

Using Laplace Transforms for Circuit Analysis

The preparatory reading for this section is [Chapter 4 \(https://ebookcentral.proquest.com/lib/swansea-ebooks/reader.action?docID=3384197&ppg=101\)](https://ebookcentral.proquest.com/lib/swansea-ebooks/reader.action?docID=3384197&ppg=101) {cite} karris which presents examples of the applications of the Laplace transform for electrical solving circuit problems.

Colophon

An annotatable worksheet for this presentation is available as [Worksheet 6 \(https://cpjobling.github.io/eg-247-textbook/laplace_transform/3/worksheet6.html\)](https://cpjobling.github.io/eg-247-textbook/laplace_transform/3/worksheet6.html).

- The source code for this page is [laplace_transform/3/circuit_analysis.ipynb \(https://github.com/cpjobling/eg-247-textbook/blob/master/laplace_transform/3/circuit_analysis.ipynb\)](https://github.com/cpjobling/eg-247-textbook/blob/master/laplace_transform/3/circuit_analysis.ipynb).
- You can view the notes for this presentation as a webpage ([HTML \(https://cpjobling.github.io/eg-247-textbook/laplace_transform/3/circuit_analysis.html\)](https://cpjobling.github.io/eg-247-textbook/laplace_transform/3/circuit_analysis.html)).
- This page is downloadable as a [PDF \(https://cpjobling.github.io/eg-247-textbook/laplace_transform/3/circuit_analysis.pdf\)](https://cpjobling.github.io/eg-247-textbook/laplace_transform/3/circuit_analysis.pdf) file.

