AC , DC

Sinusoidal Steady-State Analysis

We consider circuits energized by time-varying voltage or current sources.

Textbook:

Electric Circuits

James W. Nilsson & Susan A. Riedel

9th Edition.

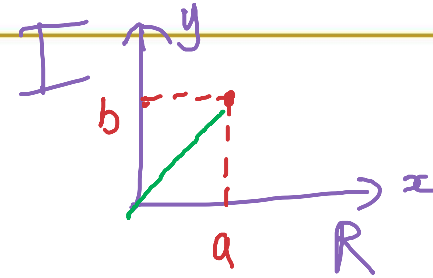
Outline

- Complex Numbers Tutorial
- Sinusoids
- Phasors
- Techniques of Circuit Analysis
- Phasor Diagrams

Sinusoid $\begin{cases} \text{sine} \\ \text{cosine} \end{cases}$

$V_0 \cos(\omega t + \theta)$

Complex Numbers Tutorial



Rectangle Form

Notation :

Rectangular form:

$$n = a + jb$$

a : real component

b : imaginary component

$$j = \sqrt{-1}$$

Polar form:

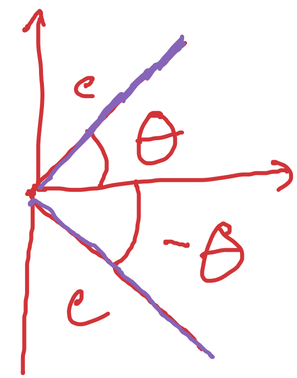
same

$$n = ce^{j\theta} \quad \text{or} \quad n = c\angle\theta^{\circ}$$

c : magnitude

θ : angle

e : natural logarithm



The conjugate of a complex number :

$$n^{*} = a - jb$$

R

$$n^{*} = c\angle-\theta^{\circ}$$

P

Complex Numbers Tutorial

Transition between rectangular and polar forms :

<u>From polar form to rectangular form :</u>

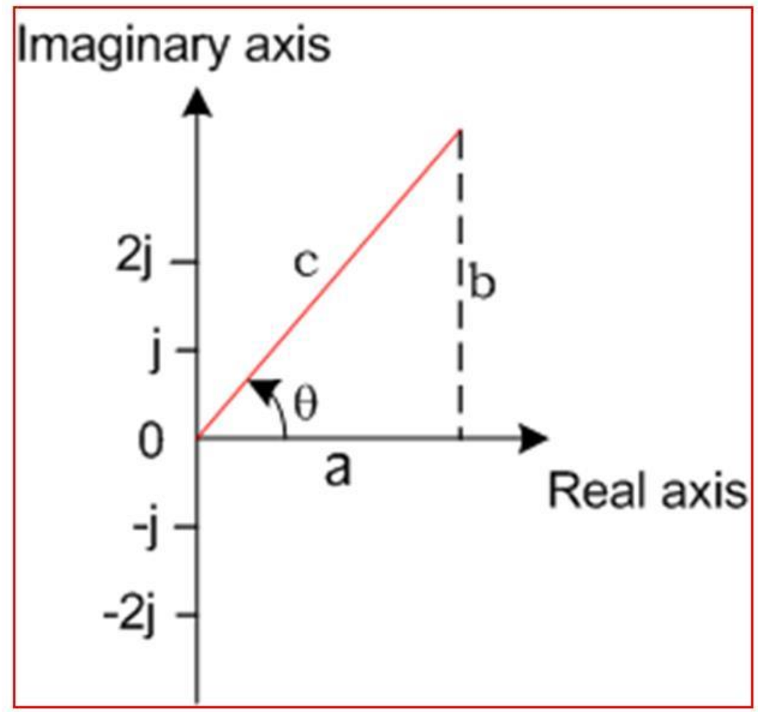
$$ce^{j\theta} = c(\cos\theta + j\sin\theta)$$
$$= c\cos\theta + jc\sin\theta$$
$$= a + jb$$

<u>From rectangular form to polar form :</u>

R $\quad a + jb = ce^{j\theta} \quad$ P

where : $\begin{cases} c = \sqrt{a^2 + b^2} \\ \tan\theta = \dfrac{b}{a} \end{cases}$

Complex plane



$$\boldsymbol{n} = \boldsymbol{a} + \boldsymbol{jb} = \boldsymbol{c}\angle\theta = \boldsymbol{c}(\cos\theta + \boldsymbol{j}\sin\theta)$$

$1 \rightarrow 1\angle 0$, e^{j0} | $-1 \rightarrow 1\angle \pm 180$, $e^{\pm j\pi}$ | $j \rightarrow 1\angle 90^\circ$, $e^{j\frac{\pi}{2}}$ | $-j \rightarrow 1\angle -90$, $e^{-j\frac{\pi}{2}}$

Complex Numbers Tutorial

Real

Imaginary

$Re(e^{j\theta})$ $Im(e^{j\theta})$

Useful Identities :

Euler's identity: $$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$j^2 = -1$

$\bar{V} = 2j\,(V) \Rightarrow \theta = 90^\circ$

$$\pm j^2 = \mp 1$$

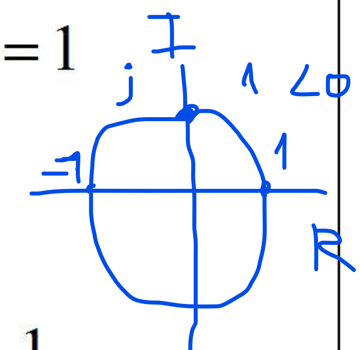
$$(-j)(j) = 1$$

$\bar{I} = -3j\,(A)$

$\Rightarrow \theta = -90^\circ$

$\Rightarrow 3\cos(\omega t - 90^\circ)$

$$j = \frac{1}{-j}$$



$$e^{\pm j\pi} = -1$$

$$e^{\pm j\pi/2} = \pm j$$

$$radian = \frac{180^\circ}{\pi}$$

$$n + n^* = 2a$$

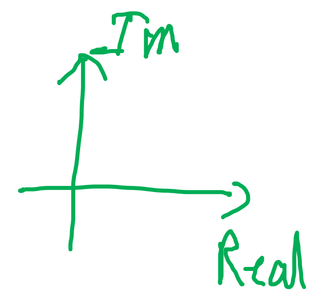
$$n - n^* = j2b$$

R

P

$$nn^* = a^2 + b^2 = c^2$$

$$\frac{n}{n^*} = 1\angle 2\theta^\circ$$



$j = -1$

Real

$\bar{V} = 10(V) \Rightarrow \theta = 0^\circ$

$\Rightarrow 10\cos(\omega t + 0^\circ)$

$\bar{I} = -3(A) \Rightarrow \theta = \pm 180^\circ$

$\Rightarrow 3\cos(\omega t \pm 180^\circ)$

so phase

Complex Numbers Tutorial

R P

Given complex numbers:
$$n_1 = a_1 + jb_1 = c_1\angle\theta_1$$
$$n_2 = a_2 + jb_2 = c_2\angle\theta_2$$

use R

Addition : $$n_1 + n_2 = (a_1 + a_2) + j(b_1 + b_2)$$

Subtraction : $$n_1 - n_2 = (a_1 - a_2) + j(b_1 - b_2)$$

If the number to be added or subtracted are given in polar form, they are first converted to rectangular form.

