TUTORIAL CLASS
(PHYSICS 3 – FINAL)

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Outline

- Chapter 4: Magnetism
- Chapter 5: Electromagnetic induction
- Chap 6: Alternating current
- Review

CHAPTER 4: MAGNETISM

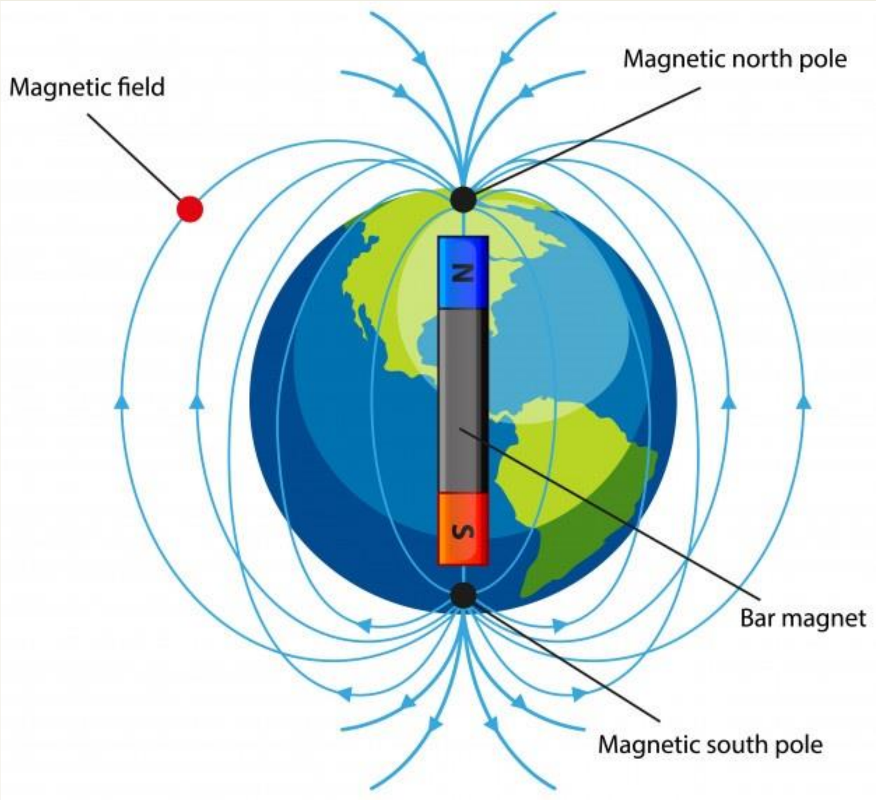
1. Definition of magnetic field

Definition: A vector field that describes magnetic influence of electric current and magnetized materials

The production of magnetic field:

+ Moving charge particles (current)

+ Intrinsic magnetic field in elementary particles (magnets)



1. Definition of magnetic field

The magnetic field force

$$\overrightarrow{F_B} = q\vec{v}\ x\ \vec{B}$$

The magnitude of magnetic force

$$F_B = |q|vBsin\phi$$

B: magnetic field (T)

F_B: magnetic field force (N)

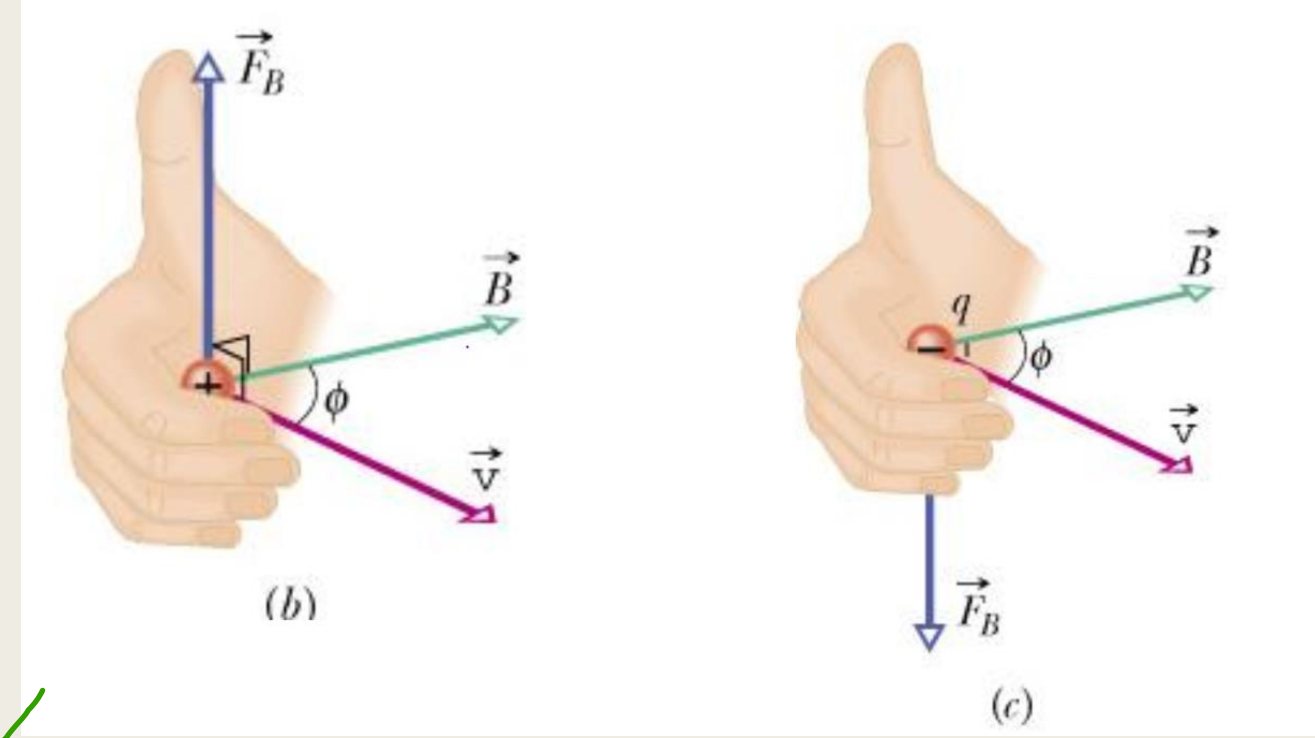
q_B : electric charge (C)

v: velocity ($m/s^{\cancel{2}}$) m/s

ϕ: the angle between $\vec{v}$ and $\vec{B}$

$(\vec{v}, \vec{B})$

1. Definition of magnetic field



$q > 0 \rightarrow$ The force is along the thumb

$q < 0 \rightarrow$ The force is opposite the thumb

Find the angle between a uniform magnetic field of 1.0 mT and the velocity of an electron if the magnetic force acting on the electron is $63.7\ x\ 10^{-19} N$ and a speed of $7\ x\ 10^{4} m/s$

B

$-1{,}6 \times 10^{-19}$

January 2018

F

v

The magnetic force acting on the electron

$F_B = |q| vB sin\phi$

$\rightarrow\ |-1.6\ x\ 10^{-19}|\ x\ 7\ x\ 10^{4}\ x\ 10^{-3}\ x\ sin\phi = 63.7\ x\ 10^{19}$

$\rightarrow sin\phi = 0.568\ \rightarrow \phi = 34.61°$

Determine the angle between a uniform magnetic field of 1mT and the velocity of a proton, if the proton has an acceleration of $3\ x\ 10^9\ m/s^2$ and a speed of $6\ x\ 10^4 m/s$?

$1{,}6 \times 10^{-19}$

B

a

v

January 2016

According to the Newton's Second Law

KL proton

$$F_B = \text{ma}$$

$$\rightarrow |q|vBsin\phi = ma \ \rightarrow sin\phi = \frac{ma}{|q|vB}$$

$$\rightarrow sin\phi = \frac{1.67\ x\ 10^{-27}\ x\ 3\ x\ 10^9}{|+1.6\ x\ 10^{-19}|\ x6\ x\ 10^4\ x\ 10^{-3}} = 0.522 \ \rightarrow \phi = 31.45°$$

The velocity has two component: parallel and perpendicular to B

$v_{||} = vcos\phi = 51177.3\ m/s$ and $v_{\perp} = vsin\phi = 31319.05\ m/s$

⇨ The velocity of a proton: $\vec{v} = 51177.3\overrightarrow{v_{||}} + 31319.05\overrightarrow{v_{\perp}}\ (m/s)$

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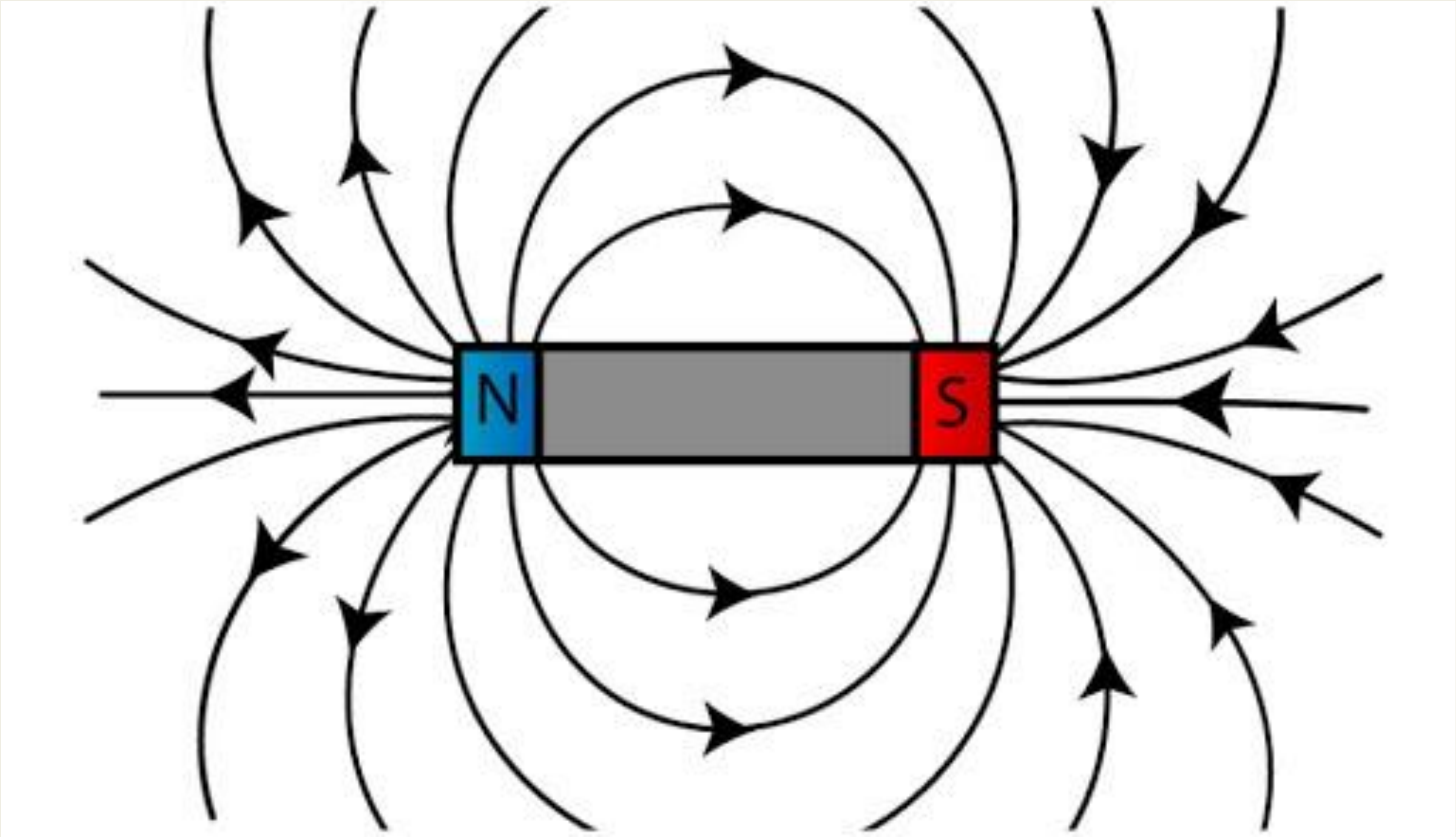
1. Definition of magnetic field



Magnetic lines ⇨ Forms a closed loop

2. Motion of Charge Particle in a Magnetic Field

For a particle moving in constant magnetic field, the magnetic force acts on a charge particle cause them moving in a circular path (or helical path depends on the angle between $\vec{v}$ and $\vec{B}$)

According to Newton's Second Law

$$\boxed{\overrightarrow{F_B} = m\overrightarrow{a_r}}$$

Since the angle between $\vec{v}$ and $\vec{B}$ is 90°

$$F = \rightarrow |q|vB = m\frac{v^2}{R}$$

F

The radius of the circular path

$$R = \frac{mv}{qB}$$

$\Rightarrow v = \ldots$

2. Motion of Charge Particle in a Magnetic Field

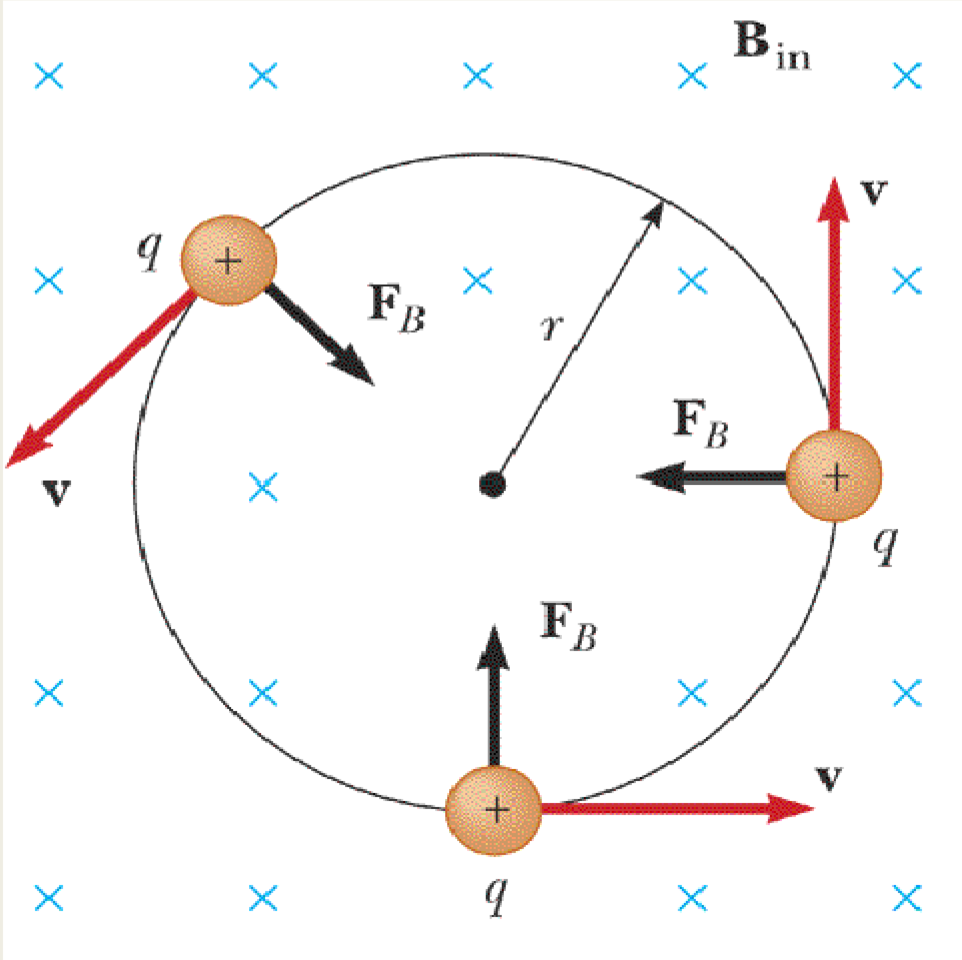
The period

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

The angular frequency

$$\omega = 2\pi f = \frac{qB}{m}$$

$$f = \frac{1}{T}$$

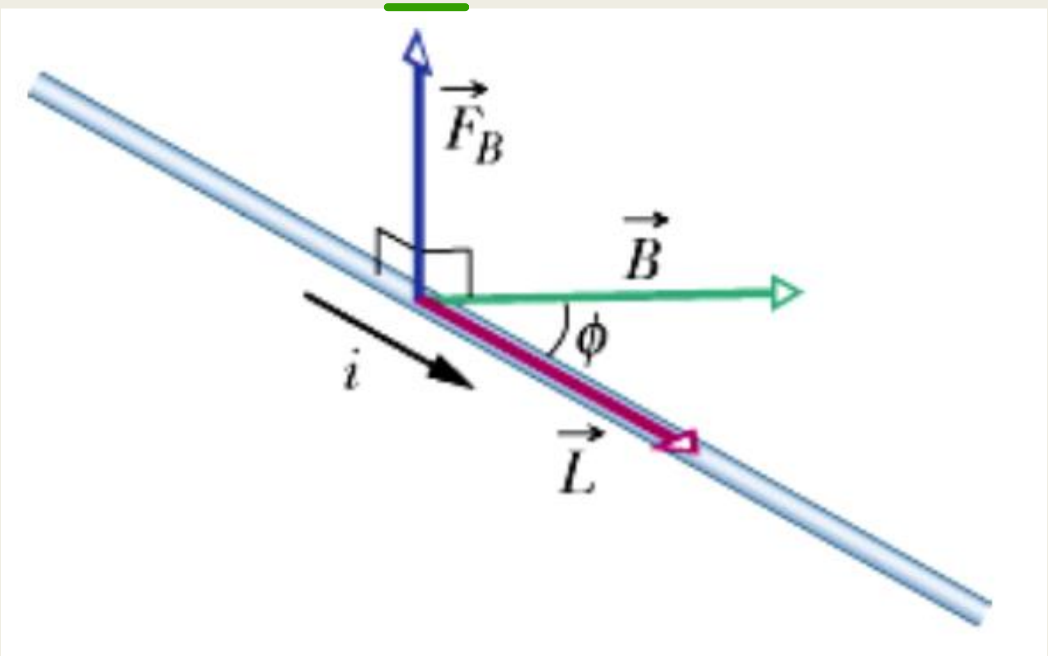


3. Magnetic Force

3.1 Magnetic force on current carrying-wire ?

The magnetic force acting on the current carrying-wire

$$\overrightarrow{F_B} = i\vec{L} \, x \, \vec{B}$$



The magnitude of the force

$$F_B = iLBsin\phi$$

ϕ: the angle between $\vec{L}$ and $\vec{B}$

A wire 230 cm long carries a current of 12.0 A is put in a uniform magnetic field of magnitude B = 3.0 T. The magnetic force on the wire is measured as 41.4 N. Find the angle of the wire with the magnetic field.

