# topic odels: An R Package for Fitting Topic Models

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#### Abstract

This article is a (slightly) modified and shortened version of Grün and Hornik (2011), published in the *Journal of Statistical Software*.

Topic models allow the probabilistic modeling of term frequency occurrences in documents. The fitted model can be used to estimate the similarity between documents as well as between a set of specified keywords using an additional layer of latent variables which are referred to as topics. The R package topicmodels provides basic infrastructure for fitting topic models based on data structures from the text mining package tm. The package includes interfaces to two algorithms for fitting topic models: the variational expectation-maximization algorithm provided by David M. Blei and co-authors and an algorithm using Gibbs sampling by Xuan-Hieu Phan and co-authors.

Keywords: Gibbs sampling, R, text analysis, topic model, variational EM.

### 1. Introduction

In machine learning and natural language processing topic models are generative models which provide a probabilistic framework for the term frequency occurrences in documents in a given corpus. Using only the term frequencies assumes that the information in which order the words occur in a document is negligible. This assumption is also referred to as the *exchangeability* assumption for the words in a document and this assumption leads to bag-of-words models.

Topic models extend and build on classical methods in natural language processing such as the unigram model and the mixture of unigram models (Nigam, McCallum, Thrun, and Mitchell 2000) as well as Latent Semantic Analysis (LSA; Deerwester, Dumais, Furnas, Landauer, and Harshman 1990). Topic models differ from the unigram or the mixture of unigram models because they are mixed-membership models (see for example Airoldi, Blei, Fienberg, and Xing 2008). In the unigram model each word is assumed to be drawn from the same term distribution, in the mixture of unigram models a topic is drawn for each document and all words in a document are drawn from the term distribution of the topic. In mixed-membership models documents are not assumed to belong to single topics, but to simultaneously belong to several topics and the topic distributions vary over documents.

An early topic model was proposed by Hofmann (1999) who developed probabilistic LSA. He assumed that the interdependence between words in a document can be explained by the latent topics the document belongs to. Conditional on the topic assignments of the words the word occurrences in a document are independent. The latent Dirichlet allocation (LDA; Blei, Ng, and Jordan 2003b) model is a Bayesian mixture model for discrete data where topics are

assumed to be uncorrelated. The correlated topics model (CTM; Blei and Lafferty 2007) is an extension of the LDA model where correlations between topics are allowed. An introduction to topic models is given in Steyvers and Griffiths (2007) and Blei and Lafferty (2009). Topic models have previously been used for a variety of applications, including ad-hoc information retrieval (Wei and Croft 2006), geographical information retrieval (Li, Wang, Xie, Wang, and Ma 2008) and the analysis of the development of ideas over time in the field of computational linguistics (Hall, Jurafsky, and Manning 2008).

C code for fitting the LDA model (http://www.cs.princeton.edu/~blei/lda-c/) and the CTM (http://www.cs.princeton.edu/~blei/ctm-c/) is available under the GPL from David M. Blei and co-authors, who introduced these models in their papers. The method used for fitting the models is the variational expectation-maximization (VEM) algorithm. Other implementations for fitting topic models—especially of the LDA model—are available. The standalone program Ida (Mochihashi 2004a,b) provides standard VEM estimation. An implementation in Python of an online version of LDA using VEM estimation as described in Hoffman, Blei, and Bach (2010) is available under the GPL from the first author's web page (http://www.cs.princeton.edu/~mdhoffma/). For Bayesian estimation using Gibbs sampling several implementations are available. GibbsLDA++ (Phan, Nguyen, and Horiguchi 2008) is available under the GPL from http://gibbslda.sourceforge.net/. The Matlab Topic Modeling Toolbox 1.3.2 (Griffiths and Steyvers 2004; Steyvers and Griffiths 2011) is free for scientific use. A license must be obtained from the authors to use it for commercial purposes. MALLET (McCallum 2002) is released under the CPL and is a Java-based package which is more general in allowing for statistical natural language processing, document classification, clustering, topic modeling using LDA, information extraction, and other machine learning applications to text. A general toolkit for implementing hierarchical Bayesian models is provided by the Hierarchical Bayes Compiler HBC (Daumé III 2008), which also allows to fit the LDA model. Another general framework for running Bayesian inference in graphical models which allows to fit the LDA model is available through Infer.NET (Microsoft Corporation 2010) The fast collapsed Gibbs sampling method is described in Porteous, Asuncion, Newman, Ihler, Smyth, and Welling (2008) and code is also available from the first author's web page (http://www.ics.uci.edu/~iporteou/fastlda/).

For R, an environment for statistical computing and graphics (R Development Core Team 2011), CRAN (http://CRAN.R-project.org) features two packages for fitting topic models: topicmodels and lda. The R package lda (Chang 2010) provides collapsed Gibbs sampling methods for LDA and related topic model variants, with the Gibbs sampler implemented in C. All models in package lda are fitted using Gibbs sampling for determining the posterior probability of the latent variables. Wrappers for the expectation-maximization (EM) algorithm are provided which build on this functionality for the E-step. Note that this implementation therefore differs in general from the estimation technique proposed in the original papers introducing these model variants, where the VEM algorithm is usually applied.