Multiplication :
$$n_1 n_2 = (a_1 + jb_1)(a_2 + jb_2)$$
$$= (a_1a_2 - b_1b_2) + j(a_1b_2 + b_1a_2)$$

minus

slow

use P
$$n_1 n_2 = (c_1\angle\theta_1)(c_2\angle\theta_2)$$
$$= c_1c_2\angle(\theta_1 + \theta_2)$$

fast

Complex Numbers Tutorial

use P

Division :

$$\frac{n_1}{n_2} = \frac{a_1 + jb_1}{a_2 + jb_2} = \frac{a_1 + jb_1}{a_2 + jb_2} \times \frac{a_2 - jb_2}{a_2 - jb_2}$$

slow

$$= \frac{(a_1 + jb_1)(a_2 - jb_2)}{a_2^2 + b_2^2}$$

$\frac{1}{3}(-40^0) \Rightarrow$

$\frac{1}{3}\cos(\omega t - 40^0)$

use P

fast

$$\frac{n_1}{n_2} = \frac{(c_1\angle\theta_1)}{(c_2\angle\theta_2)} = \frac{c_1}{c_2}\angle(\theta_1 - \theta_2)$$

Example : Find $\boxed{\frac{2\angle 90^\circ}{4\angle 75^\circ}}$

Answer:

$$\frac{2\angle 90^\circ}{4\angle 75^\circ} = \frac{2}{4}\angle(90^\circ - 75^\circ) = \boxed{\frac{1}{2}\angle 15^\circ}$$

Example : Find $\boxed{\frac{3\angle 20^\circ}{9\angle 60^\circ}}$

Answer:

$$\frac{3\angle 20^\circ}{9\angle 60^\circ} = \frac{3}{9}\angle(20^\circ - 60^\circ) = \boxed{\frac{1}{3}\angle -40^\circ}$$

Complex Numbers Tutorial

$n = c e^{j\theta} \rightarrow \sqrt{n} = (c e^{j\theta})^{1/2}$

Square root:

$$\sqrt{n} = \sqrt{c}(\theta/2) = \sqrt{c}\, e^{j\frac{\theta}{2}}$$

Complex conjugate:

$$n^{*} = \underbrace{a - jb}_{R} = \underbrace{c\angle(-\theta)}_{P} = \underbrace{ce^{-j\theta}}_{P})$$

Complex Numbers Tutorial

Q: Evaluate these complex numbers:

R

(a) $(40\angle 50^\circ + 20\angle -30^\circ)^{1/2}$

$c.(\cos\theta + j\sin\theta)$

$ce^{j\theta} =$

R

P

(b) $\dfrac{10\angle -30^\circ + (3 - j4)}{(2 + j4)(3 - j5)^*}$

P

P

$\sqrt{(43.03)^2 + (20.69)^2}$ R $\tan^{-1}\left(\dfrac{20.69}{43.03}\right)$

Sol

(a) $40\angle 50^\circ = 40(\cos 50^\circ + j\sin 50^\circ) = 25.71 + j30.64$

$20\angle -30^\circ = 20(\cos(-30^\circ) + j\sin(-30^\circ)) = 17.32 - j10$

$40\angle 50^\circ + 20\angle -30^\circ = 43.03 + j20.64 = 47.72\angle 25.63^\circ$

$(40\angle 50^\circ + 20\angle -30^\circ)^{1/2} = 6.91\angle 12.81^\circ$

$C = \sqrt{a^2 + b^2}$

$\tan\theta = \dfrac{b}{a}$

$\sqrt{47.72}$ $\dfrac{25.63}{2}$

Complex Numbers Tutorial

$10(\cos(-30) + j\sin(-30))$

(b)

$$\frac{10\angle -30^\circ + (3 - j4)}{(2 + j4)(3 - j5)^*} = \frac{8.66 - j5 + (3 - 4j)}{(2 + j4)(3 + j5)} = 11.66 - j9$$

4.66 −

$$= \frac{11.53 + j9}{-14 + j22} = \frac{14.73\angle -37.66^\circ}{26.08\angle 122.47^\circ}$$

x

$$= \underline{0.565\angle -160.13^\circ}$$

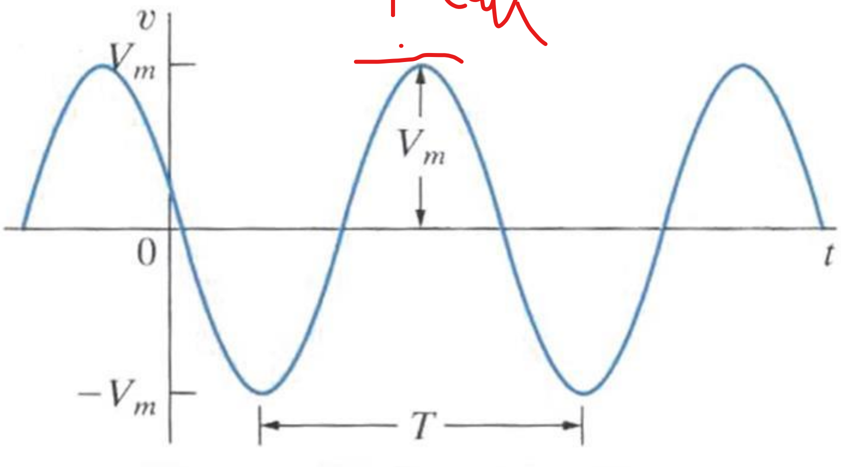
Introduction to Sinusoid

- **Sinusoid**: a signal that has the form of the sine or cosine function.
- Why sinusoidal waveforms are useful to engineers?
  1. Appears everywhere: Vibration of a string, ripples of ocean surface, and natural response of underdamped second-order systems.
  2. Easily generated.
  3. Every practical periodic signal can be represented by a linear combination of sinusoidal signals.
  4. Easily analyzed.

$V_p = V_M \rightarrow 220\sqrt{2}$ in VN

The Sinusoidal Source

Peak



$$v = V_m \cos(\omega t + \phi)$$

A sinusoidal voltage/current source produces a voltage/current that varies sinusoidally with time.

$\Omega t + \Phi$: the *argument of the sinusoid*

T : period of the function (s)

f : frequency of the function (Hz)

$$\boxed{f = \frac{1}{T}}$$

ω : angular frequency (radians/second)

$$\omega = 2\pi f = 2\pi/T$$

Φ: phase angle (degree)

$$(\text{number of degrees}) = \frac{180}{\pi}(\text{number of radians})$$

V_m : maximum amplitude (V)

V_{rms}: root mean square value (effective)

$$V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \phi)dt} = \boxed{\frac{V_m}{\sqrt{2}}}$$

thông thường này chỉ hỏi sinx cosx

chỉ dùng cho cos

The Sinusoidal Source

Phase Lead or Lag

Two sinusoids with the same frequency.

