Lecture-31
AH

classmate
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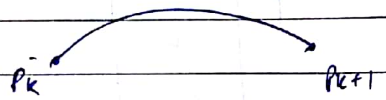
Page

Hermite spline

$$P(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Using boundary condⁿˢ.



$$\begin{bmatrix} p_k \\ p_{k+1} \\ Dp_k \\ Dp_{k+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} p_k \\ p_{k+1} \\ Dp_k \\ Dp_{k+1} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} p_k \\ p_{k+1} \\ Dp_k \\ Dp_{k+1} \end{bmatrix}$$

$$\therefore P(u) = U \cdot M_{spline} M_{geometric}$$

$$= \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_k \\ p_{k+1} \\ Dp_k \\ Dp_{k+1} \end{bmatrix}$$

$$= p_k(2u^3 - 3u^2 + 1) + p_{k+1}(-2u^3 + 3u^2) + Dp_k(u^3 - 2u^2 + u) + Dp_{k+1}(u^3 - u^2)$$

$$P(u) = p_k H_0(u) + p_{k+1} H_1(u) + Dp_k H_2(u) + Dp_{k+1} H_3(u)$$

Cardinal spline

$P(0) = p_k$

$P(1) = p_{k+1}$

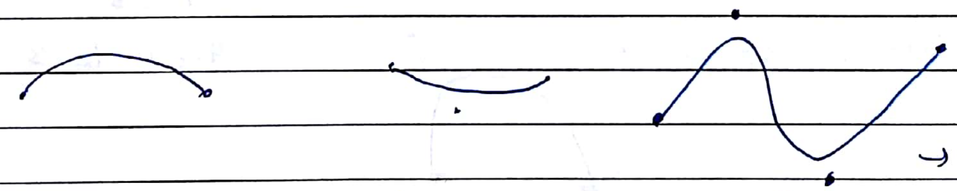
$P'(0) = \frac{1}{2}(1-t)(p_{k+1} - p_{k-1})$

$P'(1) = \frac{1}{2}(1-t)(p_{k+2} - p_k)$

t : tension parameter → controls how tightly or loosely the cardinal spline.

Bezier curve

↳ passes through first point and last control point.



→ 4 control points

more convenient using blending ~~curve~~ function.

Degree = # of control points − 1 = $n-1$

$$\boxed{P(u) = \sum_{k=0}^{n} p_k BEZ_{k,n}(u)}$$

$0 \le u \le 1$

→ The Bezier blending function $BEZ_{k,n}(u)$ are the Bernstein polynomial.

$$\boxed{BEZ_{k,n}(u) = C(n,k)\, u^k (1-u)^{n-k}}$$

$$x(u) = \sum_{k=0}^{n} x_k BEZ_{k,n}(u)$$

$$\boxed{C(n,k) = \frac{n!}{k!\,(n-k)!}}$$

$$y(u) = \sum_{k=0}^{n} y_k BEZ_{k,n}(u)$$

Similarly $z(u) =$

Properties

1) $u = 0$ to 1 → Approximation

$p(0) = p_0$

$p(1) = p_n$

2) $p'(0) = -np_0 + np_1$

$p'(1) = -np_{n-1} + np_n$

3) $p''(0) = n(n-1)[(p_2 - p_1) - (p_1 - p_0)]$

$p''(1) = n(n-1)[(p_{n-2} - p_{n-1}) - (p_{n-1} - p_n)]$

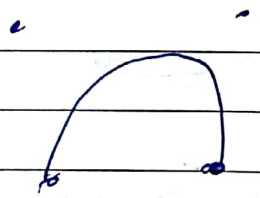
4) $\sum_{k=0}^{n} BEZ_{k,n}(u) = 1$ Convexhull property

Cubic Bezier curve

degree = 3 control points = 3+1 = 4

$n = 3$

$k = 0$ to 3



$BEZ_{k,n}(u) = C(n,k)u^k(1-u)^{n-k}$

$BEZ_{0,3}(u) = C(3,0)u^0(1-u)^{3-0} = (1-u)^3$

$BEZ_{1,3}(u) = C(3,1)u^1(1-u)^{3-1} = 3u(1-u)^2$

$BEZ_{2,3}(u) = C(3,2)u^2(1-u)^{3-2} = 3u^2(1-u)$

$BEZ_{3,3}(u) = C(3,3)u^3(1-u)^0 = u^3$

$$P(u) = [u^3 \ u^2 \ u \ 1] \ M_{BEZ} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

$$M_{BEZ} = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$P(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_4 \end{bmatrix}$$

Q. Find the eqn of the Bezier curve which passes through (1,1) and (4,2) and controls through (8,6) and (3,0).

$$\therefore P(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 8 & 6 \\ 3 & 0 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (-u^3+3u^2-3u+1) & (3u^3-6u^2+3u) & (-3u^3+3u^2) & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 8 & 6 \\ 3 & 0 \\ 4 & 2 \end{bmatrix}$$

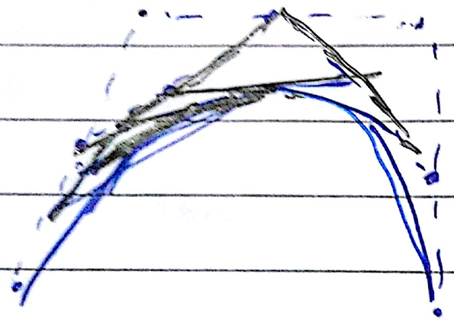
$$x(u) = -u^3+3u^2-3u+1 + 8(3u^3-6u^2+3u) + 3(-3u^3+3u^2) + 4$$

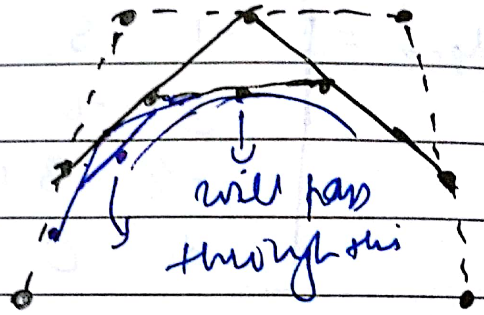
$$= -u^3 + 24u^3 - 9u^3 + 3u^2 - 48u^2 + 9u^2 - 3u^2 + 24u + 1 + 4$$

$$= 14u^3 - 39u^2 + 24u + 5$$

$$x(u) = 18u^3 - 36u^2 + 21u + 1$$

$$y(u) = 19u^3 - 33u^2 + 15u + 1$$




will pass
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