Lecture-13
Sta

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Charne's Big M method / Penalty method

General minimization problem

$$\min Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

$$\text{s.t.,} \quad a_{11}x_1 + \dots + a_{1n}x_n \geq b_1$$

$$\vdots$$

$$a_{m1}x_1 + \dots + a_{mn}x_n \geq b_m$$

$$x_1, \dots, x_n \geq 0$$

Standard form

(surplus, artificial variable)

$$a_{11}x_1 + \dots + a_{1n}x_n - s_1 + A_1 = b_1$$

$$-s_2 + A_2 = b_2$$

$$\vdots$$

$$a_{m1}x_1 + \dots + a_{mn}x_n - s_m + A_m = b_m$$

$$\min Z = c_1x_1 + c_2x_2 + \dots + c_nx_n + 0.s_1 + \dots + 0.s_m$$

$$s_i \geq 0$$

IBFS :

$$x_1 = \dots = x_n = 0$$

$$s_i = -b_i \quad \text{but } s_i \geq 0 \rightarrow \text{problem!}$$

$\therefore$ include an artificial variable

$n+m$ variables] → only m will be basic variables
m equations

Now, $x_1 = \dots = x_n = 0$

$s_1 = \dots = s_m = 0$

$\therefore A_1 = \dots = A_m \neq 0$

$A_i = b_i$

$\therefore$ now, $\min Z = c_1x_1 + c_2x_2 + \dots + c_nx_n + 0.s_1 + \dots + 0.s_m$
$+ MA_1 + \dots + MA_m$

M ↓ a large no.

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• When we consider the initial IBFS,

$$x_1 = x_2 = \dots = x_n = 0$$

which implies $s_i = -b_i$ which does not satisfy the non-negativity criteria to avoid this, we introduce artificial variables $A_1, A_2, \dots A_m$ such that

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n - s_i + A_i = b_i$$

where, $A_1, A_2, \dots A_m \geq 0$

• The system now has $n+2m$ variables with m equations. Hence there can be only m basic variables which means in a BFS $n+m$ variables should be 0.

• Thus we consider the IBFS as

$$x_1 = \dots = x_n = 0$$
$$s_1 = \dots = s_m = 0$$
$$A_1 = A_2 \dots = A_m \neq 0$$
$$A_i = b_i$$

Remarks:

• The artificial variable and the vectors associated with them have no physical significance and the goal is to remove these variables from BFS.

• This is achieved by assigning the price parameter to these variables in the following way:

i) ~~Assign~~ Assign a very high negative price, $-M$ if it is a maximization problem or a very high positive price, $+M$ if it is a minimization problem.

ii) A constraint with $\geq$ sign will always need a surplus and an artificial variable.

iii) A constraint with $=$ sign requires only artificial variables

If the optimal soln has an artificial variable as a basic variable, then there is no feasible solution to the LPP.

iv) This idea of artificial variables with large price is known as Charne's Big M method or the penalty method.

Q. Min $z = 4x_1 + 8x_2 + 3x_3$

s.t $x_1 + x_2 \geq 2$

$2x_1 + x_3 \geq 5$

$x_1, x_2 \geq 0$

Std. form

$x_1 + x_2 + 0.x_3 - S_1 + A_1 = 2$

$2x_1 + 0.x_2 + x_3 - S_2 + A_2 = 5$

min $z = 4x_1 + 8x_2 + 3x_3 + 0.S_1 + 0.S_2 + MA_1 + MA_2$

$x_1, x \geq 0 \quad S_1, S_2 \geq 0 \quad A_1, A_2 \geq 0$

min A is equiv. to ~~max~~ -A

$\therefore$ max $z^* = -4x_1 - 8x_2 - 3x_3 + 0.S_1 + 0.S_2 - MA_1 - MA_2$

IBFS : $x_1 = x_2 = x_3 = 0$

$S_1 = S_2 = 0$

$A_1 = 2, \ A_2 = 5$

$z_j - c_j$ corresponding to basic variables is always zero.

				c_j	-4	-8	-3	0	0	-M	-M	
	C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7	Ratio
	-M	a_6	A_1	2	1	1	0	-1	0	1	0	2
	-M	a_7	A_2	5	2	0	1	0	-1	0	1	2.5
			$z_j^* - c_j$		-3M+4	-M+8	-M+3	M	M	0	0	
	-4	a_1	x_1	2	1	1	0	-1	0	X	0	~~-2~~
$R_2 \leftarrow R_2 - 2R_1$	-M	a_7	A_2	1	0	-2	1	2	-1	X	1	1/2
			$z_j^* - c_j$		0	2M+4	-M+3	-2M+4	M	X	0	
② $R_1 \leftarrow R_1 + R_2$	-4	a_1	x_1	5/2	1	0	1/2	0	-1/2	X	X	
① $R_2 \leftarrow R_2/2$	0	a_4	S_1	1/2	0	-1	1/2	1	-1/2	X	X	
					0	8	1	0	2	X	X	

* once gone, we don't need to calculate again

→ All ≥ 0

$\therefore$ optimal sol^n

$\therefore x_1 = 5/2, \ S_1 = 1/2, \ x_2 = 0, \ x_3 = 0$

$(5/2, 0, 0)$ ✓

$\therefore Z_{min} = +10$

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