Using Laplace Transforms for Circuit Analysis

Using Laplace Transforms for Circuit Analysis

The preparatory reading for this section is Chapter 4 (Karris, 2012) which presents examples of the applications of the Laplace transform for electrical solving circuit problems.

An annotatable copy of the notes for this presentation will be distributed before the third class meeting as **Worksheet 6** in the **Week 3: Classroom Activities** section of the Canvas site. I will also distribute a copy to your personal **Worksheets** section of the **OneNote Class Notebook** so that you can add your own notes using OneNote.

You can also view the notes for this presentation as a webpage (HTML) and as a downloadable PDF file.

After class, the lecture recording and the annotated version of the worksheets will be made available to you through Canvas.

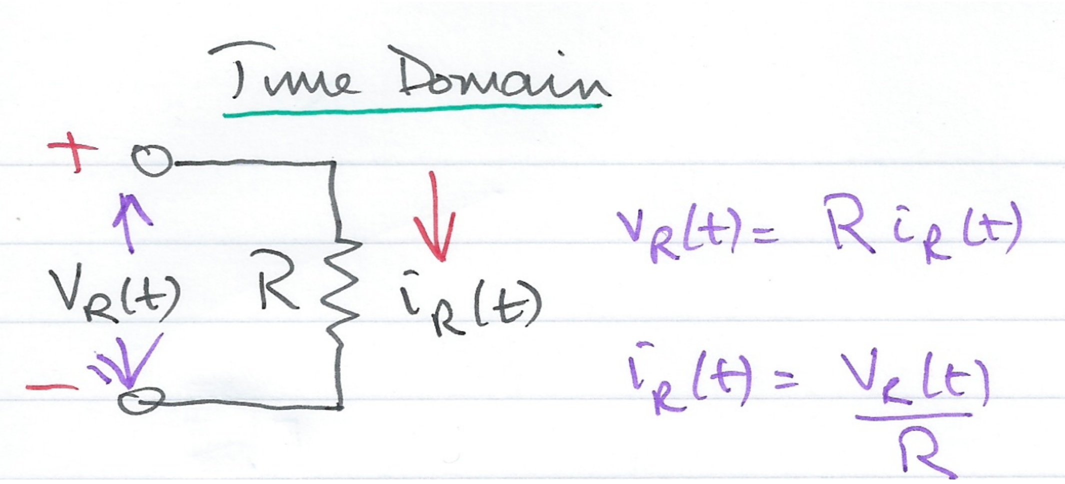
Agenda

We look at applications of the Laplace Transform for

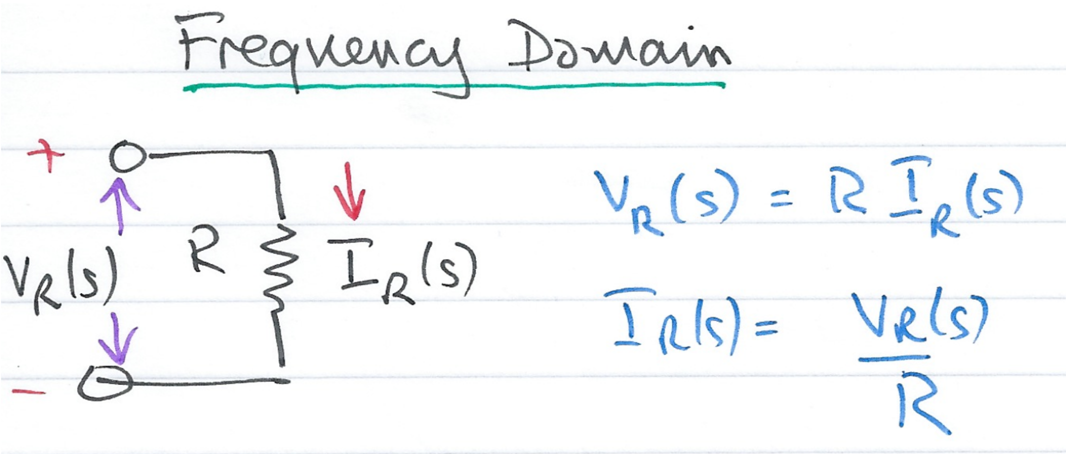
- Circuit transformation from Time to Complex Frequency
- Complex impedance
- Complex admittance

