Worksheet 7

To accompany Chapter 3.4 Transfer Functions ¶

Colophon

This worksheet can be downloaded as a PDF file. We will step through this worksheet in class.

An annotatable copy of the notes for this presentation will be distributed before the second class meeting as **Worksheet 7** in the **Week 3: Classroom Activities** section of the Canvas site. I will also distribute a copy to your personal **Worksheets** section of the **OneNote Class Notebook** so that you can add your own notes using OneNote.

You are expected to have at least watched the video presentation of Chapter 3.4 of the notes before coming to class. If you haven't watch it afterwards!

After class, the lecture recording and the annotated version of the worksheets will be made available through Canvas.

Second Hour's Agenda

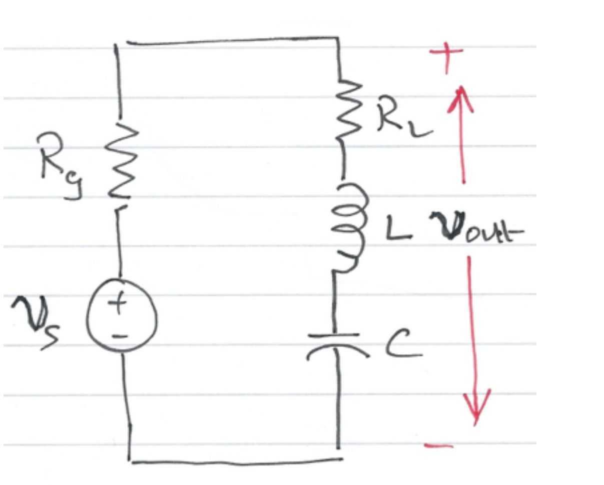
- Transfer Functions
- A Couple of Examples
- Circuit Analysis Using MATLAB LTI Transfer Function Block
- Circuit Simulation Using Simulink Transfer Function Block

In [ ]:
```
% Matlab setup
clear all
format compact
```

Transfer Functions for Circuits

Example 6

Derive an expression for the transfer function $G(s)$ for the circuit below. In this circuit R_g represents the internal resistance of the applied (voltage) source v_s, and R_L represents the resistance of the load that consists of R_L, L and C.



Sketch of Solution for Example 6

- Replace $v_s(t)$, R_g, R_L, L and C by their transformed (*complex frequency*) equivalents: $V_s(s)$, R_g, R_L, sL and $1/(sC)$
- Use the *Voltage Divider Rule* to determine $V_{\text{out}}(s)$ as a function of $V_s(s)$
- Form $G(s)$ by writing down the ratio $V_{\text{out}}(s)/V_s(s)$

Worked solution for Example 6

Pencast: ex6.pdf - open in Adobe Acrobat Reader.

Answer for Example 6

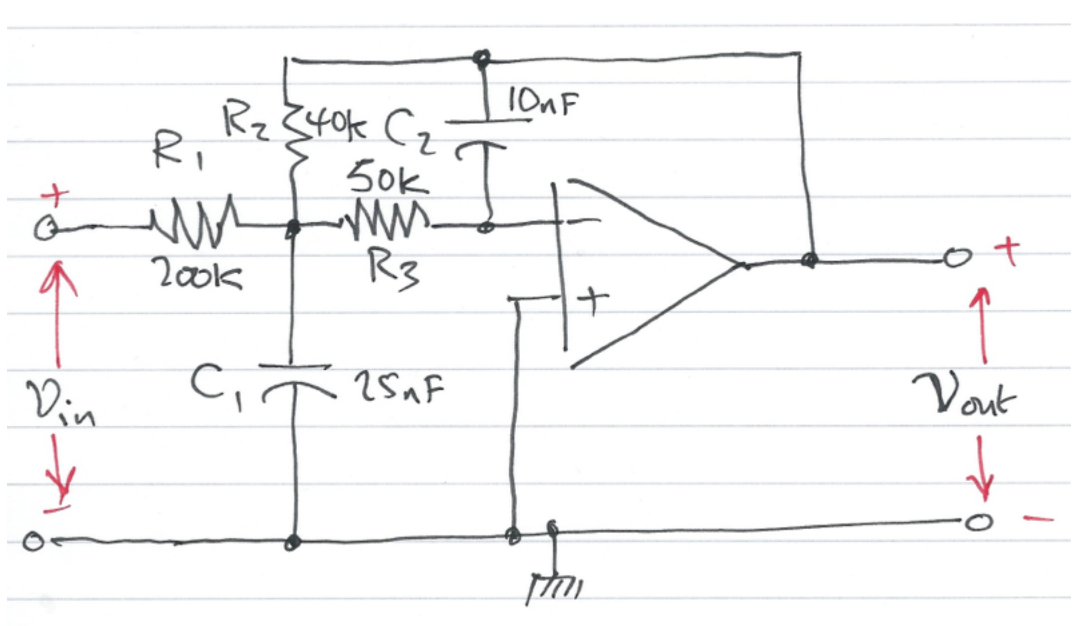
$$G(s) = \frac{V_{\text{out}}(s)}{V_s(s)} = \frac{R_L + sL + 1/sC}{R_g + R_L + sL + 1/sC}.$$

