A Causal Ablation Framework: Assessing Inferential Necessity & Sufficiency of Assumptions

13 September 2025

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- **A reasonable goal:** avoid drawing inferences that are driven by implausible or falsified assumptions

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- Asm. sets often based on convenience, not rigor
  - Under these asms., 2SLS conveniently recovers the ATE
- We often unpack $\mathcal{A}$ to debate *plausibility* of $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, but rarely for *falsifiability* or *inferential consequences*

[My] approach to empirical inference begins by asking what can be learned from the data...

... one may then ask what more can be learned given successively stronger... assumptions. This approach yields a series of successively tighter bounds on quantities of interest.

The bound is widest when no assumptions are maintained and narrows as stronger assumptions are imposed. Sufficiently strong assumptions narrow the bound to a point.

– Charles Manski (2006)

A proposed ablation framework for assessing causal inferences

Comparing assumptions

- Component asms. can be incommensurate: $\mathfrak{B} \nleq \mathfrak{C}$
  - But each component admits its own *weakening sequence*
  - Can still say $\mathfrak{B} < \mathfrak{B}' < \mathfrak{B}'' \ldots$ and $\mathfrak{C} < \mathfrak{C}' < \mathfrak{C}'' \ldots$

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- Monotonic effect of instrument Z on treatment D
  - $\mathfrak{B}$ **:** unit-level monotonicity, $D_i(z_1) \geq D_i(z_0)$ for all i
  - $\mathfrak{B}'$ **:** 1st order stochastic dominance, $z \uparrow$ shifts $D(z)$ CDF $\rightarrow$
  - $\mathfrak{B}''$ **:** monotonicity of expectations, $\mathbb{E}[D(z_1)] \geq \mathbb{E}[D(z_0)]$
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  - $\mathfrak{B}''$: monotonicity of expectations, $\mathbb{E}[D(z_1)] \geq \mathbb{E}[D(z_0)]$
  - $\mathfrak{B}'''$: no monotonicity assumption (eliminated entirely)
- Exclusion of direct Z effect on outcome Y
  - $\mathfrak{C}$: unit-level exclusion, $Y_i(z, d) = Y_i(z', d)$ for all z, z', d, i
  - $\mathfrak{C}'$: direct effect exists for at most θ fraction of units (Duarte, '24)
  - $\mathfrak{C}''$: no exclusion restriction (eliminated entirely)

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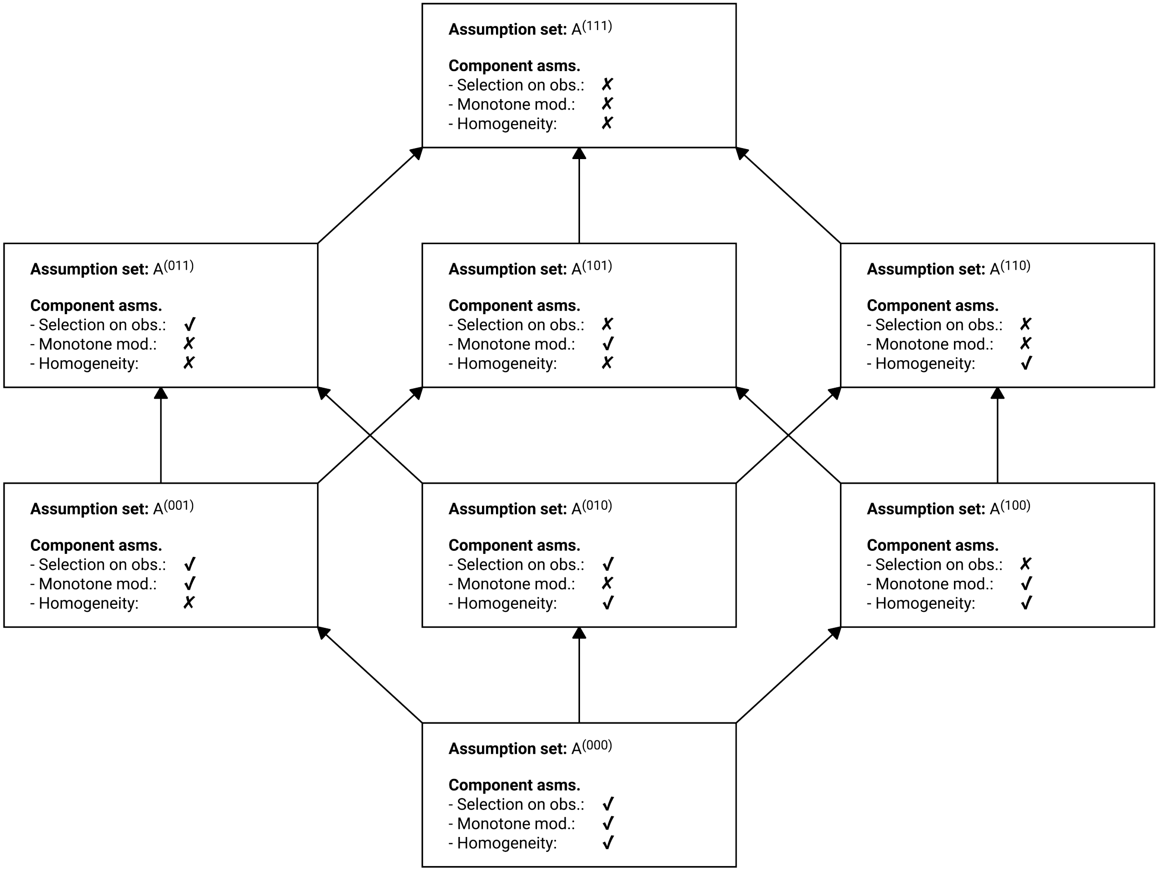
- Organize asm. sets with *Hasse diagrams* for posets
  - Facilitates presentation of results, visual diagnostics


Assumption set: A(111)
Component asms.
- Selection on obs.: ✗
- Monotone mod.: ✗
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Assumption set: A(011)
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Assumption set: A(110)
Component asms.
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Assumption set: A(001)
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- Monotone mod.: ✓
- Homogeneity: ✗
Assumption set: A(010)
Component asms.
- Selection on obs.: ✓
- Monotone mod.: ✗
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Assumption set: A(100)
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- Iterate over elements of diagram and derive results
  - Automate falsification tests & bounds analysis (Duarte et al. '23)

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- Iterate over elements of diagram and derive results
  - Automate falsification tests & bounds analysis (Duarte et al. '23)
- Evaluate inferential necessity, sufficiency, irrelevance

