Using Laplace Transforms for Circuit Analysis

The preparatory reading for this section is Chapter 4 (https://ebookcentral.proquest.com/lib/swansea-ebooks/reader.action?docID=3384197&ppg=101) {% cite karris %} which presents examples of the applications of the Laplace transform for electrical solving circuit problems.

Colophon

An annotatable worksheet for this presentation is available as **Worksheet 6 (https://cpjobling.github.io/eg-247-textbook/laplace_transform/3/worksheet6.html)**.

- The source code for this page is content/laplace_transform/3/circuit_analysis.ipynb (https://github.com/cpjobling/eg-247-textbook/blob/master/content/laplace_transform/3/circuit_analysis.ipynb).
- You can view the notes for this presentation as a webpage (HTML (https://cpjobling.github.io/eg-247-textbook/laplace_transform/3/circuit_analysis.html)).
- This page is downloadable as a PDF (https://cpjobling.github.io/eg-247-textbook/laplace_transform/3/circuit_analysis.pdf) file.
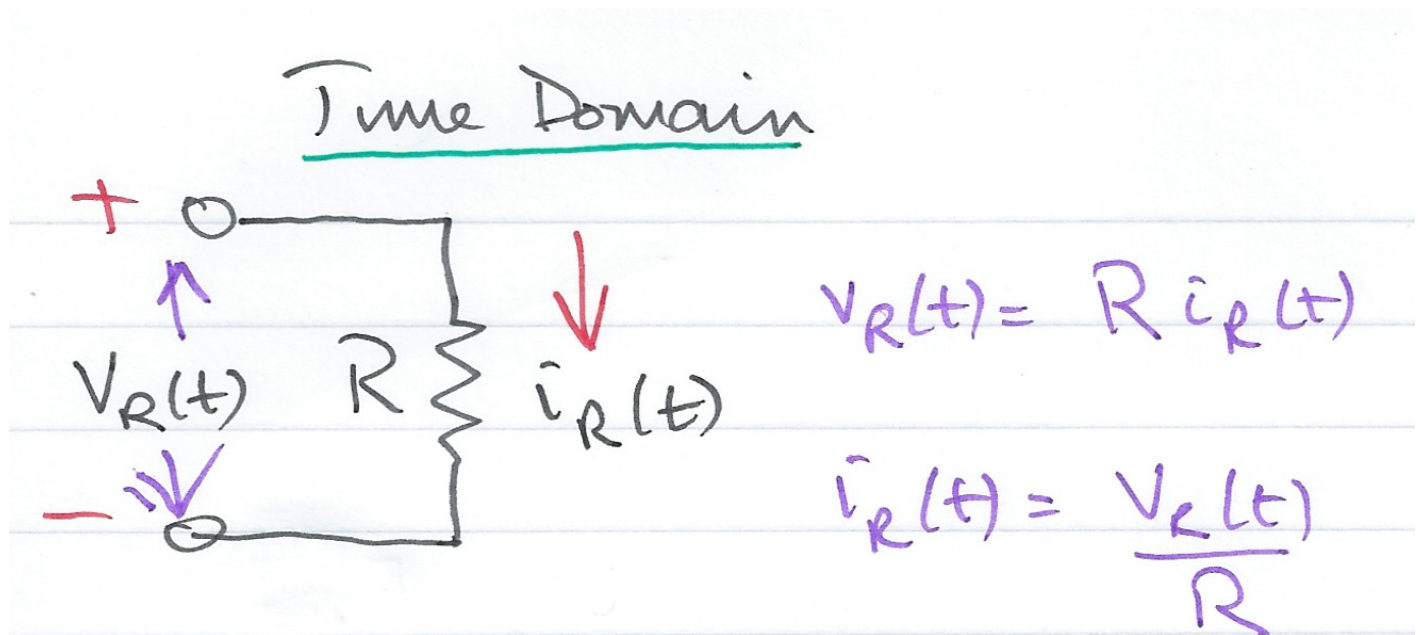
Agenda

