Comments: Seeing is Believing

- Piqueras et al., "An image is worth K topics"
- Sunderland and Morucci, "Counterfactual inpainting"
- Arnold et al., "Measuring media slant"

10 Sep 2025

Dean Knox **Penn**

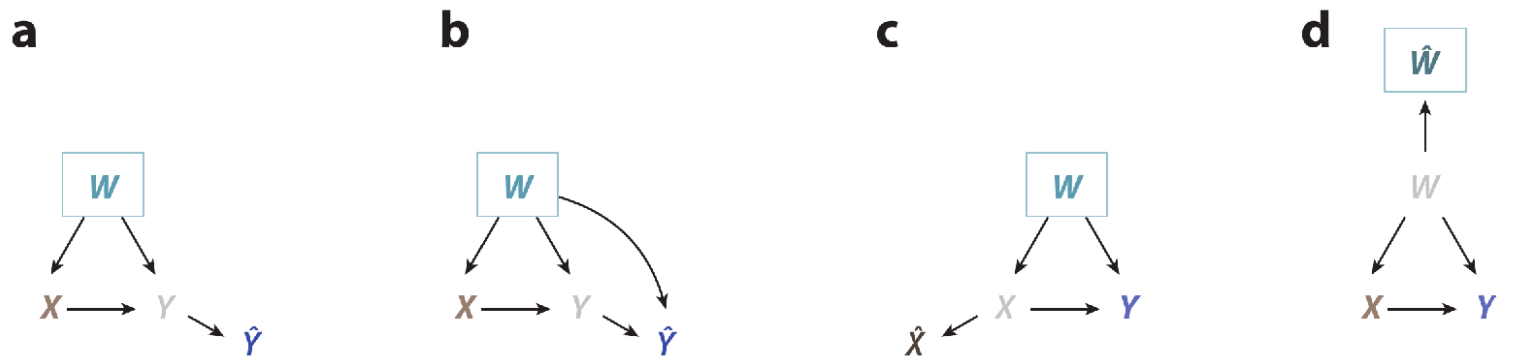

Figure 3

Causal structures of theory and measurement. Data environments correspond to the theory of **Figure 1**. Panels *a* and *b* illustrate environments in which analysts are unable to directly observe the outcome Y and thus must resort to a learned measure $\hat{Y}$ that is either (*a*) uncontaminated or (*b*) contaminated by confounders. The other two panels depict cases in which (*c*) the treatment X or (*d*) the confounders W cannot be observed, so that analysts can only adjust for learned proxies ($\hat{X}$ or $\hat{W}$).

Source: Dean Knox, Christopher Lucas, and Wendy Tam Cho. 2022. "Testing Causal Theories with Learned Proxies, "*Annual Review of Political Science*.

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- **QOI:** coefs. of multinomial logit regression $\vec{\theta}_i \sim x_i$

Some brief comments

- There are *two* relevant notions of embeddings!
  - Unobserved "true" embeddings $\vec{z}_i$ which create $\mathbf{Image}_i$
  - Estimated $\hat{\vec{z}}_i$, a projection of $\mathbf{Image}_i$ from pretrained model
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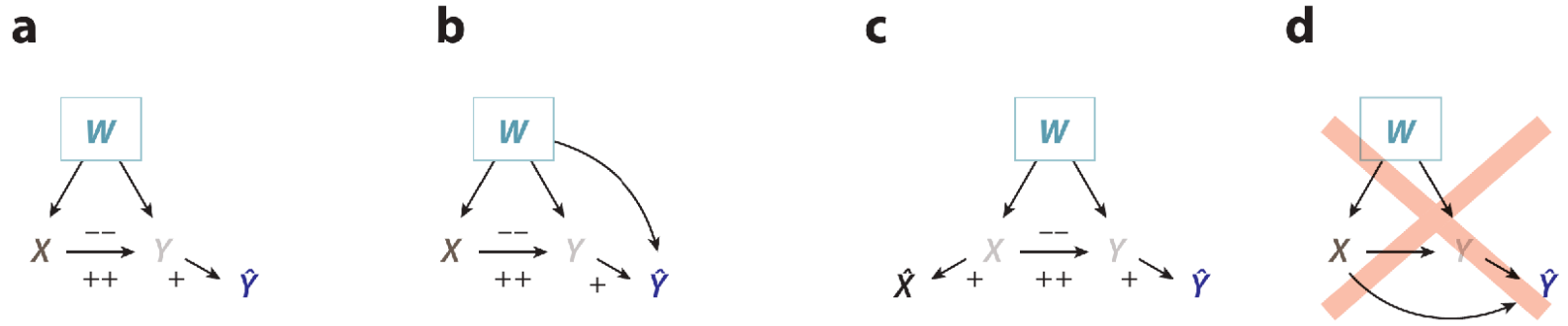

Figure 5

Learned outcomes. In the settings shown, the true outcome Y is unknown, but the proxy $\hat{Y}$ can be estimated from auxiliary information. In all cases, both confounders W and treatment X are known, and the analyst adjusts for W. (*a*) In a simple case, Y has an on-average monotonic effect on an uncontaminated proxy $\hat{Y}$, and the $X \rightarrow Y$ effect is known to exhibit distributional monotonicity of an unknown direction (indicated by the presence of both positive and negative signs). In this case, positive (negative) distributional monotonicity in $X \rightarrow Y$ is guaranteed to produce weakly positive (negative) $\text{Cov}(\hat{X}, \hat{Y}|W)$, and the sign of the $X \rightarrow Y$ effect can be identified. (*b*) This result holds even when $\hat{Y}$ is contaminated by W, so long as these contextual factors are adjusted for in a subsequent regression. (*c*) The same result still holds when X is also imperfectly, but on-average monotonically, learned. (*d*) However, if the proxy is contaminated by the treatment itself, association between X and $\hat{Y}$ cannot be interpreted as evidence for the theorized $X \rightarrow Y$ effect.

