Lecture-10
SLA

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Consider the general LPP as

unbalanced so we add a slack variable to LHS

$$\max\ z = c_1x_1 + \dots + c_nx_n$$

requirement variables

$$\text{s.t. } a_{11}x_1 + \dots + a_{1n}x_n \le b_1$$
$$\vdots$$
$$a_{m1}x_1 + \dots + a_{mn}x_n \le b_m$$
$$x_i \ge 0$$

$$\max\ Z = CX$$
$$\text{s.t. } AX \le b$$
$$x \ge 0$$

~~In~~ a LPP is said to be in the standard form if all the constraints are written as equalities. The optimal solution of the standard form and the original LPP are the same.

Standard form
$$\max\ Z = CX$$
$$\text{s.t. } AX = b$$

slack variable (≥ 0)

$$a_{11}x_1 + \dots + a_{1n}x_n + S_1 = b_1$$

Objective func^n → we keep cost parameter = 0

$$\max\ z = CX + 0.S_1$$

$$a_{21}x_1 + \dots + a_{2n}x_n + 0.S_1 + 1.S_2 = b_2$$
$$a_{31}x_1 + \dots + a_{3n}x_n + 0.S_1 + 0.S_2 + 1.S_3 = b_3$$
$$\vdots$$

If the inequality was reversed (as in minimization problem), ~~that is like~~ the we subtract a surplus variable from LHS.

→ surplus variable (≥ 0)

$$a_{11}x_1 + \dots + a_{1n}x_n - S_1 = b_2$$

Q. Solve the following LPP
$$\max\ z = 60x_1 + 50x_2$$
$$\text{s.t. } x_1 + 2x_2 \le 40$$
$$3x_1 + 2x_2 \le 60$$
$$x_1, x_2 \ge 0$$

Std. form
$$\max\ z_1 = 60x_1 + 50x_2 + 0.S_1 + 0.S_2$$
$$\text{s.t. } x_1 + 2x_2 \cancel{= 40} + S_1 + 0.S_2 = 40$$
$$3x_1 + 3x_2 + 0.S_1 + S_2 = 60$$
$$x_1, x_2, S_1, S_2 \ge 0$$

Origin : $x_1 = 0$, $x_2 = 0$ (non basic)
$S_1 = 40$, $S_2 = 60$ (Basic)

IBFS : (0, 0, 40, 60)

Simplex table :

cost parameters for basis

associated with

			c_j	60	50	0	0
C_B	B	x_B	b	a_1	a_2	a_3	a_4
0	a_3	S_1	40	1	2	1	0
0	a_4	S_2	60	3	2	0	1
		$z_i - c_i$		-60	-50	0	0

→ Pivot column (column corresponding to minimum $z_j - c_j$ value)

$$z_1 = C_B^T \cdot a_1$$

$$= (0 \quad 0)\begin{pmatrix}1\\3\end{pmatrix} = 0$$

$$z_1 - c_1 = 0 - 60 = -60$$

$$z_2 - c_2 = -50$$

$$z_3 - c_3 = 0$$

$$z_4 - c_4 = 0$$

if any one is zero ←

The solution is not optimal as long as $(z_j - c_j) < 0$ for some j.

pivot column

b/a_k — we divide only if its +ve

coeff of obj. func^n

			c_j	60	50	0	0	Ratio
C_B	B	x_B	b	a_1	a_2	a_3	a_4	
(I) 0	a_3	S_1	40	1	2	1	0	$40/1 = 40$
(II) 0	a_4	S_2	60	3	2	0	1	$60/3 = 20$
		$z_j - c_j$		-60	-50	0	0	
(I) − (II) 0	a_3	S_1	20	0	4/3	1	-1/3	$20/(4/3) = 15$
(II)/3 = (II) 60	a_1	x_1	20	1	2/3	0	1/3	$20/(2/3) = 30$
		$z_j - c_j$		0	-10	0		
50	a_2	x_2	15	0	1	3/4	-1/4	
60	a_1	x_1	10	1	0	-1/2	1/2	
		$z_j - c_j$		0	0	15/2	35/2	

3 → pivot element

select the min out of these → Pivot row (row with min ratio)

→ new pivot row

→ new pivot column

pivot element has to be zero as it is corresponding to the basic variable

we have to eliminate 2/3. ∴ we multiply upper row by 2/3 & subtract

$1 - 2/3$

→ Optimal sol^n.

→ We need to get this identity matrix

$\therefore x_1 = 10$, $x_2 = 15$, $S_1 = S_2 = 0$

$\therefore Z_{max} = 1350$

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The variable corresponding to the pivot row gets dropped from the basic variable set and the variable corresponding to the pivot column becomes the new basic variable.