Hypergraph-Nonlinear-Latency Voter Model

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***Abstract*—The Voter Model has long served as a fundamental framework for analyzing opinion dynamics; however, it often oversimplifies interactions as pairwise and assumes immediate responses. This study introduces the Hybrid Hypergraph-Nonlinear-Latency Voter Model (HNL Voter Model), which integrates group-based hypergraph structures, nonlinear adoption probabilities, and latency effects to better reflect the complexity of real-world opinion propagation. By simulating sentiment data from social media platforms, the model explores how network structures, nonlinear influences, and temporal delays collectively shape opinion diversity, cascading effects, and oscillatory behaviors. Results demonstrate that lower nonlinearity (q) values and high latency variance prolong diversity, while higher nonlinearity accelerates consensus. The model successfully replicates key behaviors observed in real-world networks, such as transient cascades and persistent oscillations, validating its robustness in capturing dynamic opinion processes. This work highlights the interplay between structural and temporal factors in social media dynamics, offering a foundation for future studies on collective decision-making and influence propagation.**

I. Introduction

The Voter Model has provided a foundational framework in understanding opinion dynamics across social networks, capturing the ways individual choices aggregate into larger social phenomena such as consensus or polarization. Traditionally, the model simplifies interactions as pairwise and assumes immediate responses, limiting its applicability to the complexity of real-world systems where social influence is often multifaceted and delayed. Recent advancements have expanded the Voter Model, introducing models that incorporate hypergraphs [1] (to capture group-based influences), nonlinear influence [2] (to account for varying susceptibility to change), and latency [3] (to simulate delayed responses).

This study proposes the Hybrid Hypergraph-Nonlinear-Latency Voter Model (HNL-Voter Model), combining these advancements to simulate opinion dynamics in complex networks. The primary question explored in this research is: How do group structures, nonlinear influences, and temporal delays interact to shape collective opinion dynamics on social media? Using a sentiment analysis dataset on Kaggle from social media platforms such as Twitter, Facebook, and Instagram, the model investigates how platform-specific structures influence opinion propagation, time to consensus, and diversity. The key contributions of this paper include: (1) Introducing a hybrid model that integrates hypergraph interactions, nonlinear influence, and latency effects. (2) Simulating opinion dynamics across platforms and identifying unique behaviors such as oscillations and cascading effects. (3) Providing insights into how network structures and temporal delays shape collective decision-making.

II. Related Work

The study of opinion dynamics in social networks has been extensively explored through various modeling approaches, reflecting the complexity and diversity of real-world interactions. This section provides an overview of the key frameworks that inform the Hybrid Hypergraph-Nonlinear-Latency Voter Model (HNL Voter Model).

The Voter Model, introduced by Holley and Liggett in 1975 [4], established a stochastic framework for studying consensus formation through pairwise interactions. The basic voter model workflow is visualized in Fig. 1. Nodes in a network adopt the opinion of a randomly chosen neighbor, leading to either full consensus or sustained coexistence of opinions, depending on the network topology. While foundational, the model's simplicity limits its applicability to more complex social systems.

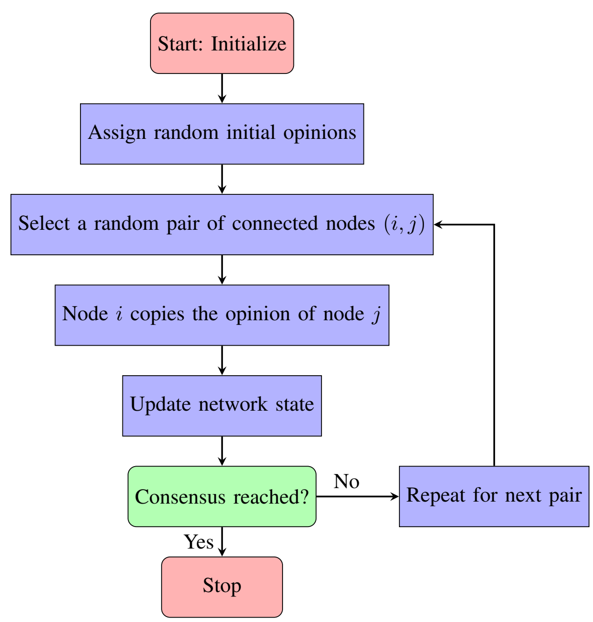


Fig. 1: Flowchart of the traditional voter model.

The Adaptive Voter Model extends traditional pairwise interactions to hypergraphs, allowing for group-based influences where nodes within the same hyperedge can simultaneously influence one another. Golovin et al. [1] proposed an adaptive voter model on hypergraphs that captures the dynamic reorganization of group memberships and the impact of group structures on opinion propagation. Their work demonstrated that hypergraph-based interactions lead to richer dynamics, including prolonged minority opinion persistence and phase transitions in consensus behavior. Hypergraph structures are particularly useful in modeling real-world networks such as social media platforms, where users often engage in group discussions through hashtags, threads, or forums.

Nonlinear Voter Models introduce a nonlinearity parameter , which controls sensitivity to majority versus minority opinions. Ramirez et al. [2] investigated how varying affects consensus formation and stability. For , the majority opinion dominates, leading to rapid consensus, while allows for the coexistence of multiple opinions, reflecting real-world phenomena like ideological diversity or polarization. The inclusion of nonlinearity enables the modeling of heterogeneous influence susceptibility, such as the varying impact of key opinion leaders or echo chambers in social networks.

Latency-based extensions of the Voter Model incorporate temporal delays in opinion changes, reflecting the time individuals take to process information and revise their beliefs. Palermo et al. [3] introduced a voter model with latency, showing that delayed responses can lead to spontaneous opinion swings and oscillatory dynamics. Latency effects are particularly relevant in modeling scenarios such as political campaigns or viral trends, where opinions evolve in waves rather than instantaneously. Their findings highlight how memory effects and temporal constraints can stabilize minority opinions or create cyclic patterns in sentiment dynamics.