The R package **topicmodels** currently provides an interface to the code for fitting an LDA model and a CTM with the VEM algorithm as implemented by Blei and co-authors and to the code for fitting an LDA topic model with Gibbs sampling written by Phan and co-authors. Package **topicmodels** builds on package **tm** (Feinerer, Hornik, and Meyer 2008; Feinerer 2011) which constitutes a framework for text mining applications within R. **tm** provides infrastructure for constructing a corpus, e.g., by reading in text data from PDF files, and transforming a corpus to a document-term matrix which is the input data for topic models. In package

topicmodels the respective code is directly called through an interface at the C level avoiding file input and output, and hence substantially improving performance. The functionality for data input and output in the original code was substituted and R objects are directly used as input and S4 objects as output to R. The same main function allows fitting the LDA model with different estimation methods returning objects only slightly different in structure. In addition the strategies for model selection and inference are applicable in both cases. This allows for easy use and comparison of both current state-of-the-art estimation techniques for topic models. Packages topicmodels aims at extensibility by providing an interface for inclusion of other estimation methods of topic models.

This paper is structured as follows: Section 2 introduces the specification of topic models, outlines the estimation with the VEM as well as Gibbs sampling and gives an overview of pre-processing steps and methods for model selection and inference. The main fitter functions in the package and the helper functions for analyzing a fitted model are presented in Section 3. An illustrative example for using the package is given in Section 4 where topic models are fitted to the corpus of abstracts in the *Journal of Statistical Software*.

## 2. Topic model specification and estimation

### 2.1. Model specification

For both models—LDA and CTM—the number of topics k has to be fixed a-priori. The LDA model and the CTM assume the following generative process for a document  $w = (w_1, \ldots, w_N)$  of a corpus D containing N words from a vocabulary consisting of V different terms,  $w_i \in \{1, \ldots, V\}$  for all  $i = 1, \ldots, N$ .

For LDA the generative model consists of the following three steps.

**Step 1:** The term distribution  $\beta$  is determined for each topic by

 $\beta \sim \text{Dirichlet}(\delta)$ .

Step 2: The proportions  $\theta$  of the topic distribution for the document w are determined by

 $\theta \sim \text{Dirichlet}(\alpha)$ .

**Step 3:** For each of the N words  $w_i$ 

- (a) Choose a topic  $z_i \sim \text{Multinomial}(\theta)$ .
- (b) Choose a word  $w_i$  from a multinomial probability distribution conditioned on the topic  $z_i$ :  $p(w_i|z_i, \beta)$ .

 $\beta$  is the term distribution of topics and contains the probability of a word occurring in a given topic.

For CTM Step 2 is modified to

**Step 2a:** The proportions  $\theta$  of the topic distribution for the document w are determined by drawing

$$\eta \sim N(\mu, \Sigma)$$
 with  $\eta \in \mathbb{R}^{(k-1)}$  and  $\Sigma \in \mathbb{R}^{(k-1) \times (k-1)}$ .  
Set  $\tilde{\eta}^{\top} = (\eta^{\top}, 0)$ .  $\theta$  is given by 
$$\theta_K = \frac{\exp\{\tilde{\eta}_K\}}{\sum_{i=1}^k \exp\{\tilde{\eta}_i\}}$$
 for  $K = 1, \dots, k$ .

#### 2.2. Estimation

For maximum likelihood (ML) estimation of the LDA model the log-likelihood of the data, i.e., the sum over the log-likelihoods of all documents, is maximized with respect to the model parameters  $\alpha$  and  $\beta$ . In this setting  $\beta$  and not  $\delta$  is in general the parameter of interest. For the CTM model the log-likelihood of the data is maximized with respect to the model parameters  $\mu$ ,  $\Sigma$  and  $\beta$ . For VEM estimation the log-likelihood for one document  $w \in D$  is for LDA given by

$$\ell(\alpha, \beta) = \log (p(w|\alpha, \beta))$$

$$= \log \int \left\{ \sum_{z} \left[ \prod_{i=1}^{N} p(w_i|z_i, \beta) p(z_i|\theta) \right] \right\} p(\theta|\alpha) d\theta$$

and for CTM by

$$\ell(\mu, \Sigma, \beta) = \log (p(w|\mu, \Sigma, \beta))$$

$$= \log \int \left\{ \sum_{z} \left[ \prod_{i=1}^{N} p(w_i|z_i, \beta) p(z_i|\theta) \right] \right\} p(\theta|\mu, \Sigma) d\theta.$$

The sum over  $z = (z_i)_{i=1,\dots,N}$  includes all combinations of assigning the N words in the document to the k topics.

The quantities  $p(w|\alpha,\beta)$  for the LDA model and  $p(w|\mu,\Sigma,\beta)$  for the CTM cannot be tractably computed. Hence, a VEM procedure is used for estimation. The EM algorithm (Dempster, Laird, and Rubin 1977) is an iterative method for determining an ML estimate in a missing data framework where the complete likelihood of the observed and missing data is easier to maximize than the likelihood of the observed data only. It iterates between an Expectation (E)-step where the expected complete likelihood given the data and current parameter estimates is determined and a Maximization (M)-step where the expected complete likelihood is maximized to find new parameter estimates. For topic models the missing data in the EM algorithm are the latent variables  $\theta$  and z for LDA and  $\eta$  and z for CTM.

