Learning and predicting with statistical models

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1 Introduction

We start, as usual, by loading the appropriate package:

> library(Modeler)

2 Simulated DataSet

In order to have something to test our models against, we simulate a dataset that has enough underlying structure to make it interesting. First, we set the random seed so that the results will be reproducible.

> set.seed(234843)

Next, we define the simulation parameters. We will simulate a dataset with nFeatures rows representing genes, only nSignif of which are significantly associated with the outcome of interest. We assume that both the training set and test set come from the same population, which is actually a mixture of two types, A and B, where the probability of belonging to type B is given by pB. The significant genes are assumed to be differentially expressed between the two types, with the difference in means following a normal distribution $(\Delta \sim \text{Norm}(\delta, \sigma))$.

```
> nFeatures <- 10000
> nSignif <- 100
> pB <- 0.4
> delta <- 1
> sigma <- 0.3
> nTrain <- 100
> nTest <- 100</pre>
```

For cleanup purposes, we specify the names of things we can safely remove later.

In addition to simulating the class assignment (A or B), we will also simulate a continuous outcome that represents a probability of belonging to class B. The continuous outcome (Figure 1) will follow a beta distribution with parameters α and β .

```
> alpha <- 0.75
> beta <- 0.95
> round(100*pbeta(seq(0.1, 0.9, 0.1), alpha, beta), 1)
[1] 17.1 28.8 39.1 48.7 57.7 66.4 74.9 83.2 91.4
> xx <- seq(0, 1, length=300)
> yy <- dbeta(xx, alpha, beta)</pre>
```

Now we can actually start the simulation. For the differentially expressed genes, we make it equally likely that they are higher in A or higher in B.

```
> signed <- -1 + 2*rbinom(nSignif, 1, 0.5)
```

As noted above, the magnitude of the difference follows a normal distibution.

```
> offsets <- c(signed*rnorm(nSignif, delta, sigma), # can change in either direction
+ rep(0, nFeatures - nSignif)) # but most don't change at all</pre>
```

2.1 Training Data

To simulate the training dataset, we first simulate the continuous outcomes (interpreted as the probability of belonging to class B). These are transformed using a logit function so they lie on the entire real line.

```
> lp <- function(p) log(p/(1-p))
> ea <- function(a) 1/(1+exp(-a))
> pOut <- rbeta(nTrain, alpha, beta)
> trainOutcome <- lp(pOut)</pre>
```

The binary classes for the simulated samples are obtained by dichotomizing the probabilities.

```
> # TODO: Fix this so it looks at correlation with the continuous outcome
> # instead of just differential expression between classes
> trainClass <- factor(c("cyan", "magenta")[1 + 1*(pOut > 0.5)])
> summary(trainClass)
```

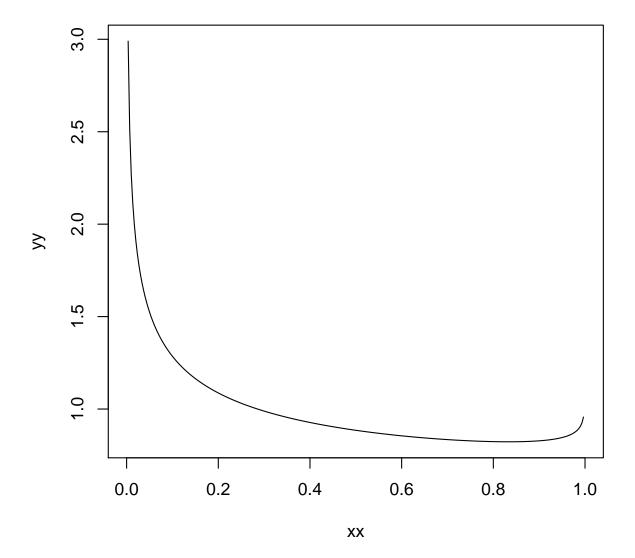
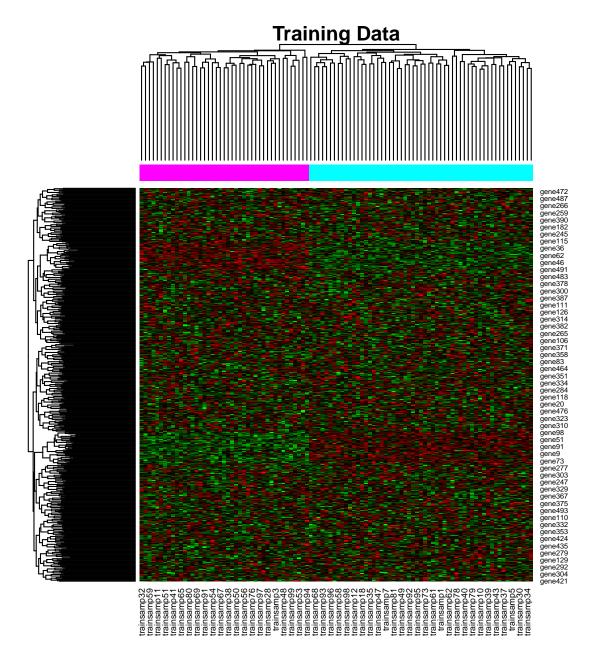