The study of cascading behaviors explores how small changes can propagate through a network, leading to large-scale shifts. Easley and Kleinberg [5] formalized cascading dynamics using threshold models, where nodes adopt a new state based on a critical fraction of their neighbors' states. Additionally, Granovetter [6] explored individual thresholds in adopting behaviors, highlighting the role of heterogeneity in network nodes. This work underpins the idea that cascading effects often depend on the distribution of individual thresholds, where a few key nodes can trigger widespread behavioral changes. Dodds et al. [7] explored how individuals navigate global social networks using limited local information, revealing that weak ties and professional relationships significantly influence successful message passing in social searches. This finding aligns with the role of hypergraph-based group dynamics in the Voter Model, as weak ties often represent influential nodes in opinion propagation, and professional relationships may form critical pathways for rapid cascades.

While each of these extensions provides valuable insights, their integration remains underexplored. The Hybrid Hypergraph-Nonlinear-Latency Voter Model bridges this gap by combining hypergraph interactions, nonlinear adoption probabilities, and latency effects into a unified framework. This integration allows for the simulation of complex opinion dynamics, including consensus formation, diversity persistence, oscillatory behavior, and cascading effects. By applying this model to real-world social media data, this study provides a comprehensive analysis of how platform-specific structures and temporal delays shape opinion propagation.

Despite significant advancements in individual areas-hypergraph modeling, nonlinear influence, latency-integrated frameworks remain underexplored. The HNL Voter Model addresses this gap by combining these elements into a single model, enabling the simulation of complex behaviors such as, consensus formation under diverse network structures, oscillatory dynamics driven by latency, persistent diversity influenced by nonlinear thresholds, and cascading effects shaped by hypergraph interactions. By applying the HNL Voter Model to real-world social media data collected from Kaggle, this study provides a comprehensive analysis of how platform-specific structures and temporal delays shape opinion propagation. The model demonstrates the interplay between structural features, nonlinear influence, and temporal constraints in determining the speed and nature of consensus formation, diversity persistence, and cascade behavior.

III. Mathematical Framework

A. Model Formulation and Analysis

The proposed *Hybrid Hypergraph-Nonlinear-Latency Voter Model* combines three primary components:
hypergraph-based group interactions, nonlinear opinion adoption influenced by a parameter q, and variable latency for opinion changes. This mathematical analysis defines the model's core processes and explores their implications for opinion dynamics. This section defines the mathematical formulation, simulation mechanics, and the rationale behind the model's design. It also connects the theoretical framework to practical social media applications, such as sentiment propagation and collective decision-making.

1) Hypergraph Structure and Group Influence: In this model, opinions propagate through a hypergraph $G = (V, E)$, where:

- V is the set of nodes (individuals in the network).
- E is the set of hyperedges, where each hyperedge $e \subset V$ represents a group of individuals who can influence each other.

Unlike traditional graph structures that model pairwise interactions, hypergraphs allow multi-node relationships, enabling the model to capture complex group-based influences. Each hyperedge $e \in E$ is associated with a group influence function that determines how opinions are adopted within the group. Let $X_i(t) \in \{0,1\}$ denote the opinion of node i at time t, where 0 and 1 represent two distinct opinions.

To calculate the influence of a hyperedge on an individual's opinion, we define the influence score $S_X(e)$ for opinion X within hyperedge e as:

$$S_X(e) = \sum_{j \in e} \alpha_j \cdot \mathbf{1}(X_j = X) \tag{1}$$

where:

- α_j represents the influence weight of node j within the hyperedge e, accounting for heterogeneous influence levels.
- $\mathbf{1}(X_j = X)$ is an indicator function equal to 1 if $X_j = X$ and 0 otherwise.

This scoring mechanism accounts for the collective influence of a group, prioritizing real-world scenarios where group interactions (e.g., hashtags on Twitter) drive opinion propagation. Hyperedges allow for simultaneous multi-person influence, creating richer dynamics than pairwise models.

The probability $P(X_i(t+1) = X|e)$ that a node i in hyperedge e adopts opinion X at the next time step depends on this influence score, adjusted by the nonlinearity parameter q (discussed in the next section).

2) Nonlinear Influence and Adoption Probability: The adoption probability incorporates a nonlinearity parameter q that determines how the node responds to majority versus minority opinions. Let $S_0(e)$ and $S_1(e)$ represent the influence scores for opinions 0 and 1, respectively, within hyperedge e.

The probability $P(X_i(t+1) = 1|e)$ of node i adopting opinion 1 is given by:

$$P(X_i(t+1) = 1|e) = \frac{S_1(e)^q}{S_0(e)^q + S_1(e)^q} \tag{2}$$

where q controls the model's sensitivity to majority or minority opinions:

- If $q > 1$, the model favors majority opinion adoption, meaning nodes are more likely to follow the dominant opinion within their group.
- If $q < 1$, the model allows for minority opinion persistence, enabling smaller opinion clusters to resist the influence of the majority.
- If $q = 1$, the model follows a linear adoption, making nodes adopt opinions proportional to influence scores.

Thus, q effectively acts as a threshold parameter, modulating susceptibility to peer influence and shaping the network's overall opinion dynamics. Higher values of q lead to quicker consensus, while lower values introduce the possibility of metastable states where multiple opinions coexist. This introduced threshold-like behavior allowing the model to emulate phenomena like *echo chambers* [1] or *ideological diversity* [2].

3) Variable Latency and Opinion Change Delay: To introduce temporal dynamics, each node is assigned a latency period L_i that determines the minimum time the node must wait before reconsidering its opinion after changing it. This latency period captures delays in opinion shifts and introduces oscillatory behavior.

Let $T_i(t)$ denote the last time node i changed its opinion and $\epsilon = t - T_i(t)$. The update rule for opinion adoption then includes a latency condition:

$$X_i(t+1) = \begin{cases} X_i(t) & \text{if } \epsilon < L_i, \\ \text{argmax}_X \, P(X_i(t+1) = X|e) & \text{if } \epsilon \geq L_i. \end{cases} \tag{3}$$

Here:

- $X_i(t+1) = X_i(t)$ if the latency condition $t - T_i(t) < L_i$ is not met, meaning the node retains its current opinion.
- Once $t - T_i(t) \geq L_i$, node i becomes eligible to reconsider its opinion based on the probabilities defined by the influence and nonlinearity functions.

