FINAL REVIEW

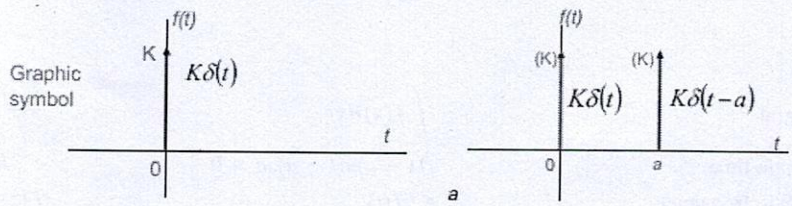
INTRODUCTION TO ELECTRICAL ENGINEERING II

I. LAPLACE TRANSFORM:

- ***Basic function:***

Step function: $ku(t) = \begin{cases} 0, t < 0 \\ k, t > 0 \end{cases}$; $ku(t-a) = \begin{cases} 0, t < a \\ k, t > a \end{cases}$; $ku(a-t) = \begin{cases} k, t < a \\ 0, t > a \end{cases}$

Impulse function: $k\delta(t), \delta(t) = 0;\ t \neq 0$



✓ An impulse is a signal of infinite amplitude and zero duration
✓ The area under the impulse function is constant.
✓ The impulse is zero everywhere except @ t = 0

- ***Functional transform pair:***

Type	*f(t) (t > 0⁻)*	*F(s)*
(impulse)	$\delta(t)$	1
(step)	$u(t)$	$\frac{1}{s}$
(ramp)	t	$\frac{1}{s^2}$
(exponential)	e^{-at}	$\frac{1}{s+a}$
(sine)	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
(cosine)	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
(damped ramp)	te^{-at}	$\frac{1}{(s+a)^2}$
(damped sine)	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
(damped cosine)	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

- ***Operational transform:***

TABLE 12.2 An Abbreviated List of Operational Transforms

Operation	$f(t)$	$F(s)$
Multiplication by a constant	$Kf(t)$	$KF(s)$
Addition/subtraction	$f_1(t) + f_2(t) - f_3(t) + \cdots$	$F_1(s) + F_2(s) - F_3(s) + \cdots$
First derivative (time)	$\frac{df(t)}{dt}$	$sF(s) - f(0^-)$
Second derivative (time)	$\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0^-) - \frac{df(0^-)}{dt}$
nth derivative (time)	$\frac{d^nf(t)}{dt^n}$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}\frac{df(0^-)}{dt} - s^{n-3}\frac{df^2(0^-)}{dt^2} - \cdots - \frac{d^{n-1}f(0^-)}{dt^{n-1}}$
Time integral	$\int_0^t f(x)\,dx$	$\frac{F(s)}{s}$
Translation in time	$f(t-a)u(t-a), a > 0$	$e^{-as}F(s)$
Translation in frequency	$e^{-at}f(t)$	$F(s+a)$
Scale changing	$f(at), a > 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
First derivative (s)	$tf(t)$	$-\frac{dF(s)}{ds}$
nth derivative (s)	$t^nf(t)$	$(-1)^n\frac{d^nF(s)}{ds^n}$
s integral	$\frac{f(t)}{t}$	$\int_s^\infty F(u)\,du$

