

Computer Graphics

Scan Conversion

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Introduction

- **Assumption:** raster display
- **How the scene is displayed?**
 - by loading the pixel arrays into the frame buffer
 - by scan converting the basic geometric-structure specifications into pixel patterns
- **Scene Description**
 - In terms of the basic geometric structures (provided by graphics package), referred to as *output primitives*
 - Group sets of output primitives into more complex structures
 - Each output primitive is specified with input coordinate data and other information about the way that object is to be displayed
 - Simplest geometric components: **Points and Straight line segments**
 - Additional output primitives: **circles and other conic sections, quadric surfaces, spline curves and surfaces, polygon color areas, and character strings**

Point Plotting

Converting a single coordinate position furnished by an application program into appropriate operations for the output device in use

- **CRT:** the electron beam is turned on to illuminate the screen phosphor at the selected location
- **A random-scan (vector) system:**
 - stores point-plotting instructions in the display list
 - coordinate values in these instructions are converted to deflection voltages
 - that position the electron beam at the screen locations to be plotted during each refresh cycle
- **Black and-white raster system:**
 - setting the bit value corresponding to a specified screen position within the frame buffer to 1
 - the electron beam sweeps across each horizontal scan line
 - it emits a burst of electrons (plots a point) whenever a value of 1 in the frame buffer
- **RGB system:** The frame buffer is loaded with the color codes for the intensities that are to be displayed at the screen pixel positions

Line Plotting

- By calculating intermediate positions along the line path between two specified endpoint positions
- **Vector pen plotter or a random-scan display:** Linearly varying horizontal and vertical deflection voltages are generated that are proportional to the required changes in the x and y directions to produce the smooth line
- **Digital devices:** by plotting discrete points between the two endpoints
 - Screen position is approximated
 - The line color (intensity) is then loaded into the frame buffer at the corresponding pixel coordinates
 - Reading from the frame buffer, the video controller "plots" the screen pixels
 - The rounding of coordinate values to integers causes lines to be displayed with a staircase appearance ("*the jaggies*")

Line Plotting: jaggies

- Noticeable on systems with low resolution
- Improve their appearance somewhat by displaying them on high-resolution systems
- More effective techniques for smoothing raster lines are based on adjusting pixel intensities along the line paths



Antialiasing

- **Jagged or stairstep appearance:** as the sampling process digitizes coordinate points on an object to discrete integer pixel positions
- This distortion of information due to low-frequency sampling (undersampling) is called *aliasing*
- Applying *antialiasing* methods compensate for the undersampling process

Antialiasing

- Set the sampling frequency to at least twice that of the highest frequency occurring in the object, referred to as the Nyquist sampling frequency (or Nyquist sampling rate): $f_s = 2f_{\max}$
- The sampling interval should be no larger than one-half the cycle interval (called the Nyquist sampling interval)
- For x-interval sampling, the Nyquist sampling interval is $\Delta x_s = \frac{\Delta x_{\text{cycle}}}{2}$ where $\Delta x_{\text{cycle}} = 1/f_{\max}$
- Unless hardware technology is developed to handle arbitrarily large frame buffers, increased screen resolution is not a complete solution to the aliasing problem

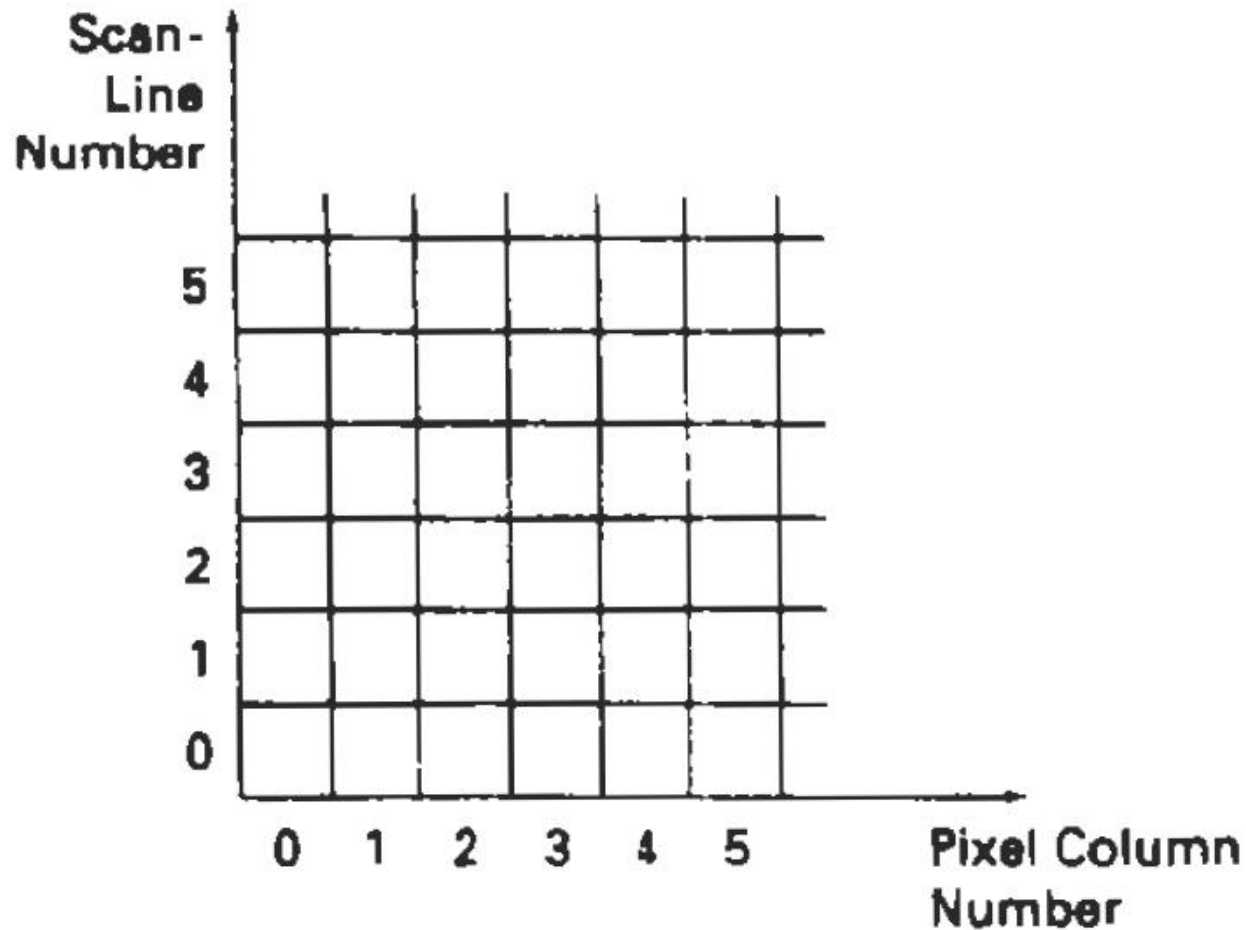
Antialiasing

- To increase sampling rate by treating the screen as if it were covered with a finer grid than is actually available
- Use multiple sample points across this finer grid to determine an appropriate intensity level for each screen pixel
- This technique of sampling object characteristics at a high resolution and displaying the results at a lower resolution is called *supersampling* (or *postfiltering*, since the general method involves computing intensities, at subpixel grid positions, then combining the results to obtain the pixel intensities)
- By supersampling, we obtain intensity information from multiple points that contribute to the overall intensity of a pixel

Antialiasing

- **Area sampling or prefiltering**
 - To determine pixel intensity by calculating the areas of overlap of each pixel with the objects to be displayed
 - The intensity of the pixel as a whole is determined without calculating subpixel intensities
- **Pixel Phasing:** Raster objects can also be antialiased by shifting the display location of pixel areas
- applied by "micropositioning" the electron beam in relation to object geometry.

Pixel positions referenced by scanline number and column number



Line Drawing Algorithms

- The Cartesian slope-intercept equation for a straight line is $y = m \cdot x + b$
 - m = slope; b = y-intercept
- Given that the two endpoints of a line segment are specified at positions (x_1, y_1) and (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}; \quad b = y_1 - mx_1$$

$|m| < 1$, Δx can be set

proportional to a small horizontal deflection voltage,

Vertical deflection is set proportional to Δy as calculated

$|m| > 1$, Δy can be set

proportional to a small horizontal deflection voltage,

Vertical deflection is set proportional to Δx as calculated

$|m| = 1$, $\Delta x = \Delta y$ horizontal and vertical deflection voltages are equal

$$m = \frac{\Delta y}{\Delta x}$$

$$\Delta x = \frac{\Delta y}{m}$$

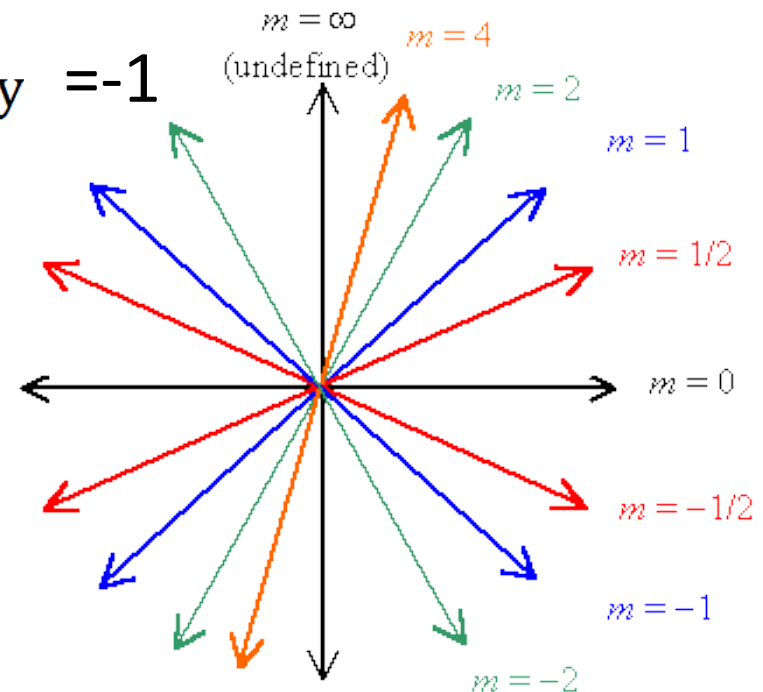
DDA Algorithm

(Digital Differential Analyzer)

- Scan conversion line drawing algorithm based on either Δy or Δx
- sample the line at unit intervals in one coordinate and determine corresponding integer values nearest the line path for the other coordinate
- Consider +ve slope and $m \leq 1$; the line is from the left endpoint to the right endpoint
 - Sample at unit x-interval $\rightarrow \Delta x = 1$
 - To compute $y_{k+1} = y_k + m$
 - Initially, $k=1$; increment 1 until the final endpoint is reached
 - m can be real no; y-value is approximated to nearest integer
- Consider +ve slope and $m > 1$; the line is from the left endpoint to the right endpoint
 - The role is reversed $\rightarrow x_{k+1} = x_k + \frac{1}{m}$

DDA Algorithm

- Consider +ve slope and $m \leq 1$; the line is from the right endpoint to the left endpoint
 - Sample at unit x-interval $\rightarrow \Delta x = -1$
 - To compute $y_{k+1} = y_k + m$
- Consider +ve slope and $m > 1$; the line is from the right endpoint to the left endpoint
 - Sample at unit x-interval $\rightarrow \Delta y = -1$
 - To compute $x_{k+1} = x_k - \frac{1}{m}$



DDA Algorithm: Example

- Draw a straight line from (2,3) to (8,7) using DDA Algorithm
- $dx=8-2=6$; $dy=7-3=4$
- $m \leq 1$ but positive
- Steps= $dx=6$; $y_{inc} = dy/steps=0.67$

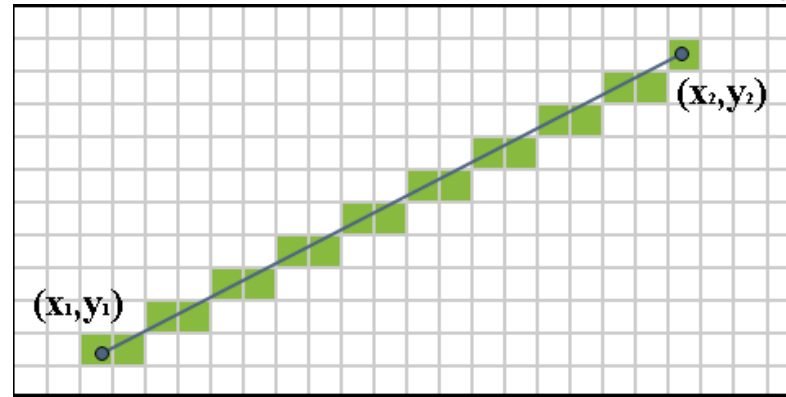
| <i>Xold</i> | <i>Yold</i> | <i>Xnew</i> | <i>Ynew</i> | <i>Round(x)</i> | <i>Round(y)</i> |
|-------------|-------------|-------------|-------------|-----------------|-----------------|
| 2 | 3 | 3 | 3.67 | 3 | 4 |
| 3 | 3.67 | 4 | 4.34 | 4 | 4 |
| 4 | 4.34 | 5 | 5.01 | 5 | 5 |
| 5 | 5.01 | 6 | 5.68 | 6 | 6 |
| 6 | 5.68 | 7 | 6.35 | 7 | 6 |
| 7 | 6.35 | 8 | 7.02 | 8 | 7 |

DDA Algorithm: Pros and Cons

- **Faster method** for calculating pixel positions than the direct use slope-intercept equation
- **Eliminates the multiplication** by making use of raster characteristics, so that appropriate increments are applied in the x or y direction to step to pixel positions along the line path
- The accumulation of **round-off error** in successive additions of the floating-point increment
 - the calculated pixel positions to **drift away** from the true line path for long line segments
- The rounding operations and floating-point arithmetic are ***time-consuming***
- **Improvement:** *by separating the increments m and $1/m$ into integer and fractional parts* so that all calculation are reduced to integer operations

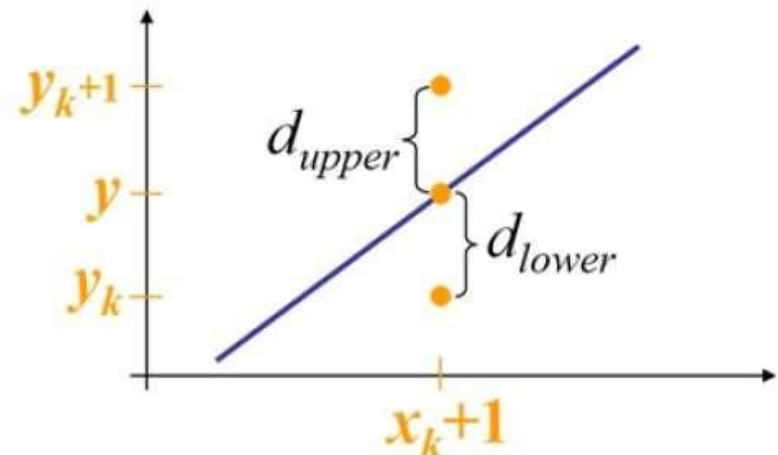
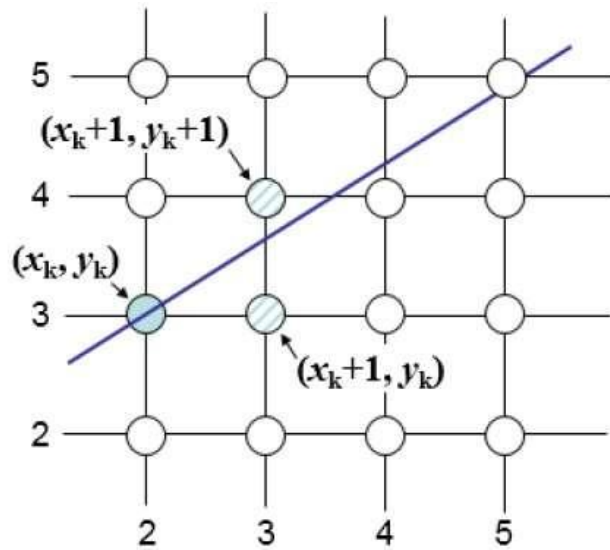
Bresenham's Algorithm

- Developed by J. E. Bresenham
- An accurate and efficient raster line-generating algorithm
- Scan converts lines using only incremental integer calculations
- $m \leq 1$ and positive
- sampling at unit x intervals
- Starting from the left endpoint (x_0, y_0) of a given line, we step to each successive column (x position) and plot the pixel whose scan-line y value is closest to the line path



Bresenham's Algorithm

- The pixel (x_k, y_k) is displayed
 - Next pixel to determine $\rightarrow (x_k+1, y_k)$ or (x_k+1, y_k+1)
- At sampling position x_k+1 , we label vertical pixel separations from the mathematical line path as **d_1 and d_2**
- The y-coordinate on the mathematical line at pixel column position x_k+1 is calculated as: $y=m(x_k+1)+b$
 - $d_1=y-y_k=m(x_k+1)+b-y_k$
 - $d_2=(y_k+1)-y=(y_k+1)-m(x_k+1)-b$
 - $d_1 - d_2=2m(x_k+1) - 2y_k + 2b - 1$



Bresenham's Algorithm

- Putting the value of m we get the following decision parameter

$$p_k = \Delta x(d_1 - d_2)$$

- Constant c is $2\Delta y + \Delta x(2b - 1)$ $= 2\Delta y \cdot x_k - 2\Delta x \cdot y_k + c$

- Δx is positive $\rightarrow p_k$ and $d_1 - d_2$ are of same sign
- y_k is closer to the line path ($d_1 < d_2$) is $p_k < 0$, plot lower pixel
 - Otherwise plot upper pixel
- Use incremental integer calculations
- The decision parameter in step k+1, $p_{k+1} = 2\Delta y \cdot x_{k+1} - 2\Delta x \cdot y_{k+1} + c$
- As $x_{k+1} = x_k + 1$ $p_{k+1} - p_k = 2\Delta y(x_{k+1} - x_k) - 2\Delta x(y_{k+1} - y_k)$
- $y_{k+1} - y_k = 0$ depending on the sign of p_k
- Initial Parameter $p_0 = 2\Delta y - \Delta x$
- The following constants are calculated once for each line to be scan converted $2\Delta y$ and $2\Delta y - 2\Delta x$

Bresenham's Line-Drawing Algorithm for $|m| < 1$

1. Input the two line endpoints and store the left endpoint in (x_0, y_0) .
2. Load (x_0, y_0) into the frame buffer; that is, plot the first point.
3. Calculate constants Δx , Δy , $2\Delta y$, and $2\Delta y - 2\Delta x$, and obtain the starting value for the decision parameter as

$$p_0 = 2\Delta y - \Delta x$$

4. At each x_k along the line, starting at $k = 0$, perform the following test:
If $p_k < 0$, the next point to plot is $(x_k + 1, y_k)$ and

$$p_{k+1} = p_k + 2\Delta y$$

Otherwise, the next point to plot is $(x_k + 1, y_k + 1)$ and

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x$$

5. Repeat step 4 Δx times.

Bresenham's Algorithm: Example

Suppose we want to draw a line starting at pixel (2,3) and ending at pixel (12,8).

