# Using Laplace Transforms for Circuit Analysis

The preparatory reading for this section is [Chapter 4 (https://ebookcentral.proquest.com/lib/swansea-ebooks/reader.action?docID=3384197&ppg=101)](https://ebookcentral.proquest.com/lib/swansea-ebooks/reader.action?docID=3384197&ppg=101) {cite} `karris` which presents examples of the applications of the Laplace transform for electrical solving circuit problems.

## Colophon

An annotatable worksheet for this presentation is available as [**Worksheet 6** (https://cpjobling.github.io/eg-247-textbook/laplace_transform/3/worksheet6.html)](https://cpjobling.github.io/eg-247-textbook/laplace_transform/3/worksheet6.html).

- The source code for this page is [laplace_transform/3/circuit_analysis.ipynb (https://github.com/cpjobling/eg-247-textbook/blob/master/laplace_transform/3/circuit_analysis.ipynb)](https://github.com/cpjobling/eg-247-textbook/blob/master/laplace_transform/3/circuit_analysis.ipynb).
- You can view the notes for this presentation as a webpage ([HTML (https://cpjobling.github.io/eg-247-textbook/laplace_transform/3/circuit_analysis.html)](https://cpjobling.github.io/eg-247-textbook/laplace_transform/3/circuit_analysis.html)).
- This page is downloadable as a [PDF (https://cpjobling.github.io/eg-247-textbook/laplace_transform/3/circuit_analysis.pdf)](https://cpjobling.github.io/eg-247-textbook/laplace_transform/3/circuit_analysis.pdf) file.