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The magnetic force acting on the current carrying-wire

$$F_B = iLBsin\phi \rightarrow sin\phi = \frac{F_B}{iLB}$$

$$\rightarrow sin\phi = \frac{41.4}{12\ x\ 230\ x\ 10^{-2}\ x\ 3} = 0.5\ \rightarrow \phi = 30°$$

$m = \frac{m}{L}$ B

A straight wire of linear mass density 0.08 kg/m is located perpendicular to a magnetic field of 0.7 T as shown in Figure 1. Find the magnitude and the direction of the current needed to balance the gravitational force

F

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ρ

For balancing the gravitational force

$$\overrightarrow{F_B} = -\overrightarrow{F_g} \rightarrow F_B = F_g = mg$$

Assume the length of a wire is 1m

$$m = \rho L = 0.08\ kg$$

$$\rightarrow F_B = F_g = mg = 0.784\ N$$

Since L and B are perpendicular

$$F_B = iLB = i\ x\ 1\ x\ 0.7 = 0.784\ \rightarrow i = 1.12\ A$$

From the right-hand rule, the direction of the current is eastward (to the right)

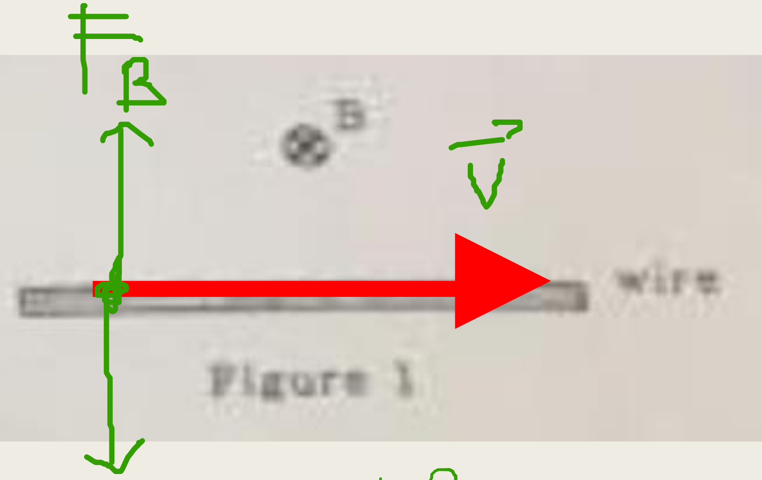


Figure 1

$iLB = mg$

$\Rightarrow i = \frac{mg}{LB}$ (A)

3. Magnetic Force
3.2 Torque on current carrying-wire

Consider a single current-carrying loop

The magnetic torque

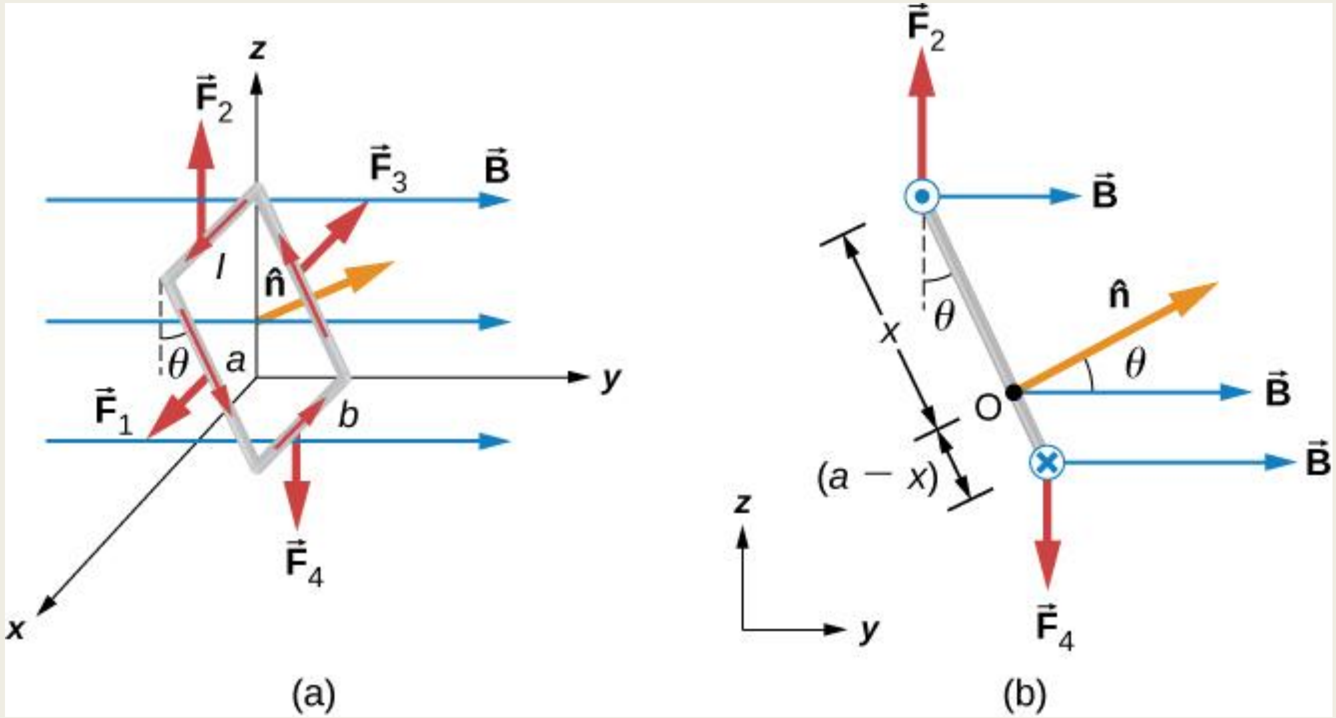
$$\tau_B = iABsin\theta$$

A: The area of the loop

θ: the angle between the force and moment arm (the angle between $\vec{B}$ and $\vec{n}$)

For N loop

$$\tau_B = NiABsin\theta$$



$\mu = NiA \quad (N\cdot m)$

A square loop of 350 turns with a side length of 7 cm carries a current of 10 A. The loop is placed in a magnetic field of 5.0 T. Calculate the magnitude of the maximum torque exerted on the loop

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$\theta = 90$

The maximum torque exerted on the loop $\rightarrow \theta = 90°$

The magnetic torque

$$\tau_B = NiAB = 350\ x\ 10\ x\ (7\ x\ 10^{-2})^2\ x\ 5 = 85.75\ (N.m)$$

$\theta = 90$ B

The plane of a circular loop wire is parallel to a 2.0-T magnetic field. The loop has a radius of 4.0 cm and carries a current of 6.0 A. Calculate the magnitude of the torque that acts on the loop

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The plane is parallel to the magnetic field → $\theta = 90°$

The magnetic torque

$$\tau_B = iAB = 6\, x\, \pi(4\, x\, 10^{-2})^2\, x\, 2 = 0.0603\ (N.m)$$

3. Magnetic Force

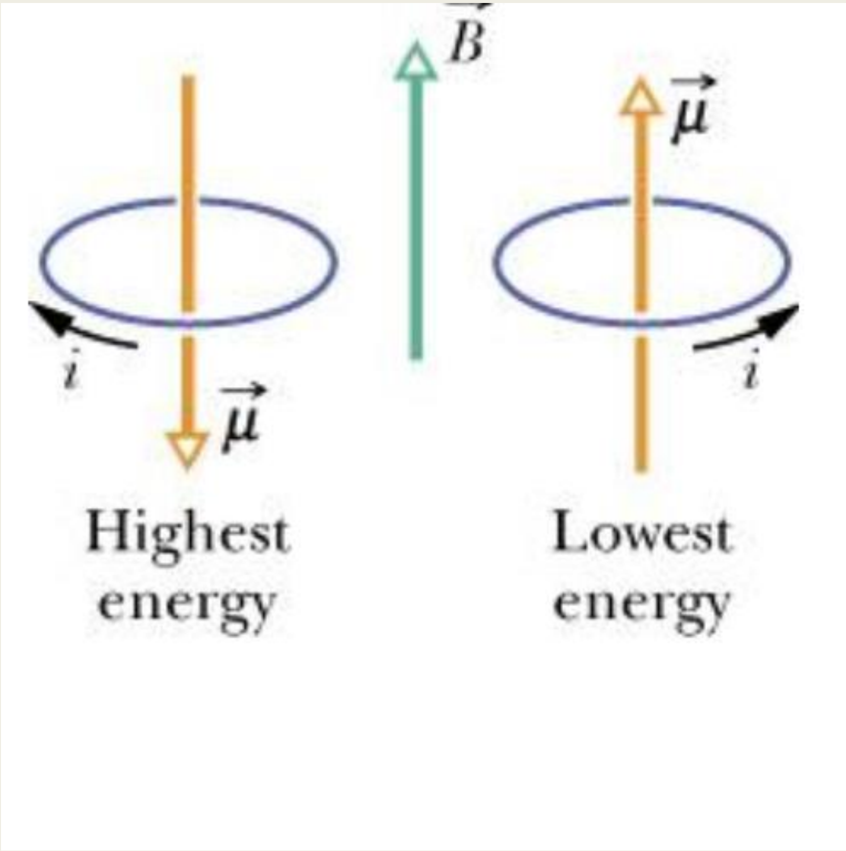
3.2 Torque on current carrying-wire

The current-carrying loop acts like a bar magnet

The magnetic dipole

$$\mu = NiA$$

$\vec{\mu}$ direction can be estimated by the right-hand rule



3. Magnetic Force

3.2 Torque on current carrying-wire

$U = -\mu B \cos\phi$

The magnetic potential energy

$$U(\theta) = -\vec{\mu}\overrightarrow{B} = -\mu B cos\theta$$

The work done on the dipole by the magnetic field

$$W = -\Delta U = -\left(U_f - U_i\right)$$

The work done by the applied force

$$W_a = -W = U_f - U_i$$

work done

(J)

A closed loop with an area of $6 \, x \, 10^{-2} m^2$ carries a current of 5.0A. The loop is placed in an external magnetic field of 0.7T. The dipole moment of the loop initially makes an angle of 60° with the magnetic field. Calculate the work done by the magnetic field as it rotates the loop from its initial orientation to a final one where the dipole moment is align with the magnetic field

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A I B ϕ

The magnetic dipole: $\mu = NiA = 1 \, x \, 5 \, x \, 6 \, x \, 10^{-2} = 0.3$

The initial magnetic potential energy

$$U_i = -\mu B cos\theta_i = -0.3 \, x \, 0.7 \, x \, cos60° = -0.105 \, (J)$$

The final magnetic potential energy

$$U_f = -\mu B cos\theta_f = -0.3 \, x \, 0.7 \, x \, cos0° = -0.21 \, (J)$$

The work done by the magnetic field

$$W = -\Delta U = -\left(U_f - U_i\right) = -(-0.21 + 0.105) = 0.105 \, (J)$$

$U_i - U_f$ | $U = -\mu B \cos\phi$

4. Magnetic field in current
4.1 Bivot-Savart Law

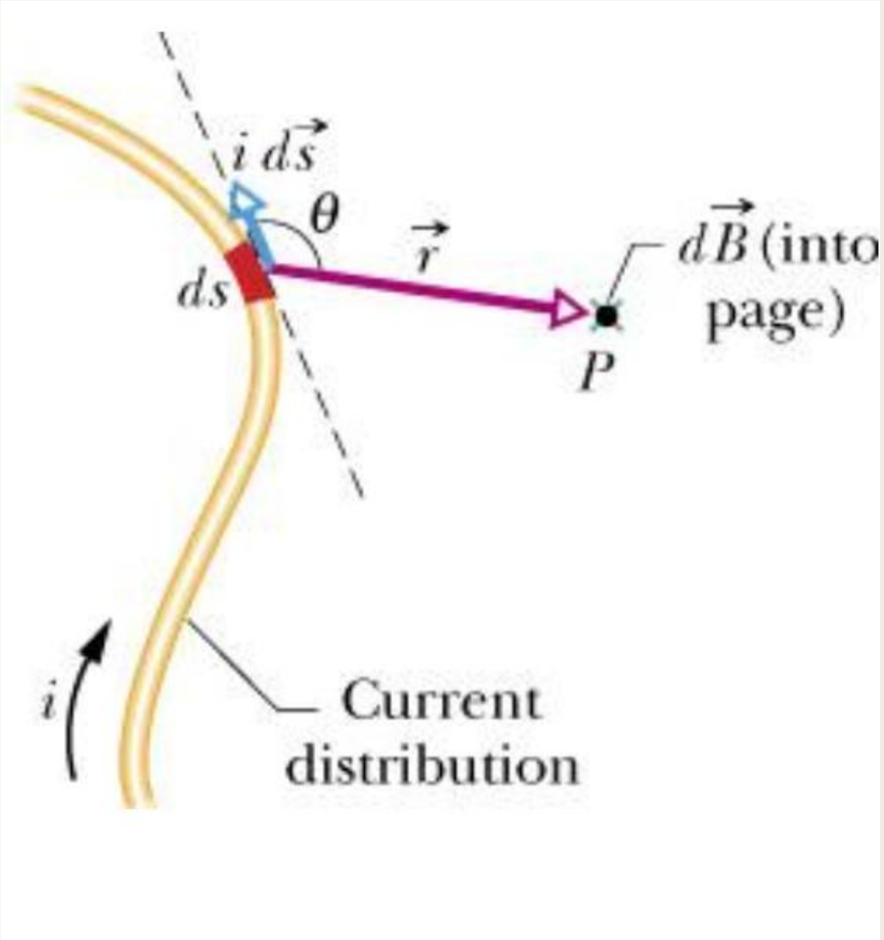
The magnetic field B at a point due to various distribution current

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{s}\ x\ \vec{r}}{r^2}$$

The magnitude

$$dB = \frac{\mu_0}{4\pi}\frac{idssin\theta}{r^2}$$

θ: the angle between $\vec{s}$ and $\vec{r}$



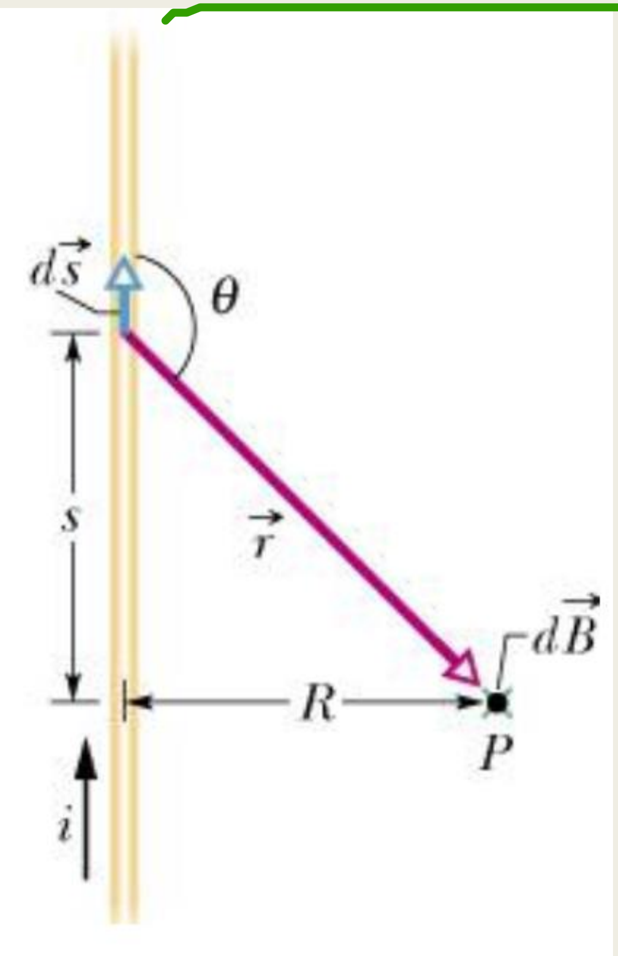
4. Magnetic field in current

4.2 Magnetic Field due to a current in a straight wire

The magnetic field due to a current in a straight wire

$$B = \frac{\mu_0 i}{2\pi R}$$

The direction of B-field is based on the right-hand rule



4. Magnetic field in current

4.3 Magnetic field due to a current in a circular wire

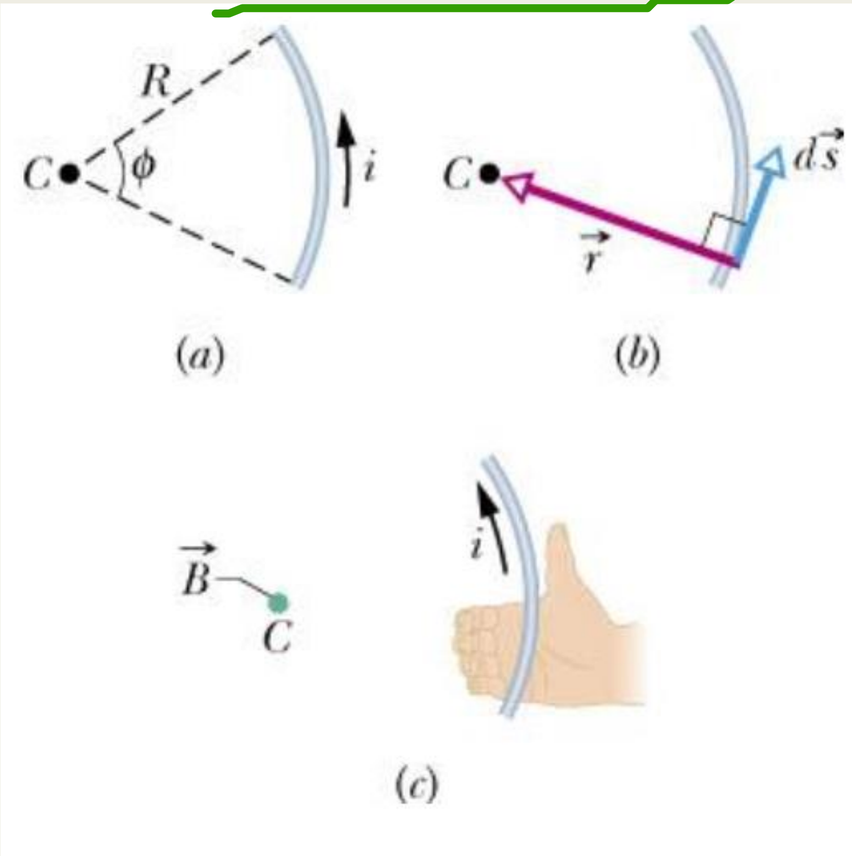
The magnetic field due to a current in a circular arc of a wire

$$B = \frac{\mu_0 i\phi}{4\pi R}$$

π

For a full circle ($\phi = 2\pi$)

$$B = \frac{\mu_0 i 2\pi}{4\pi R} = \frac{\mu_0 i}{2R}$$



Determine the magnitude and the direction of the magnetic field in the center of the circular arcs, point O (Figure 2). The current in the loop is 6.0 A, $r_1 = 2cm$, $r_2 = 4cm$, $\mu_0 = 4\pi x\ 10^{-7}\ T.m/A$

