Davies-Bouldin's index

$$DB(u) = \frac{1}{u} \sum_{r=1}^{u} \max_{s \atop s \neq s} \left(\frac{S_r + S_s}{d_{rs}} \right),$$

where: r, s = 1, ..., u – cluster number,

u – number of clusters ($u \ge 2$),

i, k = 1, ..., n – object number,

n – number of objects,

 P_r , $P_s - r$ -th, s-th cluster,

 $\mathbf{z}_r = (z_{r1}, \dots, z_{rm})$ – centroid or medoid of cluster P_r ,

j = 1, ..., m – variable number,

 $d_{rs} = \sqrt[p]{\sum_{j=1}^{m} |z_{rj} - z_{sj}|^p}$ - distance between centroids or medoids of clusters P_r and P_s (for

p = 1 Manhattan distance, for p = 2 Euclidean distance),

$$S_r = q \sqrt{\frac{1}{n_r} \sum_{i \in P_r} \sum_{j=1}^{m} \left| x_{ij}^r - z_{rj} \right|^q}$$
 - dispersion measure of a cluster P_r (for $q = 1$ the average dis-

tance of objects in cluster P_r to the centroid or medoid of cluster P_r ; for q = 2 the standard deviation of the distance of objects in cluster P_r to the centroid or medoid of cluster P_r), n_r (n_s) – number of objects in cluster P_r (P_s).

The value of u, which minimizes DB(u), is regarded as specifying the number of clusters.

The Davies-Bouldin's index have two limitations:

- index is not permitted if the number of clusters equals the number of objects (DB(u) = 0 if u = n),
- the certain number of clusters to have only one object is allowed. An unlimited number of single member clusters is not permitted.

DB(u) is a very good cluster separation measure if each cluster contain at least two objects.

References

Davies, D.L., Bouldin, D.W. (1979), *A cluster separation measure*, "IEEE Transactions on Pattern Analysis and Machine Intelligence", vol. 1, no. 2, 224-227.