Hartigan index

$$H(u) = \left(\frac{trW_u}{trW_{u+1}} - 1\right)(n - u - 1),$$

where: $X = \{x_{ij}\}, i = 1, ..., n; j = 1, ..., m - \text{data matrix},$

n – number of objects,

m – number of variables,

 $W_u = \sum_r \sum_{i \in C_r} (x_{ri} - \bar{x}_r) (x_{ri} - \bar{x}_r)^T$ — within-group dispersion matrix for data clustered into u clusters,

 x_{ri} – m-dimensional vector of observations of the i-th object in cluster r,

 \bar{x}_r – centroid or medoid of cluster r,

r = 1, ..., u – cluster number,

u – number of clusters (u = 1, ..., n - 2),

 C_r – the indices of objects in cluster r.

The estimated number of clusters is the smallest $u \ge 1$ such that $H(u) \le 10$.

References

Hartigan, J. (1975), Clustering algorithms, Wiley, New York.

Milligan, G.W., Cooper, M.C. (1985), An examination of procedures of determining the number of cluster in a data set, "Psychometrika", vol. 50, no. 2, 159-179.

Tibshirani R., Walther G., Hastie T. (2001), *Estimating the number of clusters in a data set via the gap statistic*, "Journal of the Royal Statistical Society", ser. B, vol. 63, part 2, 411-423.