

Q. min. $Z = 5x_1 + 11x_2$
 s.t $2x_1 + x_2 \leq 4$
 $3x_1 + 4x_2 \geq 24$
 $2x_1 - 3x_2 \geq 6$
 $x_1, x_2 \geq 0$

Std. form

min $Z = 5x_1 + 11x_2 + 0.s_1 + 0.s_2 + 0.s_3 + M.A_2 + M.A_3$
 $2x_1 + x_2 + s_1 = 4$
 $3x_1 + 4x_2 - s_2 + A_2 = 24$
 $2x_1 - 3x_2 - s_3 + A_3 = 6$
 $\max Z^* = -5x_1 - 11x_2 + 0.s_1 + 0.s_2 + 0.s_3 - M.A_2 - M.A_3$
 $x_1, x_2 \geq 0 \quad s_1, s_2, s_3 \geq 0 \quad A_2, A_3 \geq 0$

IBFS :

$x_1 = x_2 = 0$
 $s_2 = s_3 = 0$ (-ve ones)

B.V. $\begin{cases} s_1 = 4 \\ A_2 = 24 \\ A_3 = 6 \end{cases}$

surplus variable \leftarrow N.B
 artificial slack \leftarrow B.V

				C_j	-5	-11	0	0	0	-M	-M	
C_B	B	x_B	b	q_1	q_2	q_3	q_4	q_5	q_6	q_7		Ratio
0	q_3	s_1	4	(2)	1	1	0	0	0	0		2
-M	q_6	A_2	24	3	4	0	-1	0	1	0		8
-M	q_7	A_3	6	2	-3	0	0	-1	0	1		13
		$Z_j - C_j$		-5M	-M	0	M	M	M	M		
				+5	+11							
-5	q_1	x_1	2	1	1/2	3/2	0	0	0	0		
-M	q_2	A_2	18	0	5/2	-3/2	-1	0	1	0		
-M	q_7	A_3	2	0	-4	3	0	-1	0	1		
		$Z_j - C_j$		0	9M/2	5M/2	M	M	0	0		
					+11/2	-3/2						

Actually, we had to check $q_1 \geq 0$ but we found $q_1 = 2$ and $q_2 = 18$ and $q_7 = 2$ are ≥ 0 so we stop here.

opt: $x_1 = 2 \quad A_2 = 18 \quad A_3 = 2$

since, we got artificial variables as same opt.
 \Rightarrow No actual solⁿ.

Q. max $Z = -2x_1 + x_2 + 3x_3$
 st. $x_1 - 2x_2 + 3x_3 = 2$
 $3x_1 + 2x_2 + 4x_3 = 1$
 $x_1, x_2, x_3 \geq 0$

when constraints are given as equality, we don't need slack or surplus.

Std. form

max $Z = -2x_1 + x_2 + 3x_3 - MA_1 - MA_2$

$x_1 - 2x_2 + 3x_3 + A_1 = 2$

$3x_1 + 2x_2 + 4x_3 + A_2 = 1$

$x_1, x_2, x_3 \geq 0 \quad A_1, A_2 \geq 0$

IBFS

NB $x_1 = 0, x_2 = 0, x_3 = 0$

BV $A_1 = 2, A_2 = 1$

				C_j	-2	1	3	-M	-M	Ratio
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5		
-M	a_4	A_1	2	1	-2	3	1	0		2/3
-M	a_5	A_2	1	3	2	4	0	1		1/4
$Z_j - C_j$				-4M	-1	-7M	0	0		
				+2		-3				
$R_1 \leftarrow R_1 - 3R_2$	-M	a_4	A_1	5/4	-5/4	-7/2	0	1	-8/4	
	3	a_3	A_3	1/4	3/4	1/2	1	0	7/4	
$Z_j - C_j$				5M/4 + 3/4	7M/2	+3/2 - 1	0	0	3M/4	
				+2		-1			13/4 - M	

→ whichever went in the last step, doesn't need to be calculated anymore

optⁿ

$A_1 = 5/4 \quad a_3 = 1/4$

No optⁿ