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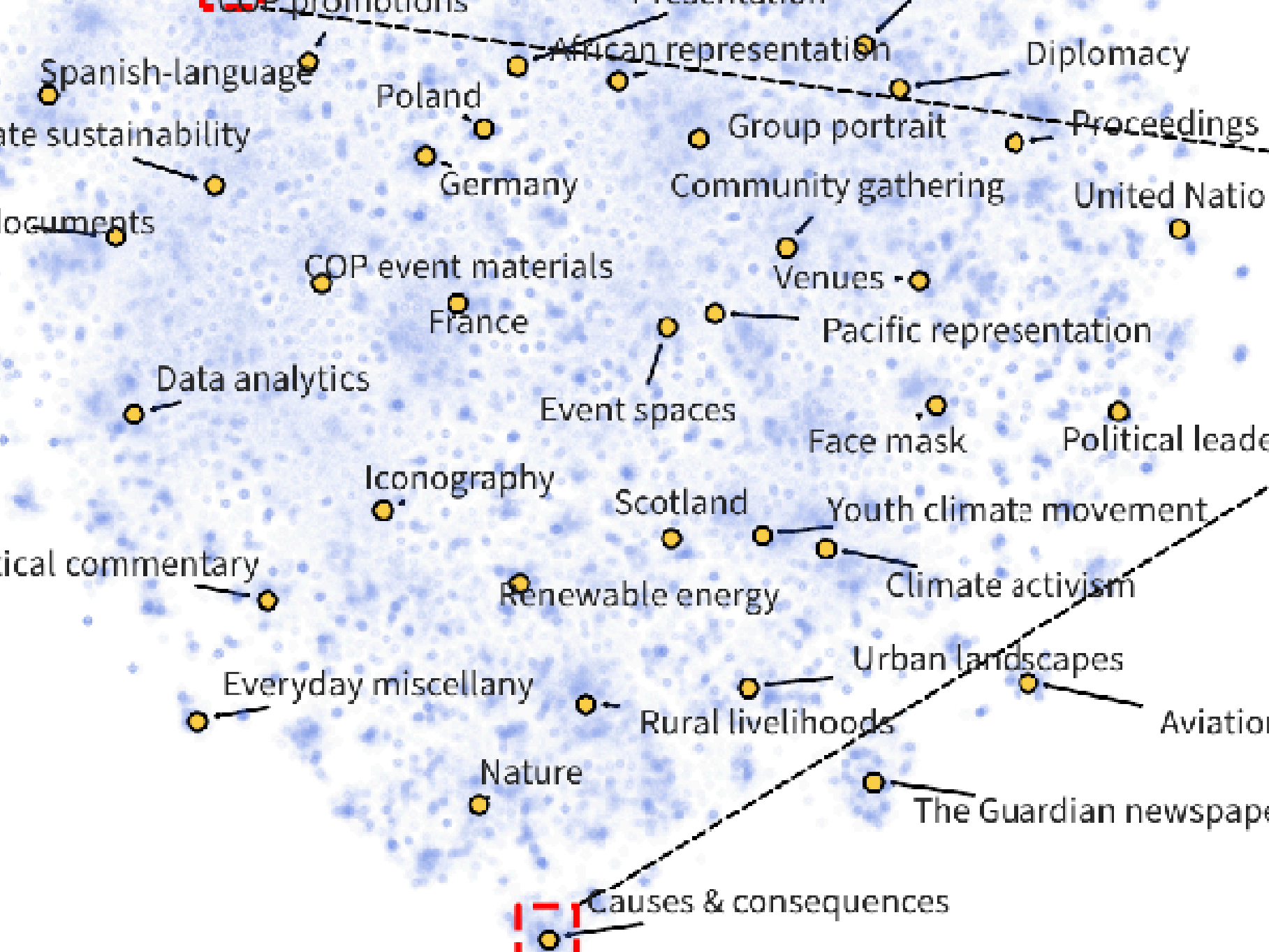
- Key estimates seem heavily shaped by regularization
  - Without it, $\vec{\beta}$ slide around freely; not constrained to lie on hull
  - Many choices for which $\tilde{D} + 1$ elements of $\vec{\theta}_i$ are non-zero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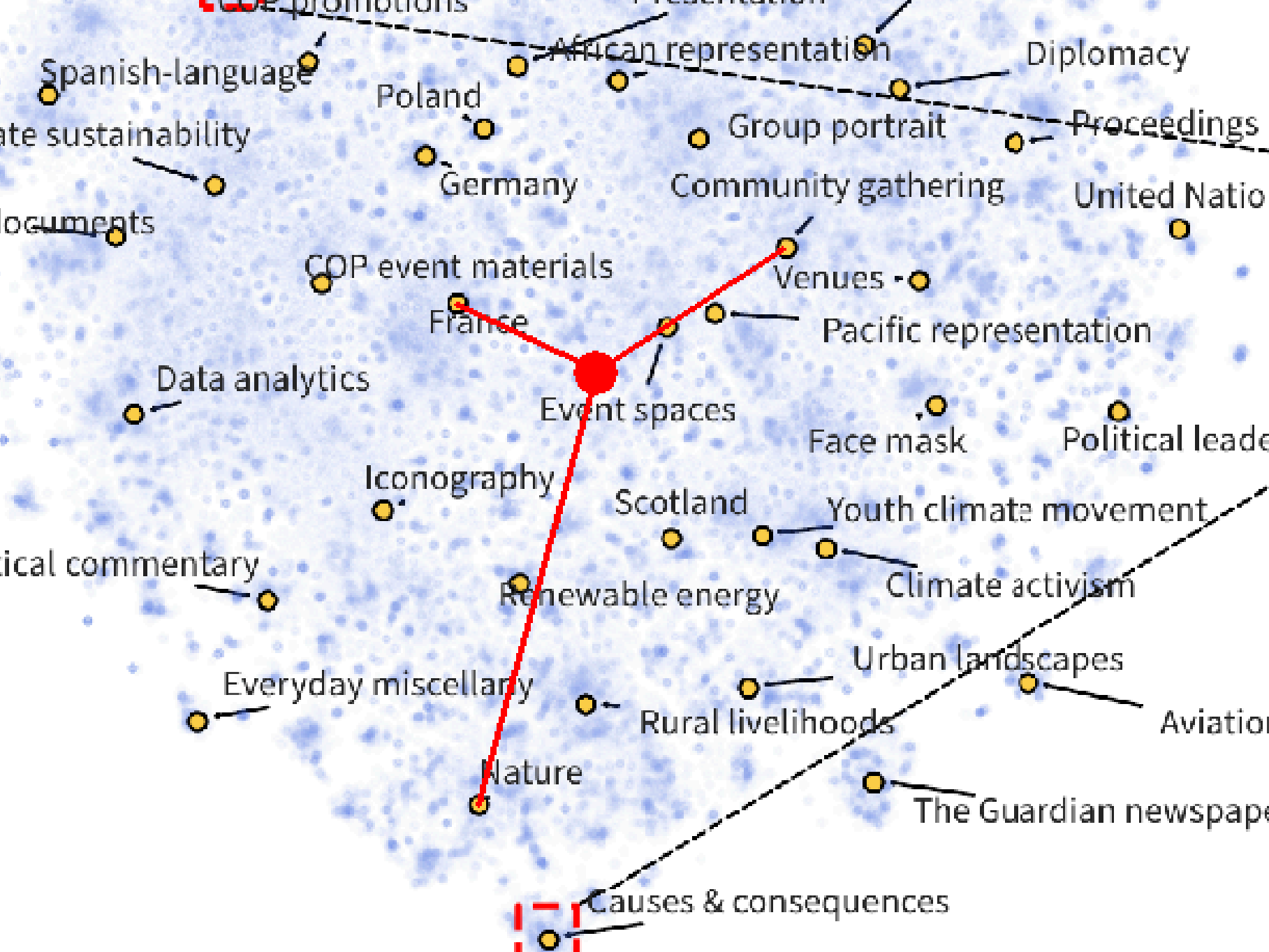
