

Q. A project consist of a series of task with the following relationships :-

$A < D, E$

$B, D < F$

$C < G$

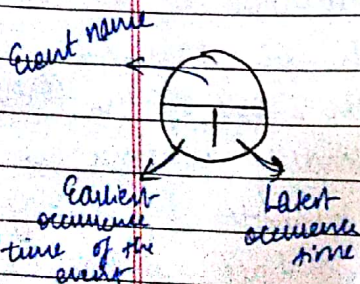
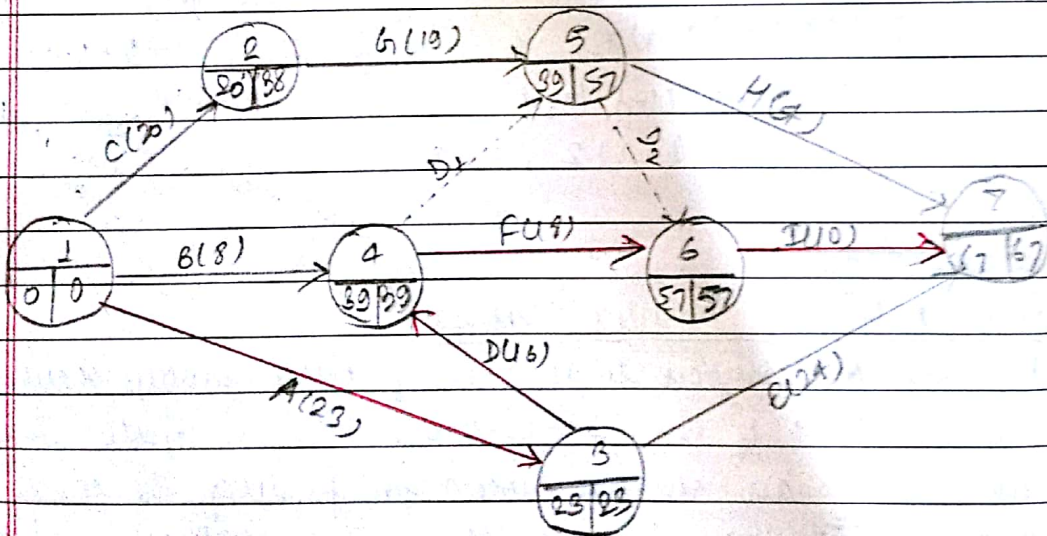
$D, B, G < H$

$F, G < I$

Time required for activity A is 23 days, for B = 8 days, C = 20 days, D = 16 days, E = 24 days, F = 18 days, G = 19 days, H = 4 days, I = 10 days.

Find the minimum time of the completion of the project

Activity	A	B	C	D	E	F	G	H	I
Predecessor activity	-	-	-	A	A	B, D	G	B, G	F, G
Time	23	8	20	16	24	18	19	4	10



Using CPM

APM

$$E_1 = 0$$

$$E_2 = E_1 + t_{12} = 20$$

$$E_3 = E_1 + t_{13} = 23$$

$$E_4 = \max \left\{ \begin{array}{l} E_1 + t_{14} = 0 + 16 = 16 \\ E_3 + t_{34} = 23 + 16 = 39 \end{array} \right.$$

$$\therefore E_4 = 39$$

$$E_5 = \max \begin{cases} E_2 + t_{25} \\ E_4 + t_{45} \end{cases} = \max \begin{cases} 19 + 20 \\ 39 + 0 \end{cases} = 39$$

$$E_6 = 39 + 18 = 57$$

$$E_7 = \max \begin{cases} 39 + 4 \\ 57 + 10 \\ 23 + 24 \end{cases} = 67$$

BPM

$$L_7 = E_7 = 67$$

$$L_6 = \min \{ L_7 - t_{67} \} = 67 - 10 = 57$$

$$L_5 = \min \begin{cases} L_7 - t_{57} \\ L_6 - t_{65} \end{cases} = \min \begin{cases} 67 - 4 \\ 57 - 0 \end{cases} = 57$$

$$L_4 = \min \begin{cases} E_5 - t_{45} \\ L_6 - t_{64} \end{cases} = \min \begin{cases} 57 - 0 \\ 57 - 18 \end{cases} = 39$$

$$L_3 = \min \begin{cases} 67 - 24 \\ 39 - 16 \end{cases} = 23$$

$$L_2 = 57 - 19 = 38$$

$$L_1 = \min \begin{cases} 38 - 20 \\ 39 - 8 \\ 23 - 23 \end{cases} = 0$$

Latent start time
→ earliest start time

Task	Duration	Earliest time		Latest time		Total float
		Start	Finish	Start	Finish	
C (1,2)	20	0	20	18	38	18
* A (1,3)	23	0	23	0	23	0 ✓
B (1,4)	8	0	8	31	39	31
G (2,5)	19	20	39	38	57	18
* D (3,4)	16	23	39	23	39	0 ✓
E (3,7)	24	23	47	43	67	20
D ₁ (4,5)	0	39	39	57	57	18
* F (4,6)	18	39	57	39	57	0 ✓
D ₂ (5,6)	0	39	39	57	57	18
H (5,7)	4	39	43	63	67	24
* I (6,7)	10	57	67	57	67	0 ✓

Total float : The difference b/w the latest start time and the earliest start time of an activity will indicate the amount of time by which the activity can be delayed without affecting total project duration. This difference is usually called the total float.

* Activities with 0 total float are known as critical activities.

* A sequence of critical activities in a network is called critical path.

* The critical path is the longest path ~~for~~ in the network from the starting event to the ending event and defines the minimum time required to complete the project.

∴ critical path

$$= A \rightarrow D \rightarrow F \rightarrow I$$

Minimum time for completion

$$= 28 + 16 + 18 + 10$$

$$= 67$$