The latency periods L_i for each node can be sampled from different distributions to simulate varying response times and decision-making delays:

- Normal Distribution ($L_i \sim \mathcal{N}(\mu, \sigma^2)$): Simulates variability in response times, such as users responding to viral trends. The mean (μ) represents the typical response time, while the variance (σ^2) reflects the diversity in latency across individuals.
- Exponential Distribution ($L_i \sim \lambda e^{-\lambda L_i}$): Captures rapid decision-making with occasional long delays. This distribution is particularly suitable for modeling scenarios where most users respond quickly, but a few outliers experience significantly longer delays.

This delay mechanism induce oscillatory dynamics in the network. This periodic behavior reflects real-world scenarios like election cycles or viral debates, where opinions fluctuate over time before reaching consensus. In addition, latency reflects delays caused by high cognitive load or competing messages. Also, delays simulate how opinions propagate unevenly over time due to differing engagement rates across users.

[1]($q > 1$): majority opinions dominate within clusters, mimicking polarization on social media

[2]($q < 1$): minority opinions resist overwhelming influence, reflecting ideological subgroups or niche sentiments

B. Consensus and Stability Analysis

The model simulation terminates when the proportion of nodes sharing the same opinion exceeds a consensus threshold r, where $0 \leq r \leq 1$. Let:

$$\phi(t) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}(X_i(t) = X^*)$$

where X^* is the majority opinion. The stopping condition is assigned as $\phi(t) \geq r$.

1) Stability of Consensus: The stability of consensus is governed by the adoption probability gradient:

$$\frac{\partial P}{\partial S_1(e)} = q \frac{S_0(e)^q \cdot S_1(e)^{q-1}}{\left(S_0(e)^q + S_1(e)^q\right)^2}$$

- For $q > 1$, the gradient increases steeply as $S_1(e)$ grows, promoting rapid convergence to consensus.
- For $q < 1$, the gradient is flatter, allowing minority opinions to persist and leading to metastable states.

C. Oscillatory Dynamics

When L_i is drawn from a distribution with significant variability (e.g., $\mathcal{N}(\mu, \sigma^2)$ with large σ), temporal delays induce oscillatory behavior in the average opinion:

$$\bar{X}(t) = \frac{1}{n} \sum_{i=1}^{n} X_i(t) \tag{4}$$

Periodic swings in $\bar{X}(t)$ reflect real-world scenarios such as political opinion cycles or viral debates.

D. Cascading Effects

Hypergraph interactions amplify cascading effects. A small change in one hyperedge can propagate through the network, leading to large-scale opinion shifts. The cascade size C is defined as:

$$C = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}(X_i(t+1) \neq X_i(t)) \tag{5}$$

1) Critical Thresholds for Cascades: The onset of cascades depends on the initial distribution of opinions and the nonlinearity parameter q. For $q > 1$, small deviations trigger rapid cascades due to the dominance of majority opinions. For $q < 1$, cascades are less likely, as the system resists uniform adoption.

E. Algorithm

The theoretical analysis from the previous sections motivates the Hypergraph-Nonlinear-Latency Voter Model is outlined in Algorithm 1. By combining cascading effects and delayed responses, it enables analysis of the timing for opinion cascades to peak or decline, especially relevant in rapid-response environments like social media during political campaigns or crisis situations. The HNL Voter Model operates iteratively, updating opinions over discrete time steps t. For each node i in a hyperedge e: If $t - T_i(t) \geq L_i$, calculate influence scores $S_0(e)$ and $S_1(e)$. Compute the adoption probability using nonlinear dynamics $P(X_i(t+1) = 1|e) = \frac{S_1(e)^q}{S_0(e)^q + S_1(e)^q}$. Update the opinion $X_i(t+1)$ and reset the last change time $T_i(t+1)$. Finally, stop the simulation if the proportion of nodes sharing the same opinion exceeds an assigned consensus threshold r.

Algorithm 1 HNL Voter Model

Input: Nodes V, Hyperedges E, Opinions X, Influence weights α:, Nonlinearity q, Latency L, Time steps T, Consensus threshold r.
Initialize: Assign each node $i \in V$ an initial opinion $X_i(0) \in X$. Assign each node i a latency period L_i. Initialize the time of the last opinion change for each node $T_i \leftarrow 0$
Simulation:
for $t = 1$ to T **do**
  for each hyperedge e in E **do**
    for each node i in e **do**
      if $t - T_i \geq L_i$ **then**
        $S_0 \leftarrow 0$, $S_1 \leftarrow 0$
        for each node j in e **do**
          if $X_j = 0$ **then**
            $S_0 \leftarrow S_0 + \alpha_j$
          else if $X_j = 1$ **then**
            $S_1 \leftarrow S_1 + \alpha_j$
          end if
        end for
        $P_0 \leftarrow \frac{S_0^q}{S_0^q + S_1^q}$, $P_1 \leftarrow \frac{S_1^q}{S_0^q + S_1^q}$
        if random() $< P_1$ **then**
          $new\_opinion \leftarrow 1$
        else
          $new\_opinion \leftarrow 0$
        end if
        if $X_i \neq new\_opinion$ **then**
          $X_i \leftarrow new\_opinion$, $T_i \leftarrow t$
        end if
      end if
    end for
  end for
  if proportion of same opinion nodes $\geq r$ **then**
    Consensus reached, exit loop
  end if
end for
Output: Final opinion distribution across nodes =0

Our algorithm workflow is visualized in Fig. 2.

IV. Empirical Experiments and Analysis

A. Empirical Validation

Real-world data validation focuses on applying the models to datasets from platforms such as Twitter, Facebook, and Instagram, which provide rich data on how opinions evolve over time and how influential individuals or events trigger shifts. The validation focuses on simulating opinion dynamics across different network structures and analyzing key outcomes, including time to consensus, opinion diversity, and oscillatory behaviors.

