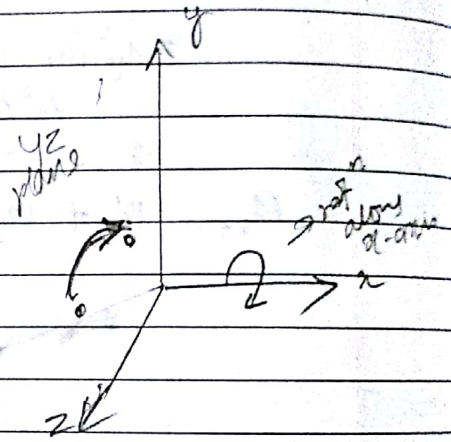# 8D transformations

Translation

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$
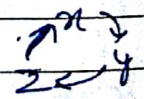
Rotation

change in
y axis means, xz plane
x " " yz "
z " " xy "

Always cyclic permutation: $x \to y \to z \to x$

$x' = x\cos\theta - y\sin\theta$
$y' = x\sin\theta + y\cos\theta$

But here x is fixed for a rot^n along x axis,
change is along yz plane
$y' = y\cos\theta - z\sin\theta$
$z' = y\sin\theta + z\cos\theta$

for
rotation
along x
axis

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Similarly
for other
axes, shift stuffs in the
matrix

Rotation about y-axis   (changes along xz plane)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & \cos\theta & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$x' = x\cos\theta - z\sin\theta$

$z' = x\sin\theta + z\cos\theta$

$y' = y$

Anti-clockwise

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

About z-axis   (change in xy plane)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}$$

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Scaling (S_x, S_y, S_z)

$- (x, y, z)$

$x' = x \cdot S_x$

$y' = y \cdot S_y$

$z' = z \cdot S_z$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$v = (0,0)$

Reflection

x-axis reflection (on the other side of yz plane)

R_{fx} ⇒ $x' = -x$

of

$y' = y$

$z' = z$

y-axis reflection

$x' = x$

$y' = -y$

$z' = z$

z-axis reflection

$x' = x$

$y' = y$

$z' = -z$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = R_{fx} \cdot P = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$
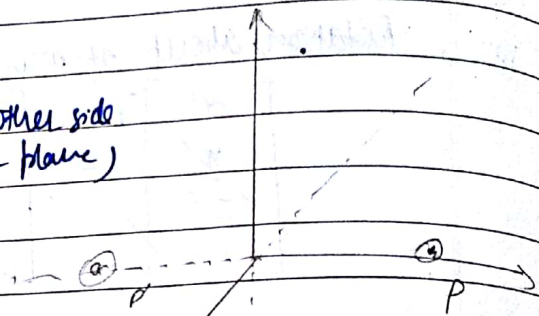
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = R_{fy} \cdot P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Shear

x-axis shear (change is only x)
y & z fixed

$x' = x + y \cdot Sh_{xy} + z \cdot Sh_{xz}$

$y' = y + x \cdot Sh_{yx} + z \cdot Sh_{yz}$

$z' = z + x \cdot Sh_{zx} + y \cdot Sh_{zy}$

x-axis shear

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_{xy} & sh_{xz} & 0 \\ 0 & sh_{xy} & 0 & 0 \\ 0 & 0 & sh_{xz} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & sh_{xy} & sh_{xz} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

y-axis shear

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ sh_{yx} & 1 & sh_{yz} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

z-axis shear

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ sh_{zx} & sh_{zy} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$