

Worksheet 7

To accompany Chapter 3.4 Transfer Functions

Colophon

This worksheet can be downloaded as a [PDF file](#). We will step through this worksheet in class.

An annotatable copy of the notes for this presentation will be distributed before the second class meeting as **Worksheet 7** in the **Week 3: Classroom Activities** section of the Canvas site. I will also distribute a copy to your personal **Worksheets** section of the **OneNote Class Notebook** so that you can add your own notes using OneNote.

You are expected to have at least watched the video presentation of [Chapter 3.4](#) of the [notes](#) before coming to class. If you haven't watch it afterwards!

After class, the lecture recording and the annotated version of the worksheets will be made available through Canvas.

Agenda

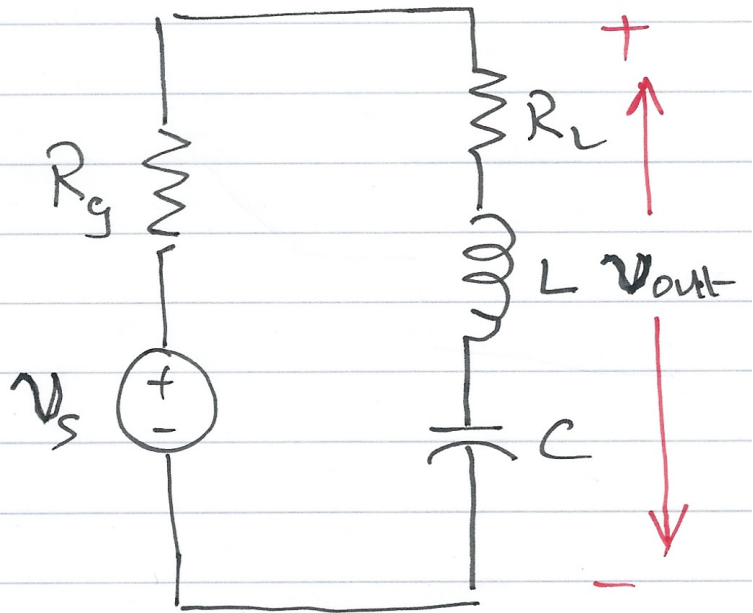
- Transfer Functions
- A Couple of Examples
- Circuit Analysis Using MATLAB LTI Transfer Function Block
- Circuit Simulation Using Simulink Transfer Function Block

```
% Matlab setup
clear all
cd ../matlab
pwd
imatlab_export_fig('print-svg') % Static svg figures.
format compact
```

Transfer Functions for Circuits

Example 6

Derive an expression for the transfer function $G(s)$ for the circuit below. In this circuit R_g represents the internal resistance of the applied (voltage) source v_s , and R_L represents the resistance of the load that consists of R_L , L and C .



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Sketch of Solution for Example 6

- Replace $v_s(t)$, R_g , R_L , L and C by their transformed (*complex frequency*) equivalents: $V_s(s)$, R_g , R_L , sL and $1/(sC)$
- Use the *Voltage Divider Rule* to determine $V_{\text{out}}(s)$ as a function of $V_s(s)$
- Form $G(s)$ by writing down the ratio $V_{\text{out}}(s)/V_s(s)$

Worked solution for Example 6

Pencast: [ex6.pdf](#) - open in Adobe Acrobat Reader.