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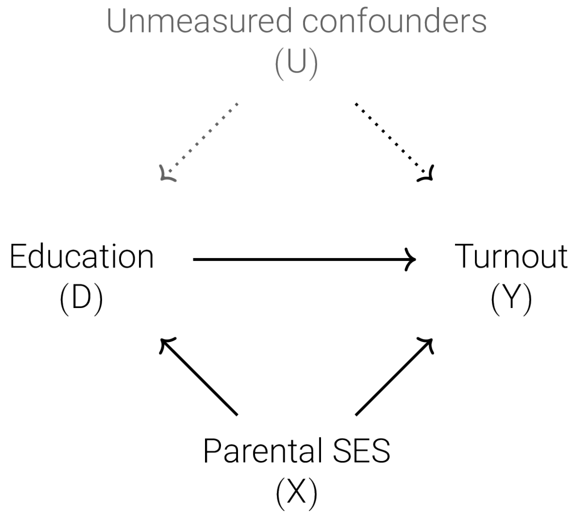
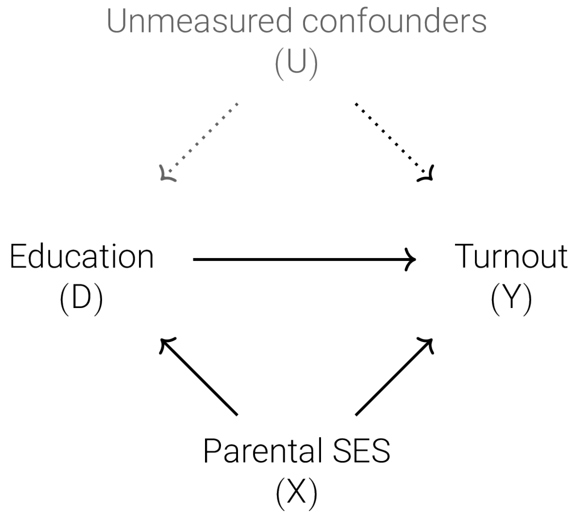
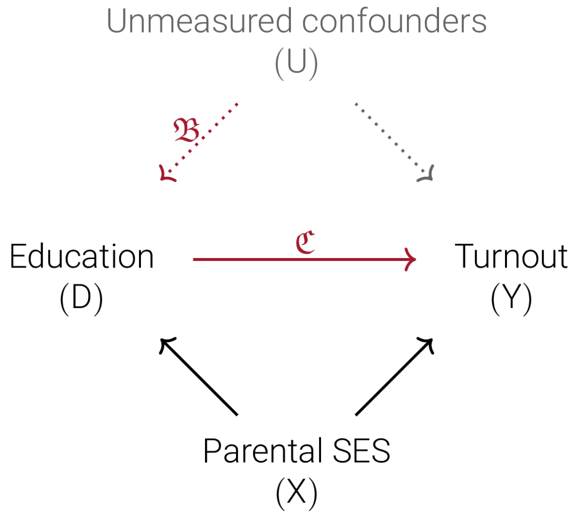
- What could we have learned from obs. data alone?

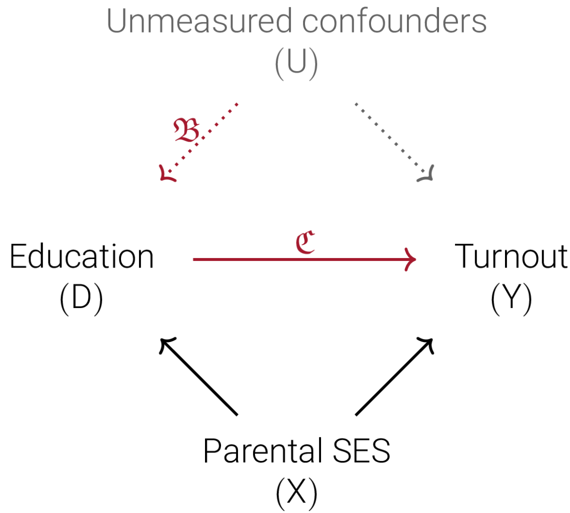

Unmeasured confounders
(U)
Education
(D)
Turnout
(Y)
Parental SES
(X)



- Common to regress $Y = \alpha + \beta X + \gamma D + \varepsilon$



- Common to regress $\boldsymbol{Y = \alpha + \beta X + \gamma D + \varepsilon}$
  - $\boldsymbol{\mathfrak{B}}$ **:** selection on observables ($\boldsymbol{U \rightarrow D}$ does not exist)



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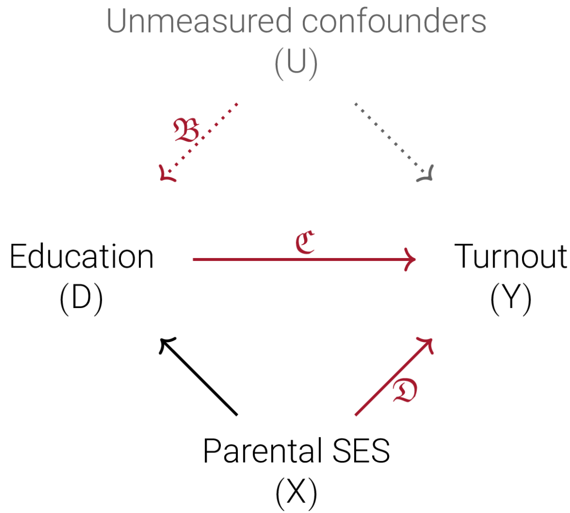

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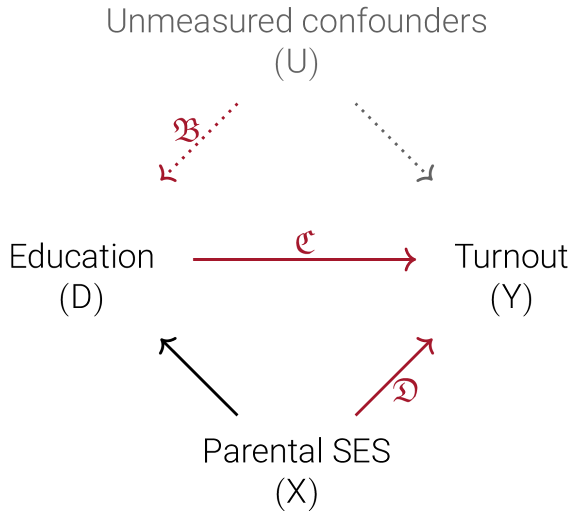
  - $\mathfrak{D}$ **:** monotone confounding (if we could hold education fixed, voting has lower relative cost for high-SES individuals)



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# asm A: selection on observables (implicit in absence of U)
problem = causalProblem("D -> Y, X -> D, X -> Y")
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# asm B: homogeneity
# E[ Y(D=1) - Y(D=0) | X=1 ] = E[ Y(D=1) - Y(D=0) | X=0 ]
with respect_to(problem):
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# asm C: monotone confounding
# E[ Y(d) | X=1 ] >= E[ Y(d) | X=0 ] for d in {0, 1}
with respect_to(problem):
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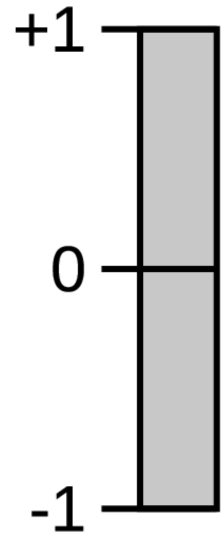

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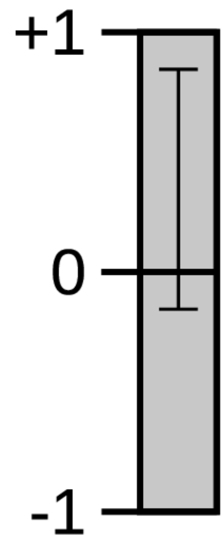

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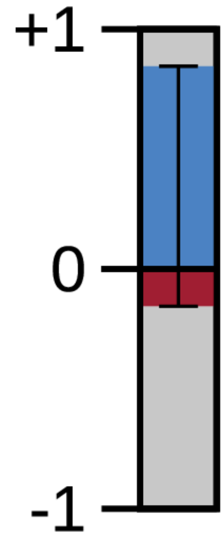

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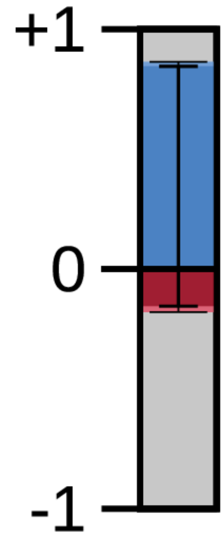