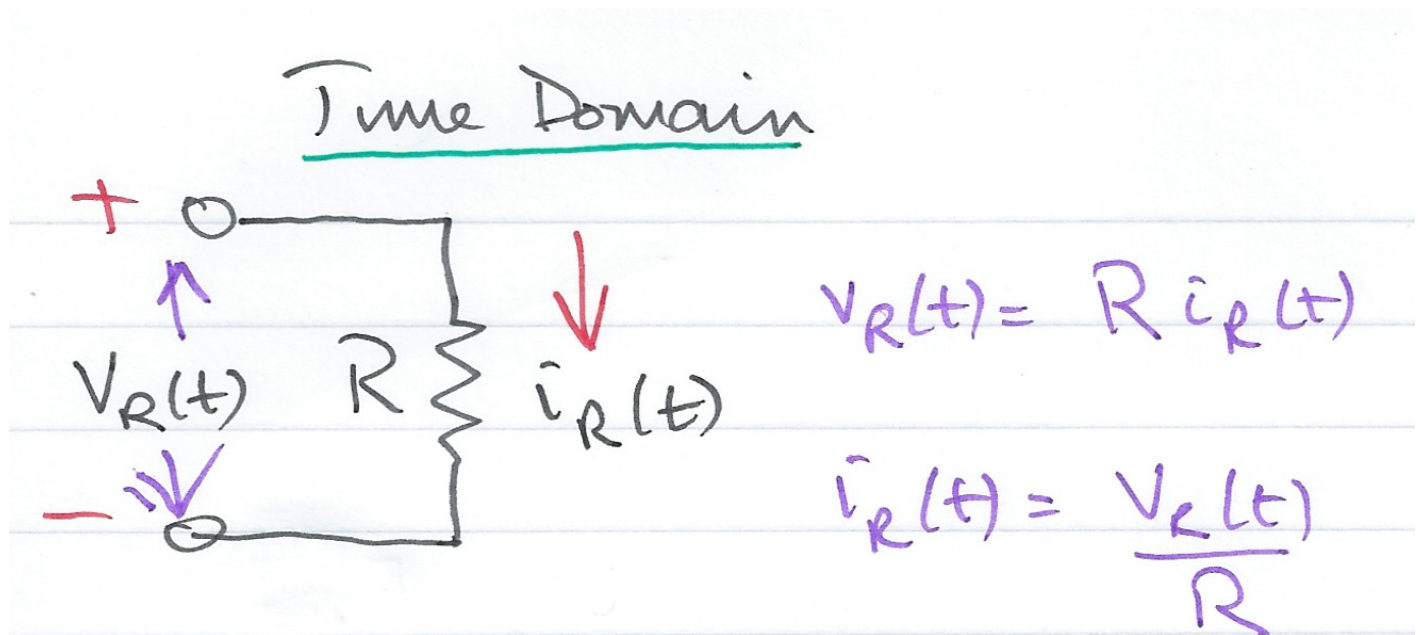
Agenda

We look at applications of the Laplace Transform for

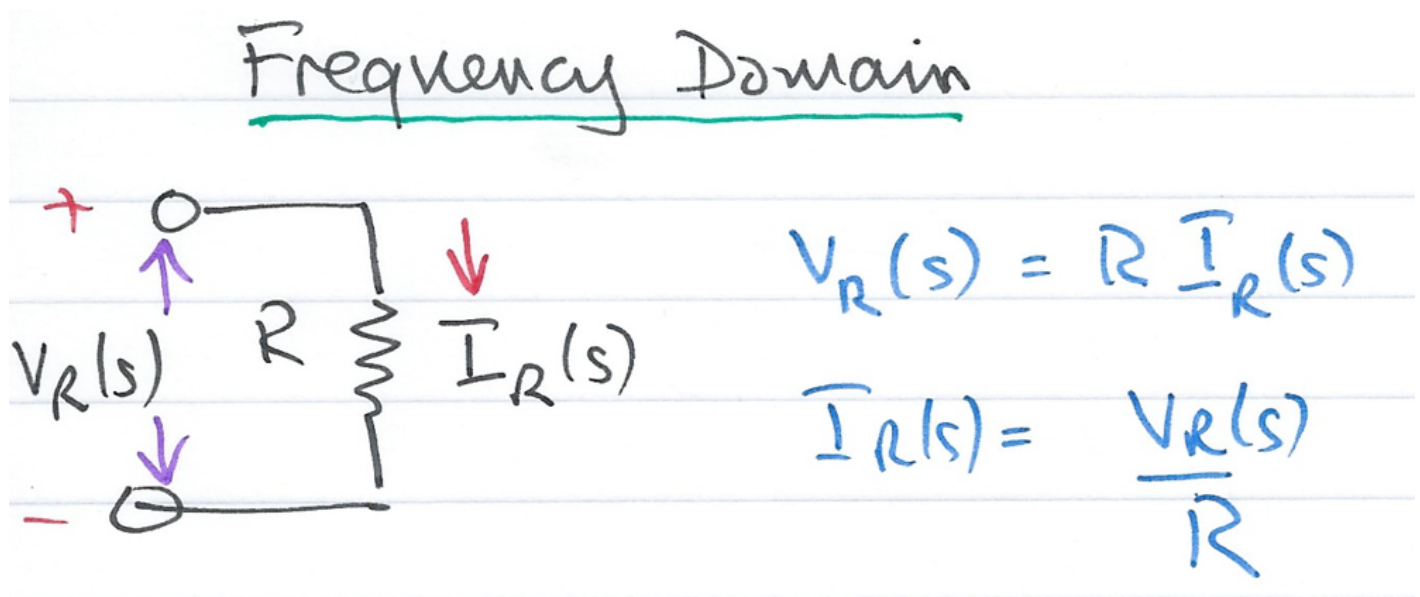
- Circuit transformation from Time to Complex Frequency
- Complex impedance
- Complex admittance

