Lecture-20
MDU

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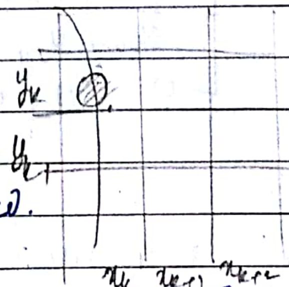
At the initial position $(0, r_y)$, the two terms evaluate to in region 1, the initial value of decision parameter is found as-

$$p1_0 = f_{ellipse}\left(1, r_y - \frac{1}{2}\right)$$
$$= r_y^2 + r_x^2\left(r_y - \frac{1}{2}\right)^2 - r_x^2 r_y^2$$
$$\boxed{p1_0 = r_y^2 - r_x^2 r_y - \frac{1}{4} r_x^2}$$

Over region 2, we sample at unit steps in the negative y direction, and midpoint is now taken b/w horizontal pixels at each step

$$p2_k = f_{ellipse}\left(x_k + \frac{1}{2}, y_k - 1\right)$$
$$= r_y^2\left(x_k + \frac{1}{2}\right)^2 + r_x^2\left(y_k - 1\right)^2 - r_x^2 r_y^2$$

If $p2_k > 0$, the midpoint is outside the ellipse boundary, x_k is selected.
If $p2_k <= 0$, x_{k+1} is selected.



We evaluate $f_{ellipse}$ at $y_{k+1} - 1 = y_k - 2$ (next sampling step)

$$p2_{k+1} = f_{ellipse}\left(x_{k+1} + \frac{1}{2}, y_{k+1} - 1\right)$$
$$= r_y^2\left(x_{k+1} + \frac{1}{2}\right)^2 + r_x^2\left[(y_k - 1) - 1\right]^2 - r_x^2 r_y^2$$

$$\boxed{p2_{k+1} = p2_k - 2r_x^2(y_k - 1) + r_x^2 + r_y^2\left[\left(x_{k+1} + \frac{1}{2}\right)^2 - \left(x_k + \frac{1}{2}\right)^2\right]}$$

When $p2_k > 0$, x_k is selected

$$\boxed{p2_{k+1} - p2_k = -2r_x^2 (y_k - 1) + r_x^2}$$

When $p2_k <= 0$, x_{k+1} is selected

$$p2_{k+1} - p2_k = -2r_x^2 (y_k - 1) + r_x^2 + r_y^2 \left[x_{k+1}^2 + x_{k+1} + \frac{1}{4} - x_k^2 - x_k - 1/4\right]$$

Now, $x_{k+1} = x_k + 1$

$$\therefore \boxed{p2_{k+1} - p2_k = -2r_x^2 (y_k - 1) + r_x^2 + r_y^2 \, 2x_{k+1}}$$

When we enter region 2, initial position (x_0, y_0) is taken as the last position selected in region 1, and the initial decision parameter

$$p2_0 = f_{ellipse}\left(x_0 + \frac{1}{2}, y_0 - 1\right)$$

$$= r_y^2 \left(x_0 + \frac{1}{2}\right)^2 + r_x^2 (y_0 - 1)^2 - r_x^2 r_y^2$$

To simplify calculation of $p2_0$, we could select pixel position in counterclockwise order starting at $(r_x, 0)$.

Constants: r_x^2, r_y^2, $2r_x^2$, $2r_y^2$ (calculate beforehand)

Midpoint ellipse algorithm

1. Input r_x, r_y and ellipse center (x_c, y_c) and obtain first point on an ellipse centered on the origin as

$$(x_0, y_0) = (0, r_y)$$

2. Calculate the initial value of the decision parameter in region 1 as

$$p1_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2$$

3. At each x_k position in region 1, starting at $k = 0$, perform the following test. If $p1_k < 0$, the next point along the ellipse centered at $(0,0)$ is (x_{k+1}, y_k) and

$$p1_{k+1} = p1_k + 2r_y^2 x_{k+1} + r_y^2$$

Otherwise the next point along the circle is (x_k+1, y_k-1) and

$$p1_{k+1} = p1_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_y^2$$

with

$$2r_y^2 x_{k+1} = 2r_y^2 x_k + 2r_y^2, \quad 2r_x^2 y_{k+1} = 2r_x^2 y_k - 2r_x^2$$

and continue until $2r_y^2 x \geq 2r_x^2 y$

4. Calculate the initial value of the decision parameter in region 2 using the last point (x_0, y_0) calculated in region 1 as

$$p2_0 = r_y^2 (x_0 + \frac{1}{2})^2 + r_x^2 (y_0 - 1)^2 - r_x^2 r_y^2$$

5. At each y_k position in region 2, starting at $k=0$, perform the following test. If $p2_k > 0$, the next point along the ellipse centered at $(0,0)$ is $(x_k, y_k - 1)$ and

$$p2_{k+1} = p2_k - 2r_x^2 y_{k+1} + r_x^2$$

Otherwise, the next point along the circle is (x_k+1, y_k-1) and

$$p2_{k+1} = p2_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_x^2$$

using the same incremental calculations for x and y as in region 1.

6. Determine symmetry points in the other three quadrants.

7. Move each calculated pixel position (x, y) onto the ellipse path centered on (x_c, y_c) and plot the coordinate values

$$x = x + x_c \quad , \quad y = y + y_c$$

8. Repeat the steps for region 1 until $2r_y^2 x \geq 2r_x^2 y$

Given input ellipse parameters $r_x = 8$ and $r_y = 6$, we illustrate the steps in the midpoint ellipse algorithm:

$$p1_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2 = -332$$

k	$p1_k$	x_{k+1}, y_{k-1}	$2r_y^2 x_{k+1}$	$2r_x^2 y_{k-1}$
0	-332	(1, 6)	72	768
1	-224	(2, 6)	144	768
2	-44	(3, 6)	216	768
3	208	(4, 5)	288	640
4	-108	(5, 5)	360	640
5	.	(6, 4)	.	512
6	244	(7, 3)	504	384

Now initial posn for region 2 $= (7, 3)$

$$p2_0 = f\left(7 + \frac{1}{2}, 2\right) = -151$$

k	$p2_k$	x_{k+1}, y_{k+1}	$2r_y^2 x_{k+1}$	$2r_x^2 y_{k+1}$
0	-151	(8, 2)	576	256
1	233	(8, 1)	576	128
2	743	(8, 0)	-	