$$dx = 12 - 2 = 10$$

$$dy = 8 - 3 = 5$$

$$p_0 = 2dy - dx = 0$$

$$2dy = 10$$

$$2dy - 2dx = -10$$

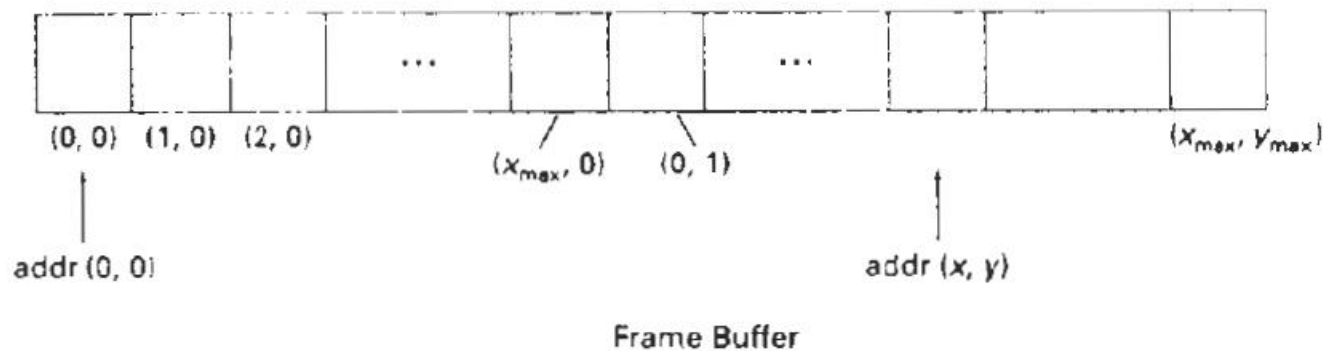
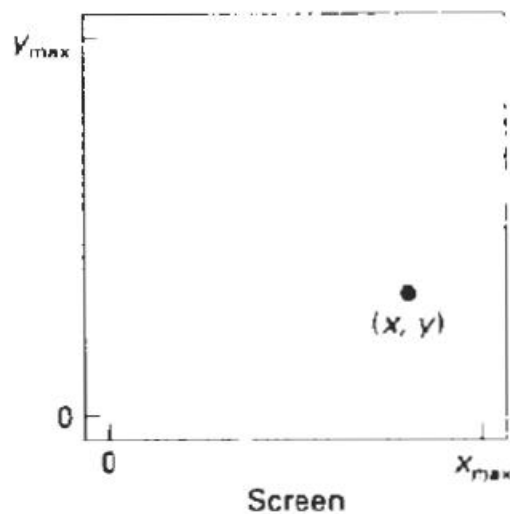
| t | p | P(x) | P(y) |
|----|-----|------|------|
| 0 | 0 | 2 | 3 |
| 1 | -10 | 3 | 4 |
| 2 | 0 | 4 | 4 |
| 3 | -10 | 5 | 5 |
| 4 | 0 | 6 | 5 |
| 5 | -10 | 7 | 6 |
| 6 | 0 | 8 | 6 |
| 7 | -10 | 9 | 7 |
| 8 | 0 | 10 | 7 |
| 9 | -10 | 11 | 8 |
| 10 | 0 | 12 | 8 |

Loading Frame Buffer

$$\text{addr}(x, y) = \text{addr}(0, 0) + y(x_{\max} + 1) + x$$

$$\text{addr}(x + 1, y) = \text{addr}(x, y) + 1$$

$$\text{addr}(x + 1, y + 1) = \text{addr}(x, y) + x_{\max} + 2$$

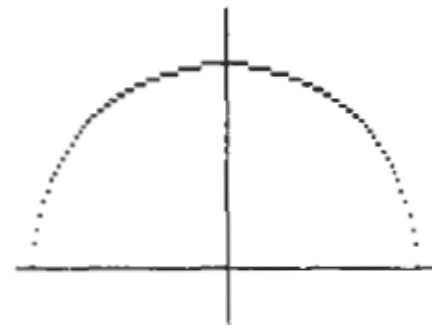


Properties of a Circle

- A circle is defined as the set of points that are all at a given distance r from a center position (x_c, y_c)
- This distance relationship is expressed by the Pythagorean theorem in Cartesian coordinates as $(x - x_c)^2 + (y - y_c)^2 = r^2$
- To calculate the position of points on a circle circumference by stepping along the x axis in unit steps from $x_c - r$ to $x_c + r$ and calculating the corresponding y values at each position as

$$y = y_c \pm \sqrt{r^2 - (x_c - x)^2}$$

- Involves considerable computation at each step
- Spacing between plotted pixel positions is not uniform
 - By interchanging x and y whenever the absolute value of the slope of the circle is greater than 1
 - increases the computation and processing



Properties of a Circle

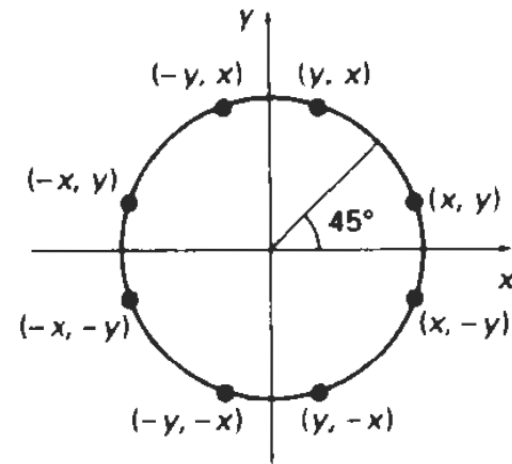
- To eliminate the unequal spacing use polar coordinates : r and Θ

$$x = x_c + r \cos \theta$$

$$y = y_c + r \sin \theta$$

- Using a fixed angular step size, a circle is plotted with equally spaced points along the circumference
 - Step size depends on the application and display device
 - Larger step size gaps are connected by straight line segment
 - If step: $1/r \rightarrow$ more continuous display
- Computation can be reduced by considering the symmetry of circles
 - Bresenham's line algorithm for raster displays is adapted

- **Direct distance comparison is to test the halfway position between two pixels to determine if this midpoint is inside or outside the circle boundary**
- **This method is more easily applied to other conics**
- **Also, the error involved in locating pixel positions along any conic section using the midpoint test is limited to one-half the pixel separation**

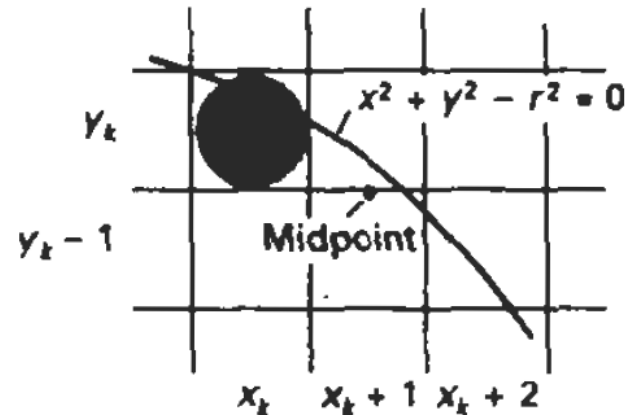


Midpoint Circle Algorithm

- Circle function: $f_{\text{circle}}(x, y) = x^2 + y^2 - r^2$

- Position of a point:

$$f_{\text{circle}}(x, y) \begin{cases} < 0, & \text{if } (x, y) \text{ is inside the circle boundary} \\ = 0, & \text{if } (x, y) \text{ is on the circle boundary} \\ > 0, & \text{if } (x, y) \text{ is outside the circle boundary} \end{cases}$$



- Current position: x_k, y_k
- Next point closer to circle: $x_k + 1, y_k$ or $x_k + 1, y_k - 1$
- The decision parameter is the circle function

$$p_k = f_{\text{circle}}\left(x_k + 1, y_k - \frac{1}{2}\right)$$

$$= (x_k + 1)^2 + \left(y_k - \frac{1}{2}\right)^2 - r^2$$

- $p_k < 0 \rightarrow$ midpoint is inside the circle
 - y_k ; closer to the circle boundary
 - $y_k - 1$; outside the circle boundary

Midpoint Circle Algorithm

- Successive decision parameters are obtained using incremental calculations

$$p_{k+1} = f_{\text{circle}}\left(x_{k+1} + 1, y_{k+1} - \frac{1}{2}\right)$$

$$= [(x_k + 1) + 1]^2 + \left(y_{k+1} - \frac{1}{2}\right)^2 - r^2$$

- Increments: $p_{k+1} = p_k + 2(x_k + 1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$
 $2x_{k+1} + 1 \quad 2x_{k+1} + 1 - 2y_{k+1}$

$$2x_{k+1} = 2x_k + 2$$

$$2y_{k+1} = 2y_k - 2$$

- Start position: $(0, r) \rightarrow 2x_{k+1}=0; 2y_{k+1}=2r$
- Rounding p_0 to an integer: $p_0=1-r$

$$p_0 = f_{\text{circle}}\left(1, r - \frac{1}{2}\right)$$

$$= 1 + \left(r - \frac{1}{2}\right)^2 - r^2$$

$$p_0 = \frac{5}{4} - r$$

Midpoint Circle Algorithm

1. Input radius r and circle center (x_c, y_c) , and obtain the first point on the circumference of a circle centered on the origin as

$$(x_0, y_0) = (0, r)$$

2. Calculate the initial value of the decision parameter as

$$p_0 = \frac{5}{4} - r$$

3. At each x_k position, starting at $k = 0$, perform the following test: If $p_k < 0$, the next point along the circle centered on $(0, 0)$ is (x_{k+1}, y_k) and

$$p_{k+1} = p_k + 2x_{k+1} + 1$$

Otherwise, the next point along the circle is $(x_k + 1, y_k - 1)$ and

$$p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$$

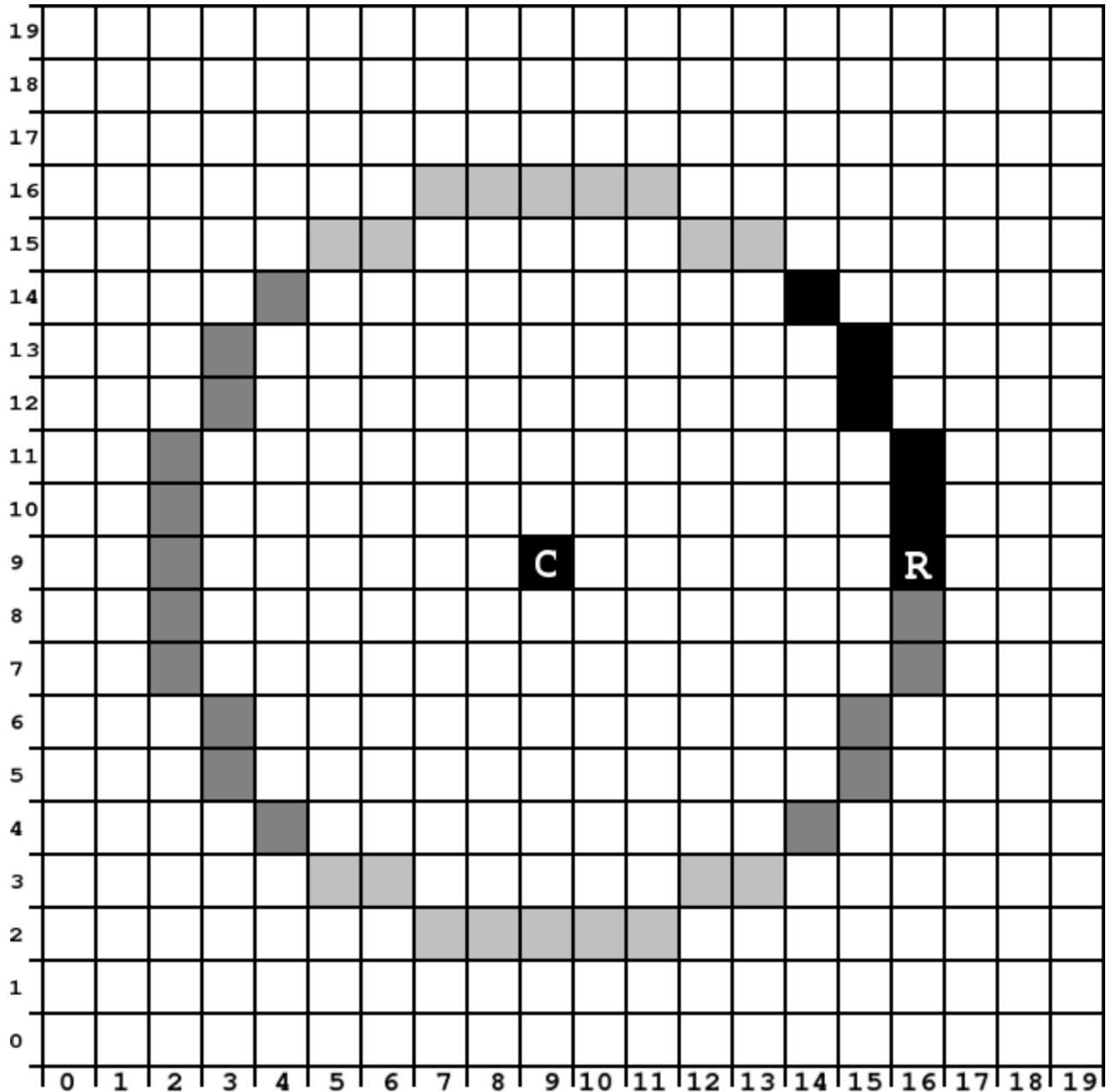
where $2x_{k+1} = 2x_k + 2$ and $2y_{k+1} = 2y_k - 2$.

4. Determine symmetry points in the other seven octants.
5. Move each calculated pixel position (x, y) onto the circular path centered on (x_c, y_c) and plot the coordinate values:

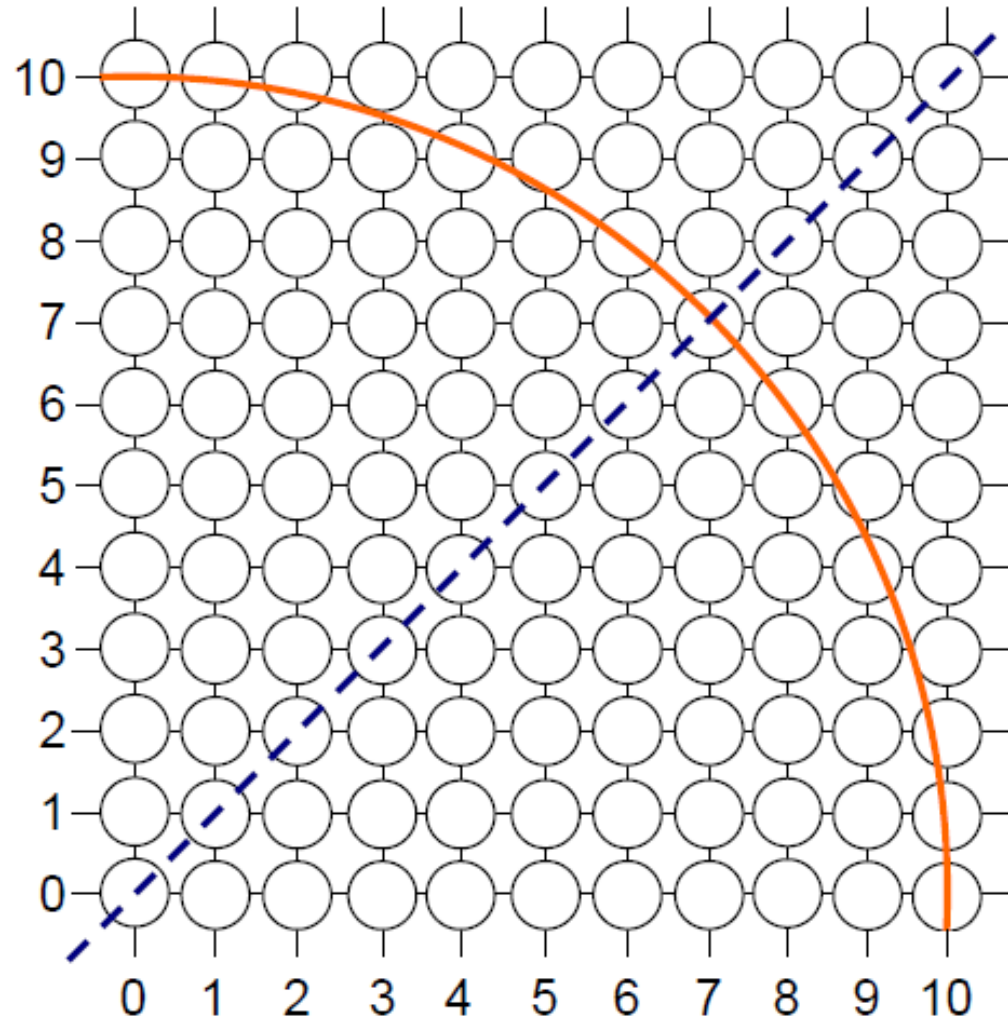
$$x = x + x_c \quad y = y + y_c$$

6. Repeat steps 3 through 5 until $x \geq y$.

Midpoint Circle Algorithm



Midpoint Circle Algorithm



| k | p_k | (x_{k+1}, y_{k+1}) | $2x_{k+1}$ | $2y_{k+1}$ |
|---|-------|----------------------|------------|------------|
| 0 | | | | |
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |

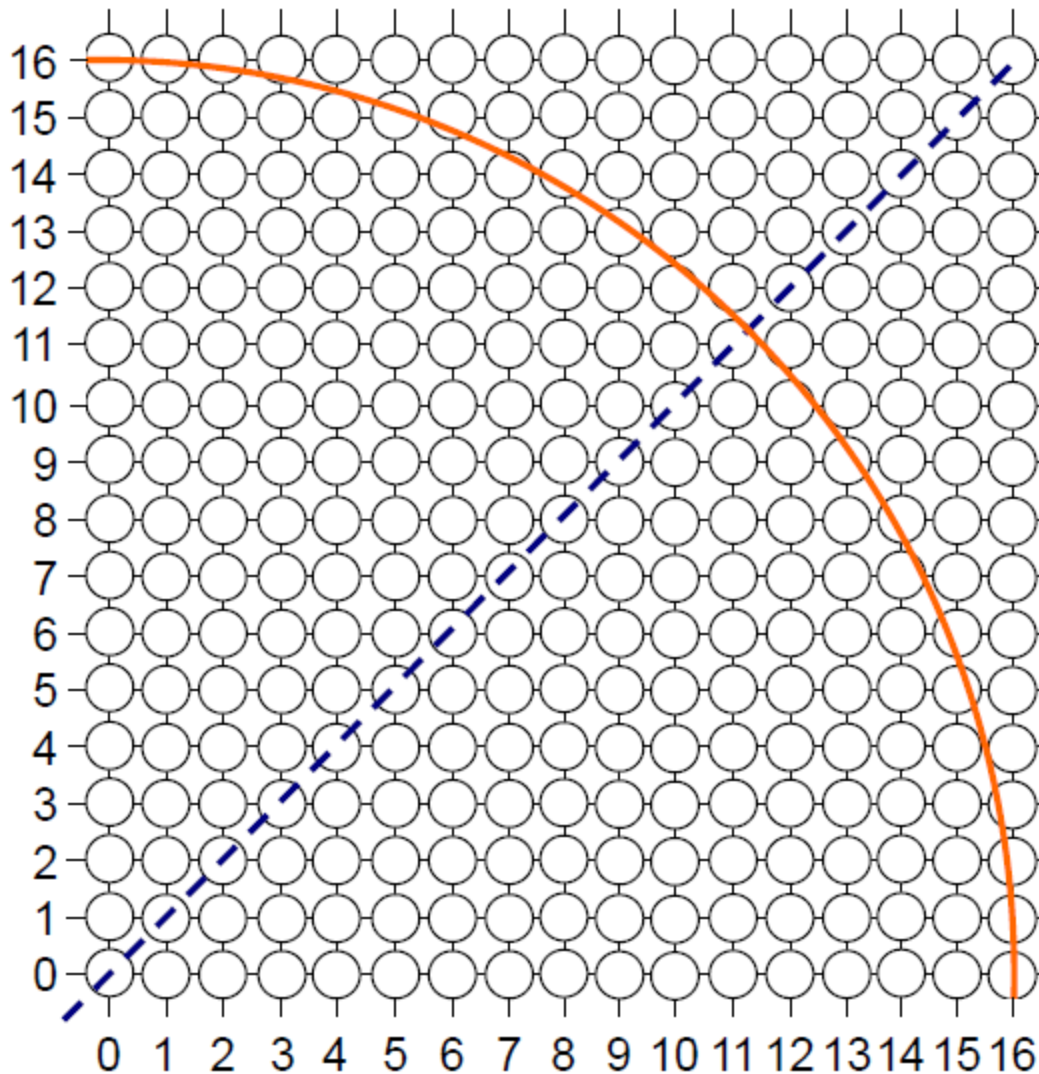
Midpoint Circle Algorithm

To see the mid-point circle algorithm in action lets use it to draw a circle centred at (0,0) with radius 10

| k | p_k | (x_{k+1}, y_{k+1}) | $2x_{k+1}$ | $2y_{k+1}$ |
|-----|-------|----------------------|------------|------------|
| 0 | -9 | (1, 10) | 2 | 20 |
| 1 | -6 | (2, 10) | 4 | 20 |
| 2 | -1 | (3, 10) | 6 | 20 |
| 3 | 6 | (4, 9) | 8 | 18 |
| 4 | -3 | (5, 9) | 10 | 18 |
| 5 | 8 | (6, 8) | 12 | 16 |
| 6 | 5 | (7, 7) | 14 | 14 |