$$v_1(t) = V_{m1}\cos(\omega t + \phi_1),\ V_{m1} > 0$$

$$v_2(t) = V_{m2}\cos(\omega t + \phi_2), V_{m2} > 0$$

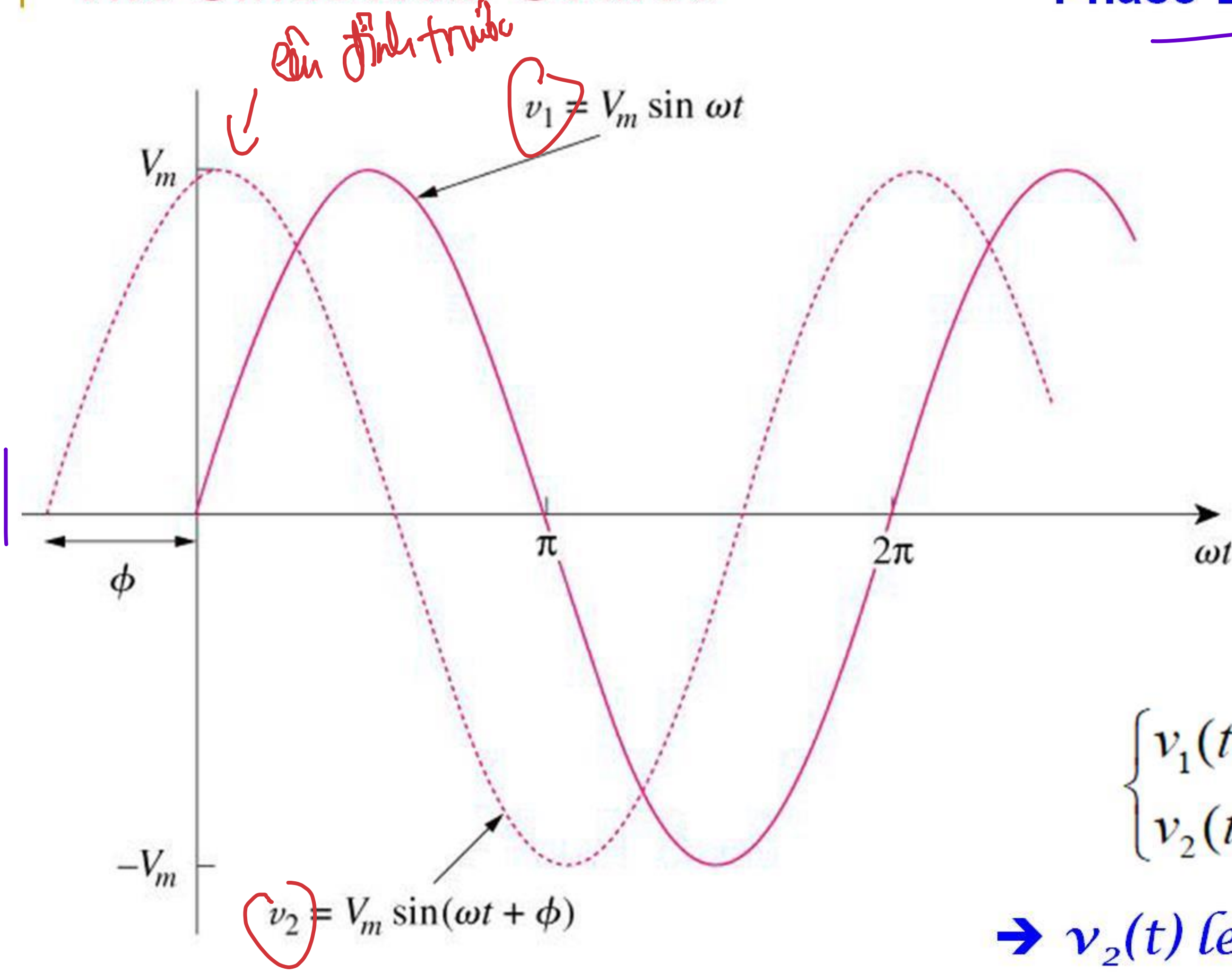
$\phi_1 - \phi_2 = 0$: $v_1(t)$ and $v_2(t)$ are *in phase*. → đồng pha

$\phi_1 - \phi_2 \neq 0$: $v_1(t)$ and $v_2(t)$ are *out of phase*.

$$\begin{cases} \phi_1 - \phi_2 > 0,\ v_1(t) \text{ leads } v_2(t) \text{ by } \phi_1 - \phi_2 \\ \phi_1 - \phi_2 < 0,\ v_1(t) \text{ lags } v_2(t) \text{ by } \phi_2 - \phi_1 \end{cases}$$

The Sinusoidal Source

Phase Lead or Lag



$$\begin{cases} v_1(t) = V_m \sin(\omega t) \\ v_2(t) = V_m \sin(\omega t + \phi) \end{cases}$$

→ $v_2(t)$ *leads* $v_1(t)$ *by* ϕ

The Sinusoidal Source

Trigonometric Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\Rightarrow \begin{cases} \sin(\omega t \pm 180^\circ) = -\sin \omega t \\ \cos(\omega t \pm 180^\circ) = -\cos \omega t \\ \sin(\omega t \pm 90^\circ) = \pm \cos \omega t \\ \cos(\omega t \pm 90^\circ) = \mp \sin \omega t \end{cases}$$

← use many times.

$$\Rightarrow A \cos \omega t + B \sin \omega t = C \cos(\omega t - \theta)$$

$$C = \sqrt{A^2 + B^2} \qquad \theta = \tan^{-1}\frac{B}{A}$$

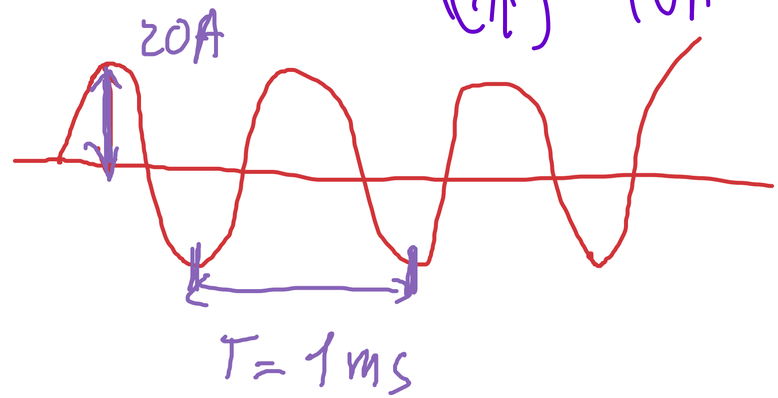
Ex. 1: Finding the Characteristics of a Sinusoidal Current

A sinusoidal current has a maximum amplitude of **20 A**. The current passes through one complete cycle in **1 ms**. The magnitude of the current at zero time is **10 A**.

$T =$

$f = \frac{1}{T} = 1000\,Hz$

$i(0) = 10A$



a) What is the frequency of the current in hertz?

b) What is the frequency in radians per second?

c) Write the expression for *i(t) using the cosine* function. Express ϕ in degrees.

d) What is the rms value of the current?

Sol. of example 1:

a) What is the frequency of the current in hertz?

From the statement of the problem, $T = 1\ ms;$

Hence f = 1/T = 1000 Hz.

f

b) What is the frequency in radians per second $\omega = 2\pi f = \frac{2\pi}{T}$

$\omega = 2\pi f = 2000\pi\ rad/s.$

c) Write the expression for *i(t) using the cosine* $i(0) = 10 \rightarrow t = 0, i = 0$

function. Express ϕ in degrees.

We have $i(t) = I_m \cos(\omega t + \phi) = 20\cos(2000\pi t + \phi)$, but $i(0) = 10$ A. Therefore

$10 = 20\cos\phi \rightarrow \phi = 60°$

Thus the expression for i(t) becomes $i(t) = 20\cos(2000\pi t + 60°)$.

d) What is the rms value of the current?

From the derivation of $V_{rms} = V_m/\sqrt{2}$, the rms value of a sinusoidal current is $20/\sqrt{2}$. Therefore the rms value is $\boxed{20/\sqrt{2}, \text{ or } 14.14\ A.}$

28.28

Ex. 2: Finding the Characteristics of a Sinusoidal Voltage

A sinusoidal voltage is given by the expression

$$v = 300\cos(\overset{\omega}{120\pi} t + 30°).$$

V ϕ

a) What is the period of the voltage in milliseconds?   T

b) What is the frequency in hertz? $f = \frac{1}{T} = 60\ Hz$

c) What is the magnitude of *v at t = 2.778 ms?*

d) What is the rms value of v?

