Worksheet 16

To accompany Chapter 6.3 The Inverse Z-Transform

Colophon

This worksheet can be downloaded as a <u>PDF file (https://cpjobling.github.io/eg-247-textbook/worksheets/worksheet16.pdf)</u>. We will step through this worksheet in class.

An annotatable copy of the notes for this presentation will be distributed before the second class meeting as **Worksheet 9** in the **Week 4: Classroom Activities** section of the Canvas site. I will also distribute a copy to your personal **Worksheets** section of the **OneNote Class Notebook** so that you can add your own notes using OneNote.

You are expected to have at least watched the video presentation of Chapter 6.3 (i z transform) of the notes (https://cpjobling.github.io/eg-247-textbook) before coming to class. If you haven't watch it afterwards!

After class, the lecture recording and the annotated version of the worksheets will be made available through Canvas.

Agenda

- Inverse Z-Transform
- Examples using PFE
- Examples using Long Division
- Analysis in MATLAB

The Inverse Z-Transform

The inverse Z-Transform enables us to extract a sequence f[n] from F(z). It can be found by any of the following methods:

- · Partial fraction expansion
- · The inversion integral
- · Long division of polynomials

Partial fraction expansion

We expand F(z) into a summation of terms whose inverse is known. These terms have the form:

$$k, \frac{r_1 z}{z - p_1}, \frac{r_1 z}{(z - p_1)^2}, \frac{r_3 z}{z - p_2}, \dots$$

where k is a constant, and r_i and p_i represent the residues and poles respectively, and can be real or complex¹.

Notes

1. If complex, the poles and residues will be in complex conjugate pairs

$$\frac{r_i z}{z - p_i} + \frac{r_i^* z}{z - p_i^*}$$

Step 1: Make Fractions Proper

- Before we expand F(z) into partial fraction expansions, we must first express it as a *proper* rational function.
- This is done by expanding F(z)/z instead of F(z)
- · That is we expand

$$\frac{F(z)}{z} = \frac{k}{z} + \frac{r_1}{z - p_1} + \frac{r_2}{z - p_2} + \cdots$$

Step 2: Find residues

· Find residues from

$$r_k = \lim_{z \to p_k} (z - p_k) \frac{F(z)}{z} = (z - p_k) \frac{F(z)}{z} \Big|_{z = p_k}$$

Step 3: Map back to transform tables form

• Rewrite F(z)/z:

$$z\frac{F(z)}{z} = F(z) = k + \frac{r_1 z}{s - p_1} + \frac{r_2 z}{s - p_2} + \cdots$$

Example 1

Karris Example 9.4: use the partial fraction expansion to compute the inverse z-transform of

$$F(z) = \frac{1}{(1 - 0.5z^{-1})(1 - 0.75z^{-1})(1 - z^{-1})}$$

MATLAB solution

See <u>example1.mlx (https://cpjobling.github.io/eg-247-textbook/dt_systems/3/matlab/example1.mlx)</u>. (Also available as <u>example1.m (https://cpjobling.github.io/eg-247-textbook/dt_systems/3/matlab/example1.m)</u>.)

Uses MATLAB functions:

- collect expands a polynomial
- sym2poly converts a polynomial into a numeric polymial (vector of coefficients in descending order of exponents)
- residue calculates poles and zeros of a polynomial
- ztrans symbolic z-transform
- iztrans symbolic inverse ze-transform
- stem plots sequence as a "lollipop" diagram

```
In [ ]: clear all
    cd matlab
    format compact
In [ ]: syms z n
```

The denoninator of F(z)

```
In [ ]: Dz = (z - 0.5)*(z - 0.75)*(z - 1);
```

Multiply the three factors of Dz to obtain a polynomial

```
In [ ]: Dz_poly = collect(Dz)
```

