Using Laplace Transforms for Circuit Analysis

The preparatory reading for this section is Chapter 4 [Kar12] which presents examples of the applications of the Laplace transform for electrical solving circuit problems.

Colophon

An annotatable worksheet for this presentation is available as **Worksheet 6**.

- The source code for this page is laplace_transform/3/circuit_analysis.ipynb.
- You can view the notes for this presentation as a webpage (HTML).
- This page is downloadable as a PDF file.

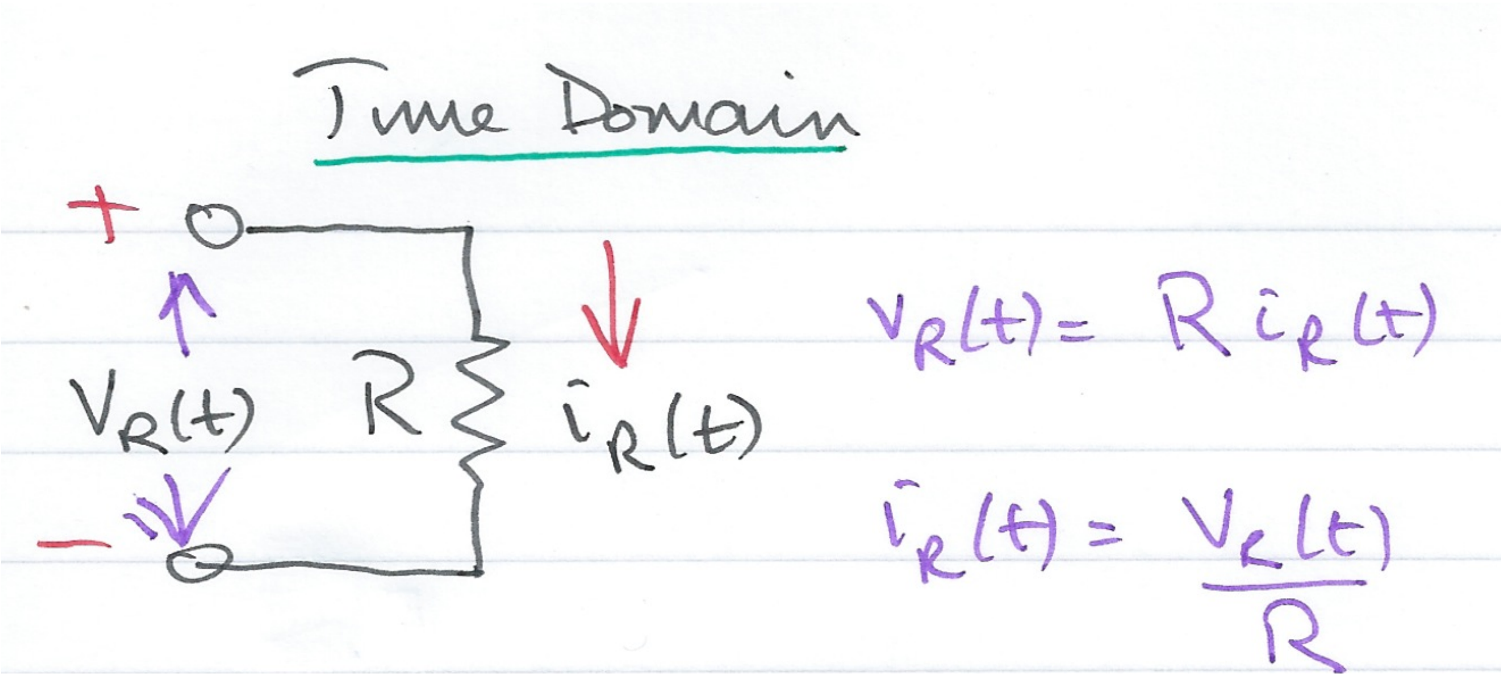
Agenda

We look at applications of the Laplace Transform for

- Circuit transformation from Time to Complex Frequency
- Complex impedance
- Complex admittance

