Applications of Statistical Simulation in the Case of EU-SILC: Using the R Package simFrame

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Abstract

This paper demonstrates the use of **simFrame** for various simulation designs in a practical application with EU-SILC data. It presents the full functionality of the framework regarding sampling designs, contamination models, missing data mechanisms and performing simulations separately on different domains. Due to the use of control objects, switching from one simulation design to another requires only minimal changes in the code. Using bespoke R code, on the other hand, changing the code to switch between simulation designs would require much greater effort. Furthermore, parallel computing with **simFrame** is demonstrated.

Keywords: R, statistical simulation, EU-SILC.

1. Introduction

This is a supplementary paper to "An Object-Oriented Framework for Statistical Simulation: The R Package **simFrame**" (Alfons, Templ, and Filzmoser 2010d) and demonstrates the use of **simFrame** (Alfons 2011) in R (R Development Core Team 2010) for various simulation designs in a practical application. It extends the example for design-based simulation in Alfons *et al.* (2010d) (Example 6.1). Different simulation designs in terms of sampling, contamination and missing data are thereby investigated to present the strengths of the framework.

Note that the paper is supplementary material and is supposed to be read after studying Alfons et al. (2010d). It does not give a detailed discussion about the motivation for the framework, nor does it describe the design or implementation of the package. Instead it is focused on showing its full functionality for design-based simulation in additional code examples with brief explanations. However, model-based simulation is not considered here.

The European Union Statistics on Income and Living Conditions (EU-SILC) is panel survey conducted in EU member states and other European countries and serves as basis for measuring risk-of-poverty and social cohesion in Europe. An important indicator calculated from this survey is the *Gini coefficient*, which is a well-known measure of inequality. In the following examples, the standard estimation method (EU-SILC 2004) is compared to two semiparametric methods under different simulation designs. The two semiparametric approaches are based on fitting a Pareto distribution (e.g., Kleiber and Kotz 2003) to the upper tail of the data. In the first approach, the classical Hill estimator (Hill 1975) is used to estimate the

shape parameter of the Pareto distribution, while the second uses the robust partial density component (PDC) estimator (Vandewalle, Beirlant, Christmann, and Hubert 2007). All these methods are implemented in the R package laeken (Alfons, Holzer, and Templ 2010a). For a more detailed discussion on Pareto tail modeling in the case of the Gini coefficient and a related measure of inequality, the reader is referred to Alfons, Templ, Filzmoser, and Holzer (2010e).

The example data set of **simFrame** is used as population data throughout the paper. It consists of 58 654 observations from 25 000 households and was synthetically generated from Austrian EU-SILC survey data from 2006 using the data simulation methodology by Alfons, Kraft, Templ, and Filzmoser (2010b), which is implemented R package **simPopulation** (Alfons and Kraft 2010).

2. Application of different simulation designs to EU-SILC

First, the required packages and the data set need to be loaded.

```
R> library("simFrame")
R> library("laeken")
R> data("eusilcP")
```

Then, the function to be run in every iteration is defined. Its argument k determines the number of households whose income is modeled by a Pareto distribution. Since the Gini coefficient is calculated based on an equivalized household income, all individuals of a household in the upper tail receive the same value.

This function is used in the following examples, which are designed to exhibit the strengths of the framework. In order to change from one simulation design to another, all there is to do is to define or modify control objects and supply them to the function runSimulation().

2.1. Basic simulation design

In this basic simulation design, 100 samples of 1500 households are drawn using simple random sampling. Note that the setup() function is not used to permanently store the samples in an object. This is simply not necessary, since the population is rather small and the sampling method is straightforward. Furthermore, the Pareto distribution is fitted to the 175 households with the largest equivalized income.

```
R> set.seed(12345)
R> sc <- SampleControl(grouping = "hid", size = 1500, k = 100)
R> results <- runSimulation(eusilcP, sc, fun = sim, k = 175)</pre>
```

Hill

In order to inspect the simulation results, methods for several frequently used generic functions are implemented. Besides head(), tail() and summary() methods, a method for computing summary statistics with aggregate() is available. By default, the mean is used as summary statistic. Moreover, the plot() method selects a suitable graphical representation of the simulation results automatically. A reference line for the true value can thereby be added as well.