Answer for Example 6

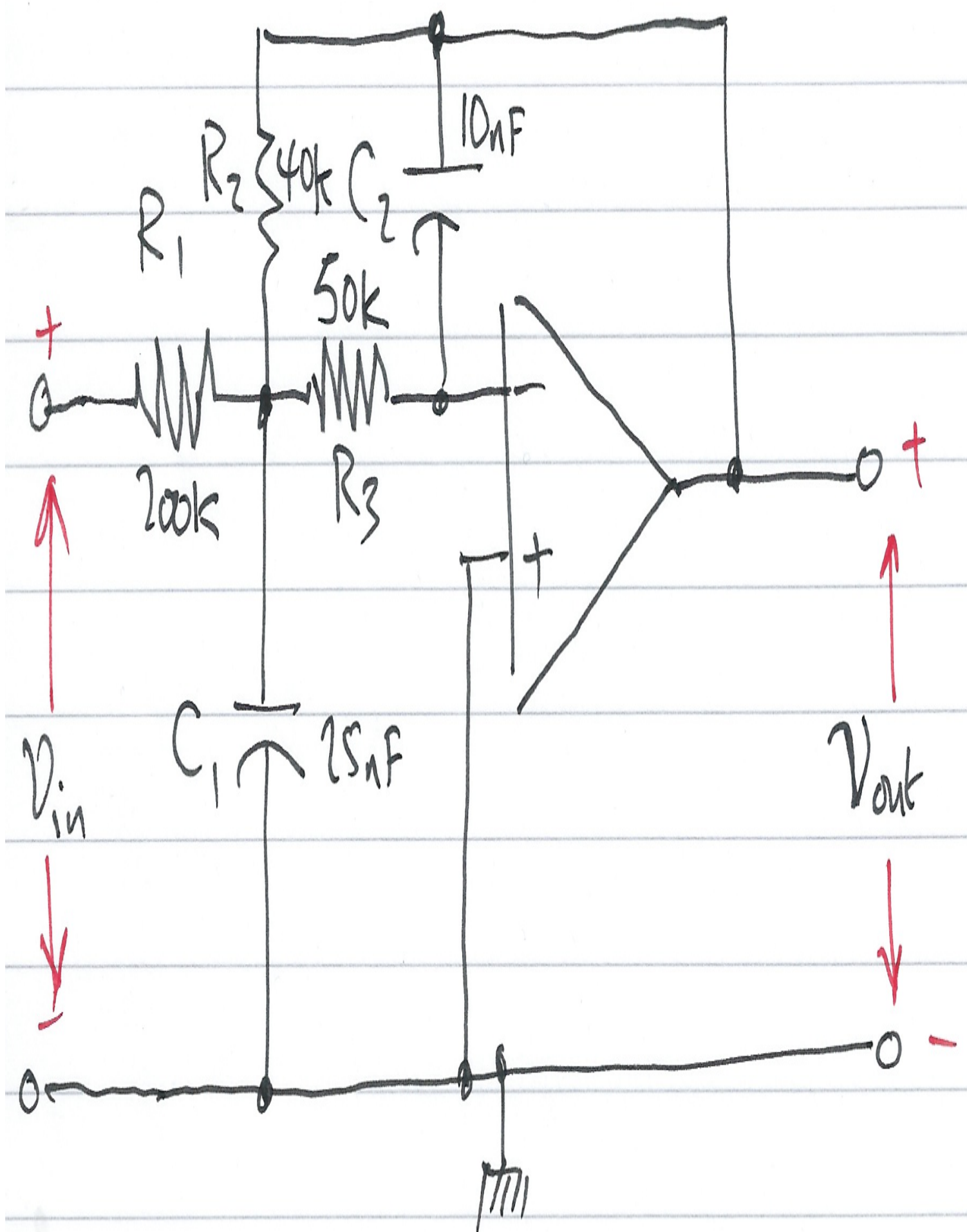
$$G(s) = \frac{V_{\text{out}}(s)}{V_s(s)} = \frac{R_L + sL + 1/sC}{R_g + R_L + sL + 1/sC}.$$

Example 7

Compute the transfer function for the op-amp circuit shown below in terms of the circuit constants R_1 , R_2 , R_3 , C_1 and C_2 . Then replace the complex variable s with $j\omega$, and the circuit constants with their numerical values and plot the magnitude

$$|G(j\omega)| = \frac{|V_{\text{out}}(j\omega)|}{|V_{\text{in}}(j\omega)|}$$

versus radian frequency ω rad/s.



Sketch of Solution for Example 7

- Replace the components and voltages in the circuit diagram with their complex frequency equivalents
- Use nodal analysis to determine the voltages at the nodes either side of the 50K resistor R_3

- Note that the voltage at the input to the op-amp is a virtual ground
- Solve for $V_{\text{out}}(s)$ as a function of $V_{\text{in}}(s)$
- Form the reciprocal $G(s) = V_{\text{out}}(s)/V_{\text{in}}(s)$
- Use MATLAB to calculate the component values, then replace s by $j\omega$.
- Plot $|G(j\omega)|$ on log-linear “paper”.