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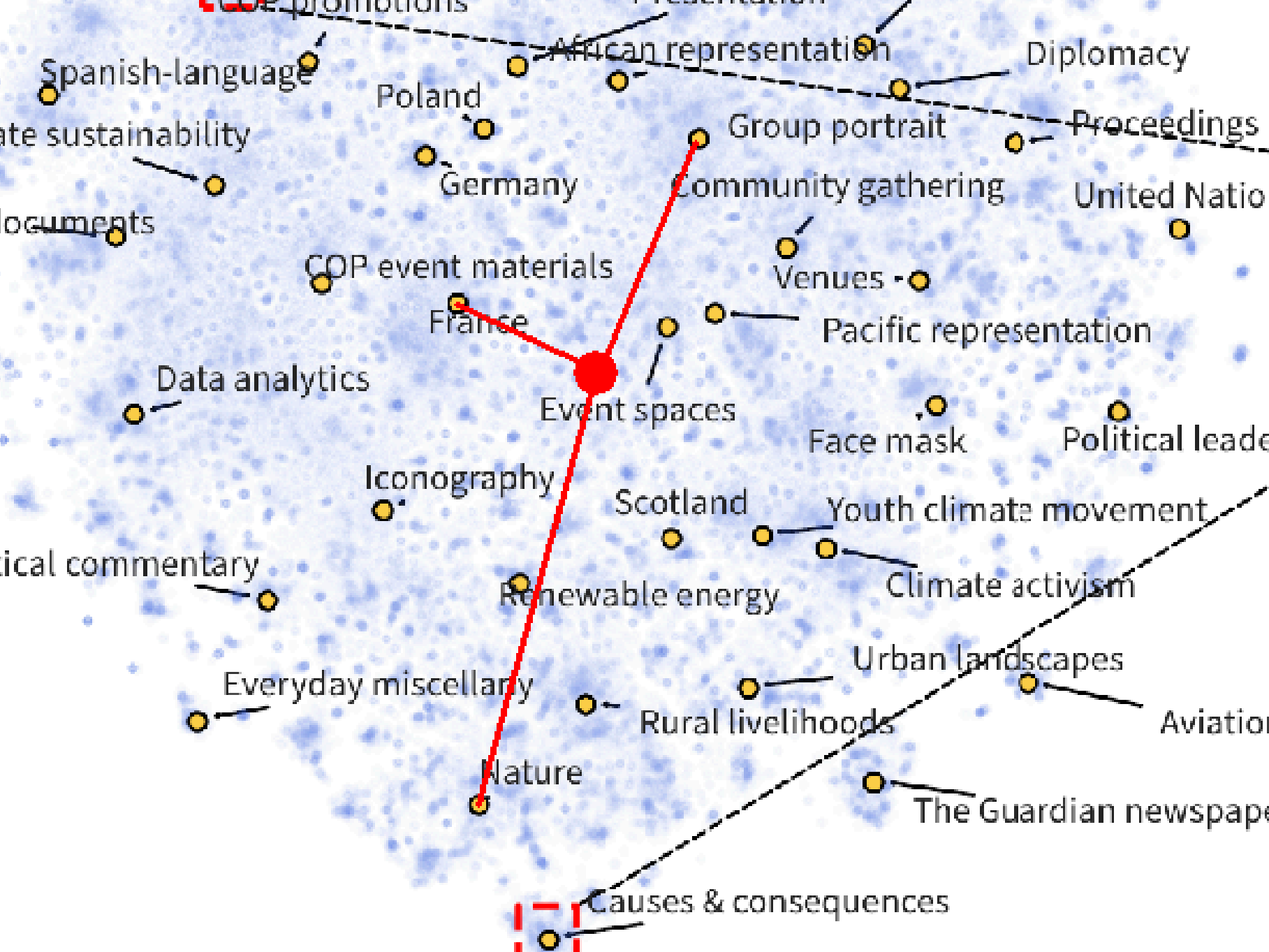
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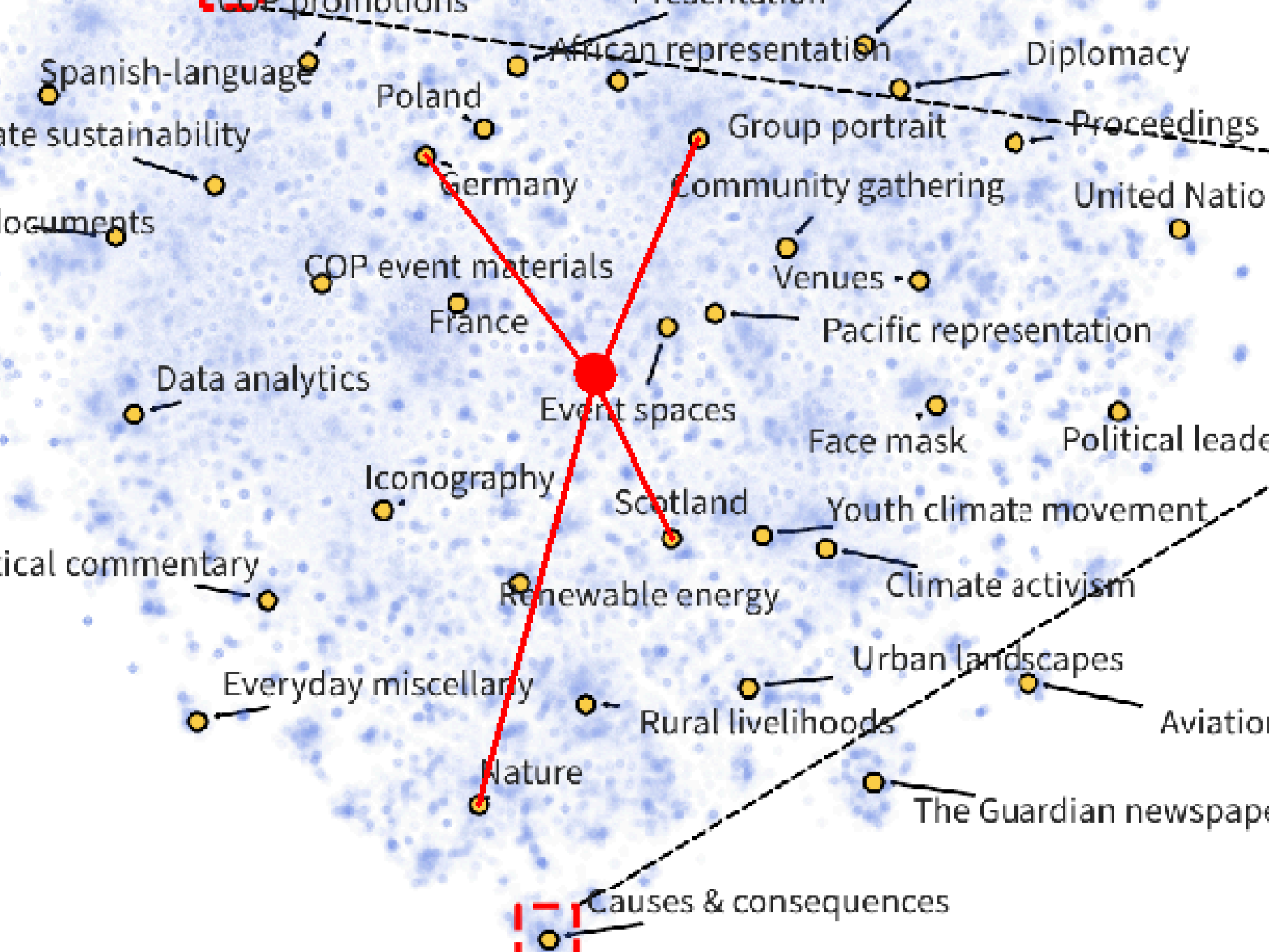
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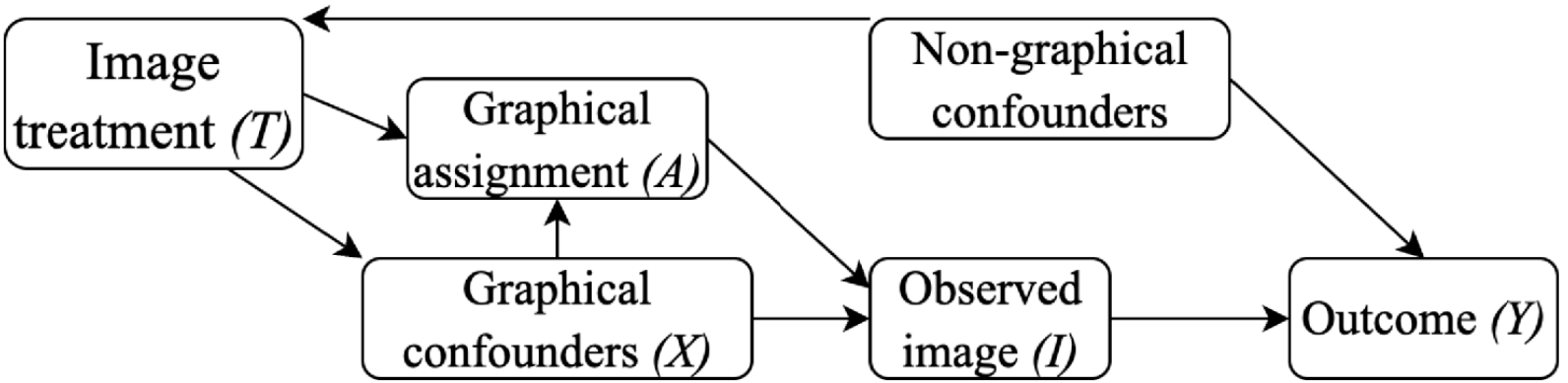
- Image I composed of *spatially separable*:
  - Background confounders X
  - High-level treatment concept T (indirectly)
- T can manifest via many graphical renderings A
- Viewers consume image I and produce outcome Y