PDC

R> head(results)

Run Sample standard

```
1
           1 26.56793 26.48025 25.66614
1
           2 26.98203 27.73124 26.39318
2
           3 27.07081 27.11886 25.52524
3
    3
4
           4 26.86841 27.70216 25.71355
    5
           5 26.43215 26.49267 25.64191
5
6
           6 26.96175 27.13876 27.17536
R> aggregate(results)
standard
             Hill
                        PDC
26.65621 26.79016 26.89564
```

R> tv <- gini(eusilcP\$eqIncome)\$value

Figure 1 shows the resulting box plots of the simulation results for the basic simulation design. While the PDC estimator comes with larger variability, all three methods are on average quite close to the true population value. This is also an indication that the choice of the number of households for fitting the Pareto distribution is suitable.

2.2. Using stratified sampling

R> plot(results, true = tv)

The most frequently used sampling designs in official statistics are implemented in **simFrame**. In order to switch to another sampling design, only the corresponding control object needs to be changed. In this example, stratified sampling by region is performed. The sample sizes for the different strata are specified by using a vector for the slot **size** of the control object.

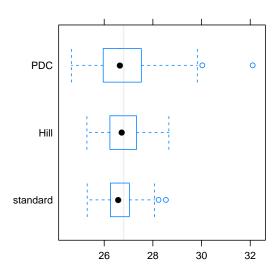


Figure 1: Simulation results for the basic simulation design.

As before, the simulation results are inspected with head() and aggregate(). A plot of the simulation results is produced as well.

R> head(results)

```
PDC
 Run Sample standard
                           Hill
    1
           1 27.08652 27.22293 27.66753
1
2
    2
           2 26.80670 27.35874 25.93378
3
    3
           3 26.68113 27.03964 26.60062
           4 25.84734 26.52346 25.18298
4
    4
5
    5
           5 26.05449 26.26848 26.60331
6
    6
           6 26.98439 27.01396 26.48090
```

R> aggregate(results)

```
standard Hill PDC 26.71792 26.85375 26.86248
```

```
R> tv <- gini(eusilcP$eqIncome)$value
R> plot(results, true = tv)
```

Figure 2 contains the plot of the simulation results for the simulation design with stratified sampling. The results are very similar to those from the basic simulation design with simple random sampling. On average, all three investigated methods are quite close to the true population value.

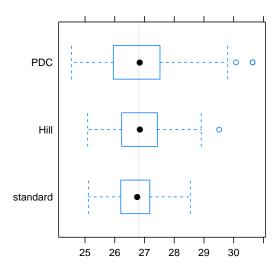


Figure 2: Simulation results for the simulation design with stratified sampling.

2.3. Adding contamination

When evaluating robust methods in simulation studies, contamination needs to be added to the data to study the influence of these outliers on the robust estimators and their classical counterparts. In **simFrame**, contamination is specified by defining a control object. Various contamination models are thereby implemented in the framework. Keep in mind that the term *contamination* is used in a technical sense here (see Alfons *et al.* 2010d; Alfons, Templ, and Filzmoser 2010c, for an exact definition) and that contamination is modeled as a two step process (see also Béguin and Hulliger 2008; Hulliger and Schoch 2009). In this example, 0.5% of the households are selected to be contaminated using simple random sampling. The equivalized income of the selected households is then drawn from a normal distribution with mean $\mu = 500\,000$ and standard deviation $\sigma = 10\,000$.

The head(), aggregate() and plot() methods are again used to take a look at the simulation results. Note that a column is added that indicates the contamination level used.

```
R> head(results)
```

```
Run Sample Epsilon standard Hill PDC
```

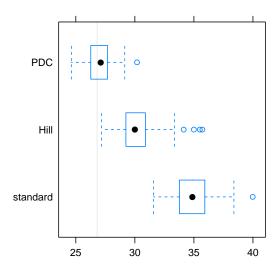


Figure 3: Simulation results for the simulation design with stratified sampling and contamination.

```
1
    1
           1
               0.005 32.71453 29.12110 27.03731
2
    2
           2
               0.005 34.22065 31.62709 26.24857
3
    3
               0.005 33.56878 28.49760 28.00937
           3
4
    4
           4
               0.005 35.26346 29.57160 26.25621
5
    5
           5
               0.005 33.79720 29.15945 25.61514
6
    6
               0.005 34.72069 28.58610 27.22342
```

