COMP4500 Assignment 1

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# Part A (30 marks total)

## Question 1: Constructing SNI and directed graph

a) My origin SNI: 9845 0048 3052

```
for i = 2 to 12
    if d[i] == d[i - 1]
        d[i] = (d[i] + 3) mod 10
```

My new SNI: 9845 0348 3052

b) 0->3 0->5

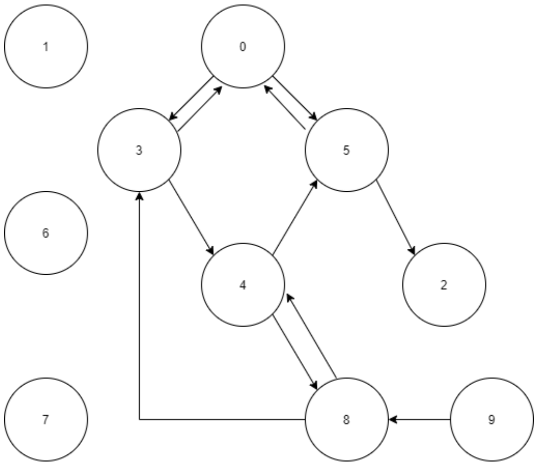
3->4 3->0

4->5 4->8

5->0 5->2

8->4 8->3

9->8




## Question 2: Strongly connected components

(a) Perform step 1 of the SCC algorithm using S as input.

Note: the purple node means on visit, gray node means finish visited, white node means still not visited.

The bold edge is the path of performing depth-first search

| x | Color[x] | Pi[x] |
|---|---|---|
| 0 | purple | undef |
| 1 | white | |
| 2 | white | |
| 3 | white | 0 |
| 4 | white | |
| 5 | white | |
| 6 | white | |
| 7 | white | |
| 8 | white | |
| 9 | white | |

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| 6 | white | |
| 7 | white | |
| 8 | white | |
| 9 | white | |

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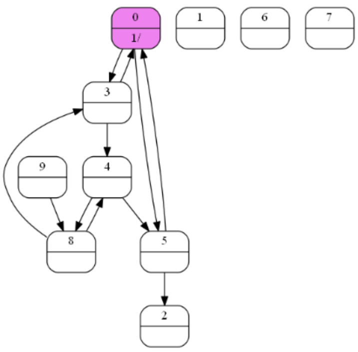

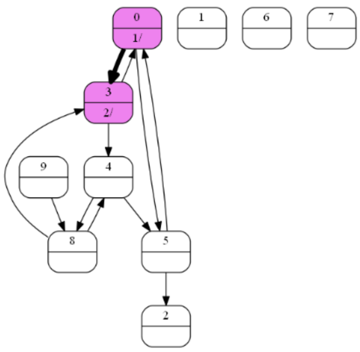

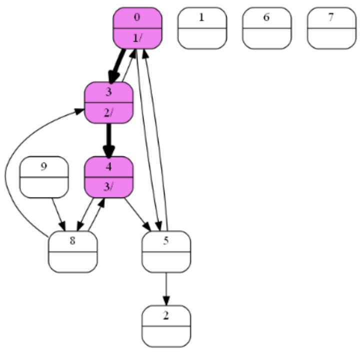

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| 3 | purple | 0 |
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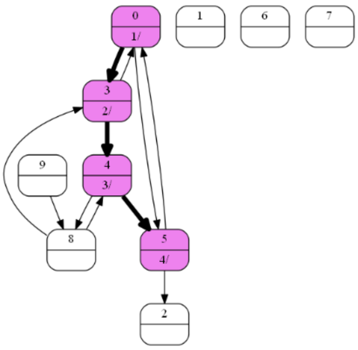
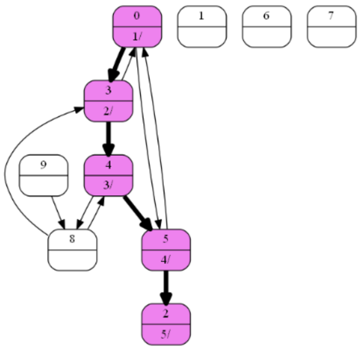
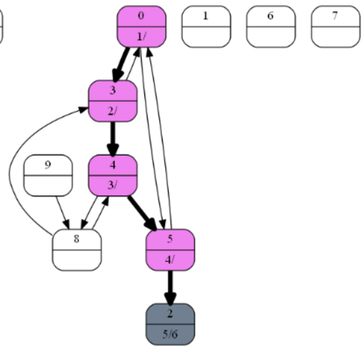
| 5 | purple | 4 |
| 6 | white | |
| 7 | white | |
| 8 | white | |
| 9 | white | |