Figure 3: Causal Graph for Inference with Graphical Treatments



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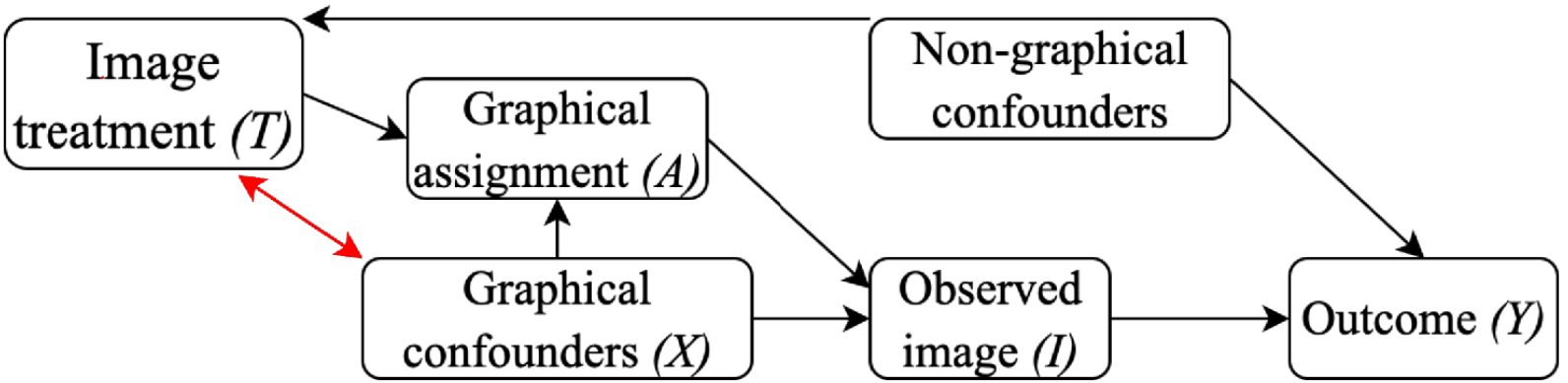

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- Quantity defined in paper is currently not an "ATE" $\int \mathbb{E}[Y(a, X)] \mathrm{d}p_{A_t}(a) - \int \mathbb{E}[Y(a, X)] \mathrm{d}p_{A_c}(a)$

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  - Also seems inconsistent with the proposed implementation

