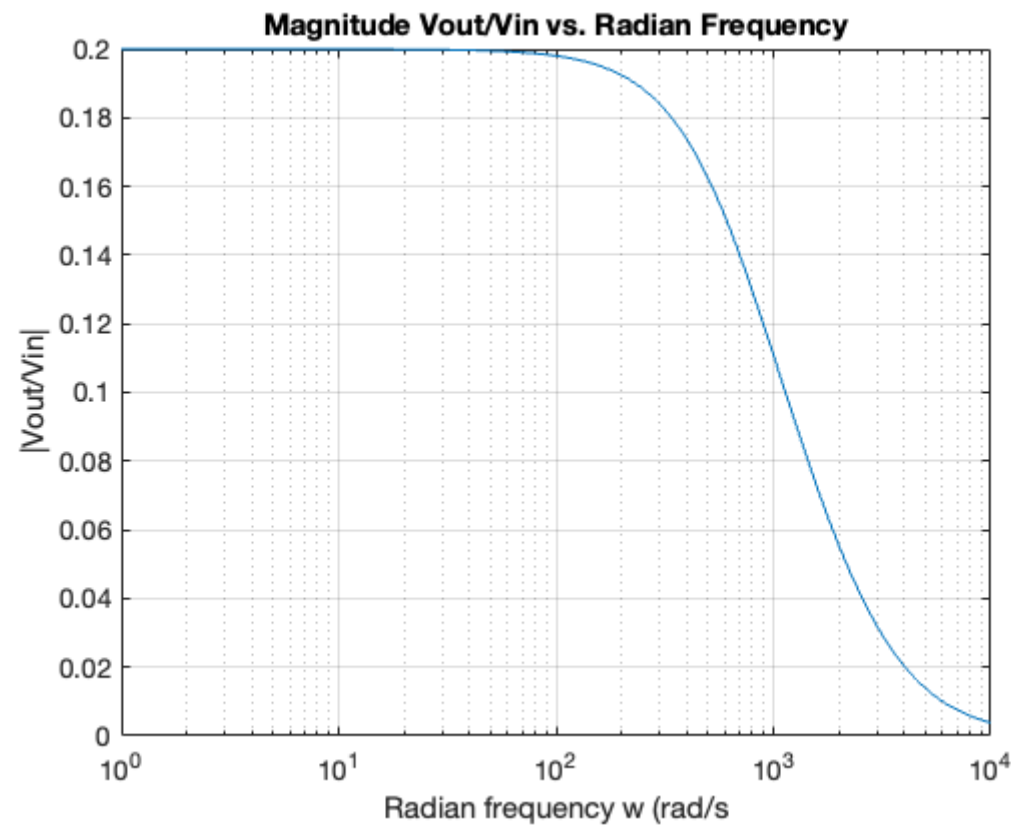
```
a = denG(1);  
b = denG(2);
```

$$G(j\omega) = \frac{-1}{a\omega^2 - jb\omega + 5}$$

```
w = 1:10:10000;  
Gs = -1./(a*w.^2 - j.*b.*w + denG(3));
```

Plot

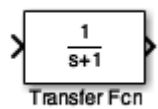
```
semilogx(w, abs(Gs))  
xlabel('Radian frequency w (rad/s)')  
ylabel('|Vout/Vin|')  
title('Magnitude Vout/Vin vs. Radian Frequency')  
grid
```



# Using Transfer Functions in MATLAB for System Analysis

Please use the file [tf\\_matlab.m](#) to explore the Transfer Function features provide by MATLAB. Open the file as a Live Script to see a nicely formatted document.

# Using Transfer Functions in Simulink for System Simulation



The Simulink transfer function (**Transfer Fcn**) block implements a transfer function

The transfer function block represents a general input output function

$$G(s) = \frac{N(s)}{D(s)}$$

and is not specific nor restricted to circuit analysis.

It can, however be used in modelling and simulation studies.

## Example

Recast Example 7 as a MATLAB problem using the LTI Transfer Function block.

For simplicity use parameters  $R_1 = R_2 = R_3 = 1 \Omega$ , and  $C_1 = C_2 = 1 \text{ F}$ .

Calculate the step response using the LTI functions.

Verify the result with Simulink.

The Matlab solution: [example8.m](#)

## MATLAB Solution

From a previous analysis the transfer function is:

$$G(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{-1}{R_1 [(1/R_1 + 1/R_2 + 1/R_3 + sC_1)(sR_3C_2) + 1/R_2]}$$

so substituting the component values we get:

$$G(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{-1}{s^2 + 3s + 1}$$

We can find the step response by letting  $v_{\text{in}}(t) = u_0(t)$  so that  $V_{\text{in}}(s) = 1/s$  then

$$V_{\text{out}}(s) = \frac{-1}{s^2 + 3s + 1} \cdot \frac{1}{s}$$

We can solve this by partial fraction expansion and inverse Laplace transform as is done in the text book with the help of MATLAB's **residue** function.

Here, however we'll use the LTI block.

