Lecture-18
MDU

classmate
Date 14/03/18
Page 59

Ellipse generating algorithm

- ellipse is an elongated circle

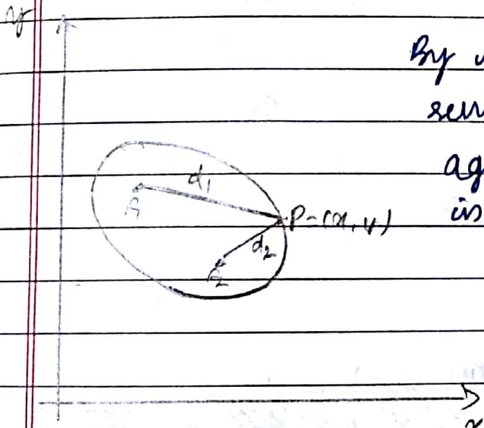
<u>Properties of an ellipse:</u>

- If the distances ~~of~~ to the two foci from any point $P=(x,y)$ on the ellipse are labelled d_1 and d_2, then the general equation of an ellipse can be stated as

$$d_1 + d_2 = \text{constant}$$

⇒ Expressing distances d_1 and d_2 in terms of the focal coordinates $F_1=(x_1,y_1)$ and $F_2=(x_2,y_2)$, we have

$$\boxed{\sqrt{(x-x_1)^2+(y-y_1)^2} + \sqrt{(x-x_2)^2+(y-y_2)^2} = \text{constant}}$$



By squaring this equation, isolating the remaining radical, and then squaring again we can write the general equation in the form

$$Ax^2 + By^2 + Cxy + Dx + Ey + F = 0$$

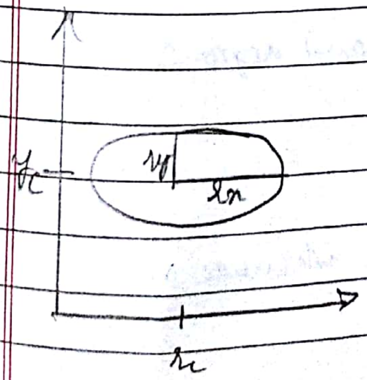
where the coefficients A, B, C, D, E, and F are evaluated in terms of the focal coordinates and the dimensions of the major and minor axes of the ellipse.

Ellipse eqns. are greatly simplified if the major & minor axes are oriented to align with the coordinate axes.

$$\left(\frac{x-x_c}{r_x}\right)^2 + \left(\frac{y-y_c}{r_y}\right)^2 = 1$$

Using polar coordinates, with parametric eqns:

$$x = x_c + r_x \cos\theta$$
$$y = y_c + r_y \sin\theta$$



An ellipse in standard position is symmetric b/w quadrants, but unlike circle not symmetric in octant of a quadrant.

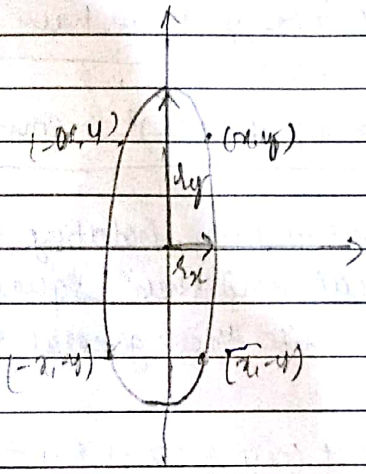
(0,B) ? Extremum

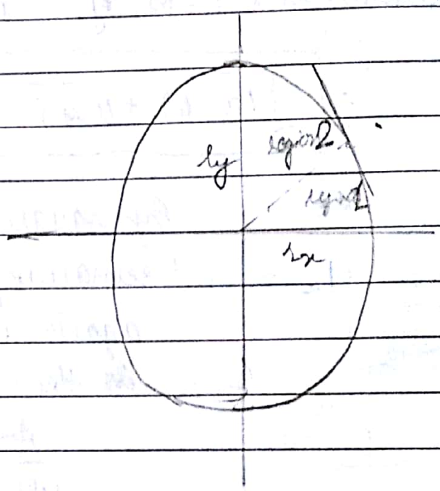
Mid-point ellipse algorithm

- Input: r_x, r_y, centre (x_c, y_c)
- Obtain the ellipse w.r.t origin, then shift the points accordingly.
- Can also be rotated → if in non standard form.
- when $r_x < r_y$

