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Worksheet 4

Contents

To accompany Unit 3.3 Computing Line Spectra

Colophon

This worksheet can be downloaded as a PDF file. We will step through this worksheet in class.

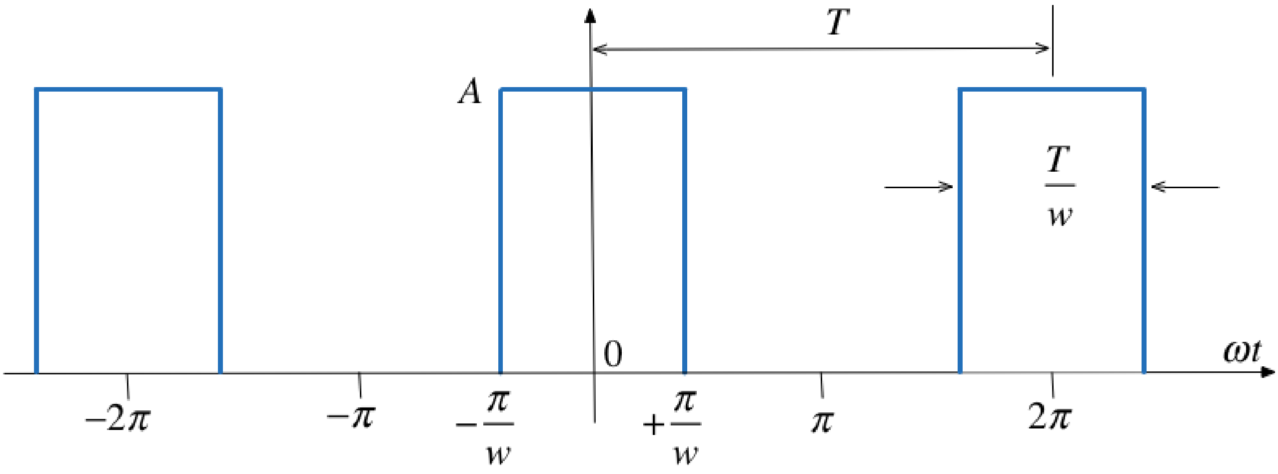
An annotatable copy of the notes for this presentation will be distributed before the second class meeting as **Worksheet 4** in the **Week 4: Classroom Activities** section of the Canvas site. I will also distribute a copy to your personal **Worksheets** section of the **OneNote Class Notebook** so that you can add your own notes using OneNote.

You are expected to have at least watched the video presentation of Unit 3.3: Computing Line Spectra of the notes before coming to class. If you haven't watch it afterwards!

After class, the lecture recording and the annotated version of the worksheets will be made available through Canvas.

Example 3

Compute the exponential Fourier series for the waveform shown below and plot its line spectra.



Solution to example 3

The recurrent rectangular pulse is used extensively in digital communication systems. To determine how faithfully such pulses will be transmitted, it is necessary to know the frequency components.

What do we know?

- The pulse duration is T/w.
- The recurrence interval T is w times the pulse duration.
- w is the ratio of pulse repetition time to the pulse duration – normally called the *duty cycle*.

Coefficients of the Exponential Fourier Series?

Given

$$C_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\Omega_0 t)e^{-jk(\Omega_0 t)}\,d(\Omega_0 t)$$

- Is the function **even** or **odd**?
- Does the signal have **half-wave symmetry**?
- What are the cosequencies of symmetry on the form of the coefficients C_k?
- What function do we actually need to integrate to compute C_k?

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DC Component?

Let $k = 0$ then perform the integral

Harmonic coefficients?

Integrate for $k \neq 0$

Exponential Fourier Series?

Effect of pulse width on frequency spectra

- Recall pulse width = T/w

We will use the provided MATLAB script sinc.mlx to explore these in class. You will also need pulse_fs.m. See Canvas/OneNote for copies of these files.

w = 2

$\Omega_0 = 1$ rad/s; $w = 2$; $T = 2\pi$ s; $T/w = \pi$ s.

w = 5

$\Omega_0 = 1$ rad/s; $w = 5$; $T = 2\pi$ s; $T/w = 2\pi/5$ s.

w = 10

$\Omega_0 = 1$ rad/s; $w = 10$; $T = 2\pi$ s; $T/w = \pi/5$ s.

