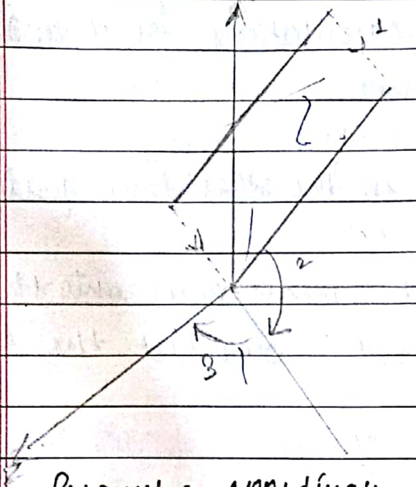
Lecture 21

AH

classmate Date 23/03/18 Page

Rotation about an arbitrary axis



(Seven matrices required)

1 → Translate to origin

2 → Rotate to xz plane (we use z axis rot^n)

3 → Rotate to z-axis

4 → Rotate object

5 → Rotate back to plane

6 → Rotate back to original axis

7 → Translate back

Parametric coordinates - x & y are changed cut of each other

$x = x_1 + Au$ $(A = (x_2 - x_1))$

$y = y_1 + Bu$ ↓ direction of vector

$z = z_1 + Cu$

$u \rightarrow$ parameter

when $u = 0$ $0 \leq u \leq 1$

$x = x_1$

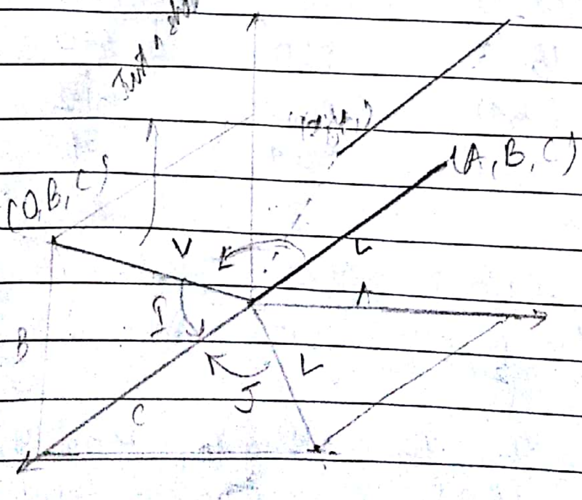
when $u = 1$

$x = x_2$

⋮

Translation matrix $T = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Just a shadow to simplify calculation



$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & x_1 \\ 0 & 1 & 0 & y_1 \\ 0 & 0 & 1 & z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$V = \sqrt{B^2 + C^2}$

Scanned by CamScanner

Not yz. Only shadow on yz.

Step 1 Translate the line to the origin

Step 2 Rotate about x until the axis of rotation is in xz plane.

Step 3 Rotate about y until the z-axis is corresponding to the axis of rotation.

Step 4 Rotate about z-axis.

Step 5 Reverse the rotation about y.

Step 6 Reverse the rotation about x

Step 7 Translate reverse back.

$V = \sqrt{B^2 + C^2}$

$\sin I = \frac{B}{V} = \frac{B}{\sqrt{B^2+C^2}}$

$\cos I = \frac{C}{V} =$

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos I & -\sin I & 0 \\ 0 & \sin I & \cos I & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C/V & -B/V & 0 \\ 0 & B/V & C/V & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_x^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C/V & B/V & 0 \\ 0 & -B/V & C/V & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$L = \sqrt{A^2 + V^2}$

$\cos J = \frac{V}{L} \qquad \sin J = \frac{A}{L}$

$$R_y = \begin{bmatrix} \cos J & 0 & \sin J & 0 \\ 0 & 1 & 0 & 0 \\ -\sin J & 0 & \cos J & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow R_y = \begin{bmatrix} \frac{V}{L} & 0 & \frac{A}{L} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{A}{L} & 0 & \frac{V}{L} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y^{-1} = \begin{bmatrix} V/L & 0 & -A/L & 0 \\ 0 & 1 & 0 & 0 \\ A/L & 0 & V/L & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_\theta = T^{-1} \cdot R_x^{-1} \cdot R_y^{-1} \cdot R_z \cdot R_y \cdot R_x \cdot T$$