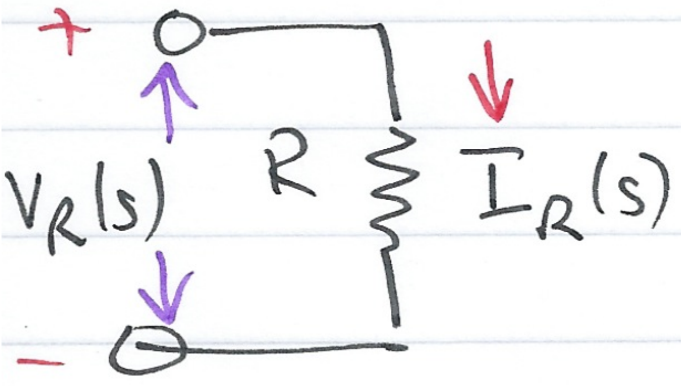
Circuit Transformation from Time to Complex Frequency

Time Domain Model of a Resistive Network



Complex Frequency Domain Model of a Resistive Circuit

Frequency Domain

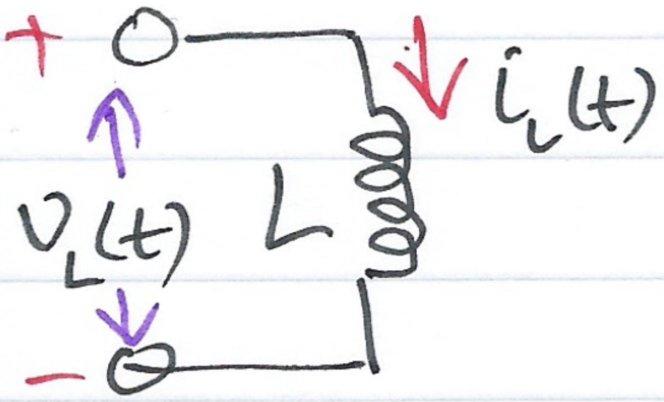


$$V_R(s) = R\,I_R(s)$$

$$I_R(s) = \frac{V_R(s)}{R}$$

Time Domain Model of an Inductive Network

Time Domain