- ***Inverse transform:***

<u>*Proper Rational function:*</u>

<u>Example:</u>

1. $F(s) = \frac{10s^2+28s+36}{(s+2)\times(s^2+2s+10)}$

$\rightarrow poles: s = -2;\ s = -1 \pm j3$

$$F(s) = \frac{K_1}{s+2} + \frac{K_2}{s-(-1+j3)} + \frac{K_2 *}{s-(-1-j3)}$$

✓ $K_1 = (s+2)F(s)|(s=-2) = 2$

✓ $K_2 = (s+1-j3)F(s)|(s=-1+j3) = 4$

$\rightarrow K_2 *= 4$

$$\rightarrow F(s) = \frac{2}{s+2} + \frac{4}{s-(-1+j3)} + \frac{4}{s-(-1-j3)}$$

$\rightarrow f(t) = (-2e^{-2t} + 8e^{-t}\cos(3t))u(t)$

2. $F(s) = \frac{5s^2+9s+4}{s^2(s+4)}$

$$F(s) = \frac{K_1}{s^2} + \frac{K_2}{s} + \frac{K_3}{s+4}$$

- ✓ $K_1 = s^2F(s)|(s=0) = 1$
- ✓ $K_2 = \frac{dF(s)}{d(s)}\Big|(s=0) = 2$
- ✓ $K_3 = (s+4)F(s)|(s=-4) = 3$

$\rightarrow K_2 *= 4$

$\rightarrow f(t) = (t + 2 + 3e^{-4t})u(t)$

3. $F(s) = \frac{768}{(s^2+6s+25)^2}$

$$F(s) = \frac{768}{(s^2+6s+25)^2} = \frac{768}{(s+3-j4)^2(s+3+j4)^2}$$

$$= \frac{K_1}{(s+3-j4)^2} + \frac{K_2}{(s+3-j4)} + \frac{K_1^*}{(s+3+j4)^2} + \frac{K_2^*}{(s+3+j4)}$$

Now we need to evaluate only K_1 and K_2, because K_1^* and K_2^* are conjugate values. The value of K_1 is

$$K_1 = \frac{768}{(s+3+j4)^2}\Big|_{s=-3+j4} = \frac{768}{(j8)^2} = -12$$

The value of K_2 is

$$K_2 = \frac{d}{ds}\left[\frac{768}{(s+3+j4)^2}\right]_{s=-3+j4} = -\frac{2(768)}{(s+3+j4)^3}\Big|_{s=-3+j4} = -\frac{2(768)}{(j8)^3}$$

$$= -j3 = 3\angle{-90^\circ}.$$

$\Rightarrow K_1^* = -12,\quad K_2^* = j3 = 3\angle 90^\circ$

We now group the partial fraction expansion by conjugate terms to obtain

$$F(s) = \left[\frac{-12}{(s+3-j4)^2} + \frac{-12}{(s+3+j4)^2}\right] + \left(\frac{3\angle{-90^\circ}}{s+3-j4} + \frac{3\angle 90^\circ}{s+3+j4}\right).$$

We now write the inverse transform of $F(s)$: $f(t) = [-24te^{-3t}\cos 4t + 6e^{-3t}\cos(4t - 90^\circ)]u(t)$.

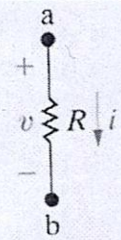
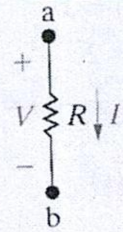
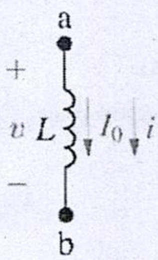
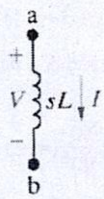
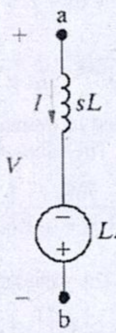
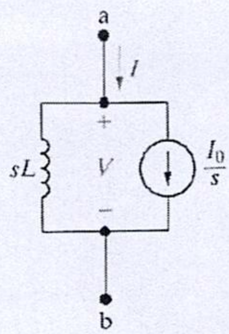
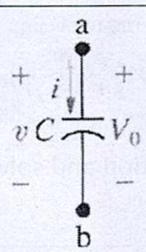
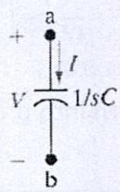
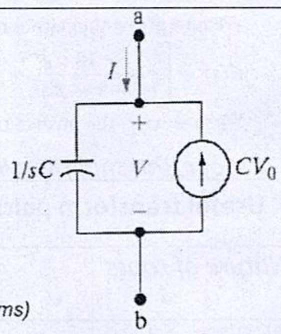
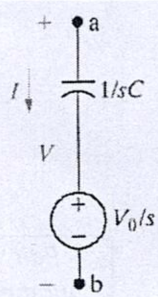
__Proper Rational function:__ turn into proper function and solve it