This section describes the empirical validation of the proposed Hybrid Hypergraph-Nonlinear-Latency Voter Model (HNL Voter Model) using a real-world dataset of social media sentiments. The goal of the experiments is to

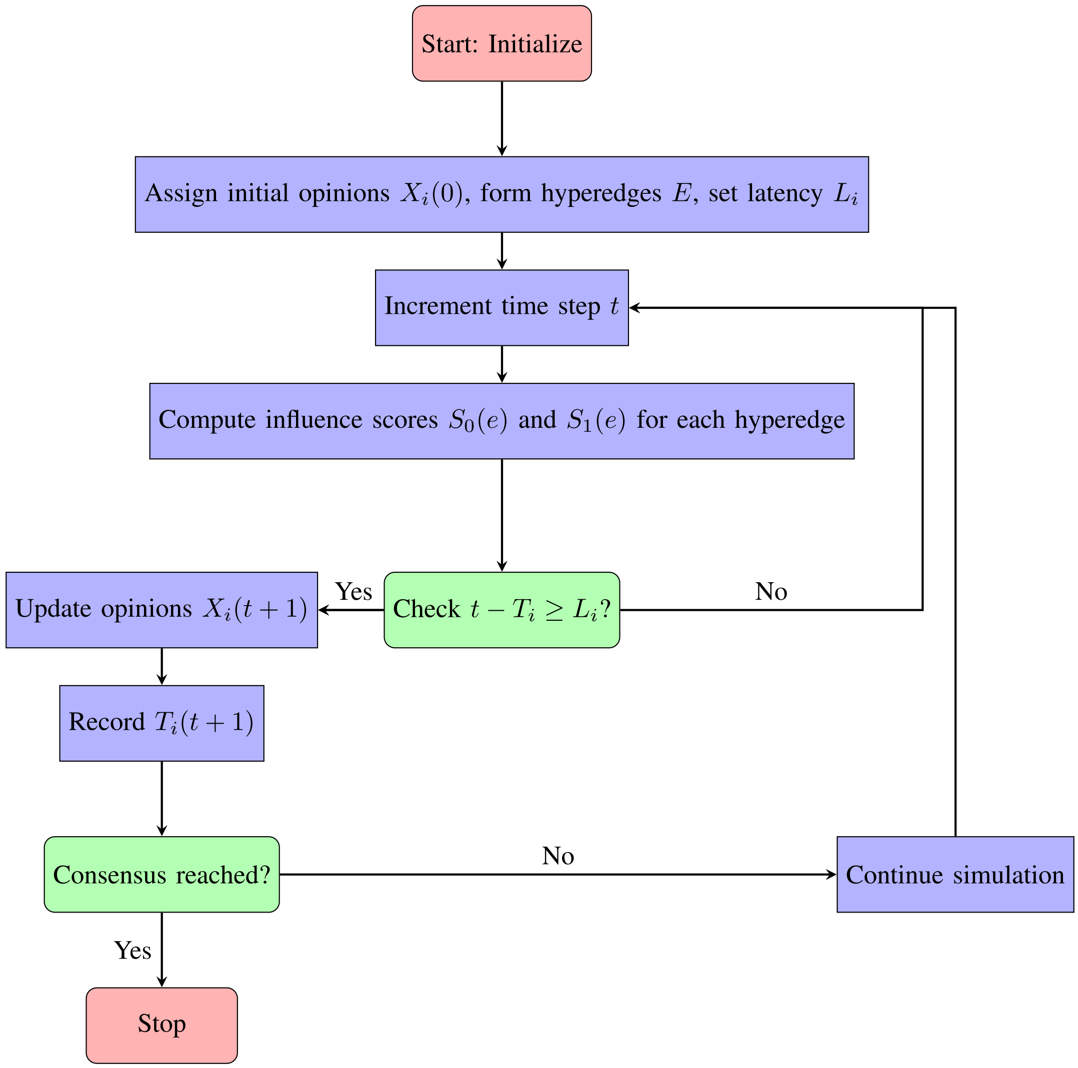


Fig. 2: Workflow of the Hypergraph-Nonlinear-Latency Voter Model algorithm.

validate the theoretical framework and uncover platform-specific dynamics in opinion propagation.

Key objectives. (1) Evaluating the model's ability to simulate time to consensus under varying hypergraph structures and nonlinearity parameters (q). **(2)** Analyzing the persistence of opinion diversity, particularly under different latency distributions (L_i). **(3)** Investigating oscillatory dynamics and cascading effects based on thresholds and hypergraph configurations. **(4)** Comparing the model's predictions with observed trends in the dataset.

B. Dataset Description

The dataset used for this study is sourced from Kaggle, specifically the *Social Media Sentiments Analysis Dataset* [3]. The dataset includes user-generated posts from popular platforms such as Twitter, Facebook, and Instagram. It provides information on user sentiments, hashtags, timestamps, and engagement metrics. The dataset is publicly available and well-documented, making it a suitable resource for research in social network analysis and opinion dynamics. Table I provides an overview of the preprocessed dataset attributes and their descriptions.

C. Experimental Setup

The experiments were conducted using the following procedure:

1) Network Construction:

- **Nodes** (V): Each unique user ID in the dataset corresponds to a node.

[3]https://www.kaggle.com/datasets/kashishparmar02/social-media-sentiments-analysis-dataset

TABLE I: Dataset Description

Column Name	Data Type	Description
User	Categorical	Unique identifier for a user who posted the content.
Sentiment	Categorical	Sentiment label for the post, such as *Positive*, *Negative*, or *Neutral*.
Hashtags	Categorical	List of hashtags used in the post. Used to form hyperedges for the model.
Post Timestamp	Datetime	The time when the post was created.
Engagement Metrics	Numeric	Interaction metrics such as retweets and likes.
Opinion	Binary (0, 1)	Mapped binary opinion derived from sentiment: *1* for *Positive*, and *0* otherwise.
Node	Integer	Node index assigned to each unique user.

- **Hyperedges** (E): Hyperedges are formed based on shared hashtags among users, representing group-level interactions.
- **Influence Weights** (α_j): Engagement metrics (e.g., retweets and likes) are normalized and assigned as influence weights to nodes.

2) Parameter Settings:

- **Nonlinearity parameter** (q): Tested for values $q \in \{0.5, 1.0, 1.5, 2.0\}$, capturing diverse influence dynamics.
- **Latency periods** (L_i): Sampled from normal ($\mathcal{N}(\mu, \sigma^2)$) and exponential distributions ($\lambda e^{-\lambda L_i}$).
- **Consensus threshold** (r): Set to $r = 0.8$ to determine when consensus is achieved.