Circuit Transformation from Time to Complex Frequency

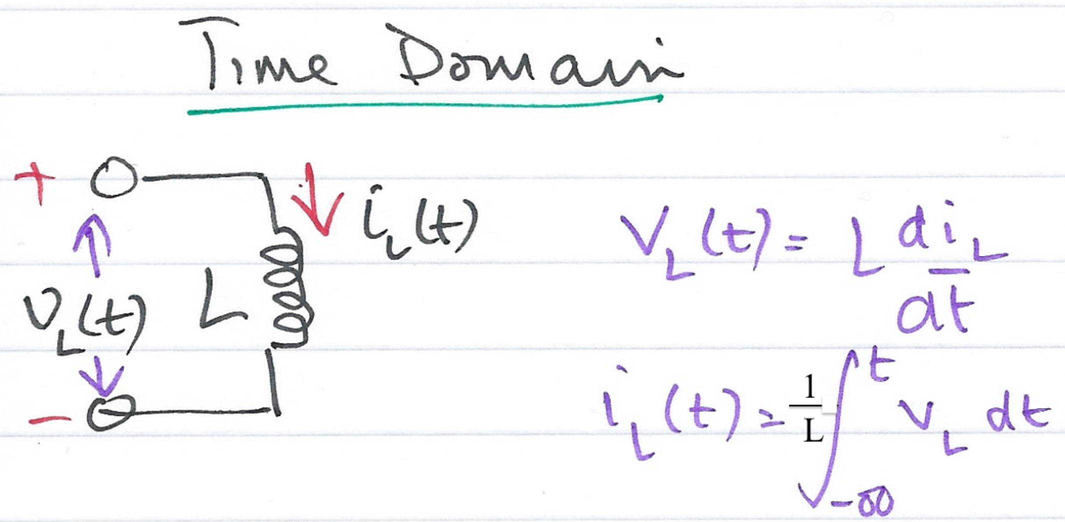
Time Domain Model of a Resistive Network