| x | Color[x] | Pi[x] |
|---|---|---|
| 0 | purple | undef |
| 1 | white | |
| 2 | grey | 5 |
| 3 | purple | 0 |
| 4 | purple | 3 |
| 5 | purple | 4 |
| 6 | white | |
| 7 | white | |
| 8 | white | |
| 9 | white | |







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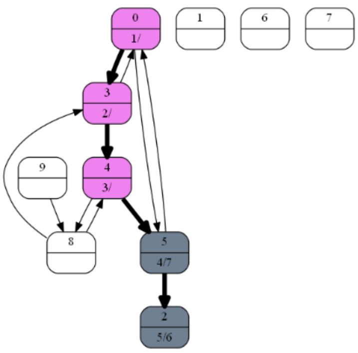

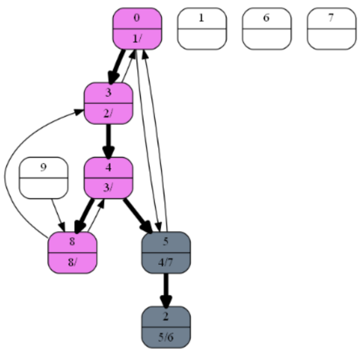

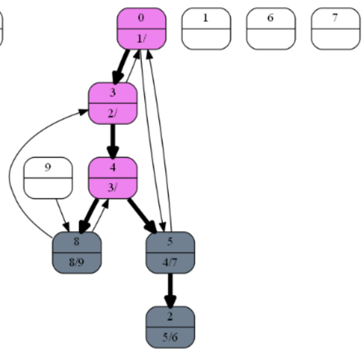

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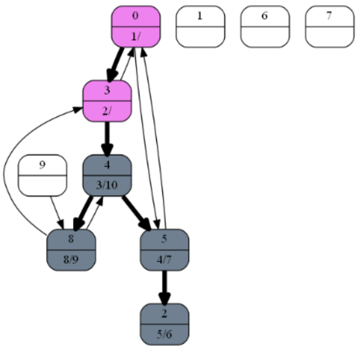

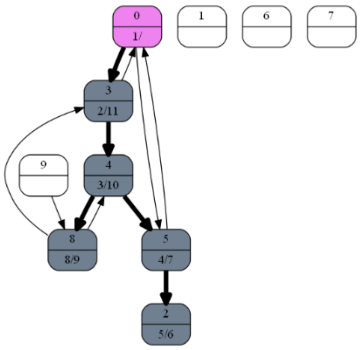

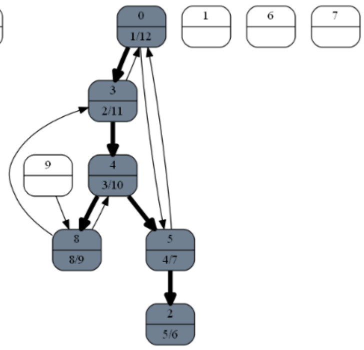

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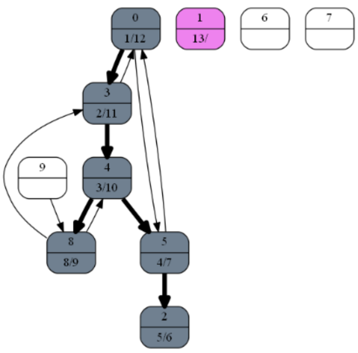