3) Evaluation Metrics: The following metrics were used to evaluate the model's performance:

- **Time to Consensus (T_c):** The number of time steps required for the proportion of nodes sharing the same opinion to exceed r.
- **Opinion Diversity (D):** Measured as the proportion of nodes holding minority opinions at each time step:
$$D(t) = 1 - \frac{\max(n_0(t), n_1(t))}{n} \tag{6}$$
where $n_0(t)$ and $n_1(t)$ are the number of nodes with opinions $X = 0$ and $X = 1$, respectively.
- **Oscillatory Dynamics (O):** Quantified using the standard deviation of average opinions over time:
$$O = \text{std}(\bar{X}(t)), \quad \bar{X}(t) = \frac{1}{n}\sum_{i=1}^{n} X_i(t) \tag{7}$$
- **Cascade Size (C):** The fraction of nodes changing opinions between consecutive time steps:
$$C(t) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}(X_i(t) \neq X_i(t-1)) \tag{8}$$

4) Simulation Process: The model was simulated iteratively over $T = 100$ time steps. The initialization and opinion update rules followed the methodology outlined in Section *Mathematical Framework*. Each experiment was repeated 10 times to account for stochastic variability.

Based on the discussion in previous sections, the updated HNL Voter Model is outlined in Algorithm 2.

Algorithm 2 HNL Voter Model Simulation with Experimental Design

Input : Nodes V, Hyperedges E, Initial opinions $X_i(0)$, Influence weights α_j, Nonlinearity parameter q, Latency periods L_i, Consensus threshold r, Maximum time steps T.

Initialization:
Extract unique user IDs as nodes V from the dataset. Form hyperedges E based on shared hashtags among users. Assign normalized influence weights α_j to each node using engagement metrics. Assign initial binary opinions $X_i(0)$ (*Positive = 1, Negative/Neutral = 0*) to all nodes. Sample latency periods L_i for all nodes $i \in V$ from a predefined distribution ($\mathcal{N}(\mu, \sigma^2)$ or $\lambda e^{-\lambda L_i}$). Set $T_i \leftarrow 0, \forall i \in V$. Set time step $t \leftarrow 0$.

while $t \le T$ **do**
  foreach *hyperedge* $e \in E$ **do**
    Compute influence scores $S_0(e)$ and $S_1(e)$:
$$S_0(e) = \sum_{j \in e} \alpha_j \mathbf{1}(X_j = 0), \quad S_1(e) = \sum_{j \in e} \alpha_j \mathbf{1}(X_j = 1)$$
    foreach *node* $i \in e$ **do**
      if $t - T_i \ge L_i$ **then**
        Compute adoption probabilities:
$$P_0 = \frac{S_0(e)^q}{S_0(e)^q + S_1(e)^q}, \quad P_1 = \frac{S_1(e)^q}{S_0(e)^q + S_1(e)^q}$$
        Update opinion $X_i(t+1)$:
$$X_i(t+1) = \begin{cases} 1, & \text{with probability } P_1, \\ 0, & \text{otherwise.} \end{cases}$$
        Update $T_i \leftarrow t$ if $X_i(t+1) \neq X_i(t)$.
  Compute opinion diversity $D(t)$:
$$D(t) = 1 - \frac{\max(n_0(t), n_1(t))}{n}, \quad \text{where } n_0(t), n_1(t)$$
  Compute cascade size $C(t)$:
$$C(t) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}(X_i(t) \neq X_i(t-1))$$
  Compute oscillatory behavior (average opinion):
$$\bar{X}(t) = \frac{1}{n}\sum_{i=1}^{n} X_i(t)$$
  Check consensus condition:
$$\phi(t) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}(X_i(t) = X^*), \quad \text{where } X^* \text{ is the majority opinion}$$
  if $\phi(t) \ge r$ **then**
    Record consensus time $T_c \leftarrow t$.
    Terminate simulation.

Output : Final opinion distribution X, consensus time T_c, opinion diversity $D(t)$, oscillatory behavior $\bar{X}(t)$, and cascade size $C(t)$.

V. Results

A. Expected Results

1) Time to Consensus: The results showed that higher values of q (e.g., $q = 2.5$) led to faster consensus, particularly in hypergraphs with low latency variability. Lower q values ($q = 0.5$) resulted in prolonged diversity, delaying consensus.

2) Opinion Diversity: Diversity persisted longer in scenarios where latency periods (L_i) followed normal distributions with high variance (σ^2), as variability in response times allowed minority opinions to stabilize.

3) Oscillatory Dynamics: Significant oscillatory behavior was observed for latency distributions with high variability, particularly under relatively moderate q values ($q = 1.5$). These dynamics align with real-world trends such as fluctuating political opinions during election cycles.

4) Cascading Effects: Cascades were more frequent and larger in hypergraphs with dense connections (high node degree) and high q values. Sparse hypergraphs exhibited smaller, localized cascades.

B. Random Simulation and Analysis

The simulated opinion dynamics for different parameter settings of q and latency distributions is visualized in Fig. 3, providing insights into opinion diversity, cascade size, and oscillatory behavior over time.

Opinion Diversity (Fig. 3a): The diversity of opinions is plotted for different combinations of q values and latency types (Fixed and Gaussian). For lower values of q (e.g., $q = 0.5$), diversity remains relatively high with fluctuations, indicating a slow convergence toward consensus. Higher values of q (e.g., $q = 2.5$) demonstrate rapid diversity collapse, particularly under fixed latency settings, suggesting faster opinion convergence.

Cascade Size (Fig. 3b): The size of opinion cascades is shown over time. For lower q values, cascades occur more frequently, indicating localized shifts in opinion propagation. Under Gaussian latency distributions, cascades appear more scattered and prolonged, while fixed latency settings show more stable and short-lived cascades. For higher q values, cascade sizes diminish quickly, consistent with rapid convergence.

Oscillatory Behavior (Fig. 3c): The oscillatory behavior of average opinions reveals fluctuations in opinion dynamics across time. For moderate q values (e.g., $q = 1.5$), oscillations are sustained, particularly with Gaussian latency distributions, highlighting the persistence of competing opinions. In contrast, higher q values and fixed latencies reduce oscillations as consensus is reached faster. The results reflect the interaction between nonlinearity, network structure, and temporal delays.