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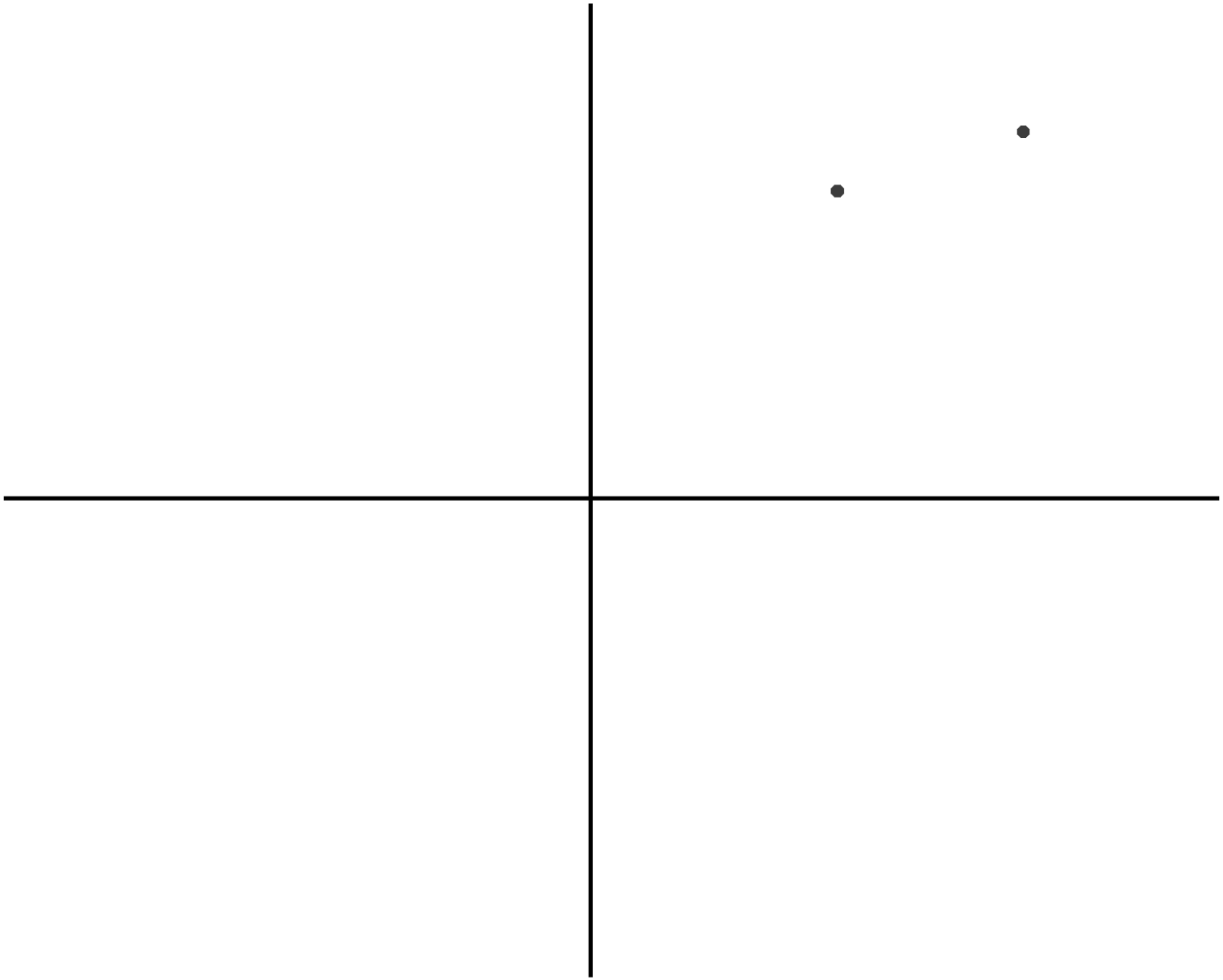
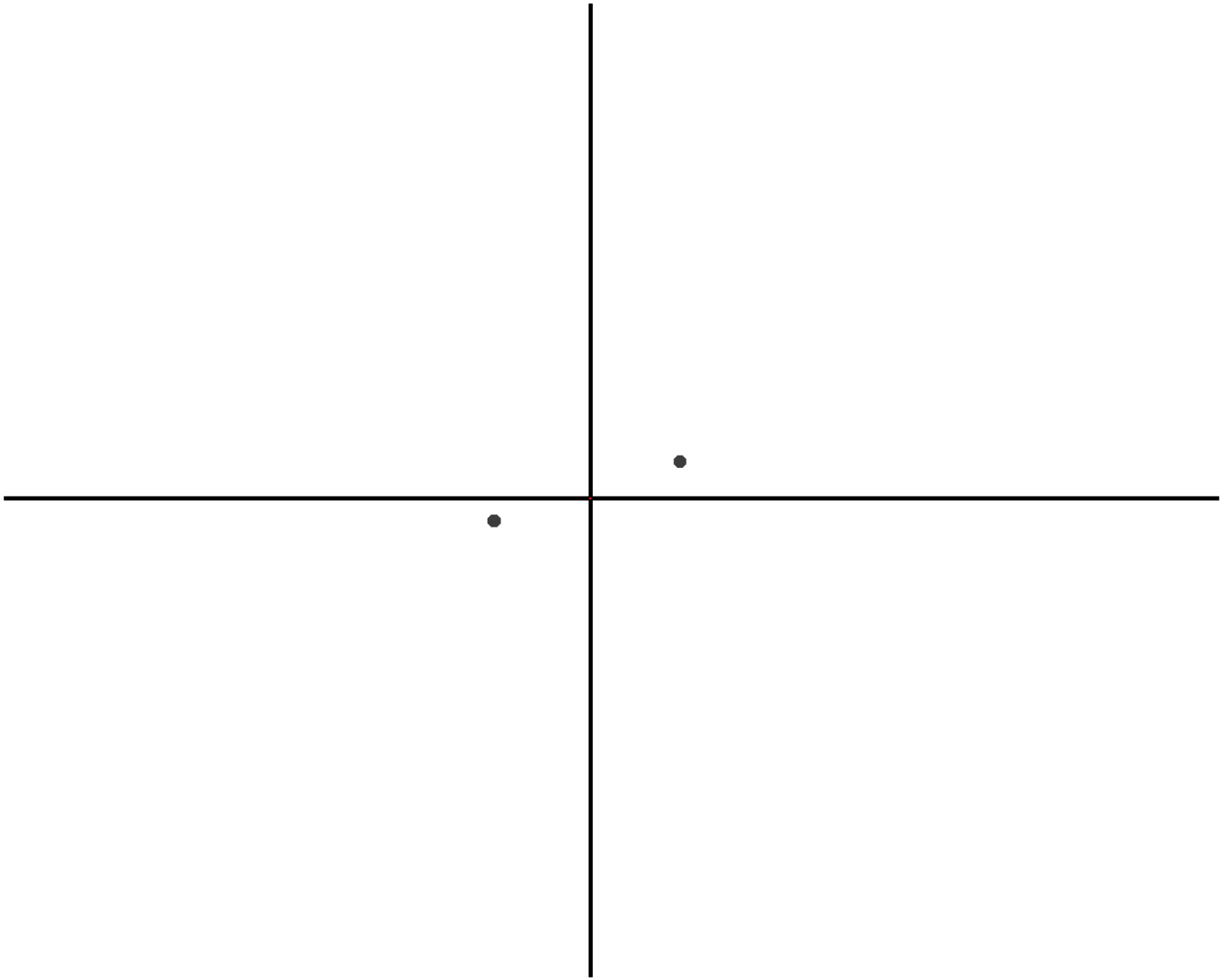