Midpoint Circle Algorithm

To see the mid-point circle algorithm in action lets use it to draw a circle centred at (0,0) with radius 16



| k | p_k | (x_{k+1}, y_{k+1}) | $2x_{k+1}$ | $2y_{k+1}$ |
|----|-------|----------------------|------------|------------|
| 0 | | | | |
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | | | | |
| 8 | | | | |
| 9 | | | | |
| 10 | | | | |
| 11 | | | | |
| 12 | | | | |

Midpoint Circle Algorithm

The key insights in the mid-point circle algorithm are:

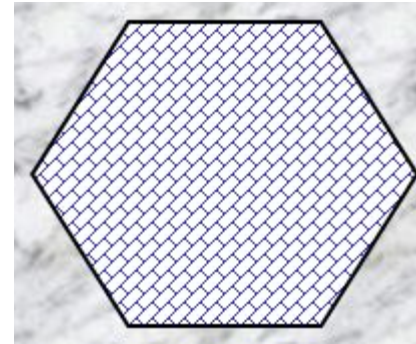
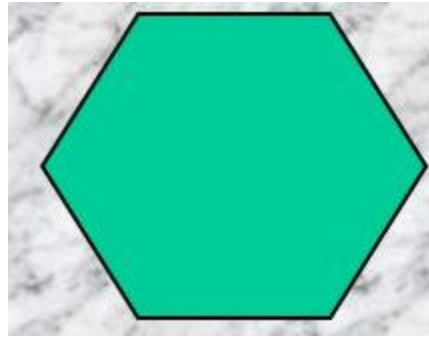
- Eight-way symmetry can hugely reduce the work in drawing a circle
- Moving in unit steps along the x axis at each point along the circle's edge we need to choose between two possible y coordinates

Filled area Primitives

- Solid-color or patterned polygon area
- Polygons are easier to process since they have linear boundaries
- There are two basic approaches for area filling
- To determine the overlap intervals for scan lines that cross the area
- To start from a given interior position and paint outward from this point until we encounter the specified boundary conditions
- The scan-line approach is typically used in general graphics packages to fill polygons circles, ellipses, and other simple curves
- All methods starting from an interior point are useful with more complex boundaries and in interactive painting systems

Filled area Primitives

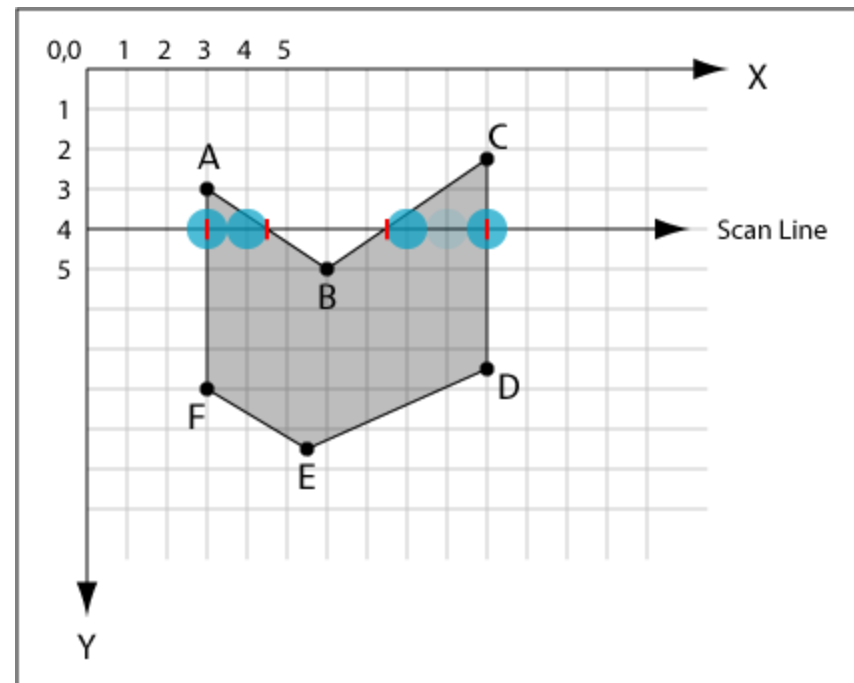
- Solid Fill
- Pattern Fill



- Polygon of n -vertices, ordered list of edges
- To learn:
 1. Scan-Line Fill Algorithm
 2. Flood-Fill Algorithm
 3. Boundary-Fill Algorithm

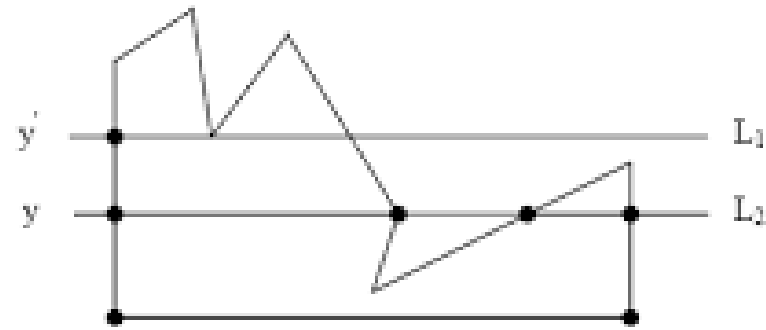
Scan Line Fill Algorithms

- Solid filling of polygonal areas
- For each scan line crossing a polygon, the area-fill algorithm locates the intersection points of the scan line with the polygon edges.
- These intersection points are then sorted from left to right, and the corresponding frame-buffer positions between each intersection pair are set to the specified fill color

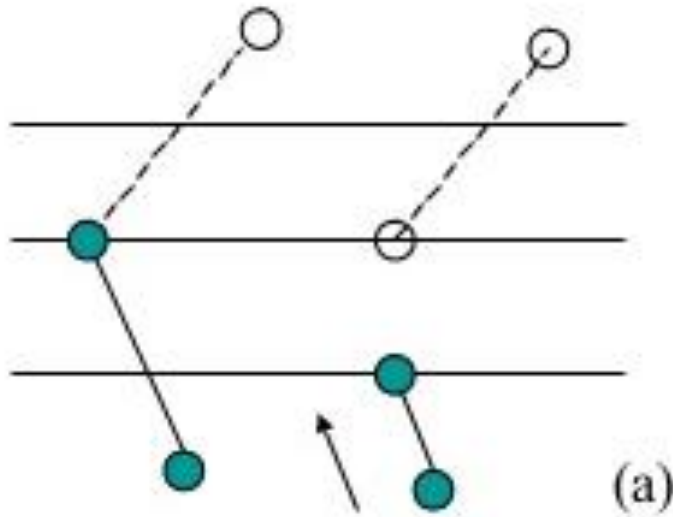


Scan Line Fill Algorithms

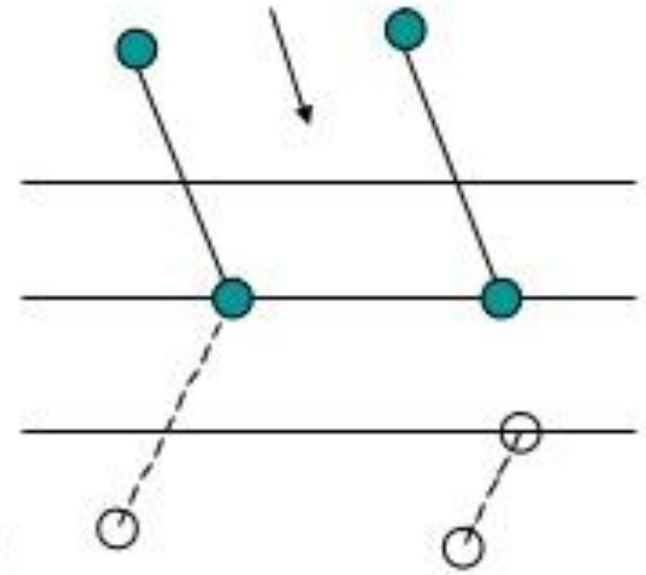
- Scan-line intersection with polygon vertices needs special handling
 - Intersects two polygon edges
- The corresponding edges either same side of the scan line or opposite side of it
- Identify these vertices by traversing the polygon boundary in clockwise or anti-clockwise manner
 - observing the relative changes in y-direction
 - y-value increases or decreases monotonically -> count single
 - otherwise, count twice



Scan Line Fill Algorithms



(a)



(b)

Adjusting endpoint values for a polygon, as we process edges in order around the polygon perimeter. The edge currently being processed is indicated as a solid line. In (a), the y coordinate of the upper endpoint of the current edge is decreased by 1. In (b), the y coordinate of the upper endpoint of the next edge is decreased by 1.

Scan Line Fill Algorithms

- Graphics algorithm takes advantage of coherence of a scene
- Properties of one part of a scene is somewhat related to other part of a scene
- Involves in incremental calculations, applied along a scan line or successive scan lines
- To determine edge intersection, incremental coordinate calculations
 - Slope of the line remains constant from one scan line to the other
 - $m = (y_{k+1} - y_k) / (x_{k+1} - x_k)$; changes in y-coordinates: $y_{k+1} - y_k = 1$
 - x-coordinate can also be determined: $x_{k+1} - x_k = 1/m$

Scan Line Fill Algorithms

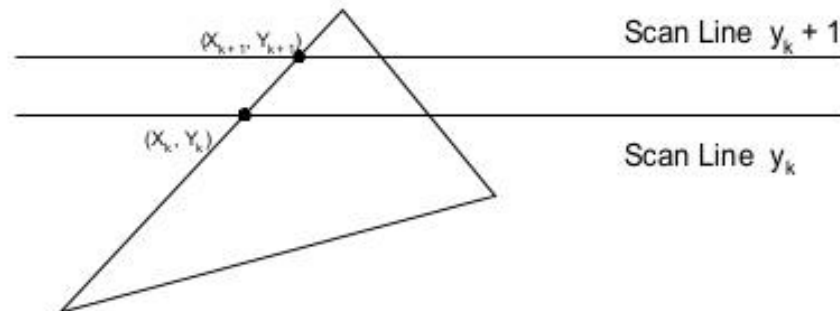
- The x_k value of k^{th} scan line of slope m is $x_k = x_0 + \frac{k}{m}$
- The increment in x-value is $1/m$ $m = \frac{\Delta y}{\Delta x}$ $x_{k+1} = x_k + \frac{\Delta x}{\Delta y}$

The scan conversion algorithm works as follows

- Intersect each scanline with all edges
- Sort intersections in x
- Calculate parity of intersections to determine in/out
- Fill the “in” pixels

Special cases to be handled:

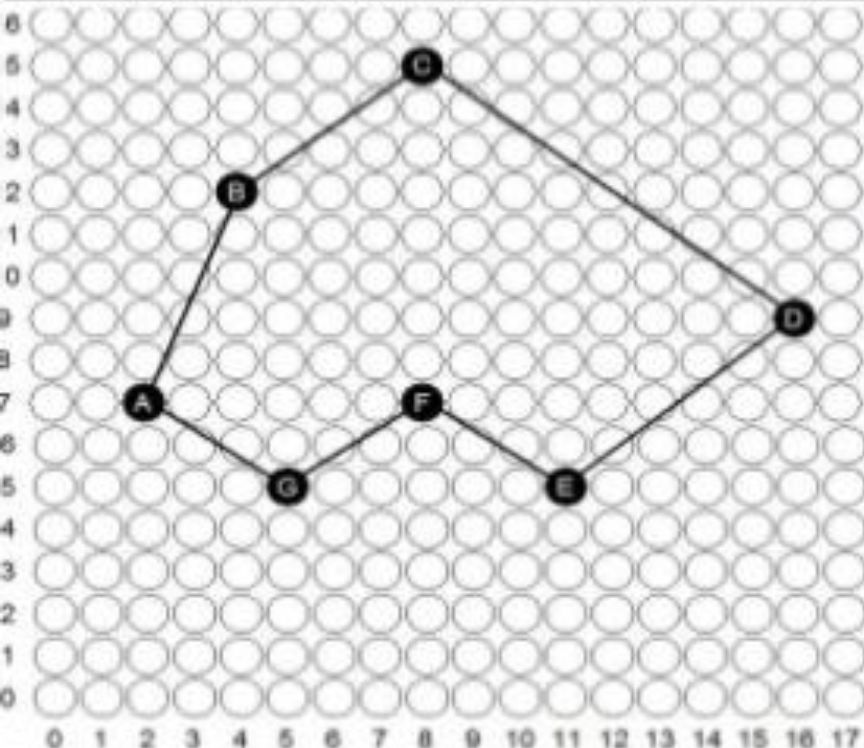
- Horizontal edges should be excluded
 - Vertices lying on scanlines handled by shortening of edges,
- Coherence between scanlines tells us that
 - Edges that intersect scanline y are likely to intersect $y + 1$
 - x changes predictably from scanline y to $y + 1$ (*Incremental Calculation Possible*)



Scan Line Fill Algorithms

(Example)

| Edge Table | | | | | | |
|------------|-----------|-----------|------|------------------|----|------------------|
| # | Edge | | 1/m | y _{min} | x | y _{max} |
| 0 | A (2, 7) | B (4, 12) | 2/5 | 7 | 2 | 12 |
| 1 | B (4, 12) | C (8, 15) | 4/3 | 12 | 4 | 15 |
| 2 | C (8, 15) | D (16, 9) | -8/6 | 9 | 16 | 15 |
| 3 | D (16, 9) | E (11, 5) | 5/4 | 5 | 11 | 9 |
| 4 | E (11, 5) | F (8, 7) | -3/2 | 5 | 11 | 7 |
| 5 | F (8, 7) | G (5, 5) | 3/2 | 5 | 5 | 7 |
| 6 | G (5, 5) | A (2, 7) | -3/2 | 5 | 5 | 7 |



Edge number 0

| # | Edge | | 1/m | y _{min} | x | y _{max} |
|---|----------|------------|-----------|------------------|---|------------------|
| 0 | A (2, 7) | B' (4, 11) | 2/5 = 0.4 | 7 | 2 | 11 |

| Scan line | x-intersection |
|-----------|---------------------|
| y = 7 | 2 |
| y = 8 | 2 + 0.4 = 2.4 ~ 2 |
| y = 9 | 2.4 + 0.4 = 2.8 ~ 3 |
| y = 10 | 2.8 + 0.4 = 3.2 ~ 3 |
| y = 11 | 4 |

Edge number 1

| # | Edge | | 1/m | y _{min} | x | y _{max} |
|---|-----------|-----------|-----------|------------------|---|------------------|
| 1 | B (4, 12) | C (8, 15) | 4/3 = 1.3 | 12 | 4 | 15 |

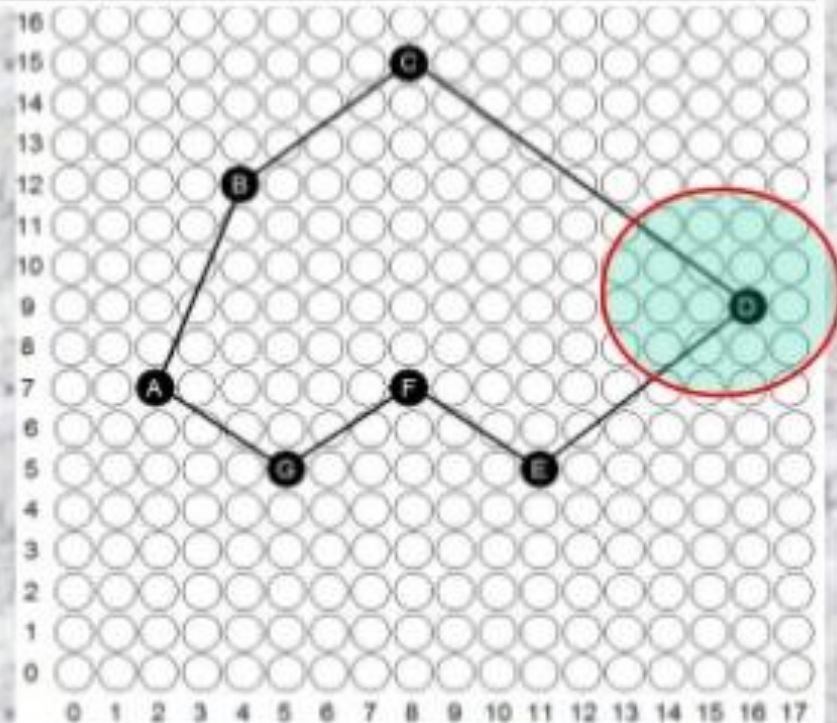
| Scan line | x-intersection |
|-----------|---------------------|
| y = 12 | 4 |
| y = 13 | 4 + 1.3 = 5.3 ~ 5 |
| y = 14 | 5.3 + 1.3 = 6.6 ~ 6 |
| y = 15 | 8 |

Scan Line Fill Algorithms

(Example)

Edge Table

| # | Edge | | 1/m | Ymin | x | Ymax |
|---|-----------|-----------|--------|------|----|------|
| 0 | A (2, 7) | B (4, 12) | 2/5 | 7 | 2 | 12 |
| 1 | B (4, 12) | C (8, 15) | 4/3 | 12 | 4 | 15 |
| 2 | C (8, 15) | D (16, 9) | $-8/6$ | 9 | 16 | 15 |
| 3 | D (16, 9) | E (11, 5) | 5/4 | 5 | 11 | 9 |
| 4 | E (11, 5) | F (8, 7) | $-3/2$ | 5 | 11 | 7 |
| 5 | F (8, 7) | G (5, 5) | 3/2 | 5 | 5 | 7 |
| 6 | G (5, 5) | A (2, 7) | $-3/2$ | 5 | 5 | 7 |



Edge number 2

| # | Edge | | 1/m | Ymin | x | Ymax |
|---|-----------|-----------|---------------|------|----|------|
| 2 | C (8, 15) | D (16, 9) | $-8/6 = -1.3$ | 9 | 16 | 15 |

| Scan line | x-intersection |
|-----------|-----------------------------|
| y = 9 | 16 |
| y = 10 | $16 - 1.3 = 14.7 \sim 15$ |
| y = 11 | $14.7 - 1.3 = 13.4 \sim 13$ |
| y = 12 | $13.4 - 1.3 = 12.1 \sim 12$ |
| y = 13 | $12.1 - 1.3 = 10.8 \sim 11$ |
| y = 14 | $10.8 - 1.3 = 9.5 \sim 10$ |
| y = 15 | 8 |

Edge number 3

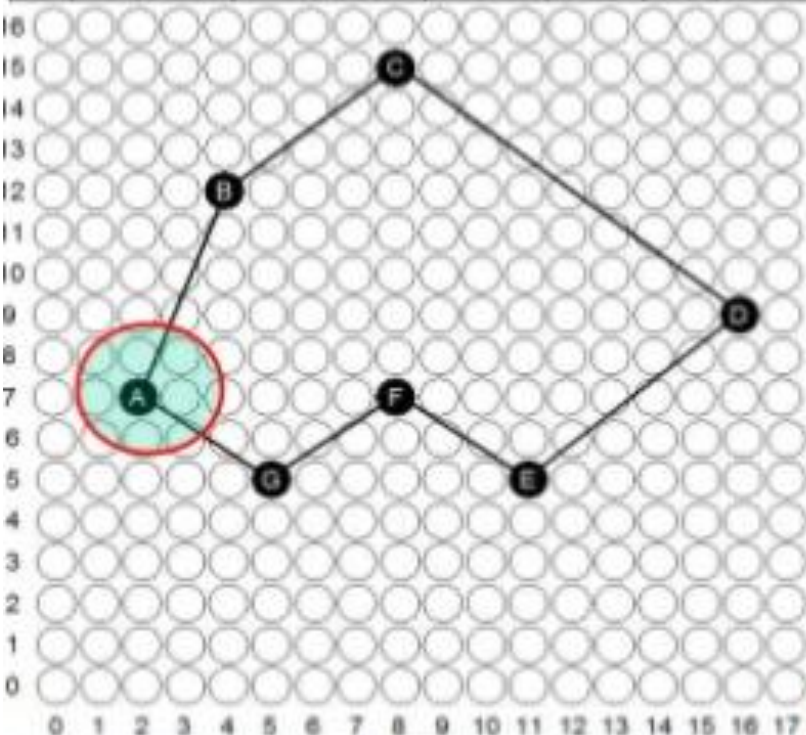
| # | Edge | | 1/m | Ymin | x | Ymax |
|---|------------|-----------|--------------|------|----|------|
| 3 | D' (15, 8) | E (11, 5) | $5/4 = 1.25$ | 5 | 11 | 8 |

| Scan line | x-intersection |
|-----------|-------------------------------|
| y = 5 | 11 |
| y = 6 | $11 + 1.25 = 12.25 \sim 12$ |
| y = 7 | $12.25 + 1.25 = 13.5 \sim 14$ |
| y = 8 | 15 |