Assumption set: $A^{(000)}$

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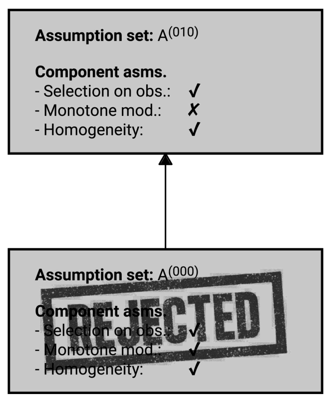
REJECTED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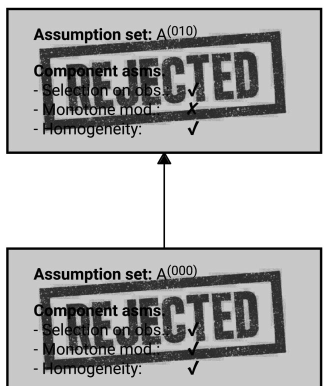
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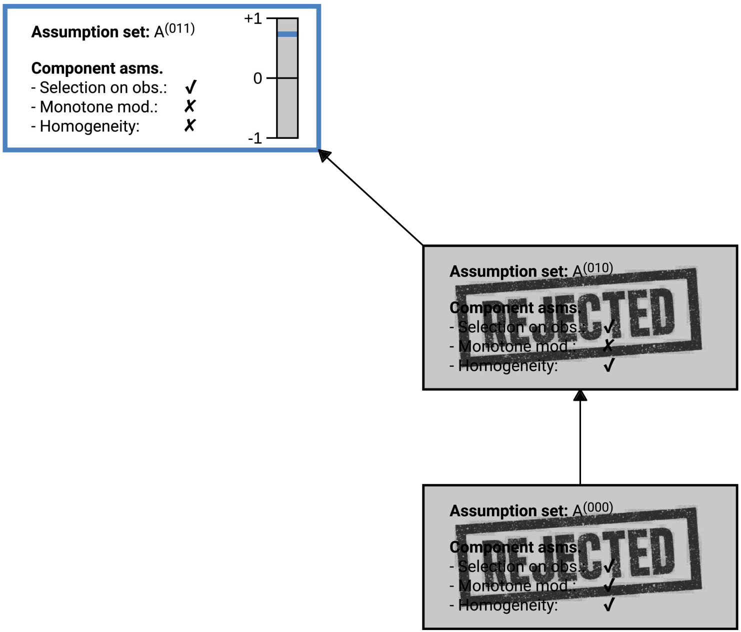
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+1
0
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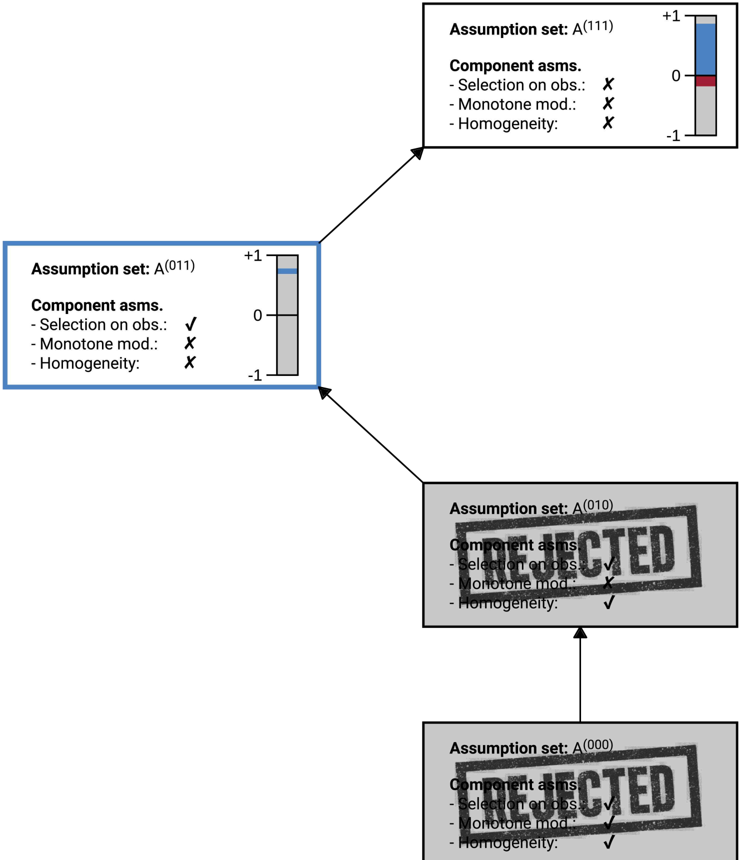