Implications

- As the width of the pulse **reduces** the width of the freqency spectra needed to fully describe the signal **increases**

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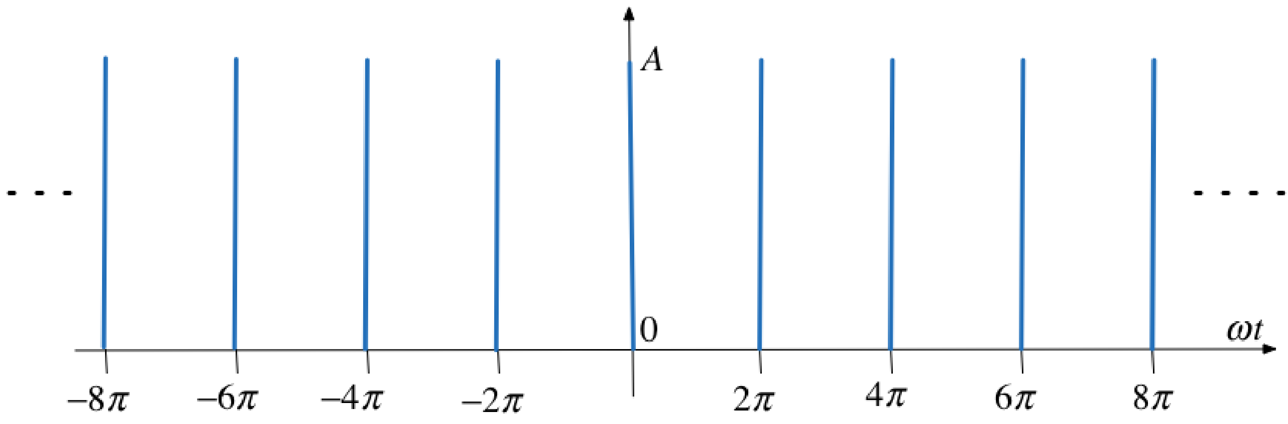
Example 4

Use the result of Example 1 to compute the exponential Fourier series of the impulse train $\delta(t \pm 2\pi k)$ shown below



Solution to example 4

To solve this we take the previous result and choose amplitude (height) A so that area of pulse is unity. Then we let width go to zero while maintaining the area of unity. This creates a train of impulses $\delta(t \pm 2\pi k)$.

$$C_k = \frac{1}{2\pi}$$

and, therefore

$$f(t) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} e^{jk\Omega_0 t}$$

Try it!

Proof!

From the previous result,

$$C_k = \frac{A}{w} \cdot \frac{\sin(k\pi/w)}{k\pi/w}$$

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$$\frac{T}{w} = \frac{2\pi}{w}$$

Let us take the previous impulse train as a recurrent pulse with amplitude

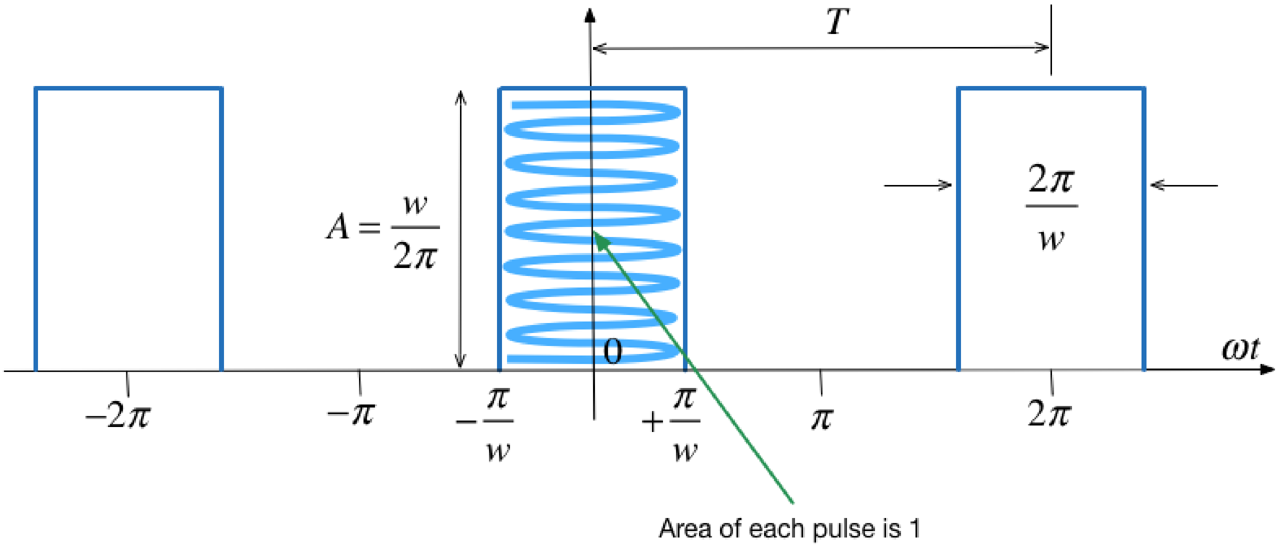
$$A = \frac{1}{T/w} = \frac{1}{2\pi/w} = \frac{w}{2\pi}.$$

Pulse with unit area

The area of each pulse is then

$$\frac{2\pi}{w} \times \frac{w}{2\pi} = 1$$

and the pulse train is as shown below:



