

Q. Show that the feasible solution for :

$$x_1 + x_2 + x_3 = 2$$

$$x_1 + x_2 - 3x_3 = 2$$

$$2x_1 + 4x_2 + 3x_3 - x_4 = 4$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$\{(1, 1, 0, 2)\}$  is not basic.

Since  $n > m$ , possible solution sets :

$$(x_1, x_2, 0, x_4)$$

Given,  $x_1 = 1, x_2 = 1, x_3 = 0, x_4 = 2$

↓  
Not basic

$$\text{LHS} = x_1 + x_2 + x_3 = 1 + 1 + 0 = 2 = \text{RHS}$$

$$\text{LHS} = x_1 + x_2 - 3x_3 = 1 + 1 - 0 = 2 = \text{RHS}$$

$$\text{LHS} = 2x_1 + 4x_2 + 3x_3 - x_4 = 2 + 4 + 0 - 2 = 4 = \text{RHS}$$

∴ the given set is a  $\text{RHS}$ .

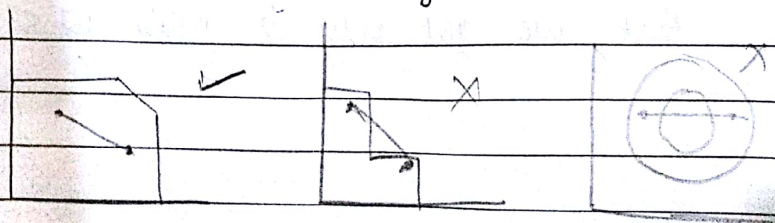
Also, since  $x_1, x_2, x_3, x_4 \geq 0$ , so it is a feasible solution.

$$A_{124} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & 4 & -1 \end{pmatrix} \rightarrow \text{rows are same, } \therefore \det = 0$$

$$|A_{124}| = 0$$

∴ linearly dependent  $\Rightarrow$  not basic

\* The solution set is always a convex set.





\* A line segment joining two points  $p_1$  and  $p_2$  such that  $p_1 = (x_1, x_2, \dots, x_n)$ ,  $p_2 = (x'_1, x'_2, \dots, x'_n)$  is the collection of points  $p$  such that

$$p = \lambda p_1 + (1-\lambda) p_2, \quad 0 \leq \lambda \leq 1$$

how far is  $p$  from  $p_2$

→ probability

line segment

$$L = \{p = (y_1, y_2, \dots, y_n) \mid p = \lambda p_1 + (1-\lambda) p_2, 0 \leq \lambda \leq 1\}$$

\* A subset  $S \subseteq \mathbb{R}^n$  is said to be convex if for any pair of points  $p_1, p_2 \in S$  the line segment joining  $p_1$  and  $p_2$  is also contained in  $S$  (LCS).

Q. Show that  $S = \{x \mid |x| \leq 2\}$  is a convex set.

We select  $x_1, x_2 \in S$

such that  $|x_1| \leq 2$ ,  $|x_2| \leq 2$

and  $|\lambda x_1 + (1-\lambda)x_2| \leq 2$

$$\leq |\lambda x_1| + |(1-\lambda)x_2|$$

$$= \lambda |x_1| + (1-\lambda) |x_2|$$

$$\leq \lambda \cdot 2 + (1-\lambda) \cdot 2 = 2$$

→ for maximization problems we convert these problems to minimization problems

## # Simplex Method

### Solving LPP by Simplex method

The optimal solution to an LPP if exists always occurs at corner points of the ~~feasible~~ feasible solution space. The simplex algorithm is an iterative process for finding the corner points and testing them for optimality. The evaluation of the corner point always starts at the origin which we call the initial basic feasible solution (IBFS) which is one of the corners of feasible region. This solution is tested for optimality whether an improvement in the objective function is possible by shifting to an adjacent corner of the feasible region. If possible, then the new corner point is again tested for optimality. This iterative search continues till the objective function cannot be further improved and the final corner point gives the optimal solution.