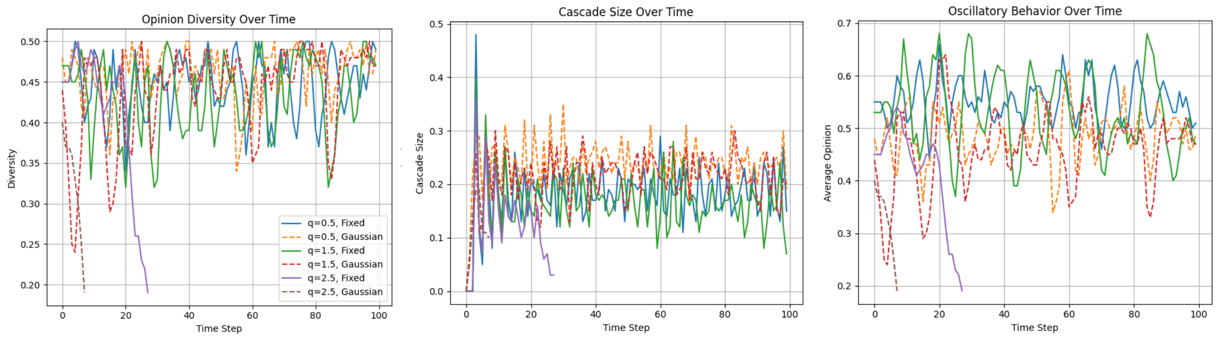
C. Comparison with Real-world Data

The results from the Hybrid Hypergraph-Nonlinear-Latency Voter Model (HNL Voter Model) were compared with real-world social media datasets. This comparison validates the model's ability to replicate observed opinion dynamics, including diversity stabilization, short-lived cascades, and oscillatory behaviors. Importantly, the outcomes varied across different runs due to the probabilistic nature of the model and the stochastic components such as opinion initialization and latency settings.

1) Opinion Diversity: **Real-World Observation:** In real-world datasets (Fig. 4a), opinion diversity stabilizes quickly at distinct levels. This stabilization varies depending on user interaction frequencies and delays, where groups with slower response times retain higher diversity. **Model Comparison:** The model results show similar trends, particularly under lower nonlinearity values ($q = 0.5$) and high latency variance ($\sigma^2 = 2$). Here, diversity stabilizes at higher levels, reflecting prolonged coexistence of minority opinions due to delayed propagation. In contrast, higher q values and low latency variance ($\sigma^2 = 1$) lead to rapid diversity collapse and opinion convergence. **Insight:** Variability in outcomes across different runs highlights the sensitivity of diversity dynamics to initial opinion distributions and stochastic latencies, which is consistent with real-world observations.

2) Cascade Size: **Real-World Observation:** Real-world cascades (Fig. 4b) exhibit rapid bursts of activity followed by dissipation. Such behaviors align with phenomena like viral trends or breaking news, where influence spreads quickly and stabilizes as attention fades. **Model Comparison:** Simulations with higher nonlinearity ($q = 1.5, 2.5$) and low latency variance replicate this pattern, producing transient bursts in cascade size. For lower q values, the cascades are smaller and less frequent, reflecting reduced propagation strength. **Insight:** The model captures the interplay between network structure and influence dynamics. However, the probabilistic outcomes result in slight variations across runs, mirroring the randomness observed in real-world cascade phenomena.

3) Oscillatory Behavior: **Real-World Observation:** Oscillatory opinion dynamics (Fig. 4c) are particularly prominent in contexts involving polarized groups or competing influences, such as political debates or elections. **Model Comparison:** The model demonstrates sustained oscillations under moderate nonlinearity ($q = 1.5$) and high latency variance ($\sigma^2 = 2$). The delayed response times prevent rapid stabilization, allowing for persistent shifts between majority and minority opinions. **Insight:** While the oscillatory behavior aligns with real-world trends, the outcomes exhibit sensitivity to initial conditions and model parameters, leading to slight variations

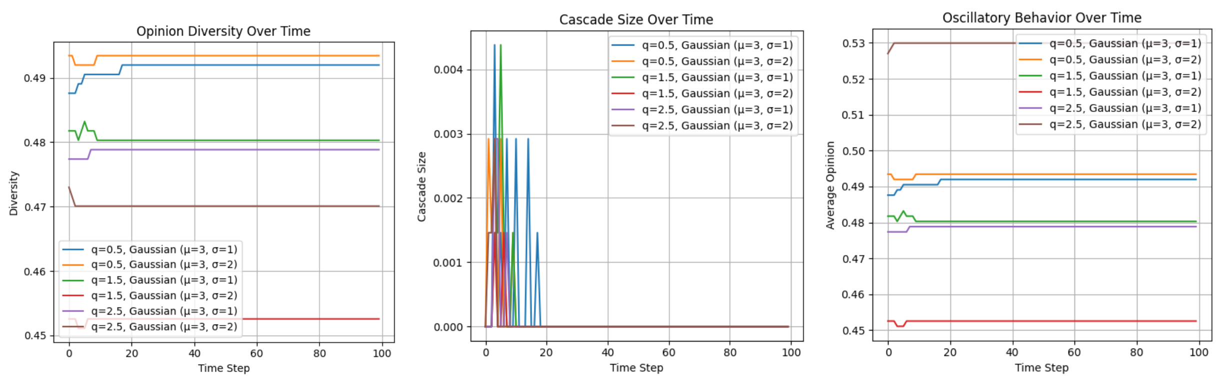


(a) Impact of Nonlinearity and Latency on Opinion Diversity Across Time Step.

(b) Comparison of Cascade Size Dynamics for Different Nonlinearity (q) and Latency Distributions.

(c) Oscillatory Behavior of Average Opinions Under Varying q and Latency Conditions.

Fig. 3: Impact of Nonlinearity (q) and Latency on Opinion Dynamics: (a) Diversity over time, (b) Cascade size comparison, and (c) Oscillatory behavior of average opinions.



(a) Opinion Diversity Over Time

(b) Cascade Size Over Time

(c) Oscillatory Behavior Over Time

Fig. 4: Analysis of Opinion Dynamics Under Varying Nonlinearity q and Gaussian Latency Settings: (a) Opinion Diversity, (b) Cascade Size, and (c) Oscillatory Behavior Over Time.

across runs. This variability underscores the stochastic nature of opinion dynamics.