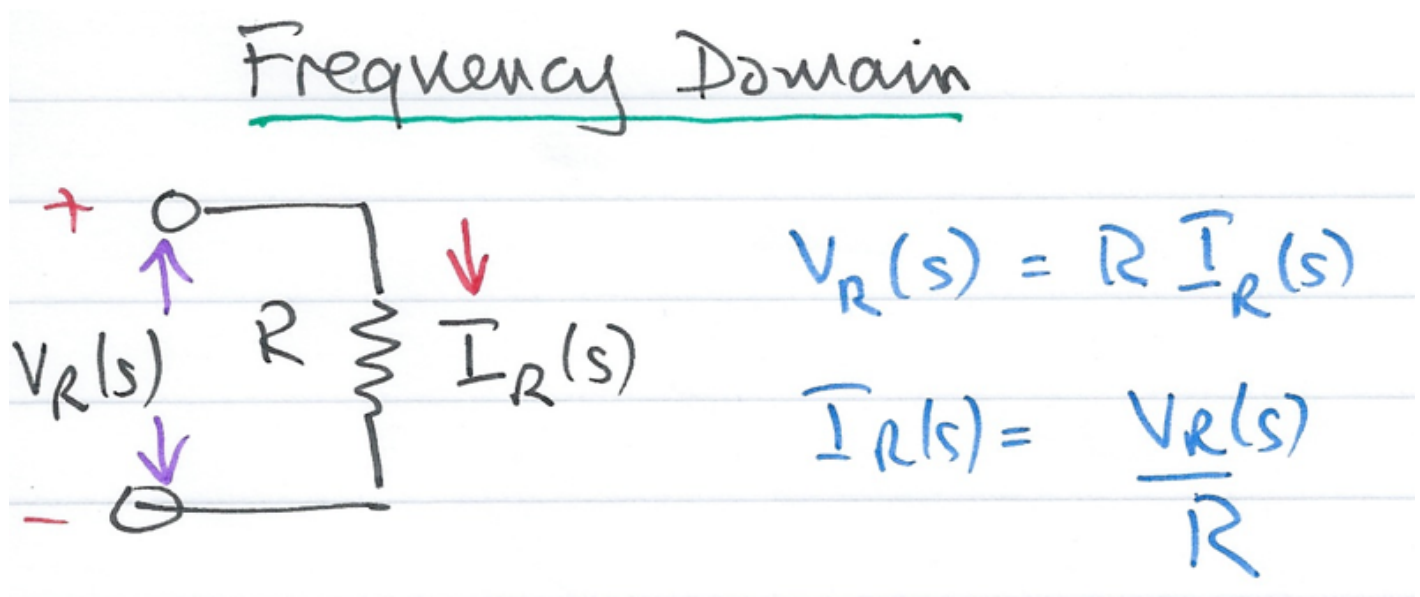We look at applications of the Laplace Transform for

- Circuit transformation from Time to Complex Frequency

- Complex impedance

- Complex admittance

Circuit Transformation from Time to Complex Frequency

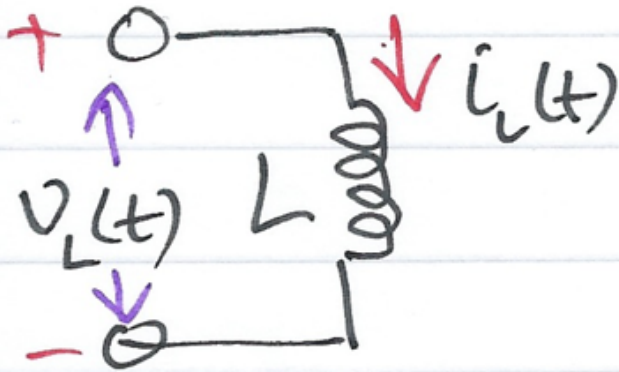## Time Domain Model of a Resistive Network

$$v_R(t) = R\, i_R(t)$$

$$i_R(t) = \frac{v_R(t)}{R}$$

Complex Frequency Domain Model of a Resistive Circuit

$$V_R(s) = R\, I_R(s)$$

$$I_R(s) = \frac{V_R(s)}{R}$$

Time Domain Model of an Inductive Network

$$\text{Time Domain}$$

$$V_L(t) = L\frac{di_L}{dt}$$

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} V_L\, dt$$

with $i_L(t)$ and $V_L(t)$, inductor L.

