Lecture-30
AH

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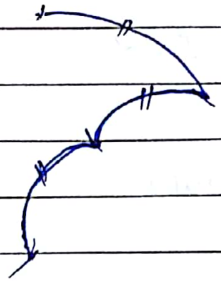
Spline curve

Spline is a flexible strip to produce a smooth curve through a dedicated set of points. We can mathematically describe such a curve with a piecewise cubic polynomial function whose first and second derivatives are continuous across the various curve section.

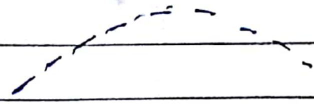


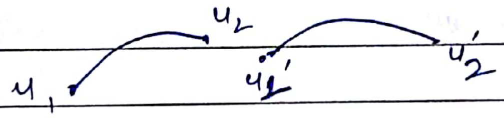
$x = x(u)$

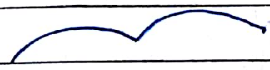
$y = y(u)$

$z = z(u)$

$u_1 \le u \le u_2$



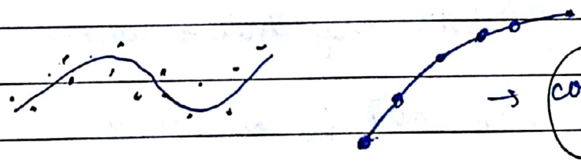




If $u_2 = u_1'$, then a single curve

Types

i) Approximation →



→ control points

ii) Interpolation →

$p_k = (x_k, y_k, z_k)$ $\quad$ n+1 control points

$k = 1$ to n

Parametric continuity equation

Spline is described with a set of parametric coordinate function of the form

$x = x(u)$

$y = y(u)$

$z = z(u)$

where $u_1 \le u \le u_2$

Parametric continuity condition

Geometric continuity condition

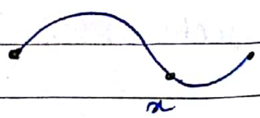
Parametric continuity condition

Only magnitude matters

$C^0, C^1, C^2, \ldots$

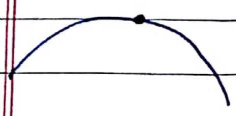
$\underline{C^0}$ $x(u)$ magnitude same

$x'(u)$



$\underline{C^1}$ → first order derivates at x are ~~same~~ same
So the curve is smooth
- ~~Direction changes~~

$x''(u)$



$\underline{C^2}$
→ second order derivates are same.
Even smoother
– Direction same.

Geometric continuity condition

↳ both magnitude + direction matters.

C_0 and G_0 are same

But C_1 and G_1 are diff.
In C_1, magnitude of derivative should be same.

In G_1, magnitudes can differ but directions should be same.

Spline specification

There are 3 equivalent methods called for specifying a particular spline representation.

i) We can state the set of boundary condns that are imposed on the spline.

ii) We can state the matrix that characterizes the spline.

iii) We can state a set of blending funcn (or basis funcn) that determines how to specify geometric constraints on the curve are ~~comp~~ combined to calculate pos^n on the curved path.

Using set of boundary cond^ns

$$x(u) = a_x u^3 + b_x u^2 + c_x u + d_x \qquad 0 \leq u \leq 1$$

we can solve for $x(0)$, $x(1)$, $x'(0)$, $x'(1)$

Similarly,

$$y(u) = a_y u^3 + \dots$$
$$z(u) = a_z u^3 + \dots$$

Using matrix

$$x(u) = \begin{bmatrix} u^3 & u^2 & u^1 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ b_x \\ c_x \\ d_x \end{bmatrix}$$

$$x(u) = U \cdot C$$

$$C = M_{spline} \cdot M_{geometric}$$

$$x(u) = U \cdot M_{spline} M_{geometric}$$

Using blending func^n

$$x(u) = \sum_{i=0}^{3} g_k \cdot BF_k(u)$$

where g_k are the constraint parameters such ~~that~~ as the control point co-ordinates or the slope of the curve of the control points.

$BF_k(u)$ → polynomial blending func^n.

Cubic spline interpolation method

n+1 control points

Points $p_k \rightarrow p_0$ to p_n

$p_k = (x_k, y_k, z_k)$

when $k = 0$ to n

Natural cubic spline

$x(u) = a_x u^3 + b_x u^2 + c_x u + d_x$

$y(u) =$

$z(u) =$

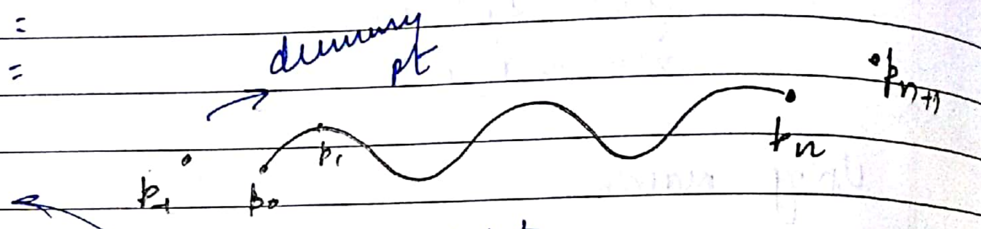
a, b, c, d

$4(n-1)$

$4n-4$

→ contraints

But total 4n contrains | ∴ we introduce 2 dummy pts.



n+1 points

n curves

(n-1) equation for boundary as n-1 internal points

Hermite equation

A hermite spline is an interpolating piece-wise cubic polynomial with a specified tangent at each control point.

Unlike the natural cubic spline, hermite spline can be adjusted locally because each control point is only dependent on its end point.

- Drawback of natural cubic spline
  - If any control point is changed, then curve is changed
  - can't be locally controlled

P(u)

If P(u) represent a parametric cubic point func^n for the curve section b/w control points P_k and P_{k+1}. Then the boundary eq^n of the hermite curve section :-

$P_0 = P_k$

$P_1 = P_{k+1}$

$P'(0) = DP_k$

$P'(1) = DP_{k+1}$

$P(u) = au^3 + bu^2 + cu + d$

$$P = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$P'(u) = \begin{bmatrix} 3u^2 & 2u & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$