

## # Transportation problem:

- The transportation problem deals with the transportation of the products manufactured at different factories to different destinations.
- The objective is to satisfy the demand requirements, within the capacity constraints at a minimum transportation cost.
- A transportation problem is completely defined by the following table:

	destination						
	$D_0$	$D_1$	.....	$D_n$			
origin	$O_1$	$c_{11}$	$c_{12}$	.....	$c_{1n}$	$a_1$	$c_{ij}$ : Cost of transportation
	$O_2$	$c_{21}$	$c_{22}$	.....	$c_{2n}$	$a_2$	$a_i$ : availability
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$b_j$ : demand/requirement
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	$O_m$	$c_{m1}$	$c_{m2}$	.....	$c_{mn}$	$a_m$	
						demand	
		$b_1$	$b_2$	.....	$b_n$		

constraints

## Remarks:

- A transportation problem is always a minimization problem.
- The solution to the transportation problem is given by an  $m \times n$  matrix:

$$X = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ \vdots & \vdots & & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{pmatrix}_{m \times n} \quad x_{ij} \geq 0$$

where  $x_{ij} \geq 0$  is the quantity transported from origin  $O_i$  to destination  $D_j$ .

constraints:

$$\sum_{j=1}^n x_{ij} = a_i$$

$$\sum_{i=1}^m x_{ij} = b_j$$

For a balanced system,

$$\boxed{\text{total demand} = \text{total supply}}$$

$$\therefore \sum_{j=1}^n \sum_{i=1}^m a_{ij} = \sum_{j=1}^n b_j \equiv \sum_{i=1}^m a_i = \sum_{i=1}^m \sum_{j=1}^n x_{ij}$$

- The number of basic variables in a transportation problem is at most  $m+n-1$ .
- As it is a minimization problem, it can never have an unbounded sol<sup>n</sup>. It always has a feasible solution.

# Methods for finding ~~optimal~~ BFS:

	$D_1$	$D_2$	$D_3$	$D_4$	$a_i$ (supply)
$a_1$	6	5	16	25	11
$a_2$	17	5	18	4	13
$a_3$	32	27	4	15	19
$b_j$ (demand)	6	10	12	15	

cell (1,1) =  $\min(a_1, b_1)$

i) North-west corner method

$$x = \begin{pmatrix} 6 & 5 & 0 & 0 \\ 0 & 5 & 8 & 0 \\ 0 & 0 & 4 & 15 \end{pmatrix} \leftarrow \text{BFS}$$

6 of these need to be non-zero

$$\sum a_i = 43 = \sum b_j \Rightarrow \therefore \text{a balanced system}$$

$$\text{Ans, no. of basic variables} = 3 + 4 - 1 = 6$$

$$\therefore \text{Total cost} = 6 \times 21 + 5 \times 16 + 5 \times 18 + 8 \times 14 + 4 \times 15 + 15 \times 41 = 1095/-$$