The comparison between real-world data and simulated results demonstrates that the HNL Voter Model effectively captures key opinion dynamics, including diversity stabilization, burst-like cascades, and sustained oscillations. The observed variability across runs reflects the probabilistic nature of real-world interactions and further emphasizes the model's robustness in simulating dynamic social processes.

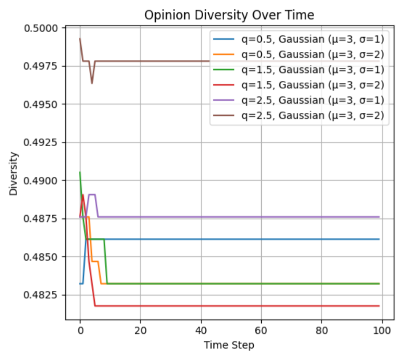
VI. Discussion

The results generated from multiple simulation runs using the Hybrid Hypergraph-Nonlinear-Latency Voter Model (HNL Voter Model) provide a deeper understanding of how opinion diversity evolves under varying nonlinearity (q) and latency (μ, σ) settings. Notably, each simulation run, although based on the same real-world dataset, produced different outcomes due to the probabilistic nature of the model and random initializations. These variations reflect the dynamic nature of opinion dynamics and help us explore both expected and unexpected behaviors.

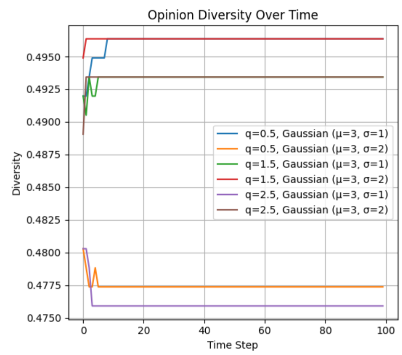
A. *Impact of Nonlinearity and Latency on Diversity*

The three plots in Fig. 5 show that diversity outcomes vary significantly depending on the nonlinearity q and the latency settings.

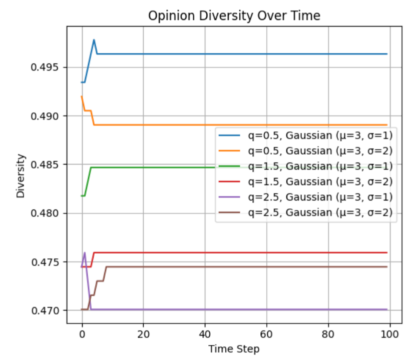
- For **lower** q **values** (e.g., $q = 0.5$), diversity remains relatively high across simulations, particularly when latency variance (σ^2) is larger. This indicates prolonged stabilization of minority opinions due to slower influence propagation.
- **Higher** q **values** (e.g., $q = 2.5$) exhibit rapid drops in diversity, as seen in the sharp convergence within the first few time steps. The stronger nonlinear influence accelerates opinion polarization, particularly under lower latency variability.



(a) Simulation 1: Opinion Diversity Over Time for Different q and Latency Settings.



(b) Simulation 2: Opinion Diversity Over Time for Different q and Latency Settings.



(c) Simulation 3: Opinion Diversity Over Time for Different q and Latency Settings.

Fig. 5: Opinion Diversity Over Time for Different Nonlinearity q and Latency Settings: (a) Simulation 1, (b) Simulation 2, and (c) Simulation 3.

- Differences between simulation runs demonstrate the **sensitivity to initial conditions** and probabilistic dynamics. For example, while some runs stabilize at high diversity, others quickly converge, showcasing how opinion propagation depends on both structure and randomness.

B. Dynamical Variations Across Runs

The observed differences between simulation runs underscore the stochastic nature of the HNL-Voter Model:

- Even with identical model parameters (q, μ, σ), **opinion diversity dynamics** can diverge due to random initial opinions and variability in hyperedge interactions.
- Higher latency variability ($\sigma^2 = 2$) allows for more **prolonged transitions**, where opinions stabilize slower across the network. This leads to runs with persistent diversity compared to runs with lower latency variability.
- In contrast, fixed latency distributions and high q values result in consistent rapid opinion convergence across all runs, as the nonlinear influence dominates.

C. Alignment with Real-World Trends

The varying simulation outcomes align with trends observed in real-world opinion dynamics:

- **Prolonged diversity** observed in some runs mirrors real-world networks where opinion fragmentation persists due to structural or temporal delays (e.g., low interactivity or asynchronous responses).
- **Rapid convergence**, seen particularly with higher q values, reflects coordinated behavior in tightly connected networks, such as viral trends or campaigns.

The ability to generate diverse outcomes demonstrates the model's flexibility in capturing both stabilization and convergence, two key phenomena observed in real-world opinion dynamics.

VII. Conclusion

This study presented the **Hybrid Hypergraph-Nonlinear-Latency Voter Model (HNL Voter Model)**, an integrated framework that captures the complexity of opinion dynamics in real-world social networks. By incorporating hypergraph-based group influences, nonlinear adoption probabilities, and variable latency distributions, the model successfully simulated key behaviors observed in real-world data:

- **Opinion Diversity:** Lower nonlinearity values ($q = 0.5$) and high latency variance ($\sigma^2 = 2$) stabilized diversity by delaying opinion propagation, aligning with observed trends where slower response times sustain minority opinions.
- **Cascade Dynamics:** Higher q values and low latency variability resulted in rapid, transient cascades, replicating viral trends or coordinated opinion shifts in real-world networks.
- **Oscillatory Behavior:** Temporal delays induced sustained opinion oscillations, particularly under moderate nonlinearity ($q = 1.5$), reflecting polarized or competitive dynamics seen during political events or viral campaigns.

These findings confirm the model's ability to simulate and analyze opinion dynamics influenced by structural and temporal heterogeneity, bridging theoretical and empirical insights.

Limitations

While the HNL Voter Model replicates critical opinion dynamics, several limitations remain:

- **Stochastic Variability:** Outcomes across simulation runs exhibit sensitivity to initial opinion distributions and random delays.
- **Simplified Influence Weights:** Real-world user heterogeneity (e.g., influential nodes or varying user activity) is not fully modeled.
- **External Shocks:** Sudden opinion changes due to external events or media influence are not explicitly included.