Figure 1: Probability of belonging to class B is simulated from this distribution, Beta(0.75, 0.95).

```
cyan magenta
     57
              43
> isB <- trainClass=="magenta"</pre>
> summary(isB)
   Mode
          FALSE
                    TRUE
                      43
logical
              57
Now we put together the training dataset.
> trainData <- matrix(rnorm(nFeatures*nTrain), ncol=nTrain) # pure noise
> trainData[,isB] <- sweep(trainData[,isB], 1, offsets, "+")</pre>
> trainData <- t(scale(t(trainData)))</pre>
> dimnames(trainData) <- list(paste("gene", 1:nFeatures, sep=''),</pre>
                                paste("trainsamp", 1:nTrain, sep=''))
>
```



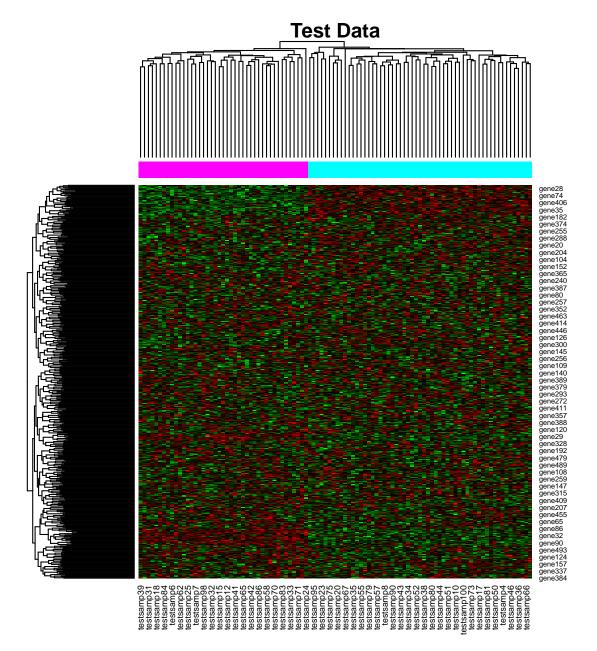
2.2 Test Data

We use the same procedure to simulate the test dataset, starting with continuous outcomes.

- > pOut <- rbeta(nTest, alpha, beta)
- > testOutcome <- lp(pOut)

We convert the continuous outcomes to binary class assignments.

```
> testClass <- factor(c("cyan", "magenta")[1 + 1*(pOut > 0.5)])
> summary(testClass)
   cyan magenta
     57
             43
> isB <- testClass=="magenta"</pre>
> summary(isB)
   Mode
          FALSE
                    TRUE
logical
             57
                      43
And we then generate the simulated microarray data.
> testData <- matrix(rnorm(nFeatures*nTest), ncol=nTest) # pure noise
> testData[,isB] <- sweep(testData[,isB], 1, offsets, "+")</pre>
> testData <- t(scale(t(testData)))</pre>
> dimnames(testData) <- list(paste("gene", 1:nFeatures, sep=''),</pre>
                                paste("testsamp", 1:nTest, sep=''))
```



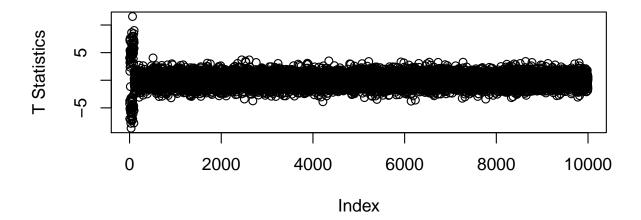
At this point, we can clean up the work space.

