$$\therefore S_B = \begin{bmatrix} B_1 & B_2 & B_3 & B_4 \\ 0 & q_1 & 0 & 1-q_1 \end{bmatrix}$$

16/4/18

$$\frac{p_1}{1-p_1} = \frac{a_{22}-a_{21}}{a_{11}-a_{12}} = \frac{2-0}{1+2} = \frac{2}{3}$$

$$3p_1 = 2-2p_1$$

$$5p_1 = 2$$

$$p_1 = \frac{2}{5}$$

$$\frac{q_1}{q_2} = \frac{a_{22}-a_{12}}{a_{11}-a_{21}} = \frac{2-(-2)}{1-0} = 4$$

$$q_1 = 4(1-q_1) = 4-4q_1$$

$$5q_1 = 4$$

$$q_1 = 4/5$$

$$v = \frac{2}{5}$$

$$\therefore S_A = \begin{pmatrix} A_1 & A_2 \\ 2/5 & 3/5 \end{pmatrix}$$

$$S_B = \begin{pmatrix} B_1 & B_2 & B_3 & B_4 \\ 0 & 4/5 & 0 & 1/5 \end{pmatrix}$$

8 Solve graphically:

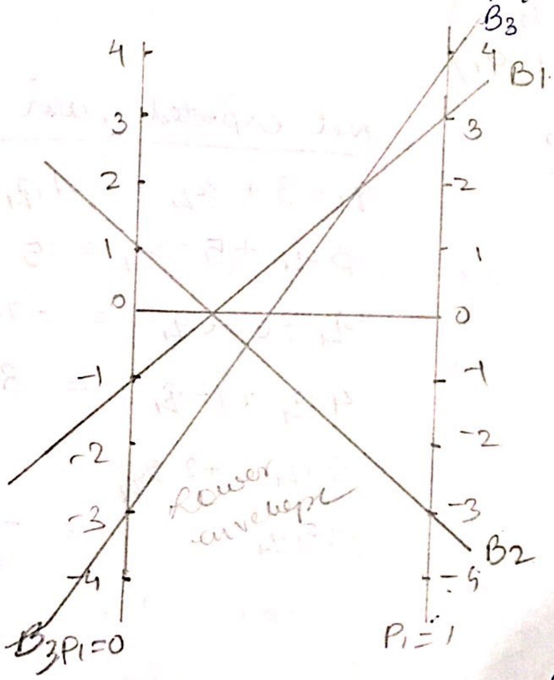
$$\begin{array}{c|ccc|c} & B_1 & B_2 & B_3 & \text{Row-min} \\ A_1 & 3 & -3 & 4 & -3 \\ A_2 & -1 & 1 & -3 & -3 \\ \hline \text{col max.} & 3 & 1 & 4 & \end{array}$$

$\bar{v} = -3$

$\underline{v} = 1$

$$S_A = \begin{pmatrix} A_1 & A_2 \\ p_1 & 1-p_1 \end{pmatrix}$$

B's pure moves	Net expected gain of A.
B_1	$(p_1 \quad 1-p_1)\begin{pmatrix} 3 \\ -1 \end{pmatrix} = 3p_1 - 1 + p_1 = 4p_1 - 1$
B_2	$-3p_1 + 1 - p_1 = -4p_1 + 1$
B_3	$4p_1 - 3 + 3p_1 = 7p_1 - 3$