Circuit Transformation from Time to Complex Frequency

Time Domain Model of a Resistive Network

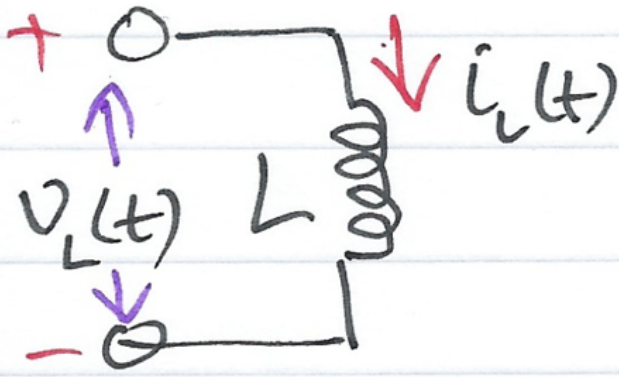


Complex Frequency Domain Model of a Resistive Circuit



Time Domain Model of an Inductive Network

Time Domain

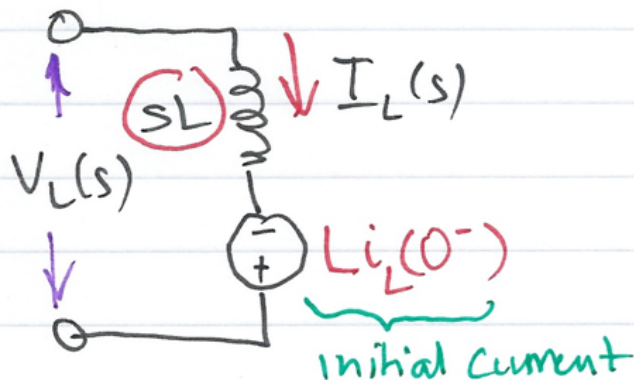


$$V_L(t) = L \frac{di_L}{dt}$$

$$i_L(t) = \frac{1}{L} \int_{-\infty}^t V_L dt$$

Complex Frequency Domain Model of an Inductive Network

Frequency Domain

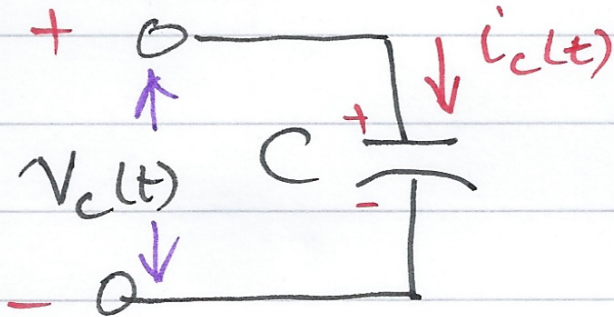


$$V_L(s) = sL I_L(s) - L i_L(0^-)$$

$$I_L(s) = \frac{V_L(s)}{Ls} + \frac{i_L(0^-)}{s}$$

Time Domain Model of a Capacitive Network

Time Domain

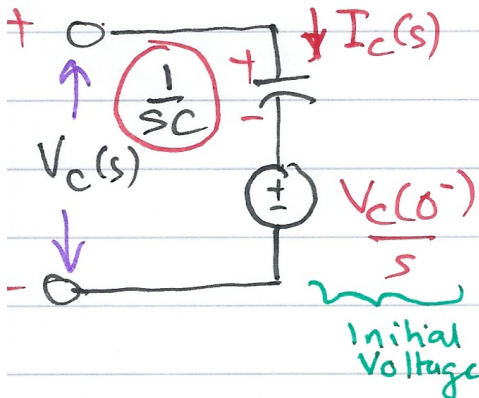


$$i_c(t) = C \frac{dv_c}{dt}$$

$$v_c(t) = \frac{1}{C} \int_{-\infty}^t i_c dt$$

Complex Frequency Domain of a Capacitive Network

Frequency Domain



$$I_c(s) = s C v_c(s) - C v_c(0^-)$$

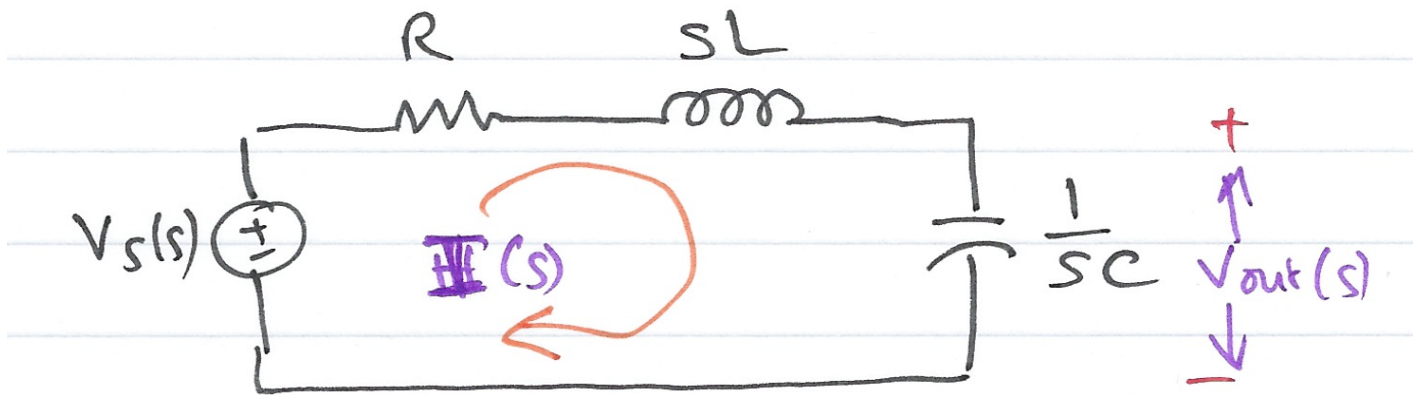
$$v_c(s) = \frac{I_c(s)}{sC} + \frac{v_c(0^-)}{s}$$