Complex Frequency Domain Model of an Inductive Network
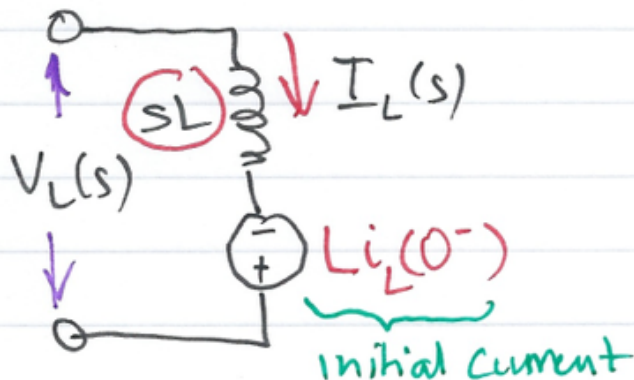
$$\text{Frequency Domain}$$

$$V_L(s) = sL\, I_L(s) - L i_L(0^-)$$

$$I_L(s) = \frac{V_L(s)}{Ls} + \frac{i_L(0^-)}{s}$$

with sL, $I_L(s)$, $V_L(s)$, source $L i_L(0^-)$ — initial current

Time Domain Model of a Capacitive Network

Time Domain

$$i_c(t) = C \frac{dv_c}{dt}$$

$$v_c(t) = \frac{1}{C} \int_{-\infty}^{t} i_c \, dt$$
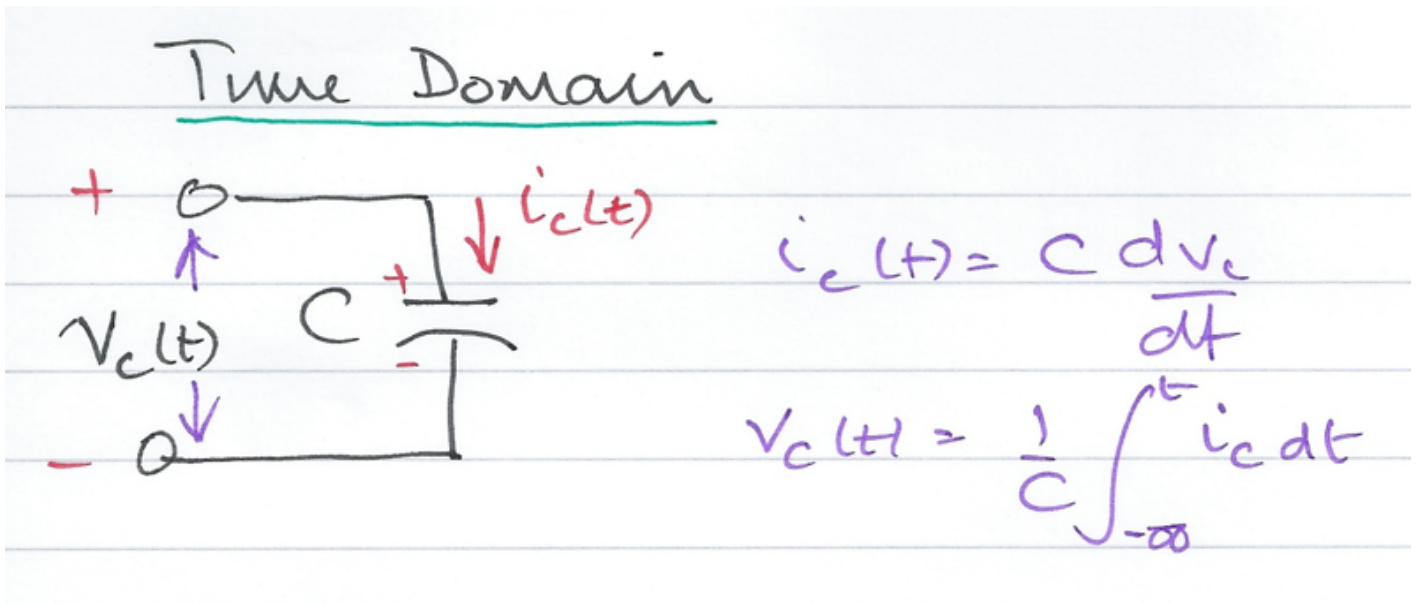
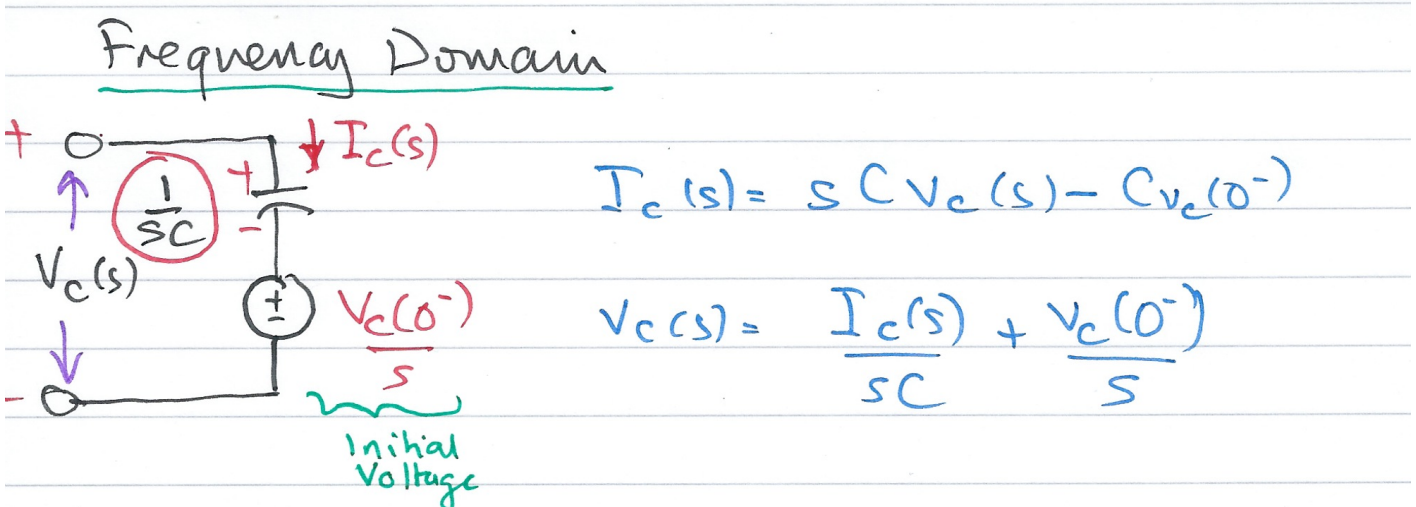## Complex Frequency Domain of a Capacitive Network

