## Rousseeuw Silhouette internal cluster quality index

$$S(u) = \sum_{i=1}^{n} S(i)/n$$
  
$$S(u) \in [-1, 1],$$

where: 
$$S(i) = \frac{b(i) - a(i)}{max\{a(i); b(i)\}}$$
,

i, k = 1, ..., n – number of object,

 $P_r$ ,  $P_s - r$ -th, s-th cluster,

u – number of clusters,

 $n_r$ ,  $n_s$  – number of objects in cluster  $P_r$ ,  $P_s$ ,

 $a(i) = \sum_{k \in \{P_r \setminus i\}} d_{ik} / (n_r - 1)$  – average dissimilarity of *i*-th object to all other objects of  $P_r$  cluster;

 $b(i) = \min_{s \neq r} \{d_{iP_s}\},\,$ 

 $d_{iP_s} = \sum_{k \in P_s} d_{ik}/n_s$  – average dissimilarity of *i*-th object to all objects of  $P_s$  cluster.

The value of u, which maximizes S(u), is regarded as specifying the number of clusters.

## References

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