Hartigan index

$$H(u) = \left(\frac{tr\mathbf{W}_u}{tr\mathbf{W}_{u+1}} - 1\right)(n - u - 1),$$

where: $\mathbf{X} = \{x_{ij}\}, i = 1,...,m$; j = 1,...,m – data matrix,

n – number of objects,

m – number of variables,

 $\mathbf{W}_{u} = \sum_{r} \sum_{i \in C_{r}} (\mathbf{x}_{ri} - \overline{\mathbf{x}}_{r}) (\mathbf{x}_{ri} - \overline{\mathbf{x}}_{r})^{T} - \text{within-group dispersion matrix for data clustered into } u$

clusters.

 \mathbf{x}_{ri} – m-dimensional vector of observations of the i-th object in cluster r,

 $\overline{\mathbf{x}}_r$ – centroid or medoid of cluster r,

 $r = 1, \dots, u$ – cluster number,

u – number of clusters (u = 1, ..., n-2),

 C_r – the indices of objects in cluster r.

The estimated number of clusters is the smallest $u \ge 1$ such that $H(u) \le 10$.

References

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