Lecture-29
SLA

classmate
Date 09/04/18
Page

- For Player A, the min value in each row represents the least gain in choosing a particular strategy - Out of these, he would select the one which maximizes his min gain. This is called the maximin value :

maximin = $\underline{v}$ = max (row-min)
↳ wrt .

	C	B	
C	-1	-3	→ -3
B	0	-2	→ -2 ✓ go betray.

- For player B, the maximum value in each column is his max loss. So, he would select the strategy that minimizes his maximum loss. This is the minimax value.

minimax = $\bar{v}$ = min (col-max)

	C	B
C	-1	-3
B	0	-2
	↓ 0	↓ -2 ✓ → go betray

- If in a game, the maximin of A and the minimax of B, are equal then the game is said to have a saddle point (or an equilibrium point) where $\underline{v} = \bar{v}$.
- The maximin or minimax value is called the value of the game. A game is said to be fair if $\underline{v} = 0 = \bar{v}$.

- A game is said to be strictly determinable if $\underline{v} = \bar{v} = v$.

Q. Determine which of the following game is strictly determinable and fair

$$\begin{array}{cc} & \text{Pl. B} \\ \text{Pl. A} & \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix} \end{array}$$

Row min

$$\begin{array}{cc} \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix} & \begin{array}{c} \to 0^* \\ \to 0^* \end{array} \\ \text{col max} \quad 5 \quad 2^* & \end{array}$$

$\therefore \underline{v} = 0$

$\bar{v} = 2$

As $\underline{v} \neq \bar{v}$, so they are not strictly determinable.

Date
Page

ii) $\begin{pmatrix} 0 & 2 \\ -1 & 4 \end{pmatrix} \begin{matrix} \to 0 ✓ \\ \to -1 \end{matrix}$

$\downarrow \quad \downarrow$
$0 \quad 4$
✓

$\underline{v} = 0$
$\bar{v} = 0$ } ⇒ fair

∴ the game is fair.

Q. Determine the range of p, q that will make 'a_{22}' a saddle point for the game, with the following payoff matrix.

$$\begin{pmatrix} 2 & 4 & 7 \\ 10 & 7 & q \\ 4 & p & 8 \end{pmatrix}$$

→ Find ignoring p & q.

$$\begin{array}{c|ccc|c} & B_1 & B_2 & B_3 & \\ \hline A_1 & 2 & 4 & 7 & \to 2 \\ A_2 & 10 & 7^* & q & \to 7 ✓ \\ A_3 & 4 & p & 8 & \to 4 \\ \hline & \downarrow & \downarrow & \downarrow & \\ & 10 & 7 ✓ & 8 & \end{array}$$

∴ for a_{22} to be saddle point $\underline{v} = \bar{v} = 7$

Already, $\underline{v} = \bar{v} = 7$
∴ $q \geq 7$
and $p \leq 7$

Q. For what values of λ the pay off matrix is strictly determinable?

$$\begin{pmatrix} \lambda & 6 & 2 \\ -1 & \lambda & -7 \\ -2 & 4 & \lambda \end{pmatrix}$$

Date 9/4/18

$$\begin{pmatrix} \lambda & 6 & 2 \\ -1 & \lambda & -7 \\ -2 & 4 & \lambda \end{pmatrix} \begin{matrix} \to 2 \\ \to -7 \\ \to -2 \end{matrix}$$

$\downarrow \quad \downarrow \quad \downarrow$

$-1 \quad 6 \quad 2$

$\underline{v} = 2 = \max\{\lambda, -7, -2\}$

$\bar{v} = -1 = \max\{\lambda, 6, 2\}$

Equating λ, $\underline{v}$ & $\bar{v}$

$-1 \leq \lambda \leq 2$

Q. For the game with payoff matrix, determine the best strategy for players A and B and find the values of the game for A and B. Is the game fair or strictly determinable?

$$\begin{array}{c|ccc} & B_1 & B_2 & B_3 \\ \hline A_1 & -1 & -2 & -2 \\ A_2 & 6 & 4 & -6 \end{array}$$

$$\begin{pmatrix} -1 & -2 & -2 \\ 6 & 4 & -6 \end{pmatrix} \begin{matrix} \to -2 \\ \to -6 \end{matrix}$$

$\downarrow \quad \downarrow \quad \downarrow$

$6 \quad 4 \quad -2$

~~maximise~~ $\underline{\ } = \max\{\text{row min}\} = \text{val of } A = -2$

$\underline{v} = \bar{v} = -2$

$\therefore$ strictly determinable (but NOT fair)

A $\to$ strategy A_1

B $\to$ strategy B_3 or B_2

Games without saddle points with mixed strategies

There is no reason to expect that the minimax and the maximin value should always lead to a unique payoff position. The optimal strategy might be determined by assigning to each strategy a ~~prob~~ probability of being chosen. Such strategies are called mixed strategies. The value of the game thus obtained represent the least probability for player A to win & least probability of B to lose.

classmate

Date
Page

- A pair of strategies (p, q) for which maximin = minimax i.e., $\underline{v} = \overline{v} = v$ is called a saddle point of $E(p, q)$.
expectation.

p_i : probability with which player A will play his move A_i, $1 \leq i \leq m$, $\sum_{i=1}^{m} p_i = 1$

q_j : probability with which player B will play his move B_j, $1 \leq j \leq n$, $\sum_{j=1}^{n} q_j = 1$

$A = (a_{ij})_{m \times n}$: pay off matrix

$$\boxed{E(p, q) = p^T A q = \sum_{i=1}^{m} \sum_{j=1}^{n} p_i a_{ij} q_j}$$

- $E(p, q)$ is the expected payoff to a player in this $m \times n$ game.

Scanned by CamScanner