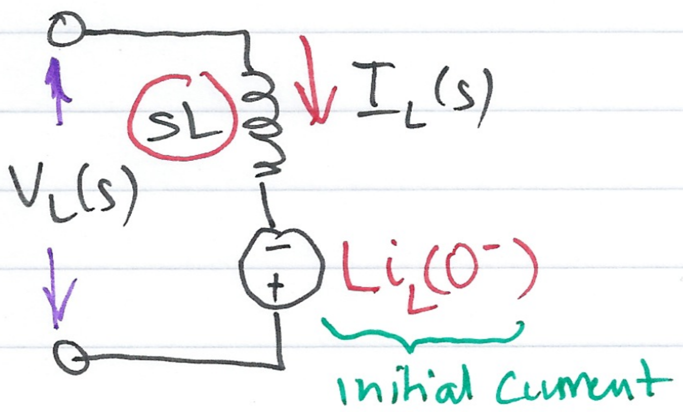


$$v_L(t) = L\frac{di_L}{dt}$$

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L\,dt$$

Complex Frequency Domain Model of an Inductive Network

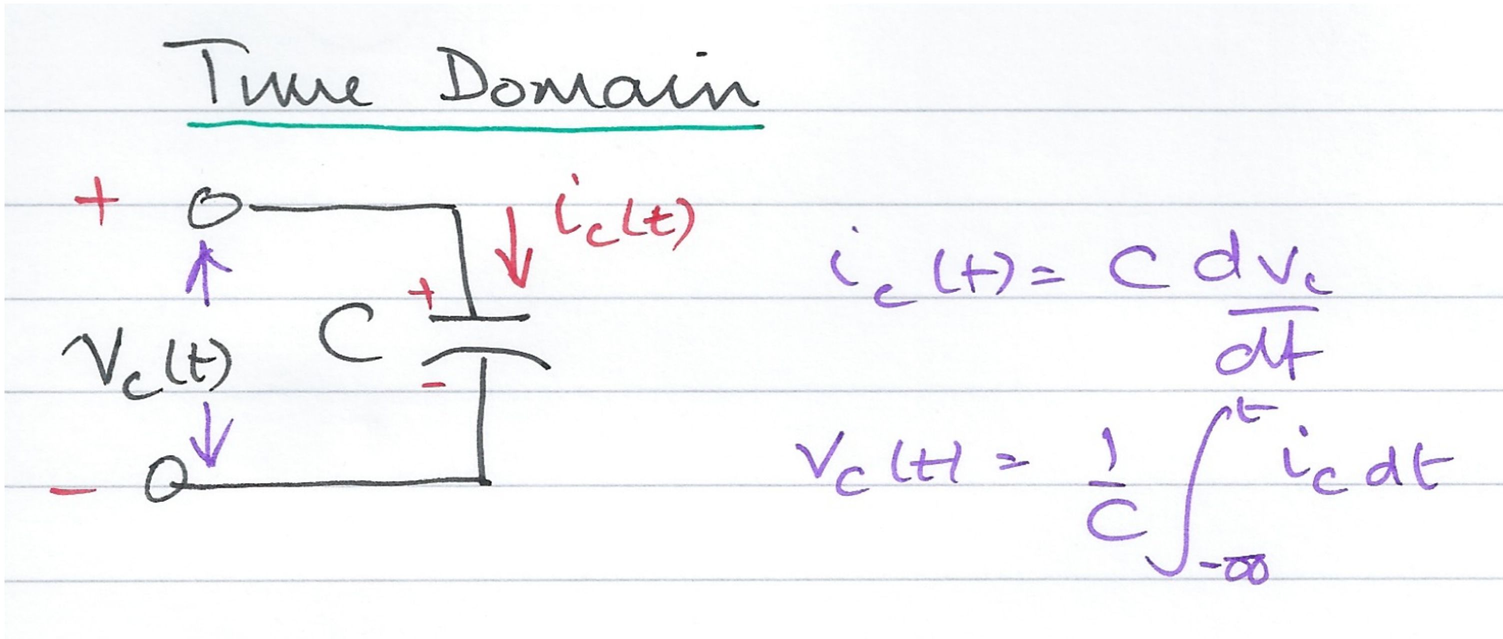
Frequency Domain



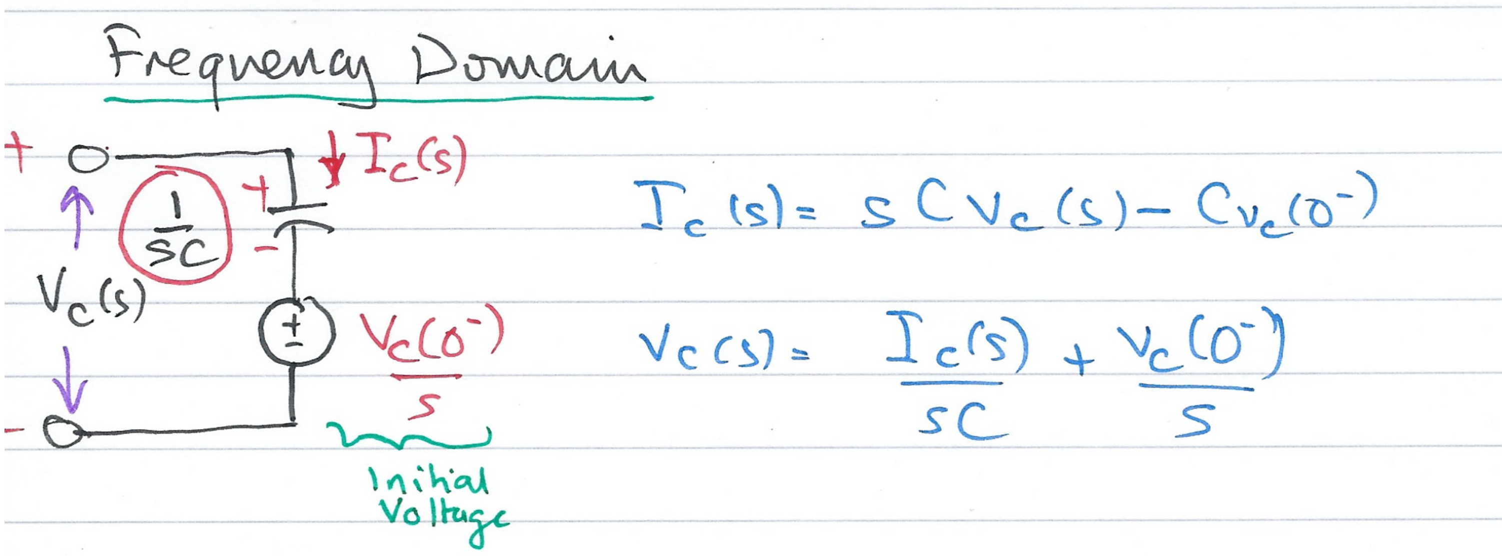
$$V_L(s) = sL\,I_L(s) - L\,i_L(0^-)$$

