Mini Project RP (dayyapp)

Part1

Question1:

- **Simulation of Random Variables:**

The below distribution of random variates were simulated using Matlab Routines and the Acceptance-Rejection Method:

- Normal with mean=2 and variance=2
- Uniform with [2,4]
- Exponential with lambda=2

Matlab Routines:

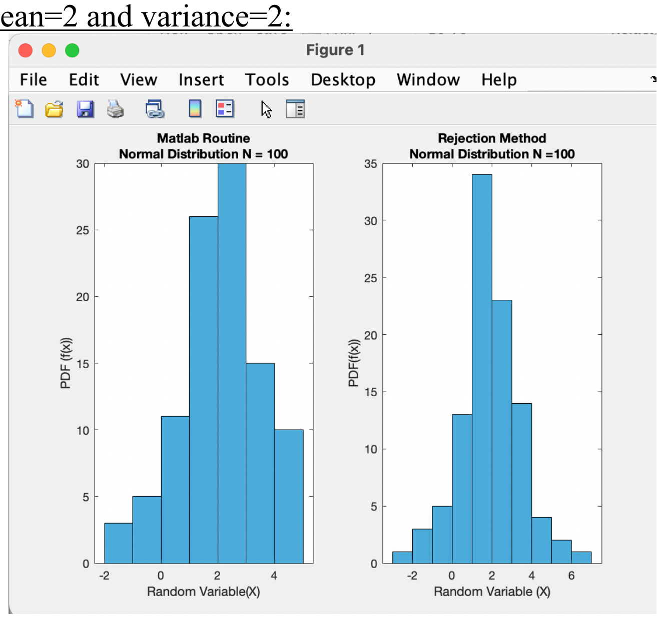
There are many in-built routines to generate the random variates for different distributions. For normal, uniform and exponential, we can use **randn, rand and exprnd built-in routines**. The random variates can also be generated for the distributions by taking the **inverse of the cumulative distribution functions** of each distribution. I have generated the variates using both the methods but have attached the plot for the former method in this report.

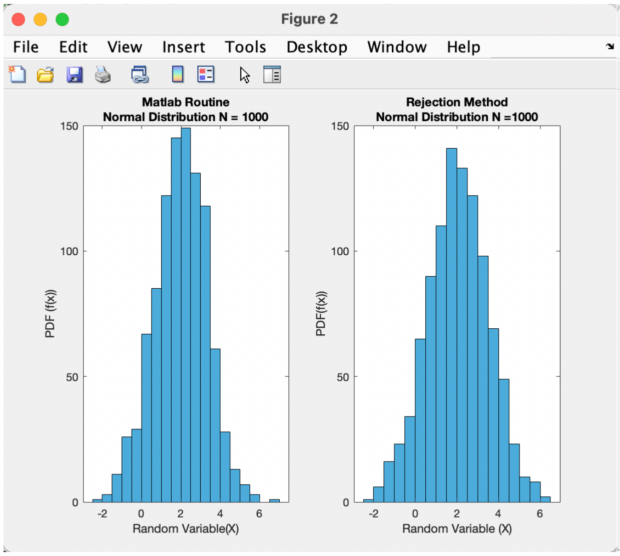
Envelope Rejection Method:

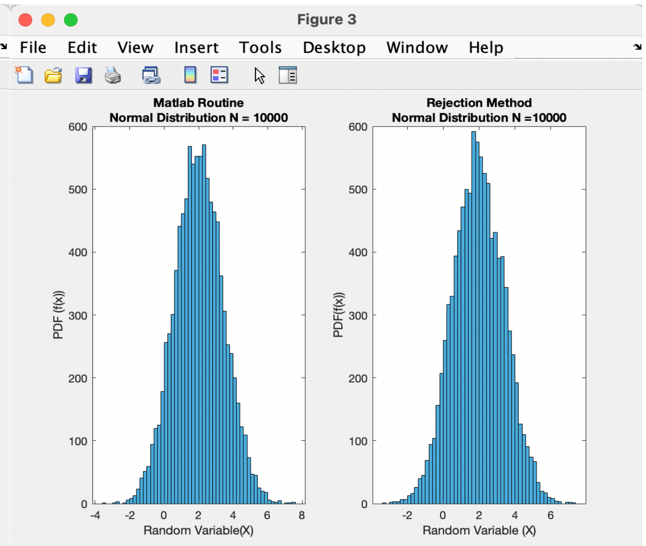
In this method, we use two distributions h and g, say exponential and normal, to generate Variates and keep only the variates that have exactly the desired distribution density function f, say normal distribution. Any variates that do not have the distribution f are discarded. For this simulation, I have taken the normal distribution as the mask distribution and the desired uniform, normal and exponential distributions are generated for different sets of sample values = [100 1000 10000] The scaling factor of the envelope distribution is adjusted based on the desired distribution to ensure it is properly encompassed.