Define the circuit as a transfer function

```
G = tf([-1],[1 3 1])
```

G =

-1

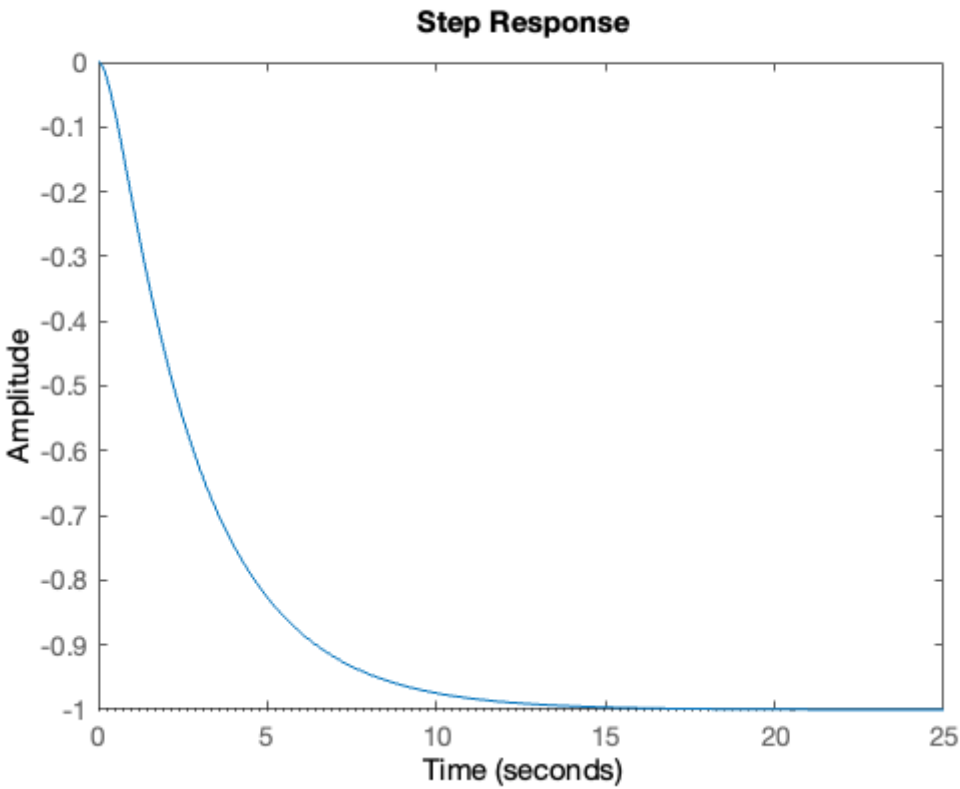
-----

$s^2 + 3 s + 1$

Continuous-time transfer function.

step response is then:

```
step(G)
```

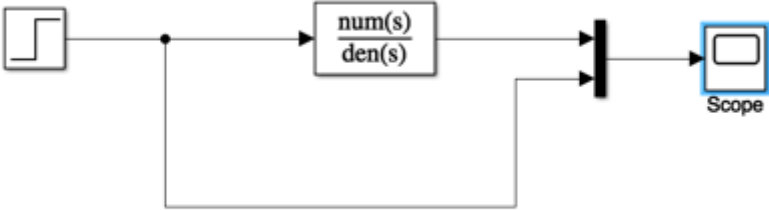


Simples!