$$I_L(s) = \frac{V_L(s)}{Ls} + \frac{L\,i_L(0^-)}{s}$$

Time Domain Model of a Capacitive Network



Complex Frequency Domain of a Capacitive Network

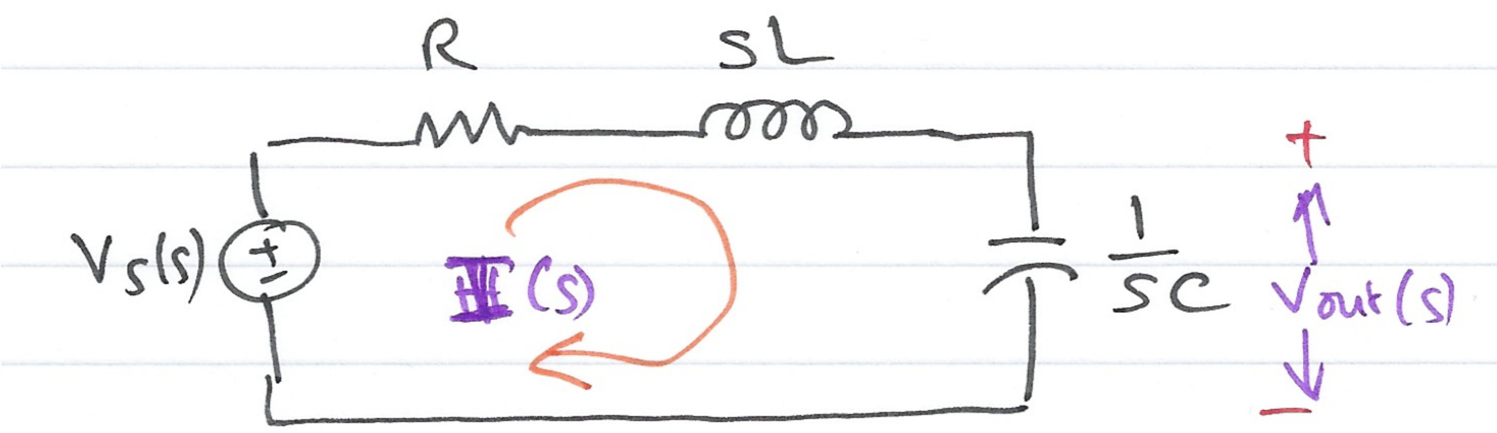


Examples

We will work through these in class. See worksheet 6.

Complex Impedance $Z(s)$

Consider the s-domain RLC series circuit, where the initial conditions are assumed to be zero.