The histograms of the different desired distributions are shown below:

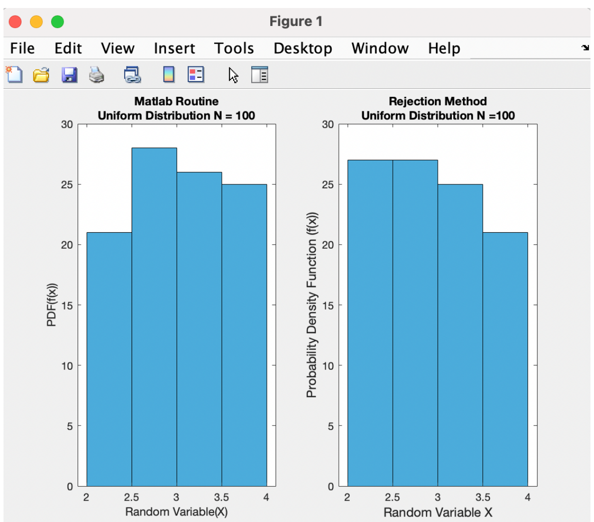
Normal with mean=2 and variance=2:

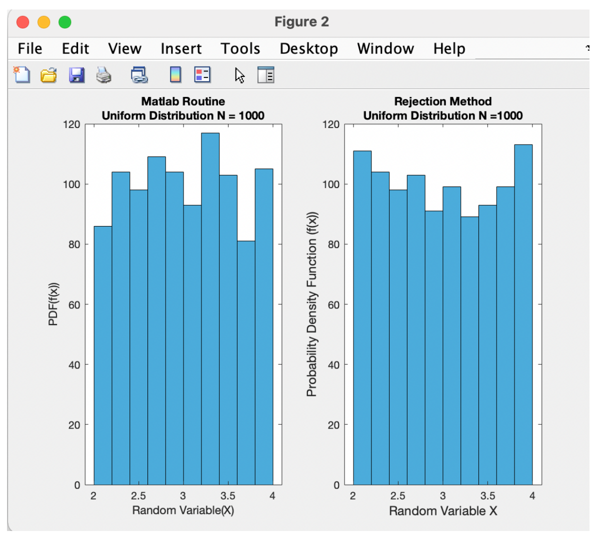




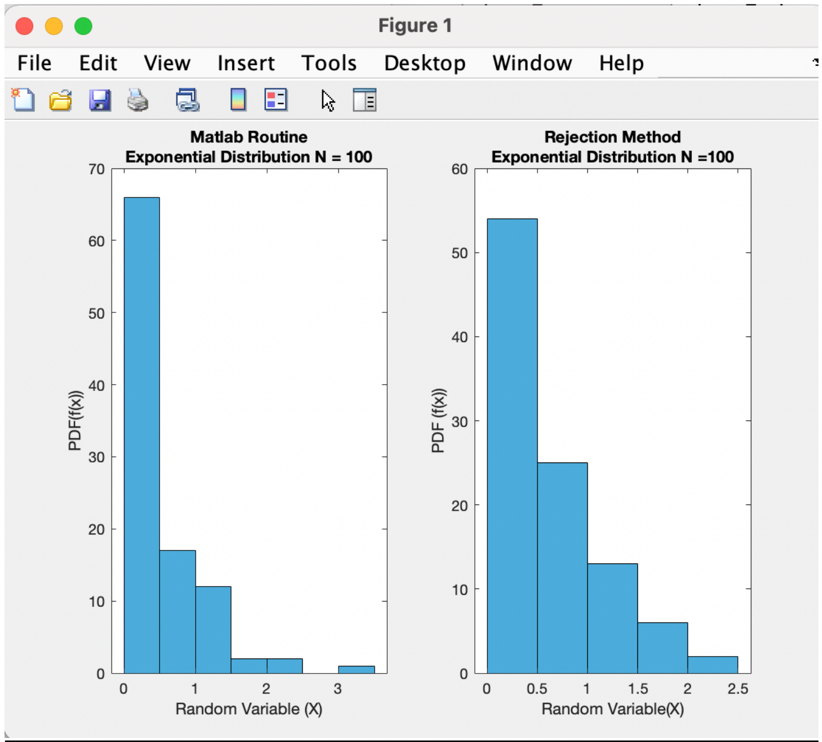


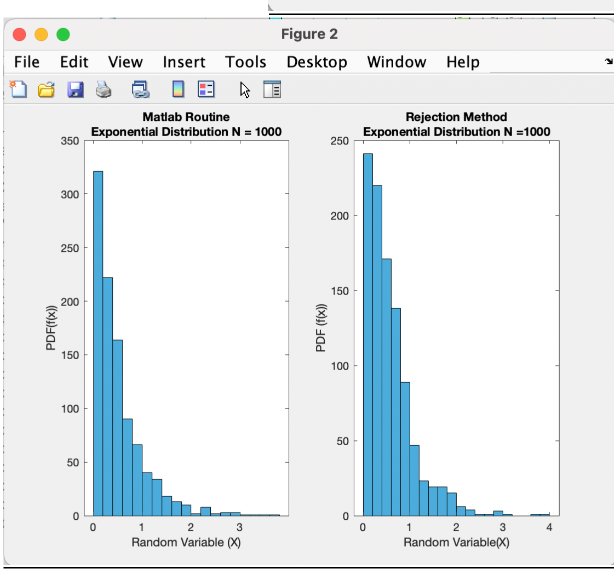
<u>Uniform on [2,4]:</u>

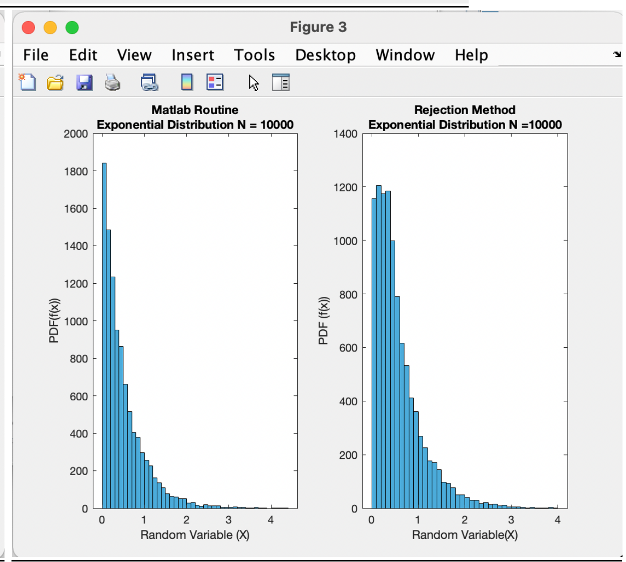




Exponential with lambda=2







- **The parameters – Mean and Variance:**

Computed parameters for above plots:

<u>For the above plotted normal distribution, we have:</u>

No. of Sample Variates	Matlab Routine		Rejection Method	
	Mean	Variance	Mean	Variance
100	2.085972e+00	2.020946e+00	1.901725e+00	2.241191e+00
1000	1.990593e+00	1.768466e+00	2.060256e+00	2.128502e+00
10000	2.024149e+00	1.979552e+00	1.968374e+00	2.012407e+00

<u>For the above plotted uniform distribution, we have:</u>

No. of Sample Variates	Matlab Routine		Rejection Method	
	Mean	Variance	Mean	Variance
100	3.022722e+00	3.264406e-01	2.960437e+00	3.177298e-01
1000	3.005365e+00	3.248574e-01	2.993827e+00	3.507295e-01
10000	2.990632e+00	3.340721e-01	2.992794e+00	3.297209e-01

<u>For the above plotted exponential distribution, we have:</u>

No. of Sample Variates	Matlab Routine		Rejection Method	
	Mean	Variance	Mean	Variance
100	5.170556e-01	2.959155e-01	5.952083e-01	2.909994e-01
1000	5.120294e-01	2.725613e-01	5.660716e-01	2.552091e-01
10000	4.995672e-01	2.552365e-01	5.691013e-01	2.610285e-01

The mean and variance given above are directly associated with the respective plots above for both Matlab routines as well as rejection method.

Theoretical Values of mean and Variance:

- **Normal distribution**: Mean=2, Variance =2
- **Uniform distribution**: a=2,b=4
  - Hence, mean = (a+b)/2 = 6/2 =3
  - Variance = ((b-a)^2)/12 = approx.(0.333)
- **Exponential distribution**: lambda=2
  - Mean = 1/lambda = ½ = 0.5
  - Variance = 1/lambda^2 = 0.25

- **Comparison of Computed and Theoretical Parameters:**

Deviation for Normal distribution: Table 1.2

No. of Sample Variates	Matlab Routine		Rejection Method	
	Mean deviation	Variance deviation	Mean deviation	Variance deviation
100	0.085972e+00	0.020946e+00	-0.098275e+00	0.241191e+00
1000	-0.009417e+00	-0.031534e+00	0.060256e+00	0.128502e+00
10000	0.0024149e+00	-0.020448e+00	-0.031626e+00	0.012407e+00

Deviation for uniform distribution: Table 1.3

No. of Sample Variates	Matlab Routine		Rejection Method	
	Mean deviation	Variance deviation	Mean deviation	Variance deviation
100	0.022722e+00	0.264406e-01	-0.039563e+00	0.177298e-01
1000	0.005365e+00	0.248574e-01	-0.002547e+00	0.507295e-01
10000	-0.009368e+00	0.340721e-01	-0.001267e+00	0.297209e-01

Deviation for exponential distribution: Table 1.4

No. of Sample Variates	Matlab Routine		Rejection Method	
	Mean deviation	Variance deviation	Mean deviation	Variance deviation
100	0.170556e-01	0.401155e-01	0.952083e-01	0.110994e-01

1000	0.120294e-01	0.225613e-01	0.660716e-01	0.052091e-01
10000	0.004322e-01	0.052365e-01	0.691013e-01	0.01108e-01

<u>Mean Comparison:</u>
From the above tables, we can see that the mean values of all the distributions are close to the theoretically calculated values of the respective distribution. I have also summarized the deviations for easy observations. For normal distribution we can see that the deviation decreases as the sample number increases. From table 1.2, For 10000 samples, the mean deviation is only 0.0024 while for that of 100 samples, we have 0.022 deviation. Hence nearly 10 times the deviation is reduced. Similarly, for the uniform and exponential distributions, we can see that the deviation is inversely proportional to the number of samples.
<u>Variance Comparison:</u>
From the table 1.4, we can see that variance deviation is only 0.00523 for the exponential distribution with 10000 samples which shows a significant reduction when compared to 0.04 deviation for the same method with 100 samples. Similar scenario is observed with respect to normal and uniform distribution. And this reduction is deviation is consistent with the increase in the number of samples for all the above three distributions.

- **Reasons for Deviations:**

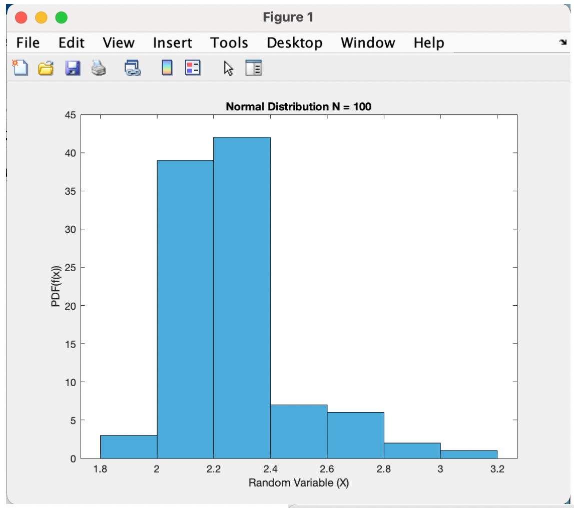
  The reasons for the variation of the computed values from the theoretical values can be due to:
    - Errors during the sample generation process – We can also see from the above tables that comparatively; the envelope rejection method has a little more deviation than the matlab routine method. This may be due to the generation of two distributions and then computing the desired distribution as it complicates the process and there is higher possibility of erroneous sample generation
    - Less number of samples taken – As we can clearly see from the above tables, with smaller samples, the deviation is higher. Hence, by taking higher number of sample variables we can reduce the variations as it will be closer to the true value.