Frequency Domain

Initial Voltage

$$I_c(s) = sC v_c(s) - C v_c(0^-)$$
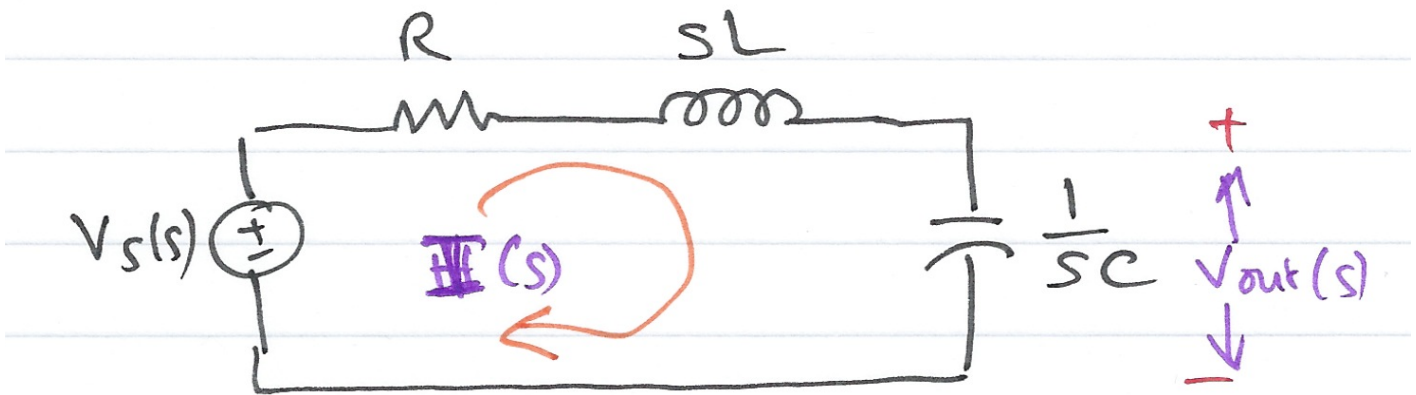
$$V_c(s) = \frac{I_c(s)}{sC} + \frac{v_c(0^-)}{s}$$

