Lecture-15
SLa

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Duality of LPP

- Associated with every LPP there always exists another LPP which is based on the same data and has the same solution. The original problem is called the primal, the associated one is called dual.
- It is important to note that either of the two LPPs can be considered as the primal and the other one as the dual.

Primal

x's are the decision variables

$$\max z = cX = c_1x_1 + \dots + c_nx_n$$

st

$$a_{11}x_1 + \dots + a_{1n}x_n \le b_1$$
$$\vdots$$
$$a_{m1}x_1 + \dots + a_{mn}x_n \le b_m$$
$$AX \le b$$
$$x_1, \dots x_n \geq 0$$

Dual

v's are the decision variables

$$\min w = b^T V = b_1v_1 + \dots + b_mv_m$$

st.

transpose

$$a_{11}v_1 + \dots + a_{m1}v_m \geq c_1$$

= → if the ... was =, then x can be -ve too

$$\vdots$$
$$a_{1n}v_1 + \dots + a_{mn}v_m \geq c_n$$
$$A^TV \geq C^T$$

- Cost parameters become the requirement parameters and the requirement parameters become the cost/price parameters.
- The number of variables in the primal is the number of constraints in the dual and vice versa.
- The requirement vector in the primal is the price vector in the dual and vice versa.
- The transpose of the coefficient matrix of the primal is the coefficient matrix of the dual.
- If the primal is a maximization problem then its dual is a minimization problem and vice versa.
- If any primal constraint is an equality, then the corresponding dual variable is unrestricted in sign and vice versa.

Q. Find the dual of the following LPP:

$$\max Z = 2x_1 + 3x_2 + 4x_3$$

s.t.

$$x_1 - 5x_2 + 3x_3 = 7 \rightsquigarrow x_1 - 5x_2 + 3x_3 \geq 7 \Rightarrow -x_1 + 5x_2 - 3x_3 \leq -7$$

$$\rightsquigarrow x_1 - 5x_2 + 3x_3 \leq 7$$

$$2x_1 - 5x_2 \leq 3 \qquad 2x_1 - 5x_2 \leq 3$$

$$3x_2 - x_3 \geq 5 \qquad -3x_2 + x_3 \leq -5$$

$$x_1, x_2, x_3 \geq 0$$

converted all to $\leq$ as it is a maximiz^n problem.

Dual: as they are coming from the same constraint

$$\min w = 7v_1' - 7v_1'' + 3v_2 - 5v_3$$

s.t, coeff of x_1 from primal constraints

$$v_1' - v_1'' + 2v_2 + 0.v_3 \geq 2$$

coeff of x_2 ← $-5v_1' + 5v_1'' - 5v_2 - 3v_3 \geq 3$

coeff of x_3 ← $3v_1' - 3v_1'' + v_3 \geq 4$

$$v_1', v_1'', v_2, v_3 \geq 0$$

$\boxed{v_1 = v_1' - v_1''}$ → unrestricted in sign

⇓

$$\min w = 7v_1 + 3v_2 - 5v_3$$

s.t,

$$v_1 + 2v_2 \geq 2$$

$$-5v_1 - 5v_2 - 3v_3 \geq 3$$

$$3v_1 + v_3 \geq 4$$

$v_2, v_3 \geq 0$ as v_1 is unrestricted in sign

Duality theorems

Theorem 1:

An LPP has a finite optimal solution iff there exists feasible solutions to both the primal and dual problems.

Theorem 2:

If an LPP has an unbounded ~~tof~~ objective function for the primal, then the dual has no feasible solution.

Theorem 3:

If the dual of an LPP has no feasible solution and the primal problem has a feasible sol^n then the primal objective func^n is unbounded.

Q. Construct the dual and solve the LPP:

$\max z = 3x_1 + 4x_2$

s.t.,

$x_1 + x_2 \leq 12$

$2x_1 + 3x_2 \leq 21$

$x_1 \leq 8$

$x_2 \leq 6$

$x_1, x_2 \geq 0$

$x_1 = 8$
$x_2 = 5/3$
$z = 92/3$

both should be same

Dual:

$\min w = 12v_1 + 21v_2 + 8v_3 + 6v_4$

s.t.,

$v_1 + 2v_2 + v_3 \geq 3$

$v_1 + 3v_2 + v_4 \geq 4$

$v_1, v_2, v_3, v_4 \geq 0$

$w = 8/3 + 28 = 92/3$

Std form: $\max w^* = -12v_1 - 21v_2 - 8v_3 - 6v_4 + 0 \cdot S_1 + 0 \cdot S_2 - MA_1 - MA_2$

$v_1 + 2v_2 + v_3 - S_1 + A_1 \geq 3$

$v_1 + 3v_2 + v_4 - S_2 + A_2 \geq 4$

$v_1, v_2, v_3, v_4 \geq 0$

Lecture-16
dd 05/03/17

simplex table will have 8 columns from a_1 to a_8

IBFS:

NB $v_4 = v_1 = v_2 = v_3 = S_1 = 0 = S_2$

BV $\begin{cases} A_1 = 3 \\ A_2 = 4 \end{cases}$

			C_j	-12	-21	-8	-6	0	0	-M	-M	
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	Ratio
-M	a_7	A_1	3	1	2	1	0	-1	0	1	0	3/2
-M	a_8	A_2	4	1	3	0	1	0	-1	0	1	4/3
			$z_j^* - c_j^*$	-2M+12	-5M+21	-M+8	-M+6	M	M	0	0	
-M	a_7	A_1	1/3	1/3	0	1	-2/3	-1	2/3	0	~~-2/3~~	1/3
-21	a_2	v_2	4/3	1/3	1	0	1/3	0	-1/3	0	~~1/3~~	✗
			$z_j^* - c_j^*$	$-\frac{M}{3}+5$	0	-M+8	2M/3+1	M	-2M/3+7	0	~~2M/3 -7+M~~	
-8	a_3	v_3	1/3	1/3	0	1	-2/3	-1	2/3	✗	✗	
-21	a_2	v_2	4/3	1/3	1	0	1/3	M	-1/3	✗	✗	
			$z_j^* - c_j^*$	7/3	0	0	13/3	8	5/3	M-8	M-5/3	

$R_1 \leftarrow R_1 - 2R_2$

Sol^n for dual

$\boxed{v_3 = 1/3, \quad v_2 = 4/3, \quad v_1 = 0, \quad v_4 = 0}$

Sol^n for primal

column^n corr. find to surplus