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0
-1
Assumption set: A(011)
Component asms.
- Selection on obs.: ✓
- Monotone mod.: ✗
- Homogeneity: ✗
+1
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Component asms.
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- Monotone mod.: ✗
- Homogeneity: ✓
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Component asms.
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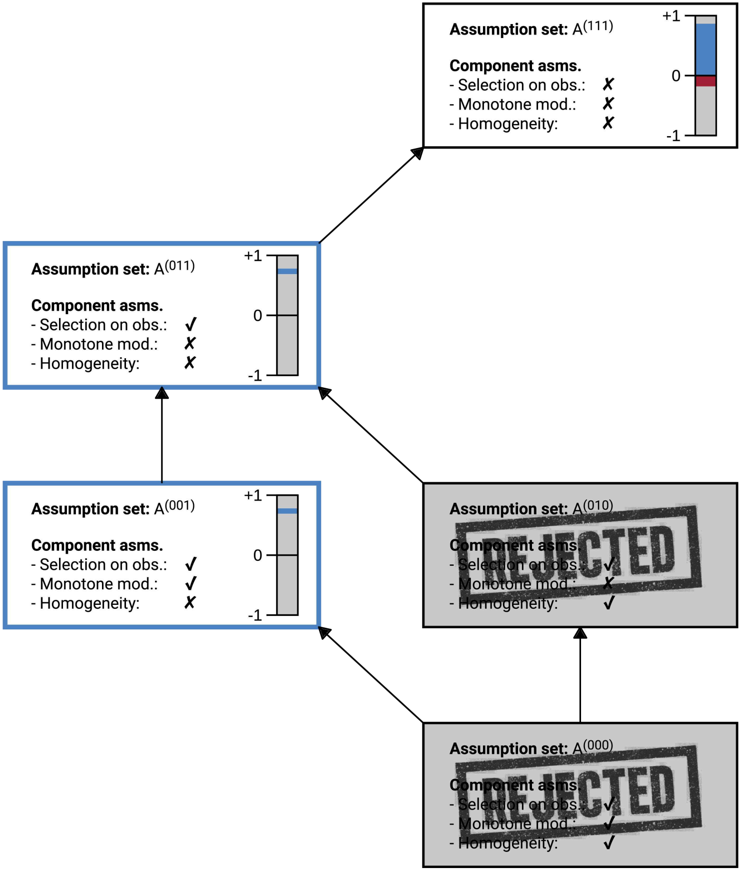
- Monotone mod.: ✓
- Homogeneity: ✓
REJECTED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- Monotone mod.: ✗
- Homogeneity: ✗
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Assumption set: A(011)
Component asms.
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-1
Assumption set: A(001)
Component asms.
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- Monotone mod.: ✓
- Homogeneity: ✗
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-1
Assumption set: A(010)
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Assumption set: A(000)
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Component asms.
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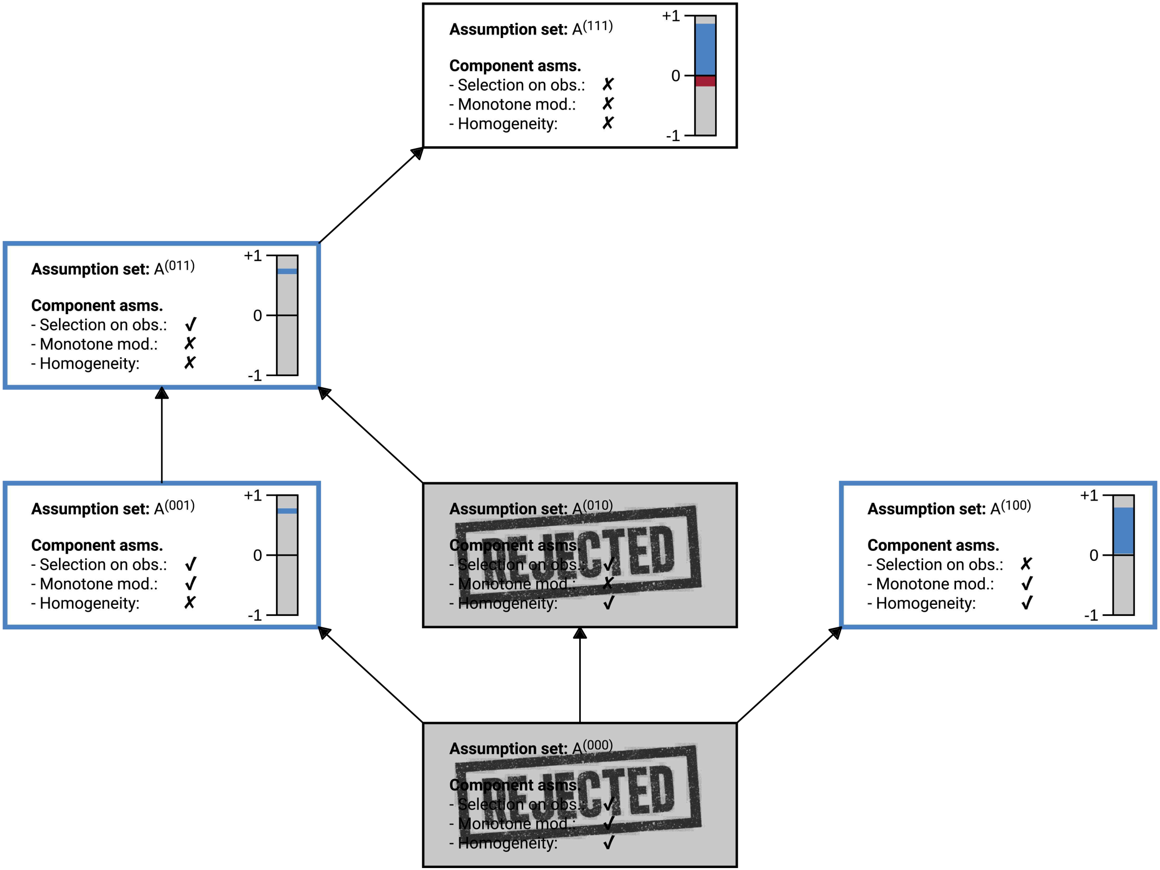
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Assumption set: A(100)
Component asms.
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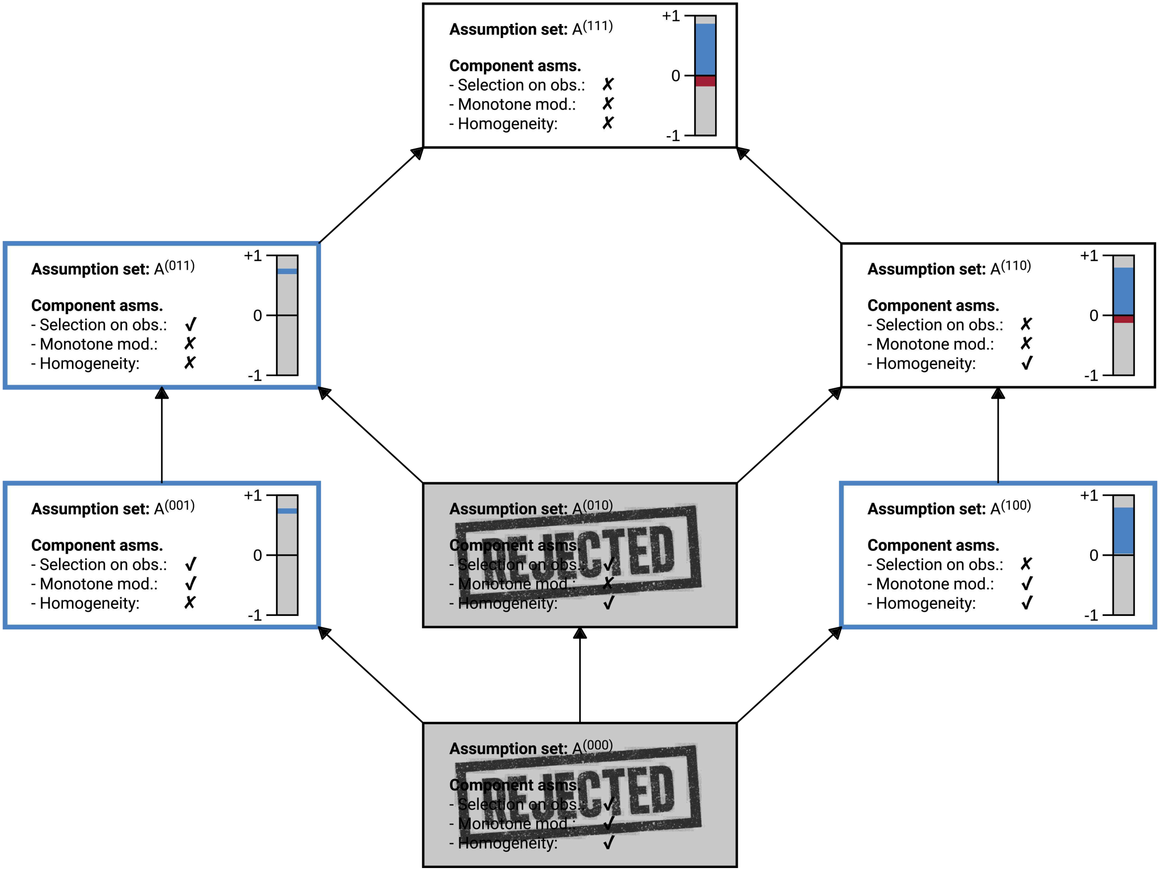