## Agenda

We look at applications of the Laplace Transform for

- Circuit transformation from Time to Complex Frequency

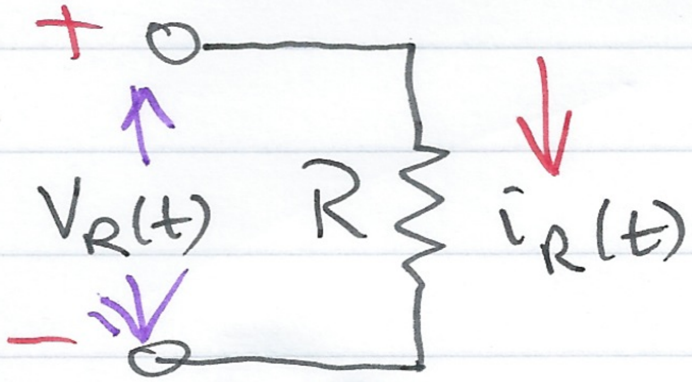
- Complex impedance

- Complex admittance

## Circuit Transformation from Time to Complex Frequency

### Time Domain Model of a Resistive Network

Time Domain

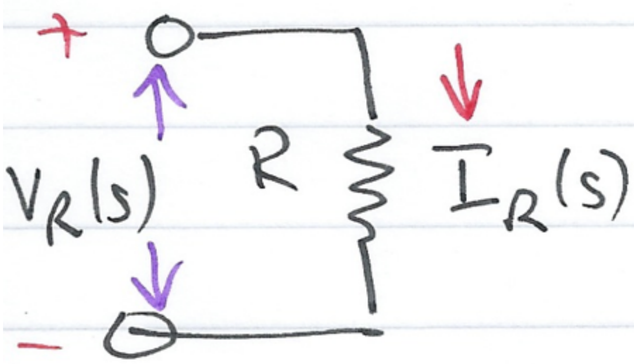



$$v_R(t) = R\, i_R(t)$$

$$i_R(t) = \frac{V_R(t)}{R}$$

### Complex Frequency Domain Model of a Resistive Circuit

Frequency Domain

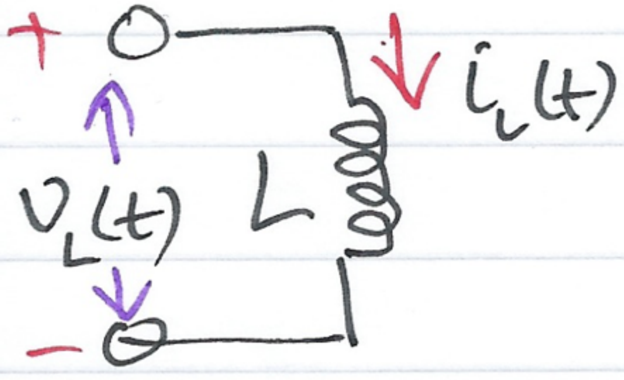



$$V_R(s) = R\, I_R(s)$$

$$I_R(s) = \frac{V_R(s)}{R}$$

### Time Domain Model of an Inductive Network

Time Domain