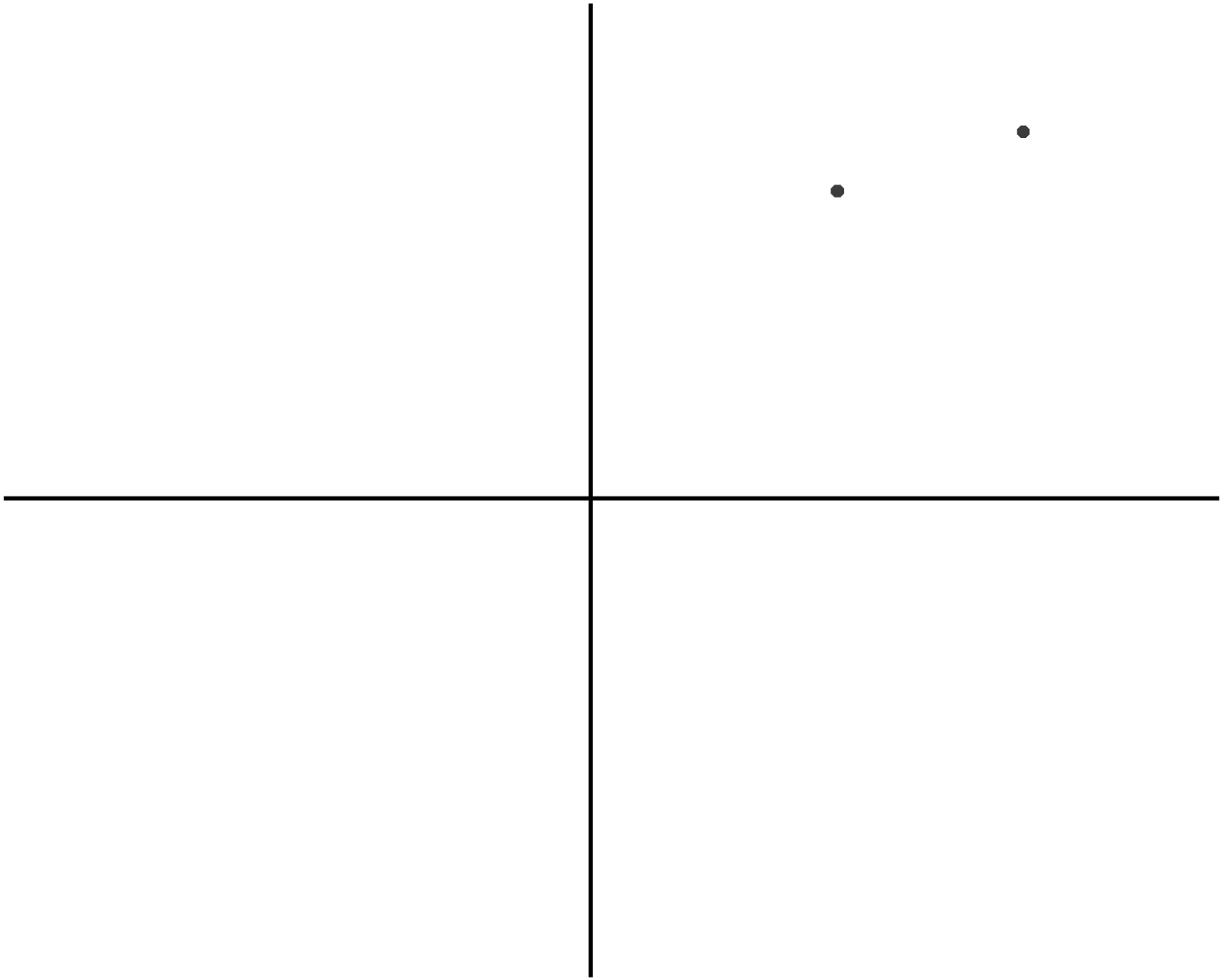
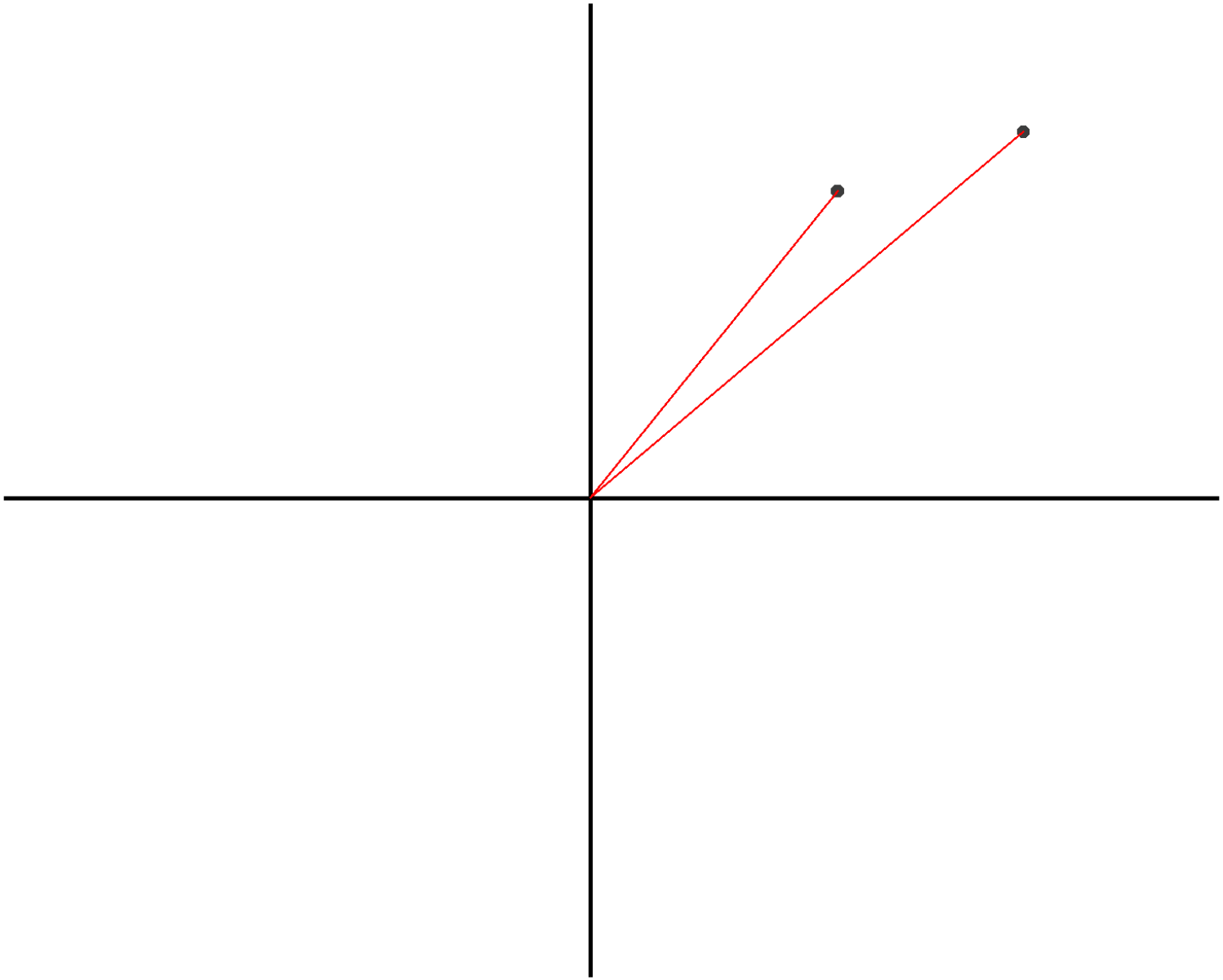