Sol. of example 2:

a) From the expression for v, $\omega = 120\pi$ rad/s. Because $\omega = 2\pi/T$, $T = 2\pi/\omega = \frac{1}{60}$ s, or 16.667 ms.

$\omega = \frac{2\pi}{T}$

b) The frequency is $1/T$, or 60 Hz.

$f = \frac{1}{T} = 60 Hz$

c) From (a), $\omega = 2\pi/16.667$; thus, at $t = 2.778$ ms, ωt is nearly 1.047 rad, or 60°. Therefore, $v(2.778 \text{ ms}) = 300 \cos(60° + 30°) = 0$ V.

given

d) $V_{rms} = 300/\sqrt{2} = 212.13$ V.

$= \frac{V}{\sqrt{2}}$

Ex. 3: Translating a Sine Expression to a Cosine Expression

We can translate the sine function to the cosine function by subtracting 90° *(π/2 rad) from the argument* of the sine function.

a) Verify this translation by showing that

$$\sin(\omega t + \theta) = \cos(\omega t + \theta - 90°).$$

sin → cos : -90^0

b) Use the result in (a) to express $\sin(\omega t + 30°)$ *as* a cosine function.

Sol. of example 3:

a) Verification involves direct application of the trigonometric identity

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta.$$

We let $\alpha = \omega t + \theta$ and $\beta = 90^\circ$. As $\cos 90^\circ = 0$ and $\sin 90^\circ = 1$, we have

$$\cos(\alpha - \beta) = \sin\alpha = \sin(\omega t + \theta) = \boxed{\cos(\omega t + \theta - 90^\circ)}.$$

b) From (a) we have

$$\sin(\omega t + 30^\circ) = \cos(\omega t + 30^\circ - 90^\circ) = \boxed{\cos(\omega t - 60^\circ)}.$$

Final

Ex. 5

Calculus the phase angle between:

$v_1 = -10\cos(\omega t + 50°)$ and

$v_2 = 12\sin(\omega t - 10°)$.

State which sinusoid is leading.

Dấu $- \Rightarrow -180^0$

$\sin \rightarrow \cos \Rightarrow -90^0$

$\rightarrow$ must same func

$$V_1 = 10\cos(wt + 50^0 - 180^0) = 10\cos(wt - 130^0)$$

and

$$V_2 = 12\cos(wt - 10 - 90^0) = 12\cos(wt - 100^0)$$

$\phi_1 = -130^0$

$\phi_2 = -100^0$

$\phi_2 - \phi_1 = 30^0$

$\rightarrow V_2$ leads V_1 by 30^0

Sol. of Ex. 5

$$v_1 = -10 \cos(\omega t + 50^\circ) = 10 \cos(\omega t + 50^\circ - 180^\circ)$$

$$v_1 = 10 \cos(\omega t - 130^\circ)$$

and

$$v_2 = 12 \sin(\omega t - 10^\circ) = 12 \cos(\omega t - 10^\circ - 90^\circ)$$

$$v_2 = 12\cos(\omega t - 100^\circ)$$

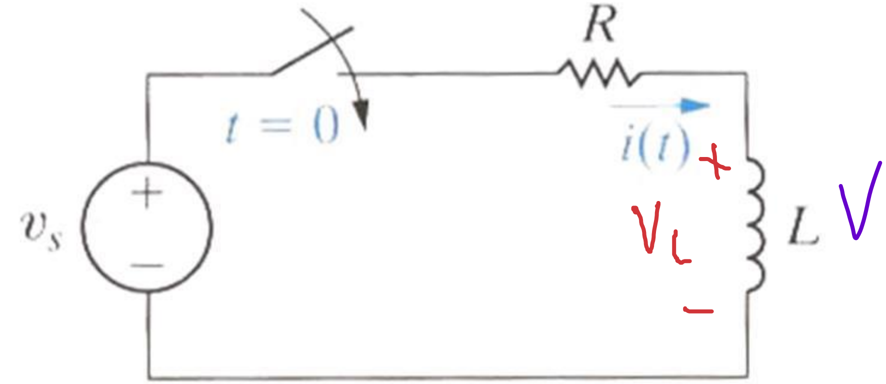
$$\phi_1 = -130^\circ,\ \phi_2 = -100^\circ;\ \phi_2 - \phi_1 = 30^\circ$$

→ v_2 leads v_1 by 30°

The Sinusoidal Response

There, v_s *is a sinusoidal voltage, or*

$$v_s = V_m \cos(\omega t + \phi)$$



Apply KVL to the circuit

$$L\frac{di}{dt} + Ri = V_m \cos(\omega t + \phi)$$

This course not study

The formal solution

& no change

$$i = \underbrace{\frac{-V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\phi - \theta) e^{-(R/L)t}}_{\textbf{Transient}} + \underbrace{\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)}_{\textbf{Steady-state}}$$

change

quá độ

mainly focus

The Sinusoidal Response

For the steady-state solution:

1. The steady-state solution is a sinusoidal function.
2. The frequency of the response signal is identical to the frequency of the source signal.
3. The maximum amplitude of the steady-state response differs from the maximum amplitude of the source.
4. The phase angle of the response signal differs from the phase angle of the source.

The Phasor

Euler's identity: $$\boxed{e^{\pm j\theta} = \cos\theta \pm j\sin\theta}$$ only used

For a given sinusoidal voltage function:

Time domain

$$v(t) = V_m \cos(\omega t + \phi) = \mathrm{Re}(V_m e^{j(\omega t+\phi)})$$

$$= \mathrm{Re}(V_m e^{j\phi} e^{j\omega t})$$

Thus $v(t)$ can be written as

$$v(t) = \mathrm{Re}(\mathbf{V} e^{j\omega t})$$

Phasor

Here $\mathbf{V} = V_m e^{j\phi} = V_m \angle\phi$ is the *phasor representation* of $v(t)$.

The phasor is a complex number that carries the amplitude and phase angle information of a sinusoidal function.

The **phasor transform** of the given sinusoidal function is:

