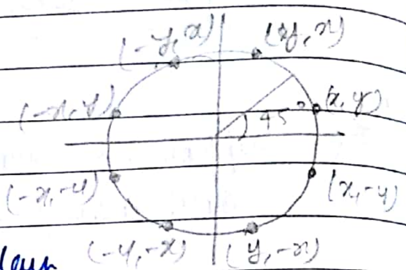
Lecture-12
MDU

Date 21/02/18
Page

Properties of a circle

- Set of points that are all at a given distance r from a center position $(x_c, y_c)$.
- Pythagorean theorem : $\boxed{(x-x_c)^2 + (y-y_c)^2 = r^2}$
- To calculate points on circle circumference by stepping along x axis in unit steps from $x_c - r$ to $x_c + r$ and calculating corresponding y values at each point as,

$$\boxed{y = y_c \pm \sqrt{r^2 - (x_c - x)^2}}$$



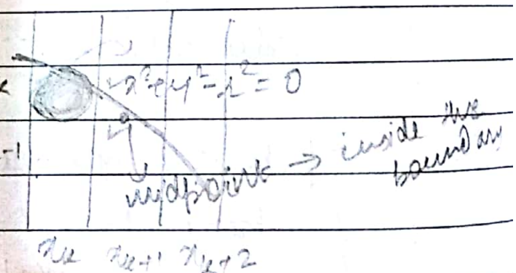

- Polar coordinates : r, θ

$$x = x_c + r\cos\theta$$
$$y = y_c + r\sin\theta$$

- Step : 1/r → more continuous display
- Computation is faster considering symmetry of the circle.
- Bresenham's line algorithm for raster display is adapted.

- Error limited to one half the pixel separation.

# Midpoint circle algorithm




- Circle function: $f_{circle}(x,y) = x^2 + y^2 - r^2$

- Position of a point:

$$f_{circle}(x,y) = \begin{cases} <0, & \text{if } (x,y) \text{ inside the circle} \\ =0, & \text{on the circle boundary} \\ >0, & \text{outside} \end{cases}$$

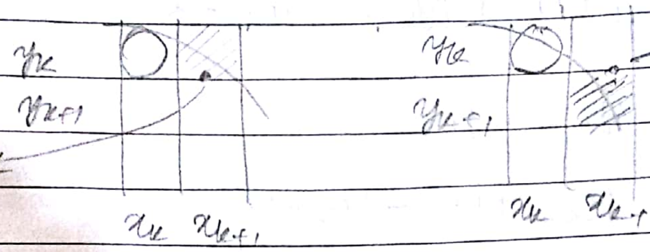
- Current pos$^n$ : $x_k, y_k$
- Next point closer to the circle : $x_k + 1, y_k$ or $x_{k+1}, y_k - 1$

- The decision parameter is the circle function

$$\boxed{\begin{aligned} p_k &= f_{circle}(x_k + 1, y_k - 1/2) \\ &= (x_k + 1)^2 + (y_k - 1/2)^2 - r^2 \end{aligned}}$$

- $P_k < 0$ : midpoint is inside the circle
  - → $y_k$ ; closer to the circle boundary
  - → $y_k - 1$ ; outside

midpoint inside boundary, ∴ $y_k$ selected




midpoint outside boundary, ∴ $y_{k+1}$ selected

$x_{k+1} = x_k + 1$

$y_{k+1} = y_k - 1$

Calculation of successive $p_k$'s

$$p_{k+1} = f_{circle}(x_{k+1}+1, y_{k+1} - 1/2)$$

$$= [(x_k+1)+1]^2 + (y_{k+1} - 1/2)^2 - r^2$$

Increments:

$$p_{k+1} = p_k + 2(x_k+1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$$

$$\Rightarrow p_{k+1} - p_k = 2x_k + 2 + (y_k - 1)^2 - y_k^2 - y_k + 1 + y_k$$

for $p_k > 0$

$$= 2x_k + 2 + y_k^2 + 1 - 2y_k - y_k^2 + 2$$

$2x_{k+1} = 2x_k + 2$

$2y_{k+1} = 2y_k - 2$

$$\boxed{p_{k+1} - p_k = 2x_k - 2y_k + 5}$$

Start position: $(0, r)$ → $2x_{k+1} = 0$ ; $2y_{k+1} = 2r$

$$p_0 = f_{circle}(1, r - 1/2)$$

$$= 1 + (r - 1/2)^2 - r^2$$

$$\Rightarrow p_0 = \frac{5}{4} - r$$

Rounding off $\boxed{p_0 = 1 - r}$

Scanned by CamScanner

~~When $p_k < 0$, $y_{k+1} = y_k$~~

$p_k = (x_k + 1)^2 + (y_k - 1/2)^2 - r^2$

$p_{k+1} = (x_{k+1} + 1)^2 + (y_{k+1} - 1/2)^2 - r^2$

When $p_k < 0$ $y_{k+1} = y_k$

$p_{k+1} - p_k = (x_{k+1} + 1)^2 - (x_k + 1)^2$

$= x_{k+1}^2 + 2x_{k+1} + 1 - x_k^2 - 1 - 2x_k$

$= 2(x_k + 1) + 1$

$= 2x_{k+1} + 1$

When $p_k > 0$, $y_{k+1} = y_k - 1$

$p_{k+1} = p_k + 2(x_k + 1) + 1 - 2(y_k - 1)$

$= p_k + 2x_{k+1} - 2y_{k+1} + 1$