Useful transform pair:

Pair number	*Nature of roots*	*F(s)*	*f(t)*
1	Distinct real	$\frac{k}{s+a}$	$Ke^{-at}u(t)$
2	Repeated real	$\frac{k}{s+a^2}$	$Kte^{-at}u(t)$
3	Distinct complex	$\frac{k}{s+a-jb} + \frac{k}{s+a+jb}$	$2\lvert K\rvert e^{-at}\cos(\beta t + \theta)\,u(t)$
4	Repeated complex	$\frac{k}{(s+a-jb)^2} + \frac{k}{(s+a+jb)^2}$	$2t\lvert K\rvert e^{-at}\cos(\beta t + \theta)\,u(t)$

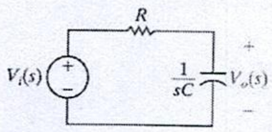
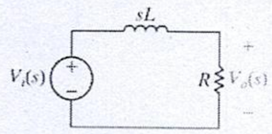
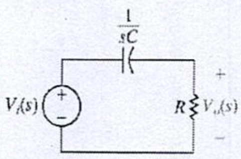
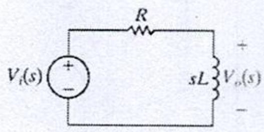
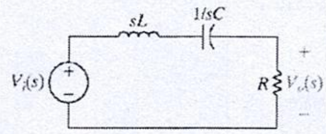
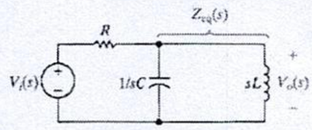
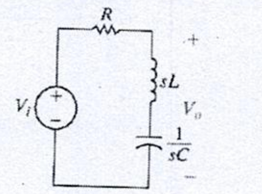
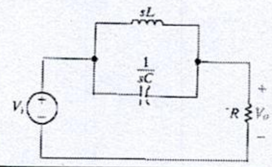
II. LAPLACE TRANSFORM IN CIRCUIT ANALYSIS:

Circuit elements in s – domain:

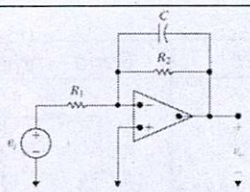
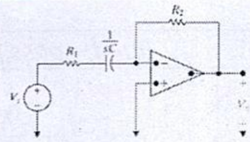
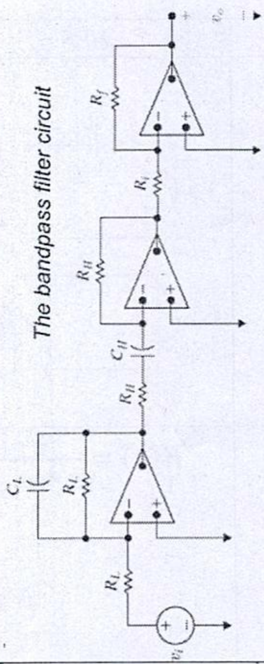
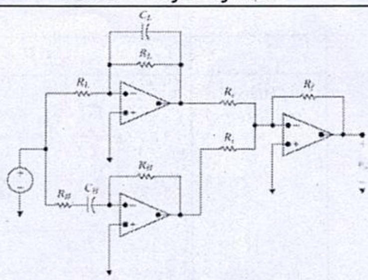
Elements	*Time – domain*	*s – domain*
Resistor	 $v = Ri$	 $V = RI$
Inductor	 $v = L \times \dfrac{dv}{dt}$ $i = \dfrac{1}{L}\int_{0-}^{t} vdx + I_0$	 *(initial current is 0 or initial energy stored = 0)*   $V = L[sI - i(0-)] = sLI - LI_0$ $\rightarrow I = \dfrac{V + LI_0}{sL} = \dfrac{V}{sL} + \dfrac{I_0}{s}$
Capacitor	 $i = C \times \dfrac{dv}{dt}$ $v = \dfrac{1}{C}\int_{0-}^{t} idx + V_0$	 *(initial voltage is 0 impedance of 1/sC Ohms)*   $I = C[sV - v(0-)]$ *or* $I = sCV - CV_0$ $\rightarrow V = \dfrac{1}{sC} \times I + \dfrac{V_0}{s}$