$$\mathbf{V} = V_m e^{j\phi} = \mathcal{P}\{V_m \cos(\omega t + \phi)\}$$

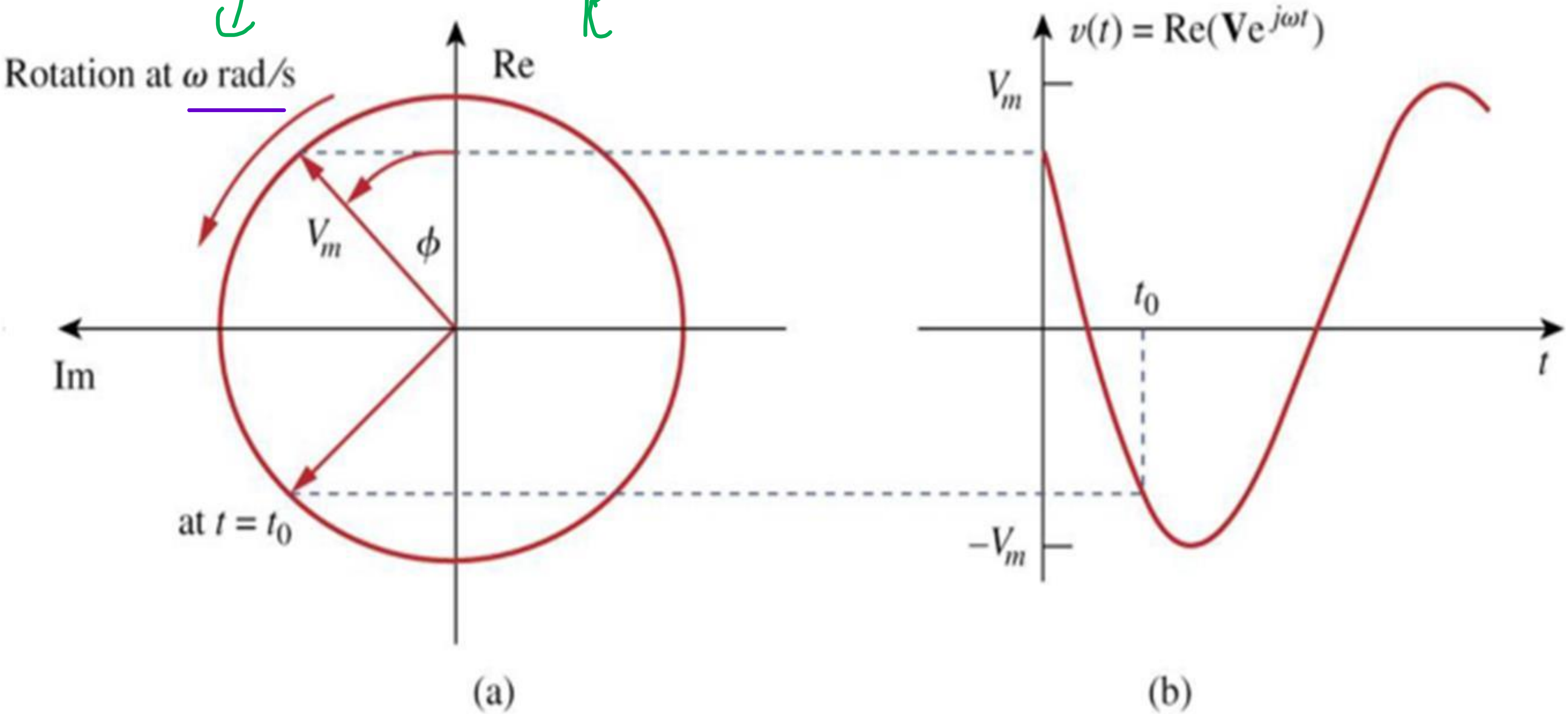
$V = 10\cos(5t + 40^0)$

$\rightarrow V = 10\angle 40$

The Phasor

Depend on time

care only Real



The Phasor

The phasor transform transfers the sinusoidal function from the ***time domain*** to the complex-number domain, called the ***frequency domain***.

$$v(t) = V_m \cos(\omega t + \phi) \Leftrightarrow \mathbf{V} = V_m \angle \phi$$

(Time-domain representation) (Phasor-domain representation)

Given the frequency ω, the phasor can equivalently represent the signal v(t).

The polar form: $\mathbf{V} = V_m e^{j\phi}$

The rectangular form: $\mathbf{V} = V_m \cos\phi + jV_m \sin\phi$

The angle notation: $V_m \angle \phi^\circ = V_m e^{j\phi}$

Inverse phasor transform is found by multiplying the phasor by e $^{j\omega t}$ and then extracting the real part of the product.

$$\mathcal{P}^{-1}\{V_m e^{j\phi}\} = \mathcal{R}\{V_m e^{j\phi} e^{j\omega t}\}$$

The Phasor

Sinusoid-PhasorTransformation

should not made this mistakes.

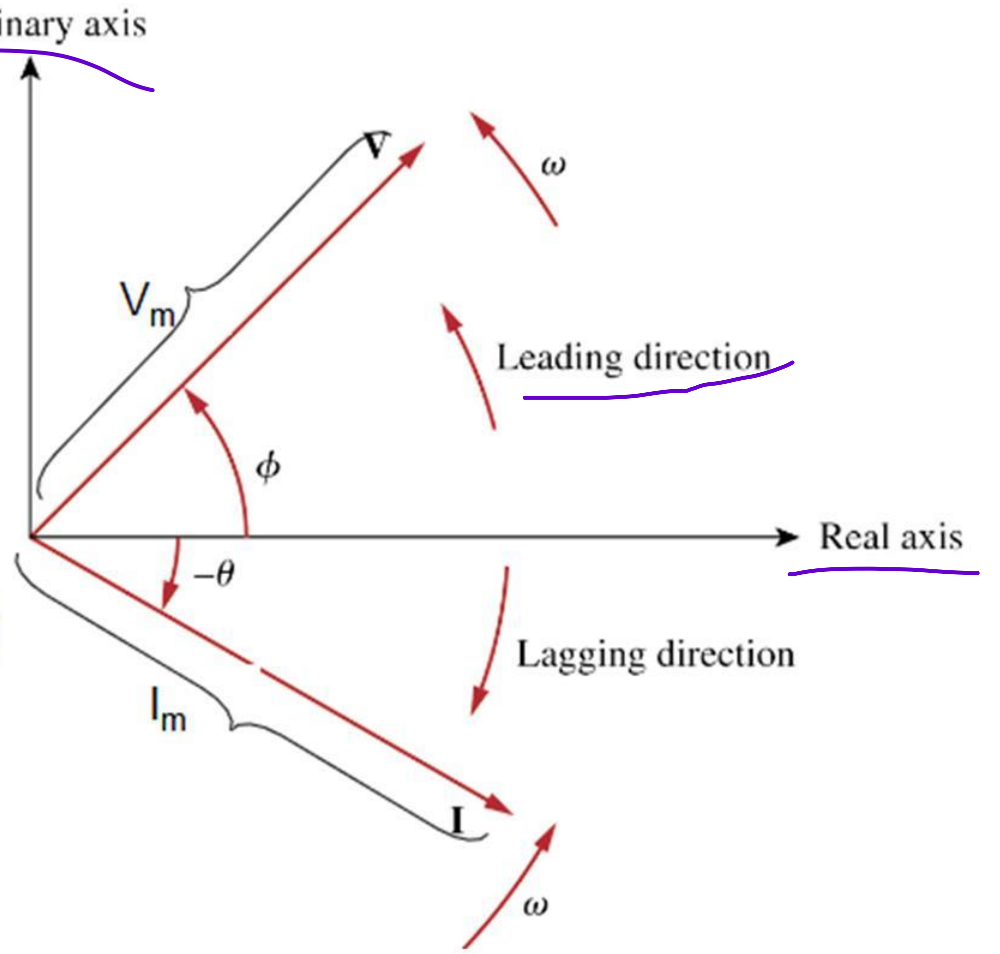
Time domain representation	**Phasor domain representation**
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$ → $V_m \cos(\omega t + \phi - 90)$	$V_m \angle(\phi - 90°)$
$I_m \cos(\omega t + \phi)$	$I_m \angle \phi$
$I_m \sin(\omega t + \phi)$	$I_m \angle(\phi - 90°)$

The Phasor

Phasor Diagram

$$\mathbf{V} = V_m \angle \phi$$

$$\mathbf{I} = I_m \angle -\theta$$



Ex. 6:

Transform these sinusoid to phasors:

