Hand in 4, part 1 of 2 - Euler methods

1. Consider the problem $y'(t) = f(y,t) = -0.5y$ with initial condition $y(0) = 1$. Inspired by the pseudo-code you did during the group-discussion, construct a python-function for solving the problem using Forward Euler until time T.
   Tip: Be aware that it is easy to make a mistake about where in the loop to update $y_{n+1} = y_n$ and $t_{n+1} = t_n$, and where/how to make the numerical solution stop at $t = T$.

2. Solve the problem until time $T = 10$ with $h = 0.1$. Plot the result together with the exact solution, which is $e^{-0.5y}$, the numerical solution should be very close to the exact one. You don't need to save this plot for the hand in, it is just for you to sanity-check your code.

3. Construct another function for solving the problem using Backward Euler, solve the problem again until $T = 10$ with $h = 0.1$. Plot the result together with the exact solution, which is $e^{-0.5y}$, the numerical solution should be very close to the exact one. You don't need to save this plot for the hand in, it is just for you to sanity-check your code.

4. Solve the problem for $h = 1$ using both methods instead. Plot the exact solution, the Forward Euler solution, and the Backward Euler Solution in one plot. Does the error of the two numerical method look comparable? Does that make sense given what you know about the order of these methods? (The order of Euler backwards was mentioned in the lecture. If you want to see how to derive the order of error of Euler backwards, see the extra practice problems with solutions on the course webpage).

5. What is the largest possible time-step h which is stable for this problem?

6. Make a plot of the solution with a h a little bit smaller than the maximum stable time-step, and a plot with a h which is a little bit bigger than the maximum stable time-step.

This should be included in this part of the hand in: Your code for Forward and Backward Euler, the plot for $h = 1$, your derivation of the maximum stable time step, and the two plots showing how Forward Euler behaves for a barely stable and barely unstable timestep h. This lab is only the first part of the hand in.