III. FIRST ORDER ACTIVE AND PASSIVE FILTER:

- ***Passive filter:***

	LPF	*HPF*	*Band – pass filter*	*Band – reject filter*
Circuit	 	 	 	 
Transfer function	$H(s) = \frac{\omega_C}{s + \omega_C}$	$H(s) = \frac{s}{s + \omega_C}$	$\boxed{H(s) = \frac{\beta s}{s^2 + \beta s + \omega_0^2}}$ $\omega_0 = \sqrt{\omega_{C1}\omega_{C2}}$ $Q = \frac{\omega_0}{\beta}$	$\boxed{H(s) = \frac{s^2 + \omega_0^2}{s^2 + \beta s + \omega_0^2}}$ $\omega_0 = \sqrt{\omega_{C1}\omega_{C2}}$ $Q = \frac{\omega_0}{\beta}$
frequency	$\omega_C = \frac{R}{L} = \frac{1}{RC}$	$\omega_C = \frac{R}{L} = \frac{1}{RC}$	Series: $\beta = \frac{R}{L};\quad \omega_0 = \frac{1}{\sqrt{LC}}$ Parallel: $\beta = \frac{1}{RC};\quad \omega_0 = \frac{1}{\sqrt{LC}}$	Series: $\beta = \frac{R}{L};\quad \omega_0 = \frac{1}{\sqrt{LC}}$ Parallel: $\beta = \frac{1}{RC};\quad \omega_0 = \frac{1}{\sqrt{LC}}$
Transfer function magnitude	$\lvert H(j\omega)\rvert = \frac{R/L}{\sqrt{\omega^2 + (R/L)^2}}$ $\lvert H(j\omega)\rvert = \frac{1/RC}{\sqrt{\omega^2 + (1/RC)^2}}$	$\lvert H(j\omega)\rvert = \frac{\omega}{\sqrt{\omega^2 + (1/RC)^2}}$		$\lvert H(j\omega)\rvert = \frac{\lvert (1/LC) - \omega^2 \rvert}{\sqrt{[(1/LC) - \omega^2]^2 + \left[\frac{\omega R}{L}\right]^2}}$ In series
			$\omega_{c1} = \omega_o \cdot \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$, $\omega_{c2} = \omega_o \cdot \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$.	$\omega_{c1} = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \left(\frac{1}{LC}\right)}$ $\omega_{c2} = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \left(\frac{1}{LC}\right)}$

- ***Active filter:***

	LPF	*HPF*	*Band – pass filter*	*Band – reject filter*
Circuit	 First order Low-pass filter		 The bandpass filter circuit	
Transfer function	$\boxed{H(s) = -K\frac{\omega_c}{s+\omega_c}}$	$\boxed{H(s) = -K\frac{s}{s+\omega_c}}$	$H(s) = \frac{-K\omega_{c2}s}{s^2 + \omega_{c2}s + \omega_{c1}\omega_{c2}}$	$H(s) = \frac{R_f}{R_i}\left(\frac{s^2+2\omega_{c1}s+\omega_{c1}\omega_{c2}}{(s+\omega_{c1})(s+\omega_{c2})}\right)$
Gain	$K = \frac{R_2}{R_1}$	$K = \frac{R_2}{R_1}$		
Cut off frequency	$\omega_c = \frac{1}{R_2 C}$	$\omega_c = \frac{1}{R_1 C}$	$\omega_{c1} = \frac{1}{R_L C_L}$ $\omega_{c2} = \frac{1}{R_H C_H}$	$\omega_{c1} = \frac{1}{R_L C_L}$ $\omega_{c2} = \frac{1}{R_H C_H}$