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$\phi = \pi$

The magnetic field at the center for a small circular arc

out

$$B_1 = \frac{\mu_0 i\phi}{4\pi r_1} = \frac{4\pi\ x\ 10^{-7}\ x\ 6\ x\ \pi}{4\pi\ x\ 2\ x\ 10^{-2}} = 9.42\ x\ 10^{-5}(T)$$

The magnetic field at the center for a large circular arc

$$B_2 = \frac{\mu_0 i\phi}{4\pi r_2} = \frac{4\pi\ x\ 10^{-7}\ x\ 6\ x\ \pi}{4\pi\ x\ 4\ x\ 10^{-2}} = 4.71\ x\ 10^{-5}(T)$$

The net magnetic field at the center

$$B = |B_1 - B_2| = 4.71\ x\ 10^{-5}T$$

The direction of the magnetic field is out of the page

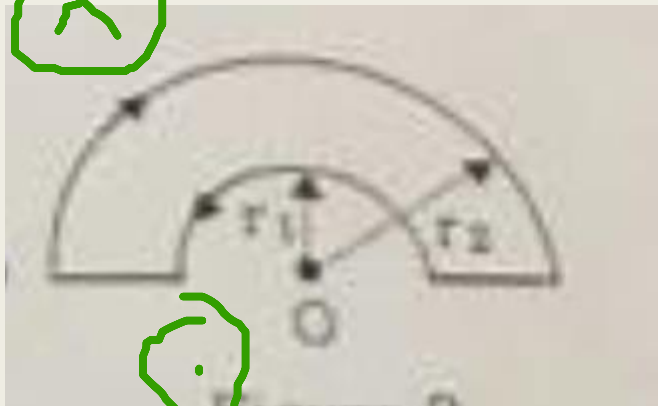

Figure 2

Figure 1 shows two concentric wire loops of radii $r_1 = 5cm$ and $r_2 = 10cm$ that are located in the vertical xy plane. The inner loop carries a current of 5.0 A, and the outer loop carries a current of 12.0 A with the direction as shown in the figure. Fin the magnitude and the direction of the net magnetic field at the center

$\phi = 2\pi$

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The magnetic field at the center for a small circle

$$B_1 = \frac{\mu_0 i}{2r_1} = \frac{4\pi\ x\ 10^{-7}\ \ x\ 5}{2\ x\ 5\ x\ 10^{-2}} = 6.28\ x\ 10^{-5}(T)$$

The magnetic field at the center for a large circle

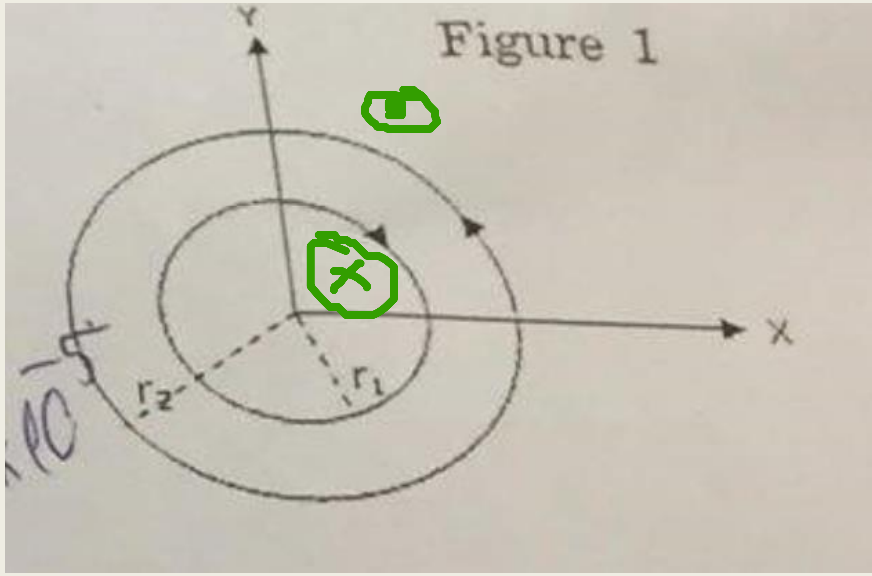
$$B_2 = \frac{\mu_0 i}{2r_2} = \frac{4\pi\ x\ 10^{-7}\ x\ 12}{2\ x\ 10\ x\ 10^{-2}} = 7.54\ x\ 10^{-5}(T)$$

out

The net magnetic field at the center

$$B = |B_1 - B_2| = 1.26\ x\ 10^{-5}T$$

The direction of the magnetic field is out of the page



A loop having two semicircles of radii a = 5.7 cm and b = 8.5 cm with a common center P. A current I = 50mA is set up in that loop (as shown in Fig.1). Find the magnitude and direction of the magnetic field at P $(\mu_0 = 4\pi x\ 10^{-7}\ T.m/A)$

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The magnetic field at point P for a small semicircle

$$B_1 = \frac{\mu_0 i\phi}{4\pi a} = \frac{4\pi\ x\ 10^{-7}\ x\ 50\ x\ 10^{-3} x\ \pi}{4\pi\ x\ 5.7\ x\ 10^{-2}} = 2.75\ x\ 10^{-7}(T)$$

The magnetic field at point P for a large semicircle

$$B_2 = \frac{\mu_0 i\phi}{4\pi b} = \frac{4\pi\ x\ 10^{-7}\ x\ 50\ x\ 10^{-3}\ x\ \pi}{4\pi\ x\ 8.5\ x\ 10^{-2}}$$

$$= 1.88\ x\ 10^{-7}(T)$$

The net magnetic field at the center

$$B = B_1 + B_2 = 4.63\ x\ 10^{-7} T$$

The direction of the magnetic field is into the page

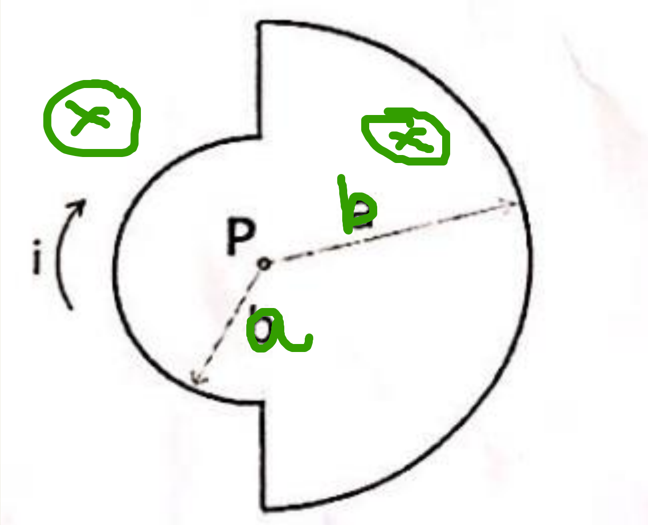

Fig. 2

A segment of wire is formed into the shape as shown in Figure 1, and carries a current I = 2.0 A. Fin the magnitude and the direction of the resulting magnetic field at point P if R = 10cm

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The magnetic field at point P for a circular arc $2R$

$$B_1 = \frac{\mu_0 i\phi}{4\pi 2R} = \frac{4\pi\ x\ 10^{-7}\ x\ 2\ x\ \frac{\pi}{2}}{4\pi\ x\ 2\ x\ 10\ x\ 10^{-2}} 1.57\ (\mu T)$$

The magnetic field at point P for a circular arc R

$$B_2 = \frac{\mu_0 i\phi}{4\pi R} = \frac{4\pi\ x\ 10^{-7}\ x\ 2\ x\ \frac{3\pi}{2}}{4\pi\ x\ 10\ x\ 10^{-2}} = 9.425\ (\mu T)$$

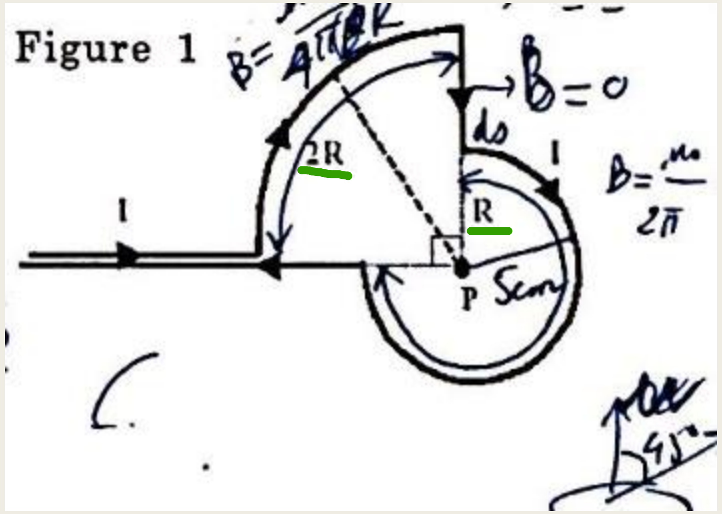


Figure 1

The net magnetic field at point P

$$B = B_1 + B_2 = 10.995\ \mu T$$

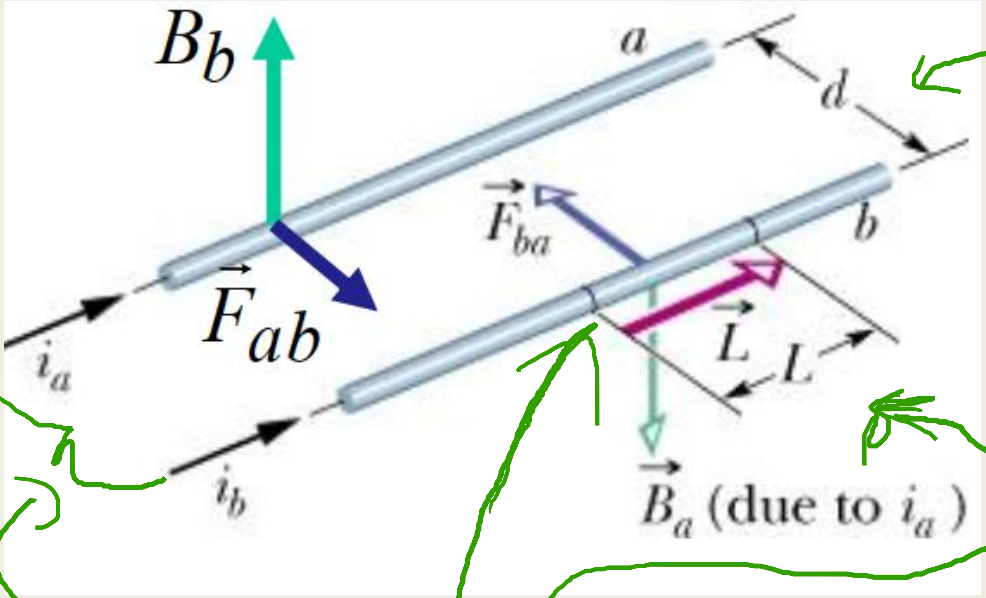
The direction of the magnetic field is into the page

4. Magnetic field in current

4.4 Force between two-parallel wire

The force between two-parallel wire

$$F_{BA} = \frac{\mu_0 L i_a i_b}{2\pi d}.$$



Two current are parallel ⇨ The force pull currents toward

Two current are anti-parallel (opposite direction) ⇨ The force push currents apart

Cùng chiều → hút

ng chiều → đẩy

$$F_{BA} = \frac{\mu_0 L i_a i_b}{2\pi d}$$

1×10^{-2} $\quad d \quad i_a \quad i_b$

Two infinite parallel wires are separated by 1.0 cm and carry currents of 5 A and 7 A in the opposite direction. Find the force (magnitude and direction) per unit length acting on each wire. ($\mu_0 = 4\pi x\ 10^{-7}\ T.m/A$)

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đi ngược

The magnetic force per unit length acting on two infinite parallel wire

$$F_{BA} = \frac{\mu_0 i_a i_b}{2\pi d} = \frac{4\pi\ x\ 10^{-7}\ x\ 5\ x\ 7}{2\pi\ x\ 10^{-2}} = 7\ x\ 10^{-4} (N)$$

The direction of the force: pull apart → parallel

repel

opposite

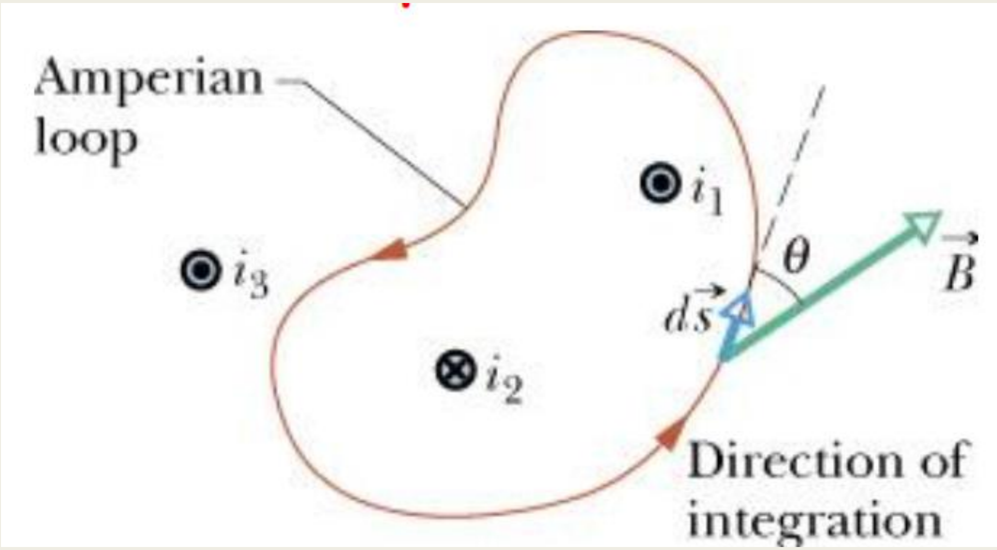
4. Magnetic field in current

4.5 Ampere's Law

The net magnetic field due to some symmetric distributions of currents

$$\oint \vec{B} d\vec{s} = \mu_0 i_{enc}$$

$$\oint B\cos\theta ds = \mu_0 i_{enc}$$



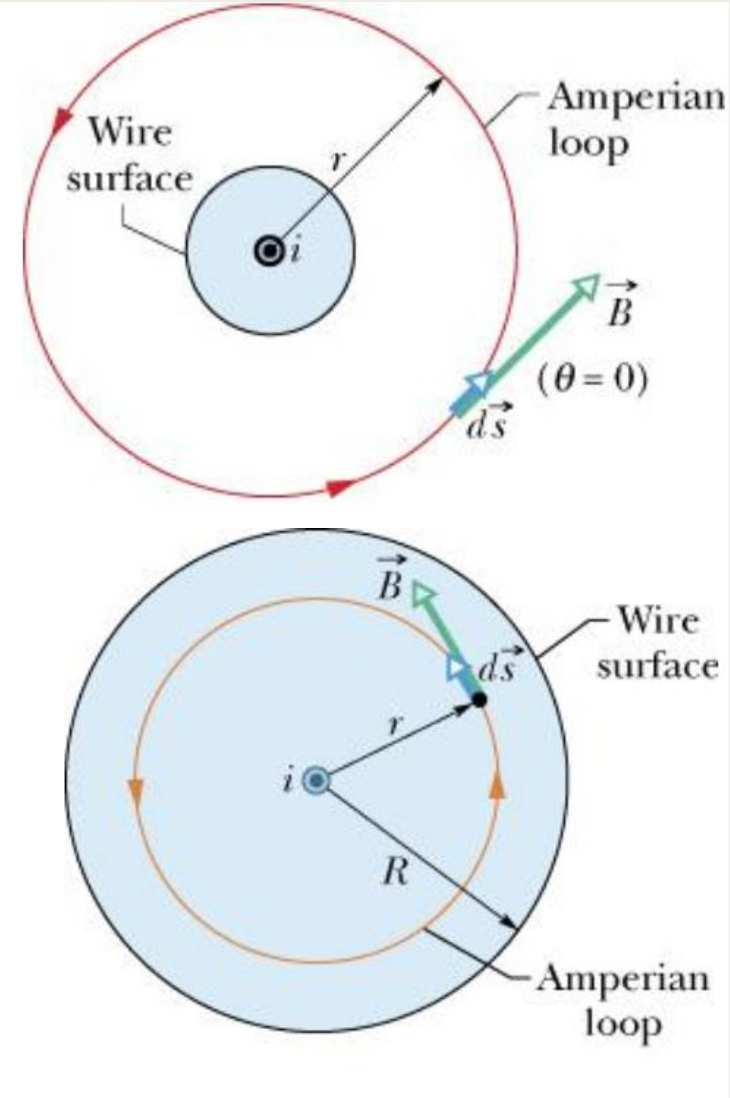
4. Magnetic field in current
4.5 Ampere's Law