For topic models a VEM algorithm is used instead of an ordinary EM algorithm because the expected complete likelihood in the E-step is still computationally intractable. For an introduction into variational inference see for example Wainwright and Jordan (2008). To facilitate

the E-step the posterior distribution  $p(\theta, z|w, \alpha, \beta)$  is replaced by a variational distribution  $q(\theta, z|\gamma, \phi)$ . This implies that in the E-step instead of

$$\mathsf{E}_p[\log p(\theta, z|w, \alpha, \beta)]$$

the following is determined

$$\mathsf{E}_q[\log p(\theta, z|w, \alpha, \beta)].$$

The parameters for the variational distributions are document specific and hence are allowed to vary over documents which is not the case for  $\alpha$  and  $\beta$ . For the LDA model the variational parameters  $\gamma$  and  $\phi$  for a given document w are determined by

$$(\gamma^*, \phi^*) = \arg\min_{(\gamma, \phi)} D_{\mathrm{KL}}(q(\theta, z | \gamma, \phi) || p(\theta, z | w, \alpha, \beta)).$$

D<sub>KL</sub> denotes the Kullback-Leibler (KL) divergence. The variational distribution is set equal to

$$q(\theta, z|\gamma, \phi) = q_1(\theta|\gamma) \prod_{i=1}^{N} q_2(z_i|\phi_i),$$

where  $q_1()$  is a Dirichlet distribution with parameters  $\gamma$  and  $q_2()$  is a multinomial distribution with parameters  $\phi_i$ . Analogously for the CTM the variational parameters are determined by

$$(\lambda^*, \nu^*, \phi^*) = \arg\min_{(\lambda, \nu, \phi)} D_{\mathrm{KL}}(q(\eta, z | \lambda, \nu^2, \phi) || p(\eta, z | w, \mu, \Sigma, \beta)).$$

Since the variational parameters are fitted separately for each document the variational covariance matrix can be assumed to be diagonal. The variational distribution is set to

$$q(\eta, z | \lambda, \nu^2, \phi) = \prod_{K=1}^{k-1} q_1(\eta_K | \lambda_K, \nu_K^2) \prod_{i=1}^N q_2(z_i | \phi_i),$$

where  $q_1()$  is a univariate Gaussian distribution with mean  $\lambda_K$  and variance  $\nu_K^2$ , and  $q_2()$  again denotes a multinomial distribution with parameters  $\phi_i$ . Using this simple model for  $\eta$  has the advantage that it is computationally less demanding while still providing enough flexibility. Over all documents this leads to a mixture of normal distributions with diagonal variance-covariance matrices. This mixture distribution allows to approximate the marginal distribution over all documents which has an arbitrary variance-covariance matrix.

For the LDA model it can be shown with the following equality that the variational parameters result in a lower bound for the log-likelihood

$$\log p(w|\alpha,\beta) = L(\gamma,\phi;\alpha,\beta) + D_{\mathrm{KL}}(q(\theta,z|\gamma,\phi)||p(\theta,z|w,\alpha,\beta))$$

where

$$L(\gamma, \phi; \alpha, \beta) = \mathsf{E}_{q}[\log p(\theta, z, w | \alpha, \beta)] - \mathsf{E}_{q}[\log q(\theta, z)]$$

(see Blei et al. 2003b, p. 1019). Maximizing the lower bound  $L(\gamma, \phi; \alpha, \beta)$  with respect to  $\gamma$  and  $\phi$  is equivalent to minimizing the KL divergence between the variational posterior probability and the true posterior probability. This holds analogously for the CTM.

For estimation the following steps are repeated until convergence of the lower bound of the log-likelihood.

**E-step:** For each document find the optimal values of the variational parameters  $\{\gamma, \phi\}$  for the LDA model and  $\{\lambda, \nu, \phi\}$  for the CTM.

**M-step:** Maximize the resulting lower bound on the log-likelihood with respect to the model parameters  $\alpha$  and  $\beta$  for the LDA model and  $\mu$ ,  $\Sigma$  and  $\beta$  for the CTM.

For inference the latent variables  $\theta$  and z are often of interest to determine which topics a document consists of and which topic a certain word in a document was drawn from. Under the assumption that the variational posterior probability is a good approximation of the true posterior probability it can be used to determine estimates for the latent variables. In the following inference is always based on the variational posterior probabilities if the VEM is used for estimation.