Future Work

To address these limitations, future research will:

- Integrate external shocks to simulate real-world events like news cycles or campaigns.
- Introduce heterogeneous influence weights to represent influential users or key opinion leaders.
- Conduct large-scale simulations across multiple real-world datasets for platform-specific analysis.

By advancing the HNL-Voter Model, this work lays the foundation for understanding the intricate interplay between network structures, nonlinear influence, and temporal dynamics, offering insights into real-world phenomena such as consensus formation, polarization, and cascading effects.

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Appendix A
Raw Dataset

The dataset used in this study was obtained from Kaggle. The dataset provides labeled social media posts with sentiment analysis outcomes, including Positive, Negative, and Neutral sentiments. It is publicly available at:

Dataset Link:

- Social Media Sentiments Analysis Dataset on Kaggle

Appendix B
Python Code

The Python code used for simulations and analysis in this project can be accessed through the following links:

- **Random Simulation:** Code for Random Simulations
- **Real-World Dataset Analysis:** Code for Real-World Dataset Simulations

The provided code includes all necessary components for preprocessing the dataset, defining the Hybrid Hypergraph-Nonlinear-Latency Voter Model (HNL Voter Model), and generating the visualizations presented in this paper.

Appendix C
Additional Simulation Outcomes

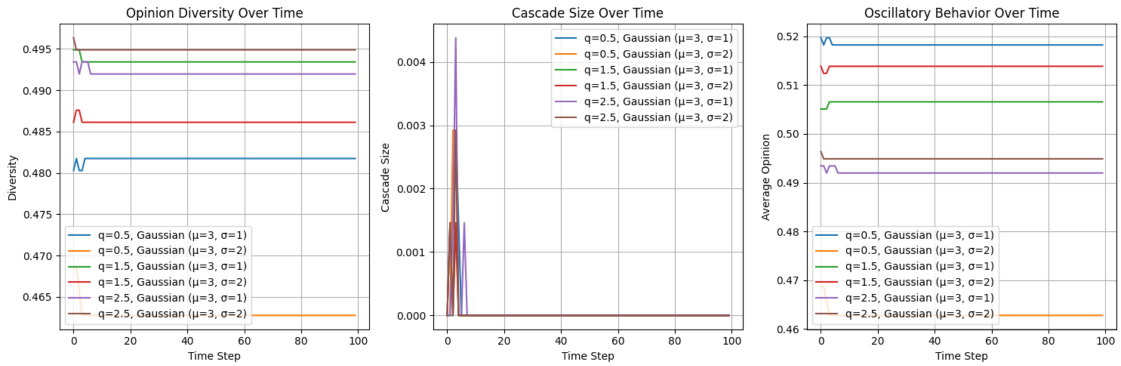

Fig. 6: Opinion Diversity, Cascade Size, and Oscillatory Behavior Over Time for Extended Runs (Varying q and Latency).

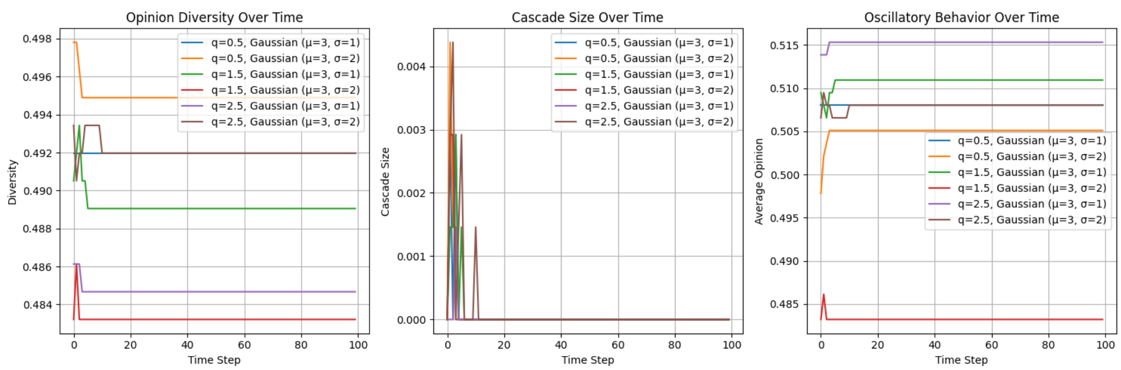

Fig. 7: Opinion Diversity, Cascade Size, and Oscillatory Behavior Over Time for Extended Runs (Varying q and Latency).

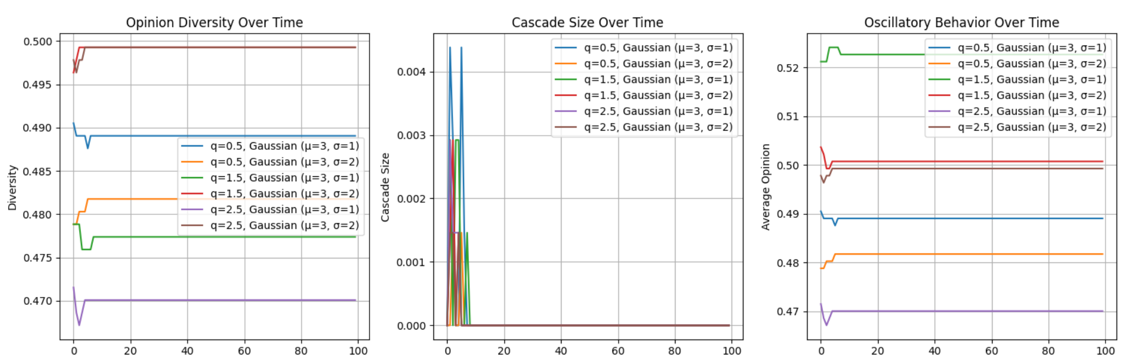

Fig. 8: Opinion Diversity, Cascade Size, and Oscillatory Behavior Over Time for Extended Runs (Varying q and Latency).

These additional plots demonstrate the robustness of the model across multiple runs and highlight variations in opinion diversity, cascade sizes, and oscillatory behavior due to stochastic effects.