R> aggregate(results)

```
Epsilon standard Hill PDC 1 0.005 34.88922 30.26179 27.02093
```

```
R> tv <- gini(eusilcP$eqIncome)$value
R> plot(results, true = tv)
```

In Figure 3, the resulting box plots are presented. The figure shows that such a small amount of contamination is enough to completely corrupt the standard estimation of the Gini coefficient. Using the classical Hill estimator to fit the Pareto distribution is still highly influenced by the outliers, whereas the PDC estimator leads to very accurate results.

2.4. Performing simulations separately on different domains

Data sets from official statistics typically contain strong heterogeneities, therefore indicators are usually computed for subsets of the data as well. Hence it is often of interest to investigate the behavior of indicators on different subsets in simulation studies. In **simFrame**, this can

be done by simply specifying the design argument of the function runSimulation(). In the case of extending the example from the previous section, the framework then splits the samples, inserts contamination into each subset and calls the supplied function for these subsets automatically. With bespoke R code, the user would need to take care of this with a loop-like structure such as a for loop or a function from the apply family.

In the following example, the simulations are performed separately for each gender. It should be noted that the value of k for the Pareto distribution is thus changed to 125. This is the same as Example 6.1 from Alfons et al. (2010d), except that a control object for sampling is supplied to runSimulation() instead of setting up the samples beforehand and storing them in an object.

Below, the results are inspected using head() and aggregate(). The aggregate() method thereby computes the summary statistic for each subset automatically. Also the plot() method displays the results for the different subsets in different panels by taking advantage of the lattice system (Sarkar 2008, 2010). In order to compute the true values for each subset, the function simSapply() is used.

R> head(results)

```
Run Sample Epsilon gender standard
                                          Hill
                                                     PDC
    1
               0.005
                       male 34.58446 29.96658 26.61415
1
           1
2
    1
           1
               0.005 female 38.82356 33.93700 28.82045
3
    2
           2
               0.005
                        male 34.34853 29.09325 27.66380
4
    2
           2
               0.005 female 36.38429 30.06097 27.42663
    3
                        male 33.39992 30.54211 23.96698
5
           3
               0.005
6
    3
               0.005 female 35.12883 30.51336 26.06518
```

R> aggregate(results)

```
Epsilon gender standard Hill PDC
1  0.005 male 33.18580 29.00265 26.21119
2  0.005 female 35.61341 31.28984 27.69054

R> tv <- simSapply(eusilcP, "gender", function(x) gini(x$eqIncome)$value)
R> plot(results, true = tv)
```

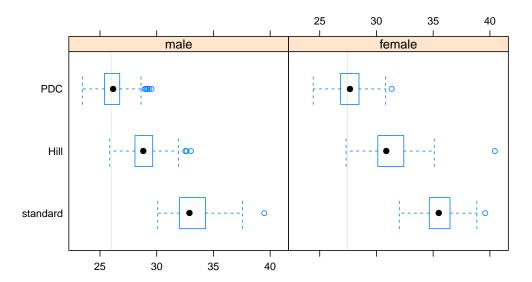


Figure 4: Simulation results for the simulation design with stratified sampling, contamination and performing the simulations separately for each gender.

The resulting plots are shown in Figure 4, which is the same as Figure 2 in Alfons *et al.* (2010d). Clearly, the PDC estimator leads to excellent results for both subsets, while the two classical approaches are in both cases highly influenced by the outliers.

2.5. Using multiple contamination levels

To get a more complete picture of the behavior of robust methods, more than one level of contamination is typically investigated in simulation studies. The only necessary modification of the code is to use a vector of contamination levels as the slot epsilon of the contamination control object. In this example, the contamination level is varied from 0% to 1% in steps of 0.25%. With bespoke R code, the user would have to add another loop-like structure to the code and collect the results in a suitable data structure. In simFrame, this is handled internally by the framework.

The simulation results are inspected as usual. Note that the aggregate() method in this case returns values for each combination of contamination level and gender.