For Gibbs sampling in the LDA model draws from the posterior distribution p(z|w) are obtained by sampling from

$$p(z_{i} = K|w, z_{-i}) \propto \frac{n_{-i,K}^{(j)} + \delta}{n_{-i,K}^{(\cdot)} + V\delta} \frac{n_{-i,K}^{(d_{i})} + \alpha}{n_{-i,L}^{(d_{i})} + k\alpha}$$

(see Griffiths and Steyvers 2004; Phan et al. 2008).  $z_{-i}$  is the vector of current topic memberships of all words without the *i*th word  $w_i$ . The index j indicates that  $w_i$  is equal to the jth term in the vocabulary.  $n_{-i,K}^{(j)}$  gives how often the jth term of the vocabulary is currently assigned to topic K without the ith word. The dot implies that summation over this index is performed.  $d_i$  indicates the document in the corpus to which word  $w_i$  belongs. In the Bayesian model formulation  $\delta$  and  $\alpha$  are the parameters of the prior distributions for the term distribution of the topics  $\beta$  and the topic distribution of documents  $\theta$ , respectively. The predictive distributions of the parameters  $\theta$  and  $\beta$  given w and z are given by

$$\hat{\beta}_{K}^{(j)} = \frac{n_{K}^{(j)} + \delta}{n_{K}^{(.)} + V\delta}, \qquad \qquad \hat{\theta}_{K}^{(d)} = \frac{n_{K}^{(d)} + \alpha}{n_{K}^{(.)} + k\alpha},$$

for j = 1, ..., V and d = 1, ..., D.

### 2.3. Pre-processing

The input data for topic models is a document-term matrix. The rows in this matrix correspond to the documents and the columns to the terms. The entry  $m_{ij}$  indicates how often the jth term occurred in the ith document. The number of rows is equal to the size of the corpus and the number of columns to the size of the vocabulary. The data pre-processing step involves selecting a suitable vocabulary, which corresponds to the columns of the document-term matrix. Typically, the vocabulary will not be given a-priori, but determined using the available data. The mapping from the document to the term frequency vector involves to-kenizing the document and then processing the tokens for example by converting them to lower-case, removing punctuation characters, removing numbers, stemming, removing stop words and omitting terms with a length below a certain minimum. In addition the final document-term matrix can be reduced by selecting only the terms which occur in a minimum number of documents (see Griffiths and Steyvers 2004, who use a value of 5) or those terms with the highest term-frequency inverse document frequency (tf-idf) scores (Blei and Lafferty

2009). The tf-idf scores are only used for selecting the vocabulary, the input data consisting of the document-term matrix uses a term-frequency weighting.

#### 2.4. Model selection

For fitting the LDA model or the CTM to a given document-term matrix the number of topics needs to be fixed a-priori. Additionally, estimation using Gibbs sampling requires specification of values for the parameters of the prior distributions. Griffiths and Steyvers (2004) suggest a value of 50/k for  $\alpha$  and 0.1 for  $\delta$ . Because the number of topics is in general not known, models with several different numbers of topics are fitted and the optimal number is determined in a data-driven way. Model selection with respect to the number of topics is possible by splitting the data into training and test data sets. The likelihood for the test data is then approximated using the lower bound for VEM estimation. For Gibbs sampling the log-likelihood is given by

$$\log(p(w|z)) = k \log\left(\frac{\Gamma(V\delta)}{\Gamma(\delta)^V}\right) + \sum_{K=1}^k \left\{ \left[ \sum_{j=1}^V \log(\Gamma(n_K^{(j)} + \delta)) \right] - \log(\Gamma(n_K^{(j)} + V\delta)) \right\}.$$

The perplexity is often used to evaluate the models on held-out data and is equivalent to the geometric mean per-word likelihood.

$$Perplexity(w) = \exp\left\{-\frac{\log(p(w))}{\sum_{d=1}^{D} \sum_{j=1}^{V} n^{(jd)}}\right\}$$

 $n^{(jd)}$  denotes how often the jth term occurred in the dth document. If the model is fitted using Gibbs sampling the likelihood is determined for the perplexity using

$$\log(p(w)) = \sum_{d=1}^{D} \sum_{j=1}^{V} n^{(jd)} \log \left[ \sum_{K=1}^{k} \theta_K^{(d)} \beta_K^{(j)} \right]$$

(see Newman, Asuncion, Smyth, and Welling 2009). The topic weights  $\theta_K^{(d)}$  can either be determined for the new data using Gibbs sampling where the term distributions for topics are kept fixed or equal weights are used as implied by the prior distribution. If the perplexity is calculated by averaging over several draws the mean is taken over the samples inside the logarithm.

In addition the marginal likelihoods of the models with different numbers of topics can be compared for model selection if Gibbs sampling is used for model estimation. Griffiths and Steyvers (2004) determine the marginal likelihood using the harmonic mean estimator (Newton and Raftery 1994), which is attractive from a computational point of view because it only requires the evaluation of the log-likelihood for the different posterior draws of the parameters. The drawback however is that the estimator might have infinite variance.

Different methods for evaluating fitted topic models on held-out documents are discussed and compared in Wallach, Murray, Salakhutdinov, and Mimno (2009). Another possibility for model selection is to use hierarchical Dirichlet processes as suggested in Teh, Jordan, Beal, and Blei (2006).