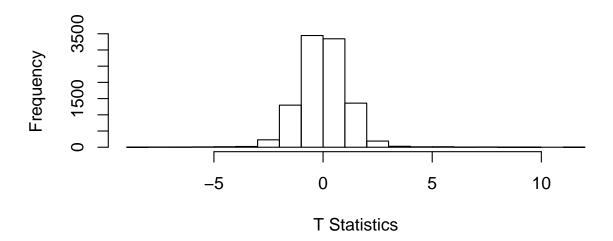
- > rm(list=paramlist)
- > rm(pOut, isB, signed, offsets)
- > rm(xx, yy, alpha, beta)
- > rm(paramlist)

3 Feature Selection

Here we implement a simple feature selection scheme. We first perform gene-by-gene t-tests on the training data to identify genes that are differentially exeprssed between the two classes.

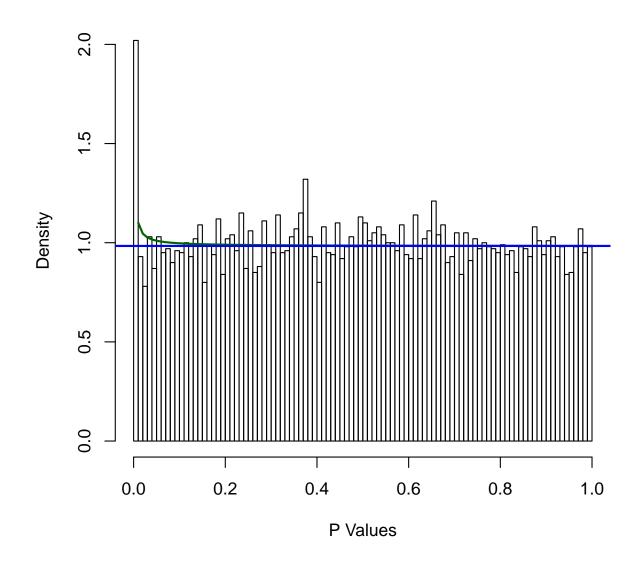
- > library(ClassComparison)
- > mtt <- MultiTtest(trainData, trainClass)</pre>





We then use a beta-uniform-mixture (BUM) model to estimate the false discover rate (FDR).

```
> bum <- Bum(mtt@p.values)
> countSignificant(bum, alpha=0.01, by="FDR")
[1] 78
> countSignificant(bum, alpha=0.05, by="FDR")
[1] 91
>
```



```
> geneset <- rownames(trainData)[selectSignificant(bum, alpha=0.05, by="FDR")]
> length(geneset)
[1] 91
> trainSubset <- trainData[geneset,]
> testSubset <- testData[geneset,]</pre>
```

4 Fitting Models and Making Predictions

4.1 K Nearest Neighbors

Note that the KNN method works for binary class prediction, but does not work for regression.

```
> knnFitted <- learn(modeler3NN, trainSubset, trainClass)</pre>
```

- > knnPredictions <- predict(knnFitted, testSubset)</pre>
- > table(knnPredictions, testClass)

testClass

knnPredictions cyan magenta cyan 57 0 magenta 0 43

>

- > knnFitted <- learn(modeler5NN, trainSubset, trainClass)
- > knnPredictions <- predict(knnFitted, testSubset)</pre>
- > table(knnPredictions, testClass)

${\tt testClass}$

knnPredictions cyan magenta cyan 57 0 magenta 0 43

4.2 Classification and regression trees

Classification

- > rpartFitted <- learn(modelerRPART, trainSubset, trainClass)
- > rpartPredictions <- predict(rpartFitted, testSubset, type='class')
- > table(rpartPredictions, testClass)