New coeffcients

The coefficients of the Exponential Fourier Series are now:

$$C_n = \frac{w/2\pi}{w}\frac{\sin(k\pi/w)}{k\pi/w} = \frac{1}{2\pi}\frac{\sin(k\pi/w)}{k\pi/w}$$

and as $\pi/w \to 0$ each recurrent pulse becomes a unit impulse, and the pulse train reduces

Also, recalling that

$$\lim_{x \to 0} \frac{sinx}{x} = 1$$

the coefficents reduce to

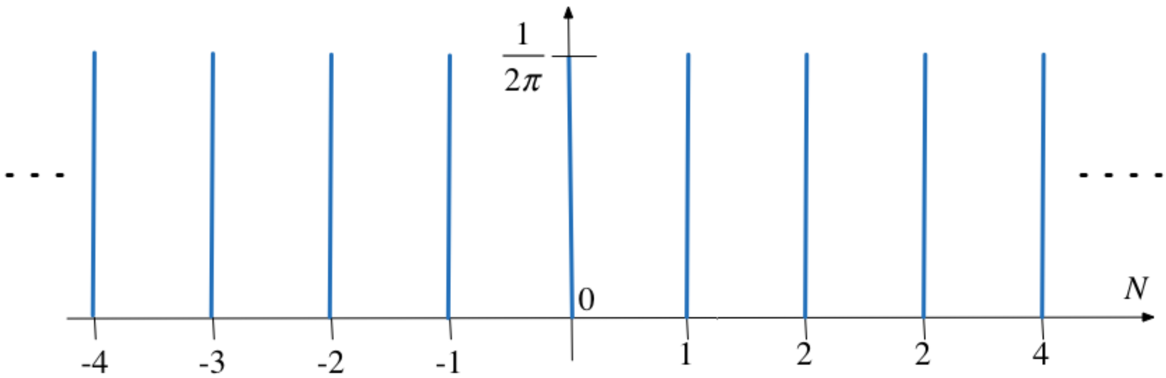
$$C_n = \frac{1}{2\pi}$$

That is all coefficients have the same amplitude and thus

$$f(t) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} e^{jk\Omega_0 t}$$

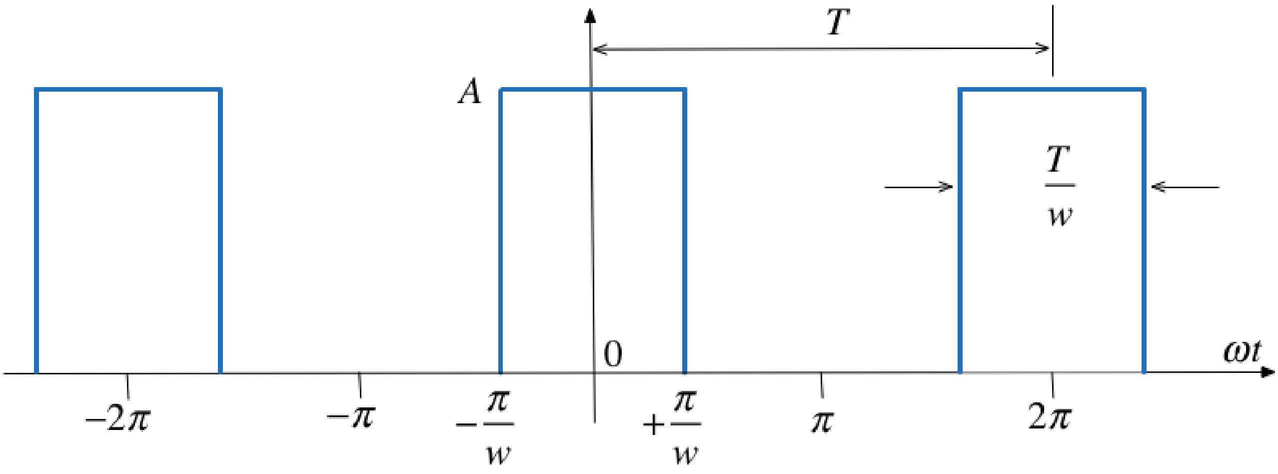
Spectrum of Unit Impulse Train

The line spectrum of a sequence of unit impulses $\delta(t \pm kT)$ is shown below:



Another Interesting Result

Consider the pulse train agin:



What happens when the pulses to the left and right of the centre pulse become less and less frequent? That is what happens when $T \rightarrow \infty$?

Well?

- As $T \rightarrow \infty$ the fundamental frequency $\Omega_0 \rightarrow 0$
- We are then left with just one pulse centred around $t = 0$.
- The frequency difference between harmonics also becomes smaller.
- Line spectrum becomes a continous function.

This result is the basis of the *Fourier Transform* which is coming soon.

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