## 3. Application: Main functions LDA() and CTM()

The main functions in package **topicmodels** for fitting the LDA and CTM models are LDA() and CTM(), respectively.

```
R> LDA(x, k, method = "VEM", control = NULL, model = NULL, ...)
R> CTM(x, k, method = "VEM", control = NULL, model = NULL, ...)
```

These two functions have the same arguments. x is a suitable document-term matrix with nonnegative integer count entries, typically a "DocumentTermMatrix" as obtained from package tm. Internally, topicmodels uses the simple triplet matrix representation of package slam (Hornik, Meyer, and Buchta 2011) (which, similar to the "coordinate list" (COO) sparse matrix format, stores the information about non-zero entries  $x_{ij}$  in the form of  $(i, j, x_{ij})$  triplets). x can be any object coercible to such simple triplet matrices (with count entries), in particular objects obtained from readers for commonly employed document-term matrix storage formats. For example the reader read\_dtm\_Blei\_et\_al() available in package tm allows to read in data provided in the format used for the code by Blei and co-authors. k is an integer (larger than 1) specifying the number of topics. method determines the estimation method used and currently can be either "VEM" or "Gibbs" for LDA() and only "VEM" for CTM(). Users can provide their own fit functions to use a different estimation technique or fit a slightly different model variant and specify them to be called within LDA() and CTM() via the method argument. Argument model allows to provide an already fitted topic model which is used to initialize the estimation.

Argument control can be either specified as a named list or as a suitable \$4 object where the class depends on the chosen method. In general a user will provide named lists and coercion to an \$4 object will internally be performed. The following arguments are possible for the control for fitting the LDA model with the VEM algorithm. They are set to their default values.

```
R> control_LDA_VEM <-
+ list(estimate.alpha = TRUE, alpha = 50/k, estimate.beta = TRUE,
+ verbose = 0, prefix = tempfile(), save = 0, keep = 0,
+ seed = as.integer(Sys.time()), nstart = 1, best = TRUE,
+ var = list(iter.max = 500, tol = 10^-6),
+ em = list(iter.max = 1000, tol = 10^-4),
+ initialize = "random")</pre>
```

The arguments are described in detail below.

estimate.alpha, alpha, estimate.beta: By default  $\alpha$  is estimated (estimate.alpha = TRUE) and the starting value for  $\alpha$  is 50/k as suggested by Griffiths and Steyvers (2004). If  $\alpha$  is not estimated, it is held fixed at the initial value. If the term distributions for the topics are already given by a previously fitted model, only the topic distributions for documents can be estimated using estimate.beta = FALSE. This is useful for example if a fitted model is evaluated on hold-out data or for new data.

verbose, prefix, save, keep: By default no information is printed during the algorithm (verbose = 0). If verbose is a positive integer every verbose iteration information

is printed. save equal to 0 indicates that no intermediate results are saved in files with prefix prefix. If equal to a positive integer, every save iterations intermediate results are saved. If keep is a positive integer, the log-likelihood values are stored every keep iteration.

seed, nstart, best: For reproducibility a random seed can be set which is used in the external code. nstart indicates the number of repeated runs with random initializations. seed needs to have the length nstart. If best=TRUE only the best model over all runs with respect to the log-likelihood is returned.

var, em: These arguments control how convergence is assessed for the variational inference step and for the EM algorithm steps by setting a maximum number of iterations (iter.max) and a tolerance for the relative change in the likelihood (tol). If during the EM algorithm the likelihood is not increased in one step, the maximum number of iterations in the variational inference step is doubled.

If the maximum number of iterations is set to -1 in the variational inference step, there is no bound on the number of iterations and the algorithm continues until the tolerance criterion is met. If the maximum number of iterations is -1 for the EM algorithm, no M-step is made and only the variational inference is optimized. This is useful if the variational parameters should be determined for new documents. The default values for the convergence checks are chosen similar to those suggested in the code available from Blei's web page as additional material to Blei *et al.* (2003b) and Blei and Lafferty (2007).

initialize: This parameter determines how the topics are initialized and can be either equal to "random", "seeded" or "model". Random initialization means that each topic is initialized randomly, seeded initialization signifies that each topic is initialized to a distribution smoothed from a randomly chosen document. If initialize = "model" a fitted model needs to be provided which is used for initialization, otherwise random initialization is used.

The possible arguments controlling how the LDA model is fitted using Gibbs sampling are given below together with their default values.

```
R> control_LDA_Gibbs <-
+ list(alpha = 50/k, estimate.beta = TRUE,
+ verbose = 0, prefix = tempfile(), save = 0, keep = 0,
+ seed = as.integer(Sys.time()), nstart = 1, best = TRUE,
+ delta = 0.1,
+ iter = 2000, burnin = 0, thin = 2000)</pre>
```

alpha, estimate.beta, verbose, prefix, save, keep, seed and nstart are the same as for estimation with the VEM algorithm. The other parameters are described below in detail.

delta: This parameter specifies the parameter of the prior distribution of the term distribution over topics. The default 0.1 is suggested in Griffiths and Steyvers (2004).

iter, burnin, thin: These parameters control how many Gibbs sampling draws are made.

The first burnin iterations are discarded and then every thin iteration is returned for iter iterations.

best: All draws are returned if best=FALSE, otherwise only the draw with the highest posterior likelihood over all runs is returned.

