- one for refreshing
- other filled with intensity values
- they can switch roles.

— Fast mechanism for generating real time ~~application~~ animations.

Lecture-5
AH
02/02/18

Two dimensional geometric transformation

changes in -

- shape
- size
- orientation

Basic transformations -

1. Translation (position)
2. Rotation (position)
3. Scaling (shape)
4. Reflection (position)
5. Shear (shape)

Translation -

~~Tac~~ Initial position of a point = (x, y)

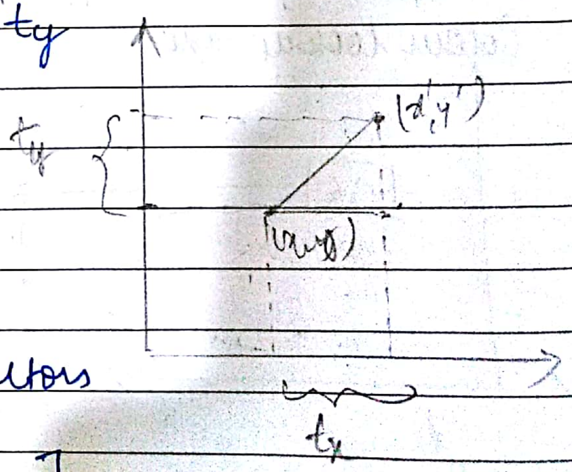
Translation vector components :

$$t_x , t_y$$

Then, new position :

$$x' = x + t_x$$

$$y' = y + t_y$$



Using matrix to represent vectors

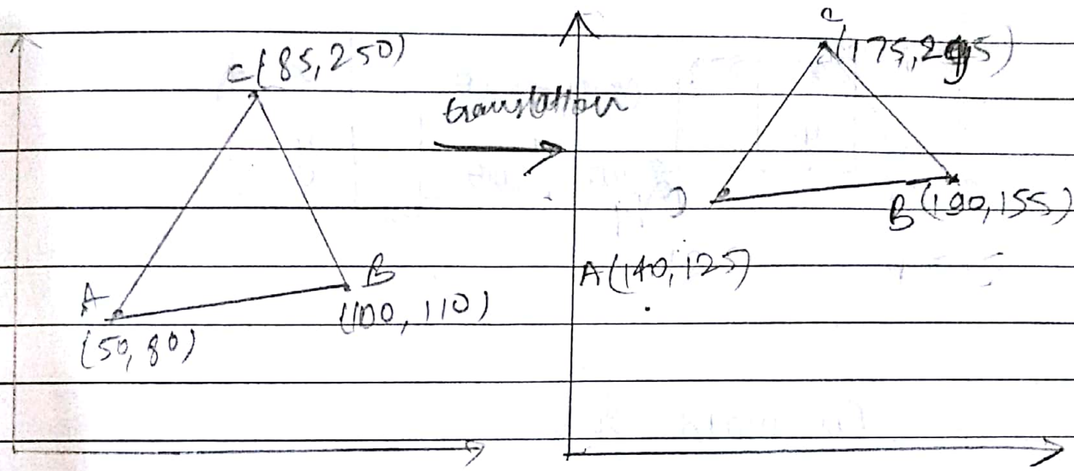
column vector repr.

$$P = \begin{bmatrix} x \\ y \end{bmatrix} \quad T = \begin{bmatrix} t_x \\ t_y \end{bmatrix} \quad P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

~~$P' = [x$~~

$\boxed{P' = P + T}$ → Only translation is additive

row vector rep^n — $P = [x \ y]$ $T = [t_x \ t_y]$ $P' = [x' \ y']$

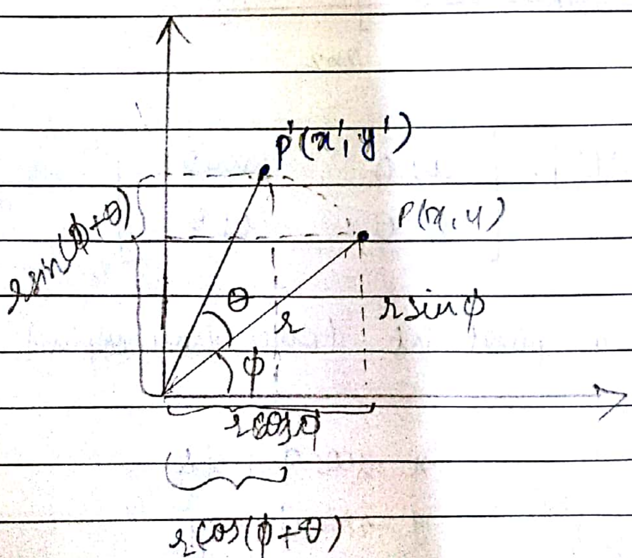


$t_x = 90$ $t_y = 45$

Rotation — (either w.r.t origin or a pivot)