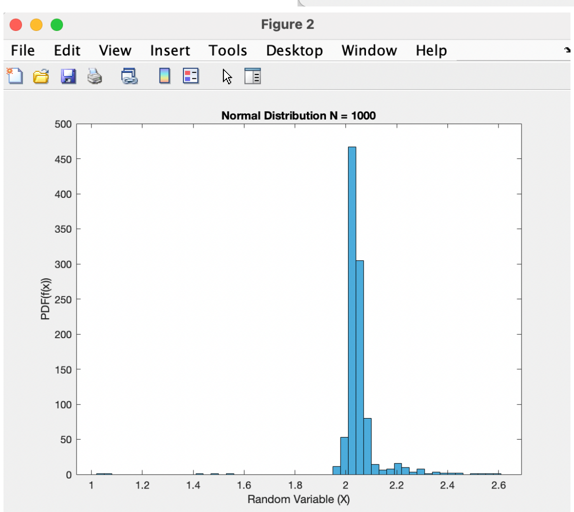
<u>Program files for question 1:</u>
Ques1_normal.m
Ques1_exp.m
Ques1_uniform.m

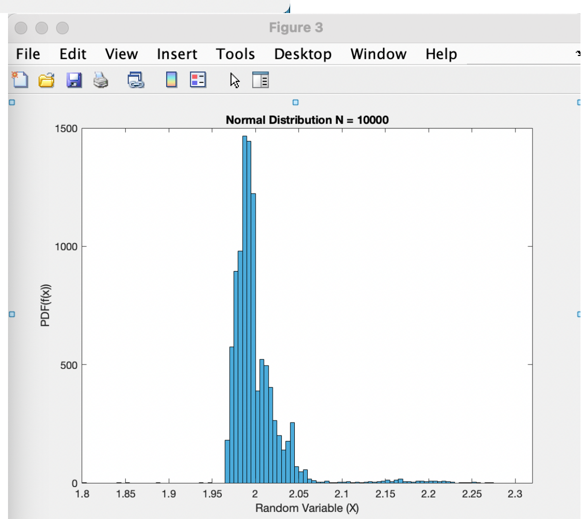
<u>**Question 2:**</u>
<u>**Transforming Random Variables**</u>

- Defining and computing histograms for Yj. Here, we define $Y_i=\_\Sigma X_{i}{}_{i}{}_{T}\ \_i=\_1\_\ldots T$. Where Xi are different distributions such as normal, uniform and exponential. Moreover, the Yj is computed for different sample sizes = 100,1000,10000. The corresponding nine different histograms are plotted below:

  Normal distribution:







Mean and Variance of Xi and Yj:

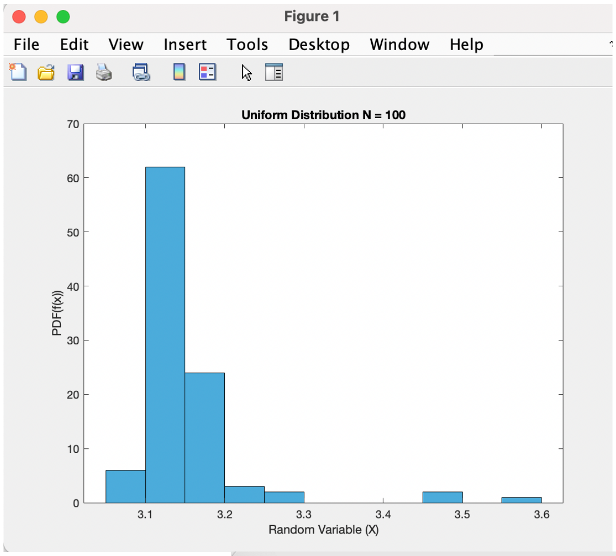
Command Window

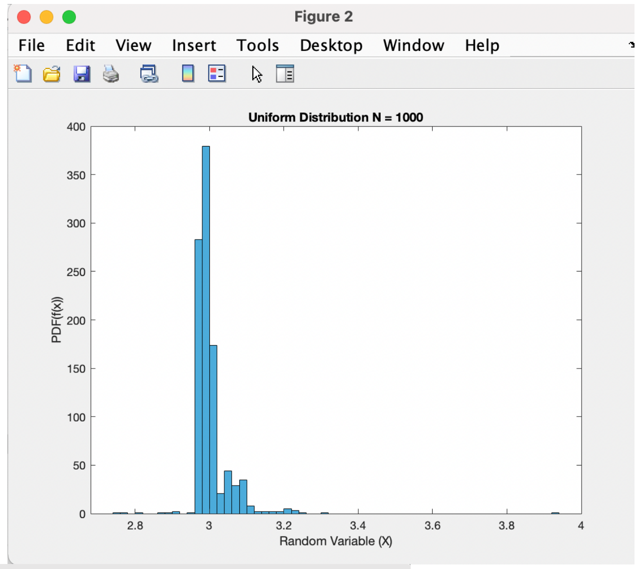
```
x_mean =

    2.0572    2.0262    1.9968

>> y_mean

y_mean =

    2.2498    2.0498    2.0000

fx >>
```