For the CTM model using the VEM algorithm the following arguments can be used to control the estimation.

```
R> control_CTM_VEM <-
+    list(estimate.beta = TRUE,
+         verbose = 0, prefix = tempfile(), save = 0, keep = 0,
+         seed = as.integer(Sys.time()), nstart = 1L, best = TRUE,
+         var = list(iter.max = 500, tol = 10^-6),
+         em = list(iter.max = 1000, tol = 10^-4),
+         initialize = "random",
+         cg = list(iter.max = 500, tol = 10^-5))</pre>
```

estimate.beta, verbose, prefix, save, keep, seed, nstart, best, var, em and initialize are the same as for VEM estimation of the LDA model. If the log-likelihood is decreased in an E-step, the maximum number of iterations in the variational inference step is increased by 10 or, if no maximum number is set, the tolerance for convergence is divided by 10 and the same E-step is continued. The only additional argument is cg.

cg: This controls how many iterations at most are used (iter.max) and how convergence is assessed (tol) in the conjugate gradient step in fitting the variational mean and variance per document.

LDA() and CTM() return S4 objects of a class which inherits from "TopicModel" (or a list of objects inheriting from class "TopicModel" if best=FALSE). Because of certain differences in the fitted objects there are sub-classes with respect to the model fitted (LDA or CTM) and the estimation method used (VEM or Gibbs sampling). The class "TopicModel" contains the call, the dimension of the document-term matrix, the number of words in the document-term matrix, the control object, the number of topics and the terms and document names and the number of iterations made. The estimates for the topic distributions for the documents are included which are the estimates of the corresponding variational parameters for the VEM algorithm and the parameters of the predictive distributions for Gibbs sampling. The term distribution of the topics are also contained which are the ML estimates for the VEM algorithm and the parameters of the predictive distributions for Gibbs sampling. In additional slots the objects contain the assignment of terms to the most likely topic and the log-likelihood which is  $\log p(w|\alpha,\beta)$  for LDA with VEM estimation,  $\log p(w|z)$  for LDA using Gibbs sampling and  $\log p(w|\mu, \Sigma, \beta)$  for CTM with VEM estimation. For VEM estimation the log-likelihood is returned separately for each document. If a positive keep control argument was given, the log-likelihood values of every keep iteration is contained. The extending class "LDA" has an additional slot for  $\alpha$ , "CTM" additional slots for  $\mu$  and  $\Sigma$ . "LDA\_Gibbs" which extends class "LDA" has a slot for  $\delta$  and "CTM\_VEM" which extends "CTM" has an additional slot for  $\nu^2$ .

Helper functions to analyze the fitted models are available. logLik() obtains the log-likelihood of the fitted model and perplexity() can be used to determine the perplexity of a fitted model also for new data. posterior() allows one to obtain the topic distributions for documents and the term distributions for topics. There is a newdata argument which

needs to be given a document-term matrix and where the topic distributions for these new documents are determined without fitting the term distributions of topics. Finally, functions terms() and topics() allow to obtain from a fitted topic model either the k most likely terms for topics or topics for documents respectively, or all terms for topics or topics for documents where the probability is above the specified threshold.

## 4. Illustrative example: Abstracts of JSS papers

The application of the package **topicmodels** is demonstrated on the collection of abstracts of the *Journal of Statistical Software* (JSS) (up to 2010-08-05).

The JSS data is available as a list matrix in the package **corpus.JSS.papers** which can be installed and loaded by

```
R> install.packages("corpus.JSS.papers",
+ repos = "http://datacube.wu.ac.at/", type = "source")
R> data("JSS_papers", package = "corpus.JSS.papers")
```

Alternatively, one can harvest JSS publication Dublin Core http://dublincore.org/meta-data (including information on authors, publication date and the abstract) from the JSS web site using the Open Archives Initiative Protocol for Metadata Harvesting (OAI-PMH), for which package **OAIHarvester** (Hornik 2011) provides an R client.

For reproducibility of results we use only abstracts published up to 2010-08-05 and omit those containing non-ASCII characters in the abstracts.

The final data set contains 348 documents. Before analysis we transform it to a "Corpus" using package tm. HTML markup in the abstracts for greek letters, subscripting, etc., is removed using package XML (Temple Lang 2010).

The corpus is exported to a document-term matrix using function DocumentTermMatrix() from package tm. The terms are stemmed and the stop words, punctuation, numbers and terms of length less than 3 are removed using the control argument. (We use a C locale for reproducibility.)