Example 7

Compute the transfer function for the op-amp circuit shown below in terms of the circuit constants R_1, R_2, R_3, C_1 and C_2. Then replace the complex variable s with $j\omega$, and the circuit constants with their numerical values and plot the magnitude

$$|G(j\omega)| = \frac{|V_{\text{out}}(j\omega)|}{|V_{\text{in}}(j\omega)|}$$

versus radian frequency ω rad/s.



Sketch of Solution for Example 7

- Replace the components and voltages in the circuit diagram with their complex frequency equivalents
- Use nodal analysis to determine the voltages at the nodes either side of the 50K resistor R_3
- Note that the voltage at the input to the op-amp is a virtual ground
- Solve for $V_{\text{out}}(s)$ as a function of $V_{\text{in}}(s)$
- Form the reciprocal $G(s) = V_{\text{out}}(s)/V_{\text{in}}(s)$
- Use MATLAB to calculate the component values, then replace s by $j\omega$.
- Plot

  $$|G(j\omega)|$$

  on log-linear "paper".

Worked solution for Example 7

Pencast: ex7.pdf - open in Adobe Acrobat Reader.

Answer for Example 7

$$G(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{-1}{R_1\left((1/R_1 + 1/R_2 + 1/R_3 + sC_1)(sC_2R_3) + 1/R_2\right)}.$$

The Matlab Bit

See attached script: solution7.m.

Week 3: Solution 7

In [ ]:
```
syms s;
```

In [ ]:
```
R1 = 200*10^3;
R2 = 40*10^3;
R3 = 50*10^3;

C1 = 25*10^(-9);
C2 = 10*10^(-9);
```

In [ ]:
```
den = R1*((1/R1+ 1/R2 + 1/R3 + s*C1)*(s*R3*C2) + 1/R2);
simplify(den)
```

Result is: `100*s*((7555786372591433*s)/302231454903657293676544 + 1/20000) + 5`

Simplify coefficients of s in denominator

In [ ]:
```
format long
denG = sym2poly(ans)
```

In [ ]:
```
numG = -1;
```

Plot

For convenience, define coefficients a and b:

In [ ]:
```
a = denG(1);
b = denG(2);
```

In [ ]:
```
w = 1:10:10000;
```

$$G(j\omega) = \frac{-1}{a\omega^2 - jb\omega + 5}$$

In [ ]:
```
Gs = -1./(a*w.^2 - j.*b.*w + denG(3));
```

In [ ]:
```
semilogx(w, abs(Gs))
xlabel('Radian frequency w (rad/s')
ylabel('|Vout/Vin|')
title('Magnitude Vout/Vin vs. Radian Frequency')
grid
```

Using Transfer Functions in Matlab for System Analysis

Please use the file tf_matlab.m to explore the Transfer Function features provide by Matlab. Use the *publish* option to generate a nicely formatted document.

Using Transfer Functions in Simulink for System Simulation

The Simulink transfer function (**`Transfer Fcn`**) block shown above implements a transfer function representing a general input output function

$$G(s) = \frac{N(s)}{D(s)}$$

that it is not specific nor restricted to circuit analysis. It can, however be used in modelling and simulation studies.

Example

Recast Example 7 as a MATLAB problem using the LTI Transfer Function block.

For simplicity use parameters $R_1 = R_2 = R_3 = 1\ \Omega$, and $C_1 = C_2 = 1$ F.

Calculate the step response using the LTI functions.

Verify the result with Simulink.

The Matlab solution: example8.m

MATLAB Solution

From a previous analysis the transfer function is:

$$G(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{-1}{R_1\left[(1/R_1 + 1/R_2 + 1/R_3 + sC_1)(sR_3C_2) + 1/R_2\right]}$$

so substituting the component values we get:

$$G(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{-1}{s^2 + 3s + 1}$$

We can find the step response by letting $v_{\text{in}}(t) = u_0(t)$ so that $V_{\text{in}}(s) = 1/s$ then

$$V_{\text{out}}(s) = \frac{-1}{s^2 + 3s + 1} \cdot \frac{1}{s}$$

We can solve this by partial fraction expansion and inverse Laplace transform as is done in the text book with the help of Matlab's `residue` function.