The magnetic field outside a long straight wire with current

$$B = \frac{\mu_0 i}{2\pi R}$$

The magnetic field inside a long straight wire with current

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r$$



4. Magnetic field in current

4.5 Ampere's Law

The magnetic field of a solenoid

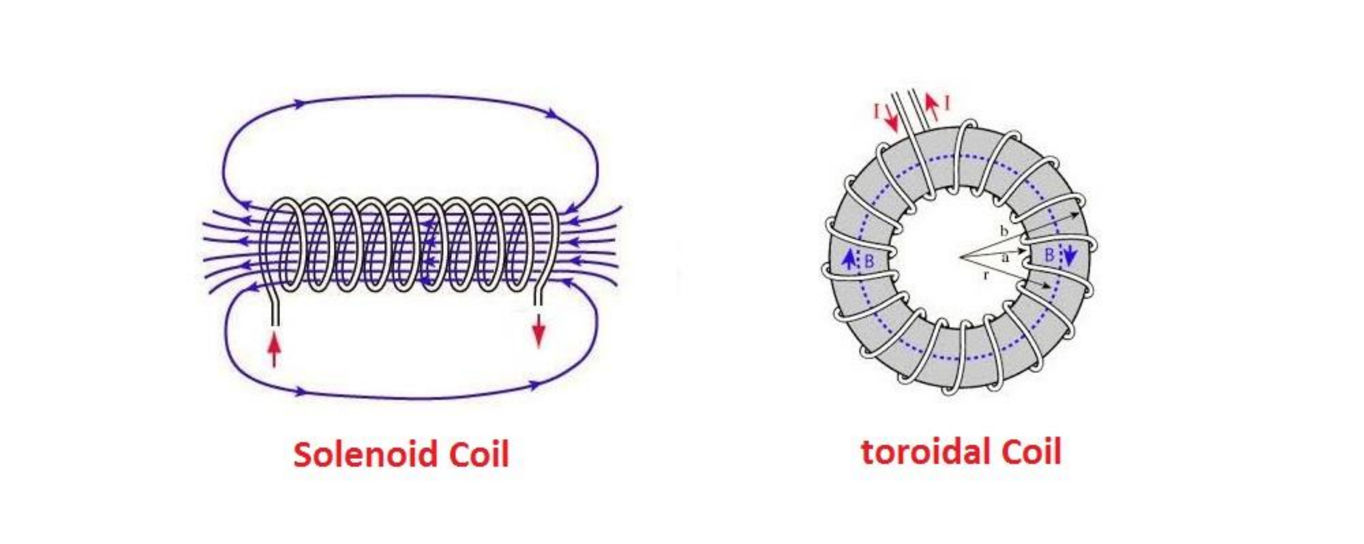
$$B = \mu_0 i n$$

n: number turns per length

The magnetic field of a toroidal

$$B = \frac{\mu_0 i \mathrm{N}}{2\pi}\frac{1}{r}$$

N:the total number of turns



Magnetic field in current

The magnetic field outside a long straight wire with current

$$B = \frac{\mu_0 i}{2\pi R}$$

The magnetic field inside a long straight wire with current

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r$$

The magnetic field due to a current in a circular arc of a wire

$$B = \frac{\mu_0 i \phi}{4\pi R}$$

The magnetic field of a solenoid

$$B = \mu_0 i n$$

n: number turns per length

The magnetic field of a toroidal

$$B = \frac{\mu_0 i \mathrm{N}}{2\pi}\frac{1}{r}$$

N:the total number of turns

The magnetic field of a coil (at a point of the central axis of the coil

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{\frac{3}{2}}}$$

Magnetic force

The magnetic force of a moving charge particle

$$F_B = |q|vBsin\phi$$

The magnetic force acting on the current-carrying wire

$$F_B = iLBsin\phi$$

The magnetic torque

$$\tau_B = iABsin\theta$$

The magnetic force between two-parallel wire

$$F_{BA} = \frac{\mu_0 L i_a i_b}{2\pi d}$$

CHAPTER 5: ELECTROMAGNETIC INDUCTION

1. Faraday's Law of Induction

The magnetic flux through the loop

$$\phi_B = \int \vec{B} d\vec{A} = \int B cos\theta dA$$

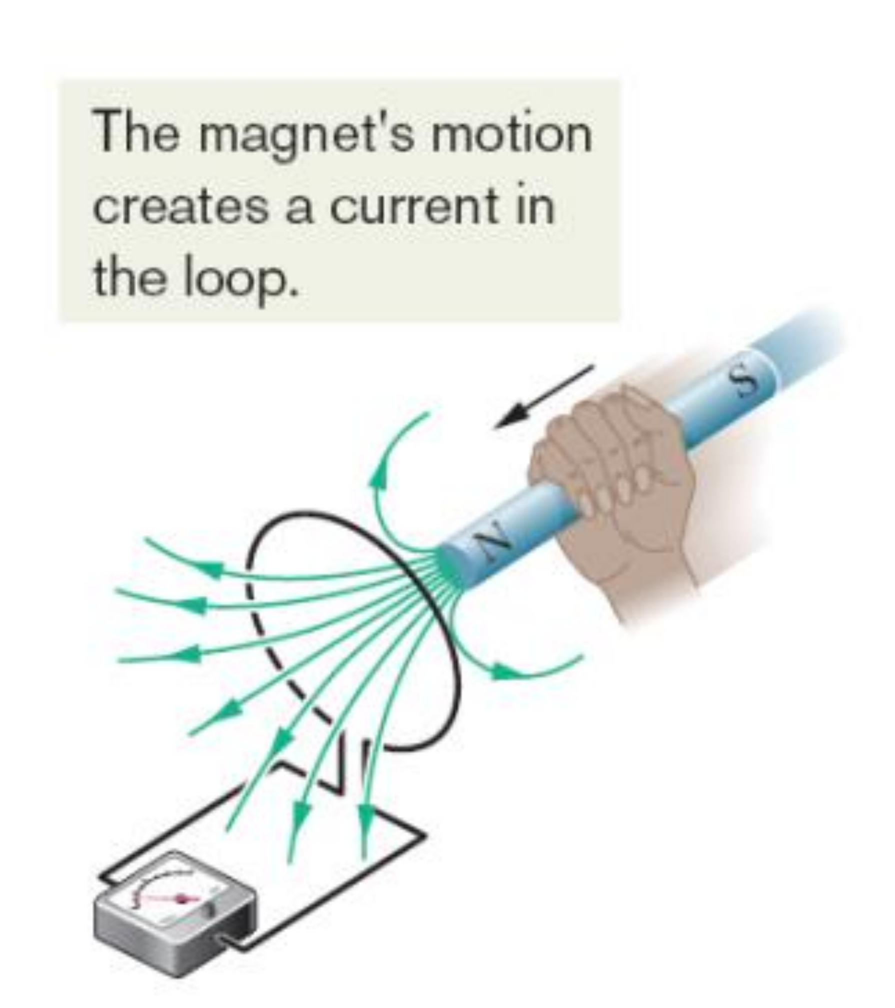
If $\theta = 0° \rightarrow \phi_B = BA$

Faraday's Law: The magnitude of the emf ε induced in a conducting loop is equal to the rate at which the magnetic flux ϕ_B through that loop will change over time

$$\varepsilon = -\frac{d\phi_B}{dt}$$

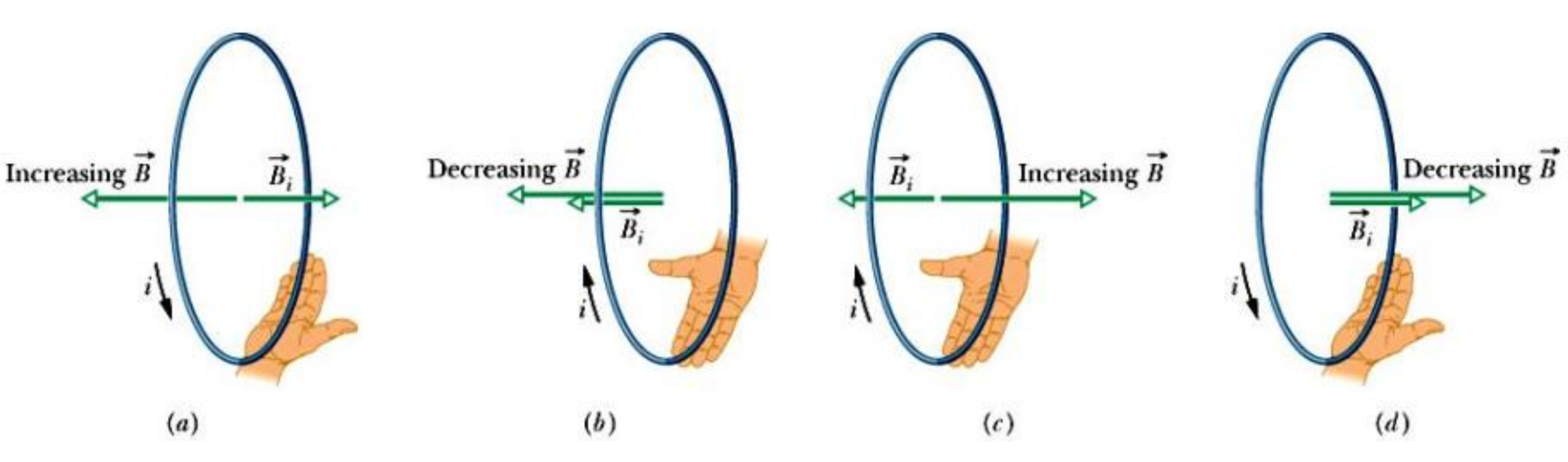
For N turns

$$\varepsilon = -\frac{dN\phi_B}{dt}$$



1. Faraday’s Law of Induction

Lenz’s law: An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current.



A conducting loop of area $50 cm^2$ is perpendicular to a magnetic field that increases uniformly in magnitude from 0.1 T to 7.5 T in 2.0 s. Find the resistance of the loop if the induced current has a value of 1.5 mA.

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$\Phi = BA$ Magnetic flux

The change of magnetic flux

$$\Delta\phi_B = \phi_{B,f} - \phi_{B,i} = (7.5 - 0.1)\ x\ 50\ x\ 10^{-4} = 0.037$$

The induced emf

$$\varepsilon = -\frac{\Delta\phi_B}{\Delta t} = -\frac{0.037}{2} = -0.0185\ (V)$$

The resistance of the loop

$$R = \frac{\varepsilon}{I} = \frac{0.0185}{1.5\ x\ 10^{-3}} = 12.33\ \Omega$$

A coil has 150 turns and each turn encloses an area of $1.0m^2$. Determine the rate of change of a magnetic field parallel to the axis of the coil in order to induce a current of 0.1A in the coil. The resistance of the coil is 150Ω

N

A

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$R = \frac{U}{I}$

The induced emf

$$\varepsilon = IR = 0.1\ x\ 150 = 15V$$

We have

$$\varepsilon = -\frac{\Delta N\phi_B}{\Delta t} = -\frac{N\Delta BA}{\Delta t} = -\frac{\Delta B}{\Delta t}\ x\ 1\ x\ 150 = 15$$

N

A

$$\rightarrow \frac{\Delta B}{\Delta t} = -0.1\ (\text{T/s})$$

rate

$\Phi = BA$

N d R

A circular coil has 100 turns of diameter of 16cm with a total resistance of 10Ω. The plane of the coil is perpendicular to a uniform magnetic field. At what rate should the magnetic field change for the power dissipated in the coil to be 1.2 W?

P

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The current

$$P = I^2 R = I^2\ x\ 10 = \ 1.2\ \rightarrow I = \frac{\sqrt{3}}{5}$$

The induced emf

$$\varepsilon = IR = \frac{\sqrt{3}}{5}\ x\ 10 = 2\sqrt{3}V$$

We have

$$\varepsilon = -\frac{\Delta N\phi_B}{\Delta t} = -\frac{N\Delta BA}{\Delta t} = -\frac{\Delta B}{\Delta t}\ x\ 100\ x\ \pi\ x\ \left(\frac{16\ x\ 10^{-2}}{2}\right)^2 = 2\sqrt{3}$$

$$\rightarrow \frac{\Delta B}{\Delta t} = -1.722\ (\mathrm{T/s})$$

A 100-turn coil is placed in a magnetic field so that the normal to the plane of the coil makes an angle of 45° with the direction of the magnetic field. An induced emf of 100 mV appears in the coil if we increase the magnetic field from $300\mu T$ to $600\mu T$ in a time interval of 1.0 s. Find the cross sectional area of the coil.

N ε Φ B t

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The induced emf

$$\varepsilon = -\frac{\Delta N\phi_B}{\Delta t} = -\frac{N\Delta B\cos\alpha A}{\Delta t} = -\frac{100\ x\ (600\ -300)\ x\ 10^{-6}\ \cos 45°\ x\ A}{1}$$

$$= 100\ x\ 10^{-3}$$

$$\rightarrow A = 4.714\ m^2$$

$\Phi = BA\cos\alpha$

2. Inductor and self-induction

An inductor is an electrical device that stores energy in a magnetic field when electric current flows through it

The inductance of the inductor

$$L = \frac{N\phi_B}{i}$$

$N\phi_B$: magnetic flux linkage

2. Inductor and self-inductor

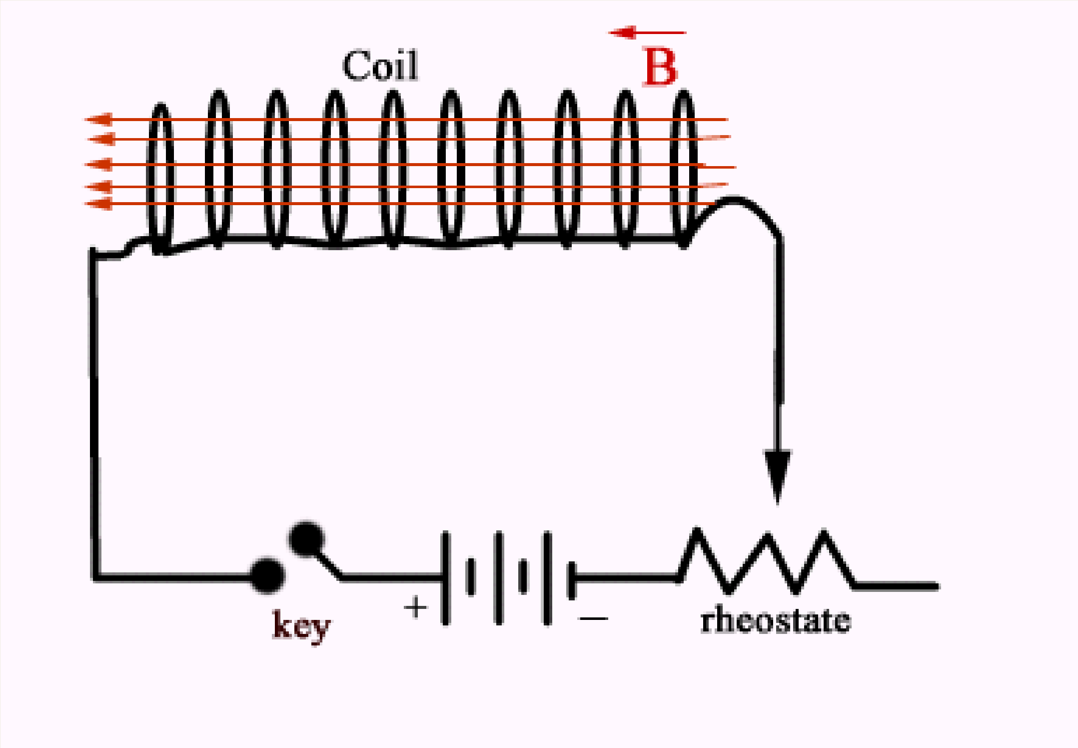
The inductor produces emf due to the change of the current

