Lecture-4
Sha

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Page

Q. The manager of an oil refinery must decide on the optimum mix of two possible blending processes of which the input and output production runs as follows:

Process	Input: Crude A	Input: Crude B	Output: Gasoline X	Output: Gasoline Y
1	6	4	6	9
2	5	6	5	5

The maximum amounts available for crude oil A is 250 units and for crude B its 200 units. Market demand shows that at least 150 units of gasoline X and 130 units of gasoline Y must be produced. The profits per production from process 1 and process 2 are Rs. 4 and Rs. 5 respectively. Formulate the LPP for maximizing the profit.

Let x_1 be the units produced by process 1 and x_2 be units produced by process 2

Profit (Z) $= 4x_1 + 5x_2$ → objective func^n.

maximize this

$6x_1 + 5x_2 \leq 250$
$4x_1 + 6x_2 \leq 200$
] constraints on amount of crude oil

$6x_1 + 5x_2 \geq 150$
$9x_1 + 5x_2 \geq 130$
] constraints on amount of gasoline

$x_1, x_2 \geq 0$] non-negativity constraint (This should always be there)

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Q. Food x contains 6 units of vitamin A and 7 units of vitamin B per gram. Food y contains 8 units/gram of vitamin A and 12 units/gram of vitamin B. The cost of food x is 12p/gram and the cost of food y is 20p/gram. The daily requirements of vitamin A and vitamin B are at least 100 units and 120 units respectively. Formulate the above as an LPP to minimize cost.

Let x_1 and x_2 be the units (grams) of food x and y produced.

$$\underset{\text{maximize}}{z} = 12x_1 + 20x_2 \quad \text{] objective func}^n$$

$$\left.\begin{aligned} \because 6x_1 + 8x_2 &\geq 100 \\ 7x_1 + 12x_2 &\geq 120 \end{aligned}\right] \text{ constraints on amount of vitamin A and B}$$

$$x_1, x_2 \geq 0 \quad \text{] non-negativity constraint}$$

Generalized representation of an LPP

In general, an LPP is represented in the following way:

$$\text{optimize } Z = c_1x_1 + c_2x_2 + \dots + c_nx_n = \sum_{i=1}^{n} c_i \cdot x_i$$

objective func^n

$$= \underbrace{(c_1 \; c_2 \; \dots \; c_n)}_{C} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}}_{X}$$

$$= CX$$

vectors

C : price parameters
X : decision variables

The objective func^n is a linear func^n of decision variables.

The space of possible solutions is limited by the constraints

$$a_{11}x_1 + \dots\dots + a_{1n}x_n \geq / \leq b_1$$

$$\vdots \qquad\qquad\qquad \vdots$$

$$a_{m1}x_1 + \dots\dots + a_{mn}x_n \geq / \leq b_m$$

$$AX \geq / \leq b_{m\times 1}$$

A: coefficient matrix
b: constants (a column vector), requirement parameter
a_{ij} : activity parameters

The coefficients a_{ij} are called activity parameters.
The m constraints are linear func^n of decision variables

$$b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix} \rightarrow \text{requirement parameter}$$

In addition, $X \geq 0$ is a requirement based on the situation.

Compact way

$$\begin{cases} \text{max/min} \quad z = CX \\ \text{s.t (subject to constraints)} \\ \quad AX \leq / \geq b \\ \quad X \geq 0 \end{cases}$$

* A set of values of the decision variables $x_1, x_2 \dots x_n$ which satisfy the set of constraints and non-negativity restriction is called a feasible solution. ~~& which optimizes the~~ & feasible sol^n that optimizes the objective func^n is called an optimized sol^n.

classmate

Date ______
Page ______

Q. Solve the following LPP :

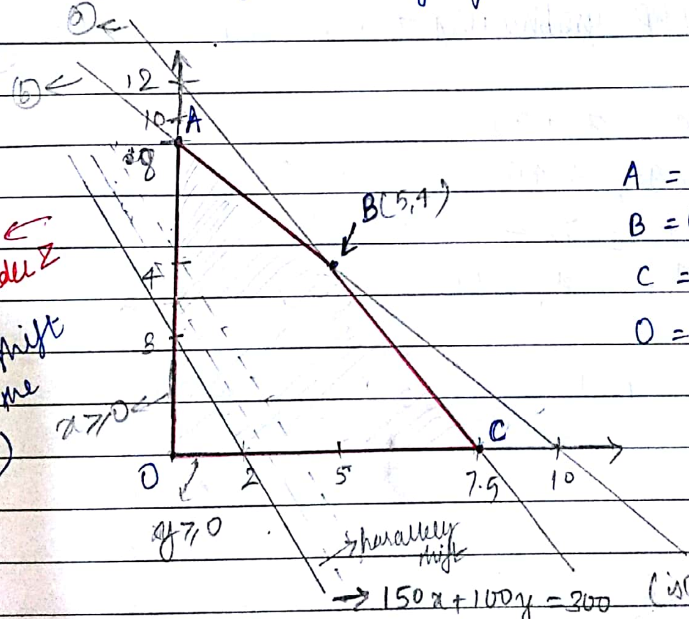
$$\max Z = 150x + 100y$$

s.t.,

$$8x + 5y \leq 60 \quad - (a)$$
$$4x + 5y \leq 40 \quad - (b)$$
$$x, y \geq 0$$

Graphical method (only for two or lesser decision variables)

- We plot the graphs by converting inequalities to equalities, and the ~~sto~~ intersection of regions covered by all constraints will be region having feasible sol^{ns}.



We have to maximize the area under Z (so we can shift parallelly the line till the end of region)

A = (0,8)
B = (5,4)
C = (7.5, 0)
O = (0,0)

} Extreme points

or

Let us take some value for Z. (LCM of x & y 'coeff.)

$\therefore$ Z = 300, say (makes it easier to plot)

check all the values and see which one is maximum.

$Z_A = 150 \times 0 + 100 \times 8 = 800$

$Z_B = 150 \times 5 + 100 \times 4 = 1150$ ⟵ max. (optimal sol^n)

$Z_C = 150 \times 7.5 + 100 \times 0 = 1125$

$Z_O = 0$

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