Assumption set: A(111)
Component asms.
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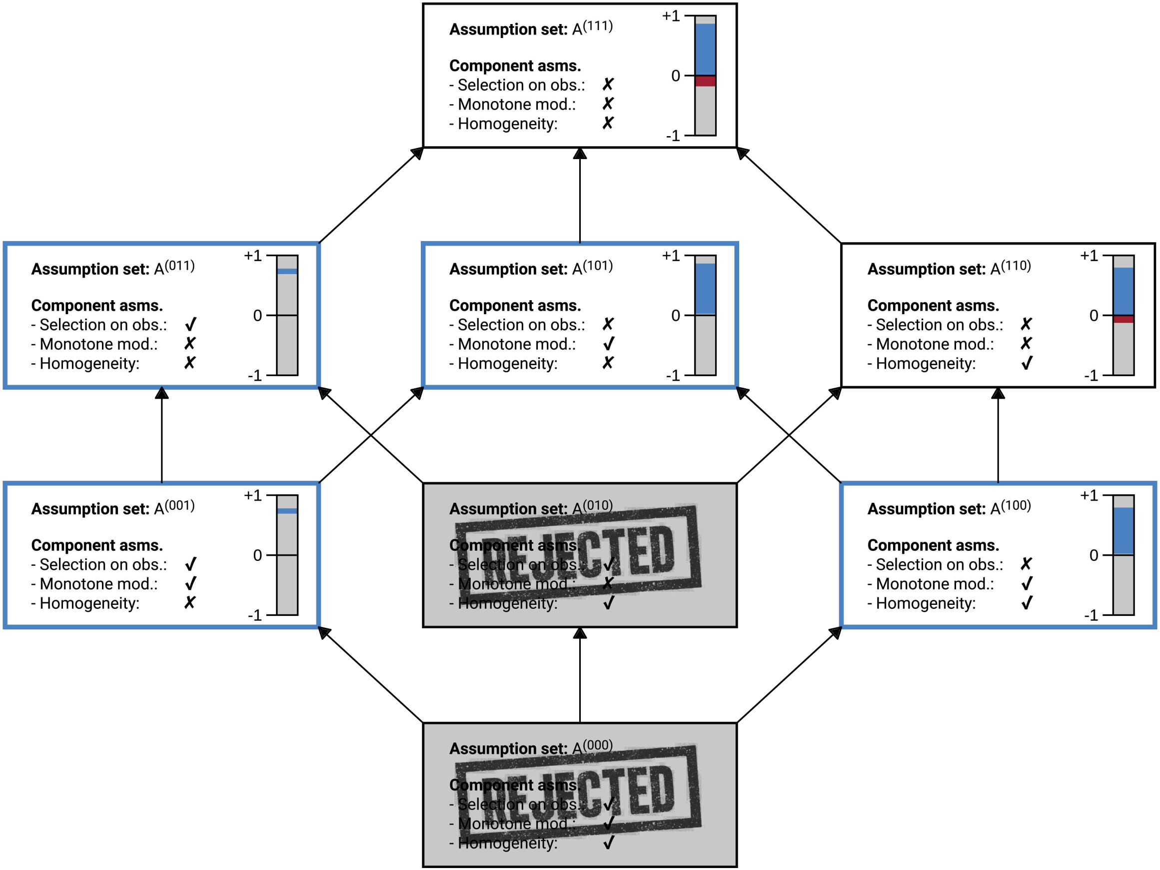

Assumption set: A(111)
Component asms.
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Component asms.
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- Homogeneity: ✗
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Component asms.
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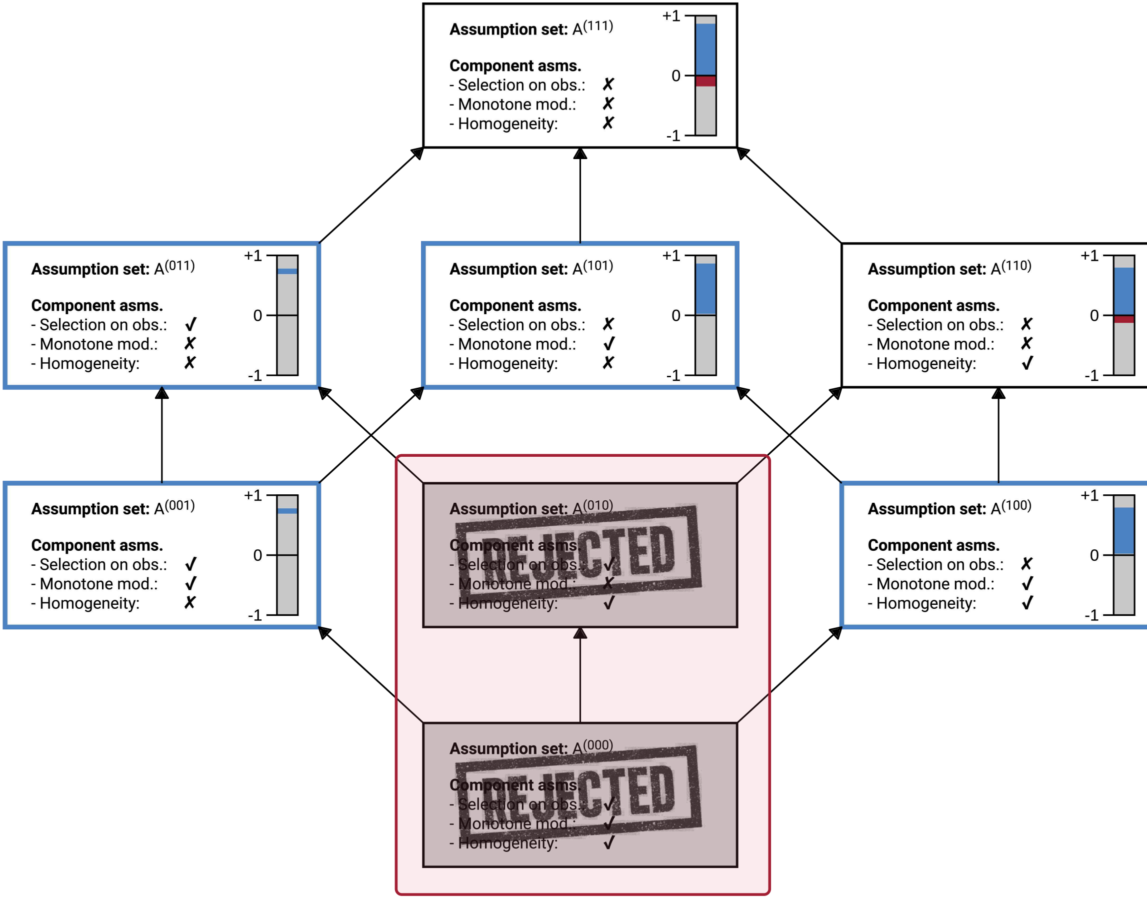
Assumption set: A(111)
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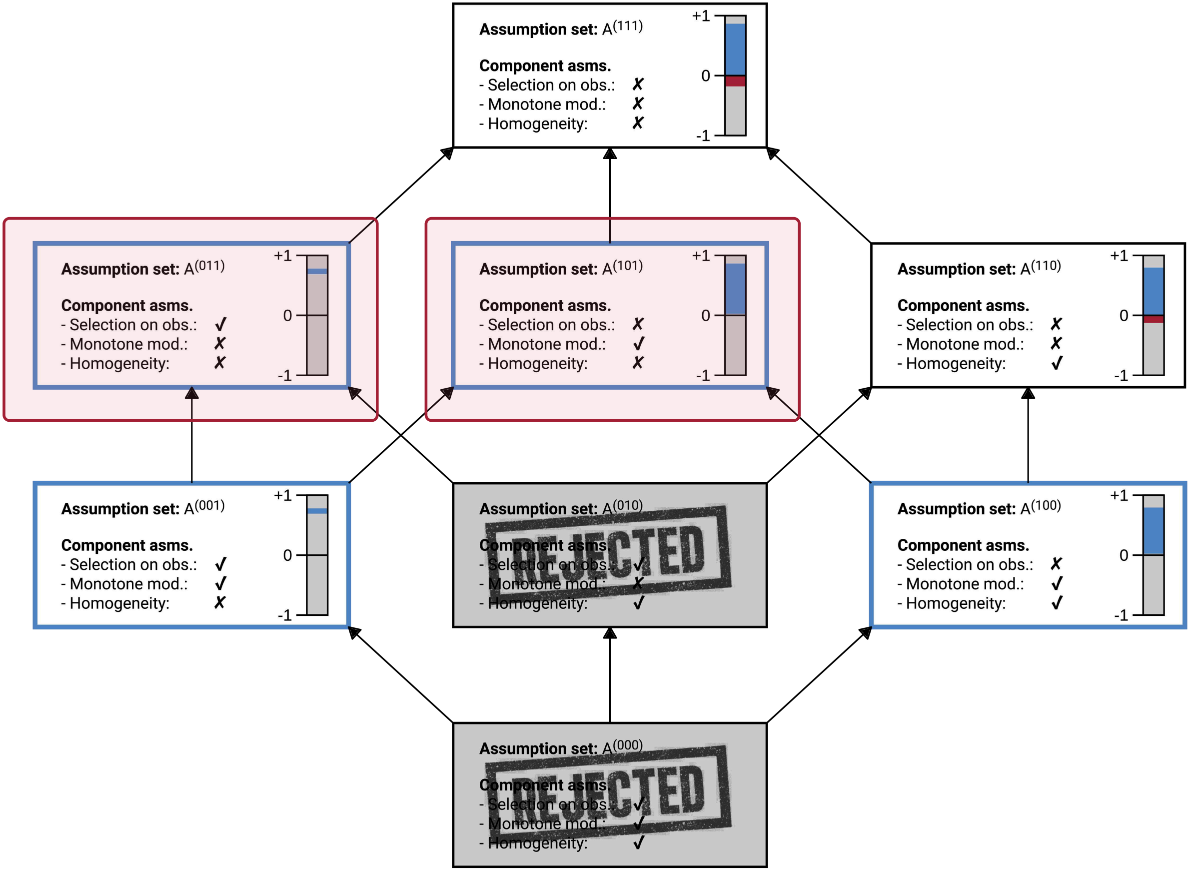
Consequentiality of asms.