- $$\varepsilon = -\frac{dN\phi_B}{dt}$$

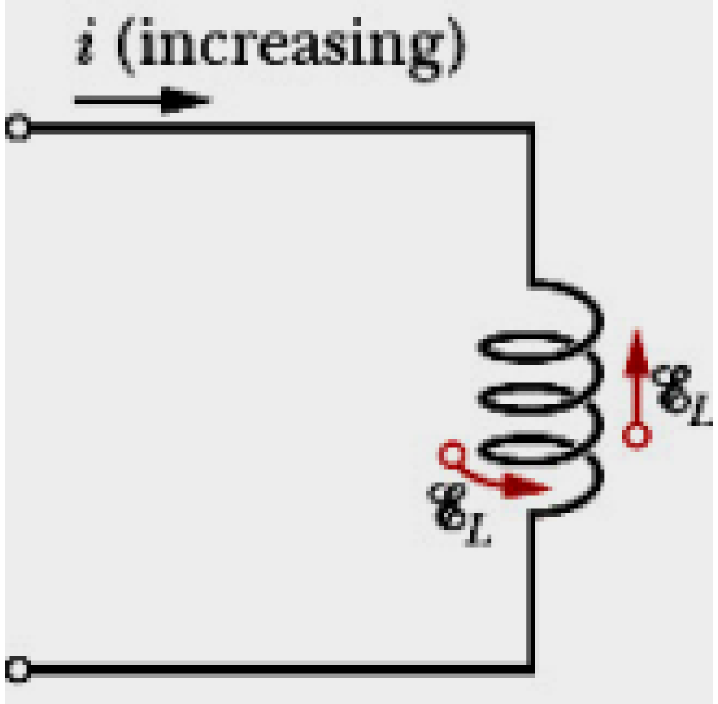
$$N\phi_B = Li$$

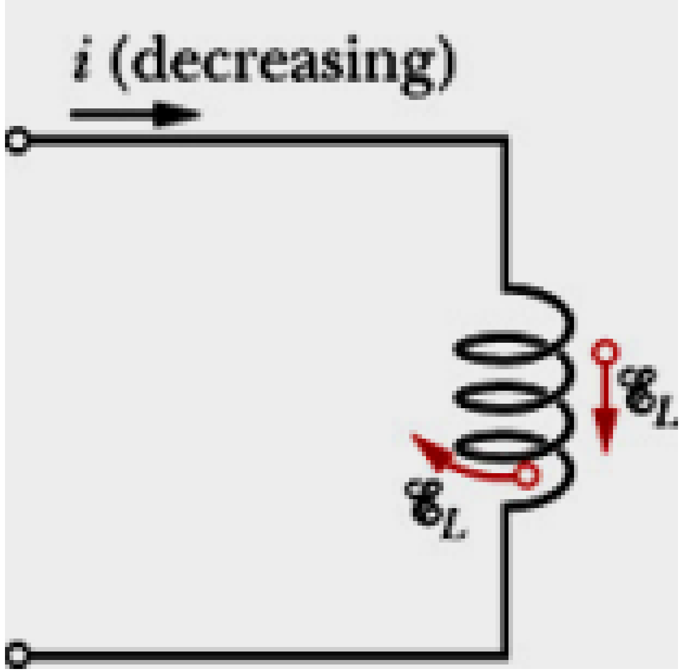
$$\rightarrow \varepsilon = -L\frac{di}{dt}$$

The direction of emf is opposite to the change of the current i



2. Inductor and self-inductor





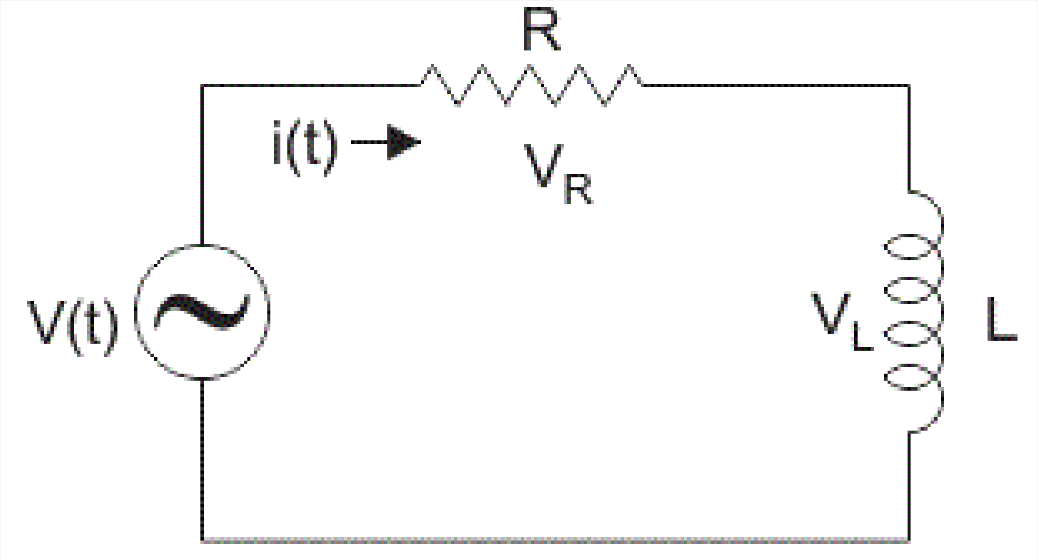
3. RL circuit

Consists of a resistor, an inductor and a battery

Consider two emf:

+ emf due to the battery

+ emf due to the inductor



3. RL circuit

According to Kirchoff's Law for the closed loop

$$-iR - L\frac{di}{dt} + \varepsilon = 0$$

$$\rightarrow L\frac{di}{dt} + iR = \varepsilon$$

The solution of the current

$$i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{tL}{R}}\right) = \frac{\varepsilon}{R}\left(1 - e^{-\frac{t}{\tau_L}}\right) = i_0\left(1 - e^{-\frac{t}{\tau_L}}\right)$$

$\tau_L = \frac{L}{R}$: inductive time constant. The time for the current increasing to 63% its maximum value.

3. RL circuit

If we remove the emf from the battery, the closed loop become

$$L\frac{di}{dt} + iR = 0$$

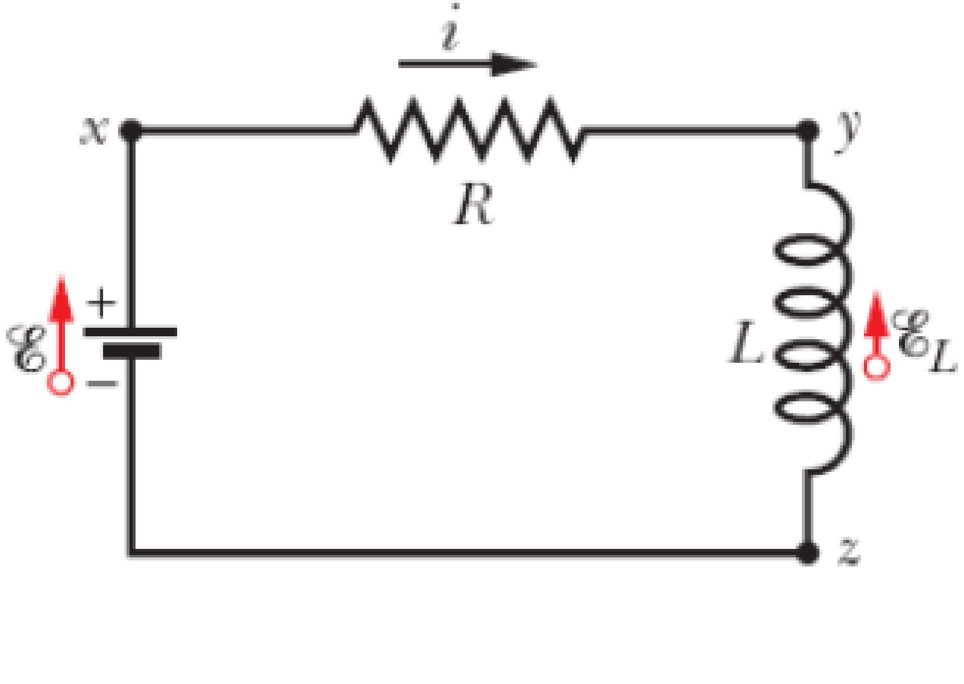
If the initial condition $i_0 = \frac{\varepsilon}{R}$. The solution of the current

$$i = \frac{\varepsilon}{R} e^{-\frac{t}{\tau_L}} = i_0 e^{-\frac{t}{\tau_L}}$$

3. RL circuit

The magnetic energy stored by an inductor carrying a current

$$U_L = \frac{1}{2} L i^2$$



I

The current in an RL circuit drops from 1.0 A to 10 mA in the first second after removing the battery from the circuit. Determine the inductance L is the resistance R is 40Ω

January 2017

The current in RL circuit (decay of current)

$$i = i_0 e^{\frac{-t}{\tau_L}} = 1\,x\,e^{\frac{-1}{\tau_L}} = 10\,x\,10^{-3} \rightarrow \ \tau_L = 0.2171\,s$$

We have

$$\tau_L = \frac{L}{R} = \frac{L}{40} = 0.2171 \ \rightarrow L = 8.685\,H$$

A battery is connected to a series RL circuit at time t = 0. If $R = 10\Omega$ and L = 200mH, at what time will the current be 47% less than its equilibrium values?

June 2017

The inductive time constant

$$\tau_L = \frac{L}{R} = \frac{200\ x\ 10^{-3}}{10} = 0.02\ (s)$$

The current in RL circuit (Rising of current)

$$i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{t}{\tau_L}}\right) = \frac{\varepsilon}{R}\left(1 - e^{-\frac{t}{0.02}}\right) = 47\% \frac{\varepsilon}{R}$$

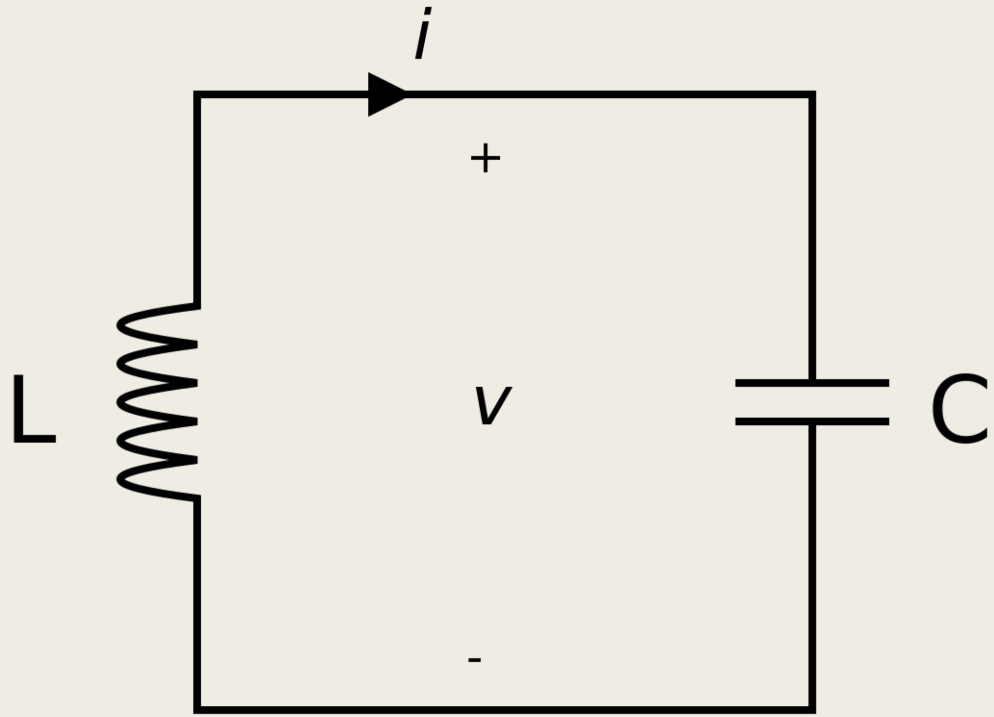
$$\rightarrow t = 0.0127\ (s)$$

CHAPTER 6: ALTERNATING CURRENT

1. LC circuit

In LC circuit, the charge current and potential difference vary sinusoidally

⇨Oscillation in the capacitor's electric field and the inductor's magnetic field



1. LC circuit

Consider a circuit with the inductor L and the capacitor C

According to the Kirchoff's law for a closed loop

$$L\frac{di}{dt} + \frac{1}{C}q = L\frac{dq^2}{dt} + \frac{1}{C}q = 0$$

The solution of the differential equation

⇨ Electromagnetic oscillation

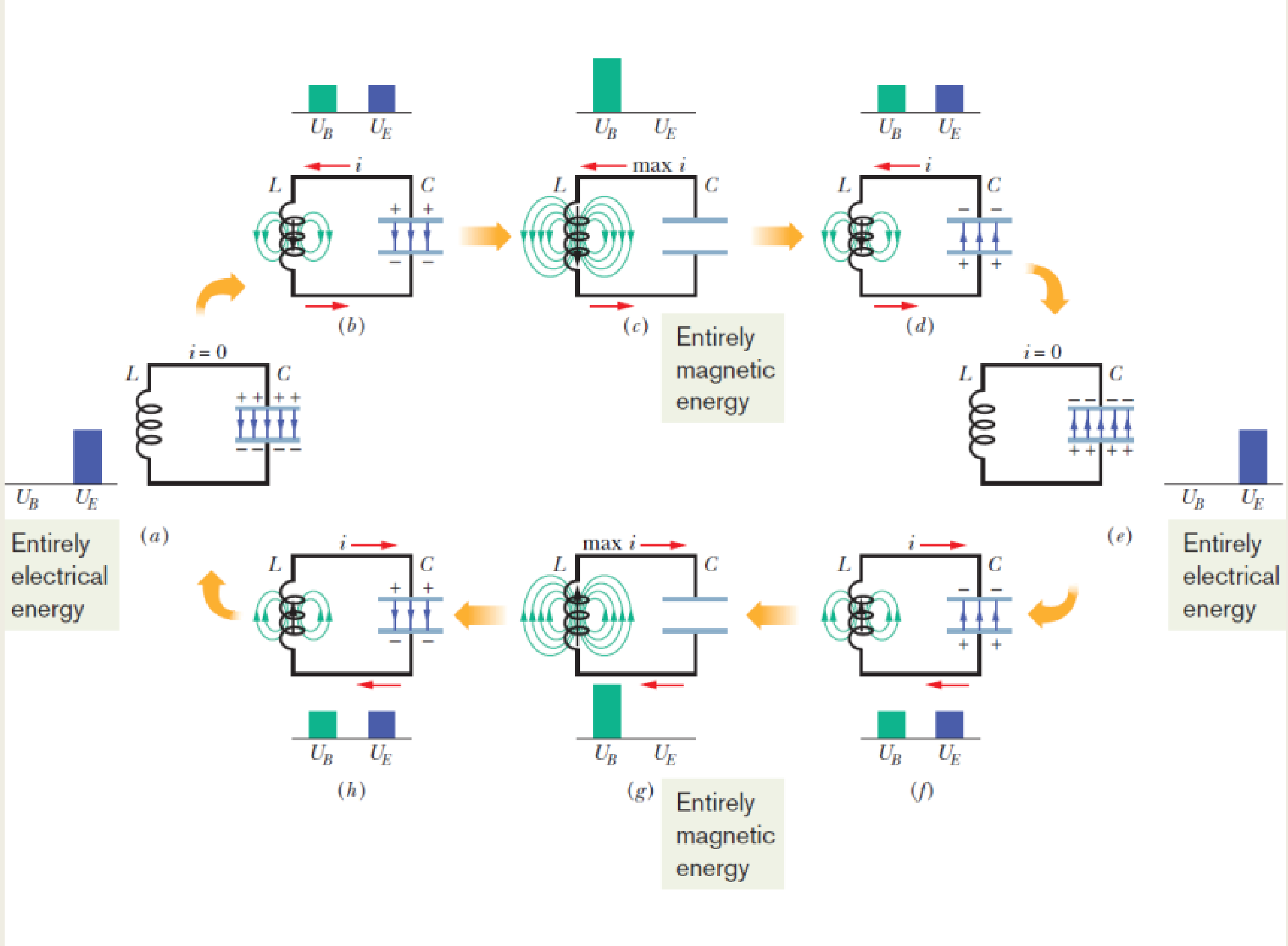
$$q = Q_{max}cos(\omega t + \phi)$$

For the current

$$i = \frac{dq}{dt} = -\omega Q sin(\omega t + \phi) = -I sin(\omega t + \phi)$$

The maximum current: $I_{max} = \omega Q_{max}$

1. LC circuit



Entirely magnetic energy
Entirely electrical energy
Entirely electrical energy
Entirely magnetic energy

1. LC circuit

For electromagnetic oscillation (LC circuit):

Angular frequency:

$$\omega = \frac{1}{\sqrt{LC}}$$

⇨ Natural angular frequency

The period

$$T = 2\pi\sqrt{LC}$$

The frequency

$$f = \frac{1}{T}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

1. LC circuit

The electric field energy in the capacitor

$$U_C = \frac{1}{2}\frac{q^2}{C}$$ stored

⇨The electric field energy at time t

$$U_C = \frac{1}{2}\frac{\left(Q_{max}cos(\omega t+\phi)\right)^2}{C} = \frac{1}{2}\frac{Q_{max}^2}{C}\cos^2(\omega t+\phi)$$

The magnetic field energy in the inductor

$$U_L = \frac{1}{2}Li^2$$ stored

⇨The magnetic field energy at time t

$$U_L = \frac{1}{2}L\left(\omega Q_{max}sin(\omega t+\phi)\right)^2 = \frac{1}{2}\frac{Q_{max}^2}{C}\sin^2(\omega t+\phi)$$