Scaling using prototype circuit:

Passive filter	*Active filter*
$R' = K_m R$ $C' = \frac{C}{K_m K_f}$ $L' = L \times \frac{K_m}{K_f}$ $\omega_C' = K_f \times \omega_C$	$R' = K_m R$ $C' = \frac{C}{K_m K_f}$ $\omega_C' = K_f \times \omega_C$

IV. HIGH ORDER ACTIVE FILTER:

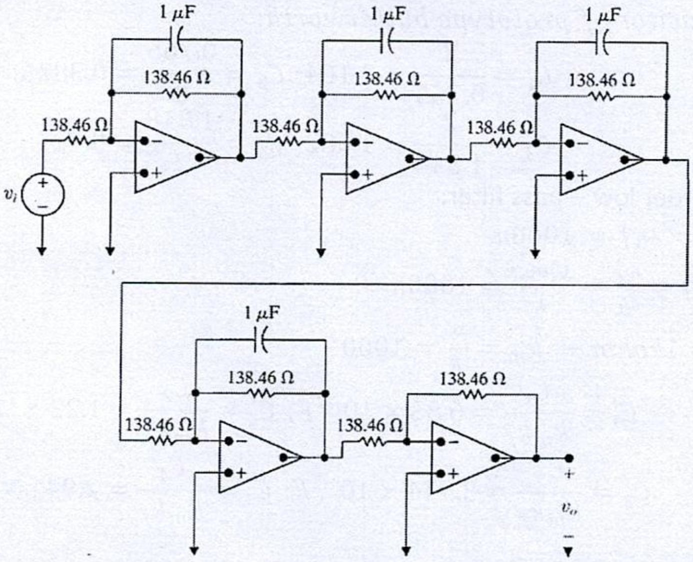
- ***Higher order op amp filter:***

$$H(s) = \left(\frac{-1}{s+1}\right)\left(\frac{-1}{s+1}\right)\cdots\left(\frac{-1}{s+1}\right) = \frac{(-1)^n}{(s+1)^n}$$

$$|H(j\omega_{cn})| = \left|\frac{1}{(j\omega_{cn}+1)^n}\right| = \frac{1}{\sqrt{2}},$$

$$\omega_{cn} = \sqrt{\sqrt[n]{2} - 1}.$$

Example fourth order low pass filter, gain = 10, Rf=1384.6



- ***Butterworth filter:*** *low – pass filter*

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_c)^{2n}}},$$

TABLE 15.1 Normalized (so that ω_c = 1 rad/s) Butterworth Polynomials up to the Eighth Order

n	nth-Order Butterworth Polynomial
1	$(s+1)$
2	$(s^2 + \sqrt{2}s + 1)$
3	$(s+1)(s^2+s+1)$
4	$(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)$
5	$(s+1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$
6	$(s^2 + 0.518s + 1)(s^2 + \sqrt{2} + 1)(s^2 + 1.932s + 1)$
7	$(s+1)(s^2 + 0.445s + 1)(s^2 + 1.247s + 1)(s^2 + 1.802s + 1)$
8	$(s^2 + 0.390s + 1)(s^2 + 1.111s + 1)(s^2 + 1.6663s + 1)(s^2 + 1.962s + 1)$

Prototype of Butterworth filter:

$$C_1 = \frac{2}{b};\ C_2 = \frac{1}{C_1}$$

b: coefficient at s

Example:

