Lecture-23
SLa

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Assignment problem

An assignment problem refers to a special case of LPP where the objective is to assign a number of resources to an equal number of activities on a one-to-one basis so as to minimize the total cost of performing the task or to maximize the total profit of allocation.

Problem of assignment arises due to varying degrees of efficiency of the available resources for performing different activities where the time or profit of performance is not the same. The problem of assignment can be rephrased as "How to assign jobs to facilities in a way so that each job is done exactly by one facility".

facilities

$F_1\ F_2 \ldots\ F_n$ → same no. of facilities & jobs

Jobs	$F_1\ F_2 \ldots F_n$
J_1	
J_2	C_{ij}
⋮	
⋮	
J_n	

C_{ij} : cost of performance

$$X = \begin{pmatrix} 1 & & & \\ & 1 & & \\ 0 & & 1 & \\ 0 & & & 1 \\ 0 & & & \end{pmatrix}$$

Total n^2 variables

Remarks

1. In an assignment problem, the number of jobs and the number of facilities have to be equal.
2. The solution $x_{ij} = \begin{cases} 1, & J_i \to F_j \\ 0, & \text{otherwise} \end{cases}$ i^{th} job assigned to j^{th} facility

   Also, $\sum_{j=1}^{n} x_{ij} = 1$, $\sum_{i=1}^{n} x_{ij} = 1$

3. An assignment problem has $(2n-1)$ basic variables out of which n variables are 1 and the rest $(n-1)$ are 0. Hence an assignment problem has a high level of degeneracy.

Hungarian method for finding optimal solution

Step 1: Subtract the smallest number in each row from every number in that row. Repeat the same for each column.

Step 2: Search for the row having exactly one zero. Box that cell and cross out all the zeroes in that column. Repeat until all rows and columns are scanned.

Step 3: If in step 2 all the zeroes are either boxed or crossed and there is exactly one assignment in each row and column, then this is the optimal solution.

Step 4: If the solution is not optimal, draw the minimum number of lines to cover all the zeroes in the following way:

i) Mark all unassigned rows.

ii) Mark all columns that have zero in the marked rows.

iii) Mark all rows that have assignments in the marked columns.

iv) Repeat till all rows or columns are marked whenever possible.

v) Draw lines through the unmarked rows and marked columns.

vi) Select the smallest number from the uncovered cells. Subtract this from the uncovered cell and add this to the cells on the intersection of the lines drawn. Repeat step 2 to check for optimality.

Q.

	I	II	III	IV	V
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

After row-wise subtraction

30	0	45	60	70
15	0	10	40	55
30	0	45	60	75
0	0	30	30	60
20	0	35	45	70

Column wise →

30	$\boxed{0}$	35	30	15
15	0	$\boxed{0}$	10	0
30	0	35	30	20
$\boxed{0}$	0	20	0	5
20	0	25	15	15

30	$\boxed{0}$	35	30	15	
15	0	$\boxed{0}$	10	0	
30	0	35	30	20	✓
$\boxed{0}$	0	20	0	5	
20	0	25	15	15	✓

Adding 15 to cells where line intersected & subtracting from uncovered cells

15	0	20	15	0
15	15	0	0	0
15	$\boxed{0}$	20	15	5
0	15	20	0	5
5	0	10	10	0