Initial position : (x, y)

Initial angle with x axis : ϕ



$x' = r\cos(\theta + \phi)$

$= \underbrace{r\cos\phi}_{x}\cos\theta - \underbrace{r\sin\phi}_{y}\sin\theta$

$= x\cos\theta - y\sin\theta$

$y' = r\sin(\theta + \phi)$

$= r\sin\phi\cos\theta + r\cos\phi\sin\theta$

$\cancel{= x\cos\theta + y}$

$= y\cos\theta + x\sin\theta$

Scanned by CamScanner

Column matrix form

$P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \qquad P = \begin{bmatrix} x \\ y \end{bmatrix}$

$\boxed{P' = R \cdot P}$ → multiplicative

↳ rotation matrix

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}_{2\times 2}}_{R} \begin{bmatrix} x \\ y \end{bmatrix}$$

Row matrix form

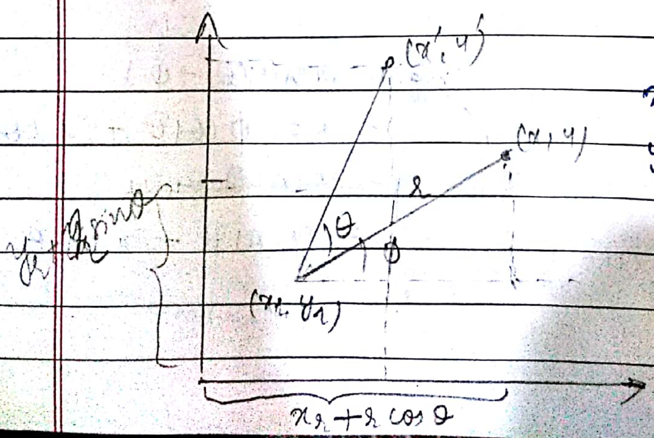
$P' = [x' \quad y'] \qquad P = [x \quad y]$

$\boxed{P' = P \cdot R}$

$$\cancel{[x' \quad y'] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}_{2\times 2} [x \quad y]}$$

$$= [x \quad y] \cdot \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}_{2\times 2}$$

Rotation w.r.t a pivot pt (other than origin)



$x = r\cos\phi + x_1$

$y = r\sin\phi + y_1$

$x' = r\cos(\theta + \phi)$

$= r\cos\theta\cos\phi - r\sin\phi\sin\theta$

$= (x - x_1)\cos\theta - (y - y_1)\sin\theta$

Similarly,

$$y_2 = (y - y_r)\cos\theta + (x - x_r)\sin\theta$$

Scaling (either w.r.t to origin or a pivot)

Translation : t_x, t_y

Rotation : θ

Scaling : (S_x, S_y) → Scaling factor

$P = \begin{bmatrix} x \\ y \end{bmatrix}$ $\quad P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$

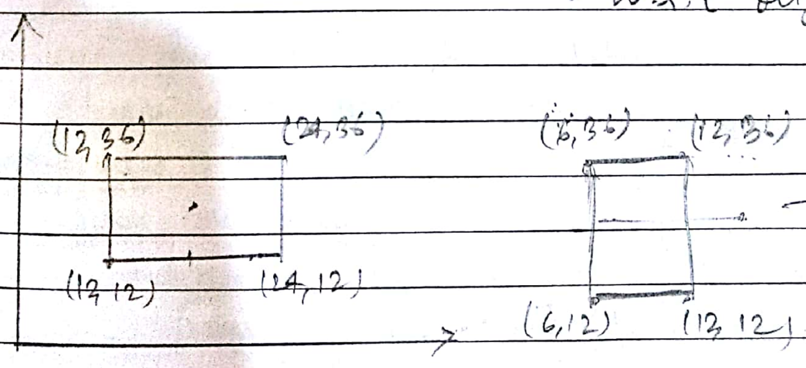
$x' = x \cdot S_x$

$y' = y \cdot S_y$

$\boxed{P' = S \cdot P}$ → multiplication

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}}_{\text{scaling matrix}} \begin{bmatrix} x \\ y \end{bmatrix}$$

↳ w.r.t origin

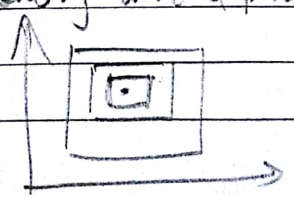


→ center point changed (unlike pivotal scaling)

$S_x = 1/2$

$S_y = 1$

Scaling w.r.t. a pivot



Date
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For composite transformation, (translation + rotation, rotation + scaling etc.) we convert the matrix operation to multiplicative form.

To do that, we introduce concept of homogeneous co-ordinates (dummy co-ordinates along x & y).

Converting to multiplication form

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} \; & \; \\ \; & \; \end{bmatrix}}_{T} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\boxed{\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix} \checkmark$$

$$\boxed{P' = T \cdot P}$$