R> head(results)

```
Run Sample Epsilon gender standard
                                           Hill
                                                     PDC
              0.0000
                        male 26.58067 26.50425 26.35969
1
    1
2
              0.0000 female 27.43355 27.03526 28.16992
    1
3
    2
              0.0025
                        male 31.63593 29.23365 27.12430
    2
              0.0025 female 31.43540 27.77698 26.85896
4
           1
5
    3
              0.0050
                        male 33.35950 31.07040 25.97415
           1
6
              0.0050 female 35.68710 34.03560 29.11359
```

R> aggregate(results)

```
Epsilon gender standard
                               Hill
                                          PDC
   0.0000
             male 25.94937 26.00769 25.85311
1
2
   0.0025
             male 30.44448 27.70155 26.01033
   0.0050
             male 33.54929 29.13202 26.16786
3
4
   0.0075
             male 36.76641 31.32342 26.49026
             male 39.42281 33.67944 26.53749
5
   0.0100
6
   0.0000 female 27.30171 27.49442 27.41323
7
   0.0025 female 31.68505 29.13643 27.61790
8
   0.0050 female 35.49976 30.92128 27.91607
9
   0.0075 female 38.51819 33.08778 28.09784
   0.0100 female 41.47137 35.32935 27.97407
```

If multiple contamination levels are used in a simulation study, the plot() method for the simulation results no longer produces box plots. Instead, the average results are plotted against the corresponding contamination levels, as shown in Figure 5. The plots show how the

R> tv <- simSapply(eusilcP, "gender", function(x) gini(x\$eqIncome)\$value)

classical estimators move away from the references line as the contamination level increases, while the values obtained with the PDC estimator remain quite accurate.

2.6. Inserting missing values

R> plot(results, true = tv)

Survey data almost always contain a considerable amount of missing values. In close-to-reality simulation studies, the variability due to missing data therefore needs to be considered. Three types of missing data mechanisms are commonly distinguished in the literature (e.g., Little and Rubin 2002): missing completely at random (MCAR), missing at random (MAR) and missing not at random (MNAR). All three missing data mechanisms are implemented in the framework.

In the following example, missing values are inserted into the equivalized household income of non-contaminated households with MCAR, i.e., the households whose values are going to be set to NA are selected using simple random sampling. In order to compare the scenario without missing values to a scenario with missing values, the missing value rates 0% and 5% are used. In the latter case, the missing values are simply disregarded for fitting the Pareto distribution

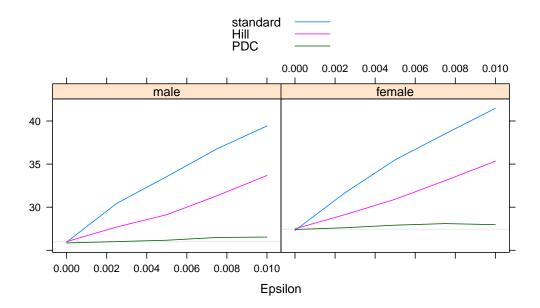


Figure 5: Simulation results for the simulation design with stratified sampling, multiple contamination levels and performing the simulations separately for each gender.

and estimating the Gini coefficient. Furthermore, the number of samples is reduced to 50 and only the contamination levels 0%, 0.5% and 1% are investigated to keep the computation time of this motivational example low.

With simFrame, only a control object for missing data needs to be defined and supplied to runSimulation(), the rest is done automatically by the framework. To apply these changes to a simulation study implemented with bespoke R code, yet another loop-like structure for the different missing value rates as well as changes in the data structure for the simulation results would be necessary.

As always, the head(), aggregate() and plot() methods are used to take a look at the simulation results. It should be noted that a column is added to the results that indicates the missing value rate used and that aggregate() in this example returns a value for each combination of contamination level, missing value rate and gender.