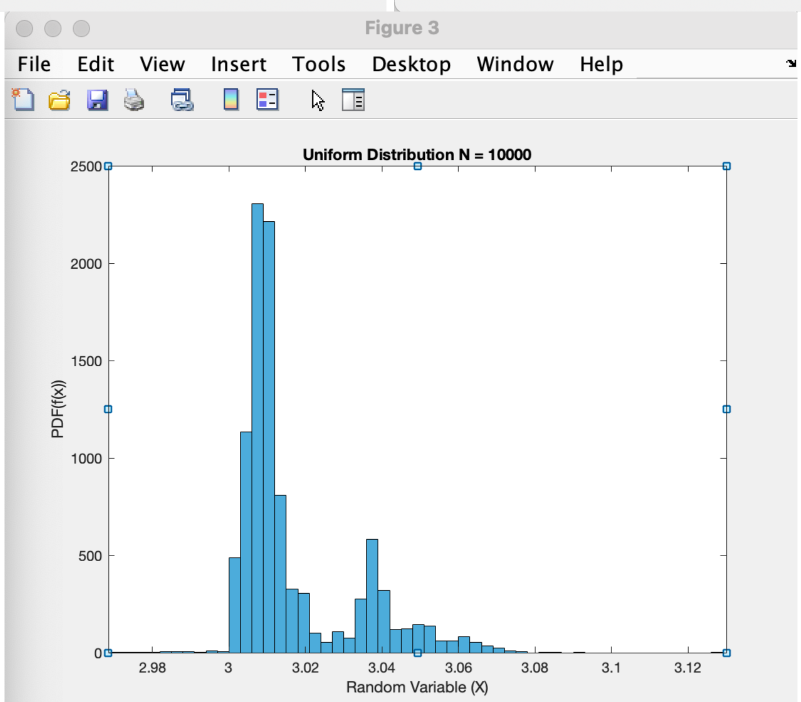
Command Window

```
x_variance =

    1.7762    2.0016    2.0479

>> y_variance

y_variance =

    0.0466    0.0073    0.0010

fx >>
```

Uniform Distribution:





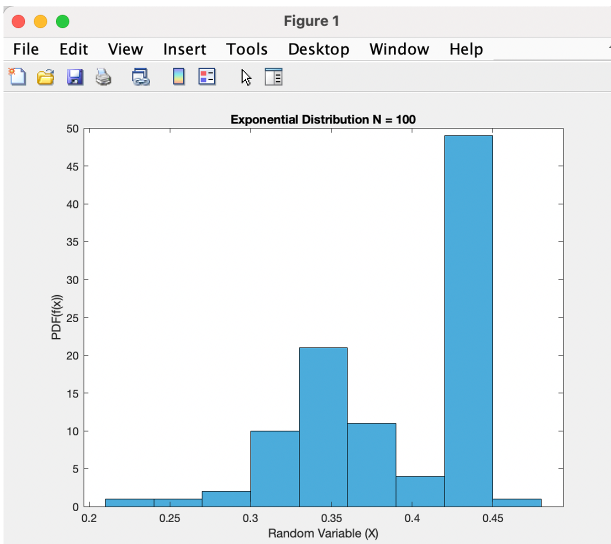


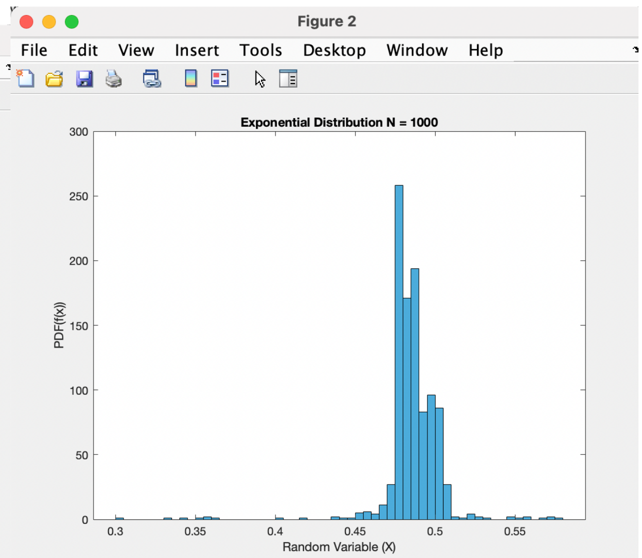
Mean and Variance of Xi and Yj:

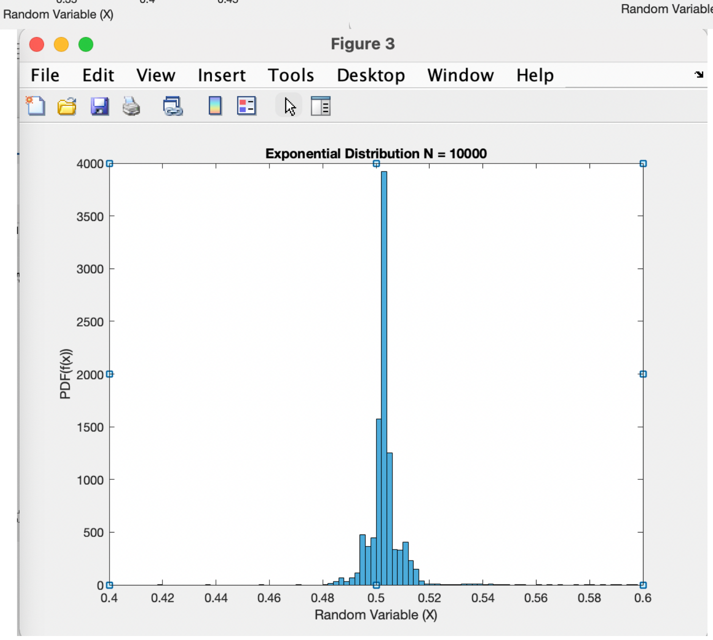
```
Command Window

x_mean =

    3.1110    2.9838    3.0014

>> y_mean

y_mean =

    3.1492    3.0025    3.0166

>>
```

```
Command Window
x_variance =

    0.3094    0.3215    0.3364

>> y_variance

y_variance =

    0.0052    0.0027    0.0004

fx >>
```