- SOO and homogeneity are *mutually incompatible* *
  - Cannot both be true, analysts must pick at most one

** with 95% confidence*



Assumption set: A(111)
Component asms.
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+1
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-1
Assumption set: A(000)
REJECTED
Component asms.
- Selection on obs.: ✓
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- SOO and homogeneity are *mutually incompatible* *
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    - Which one you choose to discard is *inferentially irrelevant*
- SOO and monotone confounding asms. are both *inferentially sufficient* to conclude ATE is positive *

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- But homogeneity asm. is *worse than irrelevant*
  - Adding/removing it never changes substantive conclusion...

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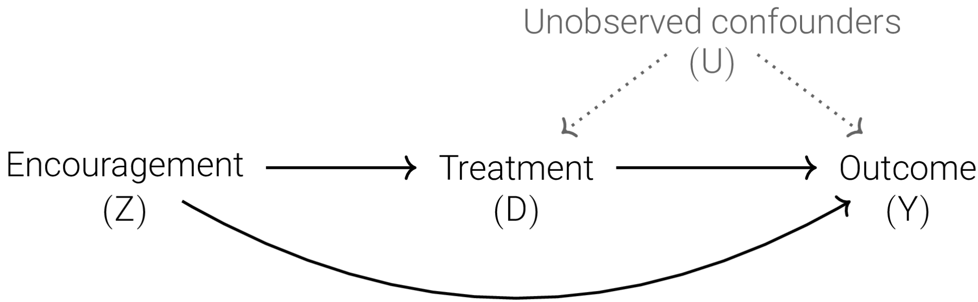
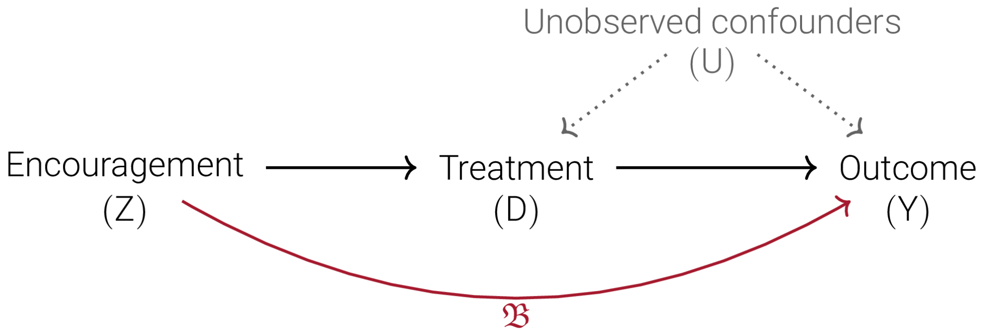
  - Cannot both be true, analysts must pick at most one
  - Which one you choose to discard is *inferentially irrelevant*
- SOO and monotone confounding asms. are both *inferentially sufficient* to conclude ATE is positive *
- But homogeneity asm. is *worse than irrelevant*
  - Adding/removing it never changes substantive conclusion... except when it leads to an asm. set contradicted by data
- Theoretical justification for monotone confounding asm. is much stronger than for traditional SOO asm.
  - Conclusion is the same, so why rely on implausible SOO?

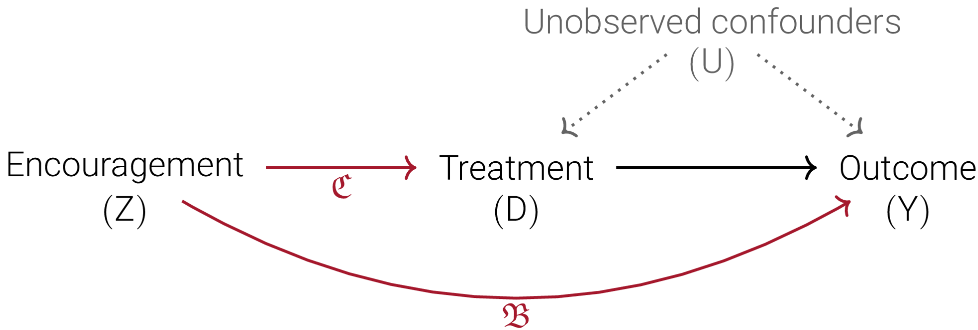
** with 95% confidence*

A second example: instrumental variables

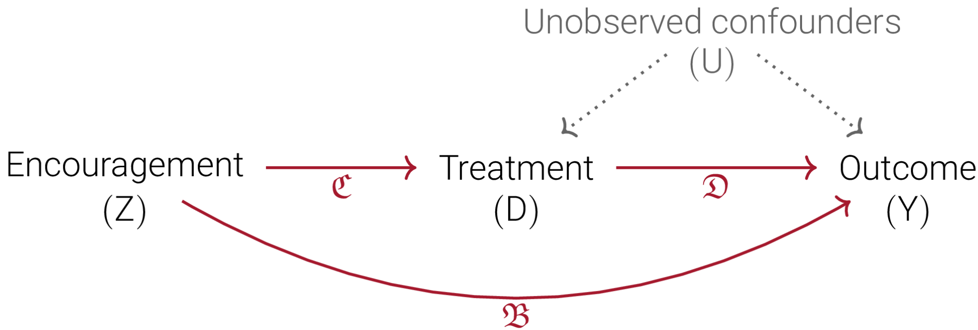

Unobserved confounders
(U)
Encouragement
(Z)
Treatment
(D)
Outcome
(Y)