Scan Line Fill Algorithms

(Example)

| Edge Table | | | | | |
|------------|-----------|-----------|------|------|--------|
| # | Edge | | 1/m | Ymin | X Ymax |
| 0 | A (2, 7) | B (4, 12) | 2/5 | 7 | 2 12 |
| 1 | B (4, 12) | C (8, 15) | 4/3 | 12 | 4 15 |
| 2 | C (8, 15) | D (16, 9) | -8/6 | 9 | 16 15 |
| 3 | D (16, 9) | E (11, 5) | 5/4 | 5 | 11 9 |
| 4 | E (11, 5) | F (8, 7) | -3/2 | 5 | 11 7 |
| 5 | F (8, 7) | G (5, 5) | 3/2 | 5 | 5 7 |
| 6 | G (5, 5) | A (2, 7) | -3/2 | 5 | 5 7 |



Edge number 4

| # | Edge | | 1/m | Ymin | X | Ymax |
|---|-----------|----------|---------------|------|----|------|
| 4 | E (11, 5) | F (8, 7) | $-3/2 = -1.5$ | 5 | 11 | 7 |

| Scan line | x-intersection |
|-----------|--------------------------|
| y = 5 | 11 |
| y = 6 | $11 - 1.5 = 9.5 \sim 10$ |
| y = 7 | 8 |

Edge number 5

| # | Edge | | 1/m | Ymin | X | Ymax |
|---|----------|----------|-------------|------|---|------|
| 5 | F (8, 7) | G (5, 5) | $3/2 = 1.5$ | 5 | 5 | 7 |

| Scan line | x-intersection |
|-----------|------------------------|
| y = 5 | 5 |
| y = 6 | $5 + 1.5 = 6.5 \sim 7$ |
| y = 7 | 8 |

Edge number 6

| # | Edge | | 1/m | Ymin | X | Ymax |
|---|----------|-----------|---------------|------|---|------|
| 6 | G (5, 5) | A' (4, 6) | $-3/2 = -1.5$ | 5 | 5 | 6 |

| Scan line | x-intersection |
|-----------|----------------|
| y = 5 | 5 |
| y = 6 | 4 |

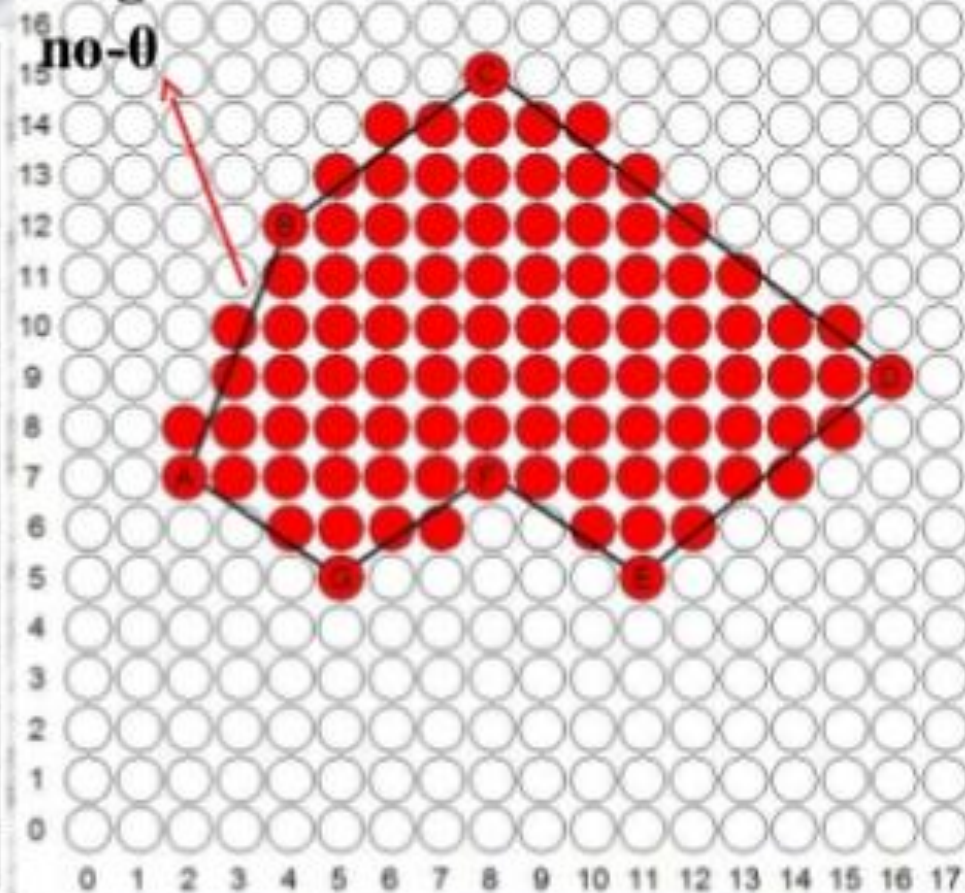
Correction: A'(3,6)

Scan Line Fill Algorithms

(Example)

| Scan line | x-intersections | | | | | | | x-intersections pair Ascending order |
|-----------|-----------------|---|----|----|----|---|---|---|
| | Edge# | | | | | | | |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | |
| 5 | | | | 11 | 11 | 5 | 5 | (5, 5), (11, 11) |
| 6 | | | | 12 | 10 | 7 | 4 | (4, 7), (10, 12) |
| 7 | 2 | | | 14 | 8 | 8 | | (2, 8), (8, 14) |
| 8 | 2 | | | 15 | | | | (2, 15) |
| 9 | 3 | | 16 | | | | | (3, 16) |
| 10 | 3 | | 15 | | | | | (3, 15) |
| 11 | 4 | | 13 | | | | | (4, 13) |
| 12 | | 4 | 12 | | | | | (4, 12) |
| 13 | | 4 | 11 | | | | | (4, 11) |
| 14 | | 6 | 10 | | | | | (6, 10) |
| 15 | | 8 | 8 | | | | | (8, 8) |

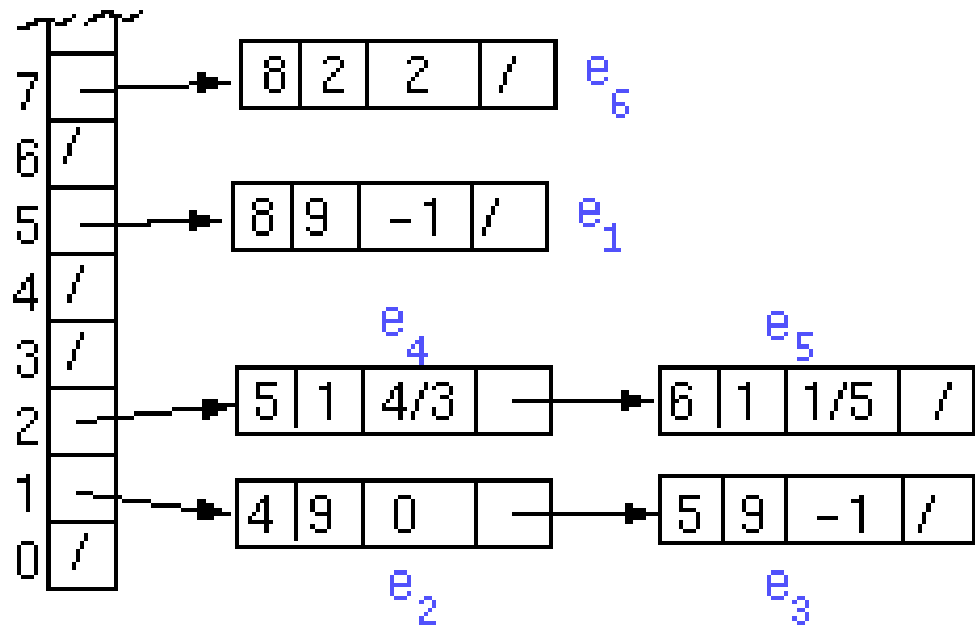
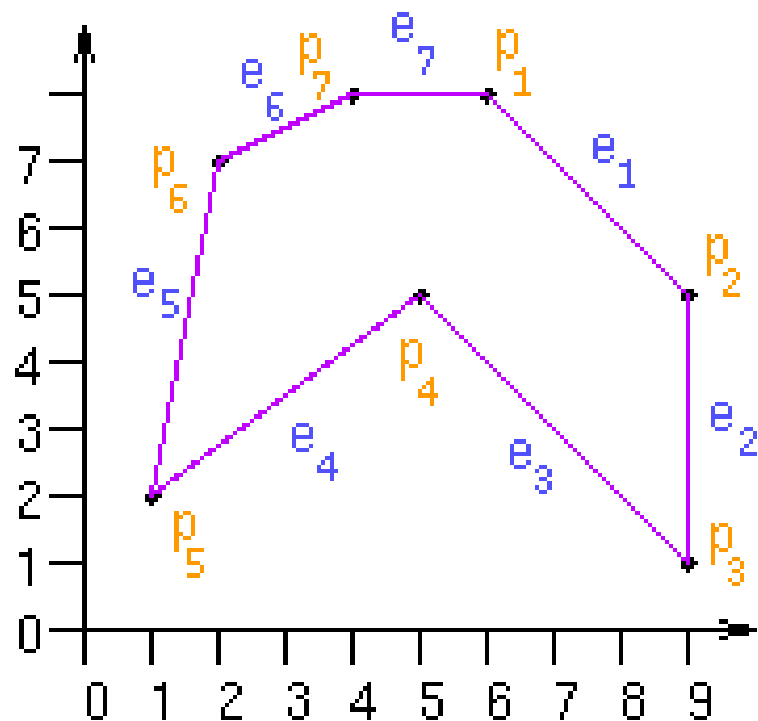
Edge
no-0



Scan Line Fill Algorithms: AEL

- Process the scan lines from bottom to top to construct Active Edge Table during scan conversion.
- Maintain an active edge list for the current scan-line.
- When the current scan line reaches the lower / upper endpoint of an edge it becomes active.
- When the current scan line moves above the upper / below the lower endpoint, the edge becomes inactive
- Use iterative coherence calculations to obtain edge intersections quickly.
- AEL is a linked list of active edges on the current scanline, y .
 - Each active edge line has the following information
 - y_{upper} : last scanline to consider
 - x_{lower} : edge's intersection with current y
 - $1/m$: x increment
 - The active edges are kept sorted by x

Scan Line Fill Algorithms: AEL



Scan Line Polygon Fill Algorithm

1. Set y to the smallest y coordinate that has an entry in the ET; *i.e.*, y for the first nonempty bucket.
2. Initialize the AET to be empty.
3. Repeat until the AET and ET are empty:
 1. Move from ET bucket y to the AET those edges whose $y_{min} = y$ (entering edges).
 2. Remove from the AET those entries for which $y = y_{max}$ (edges not involved in the next scanline), then sort the AET on x (made easier because ET is presorted).
 3. Fill in desired pixel values on scanline y by using pairs of x coordinates from AET.
 4. Increment y by 1 (to the coordinate of the next scanline).
 5. For each nonvertical edge remaining in the AET, update x for the new y .

Inside-Outside Test

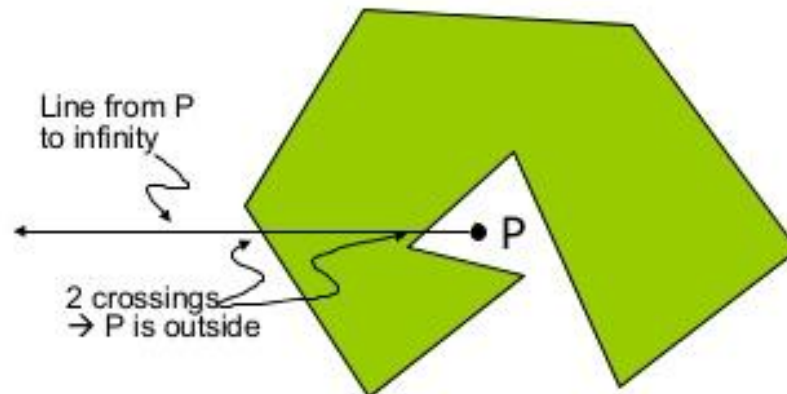
- To determine whether a point on the scan line lies inside or outside the polygon
- Mostly used to identify a point in the hollow polygon
- Two Methods:
 1. Even-Odd Rule, Odd-Even Rule, Odd Parity Rule
 2. Nonzero Winding number rule

Inside-Outside Test

Even-Odd Parity Rule

Inside-outside test for a point P:

1. Draw line from P to infinity
 - Any direction
 - Does not go through any vertex
2. Count the number of times the line crosses an edge
 - If the number of crossings is odd, P is inside
 - If the number of crossings is even, P is outside

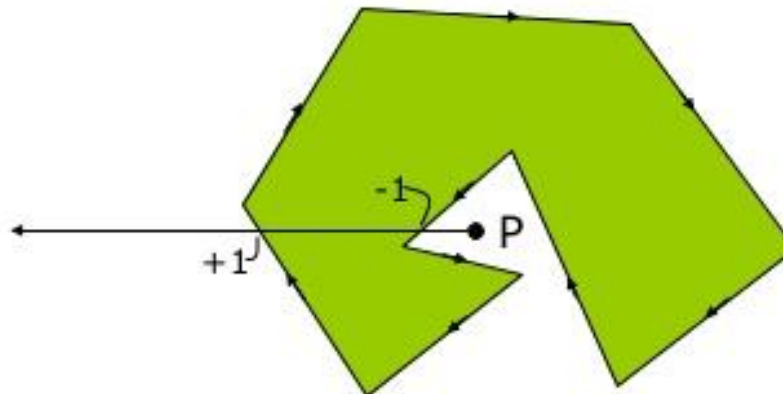


Inside-Outside Test

Non-Zero Winding Number Rule

Inside-outside test:

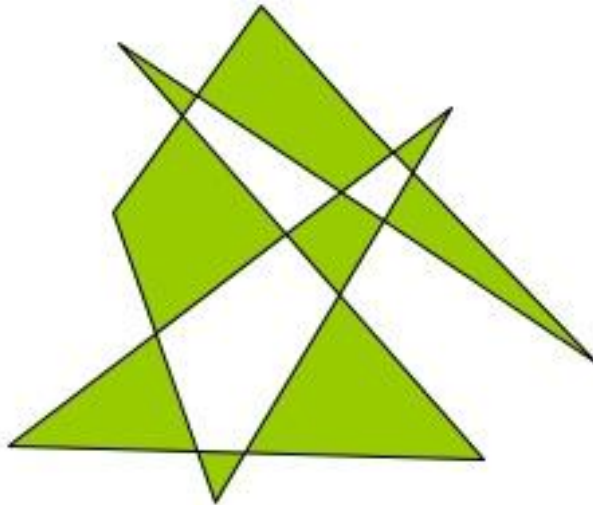
1. Determine the **winding number** W of P
 - a. Initialize W to zero and draw a line from P to infinity
 - b. If the line crosses an edge directed from bottom to top, $W++$
 - c. If the line crosses an edge directed from top to bottom, $W--$
2. If the $W = 0$, P is outside
3. Otherwise, P is inside



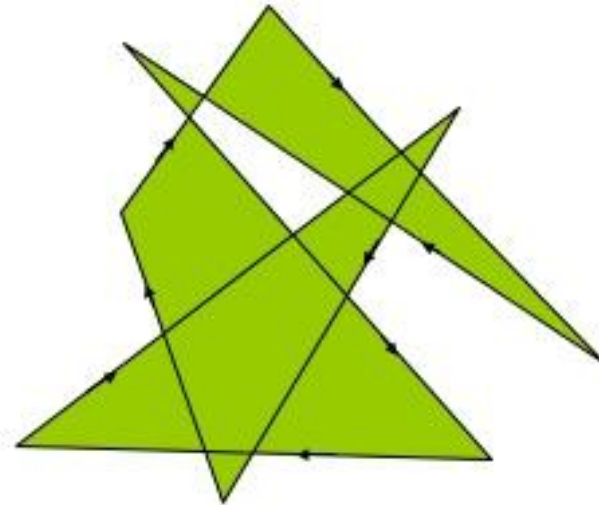
Inside-Outside Test

General polygons

Can be self intersecting
Can have interior holes



Even-odd parity



Non-zero winding

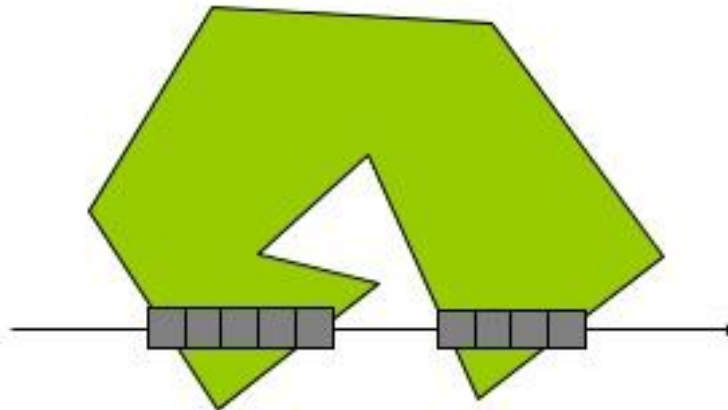
The non-zero winding number rule and the even-odd parity rule can give different results for general polygons

Inside-Outside Test

Raster-Based Filling

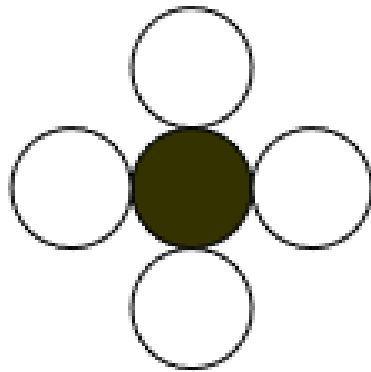
For each scan line

- Determine points where the scan line intersects the polygon
- Set pixels between intersection points (using a fill rule)
 - Even-odd parity rule: set pixels between pairs of intersections
 - Non-zero winding rule: set pixels according to the winding number

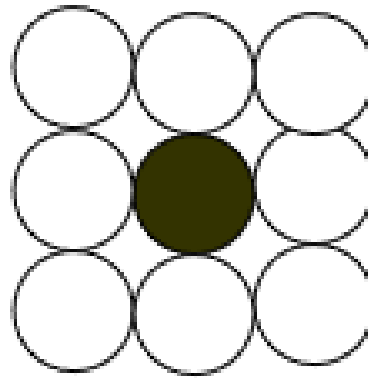


Boundary Fill Algorithms

- To start from an interior point and paint the interior outward toward the boundary
- If the boundary is specified by a single color, the fill algorithm processed outward pixel by pixel until the boundary color is encountered
- A boundary fill procedure accepts an interior point (x,y) , a fill color, and a boundary color
- Based on connectivity the filling is of two types