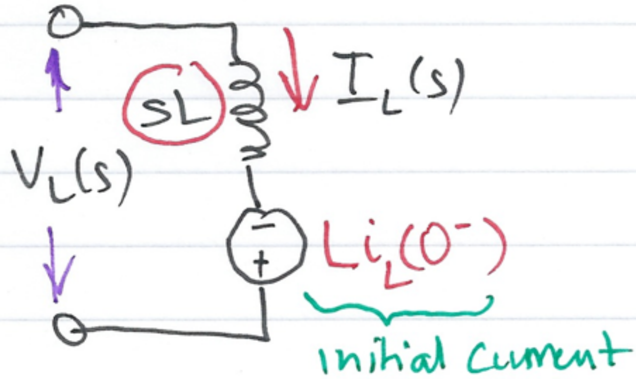



$$V_L(t) = L\frac{di_L}{dt}$$

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} V_L \, dt$$

### Complex Frequency Domain Model of an Inductive Network

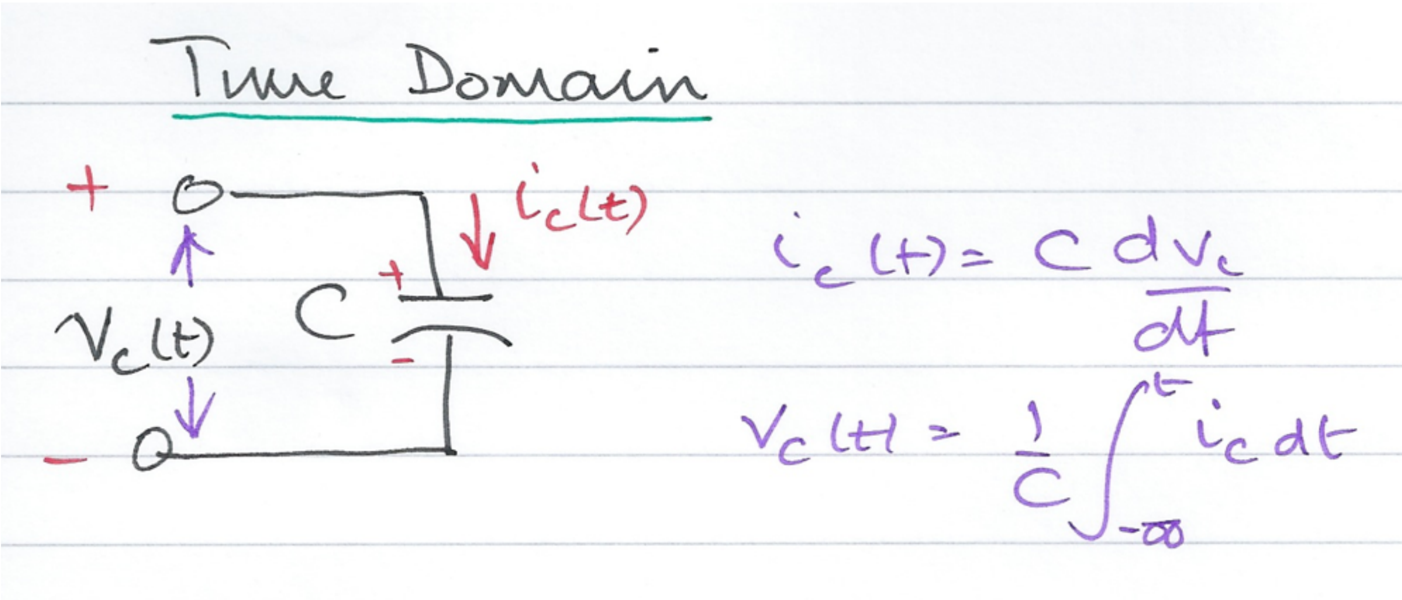
Frequency Domain



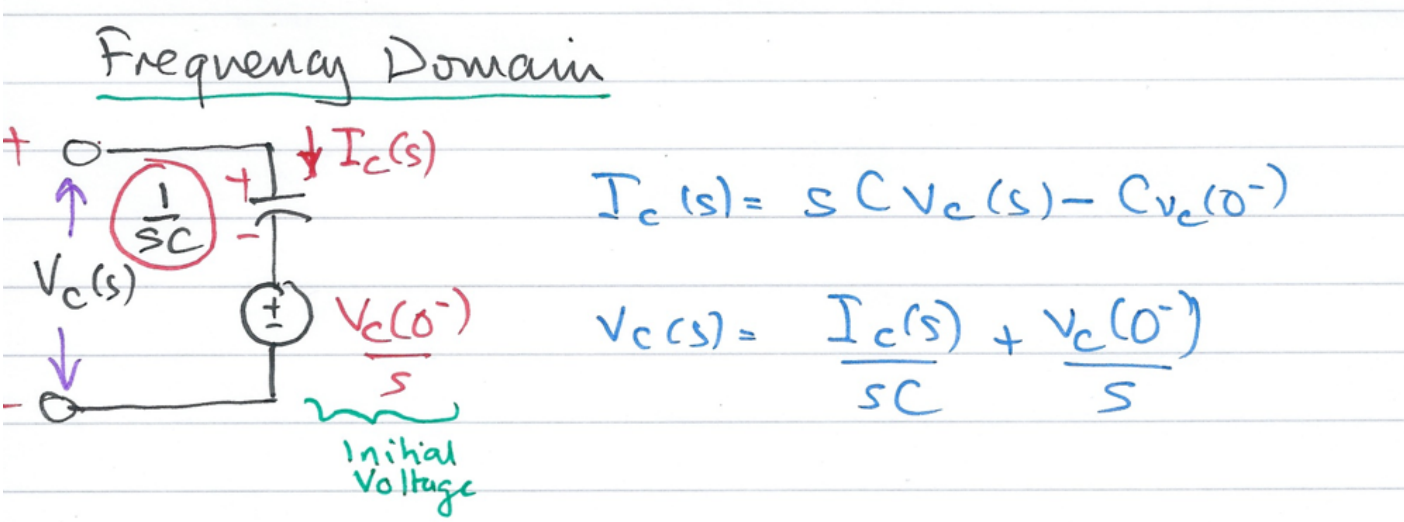

$$V_L(s) = sLI_L(s) - Li_L(0^-)$$

$$I_L(s) = \frac{V_L(s)}{Ls} + \frac{i_L(0^-)}{s}$$

## Time Domain Model of a Capacitive Network




## Complex Frequency Domain of a Capacitive Network

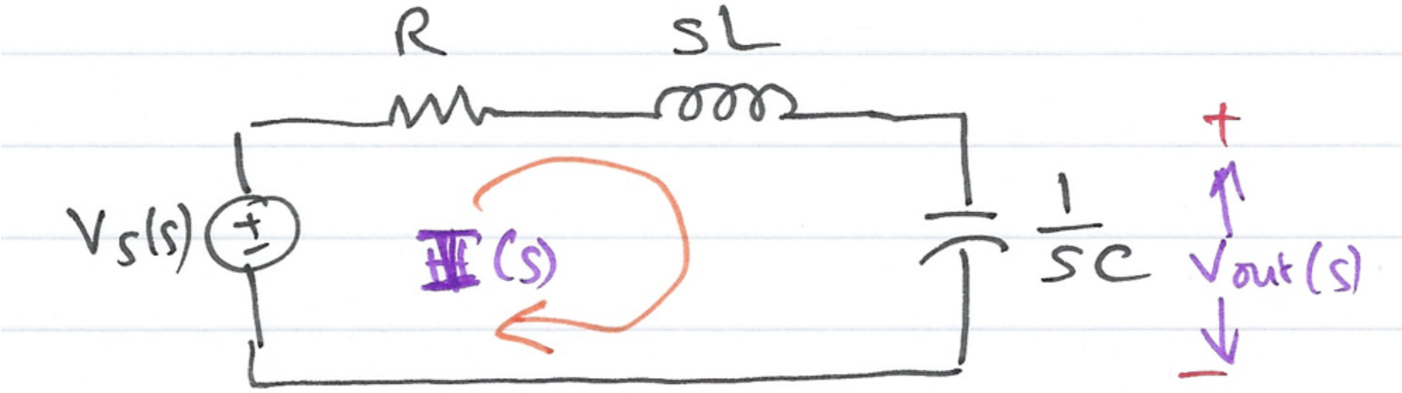



## Examples

We will work through these in class. See worksheet 6 (worksheet6).