Worked solution for Example 7

Pencast: [ex7.pdf](#) - open in Adobe Acrobat Reader.

Answer for Example 7

$$G(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{-1}{R_1 \left(\left(1/R_1 + 1/R_2 + 1/R_3 + sC_1 \right) \left(sC_2R_3 \right) + 1/R_2 \right)}$$

The Matlab Bit

See attached script: [solution7.m](#).

Week 3: Solution 7

```
syms s;

R1 = 200*10^3; % 200 kOhm
R2 = 40*10^3; % 40 kOhm
R3 = 50*10^3; % 50 kOhm

C1 = 25*10^(-9); % 25 nF
C2 = 10*10^(-9); % 10 nF

den = R1*((1/R1+ 1/R2 + 1/R3 + s*C1)*(s*R3*C2) + 1/R2);
simplify(den)
```

Result is: $100*s*((7555786372591433*s)/302231454903657293676544 + 1/20000) + 5$

Simplify coefficients of s in denominator

```
format long
denG = sym2poly(ans)

numG = -1;
```

Plot

For convenience, define coefficients a and b :

```
a = denG(1);
b = denG(2);

w = 1:10:10000;

Gw = -1./(a*w.^2 - j.*b.*w + denG(3));

semilogx(w, abs(Gw))
xlabel('Radian frequency w (rad/s)')
ylabel('|Vout/Vin|')
title('Magnitude Vout/Vin vs. Radian Frequency')
grid
```

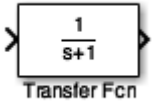
$$G(j\omega) = \frac{-1}{a\omega^2 - j b\omega + 5}$$

Using Transfer Functions in Matlab for System Analysis

Please use the file [tf_matlab.m](#) to explore the Transfer Function features provide by Matlab. Use the *publish* option to generate a nicely formatted document.



Using Transfer Functions in Simulink for System Simulation



The Simulink transfer function (**Transfer Fcn**) block shown above implements a transfer function representing a general input output function

$$G(s) = \frac{N(s)}{D(s)}$$

that it is not specific nor restricted to circuit analysis. It can, however be used in modelling and simulation studies.

Example

Recast Example 7 as a MATLAB problem using the LTI Transfer Function block.

For simplicity use parameters $R_1 = R_2 = R_3 = 1 \, \Omega$, and $C_1 = C_2 = 1 \, \text{F}$.

Calculate the step response using the LTI functions.

Verify the result with Simulink.

The Matlab solution: [example8.m](#)

MATLAB Solution

From a previous analysis the transfer function is:

$$G(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{-1}{R_1 \left[(1/R_1 + 1/R_2 + 1/R_3 + sC_1)(sR_3C_2) + 1/R_2 \right]}$$

so substituting the component values we get:

$$G(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{-1}{s^2 + 3s + 1}$$

We can find the step response by letting $v_{\text{in}}(t) = u_0(t)$ so that $V_{\text{in}}(s) = 1/s$ then

$$V_{\text{out}}(s) = \frac{-1}{s^2 + 3s + 1} \cdot \frac{1}{s}$$

We can solve this by partial fraction expansion and inverse Laplace transform as is done in the text book with the help of Matlab's **residue** function.

Here, however we'll use the LTI block that was introduced in the lecture.

Define the circuit as a transfer function

```
G = tf([-1],[1 3 1])
```

step response is then:

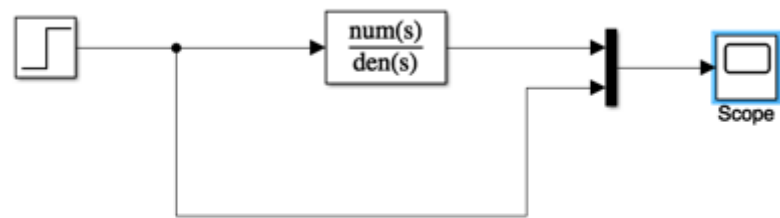
```
step(G)
```

Simples!