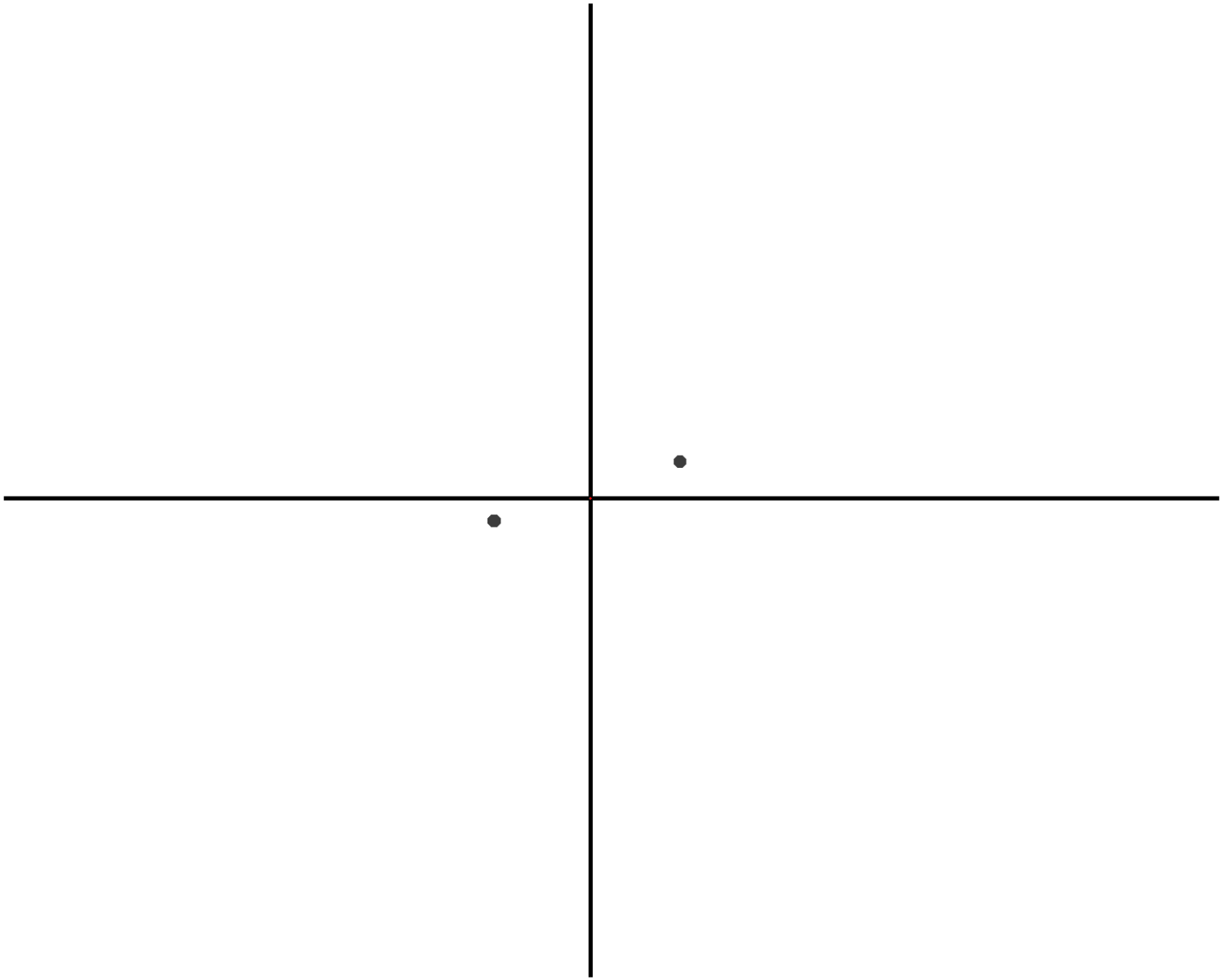
  - But this means changing all $\mathbf{sim}(d_i, d_k)$ or $\mathbf{sim}(d_j, d_k)$!