(a) $i = 6\cos(50t - 40°)$ A $\Rightarrow I = 6 \angle -40°$ A

(b) $v = -4\sin(30t + 50°)$ V

$\Rightarrow v = 4\cos(30t + 50° + 90°)$

$= 4\cos(30t + 140°)$

$\Rightarrow V = 4 \angle 140°$ V

Sol. of Ex. 6:

(a) $i = 6\cos(50t - 40^\circ)$ has the phasor

$$\mathbf{I} = 6\angle -40^\circ \ \mathrm{A}$$

(b) Since $-\sin A = \cos(A + 90^\circ)$

$$v = -4\sin(30t + 50^\circ) = 4\cos(30t + 50^\circ + 90^\circ)$$

$$= 4\cos(30t + 140^\circ)\ \mathrm{V}$$

The phasor of v is $\mathbf{V} = 4\angle 140^\circ \ \mathrm{V}$

Ex. 7: Find the sinusoid representation by these phasors:

(a) $\mathbf{I} = -3 + j4$ A $\rightarrow$ $i(t) =$

(b) $\mathbf{V} = j8e^{-j20^\circ}$ V

$\sqrt{9+16}$ $\quad \tan^{-1}\left(-\frac{4}{3}\right) + 180^\circ$

Sol of Ex. 7:

(a) $\mathbf{I} = -3 + j4 = 5\angle 126.87^\circ$ P domain

$i(t) = 5\cos(\omega t + 126.87^\circ)$ A time

(b) $j = 1\angle 90^\circ$,

$$\mathbf{V} = j8\angle -20^\circ = (1\angle 90^\circ) \times (8\angle -20^\circ)$$

$$= 8\angle 90^\circ - 20^\circ = 8\angle 70^\circ \text{ V}$$

$v(t) = 8\cos(\omega t + 70^\circ)$ V

$t \longleftrightarrow P$

Ex. 8: Adding Cosines Using Phasors

$\rightarrow \quad 20\angle -30^\circ \quad + \quad 40\angle 60^\circ = (17.32 - j10) + (20 + j34.64)$

$= 37.32 + j24.64$

$= 44.72 \angle 33.43$

If $y_1 = 20\cos(\omega t - 30^\circ)$ and $y_2 = 40\cos(\omega t + 60^\circ)$, express $y = y_1 + y_2$ as a single sinusoidal function.

a) Solve by using trigonometric identities.

b) Solve by using the phasor concept.

a) $y_1 = 20\cos\omega t \cos 30^\circ + 20\sin\omega t \sin 30^\circ$

$y_2 = 40\cos\omega t \cos 60^\circ - 40\sin\omega t \sin 60^\circ$

$y = (20\cos 30^\circ + 40\cos 60^\circ)\cos\omega t + (20\sin 30^\circ - 40\sin 60^\circ)\sin\omega t$

$= 37.32\cos\omega t - 24.64\sin\omega t$

Sol. of Ex. 8:

a) First we expand both y_1 and y_2, using the cosine of the sum of two angles, to get

$$y_1 = 20\cos\omega t\cos 30^\circ + 20\sin\omega t\ \sin 30^\circ;$$

$$y_2 = 40\cos\omega t\cos 60^\circ - 40\sin\omega t\ \sin 60^\circ.$$

Adding y_1 and y_2, we obtain

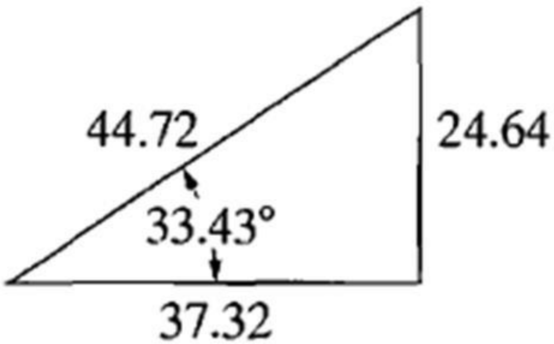
$$y = (20\cos 30 + 40\cos 60)\cos\omega t$$

$$+ (20\sin 30 - 40\sin 60)\sin\omega t$$

$y =$ $$= 37.32\cos\omega t - 24.64\sin\omega t.$$

Sol. of Ex. 8 cont.:

To combine these two terms we treat the co-efficients of the cosine and sine as sides of a right triangle (Fig. and then multiply and divide the right-hand side by the hypotenuse. Our expression for y becomes



$$y = 44.72\left(\frac{37.32}{44.72}\cos\omega t - \frac{24.64}{44.72}\sin\omega t\right)$$

$$= 44.72(\cos 33.43^\circ \cos\omega t - \sin 33.43^\circ \sin\omega t).$$

Again, we invoke the identity involving the cosine of the sum of two angles and write

$$y = 44.72\cos(\omega t + 33.43^\circ).$$

Sol. of Ex. 8 cont.:

b) We can solve the problem by using phasors as follows: Because

$$y = y_1 + y_2,$$

then, from Eq. 9.24,

$$\mathbf{Y} = \mathbf{Y}_1 + \mathbf{Y}_2$$

$C\angle(\cos\phi + j\sin\phi)$

$$= 20\angle -30^\circ + 40\angle 60^\circ$$

$$= (17.32 - j10) + (20 + j34.64)$$

$$= 37.32 + j24.64$$

$$= 44.72\angle 33.43^\circ.$$

A ϕ

Sol. of Ex. 8 cont.:

Once we know the phasor **Y**, we can write the corresponding trigonometric function for y by taking the inverse phasor transform:

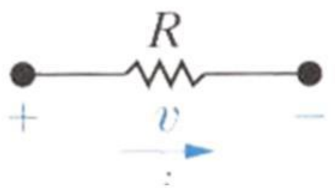
$$y = \mathcal{P}^{-1}\{44.72e^{j33.43}\} = \Re\{44.72e^{j33.43}e^{j\omega t}\}$$

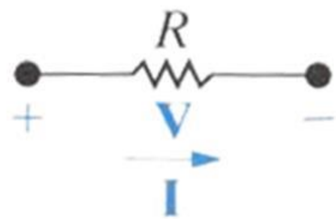
y =

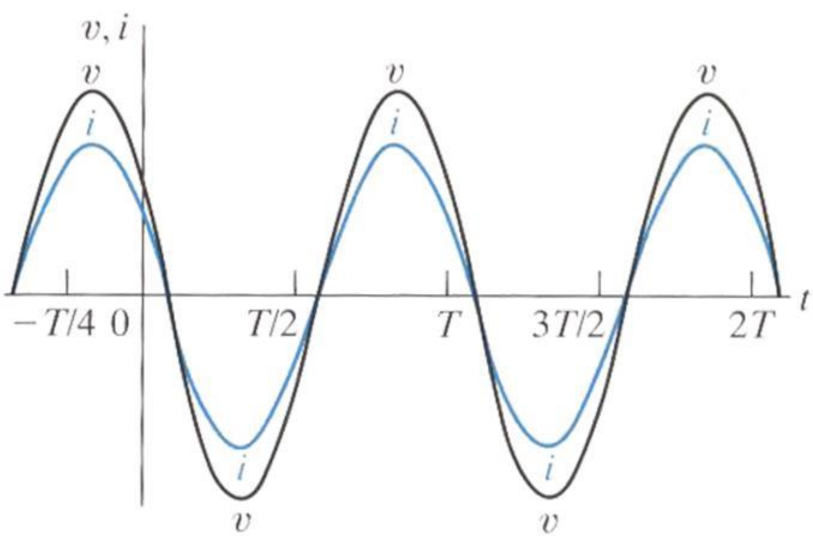
$$= 44.72\cos(\omega t + 33.43^\circ).$$

The superiority of the phasor approach for adding sinusoidal functions should be apparent. Note that it requires the ability to move back and forth between the polar and rectangular forms of complex numbers.

