Lecture-11
MDu

classmate
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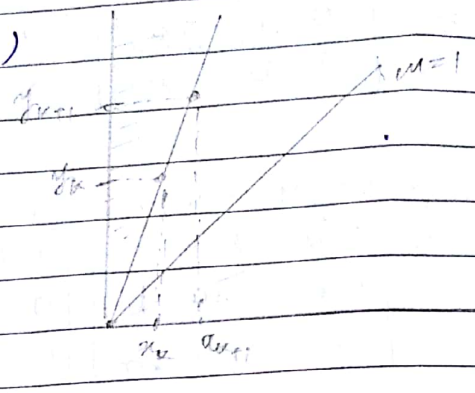
Page

Sampling along y-axis

$(m > 1)$

start : (x_1, y_1)

end : (x_2, y_2)



x_k

$x_{k+1} = x_k + 1$

$d_1 = x - x_k$

$d_2 = x_{k+1} - x$

$y_{k+1} = mx + b$

$x = (y_{k+1} - b) \times \frac{1}{m}$

$d_1 - d_2 = 2x - 2x_k - 1$

$= \frac{2}{m}(y_{k+1} - b) - 2x_k - 1$

$m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{1}{m} = \frac{\Delta x}{\Delta y}$

$p_k = \Delta y (d_1 - d_2)$

$= \Delta y \left(\frac{2}{m}(y_{k+1} - b) - 2x_k - 1 \right)$

$= \Delta y \cdot \frac{2}{m} \cdot y_{k+1} - \frac{2\Delta y\, b}{m} - 2x_k \Delta y - \Delta y$

$= 2\Delta x\, y_{k+1} - 2\Delta x\, b - 2x_k \Delta y - \Delta y$

$$p_k = 2\Delta x\, y_k + 2\Delta x - 2\Delta x b - 2x_k \Delta y - \Delta y$$

(substitute)

$\Rightarrow p_k = 2\Delta x y_k - 2x_k \Delta y + c \quad - (i)$

If $p_k < 0$, we take x_k

If $p_k > 0$, we take x_{k+1}

$p_{k+1} = 2\Delta x\, y_{k+1} - 2x_{k+1} \Delta y + c \quad - (ii)$

(ii) - (i) $= p_{k+1} - p_k = 2\Delta x (y_{k+1} - y_k) - 2\Delta y (x_{k+1} - x_k)$

~~$= 2\Delta x - 2\Delta y$~~

$= 2\Delta x - 2\Delta y (x_{k+1} - x_k)$

$$p_{k+1} - p_k = \begin{cases} 2\Delta x, & \text{if } p_k < 0 \\ 2\Delta x - 2\Delta y & \text{otherwise} \end{cases}$$

$p_k = 2\Delta x y_k + 2\Delta x - 2\Delta x b - 2x_k \Delta y - \Delta y$

$p_0 = 2\Delta x y_0 + 2\Delta x - 2\Delta x b - 2x_0 \Delta y - \Delta y$

$y_0 = mx_0 + b$

$b = y_0 - mx$

$= y_0 - \frac{\Delta y}{\Delta x} x_0$

$= 2\Delta x y_0 + 2\Delta x - 2\Delta x y_0 + 2\Delta y x_0 - 2x_0 \Delta y - \Delta y$

$\boxed{p_0 = 2\Delta x - \Delta y}$

Q. Suppose we want to draw a line starting at pixel (2,3) and ending at pixel (12,8)

$dx = 12 - 2 = 10$

$dy = 8 - 3 = 5$

$p0 = 2dy - dx = 0$

$2dy = 10$

$2dy - 2dx = -10$

k	p	P(x)	P(y)
0	0	2	3
1	-10	3	4
2	0	4	4
3	-10	5	5
4	0	6	5
5	-10	7	6
6	0	8	6
7	-10	9	7
8	0	10	7
9	-10	11	8
10	0	12	8

If $p_k < 0$, y remains same

~~$p_{k+1} = p_k + 2\Delta y$~~

~~or~~ $\boxed{p_{k+1} = p_k + 2\Delta y}$

else, y incremented

$\boxed{p_{k+1} = p_k + 2\Delta y - 2\Delta x}$

$p_1 = p_0 + 2\Delta y - 2\Delta x$

$= 0 + 10 - 20 = -10$

$p_2 = p_1 + 2\Delta y = -10 + 10 = 0$ as $y_{k+1} = y_k$, $\Delta x = 0$

$p_3 = p_2 + 2\Delta y - 2\Delta x = 0 - 10 = -10$