R> head(results)

```
PDC
 Run Sample Epsilon NArate gender standard
                                                  Hill
    1
               0.000
                        0.00
                               male 26.58067 27.00998 26.26273
1
           1
               0.000
2
    1
           1
                        0.00 female 27.43355 27.92305 26.69034
3
    2
           1
               0.000
                        0.05
                               male 26.62313 26.54198 26.01043
    2
4
           1
               0.000
                        0.05 female 27.51209 26.83574 27.25464
    3
                               male 33.71363 28.44824 26.46635
5
               0.005
                        0.00
           1
6
    3
               0.005
                        0.00 female 35.47508 28.48208 27.70783
```

R> aggregate(results)

```
Epsilon NArate gender standard
                                       Hill
                                                  PDC
1
     0.000
             0.00
                    male 25.89948 25.99777 25.74944
2
     0.005
             0.00
                    male 33.52791 29.30477 26.14659
3
     0.010
             0.00
                    male 39.45422 32.74672 26.64929
4
             0.05
                    male 25.88434 25.87824 25.80541
     0.000
5
     0.005
             0.05
                    male 33.87975 29.60079 26.18759
                    male 39.99526 33.44462 26.31274
6
     0.010
             0.05
7
     0.000
             0.00 female 27.17769 27.30586 27.19275
8
     0.005
             0.00 female 35.46414 31.37099 27.98622
9
     0.010
             0.00 female 41.28625 35.22113 28.19677
10
     0.000
             0.05 female 27.16026 27.37710 27.20892
     0.005
             0.05 female 35.85305 31.56317 27.80455
11
12
     0.010
             0.05 female 41.86453 35.44025 27.98948
```

```
R> tv <- simSapply(eusilcP, "gender", function(x) gini(x$eqIncome)$value)
R> plot(results, true = tv)
```

If multiple contamination levels and multiple missing value rates are used in the simulation study, conditional plots are produced by the plot() method for the simulation results. Figure 6 shows the resulting plots for this example. The bottom panels illustrate the scenario without missing values, while the scenario with 5% missing values is displayed in the top panels. In this case, there is not much of a difference in the results for the two scenarios.

2.7. Parallel computing

Statistical simulation is an *embarrassingly parallel* procedure, hence parallel computing can drastically reduce the computational costs. In **simFrame**, parallel computing is implemented using **snow** (Rossini, Tierney, and Li 2007; Tierney, Rossini, Li, and Sevcikova 2008). Only minimal additional programming effort due to the use of **snow** is required to adapt the code from the previous example: to initialize the computer cluster, to ensure that all packages and objects are available on each worker process, to use the function clusterRunSimulation() instead of runSimulation() and to stop the computer cluster after the simulations. In addition, random number streams (e.g., L'Ecuyer, Simard, Chen, and Kelton 2002; Sevcikova and Rossini 2009) should be used instead of the built-in random number generator.

```
R> c1 <- makeCluster(4, type = "SOCK")
R> clusterEvalQ(c1, {
```

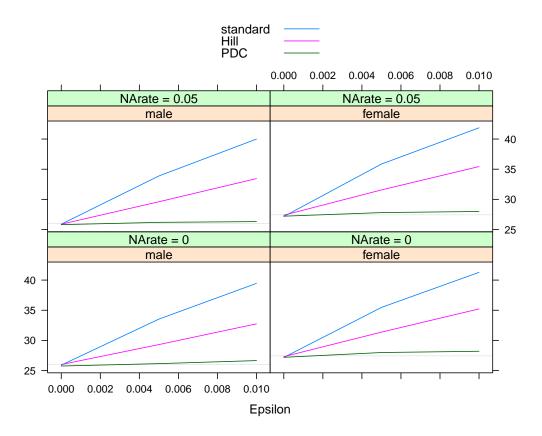


Figure 6: Simulation results for the simulation design with stratified sampling, multiple contamination levels, multiple missing value rates and performing the simulations separately for each gender.

```
+ library("simFrame")
+ data("eusilcP")
+ })
R> clusterSetupRNG(cl, seed = 12345)
R> sc <- SampleControl(design = "region", grouping = "hid",
+ size = c(75, 250, 250, 125, 200, 225, 125, 150, 100),
+ k = 50)
R> cc <- DCARContControl(target = "eqIncome", epsilon = c(0,
+ 0.005, 0.01), dots = list(mean = 5e+05, sd = 10000))
R> nc <- NAControl(target = "eqIncome", NArate = c(0, 0.05))
R> clusterExport(cl, c("sc", "cc", "nc", "sim"))
R> results <- clusterRunSimulation(cl, eusilcP, sc, contControl = cc,
+ NAControl = nc, design = "gender", fun = sim, k = 125)
R> stopCluster(cl)
```

When the parallel computations are finished and the simulation results are obtained, they can be inspected as usual.