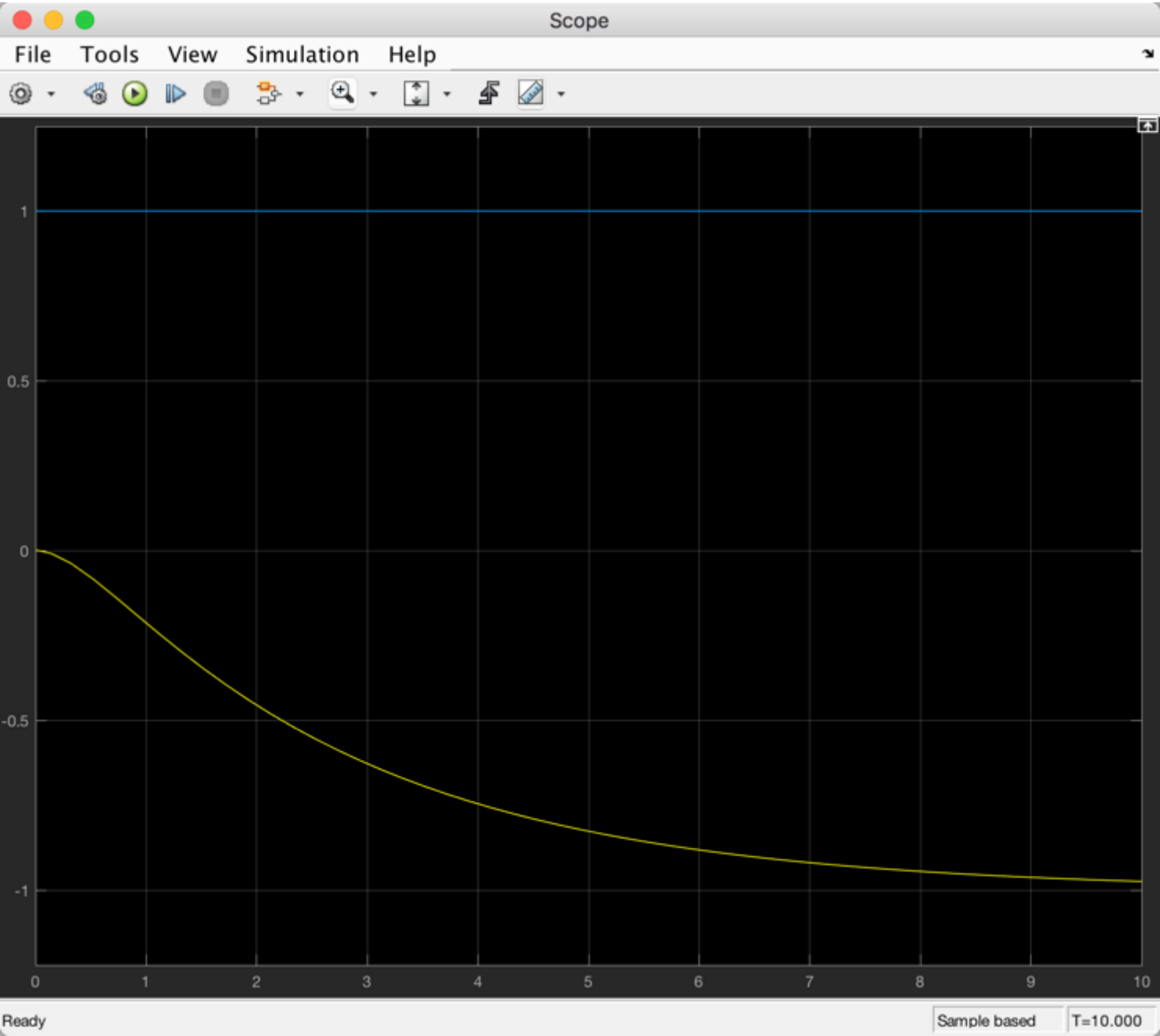
Simulink model

See [example 8.slx](#)

```
open example_8
```



Result



Let's go a bit further by finding the frequency response:

```
bode(G)
```

Matlab Solutions

For convenience, single script MATLAB solutions to the examples are provided and can be downloaded from the accompanying [MATLAB](#) folder.

- Solution 7 [\[solution7.m\]](#)
- Example 8 [\[example8.m\]](#)
- Simulink model [\[example 8.slx\]](#)

```
cd ../matlab
ls
open solution7
```