testClass

rpartPredictions cyan magenta cyan 49 4 magenta 8 39

Regression

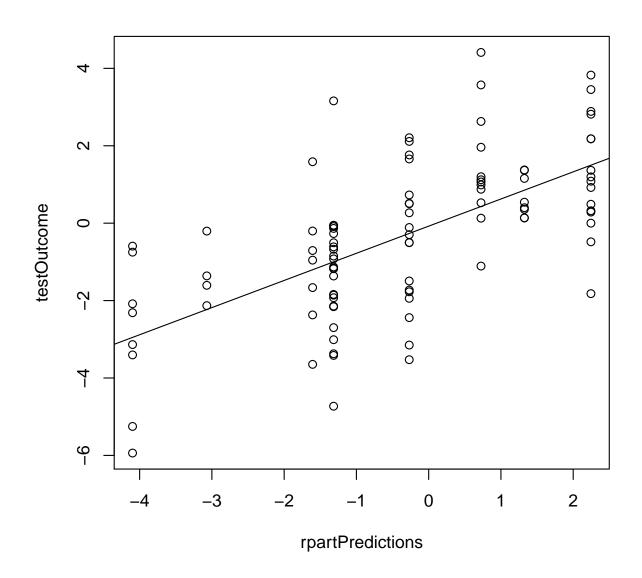
- > rpartFitted <- learn(modelerRPART, trainSubset, trainOutcome)
- > rpartPredictions <- predict(rpartFitted, testSubset)
- > table(rpartPredictions > 0, testClass)

 $\begin{array}{ccc} & \text{testClass} & \\ & \text{cyan magenta} \\ \text{FALSE} & 54 & 10 \\ \text{TRUE} & 3 & 33 \\ \end{array}$

> cor(rpartPredictions, testOutcome)

[1] 0.6316475

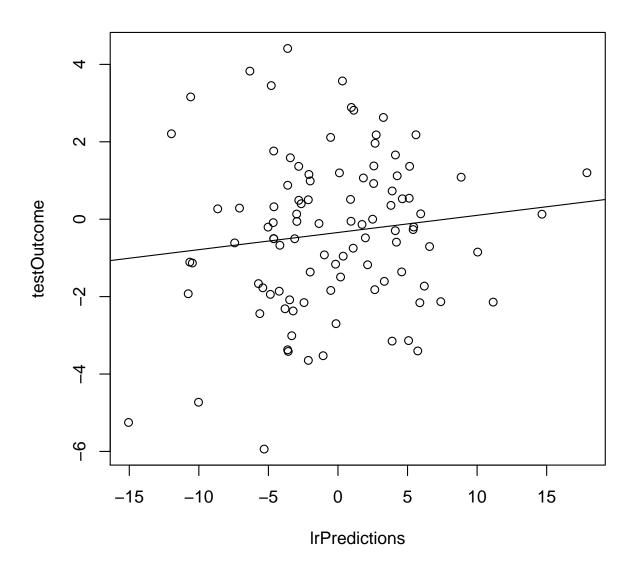
> temp <- lm(testOutcome ~ rpartPredictions)</pre>



4.3 Linear/Logistic Regression

Classification

```
> # takes too long for the vignette, because of the "step"
> # across glm fits.
> lrFitted <- learn(modelerLR, trainSubset, trainClass)</pre>
> lrPredictions <- predict(lrFitted, testSubset)
> table(lrPredictions, testClass)
   Regression
> lrFitted <- learn(modelerLR, trainSubset, trainOutcome)
> lrPredictions <- predict(lrFitted, testSubset)</pre>
> table(lrPredictions > 0, testClass)
       testClass
        cyan magenta
  FALSE
          34
                  19
  TRUE
          23
                  24
> cor(lrPredictions, testOutcome)
[1] 0.12484
> temp <- lm(testOutcome ~ lrPredictions)</pre>
```



4.4 Compound Covariate Prediction

Classification only

- > ccpFitted <- learn(modelerCCP, trainSubset, trainClass)</pre>
- > ccpPredictions <- predict(ccpFitted, testSubset)</pre>
- > table(ccpPredictions, testClass)

```
testClass
ccpPredictions cyan magenta
cyan 57 0
magenta 0 43
```

4.5 Support Vector Machines

Classification

```
> # takes too long for the vignette, because of the "step"
> # across glm fits.
> svmFitted <- learn(modelerSVM, trainSubset, trainClass)
> svmPredictions <- predict(svmFitted, testSubset)
> table(svmPredictions, testClass)
```