R> head(results)

```
Run Sample Epsilon NArate gender standard
                                                             PDC
                                                  Hill
    1
           1
               0.000
                        0.00
                               male 26.20067 27.02017 23.66565
1
2
    1
               0.000
                        0.00 female 28.79194 29.23548 27.12933
           1
    2
3
           1
               0.000
                        0.05
                               male 26.19328 24.91570 24.07906
4
    2
               0.000
                        0.05 female 28.86860 27.38585 27.80012
           1
                               male 34.46084 31.74470 24.87023
5
    3
               0.005
                        0.00
           1
    3
               0.005
                        0.00 female 36.27429 32.14269 28.06137
6
           1
```

R> aggregate(results)

```
Epsilon NArate gender standard
                                       Hill
                                                  PDC
     0.000
             0.00
                    male 25.89996 25.98977 25.86451
1
2
             0.00
     0.005
                    male 33.56743 29.36361 26.39515
3
     0.010
             0.00
                    male 39.40362 33.05926 26.68715
4
     0.000
             0.05
                    male 25.87909 25.86055 26.00109
5
     0.005
             0.05
                    male 33.94829 29.65456 26.32813
     0.010
                    male 39.95535 33.24853 26.78947
6
             0.05
7
     0.000
             0.00 female 27.38636 27.52210 27.48816
8
     0.005
             0.00 female 35.52688 31.30099 28.03385
             0.00 female 41.35311 35.81549 28.67901
9
     0.010
     0.000
             0.05 female 27.38459 27.51825 27.54063
10
11
     0.005
             0.05 female 35.87991 31.74678 28.18308
12
     0.010
             0.05 female 41.89804 36.21921 28.41367
```

```
R> tv <- simSapply(eusilcP, "gender", function(x) gini(x$eqIncome)$value)
R> plot(results, true = tv)
```

Figure 7 shows the simulation results obtained with parallel computing. The plots are, of course, very similar to the plots for the previous example in Figure 6, since the design of the simulation studies is the same.

3. Conclusions

In this paper, the use of the R package **simFrame** for different simulation designs has been demonstrated in a practical application. The full functionality of the framework for design-based simulation has been presented in various code examples. These examples showed that the framework allows researchers to make use of a wide range of simulation designs with only a few lines of code. In order to switch from one simulation design to another, only control objects need to be defined or modified. Even moving from basic to highly complex designs therefore requires only minimal changes to the code. With bespoke R code, such modifications would often need a considerable amount of programming. Furthermore, parallel computing with **simFrame** can easily be done based on package **snow**.

Besides the functionality for carrying out simulation studies, methods for several frequently used generic functions are available for inspecting or summarizing the simulation results. Most

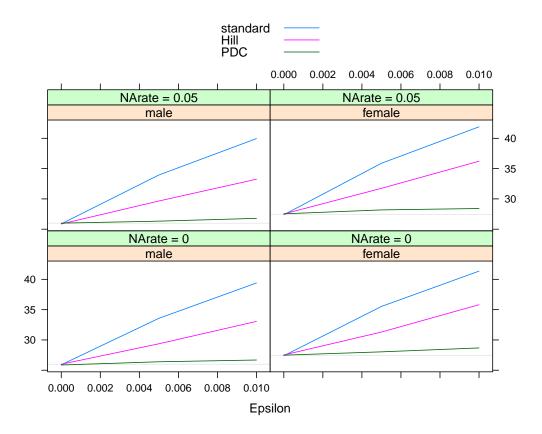


Figure 7: Simulation results obtained by parallel computing for the simulation design with stratified sampling, multiple contamination levels, multiple missing value rates and performing the simulations separately for each gender.

notably, a suitable plot method of the simulation results is selected automatically depending on their structure.

Due to this flexibility, **simFrame** is widely applicable for gaining insight into the quality of statistical methods and is a valuable addition to a researcher's toolbox.

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