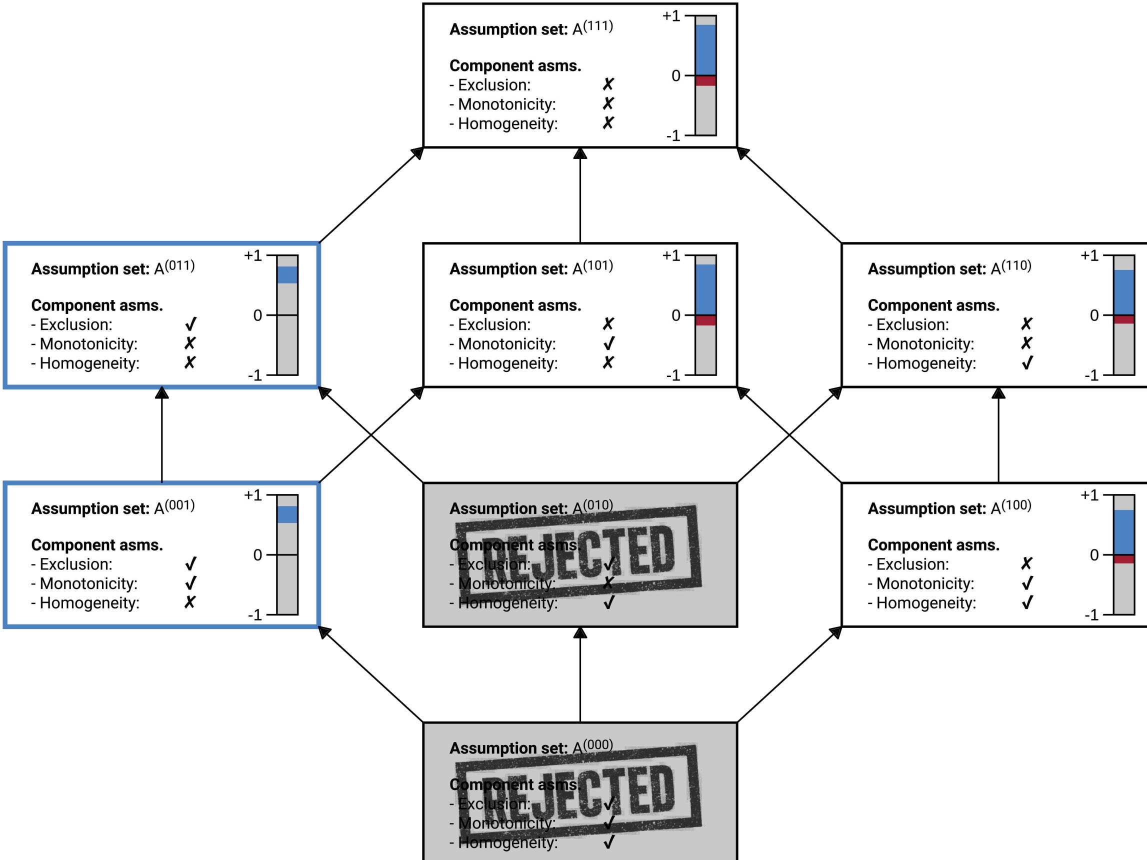
- $\mathfrak{B}$ **:** exclusion restriction, $Z \to Y$ does not exist



- $\mathfrak{B}$: exclusion restriction, $Z \to Y$ does not exist
- $\mathfrak{C}$: monotonicity, $D_i(z_1) \geq D_i(z_0)$ for all i



- $\mathfrak{B}$: exclusion restriction, $Z \rightarrow Y$ does not exist
- $\mathfrak{C}$: monotonicity, $D_i(z_1) \geq D_i(z_0)$ for all i
- $\mathfrak{D}$: homogeneity, $\mathbb{E}[Y(d_1) - Y(d_0)|\textbf{strata}]$
  conditional effect equal across all principal strata



Assumption set: $A^{(111)}$
Component asms.
- Exclusion: ✗
- Monotonicity: ✗
- Homogeneity: ✗
+1
0
-1
Assumption set: $A^{(011)}$
Component asms.
- Exclusion: ✓
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- Homogeneity: ✗
+1
0
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Assumption set: $A^{(101)}$
Component asms.
- Exclusion: ✗
- Monotonicity: ✓
- Homogeneity: ✗
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Assumption set: $A^{(110)}$
Component asms.
- Exclusion: ✗
- Monotonicity: ✗
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+1
0
-1
Assumption set: $A^{(001)}$
Component asms.
- Exclusion: ✓
- Monotonicity: ✓
- Homogeneity: ✗
+1
0
-1
Assumption set: $A^{(010)}$
REJECTED
Component asms.
- Exclusion: ✓
- Monotonicity: ✗
- Homogeneity: ✓
Assumption set: $A^{(100)}$
Component asms.
- Exclusion: ✗
- Monotonicity: ✓
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+1
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Assumption set: $A^{(000)}$
REJECTED
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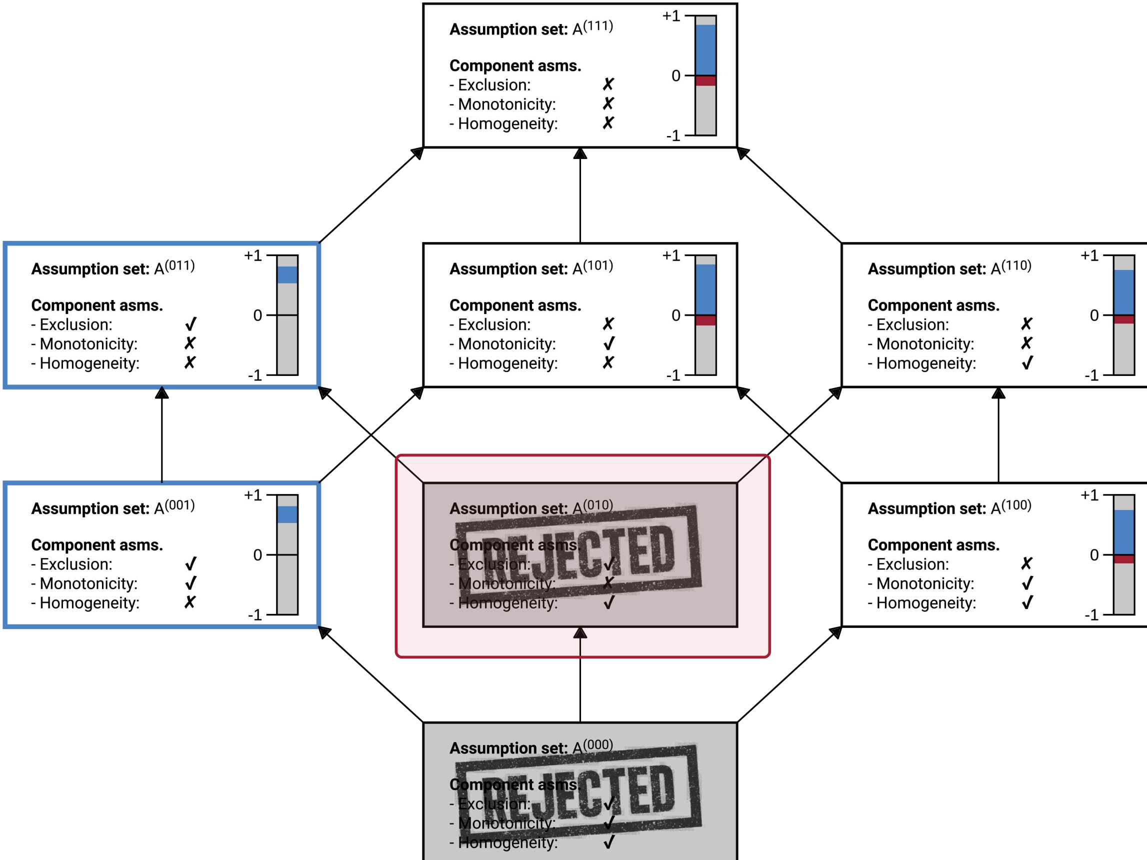