Examples

We will work through these in class. See [worksheet 6 \(worksheet6\)](#).

Complex Impedance $Z(s)$

Consider the s -domain RLC series circuit, where the initial conditions are assumed to be zero.



For this circuit, the sum

$$R + sL + \frac{1}{sC}$$

represents that total opposition to current flow. Then,

$$I(s) = \frac{V_s(s)}{R + sL + 1/(sC)}$$

and defining the ratio $V_s(s)/I(s)$ as $Z(s)$, we obtain

$$Z(s) = \frac{V_s(s)}{I(s)} = R + sL + \frac{1}{sC}$$

The s -domain current $I(s)$ can be found from

$$I(s) = \frac{V_s(s)}{Z(s)}$$

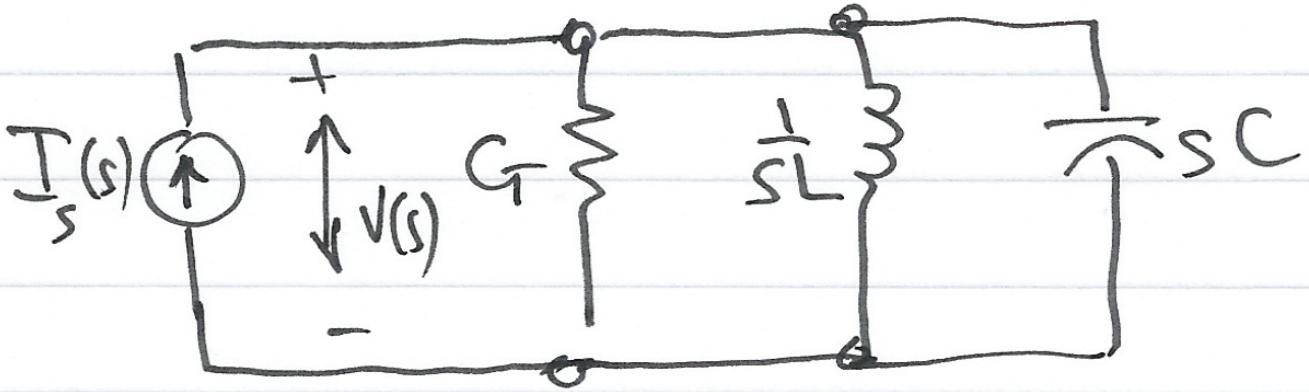
where

$$Z(s) = R + sL + \frac{1}{sC}.$$

Since $s = \sigma + j\omega$ is a complex number, $Z(s)$ is also complex and is known as the *complex input impedance* of this RLC series circuit.

Complex Admittance $Y(s)$

Consider the s -domain GLC parallel circuit shown below where the initial conditions are zero.



For this circuit

$$GV(s) + \frac{1}{sL}V(s) + sCV(s) = I_s(s)$$

$$\left(G + \frac{1}{sL} + sC\right)V(s) = I_s(s)$$

Defining the ratio $I_s(s)/V(s)$ as $Y(s)$ we obtain

$$Y(s) = \frac{I_s(s)}{V(s)} = G + \frac{1}{sL} + sC = \frac{1}{Z(s)}$$

The s -domain voltage $V(s)$ can be found from

$$V(s) = \frac{I_s(s)}{Y(s)}$$

where

$$Y(s) = G + \frac{1}{sL} + sC.$$

$Y(s)$ is complex and is known as the *complex input admittance* of this GLC parallel circuit.

Reference

See [Bibliography \(/zbib\)](#).