Examples

We will work through these in class. See worksheet 6 (worksheet6).

Complex Impedance $Z(s)$

Consider the s-domain RLC series circuit, where the initial conditions are assumed to be zero.

For this circuit, the sum

$$R + sL + \frac{1}{sC}$$

represents that total opposition to current flow. Then,

$$I(s) = \frac{V_s(s)}{R + sL + 1/(sC)}$$

and defining the ratio $V_s(s)/I(s)$ as $Z(s)$, we obtain

$$Z(s) = \frac{V_s(s)}{I(s)} = R + sL + \frac{1}{sC}$$

The s-domain current $I(s)$ can be found from
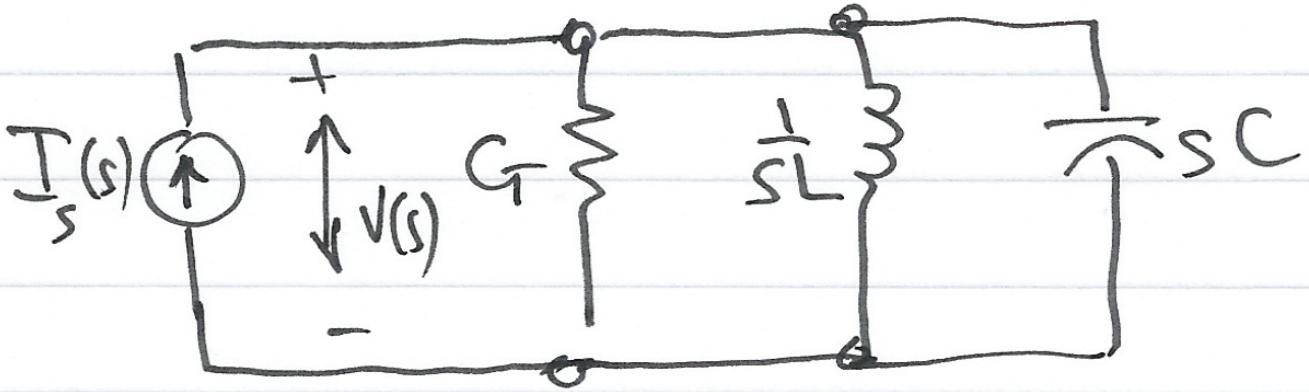
$$I(s) = \frac{V_s(s)}{Z(s)}$$

where

$$Z(s) = R + sL + \frac{1}{sC}.$$

Since $s = \sigma + j\omega$ is a complex number, $Z(s)$ is also complex and is known as the *complex input impedance* of this RLC series circuit.

Complex Admittance $Y(s)$

Consider the s-domain GLC parallel circuit shown below where the initial conditions are zero.

For this circuit

$$GV(s) + \frac{1}{sL}V(s) + sCV(s) = I_s(s)$$
$$\left(G + \frac{1}{sL} + sC\right)V(s) = I_s(s)$$

Defining the ratio $I_s(s)/V(s)$ as $Y(s)$ we obtain
$$Y(s) = \frac{I_s(s)}{V(s)} = G + \frac{1}{sL} + sC = \frac{1}{Z(s)}$$

The s-domain voltage $V(s)$ can be found from
$$V(s) = \frac{I_s(s)}{Y(s)}$$

where

$$Y(s) = G + \frac{1}{sL} + sC.$$

$Y(s)$ is complex and is known as the *complex input admittance* of this GLC parallel circuit.

Reference

{% bibliography --cited %}