Design a fourth order low – pass butterworth filter:

Fc=500Hz, gain =10, R = 1k

➔ *Solution:*

4^{th} order: polynomial of butterworth filter:

From table: $(s^2 + 0.765s + 1) \times (s^2 + 1.848s + 1)$

$$\to b_1 = 0.765, \qquad b_2 = 1.848$$

Capacitor of prototype butterworth:

$$C_1 = \frac{2}{0.765} = 2.164;\ C_2 = \frac{0.765}{2} = 0.3825;$$

$$C_3 = \frac{2}{1.848} = 1.082;\ C_4 = \frac{1.848}{2} = 0.924.$$

4^{th} order low – pass filter:

$\omega_C{}' = 2\pi f = 1000\pi$

$\to k_f = \frac{\omega_C'}{\omega_C} = \frac{1000\pi}{1} = 1000\pi$

$R' = 1kohm \to k_m = \frac{R'}{R} = 1000$

$$C_1' = \frac{C_1}{k_m k_f} = 6.83 \times 10^{-7}F;\ C_2' = \frac{C_2}{k_m k_f} = 1.22 \times 10^{-7}F$$

$$C_3' = \frac{C_3}{k_m k_f} = 3.444 \times 10^{-7}F;\ C_4' = \frac{C_4}{k_m k_f} = 2.941 \times 10^{-7}F$$

Narrow band – pass and band – reject filter: review at slide and book

V. FOURIER SERIES:

$$f(t) = a_v + \sum_{1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{1}^{\infty} b_n \sin(n\omega_0 t)$$

	a_v	a_n	b_n
Fourier series	$\frac{1}{T} \times \int_{t0}^{t0+T} f(t)dt$	$\frac{2}{T} \times \int_{t0}^{t0+T} f(t)\cos(n\omega_0 t)\,dt$	$\frac{2}{T} \times \int_{t0}^{t0+T} f(t)\sin(n\omega_0 t)\,dt$
Even function	$\frac{2}{T} \times \int_{t0}^{t0+T/2} f(t)dt$	$\frac{4}{T} \times \int_{t0}^{t0+T/2} f(t)\cos(n\omega_0 t)\,dt$	0
Odd function	0	0	$\frac{4}{T} \times \int_{t0}^{t0+T/2} f(t)\sin(n\omega_0 t)\,dt$
Half +even	0	$\frac{4}{T} \times \int_{t0}^{t0+T/2} f(t)\cos(n\omega_0 t)\,dt$ n= 1, 3, 5, 7...	0
Half +odd	0	0	$\frac{4}{T} \times \int_{t0}^{t0+T/2} f(t)\sin(n\omega_0 t)\,dt$ n= 1, 3, 5, 7...
Quarter + even	0	$\frac{8}{T} \times \int_{t0}^{t0+T/4} f(t)\cos(n\omega_0 t)\,dt$ n= 1, 3, 5, 7...	0
Quarter + odd	0	0	$\frac{8}{T} \times \int_{t0}^{t0+T/4} f(t)\sin(n\omega_0 t)\,dt$ n= 1, 3, 5, 7...

Even: đối xứng qua Oy
Odd: đối xứng qua gốc O
Haft – wave: Đảo hàm số xuống âm và lặp lại
Quarter – wave: đối xứng qua trục vuông góc Ox tại vị trí T/4

- ***Trigonometric form:***

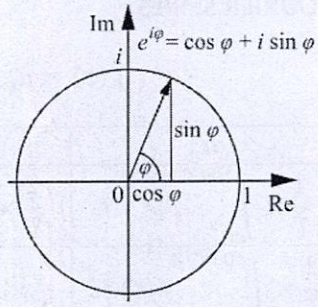
$$f(t) = a_v + \sum_{1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{1}^{\infty} b_n \sin(n\omega_0 t)$$

$$\rightarrow Trigonometric\ form\text{: } f(t) = a_v + \sum_{1}^{\infty} A_n \cos(n\omega_0 t - \theta_n)$$