  unit step in x-direction

  when slope < 1 → unit step in y-direction





Ellipse function centred at origin

$$\boxed{f_{ellipse}(x,y) = r_y^2 x^2 + r_x^2 y^2 - r_x^2 r_y^2}$$

Which has the following properties

$$f_{ellipse}(x,y) = \begin{cases} < 0 & \text{if } (x,y) \text{ is inside the ellipse boundary} \\ = 0 & \text{if on} \\ > 0 & \text{if outside} \end{cases}$$

The slope of ellipse

$$\frac{dy}{dx} = -\frac{2r_y^2 x}{2r_x^2 y}$$

At the boundary b/w region 1 and region 2

$\frac{dy}{dx} = -1$ and

$$2r_y^2 x = 2r_x^2 y$$

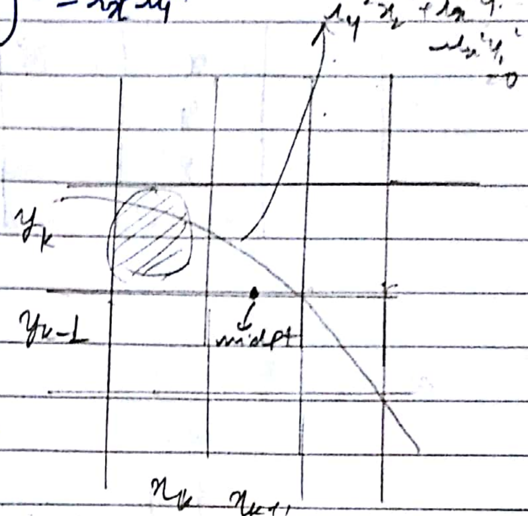
Therefore, we move out of region 1 whenever

$$2r_y^2 x \geq 2r_x^2 y$$

To determine next position

$p1_k = f_{ellipse}(x_k+1, y_k - 1/2)$

$= r_y^2 (x_k+1)^2 + r_x^2 \left(y_k - \frac{1}{2}\right)^2 - r_x^2 r_y^2$



$p1_k < 0 \rightarrow y_k$ else $y_k - 1$

Next sampling pos^n

$p1_{k+1} = f_{ellipse}\left(x_{k+1}+1, y_{k+1} - \frac{1}{2}\right)$

$= r_y^2 [(x_k+1)+1]^2 + r_x^2 \left(y_{k+1} - \frac{1}{2}\right)^2 - r_x^2 r_y^2$

$p1_{k+1} = p1_k + 2r_y^2 (x_k+1) + r_y^2 + r_x^2 \left[\left(y_{k+1} - \frac{1}{2}\right)^2 - \left(y_k - \frac{1}{2}\right)^2\right]$

$$\text{increment} = \begin{cases} 2r_y^2 x_{k+1} + r_y^2 & \text{if } p1_k < 0 \\ 2r_y^2 x_{k+1} + r_y^2 - 2r_x^2 y_{k+1} & \text{if } p1_k \geq 0 \end{cases}$$

$p1_k < 0$ y_k y_{k+1}

$p1_{k+1} - p1_k = 2r_y^2 (x_k+1) + r_y^2 + r_x^2 \left[\underbrace{y_{k+1}^2 + \frac{1}{4} - y_{k+1} - y_k^2 - \frac{1}{4} + y_k}_{} \right]$

0 if $y_{k+1} = y_k$

$2r_x^2 y_{k+1}$ if $y_{k+1} = y_k - 1$

If $p1_k < 0$

$p1_{k+1} - p1_k = 2r_y^2 (x_k+1) + r_y^2$

Else if $p1_k \geq 0$

$p1_{k+1} - p1_k = 2r_y^2 (x_k+1) + r_y^2 - 2r_x^2 y_{k+1}$