4- Connected



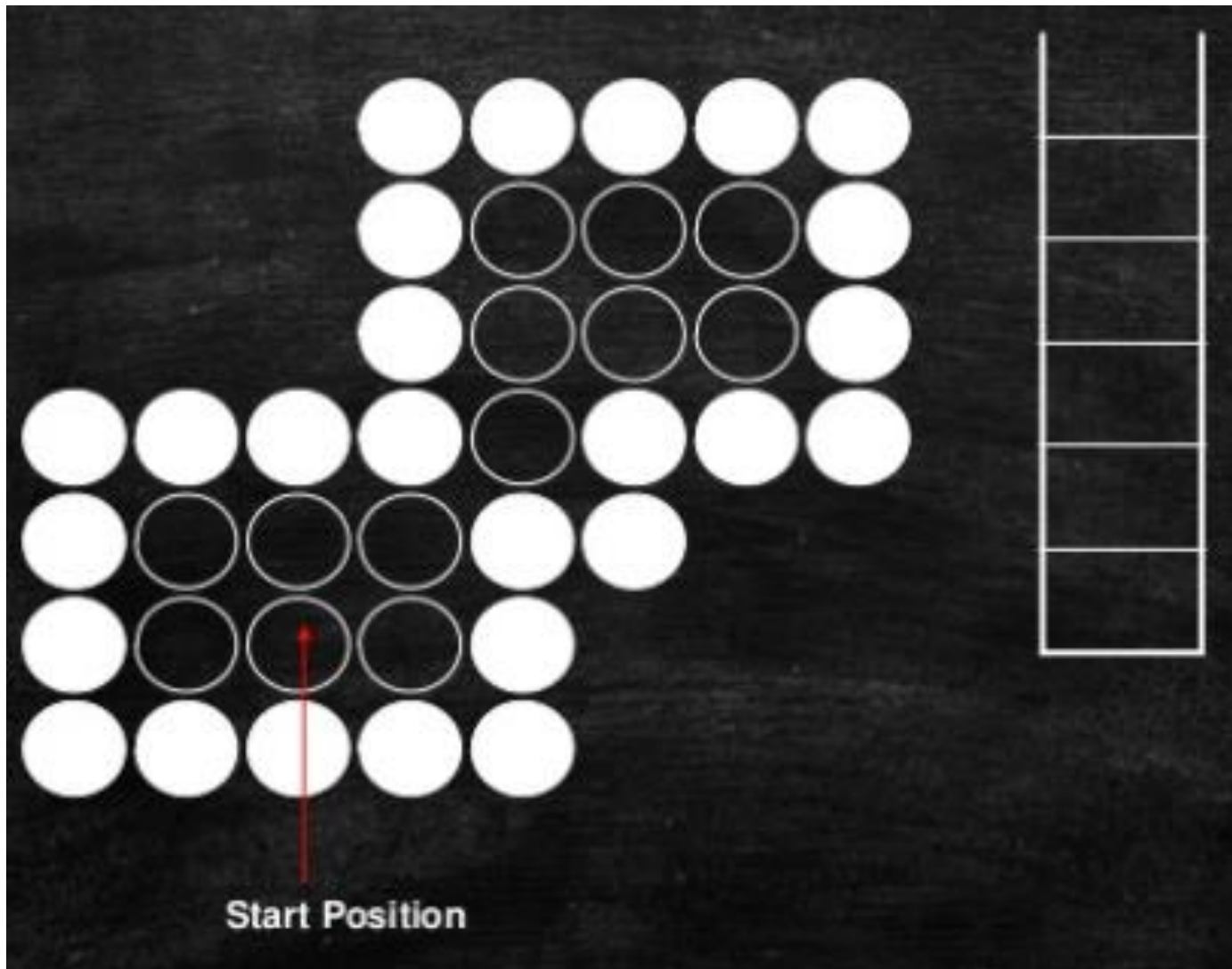
8- Connected

Boundary Fill Algorithms

```
void boundaryFill(int x, int y,  
                  int fillColor, int borderColor)  
{  
    getPixel(x, y, color);  
    if ((color != borderColor)  
        && (color != fillColor)) {  
        setPixel(x,y);  
        boundaryFill(x+1,y,fillColor,borderColor);  
        boundaryFill(x-1,y,fillColor,borderColor);  
        boundaryFill(x,y+1,fillColor,borderColor);  
        boundaryFill(x,y-1,fillColor,borderColor);  
    }  
}
```