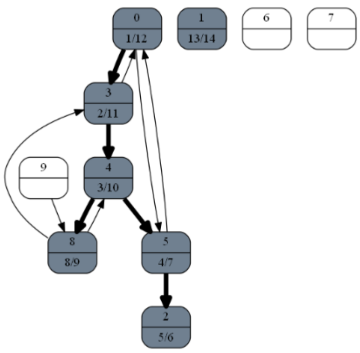

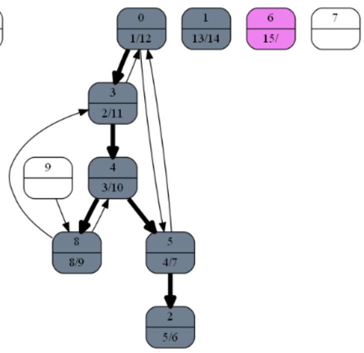

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|---|---|---|
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| 2 | grey | 5 |
| 3 | grey | 0 |
| 4 | grey | 3 |
| 5 | grey | 4 |
| 6 | grey | undef |
| 7 | white | |
| 8 | grey | 4 |
| 9 | white | |

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| 2 | grey | 5 |
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| 4 | grey | 3 |
| 5 | grey | 4 |
| 6 | grey | undef |
| 7 | purple | undef |
| 8 | grey | 4 |
| 9 | white | |

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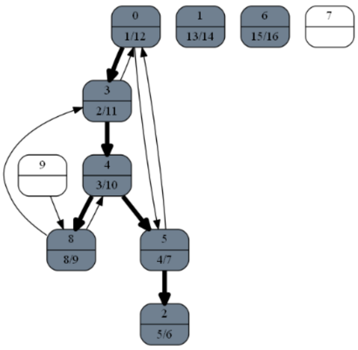
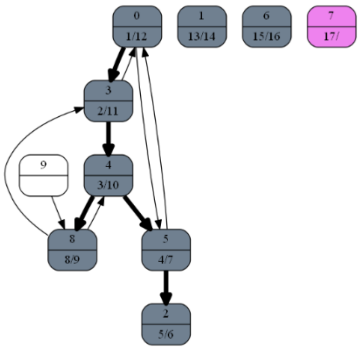
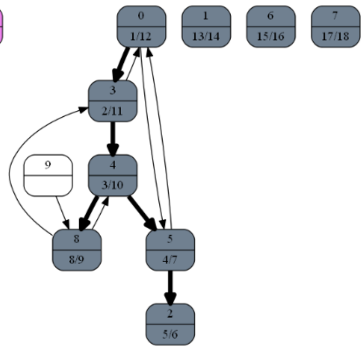
| 4 | grey | 3 |
| 5 | grey | 4 |
| 6 | grey | undef |
| 7 | grey | undef |
| 8 | grey | 4 |
| 9 | white | |

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| 1 | grey | undef |
| 2 | grey | 5 |
| 3 | grey | 0 |
| 4 | grey | 3 |
| 5 | grey | 4 |
| 6 | grey | undef |
| 7 | grey | undef |
| 8 | grey | 4 |
| 9 | purple | undef |

| x | Color[x] | Pi[x] |
|---|---|---|
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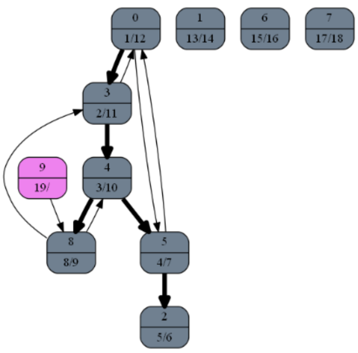
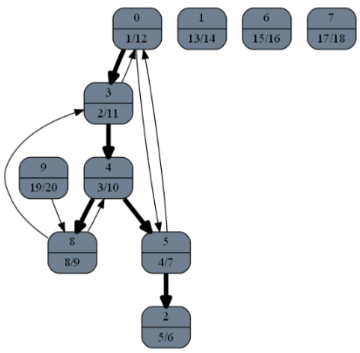
| 1 | grey | undef |
| 2 | grey | 5 |
| 3 | grey | 0 |
| 4 | grey | 3 |
| 5 | grey | 4 |
| 6 | grey | undef |
| 7 | grey | undef |
| 8 | grey | 4 |
| 9 | grey | undef |