The mean term frequency-inverse document frequency (tf-idf) over documents containing this term is used to select the vocabulary. This measure allows to omit terms which have low frequency as well as those occurring in many documents. We only include terms which have a tf-idf value of at least 0.1 which is a bit more than the median and ensures that the very frequent terms are omitted.

```
R> library("slam")
R> summary(col_sums(JSS_dtm))
   Min. 1st Qu. Median
                           Mean 3rd Qu.
                                            Max.
   1.00
           1.00
                   2.00
                           5.98
                                   4.00
                                         450.00
R> term_tfidf <-</pre>
     tapply(JSS_dtm$v/row_sums(JSS_dtm)[JSS_dtm$i], JSS_dtm$j, mean) *
       log2(nDocs(JSS_dtm)/col_sums(JSS_dtm > 0))
R> summary(term_tfidf)
   Min. 1st Qu. Median
                           Mean 3rd Qu.
                                            Max.
0.01520 0.07472 0.09817 0.12210 0.13840 1.16500
R> JSS_dtm <- JSS_dtm[,term_tfidf >= 0.1]
R> JSS_dtm <- JSS_dtm[row_sums(JSS_dtm) > 0,]
R> summary(col_sums(JSS_dtm))
   Min. 1st Qu. Median
                           Mean 3rd Qu.
                                            Max.
  1.000
          1.000
                 1.000
                          2.763
                                  3.000 47.000
```

After this pre-processing we have the following document-term matrix with a reduced vocabulary which we can use to fit topic models.

```
R> dim(JSS_dtm)
```

#### [1] 348 2086

In the following we fit an LDA model with 30 topics using (1) VEM with  $\alpha$  estimated, (2) VEM with  $\alpha$  fixed and (3) Gibbs sampling with a burn-in of 1000 iterations and recording every 100th iterations for 1000 iterations. The initial  $\alpha$  is set to the default value. By default only the best model with respect to the log-likelihood  $\log(p(w|z))$  observed during Gibbs sampling is returned. In addition a CTM is fitted using VEM estimation.

We set the number of topics rather arbitrarily to 30 after investigating the performance with the number of topics varied from 2 to 200 using 10-fold cross-validation. The results indicated that the number of topics has only a small impact on the model fit on the hold-out data. There is only slight indication that the solution with two topics performs best and that the performance deteriorates again if the number of topics is more than 100. For applications a model with only two topics is of little interest because it enables only to group the documents very coarsely. This lack of preference of a model with a reasonable number of topics might be due to the facts that (1) the corpus is rather small containing less than 500 documents and (2) the corpus consists only of text documents on statistical software.

```
R> library("topicmodels")
R> k <-30
R> SEED <- 2010
R> jss\_TM <-
+ list(VEM = LDA(JSS\_dtm, k = k, control = list(seed = SEED)),
+ VEM\_fixed = LDA(JSS\_dtm, k = k,
+ control = list(estimate.alpha = FALSE, seed = SEED)),
+ Gibbs = LDA(JSS\_dtm, k = k, method = "Gibbs",
+ control = list(seed = SEED, burnin = 1000,
+ thin = 100, iter = 1000)),
+ CTM = CTM(JSS\_dtm, k = k,
+ control = list(seed = SEED,
+ var = list(tol = 10^-4), em = list(tol = 10^-3))))
```

To compare the fitted models we first investigate the  $\alpha$  values of the models fitted with VEM and  $\alpha$  estimated and with VEM and  $\alpha$  fixed.

We see that if  $\alpha$  is estimated it is set to a value much smaller than the default. This indicates that in this case the Dirichlet distribution has more mass at the corners and hence, documents

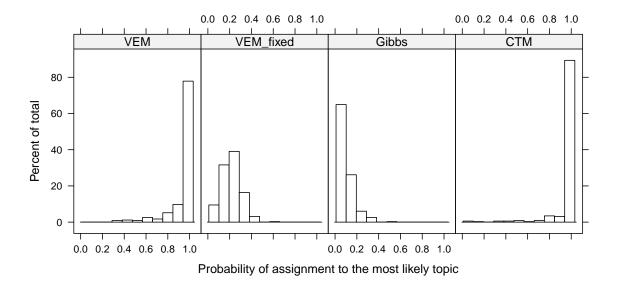


Figure 1: Histogram of the probabilities of assignment to the most likely topic for all documents for the different estimation methods.

consist only of few topics. The influence of  $\alpha$  on the estimated topic distributions for documents is illustrated in Figure 1 where the probabilities of the assignment to the most likely topic for all documents are given. The lower  $\alpha$  the higher is the percentage of documents which are assigned to one single topic with a high probability. Furthermore, it indicates that the association of documents with only one topic is strongest for the CTM solution.

The entropy measure can also be used to indicate how the topic distributions differ for the four fitting methods. We determine the mean entropy for each fitted model over the documents. The term distribution for each topic as well as the predictive distribution of topics for a document can be obtained with posterior(). A list with components "terms" for the term distribution over topics and "topics" for the topic distributions over documents is returned.

Higher values indicate that the topic distributions are more evenly spread over the topics.