The V-I Relationship for a Resistor







Given a current in a resistor:

$$i = I_m \cos(\omega t + \theta_i)$$

The voltage of the resistor is:

$$v = Ri = RI_m \cos(\omega t + \theta_i)$$

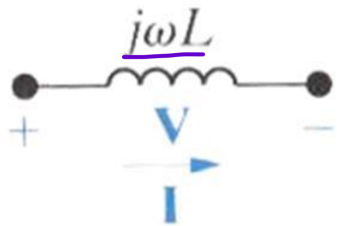
Phasor presentation:

$$\mathbf{V} = RI_m e^{j\theta_i} = R\mathbf{I}$$

Voltage and current of a resistor are ***in phase*** .

The V-I Relationship for an Inductor

L



leads

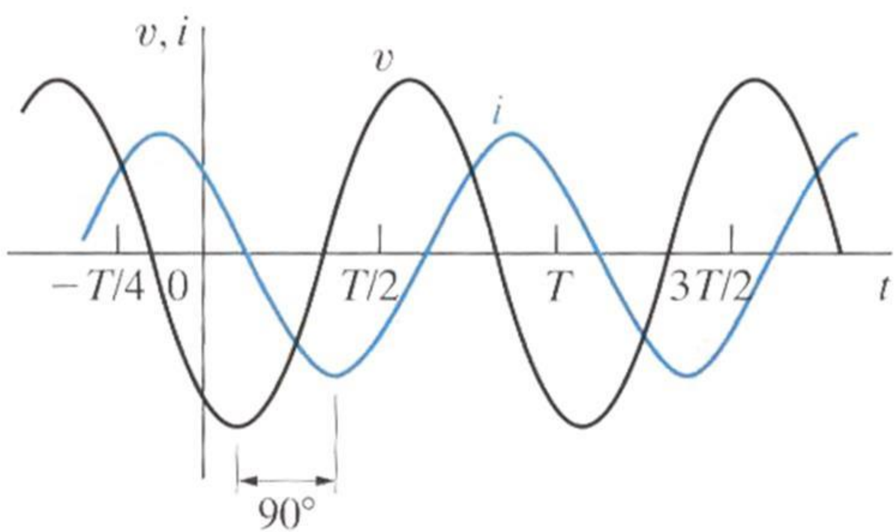
Given a current in an inductor:

$$i = I_m \cos(\omega t + \theta_i)$$

The voltage is:

$$v = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \theta_i)$$

$$= -\omega L I_m \cos(\omega t + \theta_i - 90°)$$



The phasor presentation:

$$\mathbf{V} = -\omega L I_m e^{j(\theta_1 - 90°)}$$

$$= -\omega L I_m e^{j\theta_i} e^{-j90°}$$

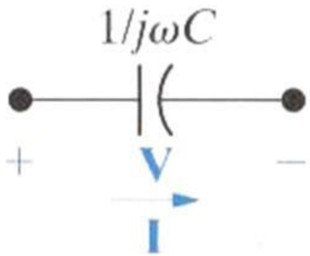
$$= j\omega L I_m e^{j\theta_i}$$

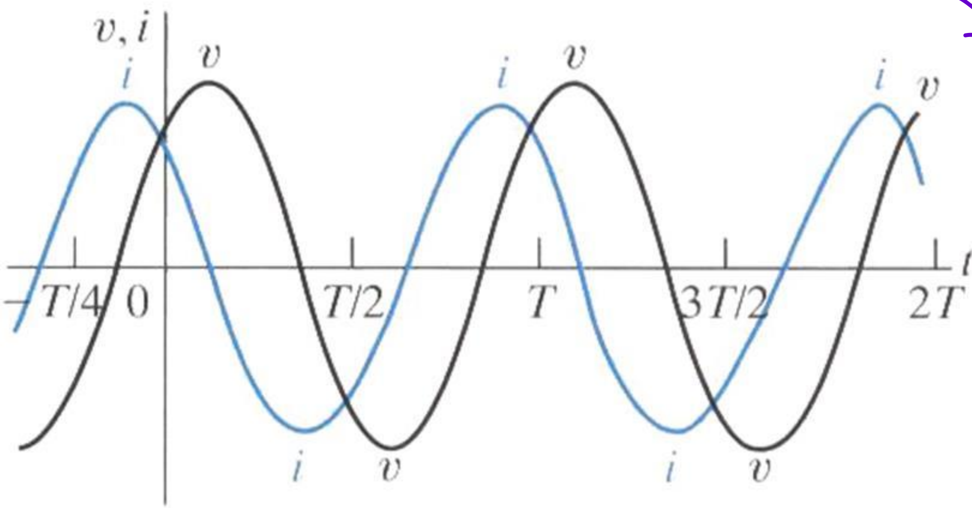
$$= j\omega L \mathbf{I}$$

Phasor Form

In an inductor, the ***voltage leads*** the current by 90° or the ***current lags*** behind the voltage by 90°.

The V-I Relationship for a Capacitor





Given a voltage in a capacitor:

$$v = V_m \cos(\omega t + \theta_v)$$

leads

The current is:

$$i = C\frac{dv}{dt} = -\omega C V_m \cos(\omega t + \theta_v - 90°)$$

The phasor presentation:

$$\mathbf{I} = -\omega C V_m e^{j(\theta_v - 90°)} = j\omega C\mathbf{V}$$

or :

$$\mathbf{V} = \frac{1}{j\omega C}\mathbf{I}$$

In a capacitor, the ***voltage lags*** behind the current by 90^0 or the ***current leads*** the voltage by 90^0 .

Summary of Voltage-Current Relationships

Element	Time domain	Frequency domain
R	$V = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\dfrac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\dfrac{dv}{dt}$	$\mathbf{V} = \dfrac{\mathbf{I}}{j\omega C}$

Ex. 9

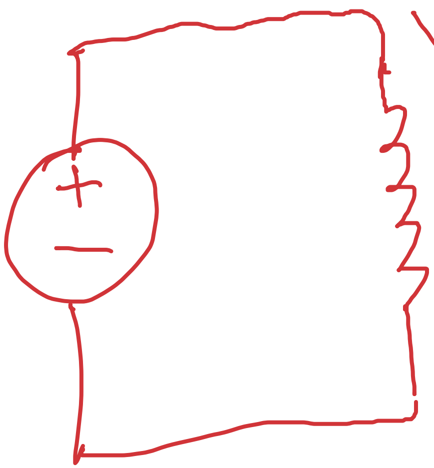
$V = 12 \angle 45^\circ$

$j\omega L I$

The voltage *v= 12 cos(60t+ 45°) is applied to a 0.1-H inductor. Find the ~~steady-state current~~ through the inductor.*

$i(t) = ?$

$12\cos(60t+45^\circ)$



$L = 0.1H$

Sol. of Ex. 9

$V = j\omega L I$

$\mathbf{V} = 12\angle 45^\circ \text{ V} = j\omega L\mathbf{I}$, where $\omega = 60\text{rad/s}$.