# Complex Impedance $Z(s)$

Consider the $s$-domain RLC series circuit, where the initial conditions are assumed to be zero.




For this circuit, the sum

$$R + sL + \frac{1}{sC}$$

represents that total opposition to current flow. Then,

$$I(s) = \frac{V_s(s)}{R + sL + 1/(sC)}$$

and defining the ratio $V_s(s)/I(s)$ as $Z(s)$, we obtain

$$Z(s) = \frac{V_s(s)}{I(s)} = R + sL + \frac{1}{sC}$$

The $s$-domain current $I(s)$ can be found from

$$I(s) = \frac{V_s(s)}{Z(s)}$$

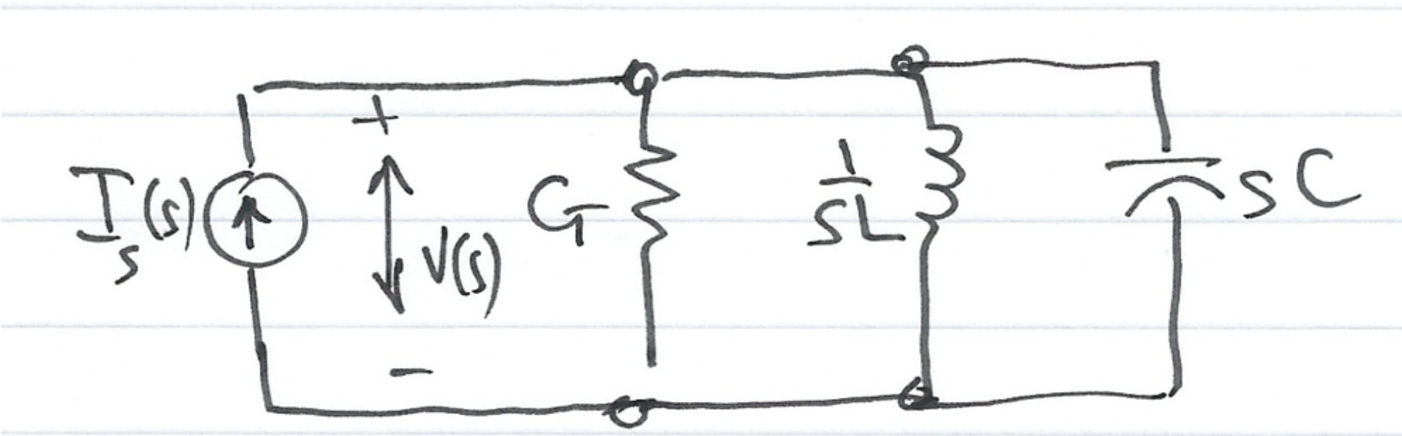
where

$$Z(s) = R + sL + \frac{1}{sC}.$$

Since $s = \sigma + j\omega$ is a complex number, $Z(s)$ is also complex and is known as the *complex input impedance* of this RLC series circuit.

## Complex Admittance $Y(s)$

Consider the $s$-domain GLC parallel circuit shown below where the initial conditions are zero.




For this circuit

$$GV(s) + \frac{1}{sL}V(s) + sCV(s) = I_s(s)$$
$$\left(G + \frac{1}{sL} + sC\right)V(s) = I_s(s)$$

Defining the ratio $I_s(s)/V(s)$ as $Y(s)$ we obtain

$$Y(s) = \frac{I_s(s)}{V(s)} = G + \frac{1}{sL} + sC = \frac{1}{Z(s)}$$

The $s$-domain voltage $V(s)$ can be found from

$$V(s) = \frac{I_s(s)}{Y(s)}$$

where

$$Y(s) = G + \frac{1}{sL} + sC.$$

$Y(s)$ is complex and is known as the *complex input admittance* of this GLC parallel circuit.

## Reference

See [Bibliography (/zbib)](/zbib).