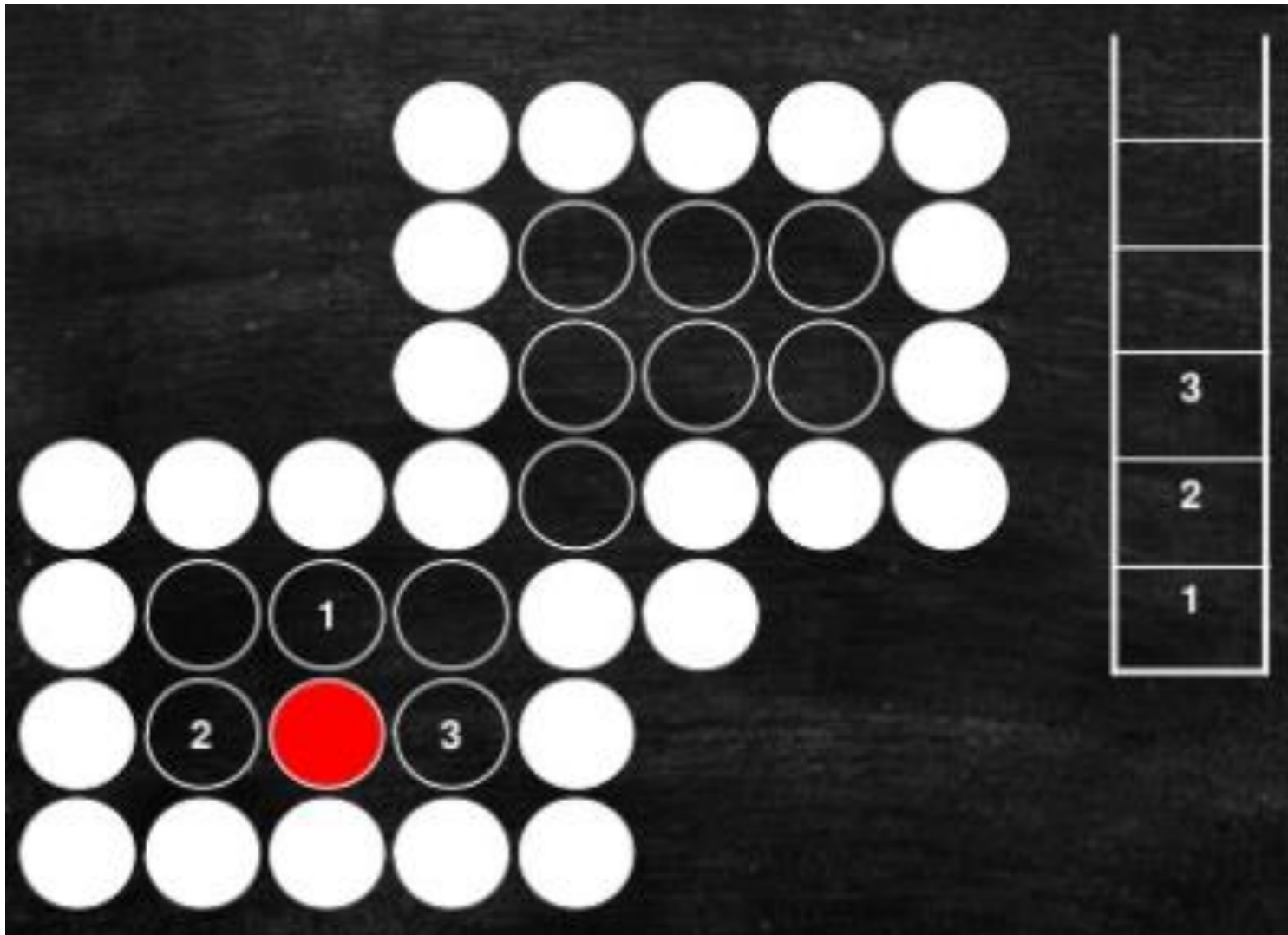
Boundary Fill Algorithms

Example: 4-connected



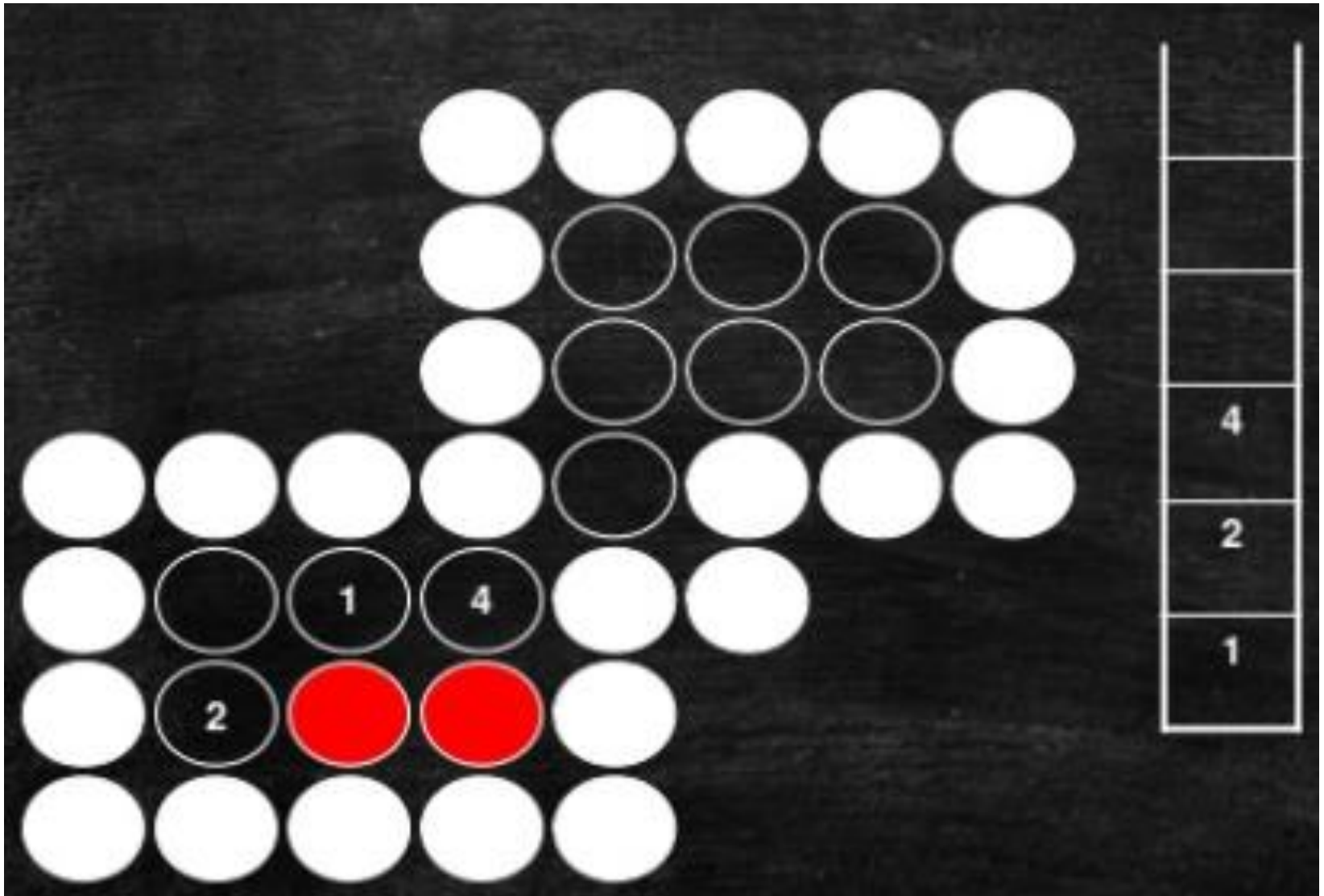
Boundary Fill Algorithms

Example: 4-connected

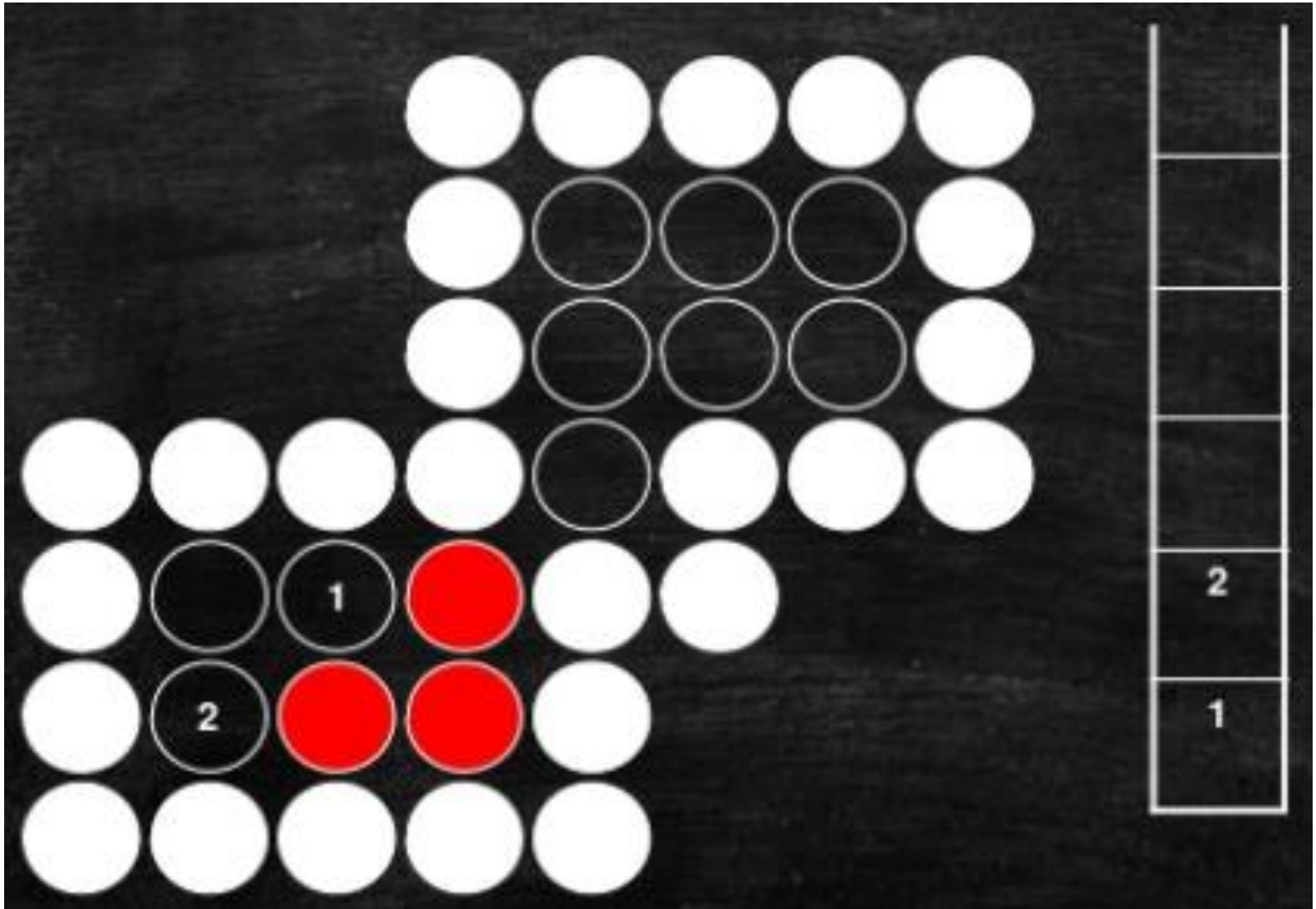


Boundary Fill Algorithms

Example: 4-connected

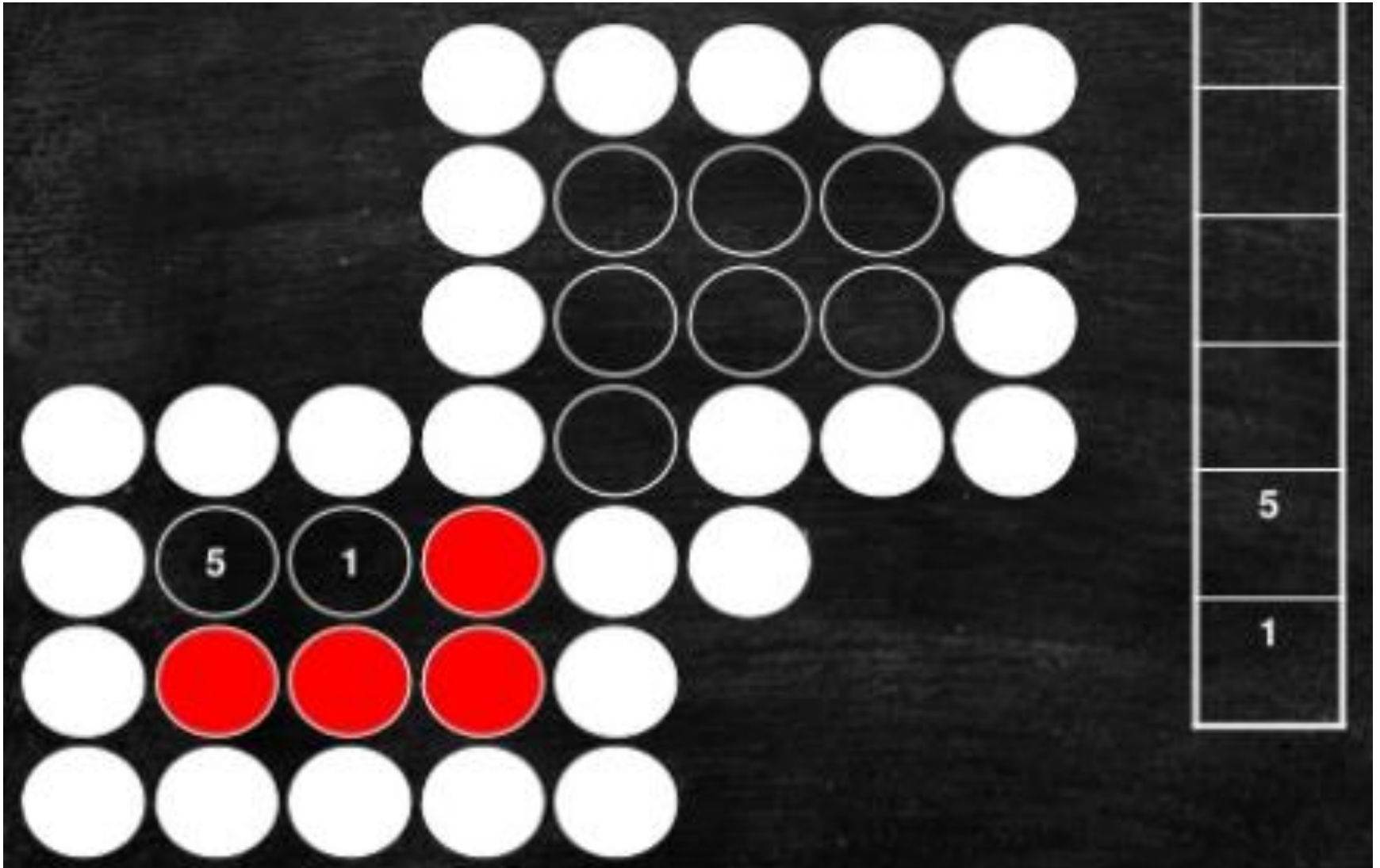


Example: 4-connected



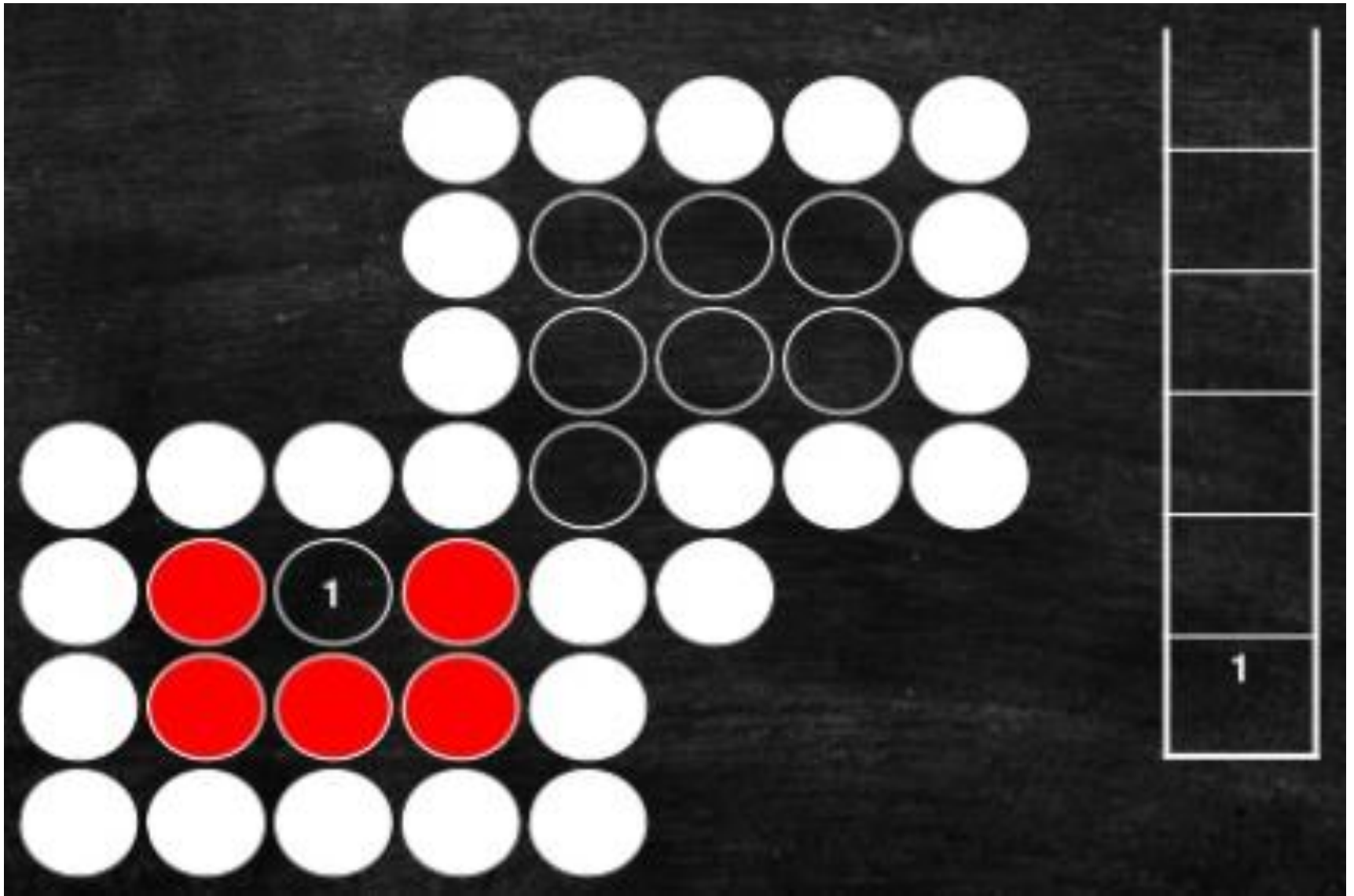
Boundary Fill Algorithms

Example: 4-connected

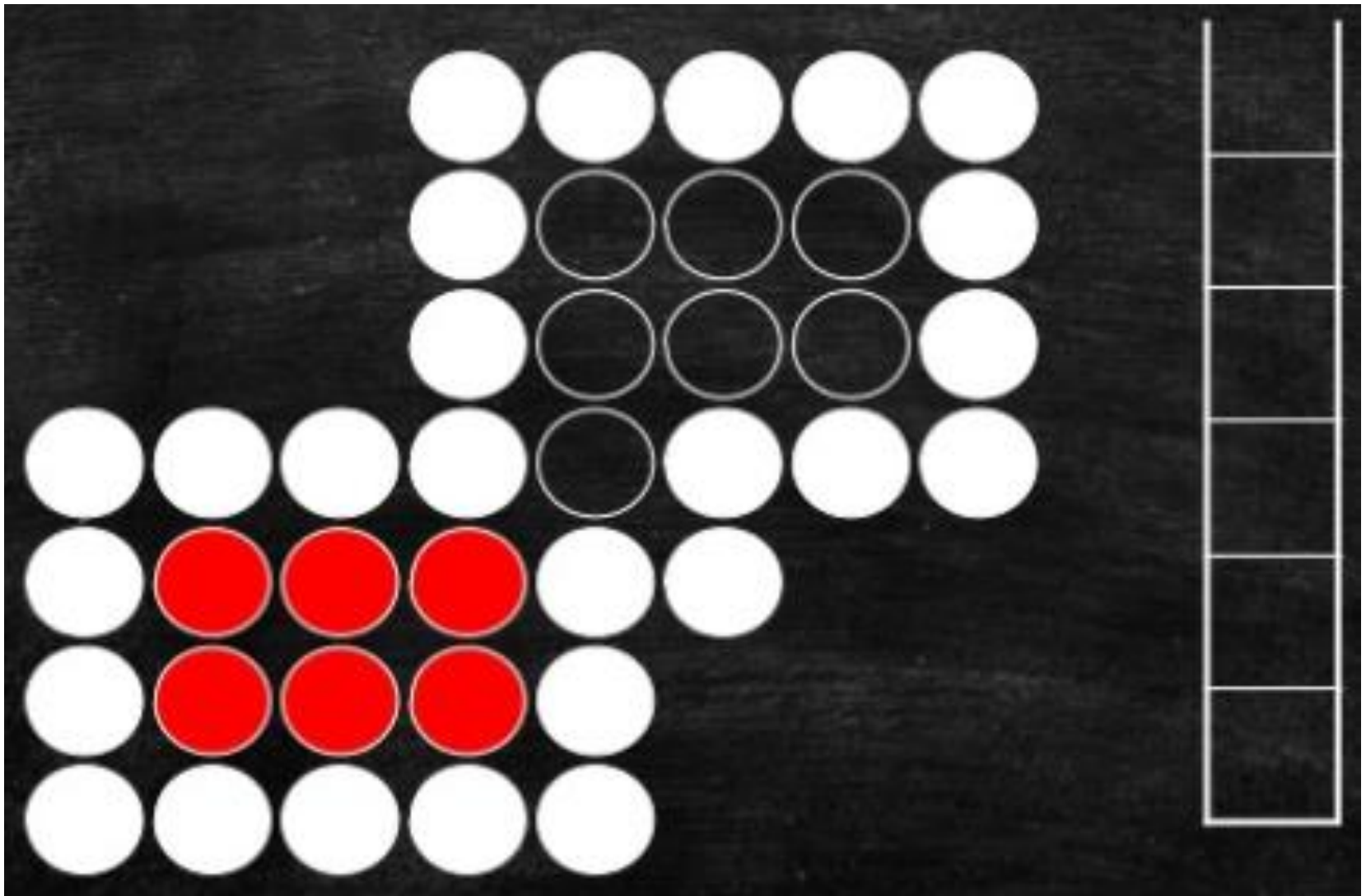


Boundary Fill Algorithms

Example: 4-connected

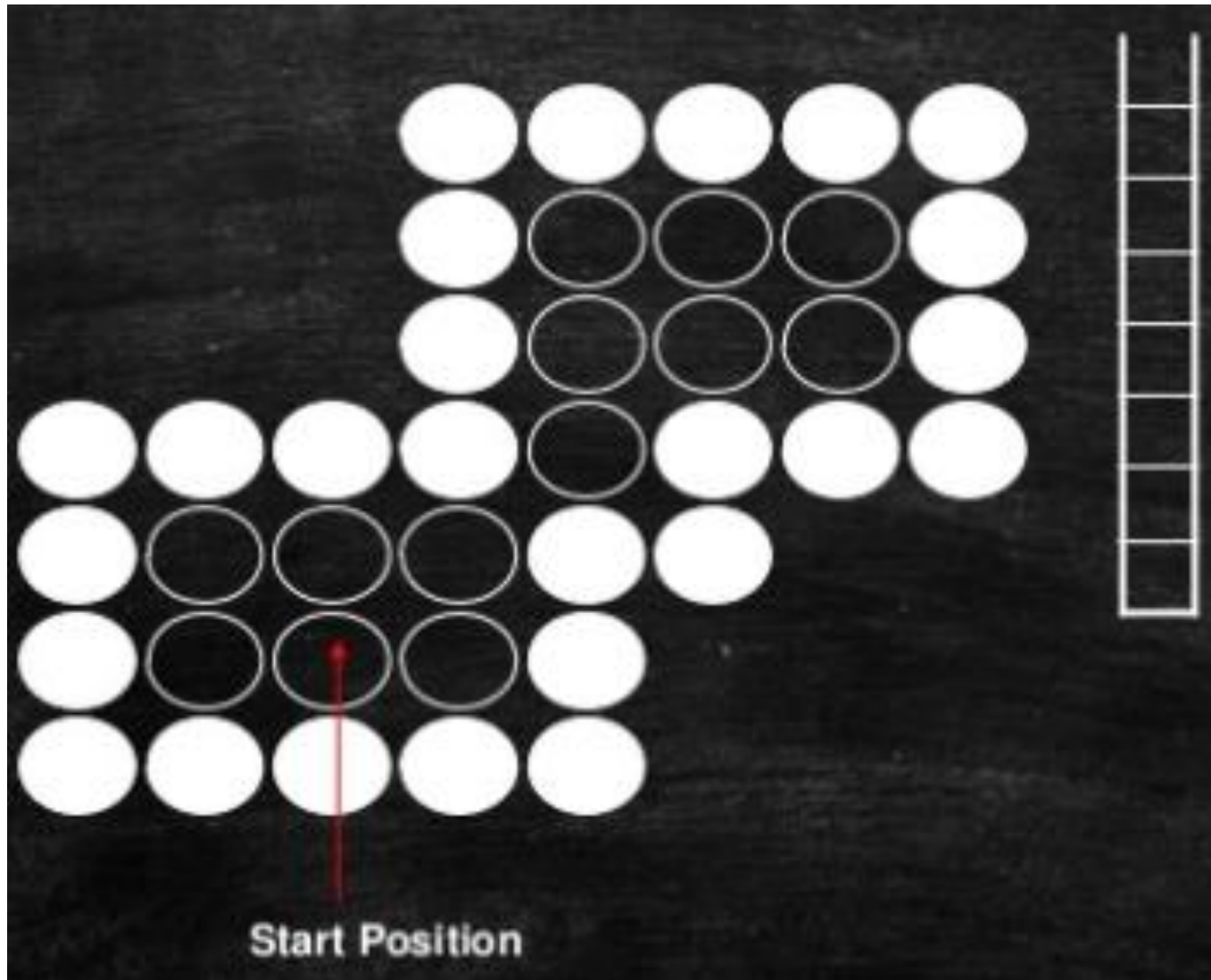


Example: 4-connected



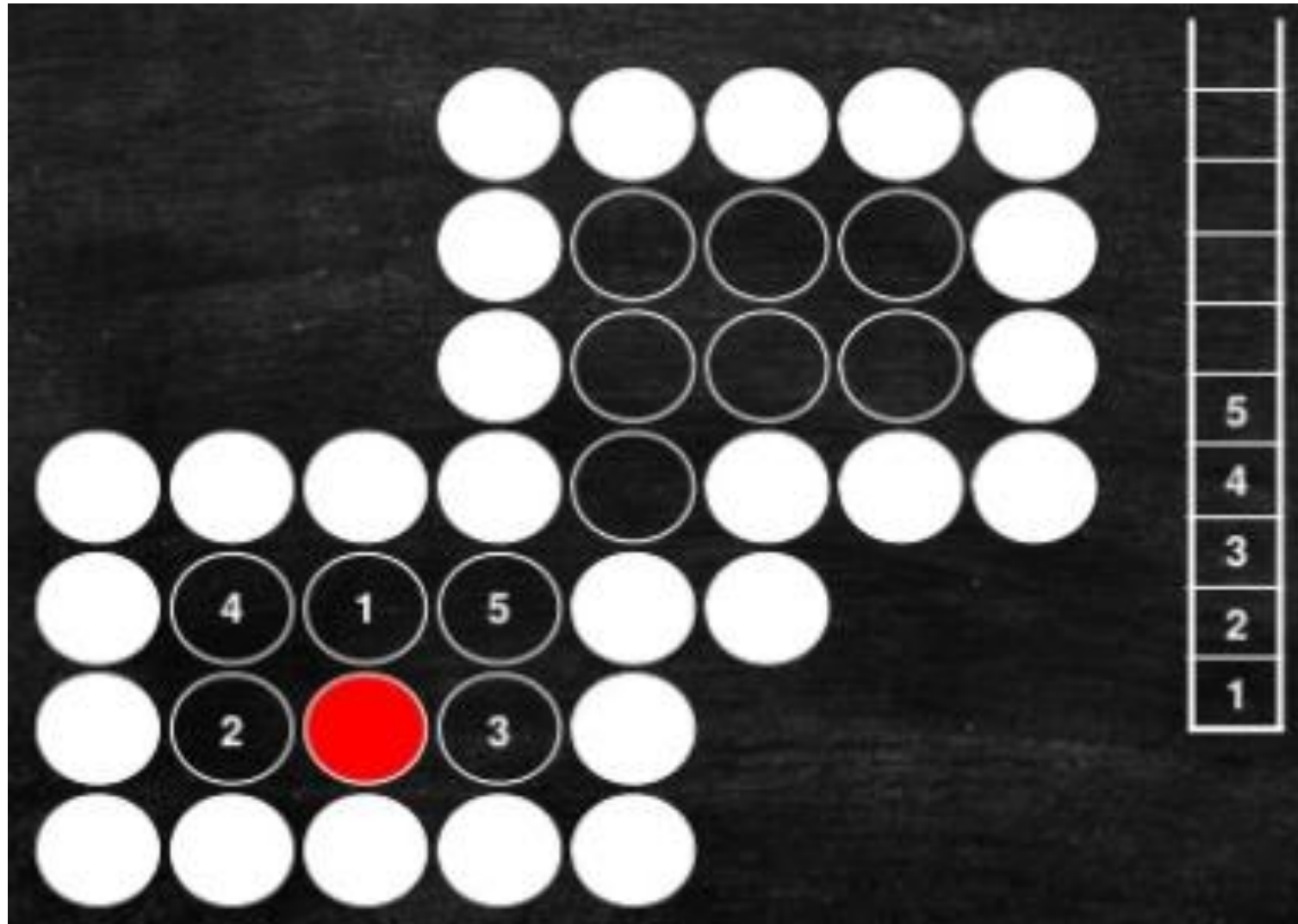
Boundary Fill Algorithms

Example: 8-connected



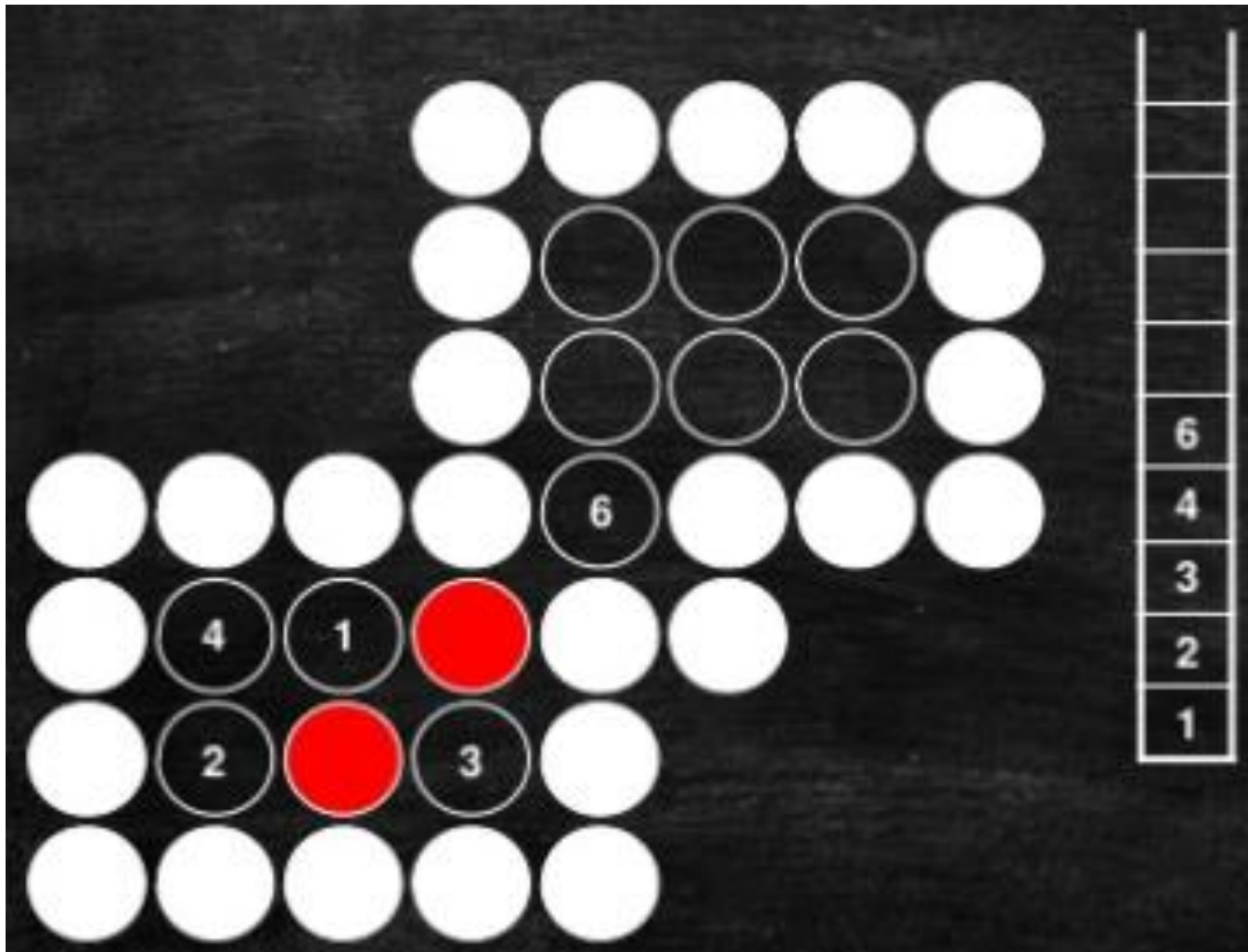
Boundary Fill Algorithms

Example: 8-connected



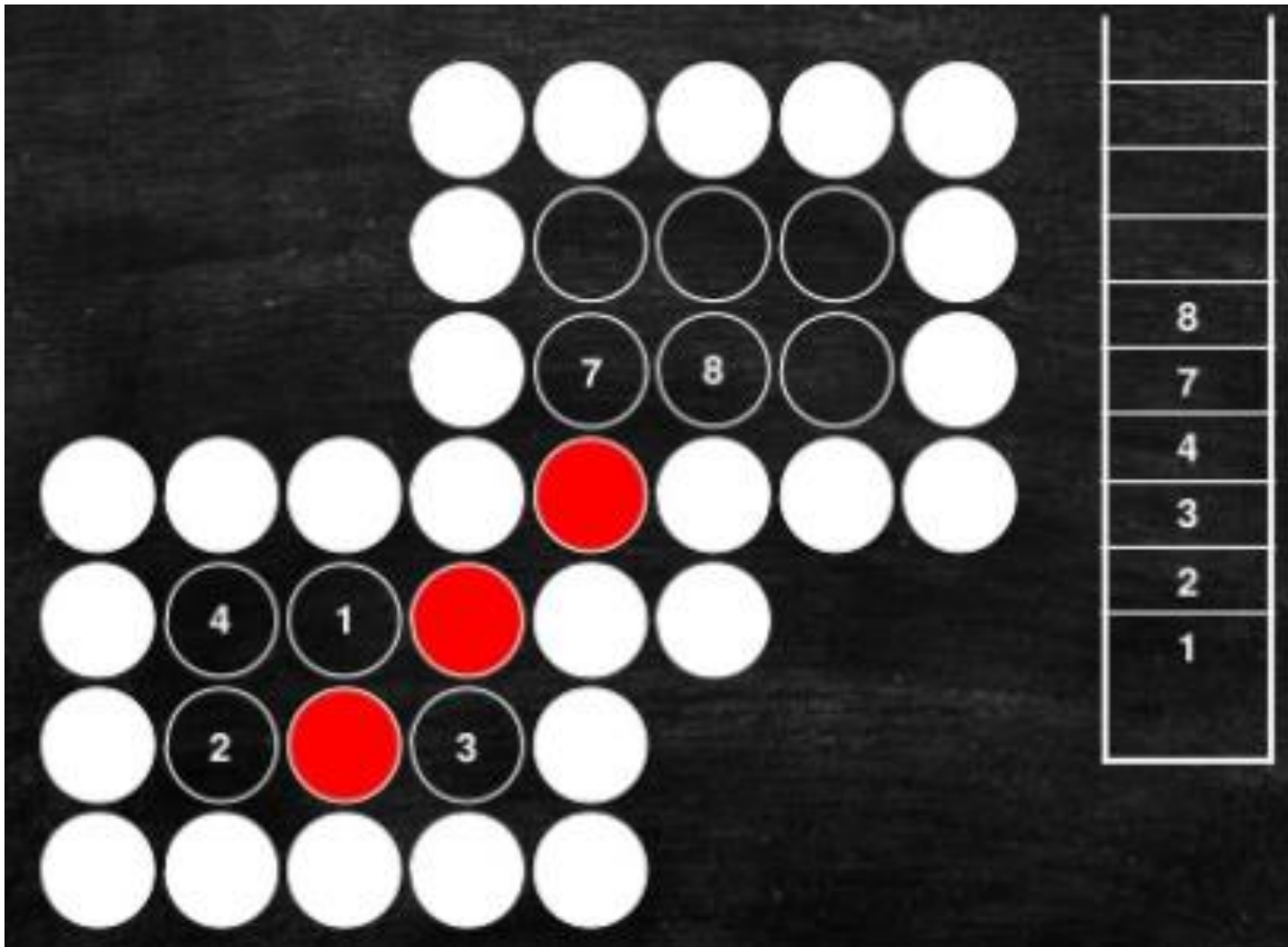