Second last graph: Finishing times for the original graph G

Last graph: Strongly Connected Components

(b) 0 -> 5 0 -> 3

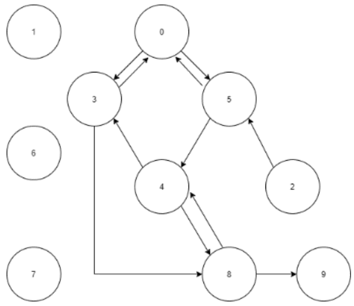
2 -> 5

3 -> 0 3 -> 8

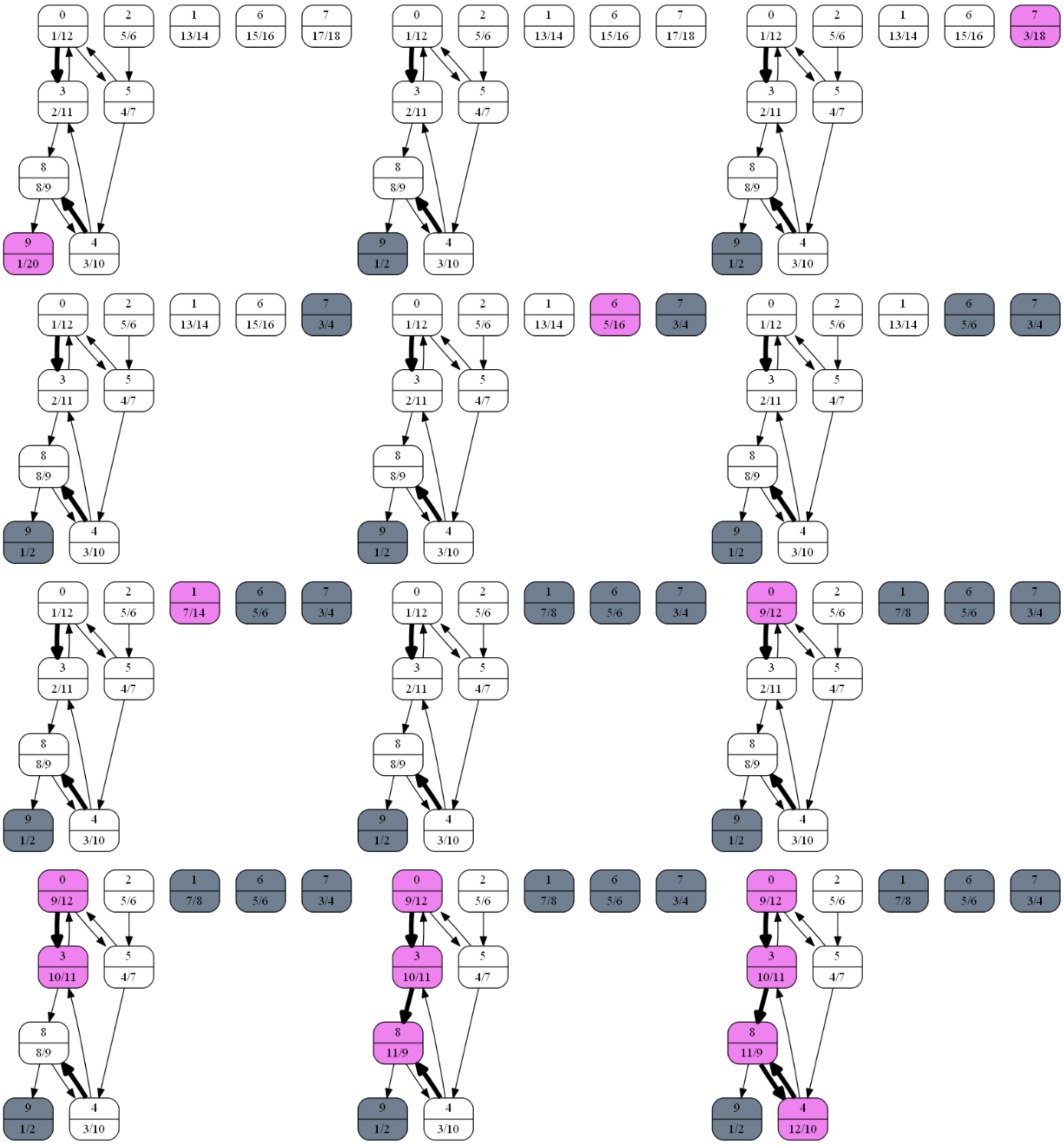