Assumption set: A(111)
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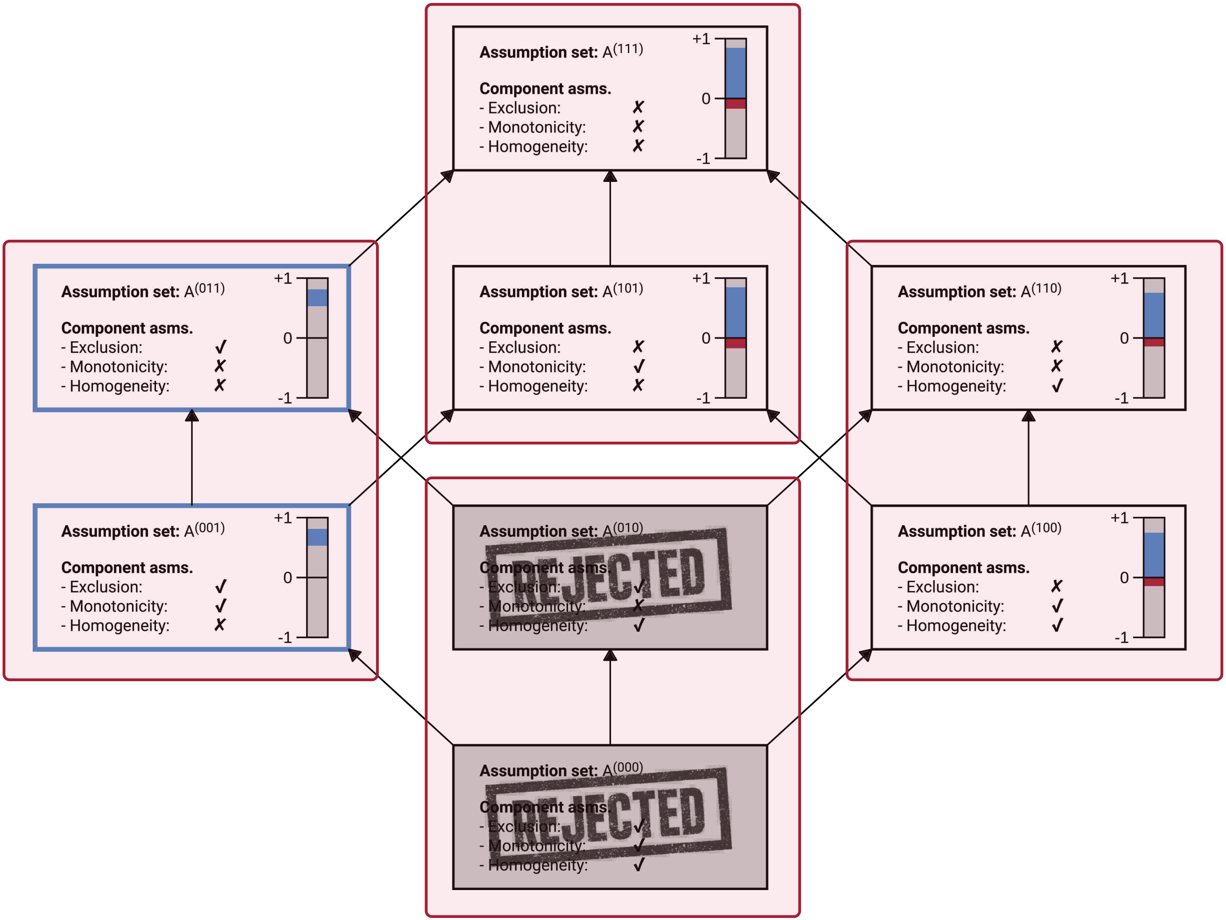
Consequentiality of asms.

- Exclusion and homogeneity *mutually incompatible* *

** with 95% confidence*



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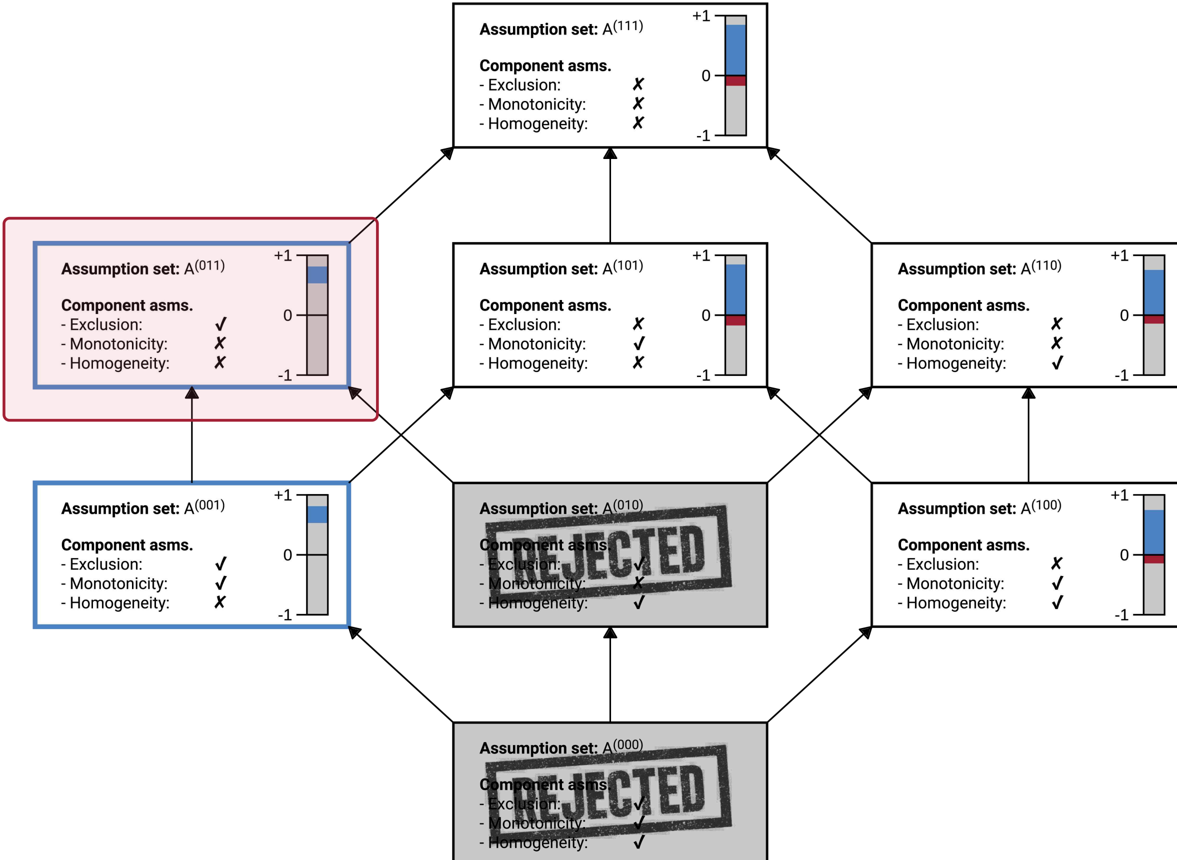
Consequentiality of asms.

- Exclusion and homogeneity *mutually incompatible* *
- Monotonicity is *inferentially irrelevant*
  - Substantive conclusion unaffected by adding/removing

** with 95% confidence*



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- Exclusion and homogeneity *mutually incompatible* *
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    - Substantive conclusion unaffected by adding/removing
- Exclusion is *necessary & sufficient* to say ATE > 0 *

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- Exclusion is *necessary & sufficient* to say ATE > 0 *
  - Why defend monotonicity if you've already made your point?

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  - Substantive conclusion unaffected by adding/removing
- Exclusion is *necessary & sufficient* to say ATE > 0 *
  - Why defend monotonicity if you've already made your point?
- Just because asm. is in the "standard package" for a design doesn't mean we need to blindly adopt it

** with 95% confidence*

Conclusion

- Framework for inferential consequences of asms.
  - Focus on substantive claims about direction of effects
  - Formalize inferential necessity, sufficiency, irrelevance
  - Should be paired with careful substantive evaluation of asms.
- Demonstrate robustness/detect fragility in claims
  - What do readers need to believe before buying this claim?
- Guide efforts to justify asms. with domain expertise
- Suggest directions for future work