Make into a rational polynomial

```
z^2
```

```
In [ ]: num = [0, 1, 0, 0];
```

$$z^3 - 9/4z^2 - 13/8z - 3/8$$

```
In [ ]: den = sym2poly(Dz_poly)
```

Compute residues and poles

```
In [ ]: [r,p,k] = residue(num,den);
```

Print results

fprintf works like the c-language function

```
In [ ]: fprintf('\n')
    fprintf('r1 = %4.2f\t', r(1)); fprintf('p1 = %4.2f\n', p(1));...
    fprintf('r2 = %4.2f\t', r(2)); fprintf('p2 = %4.2f\n', p(2));...
    fprintf('r3 = %4.2f\t', r(3)); fprintf('p3 = %4.2f\n', p(3));
```

Symbolic proof

$$f[n] = 2\left(\frac{1}{2}\right)^n - 9\left(\frac{3}{4}\right)^n + 8$$

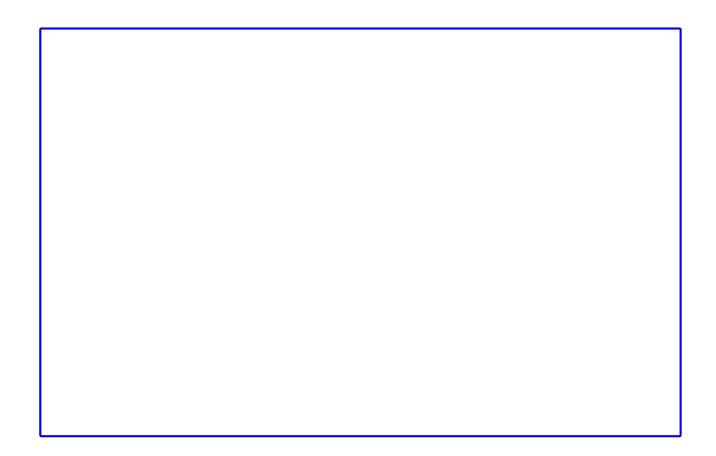
Sequence

```
In [ ]: n = 1:15;
    sequence = subs(fn,n);
    stem(n,sequence)
    title('Discrete Time Sequence f[n] = 2*(1/2)^n-9*(3/4)^n + 8');
    ylabel('f[n]')
    xlabel('Sequence number n')
```

Example 2

Karris example 9.5: use the partial fraction expansion method to to compute the inverse z-transform of

$$F(z) = \frac{12z}{(z+1)(z-1)^2}$$



MATLAB solution

See <u>example2.mlx</u> (https://cpjobling.github.io/eg-247-textbook/dt_systems/3/matlab/example2.mlx). (Also available as <u>example2.m</u> (https://cpjobling.github.io/eg-247-textbook/dt_systems/3/matlab/example2.m).)

Uses additional MATLAB functions:

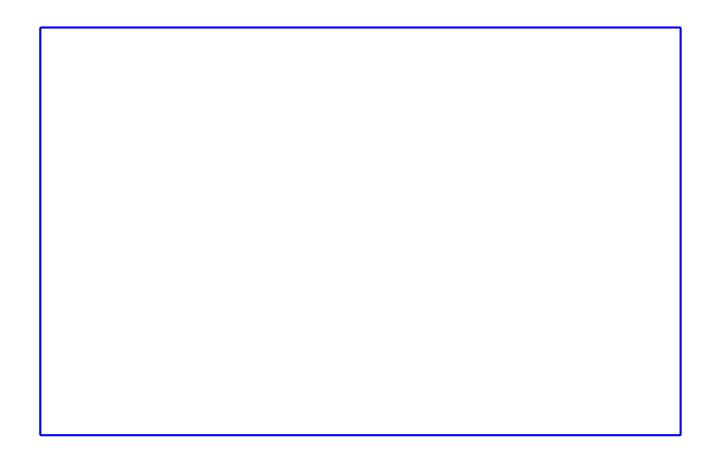
• dimpulse – computes and plots a sequence f[n] for any range of values of n

In []: open example2

Example 3

Karris example 9.6: use the partial fraction expansion method to to compute the inverse z-transform of

$$F(z) = \frac{z+1}{(z-1)(z^2+2z+2)}$$



MATLAB solution

See <u>example3.mlx</u> (https://cpjobling.github.io/eg-247-textbook/dt_systems/3/matlab/example3.mlx). (Also available as <u>example3.m</u> (https://cpjobling.github.io/eg-247-textbook/dt_systems/3/matlab/example3.m).)