Exponential Distribution:







Mean and Variance of Xi and Yj:

Command Window

```
x_mean =

    0.4423    0.4889    0.5020

>> y_mean

y_mean =

    0.3916    0.4859    0.5033

>>
```

Command Window

```
x_variance =

    0.1825    0.2651    0.2477

>> y_variance

y_variance =

    0.0026    0.0003    0.0001

>>
```

- The pdf that is closest in resemblance to the distribution of Y is the Normal distribution irrespective of the distribution of X. Thus, we can see that, for different distributions of X be it exponential, uniform, or normal, the distribution of Y is always normal. Also, we can see that the Mean of Y is same as that of the mean of the X distribution that was used to generate the respective Y distribution. But the Variance of Y = Variance of X/ (sample number^2) and, we can say the variance of Y= $\sigma\sqrt{n}$, where σ is the standard deviation for the distribution of X.
- Also, we can see that as the sample size increases, the sampling distribution of the mean of Y can be approximated by the mean and standard deviation of the normal distribution. That is if we repeatedly perform the same experiment by taking random sample sizes, the when the size is large, the distribution will approach a normal distribution. Also, clearly from the above images, mean and variances, we can see that higher samples are present around the mean for larger distributions.
- We can also state that this is an example of the central limit theorem.

<u>Program files for question 2:</u>
Ques2_normal.m
Ques2_exp.m
Ques2_uniform.m

Question 3

Convergence of Random variables

From the given paper, “Understanding Convergence Concepts: A Visual-Minded and Graphical Simulation-Based Approach”, the below GUI was created to showcase the below different convergences of Yj:

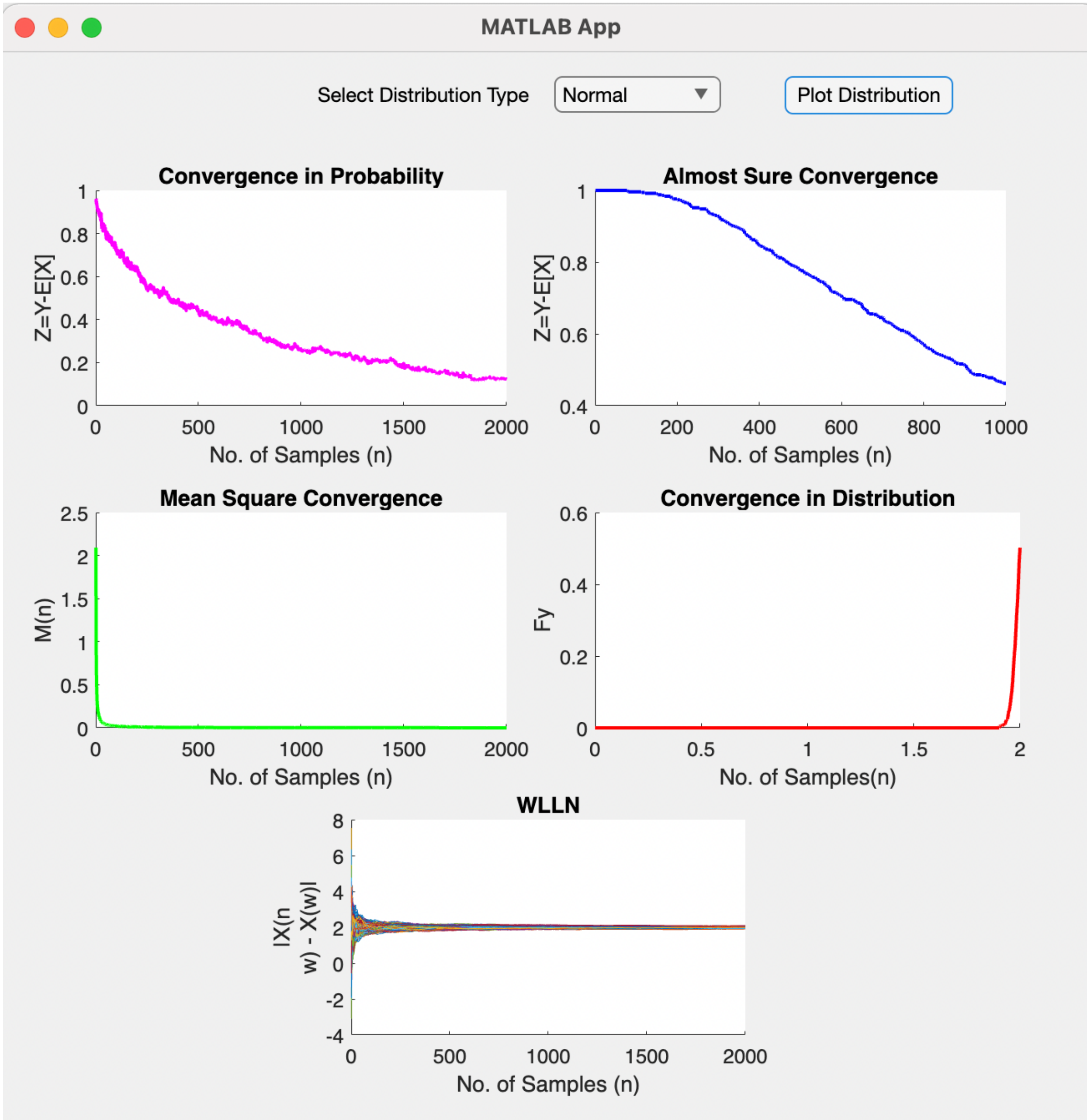
$$Y_T \xrightarrow{P} \boxed{\text{M}}$$
$$Y_T \xrightarrow{a.s.} \boxed{\text{M}}$$
$$Y_T \xrightarrow{2} \boxed{\text{M}}$$
$$Y_T \xrightarrow{L} X$$