For this circuit, the sum

$$R + sL + \frac{1}{sC}$$

represents that total opposition to current flow. Then,

$$I(s) = \frac{V_s(s)}{R + sL + 1/(sC)}$$

and defining the ratio $V_s(s)/I(s)$ as $Z(s)$, we obtain

$$Z(s) = \frac{V_s(s)}{I(s)} = R + sL + \frac{1}{sC}$$

The s-domain current $I(s)$ can be found from

The s-domain current $I(s)$ can be found from

$$I(s) = \frac{V_s(s)}{Z(s)}$$

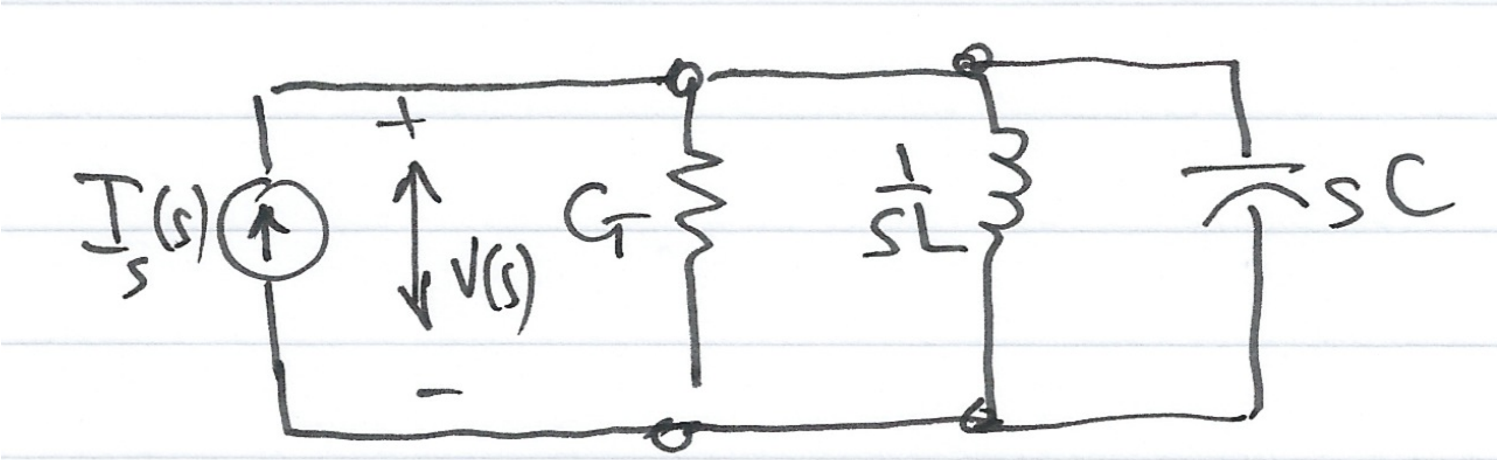
where

$$Z(s) = R + sL + \frac{1}{sC}.$$

Since $s = \sigma + j\omega$ is a complex number, $Z(s)$ is also complex and is known as the *complex input impedance* of this RLC series circuit.

Complex Admittance $Y(s)$

Consider the s-domain GLC parallel circuit shown below where the initial conditions are zero.



For this circuit

$$GV(s) + \frac{1}{sL}V(s) + sCV(s) = I_s(s)$$

$$\left(G + \frac{1}{sL} + sC\right)V(s) = I_s(s)$$

Defining the ratio $I_s(s)/V(s)$ as $Y(s)$ we obtain

$$Y(s) = \frac{I_s(s)}{V(s)} = G + \frac{1}{sL} + sC = \frac{1}{Z(s)}$$

The s-domain voltage $V(s)$ can be found from

$$V(s) = \frac{I_s(s)}{Y(s)}$$

where

$$Y(s) = G + \frac{1}{sL} + sC.$$

$Y(s)$ is complex and is known as the *complex input admittance* of this GLC parallel circuit.

Reference

See [Bibliography](#).

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