In []: open example3

Inverse Z-Transform by the Inversion Integral

The inversion integral states that:

$$f[n] = \frac{1}{i2\pi} \oint_C F(z) z^{n-1} dz$$

where C is a closed curve that encloses all poles of the integrant.

This can (apparently) be solved by Cauchy's residue theorem!!

Fortunately (:-), this is beyond the scope of this module!

See Karris Section 9.6.2 (pp 9-29-9-33) if you want to find out more.

Inverse Z-Transform by the Long Division

To apply this method, F(z) must be a rational polynomial function, and the numerator and denominator must be polynomials arranged in descending powers of z.

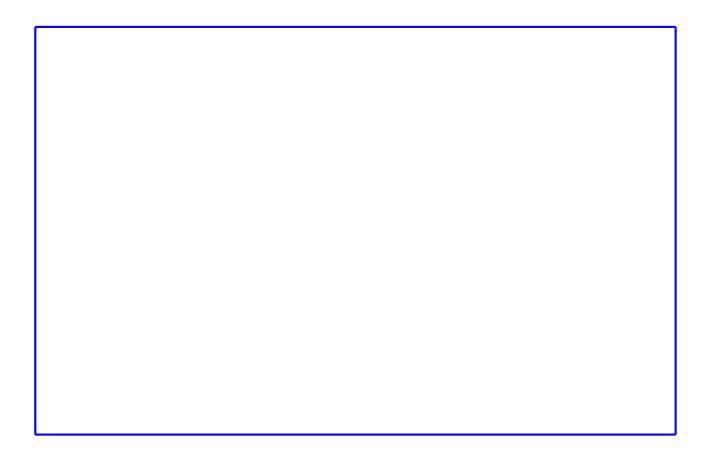
We will work through an example in class.

[Skip next slide in Pre-Lecture]

Example 4

Karris example 9.9: use the long division method to determine f[n] for n = 0, 1, and 2, given that

$$F(z) = \frac{1 + z^{-1} + 2z^{-2} + 3z^{-3}}{(1 - 0.25z^{-1})(1 - 0.5z^{-1})(1 - 0.75z^{-1})}$$



MATLAB

See <u>example4.mlx (https://cpjobling.github.io/eg-247-textbook/dt_systems/3/matlab/example4.mlx)</u>. (also available as <u>example4.m (https://cpjobling.github.io/eg-247-textbook/dt_systems/3/matlab/example4.m)</u>.)

```
In [ ]: open example4
```

Methods of Evaluation of the Inverse Z-Transform

Partial Fraction Expansion

Advantages

- · Most familiar.
- Can use MATLAB residue function.

Disadvantages

• Requires that F(z) is a proper rational function.

Inversion Integral

Advantage

• Can be used whether F(z) is rational or not

Disadvantages

• Requires familiarity with the Residues theorem of complex variable analaysis.

Long Division

Advantages

- Practical when only a small sequence of numbers is desired.
- Useful when z-transform has no closed-form solution.

Disadvantages

- Can use MATLAB dimpulse function to compute a large sequence of numbers.
- Requires that F(z) is a proper rational function.
- Division may be endless.

Summary

- Inverse Z-Transform
- Examples using PFE
- Examples using Long Division
- Analysis in MATLAB

Coming Next

• DT transfer functions, continuous system equivalents, and modelling DT systems in Matlab and Simulink.