The convergence of Y is demonstrated for the three different distributions which are normal, uniform and exponential. Based on the paper, I have taken the values of the sample size to be 2000 and the number of realizations as 500. Also, the epsilon value is taken as 0.05. The convergence is predominantly based on the law of large numbers where, as the number of random variables increases, the probability that the sample mean falls away from the true mean by more than epsilon converges to zero. This has been demonstrated using the GUI in MATLAB.

Based on the simulations, we can conclude that incase of the convergence in probability, mean square convergence and almost sure convergence, the Yt converge towards the mean value of the distribution of X. And the distribution itself converges towards X. Here Y converges to Mean implies that (Y-Mean) converges to zero. Thus, **taking Z=Y-Mean, we have constructed the GUI to plot the convergence of Z towards zero, which means Y converges towards E[X].**

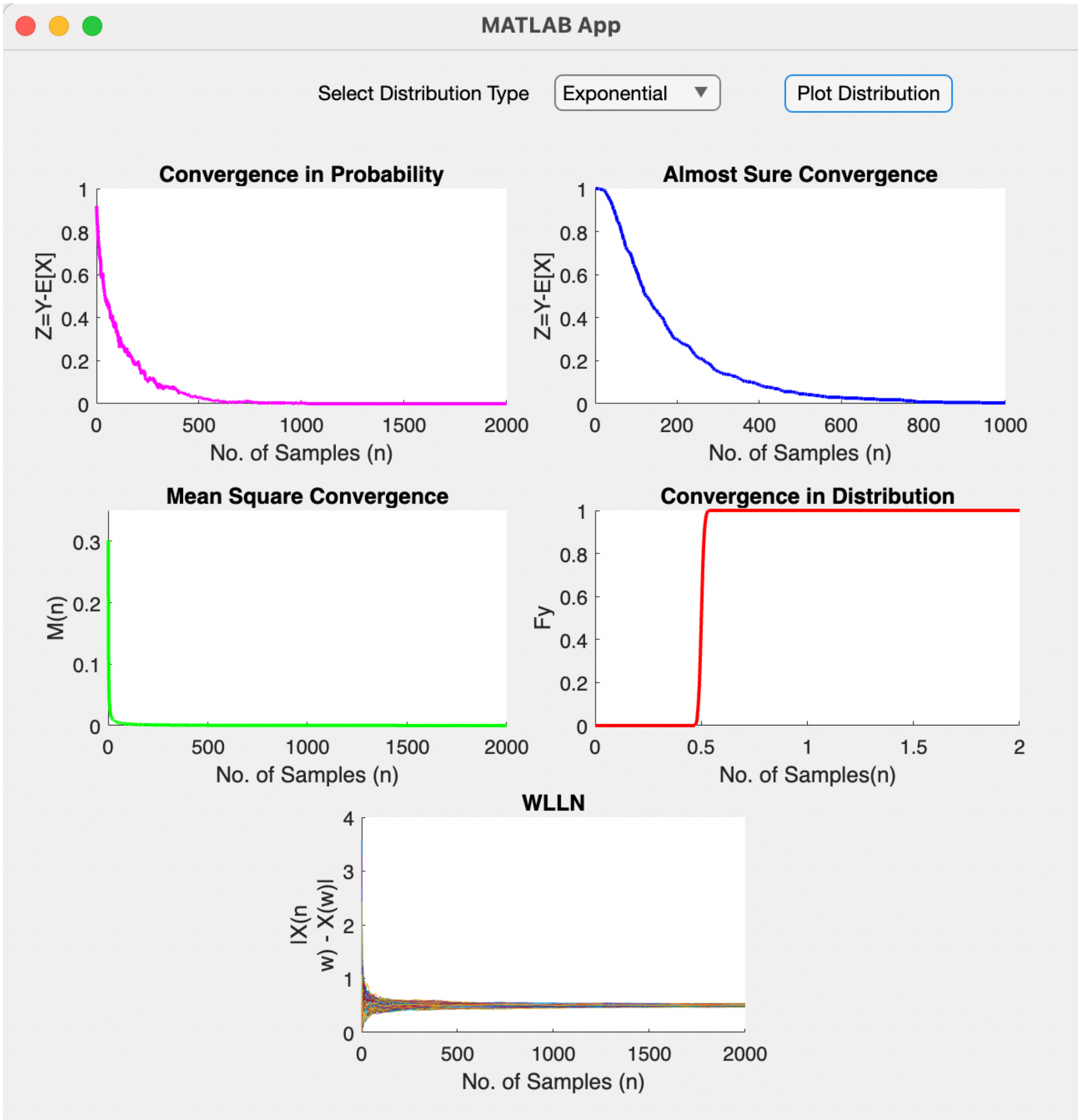
The below figure depicts the GUI of the convergence of a normal distribution. We can see that the In-probability convergence, almost sure convergence, and the mean square converge towards zero. This is because we are converging the Z variables which is Y-E[X] and the convergence of the Y distribution is towards the mean value which is 2 for 500 realizations. Here E[X] is 2 and the probability that Y falls away from the mean by epsilon is zero. Thus, the convergence is towards zero.

