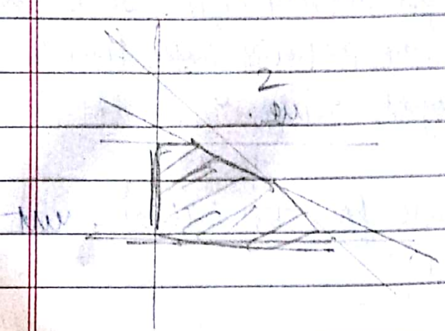
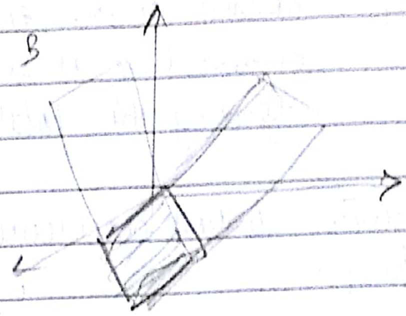
Lecture-7
SLa

classmate
Date 07/02/18
Page

* If we have n variables, we get a (n-1) dimensional hyper-space.
* For 3 variables we get 2 dimensional plane.
* For $n > 2$, the graphical method starts getting complex.





General LPP form

$$\max z = CX$$
st $AX \leq b$
$X \geq 0$

$X = \begin{pmatrix} \vdots \end{pmatrix}_{n\times 1}$ → column vector

or, $\min z = CX$
st. $AX \geq b$
$X \geq 0$

$A = \begin{pmatrix} \cdots \\ \vdots \end{pmatrix}_{m\times n}$

when $m=n$, ⇒ unique sol^n

* Finding the sol^n of an LPP is same as solving m equations with n variables, which implies when
  - $m=n$ ⇒ we have a unique sol^n
  - $m>n$ ⇒ has solutions. ($m-n$ equations are redundant)
  - $m<n$ ⇒ infinite solutions

3 dimensional space ← $\mathbb{R}^3 = \underset{\text{span}}{sp}\{\underset{v_1}{(1,0,0)}, \underset{v_2}{(0,1,0)}, \underset{v_3}{(0,0,1)}\}$ → These three vectors can be used to span whole 3 dimensional space (i.e, each point can expressed using these)

A point
$(4,-2,3)$ can be represented by $4v_1 - 2v_2 + 3v_3$

Scanned by CamScanner

classmate

Date ______
Page ______

* For n dimensional space, n vectors can span the whole space.

$$\mathbb{R}^n = \mathrm{sp}\{(1,0,0,\dots),\ (0,1,0,\dots),\ \dots\}$$

* A set of vectors $\{v_1, v_2, \dots v_n\}$ in an n-dimensional space is said to be linearly dependent iff some of the vectors can be written as a linear combination of the others. That is, there exists constants $c_1, c_2, \dots c_n$ such that for some k,

$$c_1v_1 + c_2v_2 + \dots + c_nv_n = 0$$

and $c_k \neq 0$

$\Rightarrow c_1v_1 = -(\dots\dots)$

$\Rightarrow v_1 = \frac{-1}{c_1}(\dots\dots)$ } linear dependence

* A set of vectors $(v_1, v_2 \dots v_n)$ in an n-dimensional space is said to be linearly independent if for the constants $c_1, c_2 \dots c_n$ the linear combination $c_1v_1 + c_2v_2 + \dots + c_nv_n = 0$ iff $c_1 = c_2 = \dots = c_n = 0$

$c_1(1,0,0) + c_2(0,1,0) + c_3(0,0,1) = (0,0,0)$ } linear independence

* In an LPP with m equations and n variables where $n > m$, a solution which has at least $n-m$ variables as zero (0) and the m vectors associated with m non-zero solutions are linearly independent is called a basic feasible solution (BFS).

$\{x_1, x_2, x_3, \dots x_m, \dots x_n\}$ $\underbrace{\qquad}_{n-m \text{ consider as } 0}$

Max No. of bfs possible $= {}^nC_m$

$\{\underbrace{x_1, x_2}_{m,2}, \underbrace{x_3, x_4}_{=0,\ =0}\}$

these two we consider as 0 and solve the rest

basic variables — these two only can give us unique sol^n

* A BFS must not contain more than m non-zero sol^ns.
* The non-zero variables are called the basic variables.

* A BFS in which at least one basic variable is zero, is called a degenerate BFS. Otherwise, it is a non-degenerate BFS.

$AX = \begin{pmatrix} & & \\ & & \end{pmatrix}$

if two columns are linearly dependent, then one can expressed in terms of the other means one column becomes a constant times other. So determinant = 0.

Q. Find the BFS of the following system:

$$x_1 + x_2 + x_3 = 8$$
$$3x_1 + 2x_2 = 18$$

Since, $n > m$, possible sol^n sets:

$(x_1, x_2, 0)$ $(x_1, 0, x_3)$, $(0, x_2, x_3)$

$$X_{12} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad A_{12} = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$$

$$|A_{12}| = -1$$

$$A_{12} X_{12} = \begin{pmatrix} 8 \\ 18 \end{pmatrix}$$

$$\Rightarrow X_{12} = A_{12}^{-1} \begin{pmatrix} 8 \\ 18 \end{pmatrix}$$

$$= -\begin{pmatrix} 2 & -1 \\ -3 & 1 \end{pmatrix}_{2\times 2} \begin{pmatrix} 8 \\ 18 \end{pmatrix}_{2\times 1} = -\begin{pmatrix} 16-18 \\ -24+18 \end{pmatrix}_{2\times 1} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}_{2\times 1}$$

$$\therefore \text{ One of the BFS } = \overset{x_1 \; x_2 \; x_4}{(2, 6, 0)}$$

$$X_{23} = \begin{pmatrix} x_2 \\ x_3 \end{pmatrix} \qquad A_{23} = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}$$

$$|A_{23}| = -2$$

$$X_{23} = \frac{1}{-2} \begin{pmatrix} 0 & -2 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 8 \\ 18 \end{pmatrix}$$

$$= \begin{pmatrix} 9 \\ -1 \end{pmatrix}_{2\times 1}$$

Since, x_3 is coming out to be -ve.
$\therefore$ $(0, 9, -1)$ is not a BFS.

$$X_{13} = \begin{pmatrix} x_1 \\ x_3 \end{pmatrix} \qquad A_{13} = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}$$

$$|A_{13}| = -3$$

$$X_{13} = \frac{1}{-3} \begin{pmatrix} 0 & -1 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 8 \\ 18 \end{pmatrix}$$

$$= \frac{1}{-3} \begin{pmatrix} 0 - 18 \\ -24 + 18 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

$\therefore$ $(6, 0, 2)$ is a BFS.

So, we observe max possible BFS = 3.
But, we got only 2 valid BFS.