testClass

symPredictions cyan magenta cyan 57 0 magenta 0 43

Regression

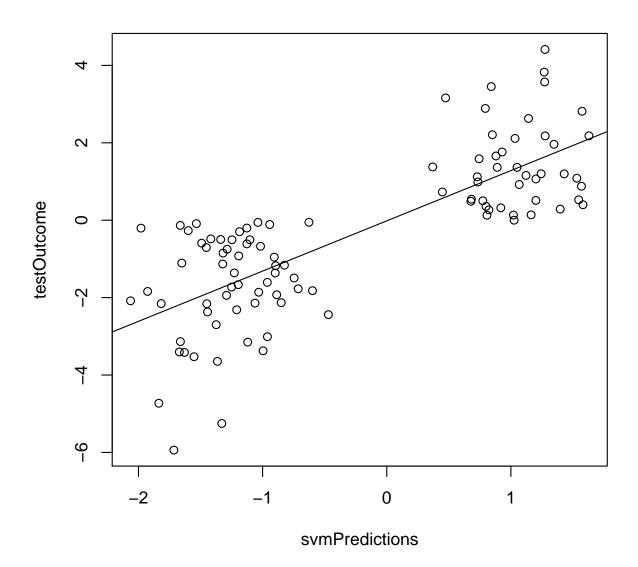
- > svmFitted <- learn(modelerSVM, trainSubset, trainOutcome)</pre>
- > svmPredictions <- predict(svmFitted, testSubset)</pre>
- > table(svmPredictions > 0, testClass)

 ${\tt testClass}$

cyan magenta

FALSE 57 0 TRUE 0 43

- > cor(svmPredictions, testOutcome)
- [1] 0.7775552
- > temp <- lm(testOutcome ~ svmPredictions)</pre>



4.6 Neural Networks

Classification

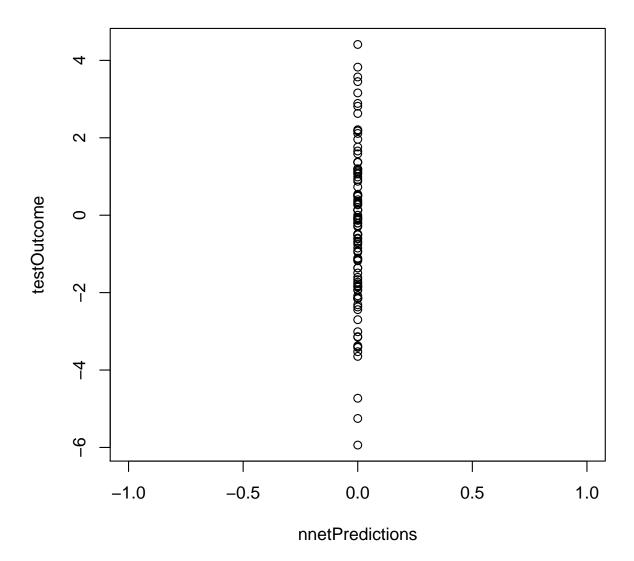
> nnetFitted <- learn(modelerNNET, trainSubset, trainClass)</pre>

weights: 466

initial value 77.599783 iter 10 value 0.002918

```
final value 0.000080
converged
> nnetPredictions <- predict(nnetFitted, testSubset)</pre>
> table(nnetPredictions, testClass)
                      testClass
nnetPredictions
                       cyan magenta
  1.40115426339924e-06
                        1
                                  0
  1.40129403562468e-06
                                  0
 1.4017263397493e-06
                          1
  1.40181638348734e-06
                       1
                                  0
  1.4018815155702e-06
                        1
  1.40188357795567e-06
                        1
  1.40188639659736e-06
                                  0
                        1
                         50
  1.40188891908656e-06
                                  0
  1
                         0
                                 43
  Regression
> nnetFitted <- learn(modelerNNET, trainSubset, trainOutcome)</pre>
# weights: 466
initial value 528.161386
final value 484.616805
converged
> nnetPredictions <- predict(nnetFitted, testSubset)</pre>
> table(nnetPredictions > 0, testClass)
       testClass
        cyan magenta
 FALSE 57
                  43
> cor(nnetPredictions, testOutcome)
     [,1]
[1,] NA
```

> temp <- lm(testOutcome ~ nnetPredictions)</pre>



4.7 Random Forests

Classification

- > rfFitted <- learn(modelerRF, trainSubset, trainClass)</pre>
- > rfPredictions <- predict(rfFitted, testSubset)
- > table(rfPredictions, testClass)

```
testClass
```

rfPredictions cyan magenta

 cyan
 57
 0

 magenta
 0
 43

Regression

- > rfFitted <- learn(modelerRF, trainSubset, trainOutcome)</pre>
- > rfPredictions <- predict(rfFitted, testSubset)</pre>
- > table(rfPredictions > 0, testClass)

testClass

cyan magenta

FALSE 57 3 TRUE 0 40

> cor(rfPredictions, testOutcome)

[1] 0.73546

> temp <- lm(testOutcome ~ rfPredictions)</pre>