$$S_B = \begin{pmatrix} B_2 & B_3 \\ q_1 & 1-q_1 \end{pmatrix}$$

Now, $A = \begin{pmatrix} -3 & 4 \\ 1 & -3 \end{pmatrix}$

$$\frac{p_1}{p_2} = \frac{-3-1}{-3-4} = \frac{-4}{-7} = \frac{4}{7}$$

$7p_1 = 4 - 4p_1$

$p_1 = \frac{4}{11}$

$$\frac{q_1}{q_2} = \frac{-3-4}{-3-1} = \frac{+7}{+4}$$

$4q_1 = 7 - 7q_1$

$11q_1 = 7$

$q_1 = \frac{7}{11}$

$$v = \frac{9-4}{-6-5} = \frac{5}{-11} = \frac{-5}{11}$$

$$S_A = \begin{pmatrix} A_1 & A_2 \\ 4/11 & 7/11 \end{pmatrix}$$

$$S_B = \begin{pmatrix} B_1 & B_2 & B_3 \\ 0 & 7/11 & 4/11 \end{pmatrix}$$

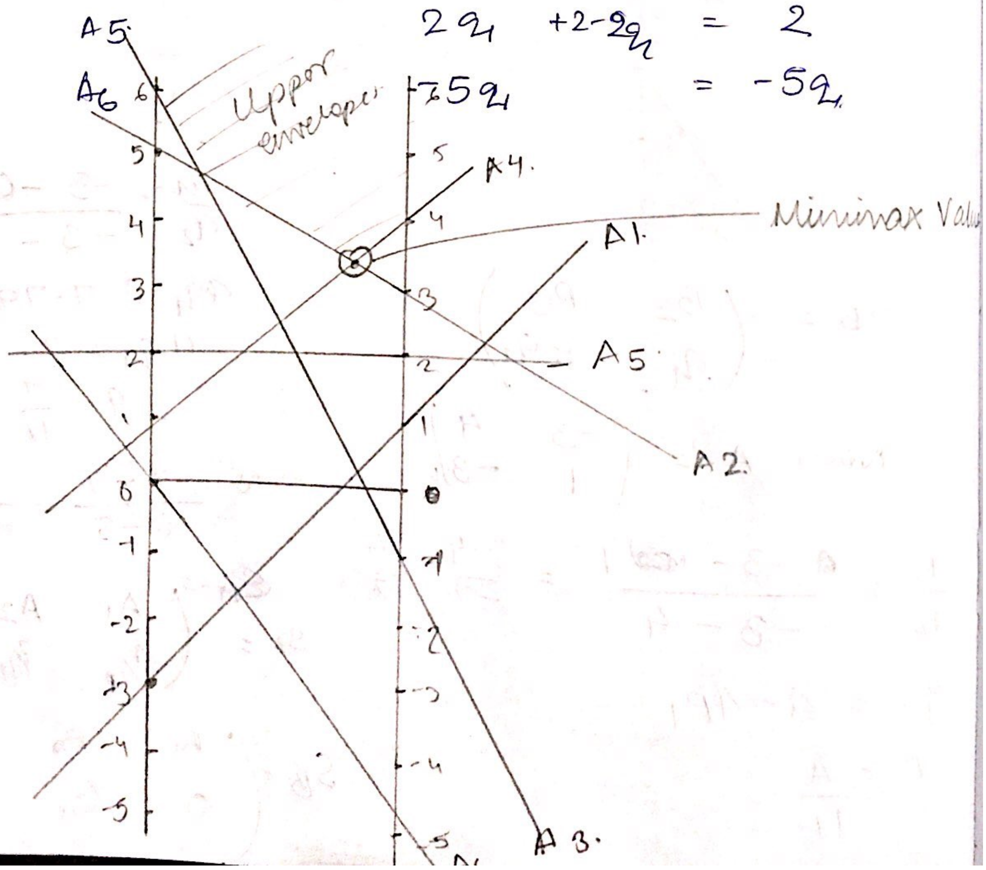
Q. Obtain the optimal strategies for the 2 persons and the value of the game whose payoff matrix is given by :

Here, from perspective of B.

$$\begin{array}{c c} & \begin{matrix} B_1 & B_2 \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{matrix} & \begin{pmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{pmatrix} \end{array}$$

$$S_B = \begin{pmatrix} B_1 & B_2 \\ q_1 & 1-q_1 \end{pmatrix}$$

A's pure moves	Net expected gain of B
A1	$q_1 - 3 + 3q_1 = 4q_1 - 3$
A2	$3q_1 + 5 - 5q_1 = 5 - 2q_1$
A3	$-q_1 + 6 - 6q_1 = -7q_1 + 6$
A4	$4q_1 + 1 - q_1 = 3q_1 + 1$
A5	$2q_1 + 2 - 2q_1 = 2$
A6	$-5q_1 = -5q_1$



A $\quad S_A = \begin{pmatrix} A_2 & A_4 \\ p_1 & 1-p_1 \end{pmatrix}$

$$A = \begin{pmatrix} 3 & 5 \\ 4 & 1 \end{pmatrix}$$

$$\frac{p_1}{p_2} = \frac{1-\cancel{8}4}{3-\cancel{6}5} = \frac{-3}{-2} = \frac{3}{2} \qquad \frac{q_1}{q_2} = \frac{1-5}{3-4} = 4.$$