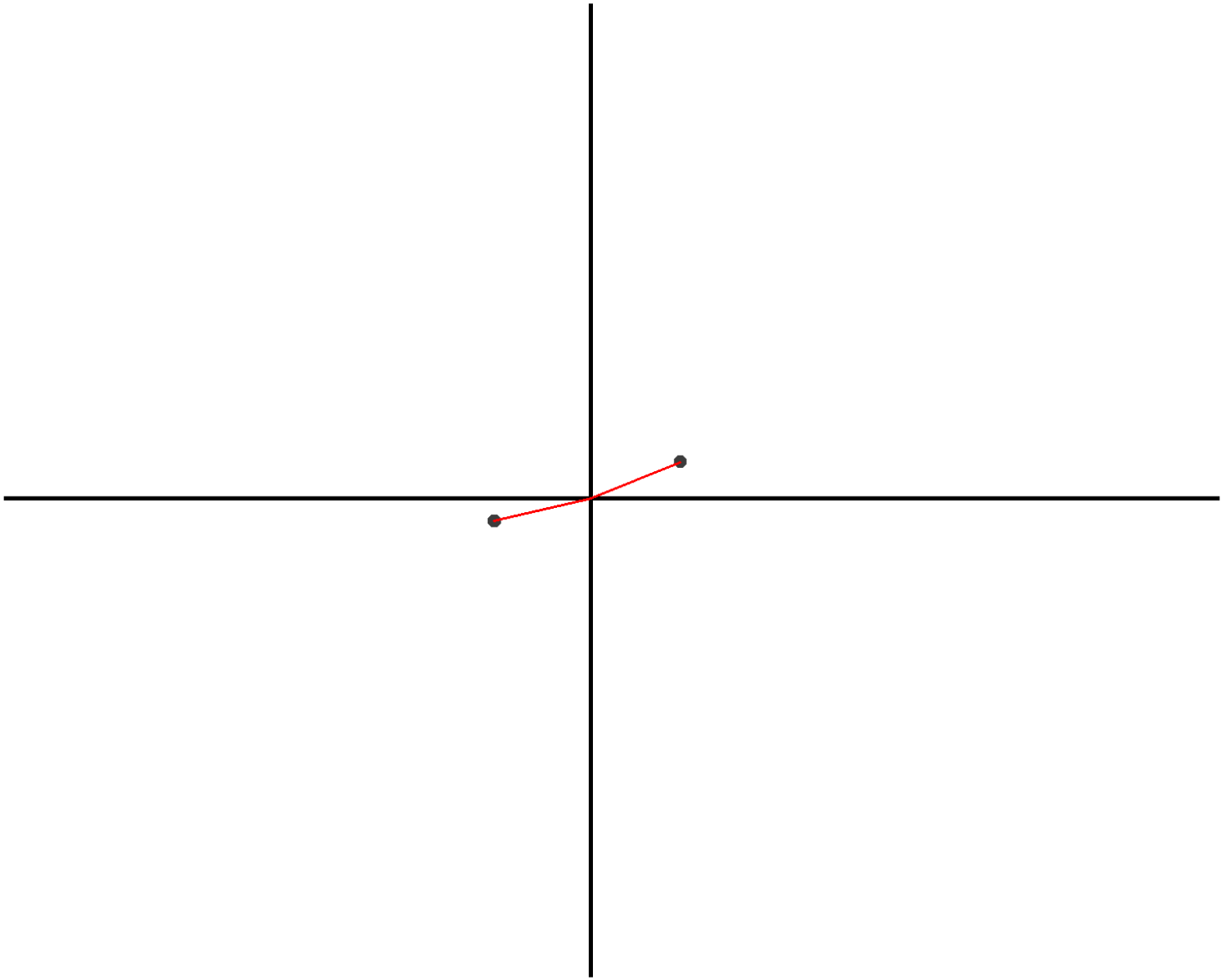












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