Lecture - 33

Date 18/04/18

Principle of dominance

- Sometimes it is observed that one of the pure strategies of either players is always inferior to at least one of the remaining strategies. This strategy is said to be dominated by the others. In such a situation the player would never select the inferior strategy so the size of the payoff matrix can be reduced by deleting such inferior strategies. The value of the game and the nonzero probabilities remain unchanged after such deletions.

General rule of dominance -

- If all elements of the k-th row is less than equal to the corresponding elements of the r^{th} row then the k^{th} row is dominated by the r^{th} row.
- If all elements of k^{th} column is greater than equal to the corresponding elements of the r^{th} column, then the k^{th} column is dominated by r^{th} column.
- The dominated rows and columns may be deleted to reduce the size of the payoff matrix.
- Modified dominance - If a convex linear combination of some row dominate the k^{th} row then k^{th} row gets deleted.

Note -

- For the row player, the dominated row is the inferior one. and for the column player, the dominated column is the superior one.

Q. Solve the game using principle of dominance -

$$\begin{array}{c|cccc} & B_1 & B_2 & B_3 & B_4 \\ \hline A_1 & 4 & -2 & 3 & -1 \\ A_2 & -1 & 2 & 0 & 1 \\ A_3 & -2 & 1 & -2 & 0 \end{array}$$

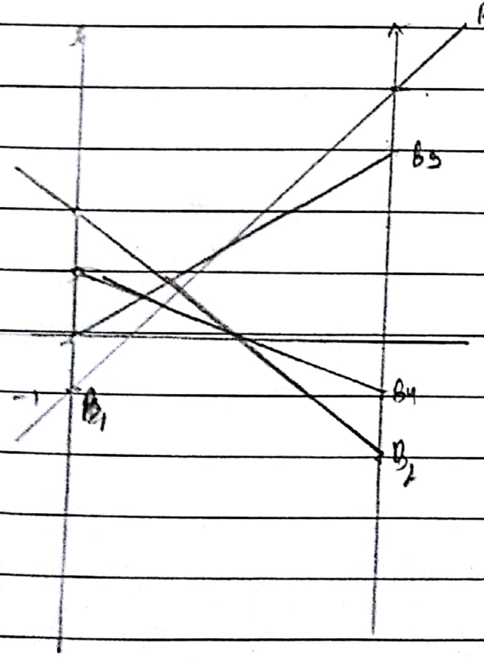
From the payoff matrix, A_3 is inferior to A_2.
Hence ~~by it~~ after deletion

$$\begin{array}{c|cccc} & B_1 & B_2 & B_3 & B_4 \\ \hline A_1 & 4 & -2 & 3 & -1 \\ A_2 & -1 & 2 & 0 & 1 \end{array}$$

$$S_A = \begin{pmatrix} A_1 & A_2 & A_3 \\ p_1 & 1-p_1 & 0 \end{pmatrix}$$

$$S_B = \begin{pmatrix} B_1 & B_2 & B_3 & B_4 \\ & & & \end{pmatrix}$$

B's pure moves	Expected ~~g~~ gain of A
B_1	$4p_1 - 1 + p_1 = 5p_1 - 1$
B_2	$-2p_1 + 2 - 2p_1 = -4p_1 + 2$
B_3	$3p_1$
B_4	$-p_1 + 1 - p_1 = -2p_1 + 1$



$$\begin{array}{c|cc} & B_1 & B_4 \\ \hline A_1 & 4 & -1 \\ A_2 & -1 & 1 \end{array}$$

$\frac{p_1}{p_2} = \frac{2}{5} \Rightarrow 5p_1 = 2 - 2p_1$

$\Rightarrow 7p_1 = 2 \Rightarrow p_1 = 7/2$

$\frac{q_1}{q_2} = \frac{2}{5} \Rightarrow q_1 = \frac{2}{7}$

$v = \frac{4 - 1}{5+2} = \frac{3}{7}$.

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Q.

$$\begin{array}{c|cccc} & B_1 & B_2 & B_3 & B_4 \\ A_1 & 3 & 2 & 4 & 0 \\ A_2 & 3 & 4 & 2 & 4 \\ A_3 & 4 & 2 & 4 & 0 \\ A_4 & 0 & 4 & 0 & 8 \end{array}$$

A_3 dominates A_1 → delete inferior row

Now, B_1 dominates B_3 → delete dominant column

$c_1 v_1 + c_2 v_2$

$\sum c_i = 1$

$\frac{1}{2} v_1 + \frac{1}{2} v_2$

$= \frac{1}{2}(2,4,0) + \frac{1}{2}(4,0,8) = (3,2,4)$

$\frac{1}{2}(4,0) + \frac{1}{2}(0,8) = (2,4)$

∴ Now we are left with

$$\begin{array}{c|cc} & B_3 & B_4 \\ A_3 & 4 & 0 \\ A_4 & 0 & 8 \end{array}$$