1. LC circuit

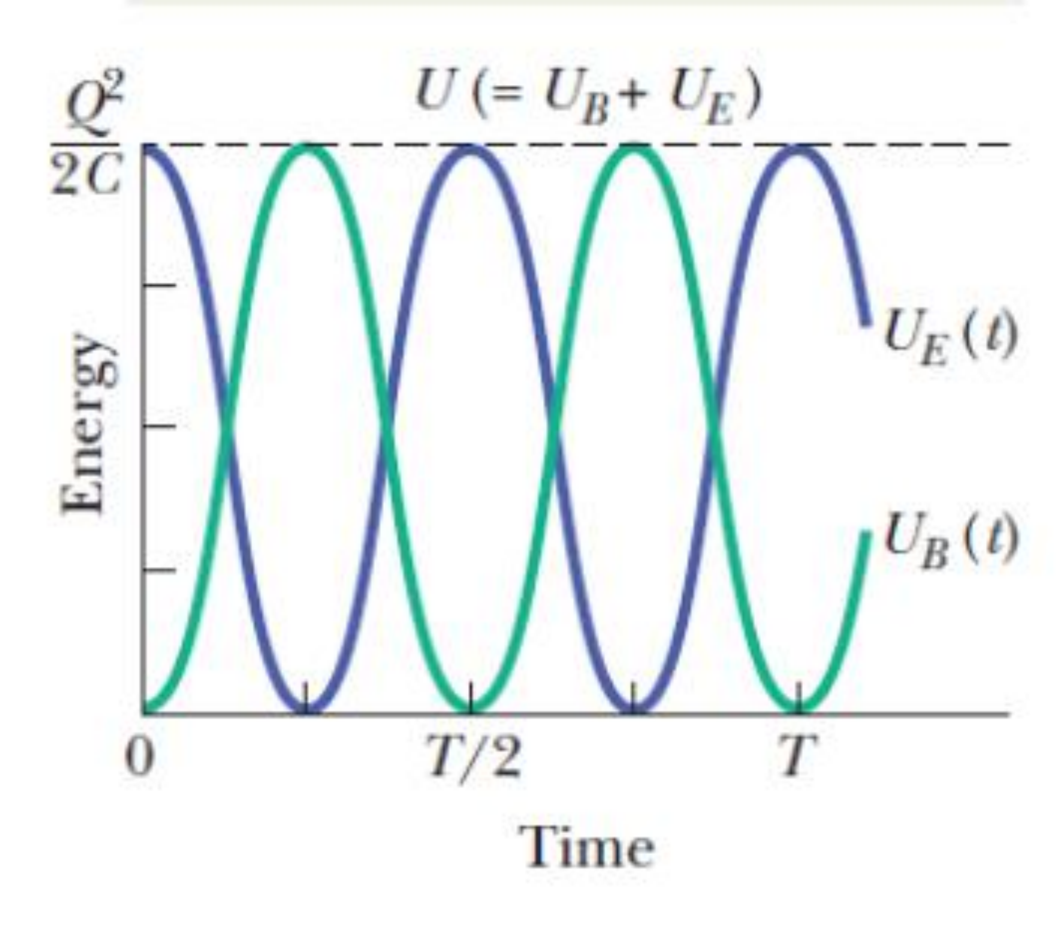
The electromagnetic energy

$$U = U_C + U_L$$

The maximum energy

$$U_{max} = \frac{1}{2}\frac{Q_{max}^2}{C}$$

If $U_C \downarrow$, $U_L \uparrow$ and vice versa



An LC circuit includes a capacitor of $25\mu F$. The circuit has a period of 5.0 ms. The peak current (the amplitude) is 25 mA. Determine (a) the inductance; (b) the peak voltage

C T L V_{max}

January 2016

a) We have: The period of a LC circuit

$$T = 2\pi\sqrt{LC} = 2\pi\sqrt{L\ x\ 25\ x\ 10^{-6}} = 5\ x\ 10^{-3}$$
$$\rightarrow L = 0.0253\ H$$

b) The peak electric charge

$$I_{max} = \omega Q_{max} = \frac{Q_{max}}{\sqrt{LC}} = 25\ x\ 10^{-3} \rightarrow Q_{max} = 1.989\ x\ 10^{-5}(C)$$

The peak voltage

$$V_{max} = \frac{Q_{max}}{C} = \frac{1.989\ x\ 10^{-5}}{25\ x\ 10^{-6}} = 0.796\ V$$

In an oscillating LC circuit with $C = 64.0\ mF$, the current is given by $i = \underset{I_0}{1.6}\sin(2500t + 0.68)$ where t is in seconds, I in amperes, and the phase constant in radians.

a) How soon after t = 0 will the current reach its maximum value

b) Find the inductance L and the total energy

January 2019

a) The current reach its maximum value

$i = 1.6\sin(2500t + 0.68) = 1.6 \rightarrow \sin(2500t + 0.68) = 1$

$\rightarrow 2500t + 0.68 = \frac{\pi}{2} + k2\pi\ (k \in \{0; 1; 2; 3; \dots\})$

$\rightarrow t = 3.563\ x\ 10^{-4} + k2.513\ x\ 10^{-3}$

$\rightarrow t = \{0.3563(ms); 2.8695\ (ms); 5.382\ (ms); \dots\}$

b) The angular frequency

$\omega = \frac{1}{\sqrt{LC}} = 2500 \rightarrow L = 2.5\ \mu H$

When the current reach its maximum value $\rightarrow\ U_C = 0$

$U_L = \frac{1}{2}LI_{max}^2 = \frac{1}{2}\ x\ 2.5\ x\ 10^{-6}\ x\ 1.6^2 = 3.2\ x\ 10^{-6}\ J$

The total energy: $U = U_C + U_L = 3.2\ x\ 10^{-6} J$

2. Alternating current
2.1 Definition

Alternating current is an electric current which periodical direction

The basic mechanism of an alternating-current generator is a conducting loop rotated in an external magnetic field

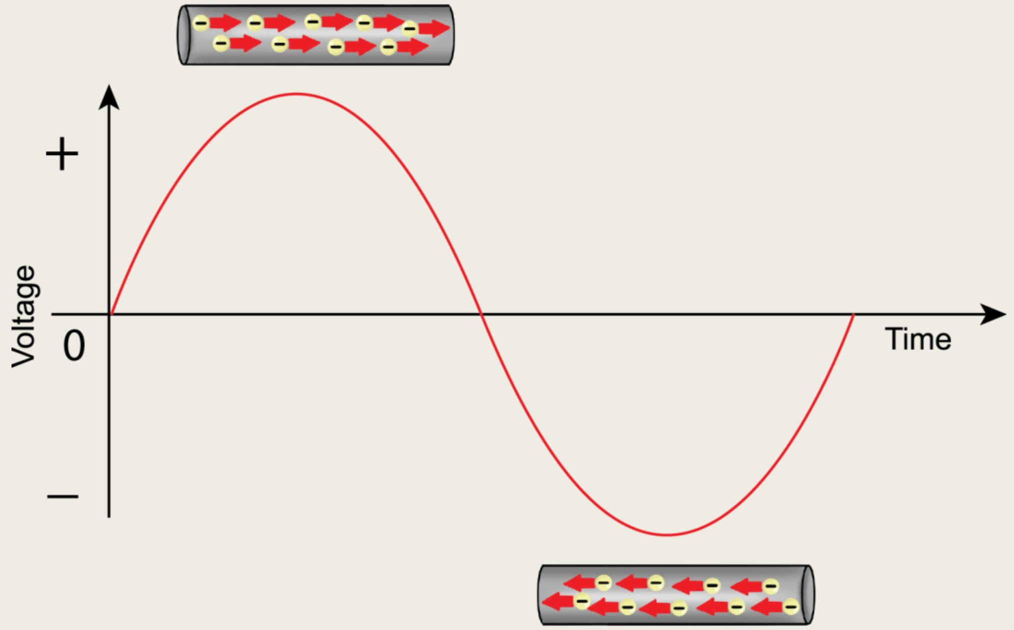
The emf

$$\varepsilon = \varepsilon_m \sin(\omega_d t)$$

The current

$$i = I sin(\omega_d t - \phi)$$

ω_d: driving angular frequency



2. Alternating current
2.2 Resistive load

Consider the circuit with alternating generator and resistor

$$\varepsilon - iR = 0$$

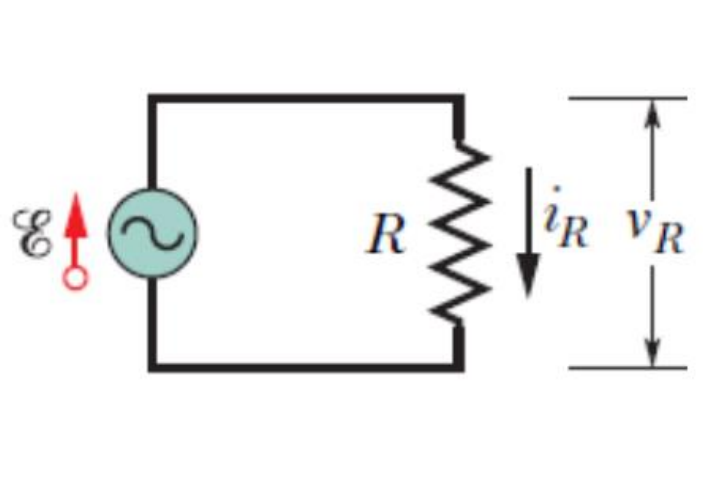
The voltage

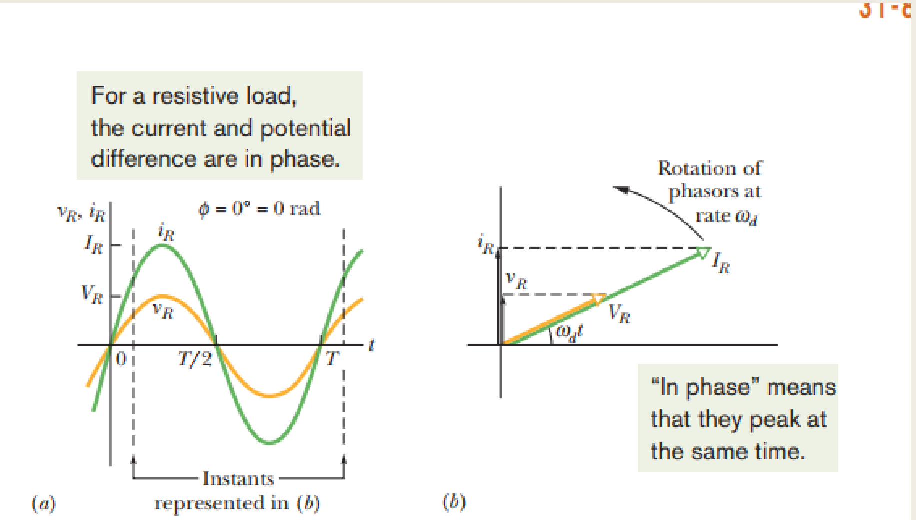
$$v_R = \varepsilon_m \sin(\omega_d t) = V_R \sin(\omega_d t)$$

The current

$$i_R = \frac{v_R}{R} = \frac{V_R}{R} \sin(\omega_d t)$$

⇨ v_R and i_R are in phase ($\phi = 0$)





2. Alternating current
2.3 Capacitive load

Consider the circuit with alternating generator and capacitor

$$\varepsilon - v_C = 0$$

The voltage

$$v_C = \varepsilon_m \sin(\omega_d t) = V_C \sin(\omega_d t)$$

The electric charge

$$q_C = C v_C = C V_C \sin(\omega_d t)$$

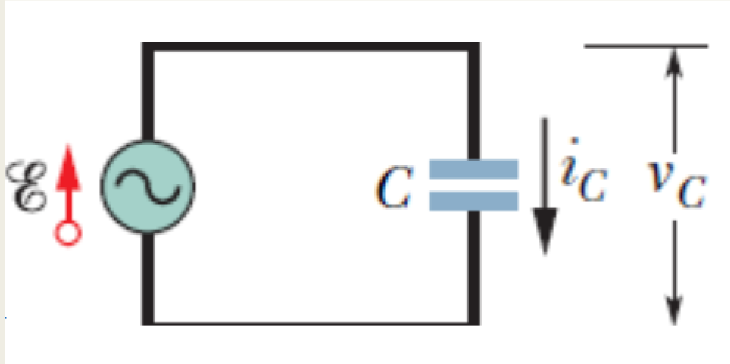
The current

$$i_C = \frac{dq_C}{dt} = \omega_d C V_C \cos(\omega_d t) = I_C \cos(\omega_d t)$$

⇨ i_C leads v_C by $90\,°\left(\phi = -\frac{\pi}{2}\right)$

The capacitive reactance: $X_C = \dfrac{1}{\omega_d C}$

$$\rightarrow V_C = I_C X_C$$



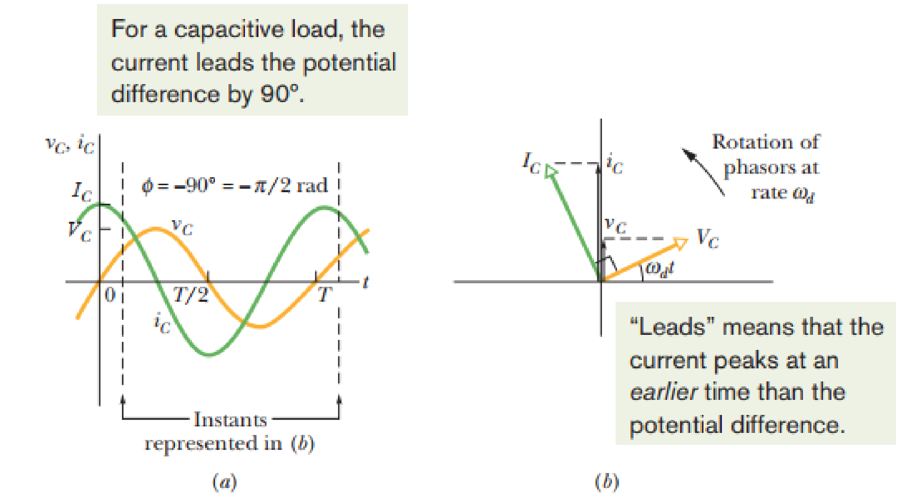


Fig. 31-11 (*a*) The current in the capacitor leads the voltage by 90° (= $\pi/2$ rad). (*b*) A phasor diagram shows the same thing.

2. Alternating current
2.4 Inductive load

Consider the circuit with alternating generator and inductor

$$\varepsilon - v_L = 0$$

The voltage

$$v_L = \varepsilon_m \sin(\omega_d t) = V_L \sin(\omega_d t)$$

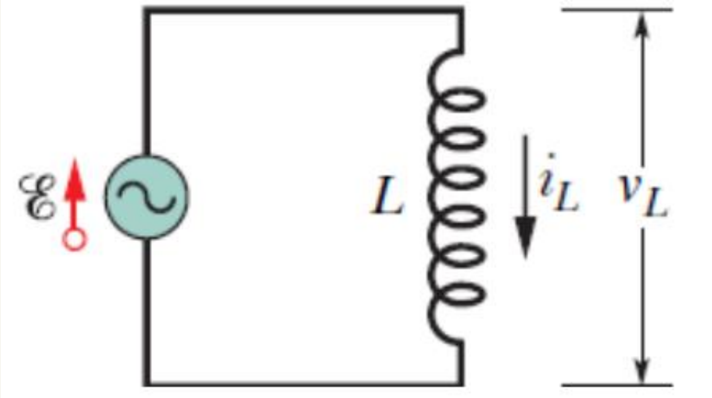
The current

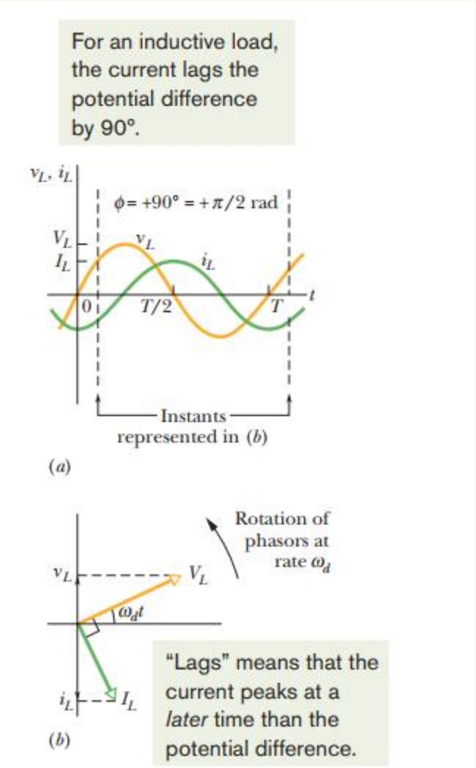
$$i_L = -\left(\frac{V_L}{\omega_d L}\right)\cos(\omega_d t) = I_L \cos(\omega_d t)$$

⇨ i_L lags v_L by $90\,°\left(\phi = +\frac{\pi}{2}\right)$

The inductive reactance: $X_L = \omega_d L$

$$\rightarrow V_L = I_L X_L$$





2. Alternating current
2.5 The series RLC circuit

Consider a circuit with a resistor, a capacitor and an inductor in series

$$\varepsilon = \varepsilon_m \sin(\omega_d t)$$
$$i = I sin(\omega_d t - \phi)$$

Since it is a series circuit

$$\varepsilon_m{}^2 = V_R^2 + (V_L - V_C)^2$$
$$\rightarrow \varepsilon_m = \sqrt{V_R^2 + (V_L - V_C)^2}$$