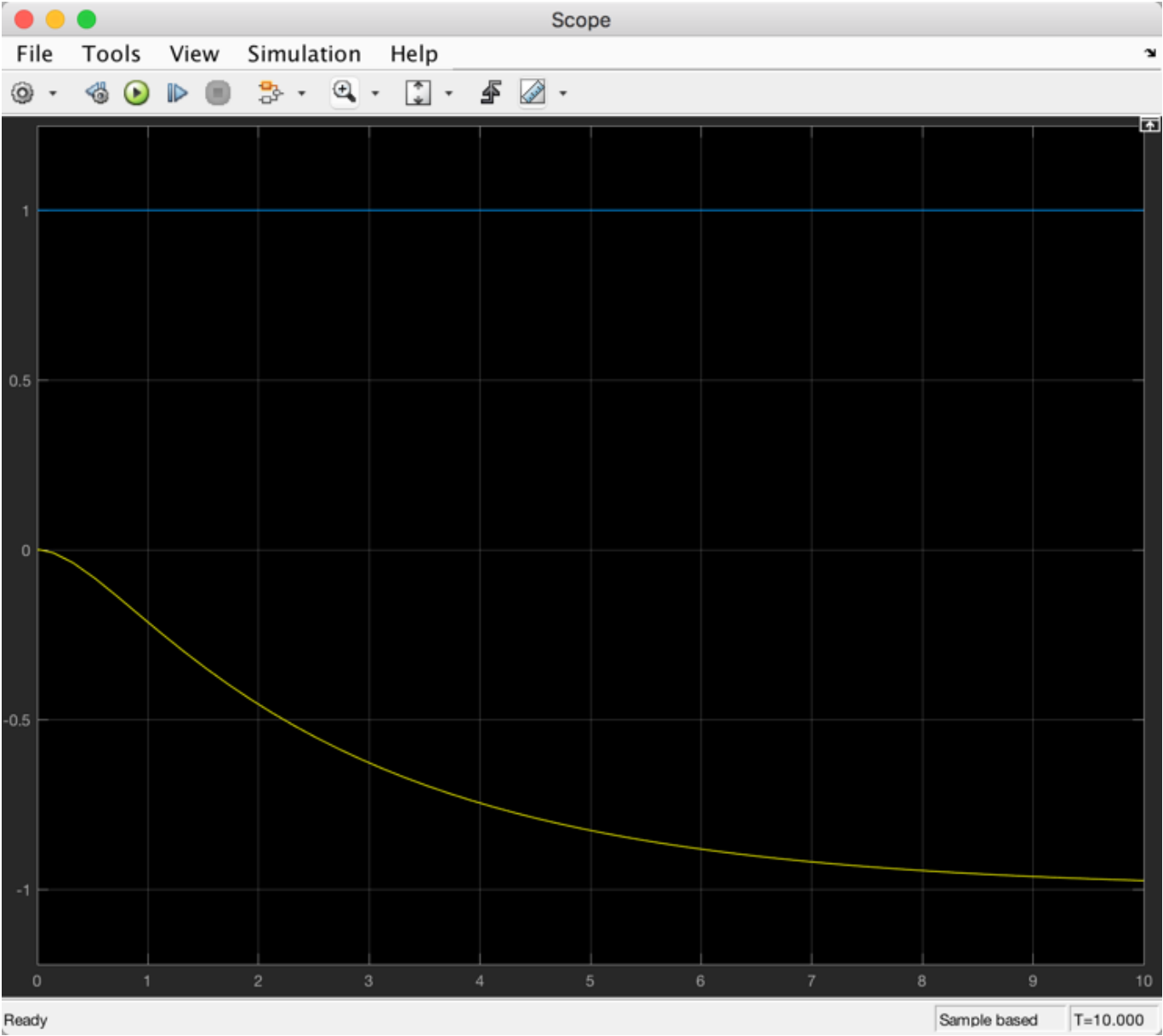
Simulink model

See [example 8.slx](#)

```
open example_8
```

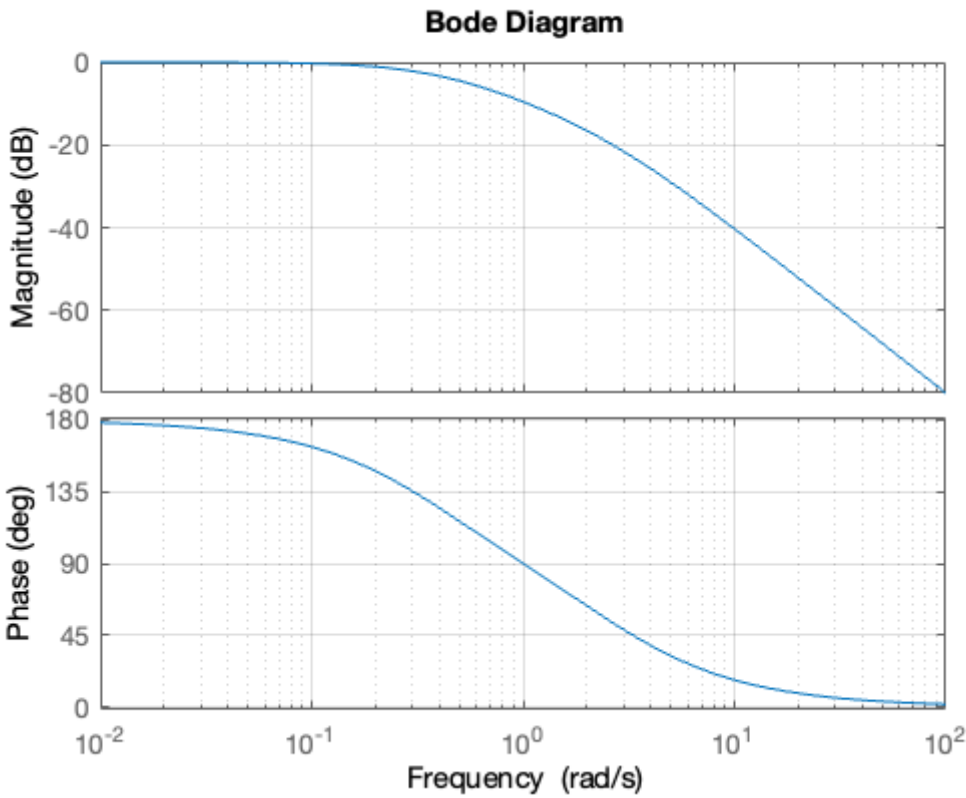


Result



Let’s go a bit further by finding the frequency response:

```
bode(G), grid
```



# Reference

See [Bibliography](#).

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