4 -> 8 4 -> 3

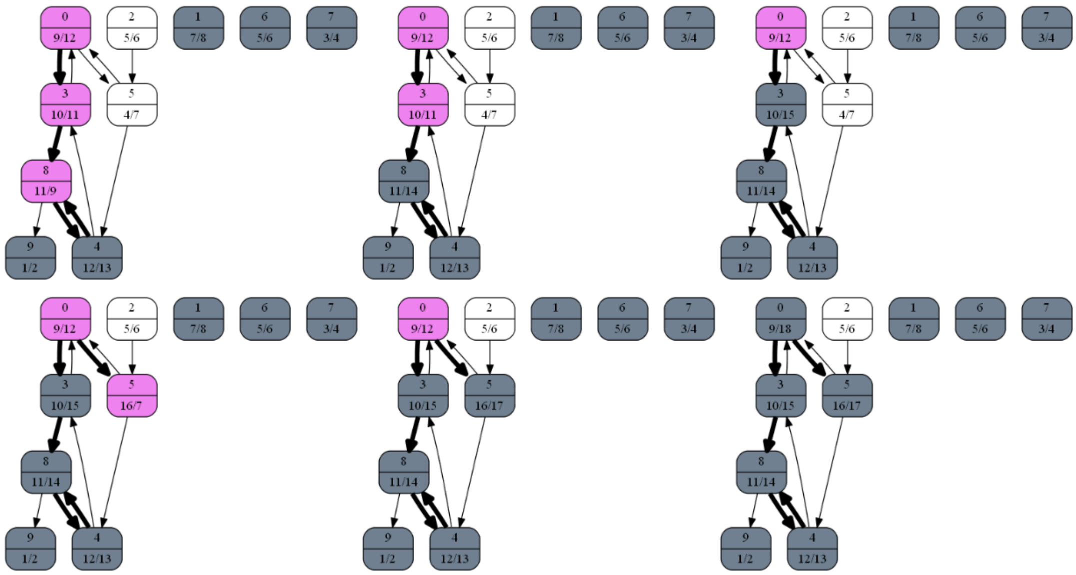
5 -> 4 5 -> 0

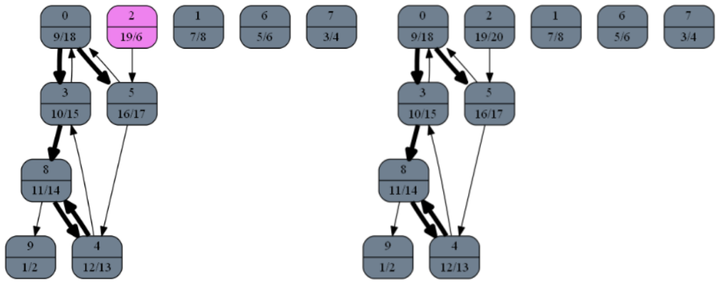
8 -> 9 8 -> 4



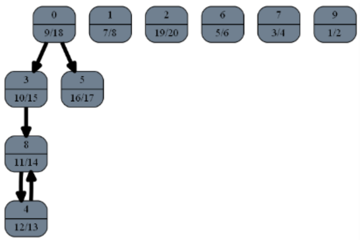

(c) Perform steps 3, 4 of the SCC algorithms.