$2p_1 = \cancel{400}\ 3-3p_1 \qquad q_1 = 4-4q_1$

$5p_1 = 3 \qquad 5q_1 = 4$

$p_1 = \frac{3}{5} \qquad q_1 = \frac{4}{5}$

$$v = \frac{3-20}{4-9} = \frac{-17}{-5} = \frac{17}{5}$$

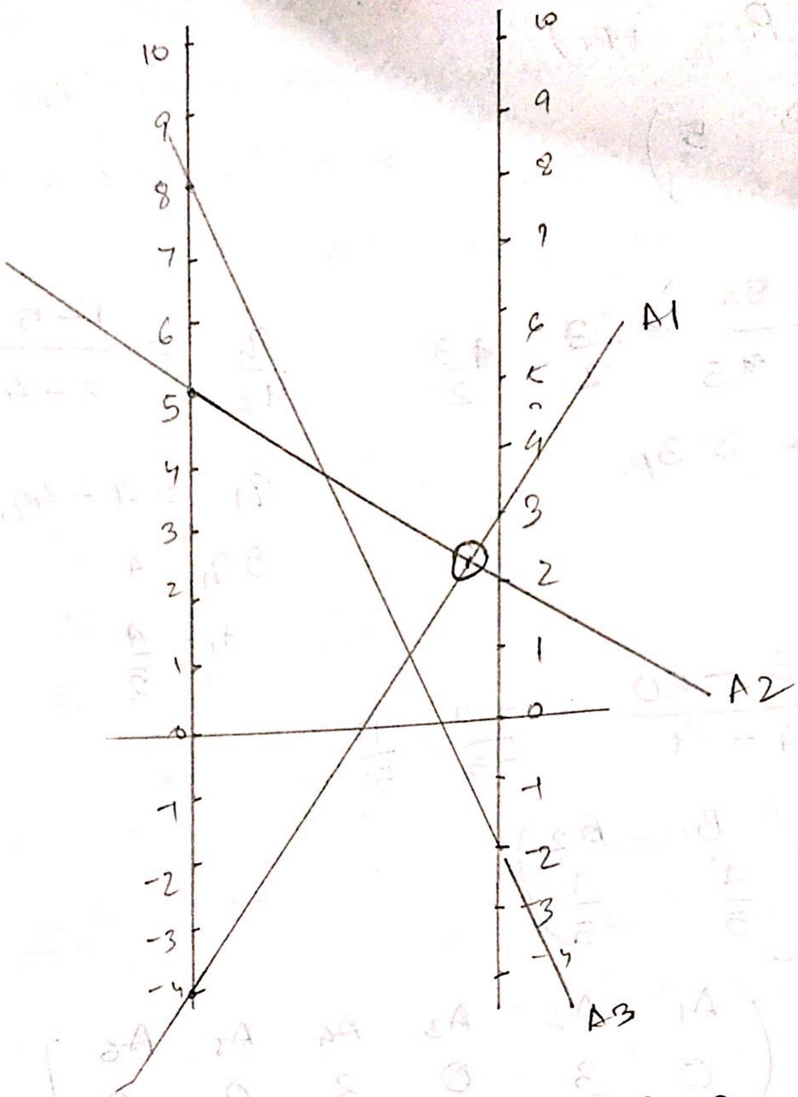
$$S_B = \begin{pmatrix} B_1 & B_2 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$$

$$S_A = \begin{pmatrix} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 \\ 0 & \frac{3}{5} & 0 & \frac{2}{5} & 0 & 0 \end{pmatrix}$$

B

$$\begin{array}{c c} & \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{pmatrix} 3 & -4 \\ 2 & 5 \\ -2 & 8 \end{pmatrix} \end{array} \qquad S_B = \begin{pmatrix} B_1 & B_2 \\ q_1 & 1-q_1 \end{pmatrix}$$

A's pure moves	Net expected gain of B
A_1	$3q_1 - 4 + 4q_1 = 7q_1 - 4$
A_2	$2q_1 + 5 - 5q_1 = -3q_1 + 5$
A_3	$-2q_1 + 8 - 8q_1 = -10q_1 + 8.$



$$A = \begin{pmatrix} 3 & -4 \\ 2 & 5 \end{pmatrix}$$

$$S_A = \begin{pmatrix} A_1 & A_2 \\ p_1 & 1-p_1 \end{pmatrix}$$

$$\frac{p_1}{p_2} = \frac{5-2}{3+4} = \frac{3}{7} \qquad \frac{q_1}{q_2} = \frac{5+4}{3-2} = \frac{9}{1}$$

$$7p_1 = 3 - 3p_1 \qquad q_1 = 9 - 9q_1$$

$$10p_1 = 3 \qquad 10q_1 = 9$$

$$p_1 = \frac{3}{10} \qquad q_1 = \frac{9}{10}$$

$$v = \frac{15+8}{8+2} = \frac{23}{10}$$

$$\therefore S_B = \begin{pmatrix} B_1 & B_2 \\ 9/10 & 1/10 \end{pmatrix}$$

$$S_A = \begin{pmatrix} A_1 & A_2 & A_3 \\ \frac{3}{10} & \frac{7}{10} & 0 \end{pmatrix}$$