Để yêu cầu tính nào thì sài, ko yêu cầu cái nào có thể bỏ

The impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

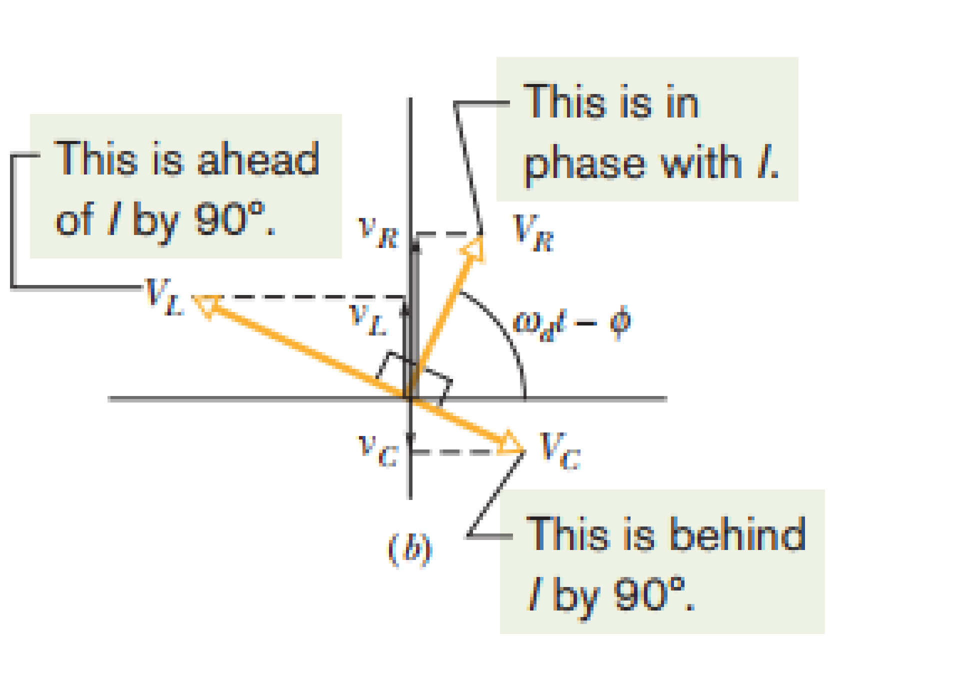
The current

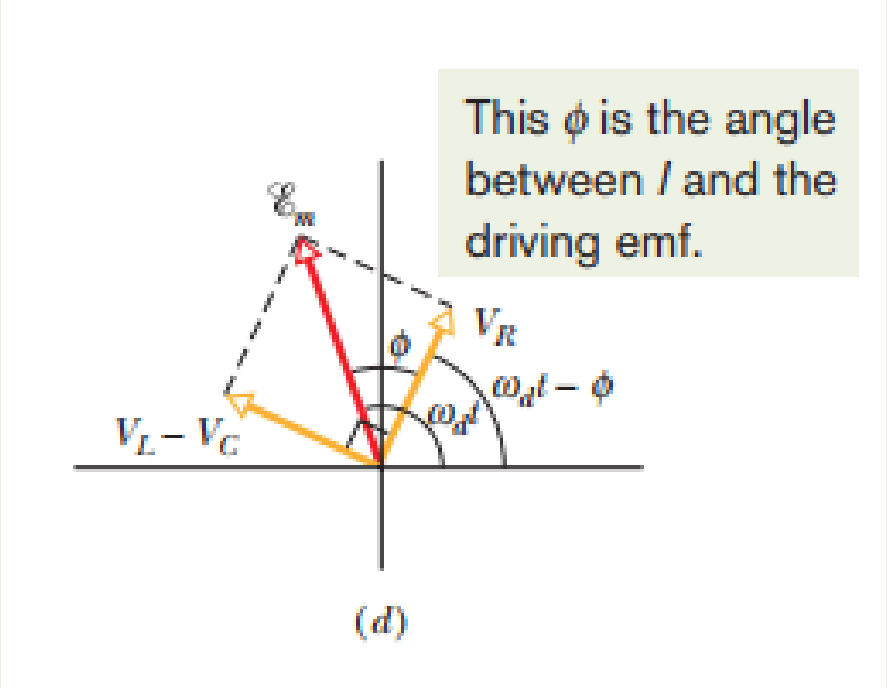
$$I = \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

The phase constant

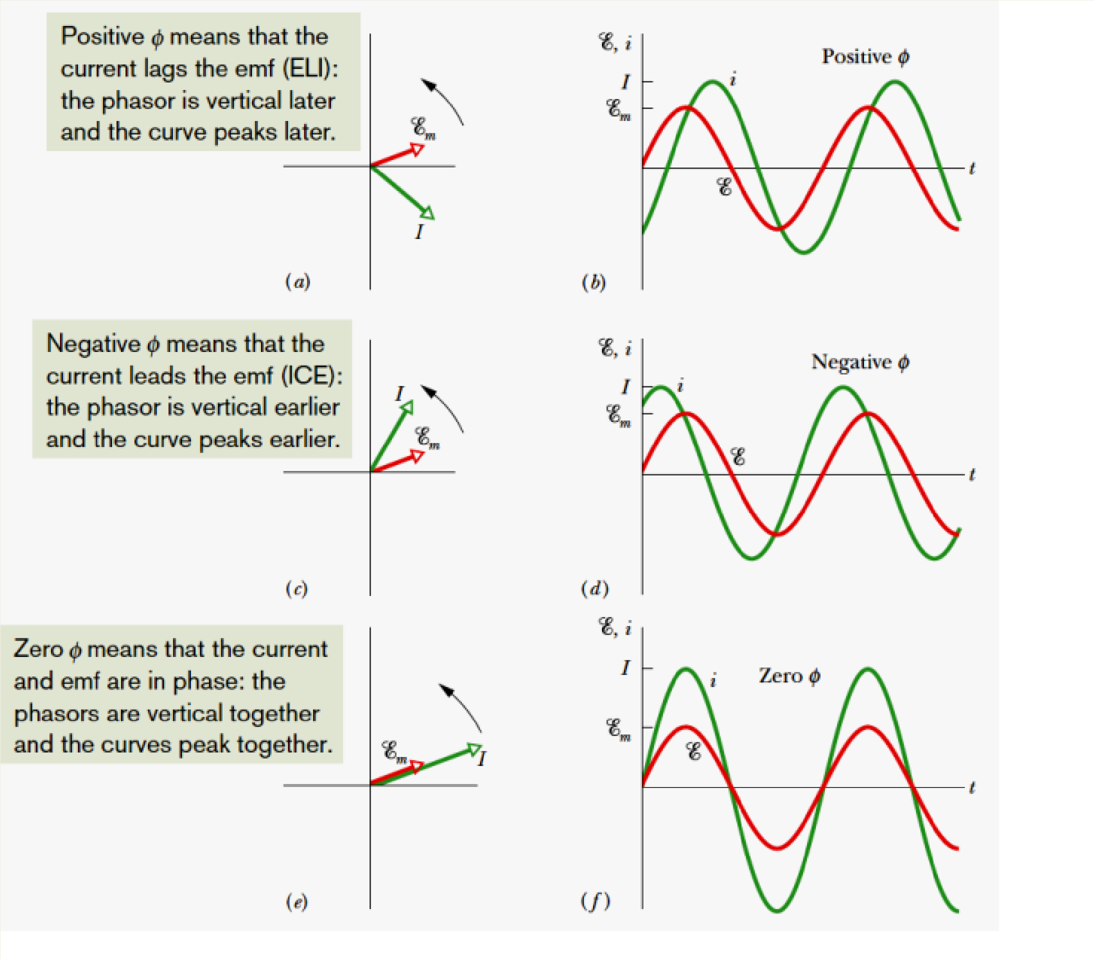
$$tan\phi = \frac{X_L - X_C}{R}$$

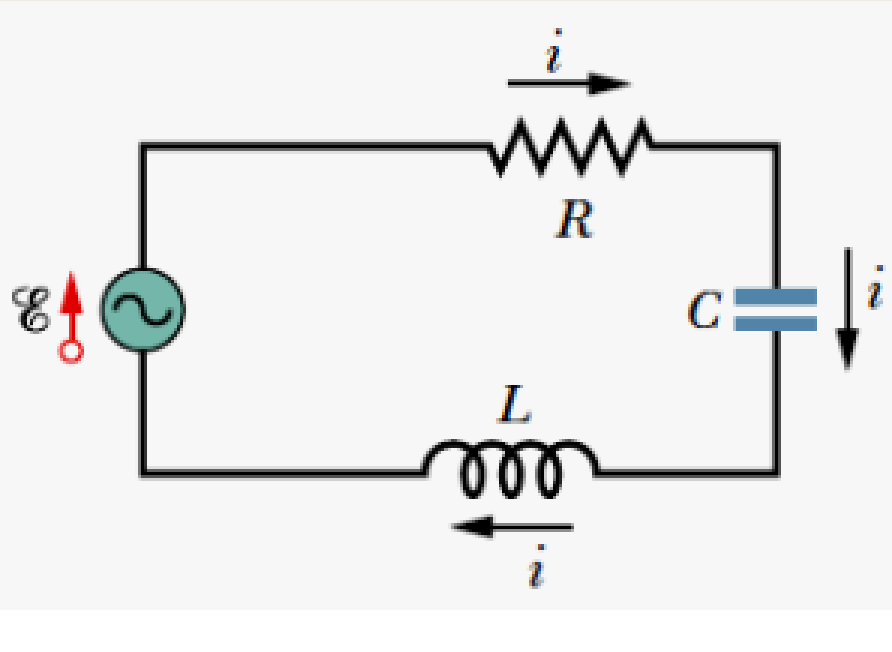
2. Alternating current
2.5 The series RLC circuit





2. Alternating current
2.5 The series RLC circuit





2. Alternating current
2.5 The series RLC circuit

Resonant condition: The driving angular frequency is equal to the natural angular frequency

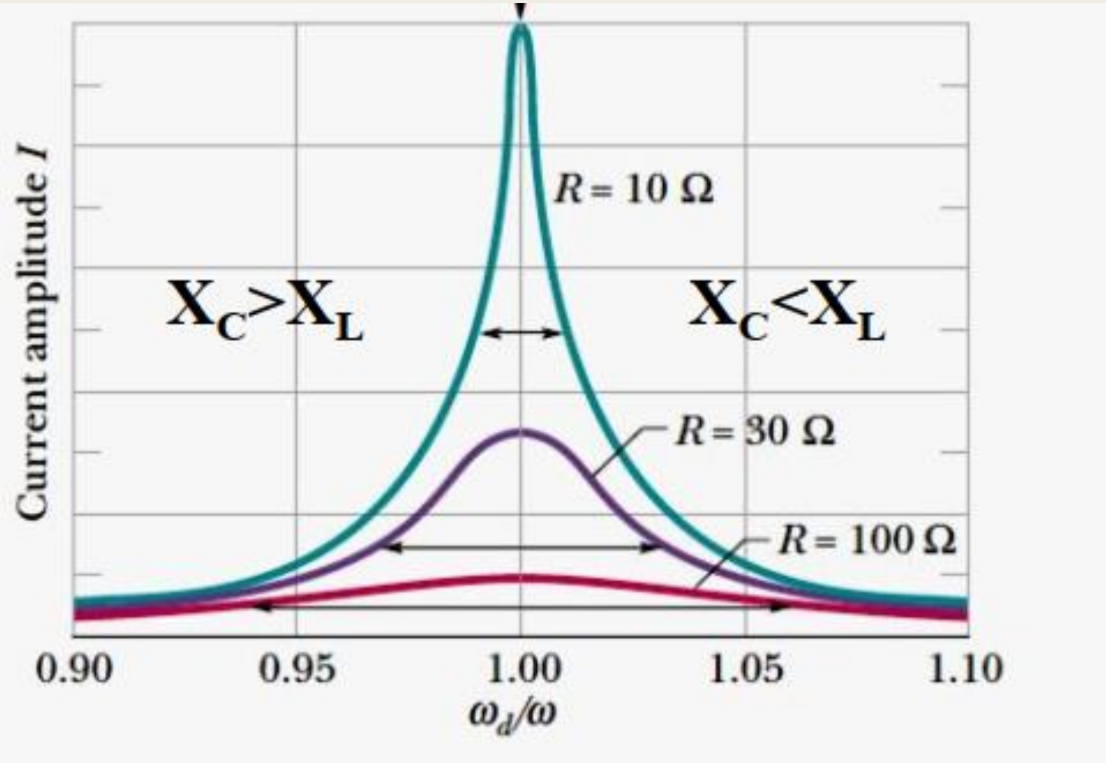
$$\omega_d = \omega = \frac{1}{\sqrt{LC}}$$

$$\rightarrow X_L = X_C$$

For driving angular frequency

$\omega_d < \omega \rightarrow X_L < X_C \rightarrow$ Capacitive

$\omega_d > \omega \rightarrow X_L > X_C \rightarrow$ Inductive



2. Alternating current
2.6 Power

The instantaneous rate at which energy is dissipated in the resistor

$$P = i^2R = I^2Rsin^2(\omega_d t - \phi)$$

The average power

$$P_{avg} = \frac{I^2R}{2} = \left(\frac{I}{\sqrt{2}}\right)^2 R$$

The rms voltage, emf and current

$$V_{rms} = \frac{V}{\sqrt{2}}; \varepsilon_{rms} = \frac{\varepsilon_m}{\sqrt{2}}; I_{rms} = \frac{\varepsilon_{rms}}{Z} = \frac{I}{\sqrt{2}}$$

$$\rightarrow P_{avg} = \varepsilon_{rms}I_{rms}\frac{R}{Z} = \varepsilon_{rms}I_{rms}cos\phi$$

$I^2.R$

3. Transformer

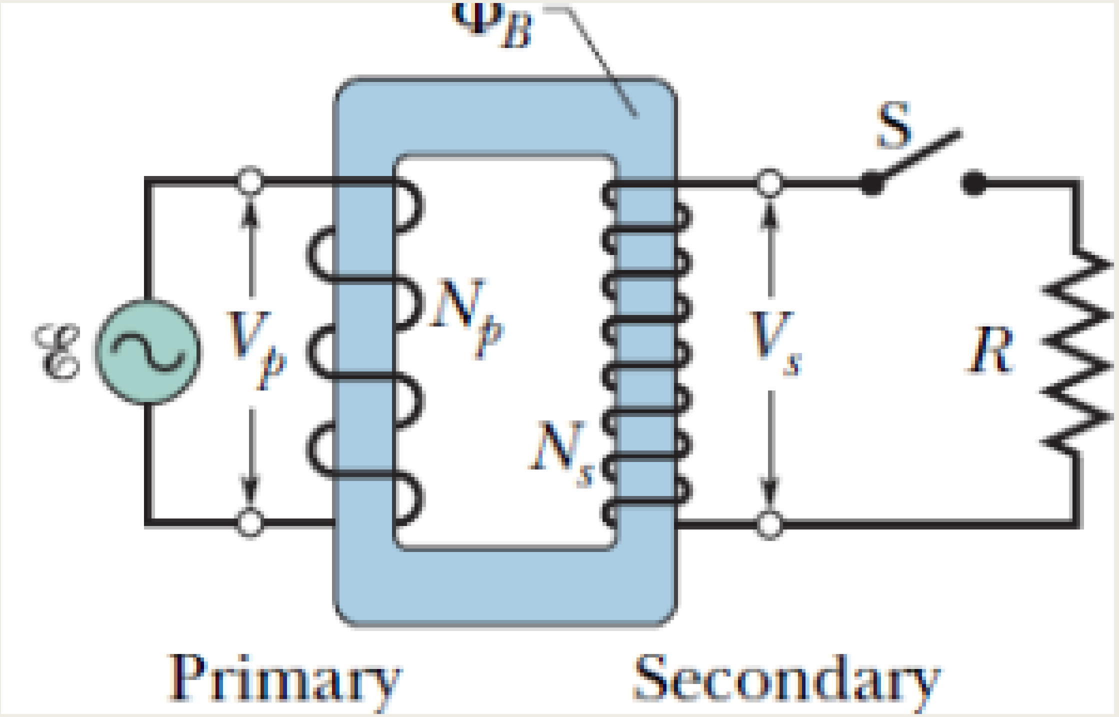
A device with which we can raise and lower the ac voltage in a circuit, keeping the product current voltage essentially constant

Transformation of the voltage

$$V_S = V_P \frac{N_S}{N_P}$$

If $N_S > N_P \rightarrow$ step-up transformer

If $N_S < N_P \rightarrow$ step-down transformer



L C R

A series RLC circuit with L = 300mH, $C = 15\mu F, R = 50\Omega$ is connected to an AC voltage source with amplitude 12.8V and frequency 50Hz. Find (a) the current amplitude (b) the phase difference between the voltage and the current (c) sketch the phasor diagram of the circuit.

January 2016

V f I ϕ

$X_L = \omega L \ (\Omega)$; $X_C = \frac{1}{\omega C} \ (\Omega)$

a) The angular frequency: $\omega_d = 2\pi f_d = 100\pi \ (rad/s)$

The impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2} = \sqrt{50^2 + \left(100\pi \ x \ 300 \ x 10^{-3} - \frac{1}{100\pi \ x \ 15 \ x \ 10^{-6}}\right)^2}$$

$= 128.12 \ \Omega$

The current amplitude

$$I = \frac{\varepsilon_m}{Z} = \frac{12.8}{128.12} \approx 0.1 \ A$$

b) The phase difference between the voltage and the current

$$tan\phi = \frac{X_L - X_C}{R} = -2.359 \ \rightarrow \phi = -1.169 \ rad$$

$f \qquad\qquad f$

The resonant frequency of a series RLC circuit is 5.0 kHz. When it is driven at a frequency of 7.0 kHz, it has an impedance of 850Ω and a phase constant 45°. Find R,L and C for this circuit.

