Lecture-11
SLA
19/02/18

Q. Solve by simplex method

$\max Z = 5x_1 + 6x_2 + x_3$

s.t, $9x_1 + 3x_2 - 2x_3 \leq 5$

$4x_1 + 2x_2 - x_3 \leq 2$

$x_1 - 4x_2 + x_3 \leq 3$

$x_1, x_2, x_3 \geq 0$

Standard form -

$\max Z = 5x_1 + 6x_2 + x_3 + 0 \cdot s_1 + 0 s_2 + 0 s_3$

$9x_1 + 3x_2 - 2x_3 + s_1 = 5$

$4x_1 + 2x_2 - x_3 + s_2 = 2$

$x_1 - 4x_2 + x_3 + s_3 = 3$ → requirement → 0

$x_1, x_2, x_3 \geq 0$ $\quad s_1, s_2, s_3 \geq 0$

BFS: non-basic

$x_1 = x_2 = x_3 = 0$

$s_1 = 5, s_2 = 2, s_3 = 3$

Basic variables

			C_j	5	6	1	0	0	0	→ coeff of obj func	Ratio b/a_{ik}
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6		
0	a_4	s_1	5	9	3	-2	1	0	0		5/3
0	a_5	s_2	2	4	2	-1	0	1	0		2/2=1 → min.
0	a_6	s_3	3	1	-4	1	0	0	1		3/1 → can't take -ve
		$z_j - C_j$		-5	-6	-1	0	0	0		

pivot column → basic var
pivot row → non-b. va
pivot element

$z_j = C_B \cdot a_j$

$z_j - C_j = 0 - 5 = -5$ → not optimal -ve

↳ want to bring 1 here.
∴ divide throughout by 2.

all are -ve so we can't find any ratio. So unbounded

			c_j						
0	a_4	s_1	2	3	0	-1/2	1	-3/2	0
6	a_2	a_2	1	2	1	-1/2	0	1/2	0
0	a_6	s_3	7	9	0	-1	0	2	1
		$z_j - c_j$		7	0	-4	0	8	0

$R_2 \to 2R_2$

$R_1 \to R_1 - R_2$

$4 \times R_2 + R_3$

* If at any iteration in a simplex algorithm corresponding to the pivot column j, $a_{ij} \leq 0 \; \forall i$ then the LPP has an unbounded soln if its a maximization problem.

$z_j - c_j \geq 0 \quad \forall j \rightarrow$ optimal

$z_j - c_j < 0$ for any $j \rightarrow$ not optimal

Q. Solve by simplex method -

~~Max $Z = 2x_2 + x_3$~~

~~s.t. $x_1 + x_2 - 2x_3 \leq 7$~~

Max $Z = 45x_1 + 80x_2$

s.t. $5x_1 + 20x_2 \leq 400$

$10x_1 + 15x_2 \leq 450$

$x_1, x_2 \geq 0$

Standard form

Max $Z = 45x_1 + 80x_2 + 0 \cdot s_1 + 0 \cdot s_2$

$10x_1 + 15x_2 + s_2 = 450$

$5x_1 + 20x_2 + s_1 = 400$

IBFS

$x_1 = 0, x_2 = 0$

$s_2 = 450, \; s_1 = 400$

$x_1, x_2 \geq 0 \quad s_1, s_2 \geq 0$

$Z_j - C_j$

			C_j	45	80	0	0	
C_B	B	x_B	b	a_1	a_2	a_3	a_4	Ratio
0	a_3	s_1	400	5	20	1	0	20
0	a_4	s_2	450	10	15	0	1	30
		$Z_j - C_j$		-45	-80	0	0	
80	a_2	x_2	20	1/4	1	1/20	0	80
0	a_4	s_2	150	25/4	0	-3/4	1	24
		$Z_j - C_j$		-25	0	4	0	
80	a_2	x_2	14	0	1	2/25	-1/25	
45	a_1	a_1	24	1	0	-3/25	4/25	
		$Z_j - C_j$	0	0	0	1	4	

$a_1 = 24$ $\quad s_1 = 0$

$x_2 = 14$ $\quad s_2 = 0$

$Z = 45 \times 24 + 80 \times 14$

$= 2200$