Boundary Fill Algorithms

Example: 8-connected



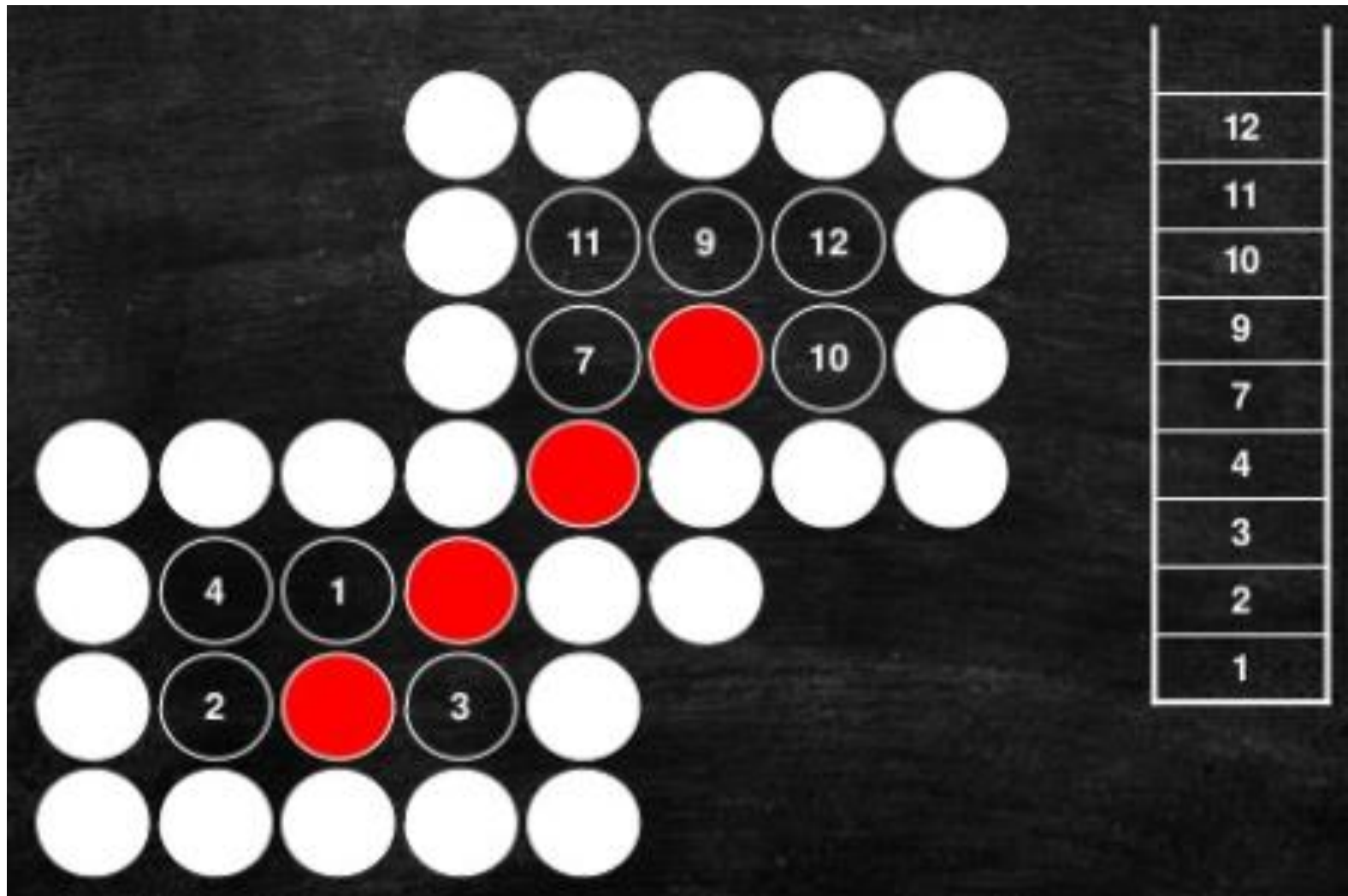
Boundary Fill Algorithms

Example: 8-connected



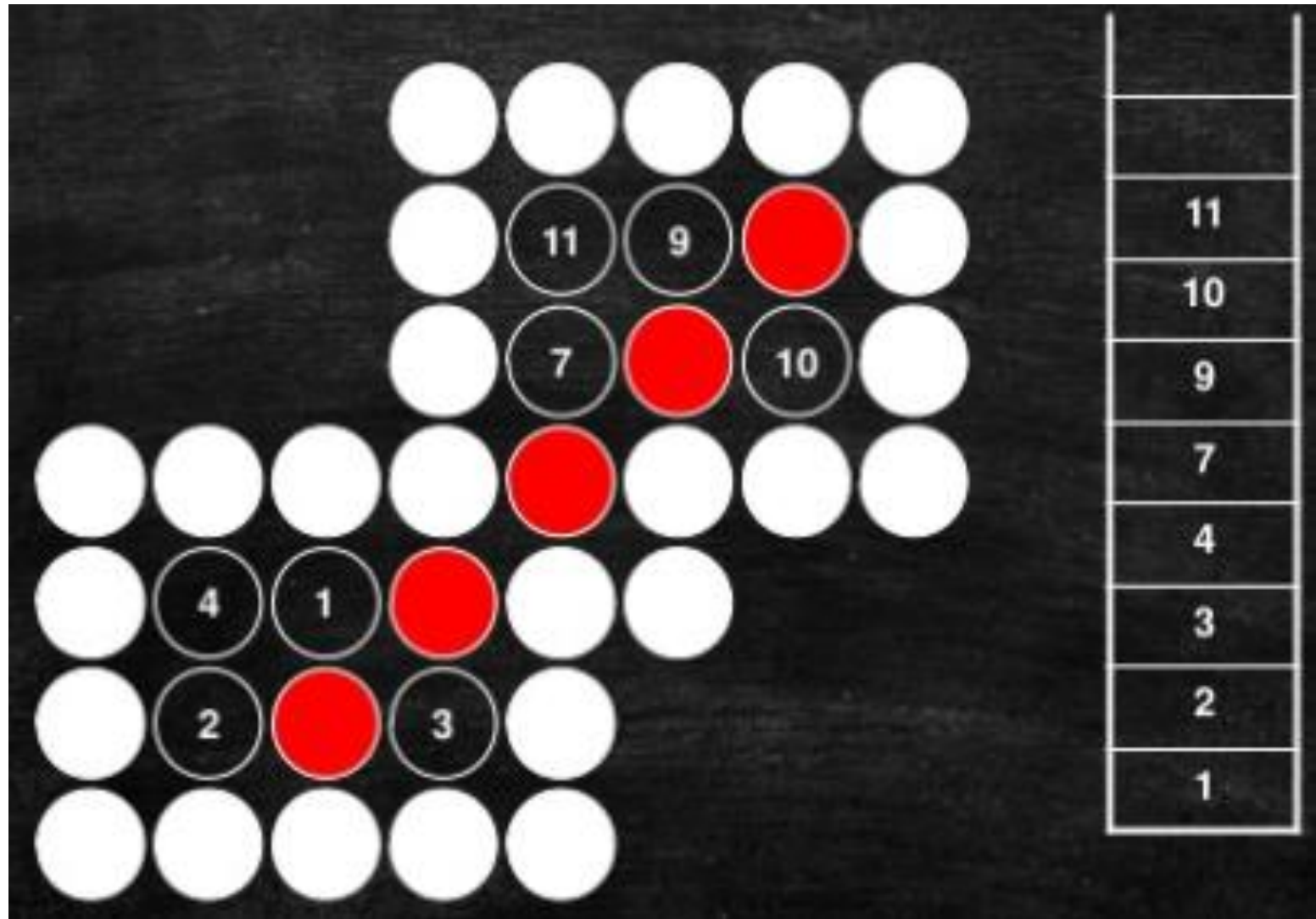
Boundary Fill Algorithms

Example: 8-connected



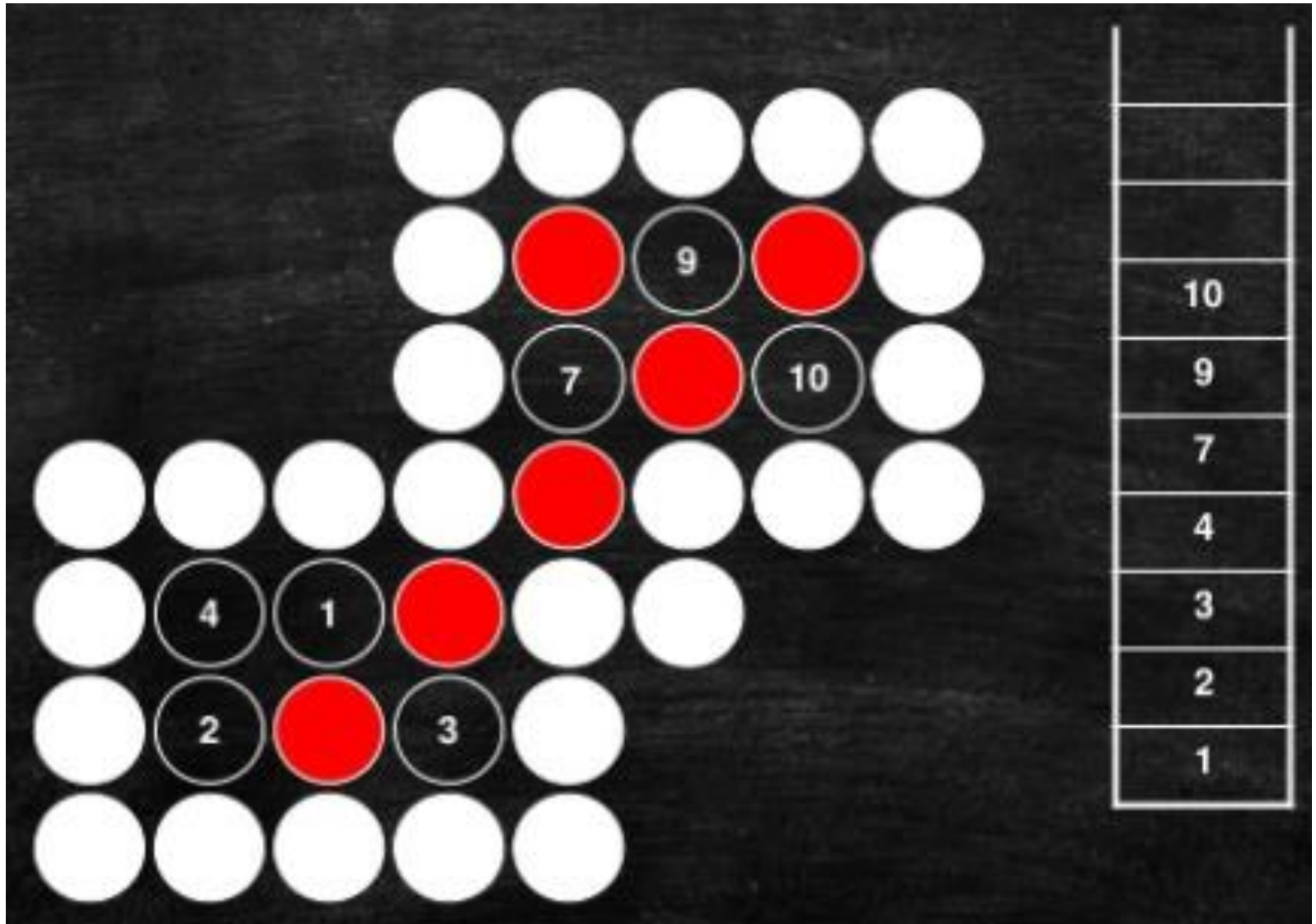
Boundary Fill Algorithms

Example: 8-connected



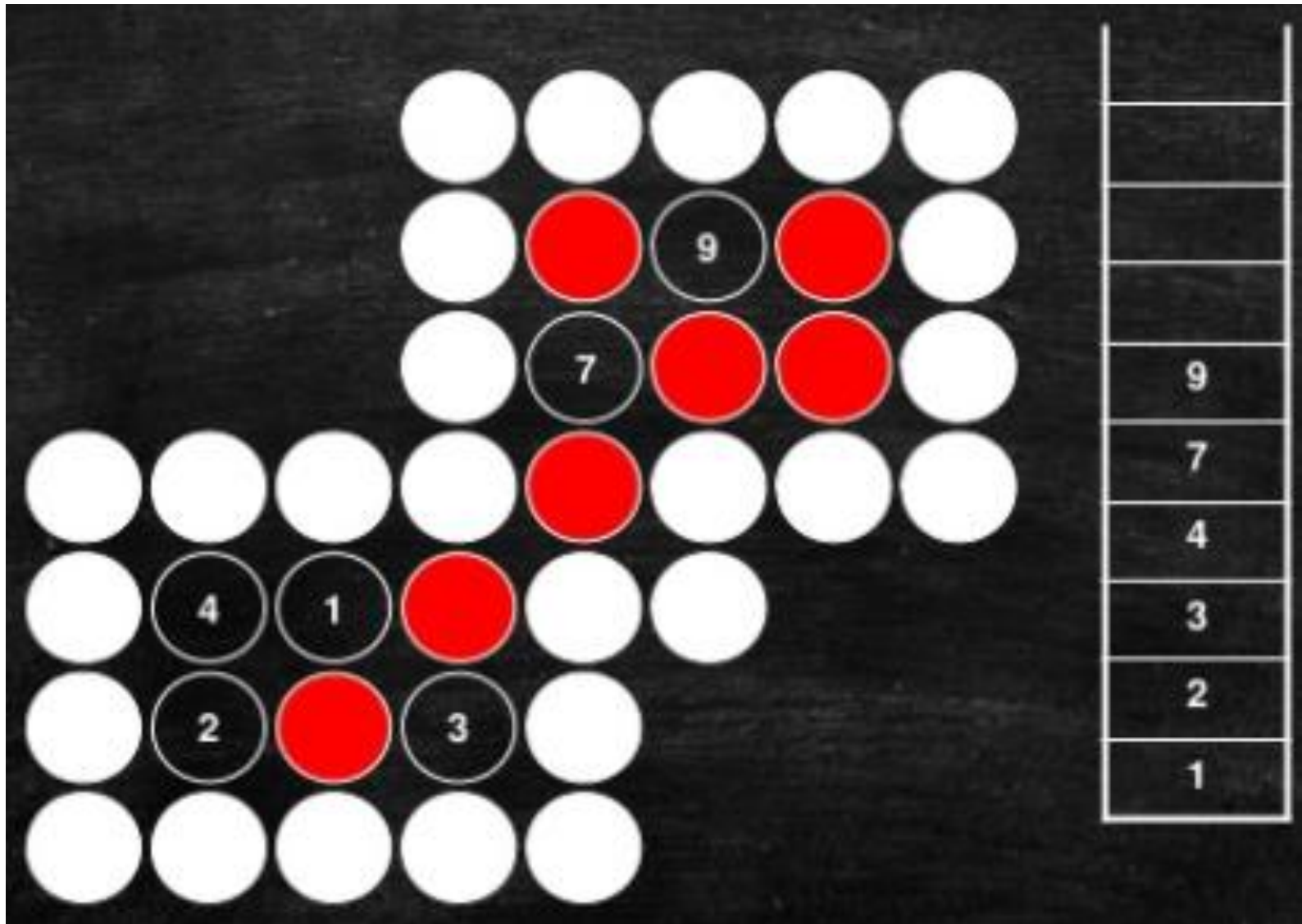
Boundary Fill Algorithms

Example: 8-connected



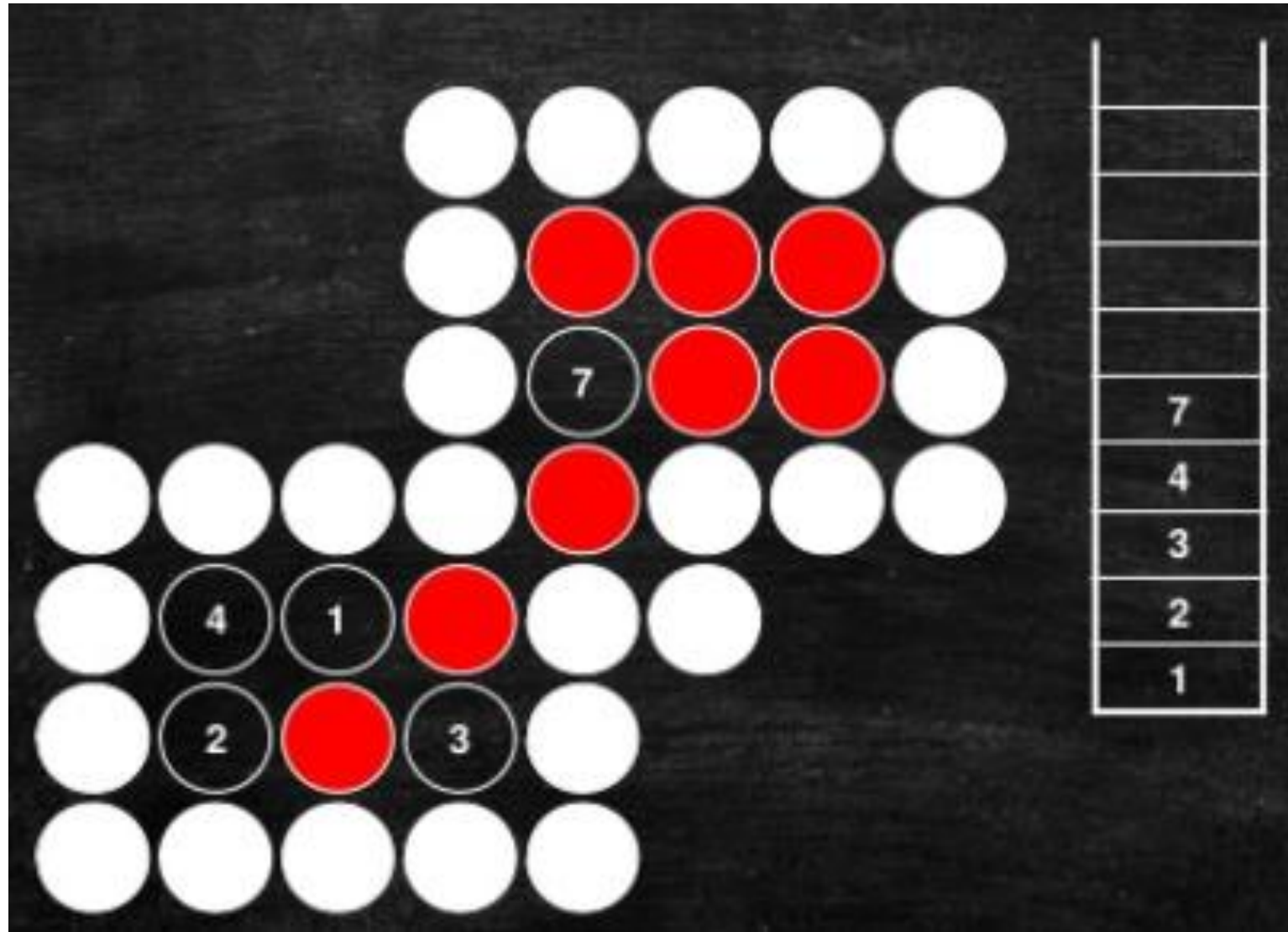
Boundary Fill Algorithms

Example: 8-connected



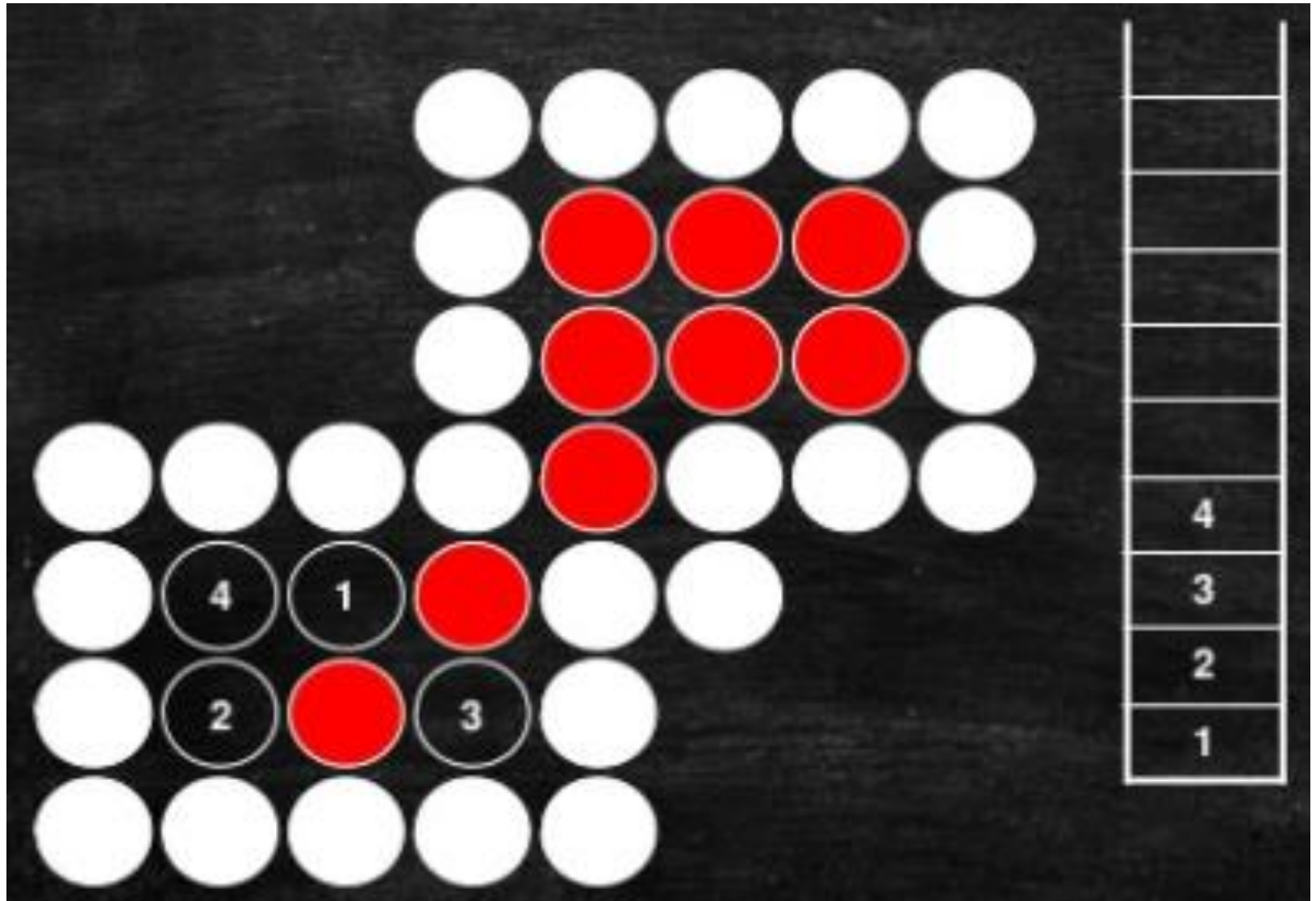
Boundary Fill Algorithms

Example: 8-connected



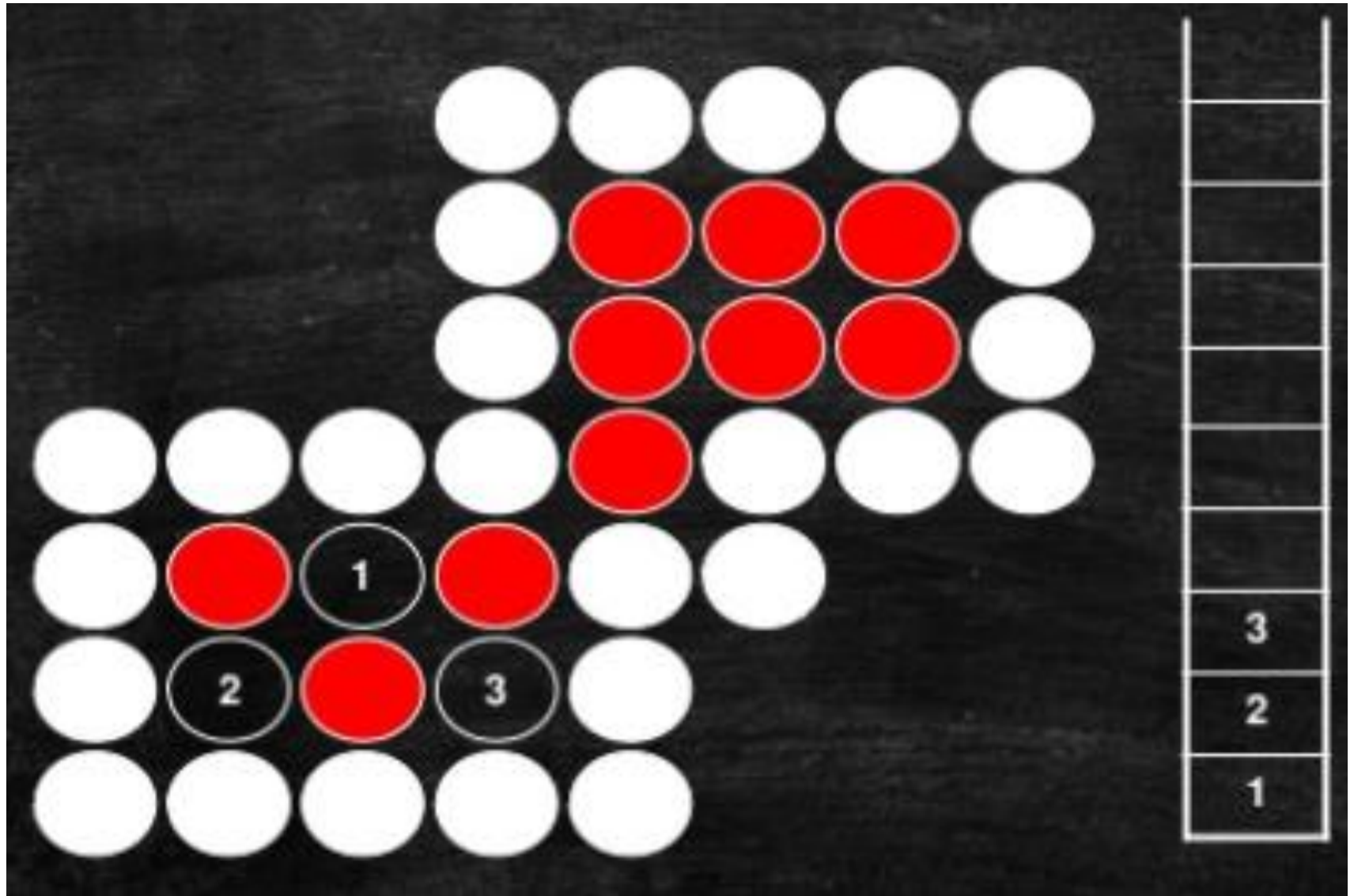
Boundary Fill Algorithms

Example: 8-connected



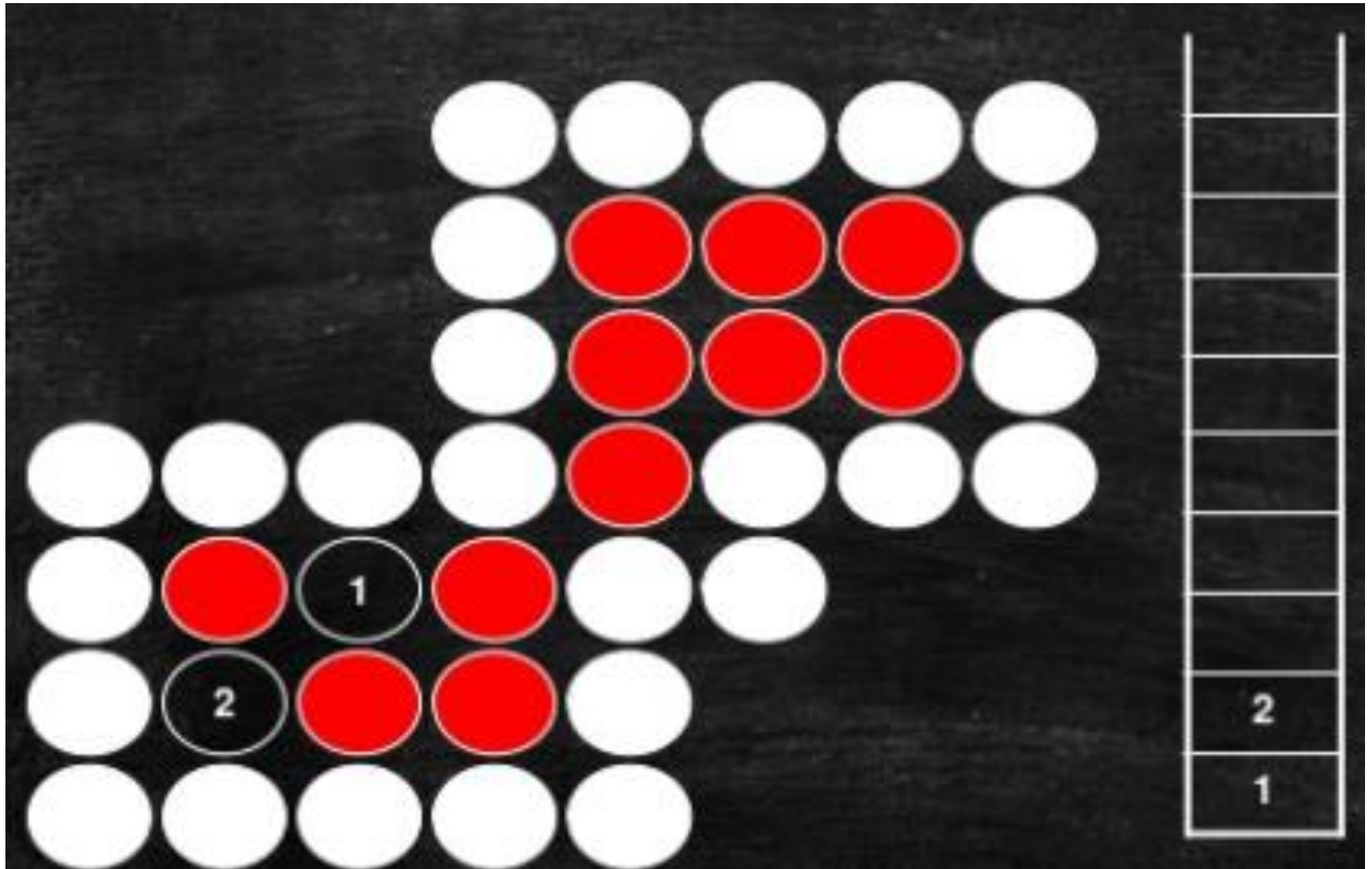
Boundary Fill Algorithms

Example: 8-connected

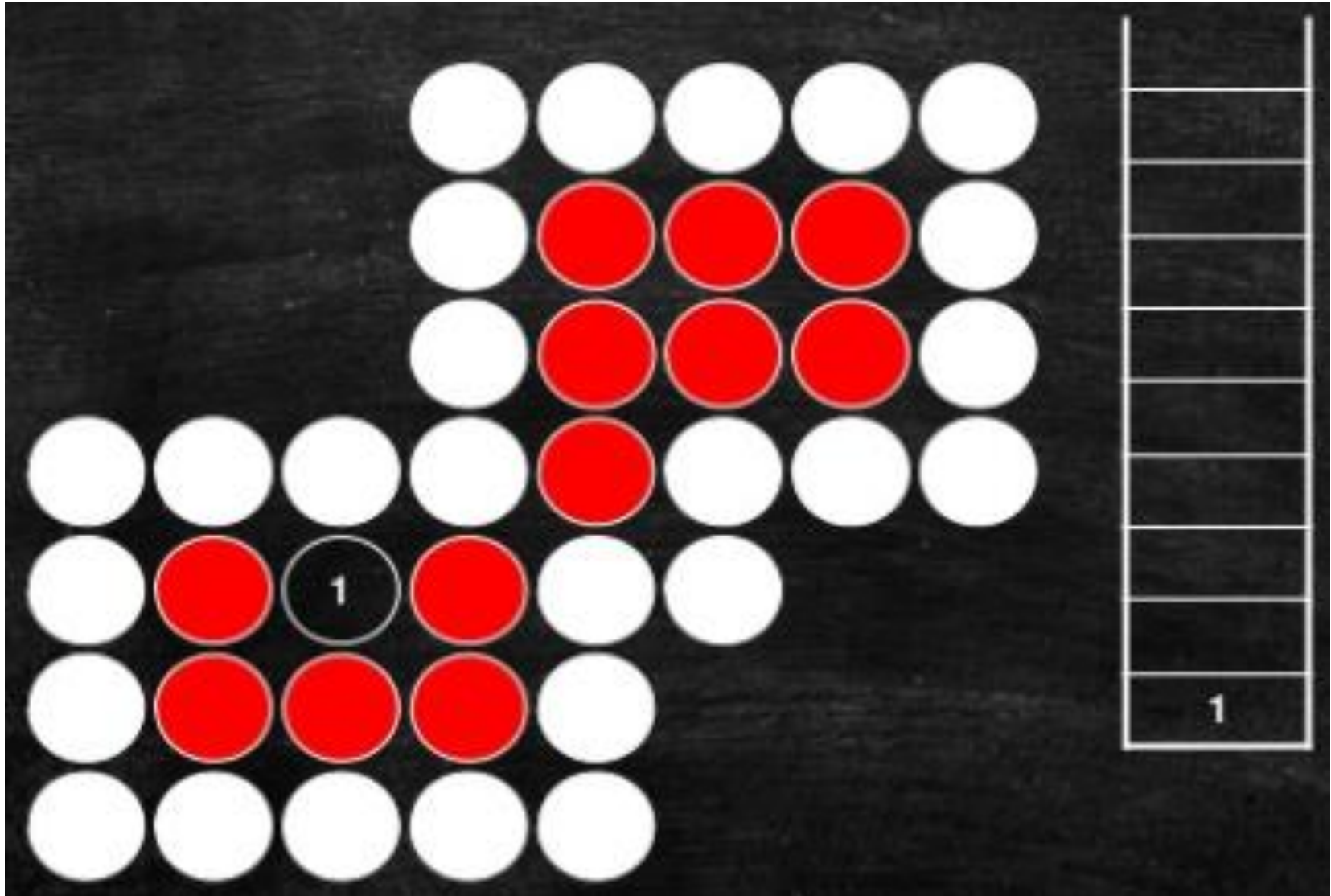