June 2018

$Z \qquad\qquad \phi$

The angular driving frequency: $\omega_d = 2\pi f_d = 14000\pi \ (rad/s)$

The resonant frequency: $\omega = \frac{1}{\sqrt{LC}} = 2\pi f = 10000\pi \ \rightarrow LC = \frac{1}{(10000\pi)^2}$

The phase constant

$$tan\phi = \frac{X_L - X_C}{R} = 1 \ \rightarrow X_L - X_C = R$$

The impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + R^2} = R\sqrt{2} = 850 \ \rightarrow R = 601.04\Omega$$

We have: $X_L - X_C = R \rightarrow \omega_d L - \frac{1}{\omega_d C} = R \ \rightarrow \omega_d^2 LC - 1 = R\omega_d C$ (Quy đồng)

$\rightarrow (14000\pi)^2 \ x \frac{1}{(10000\pi)^2} - 1 = 601.04 \ x \ 14000\pi \ x \ C \ \rightarrow C = 0.363 \ nF$

The inductance: $L = 0.028 \ H$

$R = 601.04 \ \Omega \ ; C = 0.363 nF ; L = 0.028H$

In Figure 3, $R = 20.0\Omega$, $C = 10\ \mu F$ and L = 50.0 mH. The generator provides an emf with rms voltage 100.0V and frequency 500 Hz

V

a) Find the rms current $\rightarrow I = \frac{\varepsilon}{Z}$

b) What is the rms voltage across R and C together

c) At what average rate is energy dissipated in R, in C and in L

June 2017

a) The angular driving frequency: $\omega_d = 2\pi f_d = 1000\pi\ (rad/s)$

Inductive reactance: $X_L = \omega_d L = 1000\pi \times 50 \times 10^{-3} = 157.08\ \Omega$

Capacitive reactance: $X_C = \frac{1}{\omega_d C} = \frac{1}{1000\pi \times 10 \times 10^{-6}} = 31.83\ \Omega$

The impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{20^2 + (157.08 - 31.83)^2} = 126.84\ \Omega$$

The rms current

$$I_{rms} = \frac{\varepsilon_{rms}}{Z} = \frac{100}{126.84} = 0.788\ A$$

b) The rms voltage in R: $V_{R,rms} = I_{rms} R = 15.76\ V$

$\rightarrow$ The rms voltage in C: $V_{C,rms} = I_{rms} X_C = 25.08\ V$

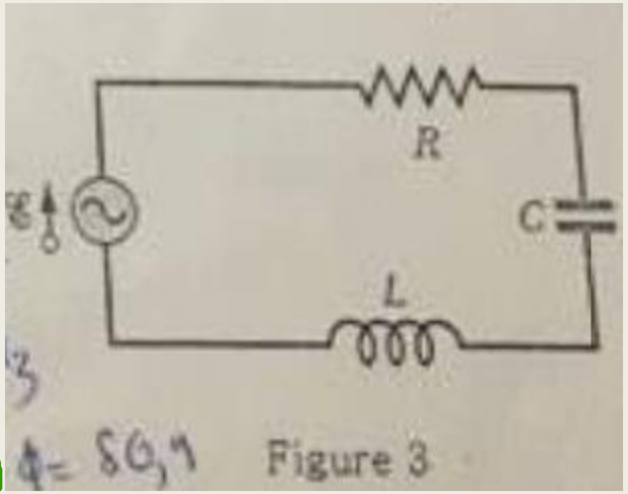
The rms voltage across R and C together

$$\rightarrow V_{RC,rms} = \sqrt{V_{R,rms}^2 + V_{c,rms}^2} = 29.62\ V$$

Since it is a series circuit

$$\varepsilon_m{}^2 = V_R^2 + (V_L - V_C)^2$$

$$\rightarrow \varepsilon_m = \sqrt{V_R^2 + (V_L - V_C)^2}$$



c) The average rate is energy dissipated in RLC

$P_{C,avg} = I_{RMS}^2 X_C\ (W)$, $P_{R,avg} = I_{rms}^2 R = 12.42W$, $P_{L,avg} = I_{RMS}^2 X_L\ (W)$

REVIEW

A potential difference of 1000 V is applied to accelerate an electron from rest. The accelerated electron then enters a uniform magnetic field and completes one revolution in 10 ns. Determine the radius of the orbit of the electron.

T

January 2017

The work done acting on the electron by electric field

$$W = q\Delta V = \Delta E_{kin} = \frac{1}{2}mv^2 - \frac{1}{2}m0^2 = \frac{1}{2}mv^2$$

$$\rightarrow v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2 \ x \ 1.6 \ x \ 10^{-19} \ x \ 1000}{9.1 \ x \ 10^{-31}}} = 18755373.03 \text{ m/s}$$

$$q\Delta V = \frac{1}{2}mv^2$$

We have:

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{18755373.03} = 10 \ x \ 10^{-9} \rightarrow r = 29.85 \ mm$$

Chu kì (s)

Two wires carrying current in the direction as shown in Figure 2. Wire I with $i_1 = 2.0\ A$ consist of a circular arc of radius R and two radial lengths. Wire 2 with $i_2 = 0.5\ A$ is long and straight at a distance R/2 from the center of the arc. For what value of arc angle ϕ (in degree) the net magnetic field B at point B due to the two current is zero?

January 2016

$B_1 = B_2$

The magnetic field at point P for a wire 1

$$B_1 = \frac{\mu_0 i_1 \phi}{4\pi R}$$

The magnetic field at point P for a wire 2

$$B_2 = \frac{\mu_0 i_2}{2\pi \frac{R}{2}} = \frac{\mu_0 i_2}{\pi R}$$

The net magnetic field B at point B due to the two current is zero

$$B_1 = B_2 \rightarrow \frac{\mu_0 i_1 \phi}{4\pi R} = \frac{\mu_0 i_2}{\pi R} \rightarrow \frac{i_1 \phi}{4} = i_2 \rightarrow \phi = 1\,rad = 57.2\,°$$

distance

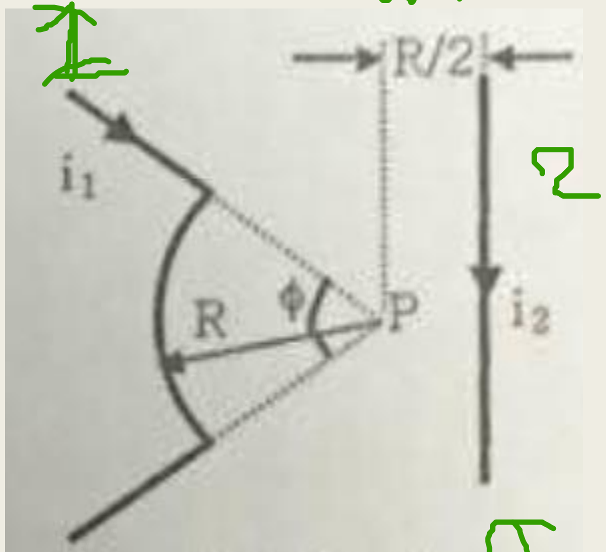


Figure 2

A metal rod is forced to move with constant velocity v = 65 cm/s along two parallel metal rails (Fig.2). A magnetic field with magnitude B = 0.35 T points out of the page. The rails are separated by L = 20cm

a) What emf is generated? ε

b) The rod has a resistance of 18.5 Ω (resistance of the rails and connector are negligible). What is the current in the rod?

January 2019

$\Delta\phi_B = B\Delta A = Bv\Delta tL$

a) We have

$\Delta\phi_B = B\Delta A = Bv\Delta tL$

The induced emf

$$\varepsilon = -\frac{\Delta\phi_B}{\Delta t} = -\frac{Bv\Delta tL}{\Delta t} = -BvL$$

$$= -0.35 \, x \, 65 \, x \, 10^{-2} \, x \, 20 \, x \, 10^{-2} = -0.042 \, V$$

b) The current in the rod

$$I = \frac{\varepsilon}{R} = \frac{0.042}{18.5} = 2.27 \, mA$$

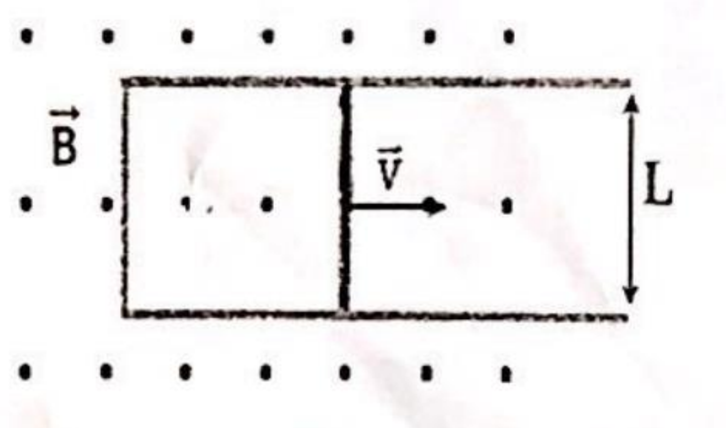


Fig. 2

L R LR circuit

An inductor with inductance $6.2\mu H$ is connected in series with a $1.25k\Omega$ resistor.

a) If a 12.0 V battery is inserted in the circuit, how long will it take for the current through the resistor to reach 75% of its final value?

b) Find the current through the resistor at time $t = 1.0\tau_L$

January 2018

$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right)$

a) The inductive time constant: $\tau_L = \frac{L}{R} = 4.96\,ns$

The current in RL circuit

$$i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{t}{\tau_L}}\right) = \frac{\varepsilon}{R}\left(1 - e^{\frac{-t}{4.96\,x\,10^{-9}}}\right) = 75\%\frac{\varepsilon}{R} \rightarrow t = 6.876\,ns$$

b) The current through the resistor at time $t = 1.0\tau_L$

$$i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{t}{\tau_L}}\right) \Rightarrow i = \frac{\varepsilon}{R}(1 - e^{-1}) = 0.63\frac{\varepsilon}{R} = 6.06\,mA$$

In an oscillating LC circuit, L = 3mH and $C = 2.7\mu F$. At t = 0, the charge on the capacitor is zero and the current is 2.0 A. (a) Find the maximum charge that will appear on the capacitor (b) At what earliest time t > 0 is the rate at with energy is stored in the capacitor $\frac{dU_C}{dt}$ greatest and (c) what is that greatest rate?

January 2017

C

I

The angular frequency: $\omega = \frac{1}{\sqrt{LC}} = 11111.11$ rad/s

a) At t = 0, the charge on the capacitor is zero ⇨ The current is maximum

$$I_{max} = \omega Q_{max} = 11111.11 Q_{max} = 2 \rightarrow Q_{max} = 1.8 \ x\ 10^{-4}\ (C)$$

The electric charge at t = 0 : Q = $Q_{max}cos(\omega t + \phi) = Q_{max}cos(\phi) = 0 \rightarrow \phi = -\frac{\pi}{2}$ $(Since\ I > 0)$

$\rightarrow$ Q = $Q_{max}cos\left(\omega t - \frac{\pi}{2}\right)$

b) The electric field energy at the capacitor

$$U_C = \frac{1}{2}\frac{\left(Q_{max}cos\left(\omega t - \frac{\pi}{2}\right)\right)^2}{C} = \frac{1}{2}\frac{Q_{max}^2}{C}\cos^2\left(\omega t - \frac{\pi}{2}\right)$$

In an oscillating LC circuit, L = 3mH and $C = 2.7\mu F$. At t = 0, the charge on the capacitor is zero and the current is 2.0 A. (a) Find the maximum charge that will appear on the capacitor (b) At what earliest time t > 0 is the rate at with energy is stored in the capacitor $\frac{dU_C}{dt}$ greatest and (c) what is that greatest rate?

January 2017

We have

$$\frac{dU_C}{dt} = \frac{-\omega}{2}\frac{Q_{max}^2}{C} 2\cos\left(\omega t - \frac{\pi}{2}\right)\sin\left(\omega t - \frac{\pi}{2}\right) = \frac{-\omega Q_{max}^2}{2C}\sin\left(2\left(\omega t - \frac{\pi}{2}\right)\right)$$

$\frac{dU_C}{dt}$ greatest ⇨ $\frac{dU_C}{dt} = \frac{-\omega Q_{max}^2}{2C}\sin\left(2\left(\omega t - \frac{\pi}{2}\right)\right) = \frac{-\omega Q_{max}^2}{2C} \rightarrow \sin\left(2\left(\omega t - \frac{\pi}{2}\right)\right) = 1$

$\rightarrow 2\left(\omega t - \frac{\pi}{2}\right) = \frac{\pi}{2} + k2\pi \rightarrow 2\omega t - \pi = \frac{\pi}{2} + k2\pi \rightarrow \omega t = \frac{3\pi}{2} + k2\pi \rightarrow t = 4.24\ x\ 10^{-4} + k5.65\ x10^{-4}$

$$k\ \epsilon\ \{0; 1; 2; \dots\}$$

The earliest time: $k = 0 \rightarrow t = 42.4\ (ns)$

c) The greatest rate at which energy is stored in the capacitor

$$\frac{dU_C}{dt} = \frac{-\omega Q_{max}^2}{2C} = \frac{-11111.11\ x\ (1.8\ x\ 10^{-4})^2}{2\ x\ 2.7\ x\ 10^{-6}} = -66.67\ (J/s)$$

The AC generator in Fig.3 supplies 120V at 60 Hz. When the switch S opens, the current leads the generator emf by 20° When S is in position 1, the current lags the generator emf by 10° . When S is in position 2, the current amplitude is 2 A. Find R,L and C

January 2019

The driving angular frequency: $\omega_d = 2\pi f_d = 120\pi\ (rad/s)$
Inductive reactance: $X_L = \omega_d L = 120\pi L$
Capacitive reactance: $X_C = \frac{1}{\omega_d C} = \frac{1}{120\pi C}$
When the switch S is open ⇨ The circuit consists of L, C and R in series
Since the current leads the generator emf by 20°

$$\rightarrow tan\phi_0 = \frac{X_L - X_{C_0}}{R} = \tan(-20°) = -0.364(1)$$

When S is in position 1⇨ The circuit consists of L, 2 capacitor in parallel and R in series
The capacitance: $C_1 = 2C$
Since the current lags the generator emf by 20°

$$\rightarrow tan\phi_1 = \frac{X_L - X_{C_1}}{R} = \tan(10°) = 0.176\ (2)$$

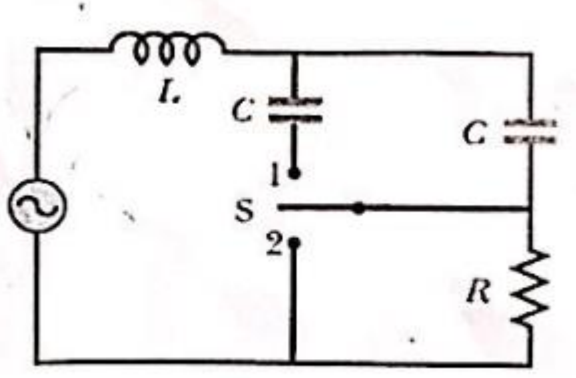


Fig. 3

The AC generator in Fig.3 supplies 120V at 60 Hz. When the switch S opens, the current leads the generator emf by 20° When S is in position 1, the current lags the generator emf by 10° . When S is in position 2, the current amplitude is 2 A. Find R,L and C

January 2019

When S is in position 2 ⇨ The circuit consists of L and C

$$\rightarrow Z_2 = \frac{\varepsilon}{I_2} = \frac{120}{2} = 60\ \Omega$$

$$\rightarrow Z_2 = \sqrt{(X_L - X_C)^2} = 3600 \rightarrow\ |X_L - X_C| = 60$$

For (1) we have

$$tan\phi_0 = \frac{X_L - X_{C_0}}{R} = \tan(-20°)$$

Since $\phi_0 < 0 \rightarrow X_L - X_{C_0} = -60\ \rightarrow \frac{-60}{R} = \tan(-20°) \rightarrow R = 164.85\ \Omega$

For (2) we have

$$tan\phi_1 = \frac{X_L - X_{C_1}}{R} = \tan(10°)$$

$$\rightarrow X_L - X_{C_1} = 29.067$$

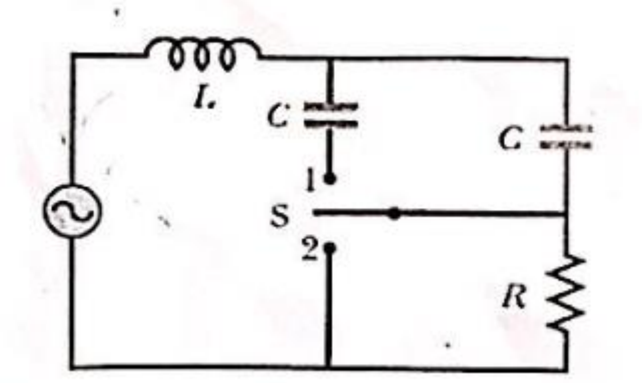


Fig. 3

V f

The AC generator in Fig.3 supplies 120V at 60 Hz. When the switch S opens, the current leads the generator emf by 20° When S is in position 1, the current lags the generator emf by 10° . When S is in position 2, the current amplitude is 2 A. Find R,L and C

January 2019

The equation for L and C

$$\begin{cases} X_L - X_{C_0} = -60 \\ X_L - X_{C_1} = 29.067 \end{cases} \rightarrow \begin{cases} \omega_d L - \dfrac{1}{\omega_d C} = -60 \\ \omega_d L - \dfrac{1}{\omega_d 2C} = 29.067 \end{cases}$$

$$\rightarrow \begin{cases} 120\pi L - \dfrac{1}{120\pi C} = -60 \\ 120\pi L - \dfrac{1}{240\pi C} = 29.067 \end{cases} \rightarrow \begin{cases} L = 0.313\ H \\ C = 14.89\ \mu F \end{cases}$$

The equation for L and C

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THANK YOU