- ***Exponential form:***

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$e^{j\beta x} = \cos\beta x + j\sin\beta x$
$e^{-j\beta x} = \cos\beta x - j\sin\beta x$



$z = a + jb = r(\cos\varphi + j\sin\varphi) = re^{j\varphi} = r\angle\varphi$

$$r = \sqrt{a^2 + b^2}$$

$$tg\varphi = \frac{b}{a}$$

$$\cos\varphi = \frac{a}{\sqrt{a^2+b^2}} \quad ; \quad \sin\varphi = \frac{b}{\sqrt{a^2+b^2}}$$

- ***Average Power:***

$$v(t) = V_{dc} + \sum_{1}^{\infty} V_n \cos(n\omega_0 t - \theta_{v_n})$$

$$i(t) = I_{dc} + \sum_{1}^{\infty} I_n \cos(n\omega_0 t - \theta_{i_n})$$

$$\rightarrow P_{av} = V_{dc} I_{dc} + \sum_{1}^{\frac{V_n I_n}{2}} \cos(\theta_{v_n} - \theta_{i_n})$$

$$P = \frac{V_{rms}^2}{R} = I_{rms}^2 R$$

- ***RMS value:***

$$F_{rms} = \sqrt{\frac{1}{T} \times \int_0^T f^2 dt}$$

$$F_{rms} = \sqrt{a_v^2 + \sum \left(\frac{A_n}{\sqrt{2}}\right)^2}$$

VI. TWO – PORT CIRCUIT:

$$V_1 = z_{11}I_1 + z_{12}I_2, \qquad V_2 = b_{11}V_1 - b_{12}I_1,$$
$$V_2 = z_{21}I_1 + z_{22}I_2; \qquad I_2 = b_{21}V_1 - b_{22}I_1;$$

$$I_1 = y_{11}V_1 + y_{12}V_2, \qquad V_1 = h_{11}I_1 + h_{12}V_2,$$
$$I_2 = y_{21}V_1 + y_{22}V_2; \qquad I_2 = h_{21}I_1 + h_{22}V_2;$$

$$V_1 = a_{11}V_2 - a_{12}I_2, \qquad I_1 = g_{11}V_1 + g_{12}I_2,$$
$$I_1 = a_{21}V_2 - a_{22}I_2; \qquad V_2 = g_{21}V_1 + g_{22}I_2.$$

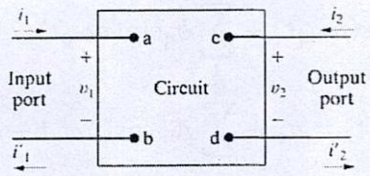


Figure 18.1 ▲ The two-port building block.

Unit: A/V → S
V/I → ohm
Others: none.

TABLE 18.1 Parameter Conversion Table

$$z_{11} = \frac{y_{22}}{\Delta y} = \frac{a_{11}}{a_{21}} = \frac{b_{22}}{b_{21}} = \frac{\Delta h}{h_{22}} = \frac{1}{g_{11}}$$

$$z_{12} = -\frac{y_{12}}{\Delta y} = \frac{\Delta a}{a_{21}} = \frac{1}{b_{21}} = \frac{h_{12}}{h_{22}} = -\frac{g_{12}}{g_{11}}$$

$$z_{21} = \frac{-y_{21}}{\Delta y} = \frac{1}{a_{21}} = \frac{\Delta b}{b_{21}} = -\frac{h_{21}}{h_{22}} = \frac{g_{21}}{g_{11}}$$

$$z_{22} = \frac{y_{11}}{\Delta y} = \frac{a_{22}}{a_{21}} = \frac{b_{11}}{b_{21}} = \frac{1}{h_{22}} = \frac{\Delta g}{g_{11}}$$