Boundary Fill Algorithms

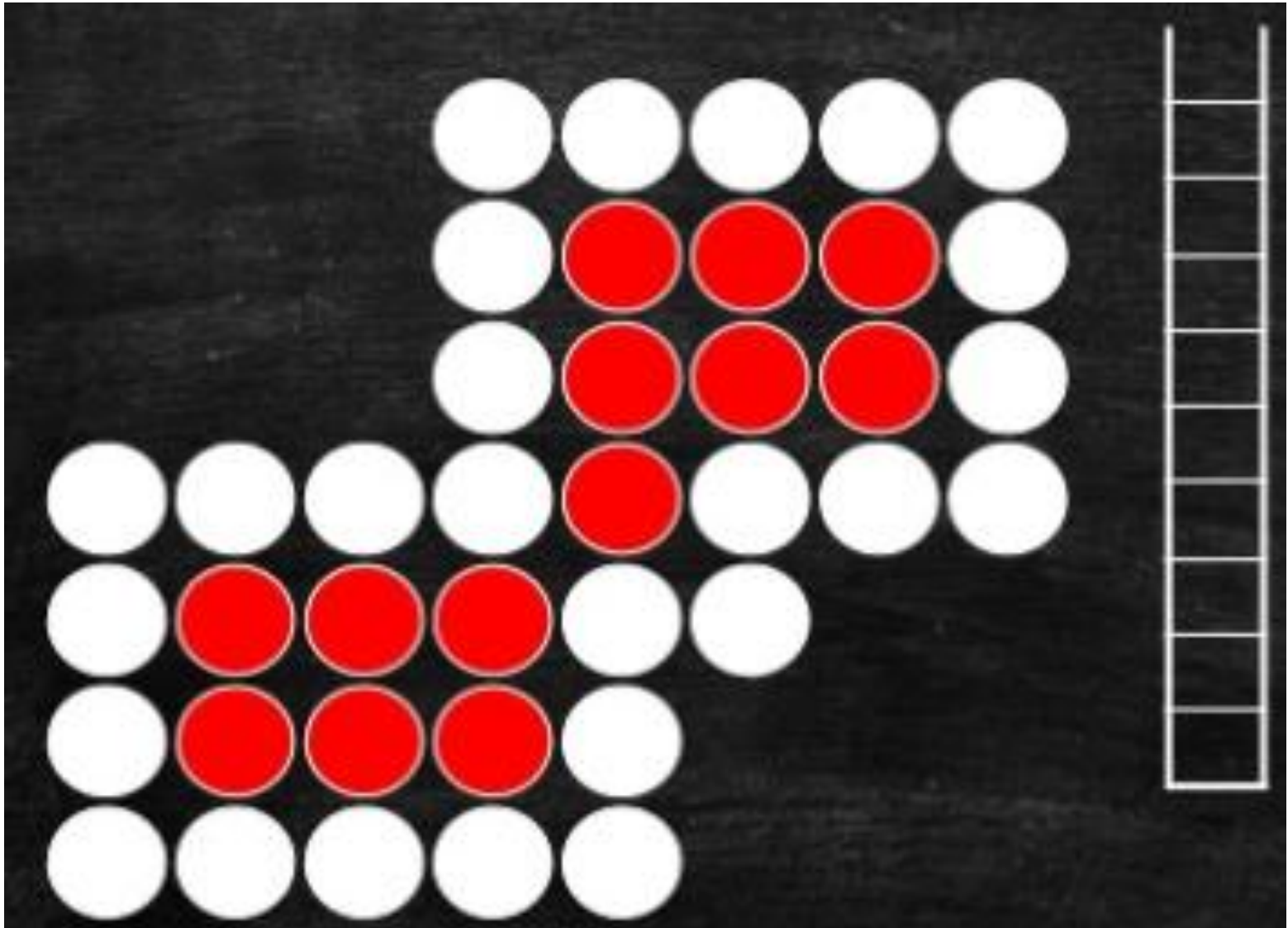
Example: 8-connected



Example: 8-connected



Example: 8-connected



Flood Fill Algorithms

- To fill an area or recolor it whose boundary is not defined by a single color
- Paint by replacing color instead of checking for boundary color
- If more than one interior color, first reassign to a single color
- 4-connected or 8-connected approach

```
void floodFill4 (int x, int y, int fillColor, int oldColor)
{
    if (getPixel (x, y) == oldColor) {
        setColor (fillColor);
        setPixel (x, y);
        floodFill4 (x+1, y, fillColor, oldColor);
        floodFill4 (x-1, y, fillColor, oldColor);
        floodFill4 (x, y+1, fillColor, oldColor);
        floodFill4 (x, y-1, fillColor, oldColor);
    }
}
```

Flood Fill Algorithms