Here, however we'll use the LTI block that was introduced in the lecture.

Define the circuit as a transfer function

In [ ]:
```
G = tf([-1],[1 3 1])
```

step response is then:

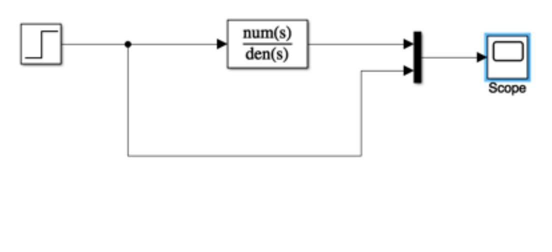
In [ ]:
```
step(G)
```

Simples!

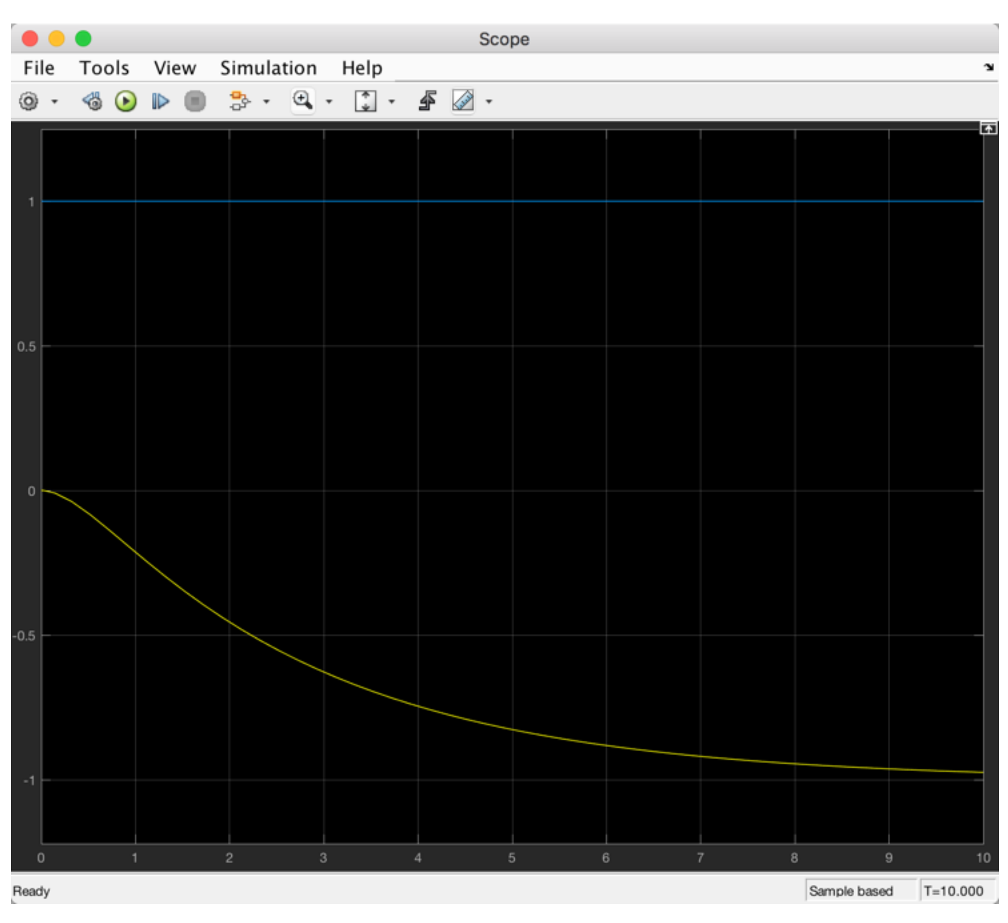
Simulink model

See example_8.slx

In [ ]:
```
open example_8
```



Result



Let's go a bit further by finding the frequency response:

In [ ]:
```
bode(G)
```

Matlab Solutions

For convenience, single script MATLAB solutions to the examples are provided and can be downloaded from the accompanying MATLAB folder.

- Solution 7 [solution7.m]
- Example 8 [example8.m]
- Simulink model [example_8.slx]

In [ ]:
```
cd ../matlab
ls
open solution7
```