Hence,

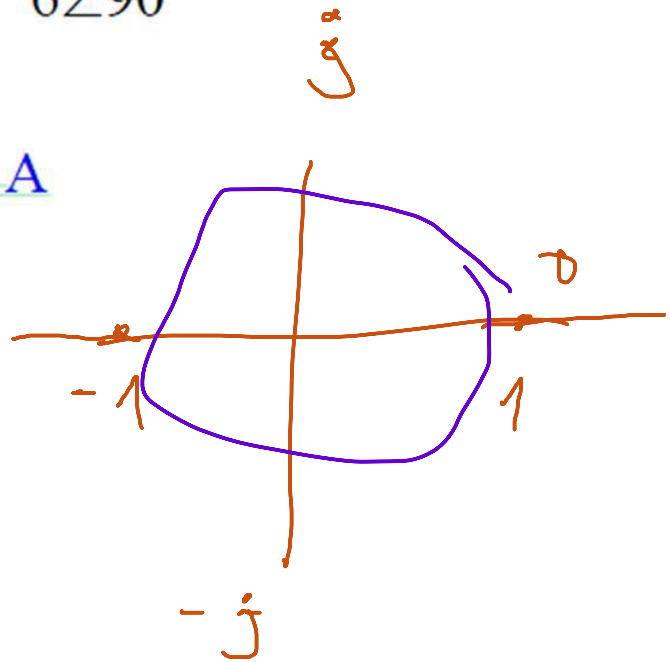
$$\mathbf{I} = \frac{\mathbf{V}}{j\omega L} = \frac{12\angle 45^\circ}{j60 \times 0.1} = \frac{12\angle 45^\circ}{6\angle 90^\circ} = 2\angle -45^\circ \text{ A}$$

$j6$

Converting,

$i(t) = 2\cos(60t - 45^\circ) \text{ A}$

$V = j\omega L I$



Impedance and Reactance

$V = ZI$

Appy Ohm's law in frequency domain:

$$\boxed{\mathbf{V} = Z\mathbf{I}}$$

Z is the **impedance** of the circuit element, which is measured in ohms.

The imaginary part of the impedance is the **reactance** .

$$Y = \frac{1}{Z} = G + jB$$

Y is the **admittance** of the circuit element, which is measured in siemens.

Admittance is a complex number, whose real part, *G*, is called **conductance**, and whose impaginary part, *B*, is called **susceptance**.

Circuit Element	*Impedance (Z)*	*Reactance*	*Admittance (Y)*	*Susceptance*
Resistor	R	--	G	--
Inductor	jωL	ωL	j(-1/ ωL)	-1/ ωL
Capacitor	j(-1/ωC)	-1/ωC	jωC	ωC

General Passive Circuit In Phasor Domain

$$\boxed{\mathbf{Z} = R + jX = |\mathbf{Z}|\angle\theta}$$

where

$$|\mathbf{Z}| = \sqrt{R^2 + X^2}, \quad \theta = \tan^{-1}\frac{X}{R}$$

φ

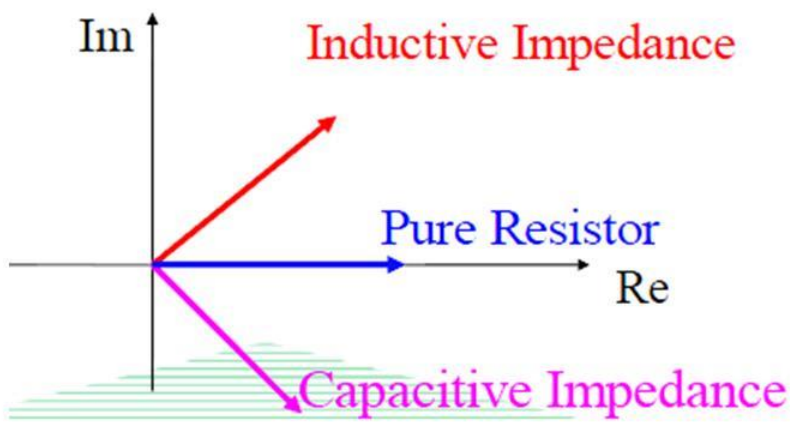
and

$$R = |\mathbf{Z}|\cos\theta = \mathrm{Re}(\mathbf{Z}): \text{ Resistance of } \mathbf{Z},$$

$$X = |\mathbf{Z}|\sin\theta = \mathrm{Im}(\mathbf{Z}): \text{ Reactance of } \mathbf{Z}$$

$$\boxed{\mathbf{Z} = R + jX = |\mathbf{Z}|\angle\theta}$$

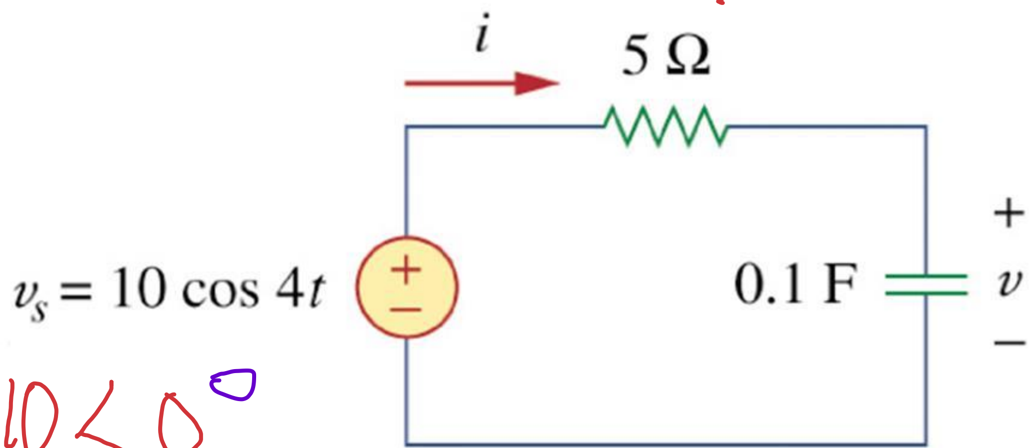
- $X > 0$: inductive impedance
- $X = 0$: pure resistor
- $X < 0$: capacitive impedance



Ex. 10

Find *v(t) and i(t) in the circuit*

$\rightarrow 5$



$10\angle 0^\circ$

$\frac{1}{j\omega C} = \frac{1}{j4 \times 0.1}$

Change to Phasor form

Sol. of Ex. 10

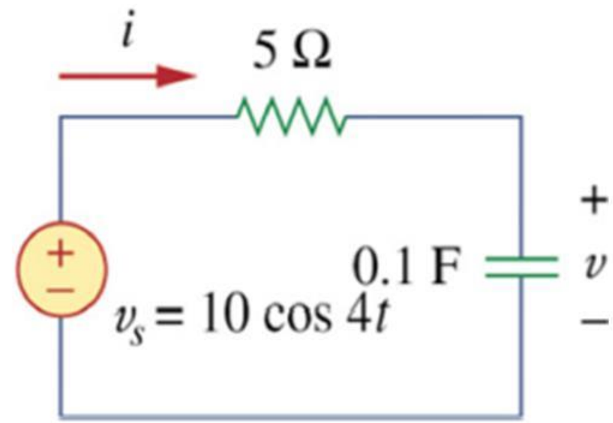
Vs w

- From the voltage source 10 cos 4t, $\omega = 4$,

$$\mathbf{V}_s = 10\angle 0^\circ \text{ V}$$

- The impedance is

$$\mathbf{Z} = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{j4 \times 0.1}$$
$$= 5 - j2.5\Omega$$



- Hence the current

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{10\angle 0^\circ}{5 - j2.5} = \frac{10(5 + j2.5)}{5^2 + 2.5^2}$$
$$= 1.6 + j0.8 = 1.789\angle 26.57^\circ \text{ A}$$