The estimated topics for a document and estimated terms for a topic can be obtained using the convenience functions topics() and terms(). The most likely topic for each document is obtained by

```
R> Topic <- topics(jss_TM[["VEM"]], 1)</pre>
```

The five most frequent terms for each topic are obtained by

```
R> Terms <- terms(jss_TM[["VEM"]], 5)
R> Terms[,1:5]
```

```
Topic 1
                 Topic 2
                            Topic 3
                                       Topic 4
                                                   Topic 5
[1,] "network"
                 "confid"
                            "cluster"
                                       "random"
                                                   "beta"
                 "formula"
                                                   "captur"
[2,] "lispstat"
                            "correl"
                                       "variat"
[3,] "mathemat"
                 "interv"
                            "gee"
                                       "mantel"
                                                   "tables"
[4,] "learn"
                 "recurs"
                            "region"
                                       "matrices"
                                                   "rcaptur"
[5,] "text"
                 "cumul"
                            "boost"
                                       "uniform"
                                                   "file"
```

If any category labelings of the documents were available, these could be used to validate the fitted model. Some JSS papers should have similar content because they appeared in the same special volume. The most likely topic of the papers which appeared in Volume 24 called "Statistical Modeling of Social Networks with 'statnet" is given by

```
R> (topics_v24 <-
+ topics(jss_TM[["VEM"]])[grep("/v24/", JSS_papers[, "identifier"])])
243 244 245 246 247 248 249 250 251
21 25 21 21 27 21 21 23 21</pre>
```

The similarity between these papers is indicated by the fact that the majority of the papers have the same topic as their most likely topic. The ten most likely terms for topic 21 are given by

```
R> terms(jss_TM[["VEM"]], 10)[, most_frequent_v24]
```

R> most\_frequent\_v24 <- which.max(tabulate(topics\_v24))</pre>

```
[1] "network" "ergm" "graph"
[4] "format" "econometr" "brief"
[7] "hydra" "statnet" "imag"
[10] "exponentialfamili"
```

Clearly this topic is related to the general theme of the special issue. This indicates that the fitted topic model was successful at detecting the similarity between papers in the same special issue without using this information.

## 5. Summary

Package **topicmodels** provides functionality for fitting the topic models LDA and CTM in R. It builds on and complements functionality for text mining already provided by package **tm**. Functionality for constructing a corpus, transforming a corpus into a document-term matrix and selecting the vocabulary is available in **tm**. The basic text mining infrastructure provided by package **tm** is hence extended to allow also fitting of topic models which are seen nowadays as state-of-the-art techniques for analyzing document-term matrices. The

advantages of package **topicmodels** are that (1) it gives access within R to the code written by David M. Blei and co-authors, who introduced the LDA model as well as the CTM in their papers, and (2) allows different estimation methods by providing VEM estimation as well Gibbs sampling. Extensibility to other estimation techniques or slightly different model variants is easily possible via the method argument.

Packages **Snowball** (Hornik 2009) and **tm** provide stemmers and stop word lists not only for English, but also for other languages. To the authors' knowledge topic models have so far only been used for corpora in English. The availability of all these tools in R hopefully does not only lead to an increased use of these models, but also facilitates to try them out for corpora in other languages as well as in different settings. In addition different modeling strategies for model selection, such as cross-validation, can be easily implemented with a few lines of R code and the results can be analyzed and visualized using already available tools in R.

Due to memory requirements package **topicmodels** will for standard hardware only work for reasonably large corpora with numbers of topics in the hundreds. Gibbs sampling needs less memory than using the VEM algorithm and might therefore be able to fit models when the VEM algorithm fails due to high memory demands. In order to be able to fit topic models to very large data sets distributed algorithms to fit the LDA model were proposed for Gibbs sampling in Newman *et al.* (2009). The proposed Approximate Distributed LDA (AD-LDA) algorithm requires the Gibbs sampling methods available in **topicmodels** to be performed on each of the processors. In addition functionality is needed to repeatedly distribute the data and parameters to the single processors and synchronize the results from the different processors until a termination criterion is met. Algorithms to parallelize the VEM algorithm for fitting LDA models are outlined in Nallapati, Cohen, and Lafferty (2007). In this case the processors are used in the E-step such that each processor calculates only the sufficient statistics for a subset of the data. We intend to look into the potential of leveraging the existing infrastructure for large data sets along the lines proposed in Nallapati *et al.* (2007) and Newman *et al.* (2009).

The package allows us to fit topic models to different corpora which are already available in R using package **tm** or can easily be constructed using tools such as the package **OAIHarvester**. We are also interested in comparing the performance of topic models for clustering documents to other approaches such as using mixtures of von Mises-Fisher distributions to model the term distributions of the documents (Banerjee, Dhillon, Ghosh, and Sra 2005) where the R package **movMF** (Hornik and Grün 2011) is available on CRAN.

Different variants of topic models have been recently proposed. Some models aim at relaxing the assumption of independence of topics which is imposed by LDA such as the CTM, hierarchical topic models (Blei, Griffiths, Jordan, and Tenenbaum 2003a) or Pachinko allocation (Li and McCallum 2006) and hierarchical Pachinko allocation (Mimno, Li, and McCallum 2007). Another possible extension of the LDA model is to include additional information. Using the time information leads to dynamic topic models (Blei and Lafferty 2006) while using the author information of the documents gives the author-topic model (Rosen-Zvi, Chemudugunta, Griffiths, Smyth, and Steyvers 2010). We are interested in extending the package to cover at least a considerable subset of the different proposed topic models. As a starting point we will use Heinrich (2009) and Heinrich and Goesele (2009) who provide a common framework for topic models which only consist of Dirichlet-multinomial mixture "levels". Examples for such topic models are LDA, the author-topic model, Pachinko allocation and hierarchical Pachinko allocation.

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