Complex Frequency Domain Model of a Resistive Circuit

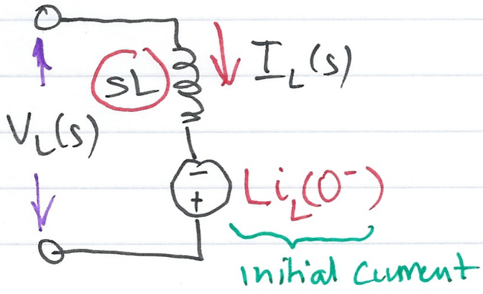


Time Domain Model of an Inductive Network



Complex Frequency Domain Model of an Inductive Network

Frequency Domain

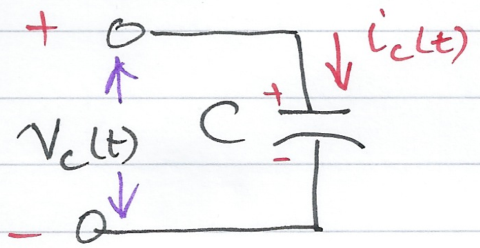


$$V_L(s) = sLI_L(s) - Li_L(0^-)$$

$$I_L(s) = \frac{V_L(s)}{Ls} + \frac{i_L(0^-)}{s}$$

Time Domain Model of a Capacitive Network

Time Domain

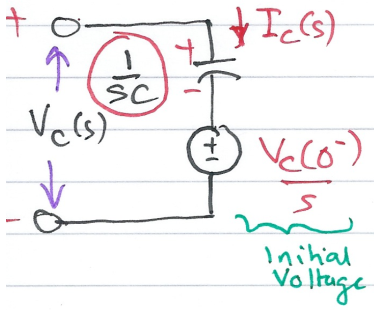


$$i_c(t) = C\frac{dv_c}{dt}$$

$$v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c\,dt$$

Complex Frequency Domain of a Capacitive Network

Frequency Domain



$$I_c(s) = sCV_c(s) - Cv_c(0^-)$$

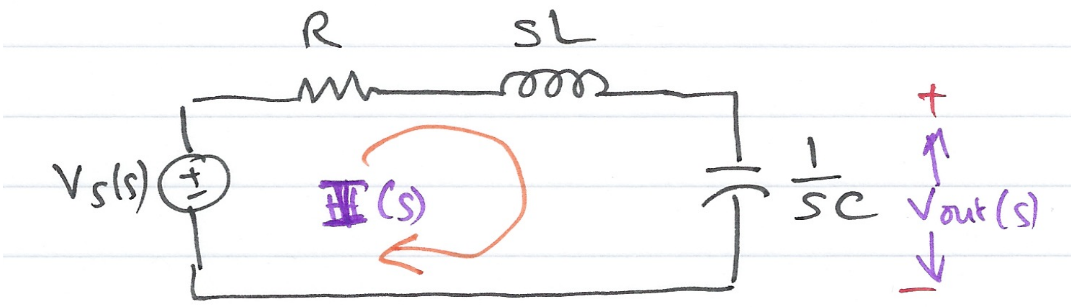
$$V_c(s) = \frac{I_c(s)}{sC} + \frac{V_c(0^-)}{s}$$

Examples

We will work through these in class. See worksheet6.

Complex Impedance Z(s)

Consider the s-domain RLC series circuit, where the initial conditions are assumed to be zero.



For this circuit, the sum

$$R + sL + \frac{1}{sC}$$

represents that total opposition to current flow. Then,

$$I(s) = \frac{V_s(s)}{R + sL + 1/(sC)}$$

and defining the ratio $V_s(s)/I(s)$ as $Z(s)$, we obtain

$$Z(s) = \frac{V_s(s)}{I(s)} = R + sL + \frac{1}{sC}$$

The s-domain current $I(s)$ can be found from

$$I(s) = \frac{V_s(s)}{Z(s)}$$

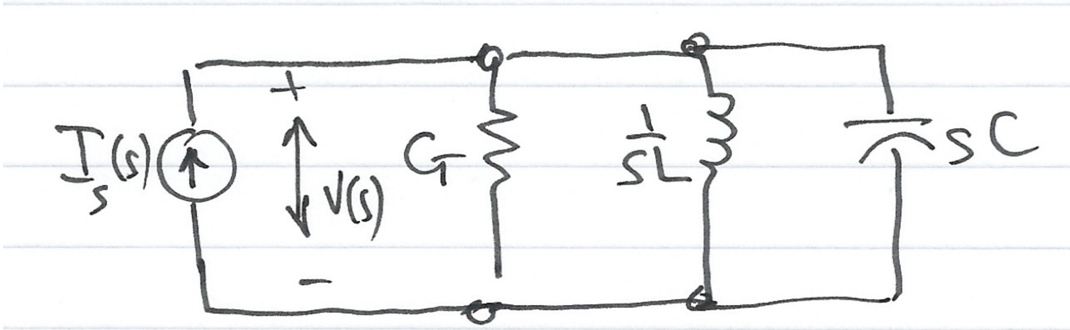
where

$$Z(s) = R + sL + \frac{1}{sC}.$$

Since $s = \sigma + j\omega$ is a complex number, $Z(s)$ is also complex and is known as the *complex input impedance* of this RLC series circuit.

Complex Admittance Y(s)

Consider the s-domain GLC parallel circuit shown below where the initial conditions are zero.



For this circuit

$$GV(s) + \frac{1}{sL}V(s) + sCV(s) = I_s(s)$$

$$\left(G + \frac{1}{sL} + sC\right)V(s) = I_s(s)$$

Defining the ratio $I_s(s)/V(s)$ as $Y(s)$ we obtain

$$Y(s) = \frac{I_s(s)}{V(s)} = G + \frac{1}{sL} + sC = \frac{1}{Z(s)}$$

The s-domain voltage $V(s)$ can be found from

$$V(s) = \frac{I_s(s)}{Y(s)}$$

where

$$Y(s) = G + \frac{1}{sL} + sC.$$

$Y(s)$ is complex and is known as the *complex input admittance* of this GLC parallel circuit.

Reference

1. Karris, S. T. (2012). *Signals and systems with MATLAB computing and Simulink modeling*. Fremont, CA.: Orchard Publishing. Retrieved from https://ebookcentral.proquest.com/lib/swansea-ebooks/reader.action?docID=3384197