Figure 3.1 GUI Depicting convergence in Normal distribution



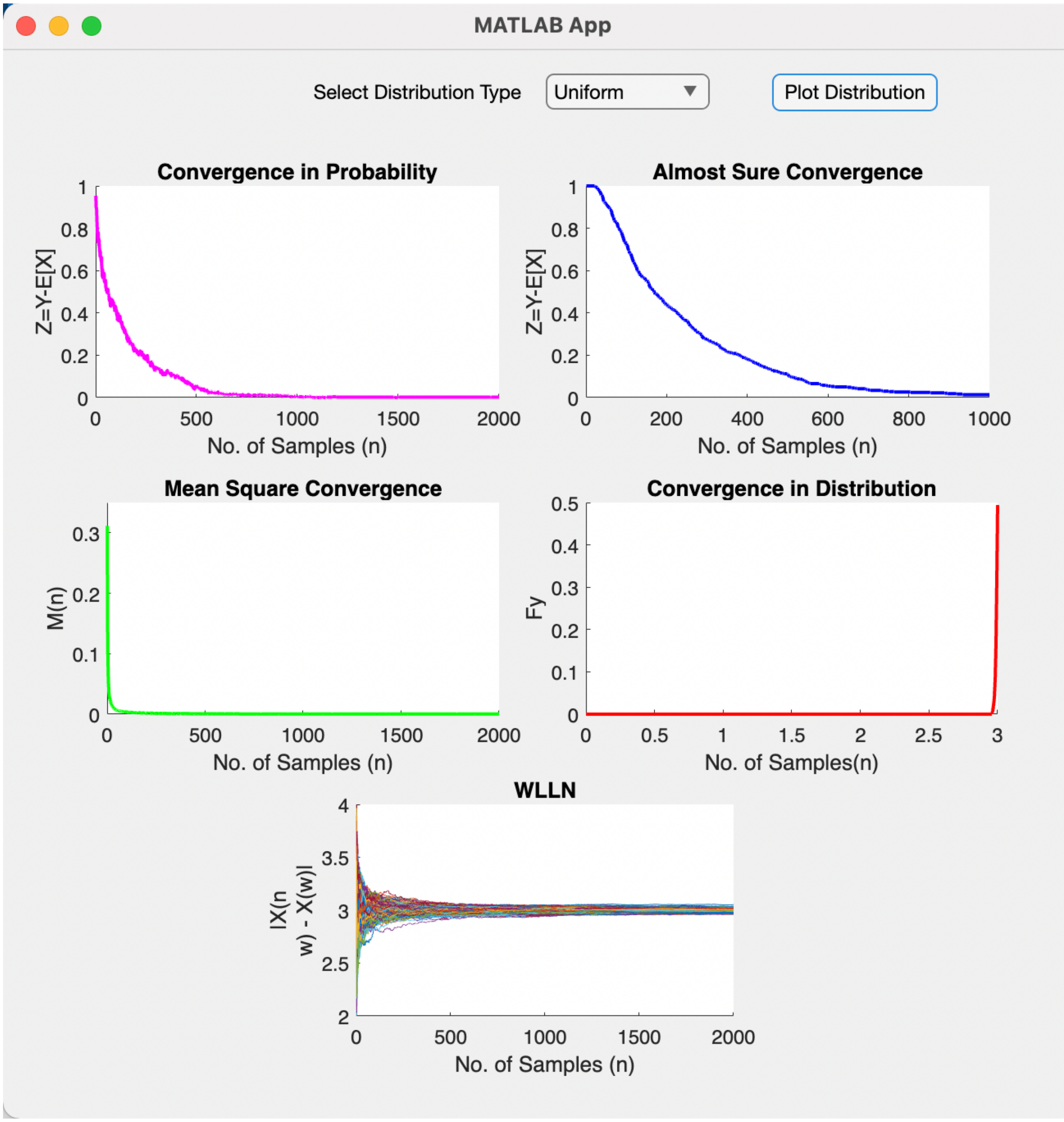
The GUI for the uniform distribution and the exponential distribution are given below. Here also, as in the above cases, Z converges towards zero which is depicted below. And Y converges towards E[X] which is 0.5 for exponential distribution and 3 in case of uniform distribution. The convergence in distribution for each cases is towards the mean though for M realizations.

Figure 3.2 GUI Depicting convergence in Exponential distribution



Here, lambda=2 which implies, mean = 0.5.
We can clearly see that the convergence in probability and almost sure convergence tends to zero as Y tends to expectation of X.

Figure 3.1 GUI Depicting convergence in Uniform distribution



Here, mean is 3. Hence, the convergence in probability and almost sure convergence is towards zero as Y converges towards the mean.

<u>Program files for Ques3:</u>

App1.mlapp
Exponential_convergence.m
Normal_convergence.m
Uniform_convergence.m
Transformation.m