Finishing times for $G^T$



Strongly Connected Component

1. 9
2. 7
3. 6
4. 1
5. 0 3 8 4 5
6. 2

## Part B (70 marks total)

Question 4: Worst-case time complexity analysis

a) In my method, I treat each interaction as a node in the graph, and the edges represent the two element sub paths of possible route of transmission from personFrom to personTo.

```
List<Interaction> FindTransmissionPath(int start, int end, List<Interaction> interactions) {
    personToInteractions: HashMap<Integer, HashSet<Interaction>> = new
HashMap().onLookupFail(new HashSet())
    endInteractions: HashSet<Interaction> = new HashSet()
    // I (number of interactions) iterations with O(1) loop body
    // with O(P) (number of person) Hashset construction cost
    // Overall: O(I + P)
    // Worst-case: |P| = 2 * |I|, if each interaction transmit between two unique person
    // 3 * |I| give us Overall O(I)
    for interaction in interactions:
        startInteractions: HashSet<Interaction> = personToInteractions[interaction.PersonFrom]
        startInteractions.add(interaction) // O(1)
        if interaction.personTo == end -> endInteractions.add(interaction)
    // O(I) due to implementation limitations
    sources: HashSet<Interaction> = personToInteractions[start]

    adjacency: HashMap<Interaction, HashSet<Interaction>> = new
HashMap().onLookupFail(new HashSet())
    // I iterations with worst-case: O(I) loop body, if successors' time >= predecessors' time
    //       and successors' personFrom don't equal to predecessors' personTo
    // Overall: O(I^2)
    for interaction in interactions:
        neighbors: HashSet<Interaction> = personToInteractions[interaction.personTo]
            .filter(i -> interaction.time <= i.time && interaction.PersonFrom != i.PersonTo)//O(I)
        adjacency[interaction] = neighbors // O(1)

    // Running time for Dijkstra's algorithm using a Java Heap as a priority queue is
    // O((|E| + |V|) * log|V|).
    // Worst-case: |E| = |V^2|, we get O((|V^2| + |V|) * log|V|).
    // as we use Interaction I as our Vertex
    // so O((|V^2| + |V|) * log|V|) give us Overall O(I^2 * logI)
    Dijkstra(adjacency, sources);
    maximumProb: Double = personToInteractions[end].
        .filter(v -> v.prob != Double.MAX_VALUE).maxBy(v -> v.prob).prob
    finalInteractionList: List<Interaction> = new ArrayList<>();
    finalInteractionList.add(lowestDDestinationVertex.get().element);
    while True // O(I)
        if head is not null
            finalInteractionList.add(0, head.element);
            head = head.predecessor;
        else // head is null (finish)
            return finalInteractionList;
```

b) The first part of my algorithm to prepare the parameters of Dijkstra's algorithm, which are a HashMap of one interaction to its valid neighbors and a HashSet of source interactions.
We assume that when HashSet and HashMap execute put, add, and get, its worst-case time complexity will be O(1) instead of O(p).
For doing a graph search, I first convert all interaction to a vertex and build an endInteraction HashSet where the path finish, which is Overall O(i);
For creating the HashSet of sources, it is O(i) in worst-case;
For creating the HashMap, I run through all interactions and filter its time elapsed and check if there is a loop in interactions, which is overall O(i^2).
Then the Running time for Dijkstra's algorithm using a Java Heap as a priority queue is Overall O(i^2 * log i) in worst-case. We use a Binary Heap in the implementation of Dijskstra’s, with O(lgV) Extract-Mins and Decrease-Keys.
In the last part, my algorithm is to find the path with the highest probability through all endInteraction HashSets, and then add it to an ArrayList, both of which are O(i).
Thus, the time complexity is O(i^2 * log i) which describes an asymptotic upper bound on the worst-case time complexity of this algorithm.