$$y_{11} = \frac{z_{22}}{\Delta z} = \frac{a_{22}}{a_{12}} = \frac{b_{11}}{b_{12}} = \frac{1}{h_{11}} = \frac{\Delta g}{g_{22}}$$

$$y_{12} = -\frac{z_{12}}{\Delta z} = -\frac{\Delta a}{a_{12}} = -\frac{1}{b_{12}} = -\frac{h_{12}}{h_{11}} = \frac{g_{12}}{g_{22}}$$ ✓

$$y_{21} = -\frac{z_{21}}{\Delta z} = -\frac{1}{a_{12}} = -\frac{\Delta b}{b_{12}} = \frac{h_{21}}{h_{11}} = -\frac{g_{21}}{g_{22}}$$

$$y_{22} = \frac{z_{11}}{\Delta z} = \frac{a_{11}}{a_{12}} = \frac{b_{22}}{b_{12}} = \frac{\Delta h}{h_{11}} = \frac{1}{g_{22}}$$ ✓

$$a_{11} = \frac{z_{11}}{z_{21}} = -\frac{y_{22}}{y_{21}} = \frac{b_{22}}{\Delta b} = -\frac{\Delta h}{h_{21}} = \frac{1}{g_{21}}$$

$$a_{12} = \frac{\Delta z}{z_{21}} = -\frac{1}{y_{21}} = \frac{b_{12}}{\Delta b} = -\frac{h_{11}}{h_{21}} = \frac{g_{22}}{g_{21}}$$

$$a_{21} = \frac{1}{z_{21}} = -\frac{\Delta y}{y_{21}} = \frac{b_{21}}{\Delta b} = -\frac{h_{22}}{h_{21}} = \frac{g_{11}}{g_{21}}$$

$$a_{22} = \frac{z_{22}}{z_{21}} = -\frac{y_{11}}{y_{21}} = \frac{b_{11}}{\Delta b} = -\frac{1}{h_{21}} = \frac{\Delta g}{g_{21}}$$

$$b_{11} = \frac{z_{22}}{z_{12}} = -\frac{y_{11}}{y_{12}} = \frac{a_{22}}{\Delta a} = \frac{1}{h_{12}} = -\frac{\Delta g}{g_{12}}$$

$$b_{12} = \frac{\Delta z}{z_{12}} = -\frac{1}{y_{12}} = \frac{a_{12}}{\Delta a} = \frac{h_{11}}{h_{12}} = -\frac{g_{22}}{g_{12}}$$

$$b_{21} = \frac{1}{z_{12}} = -\frac{\Delta y}{y_{12}} = \frac{a_{21}}{\Delta a} = \frac{h_{22}}{h_{12}} = -\frac{g_{11}}{g_{12}}$$

$$b_{22} = \frac{z_{11}}{z_{12}} = \frac{y_{22}}{y_{12}} = \frac{a_{11}}{\Delta a} = \frac{\Delta h}{h_{12}} = -\frac{1}{g_{12}}$$

$$h_{11} = \frac{\Delta z}{z_{22}} = \frac{1}{y_{11}} = \frac{a_{12}}{a_{22}} = \frac{b_{12}}{b_{11}} = \frac{g_{22}}{\Delta g}$$

$$h_{12} = \frac{z_{12}}{z_{22}} = -\frac{y_{12}}{y_{11}} = \frac{\Delta a}{a_{22}} = \frac{1}{b_{11}} = -\frac{g_{12}}{\Delta g}$$

$$h_{21} = -\frac{z_{21}}{z_{22}} = \frac{y_{21}}{y_{11}} = -\frac{1}{a_{22}} = -\frac{\Delta b}{b_{11}} = -\frac{g_{21}}{\Delta g}$$

$$h_{22} = \frac{1}{z_{22}} = \frac{\Delta y}{y_{11}} = \frac{a_{21}}{a_{22}} = \frac{b_{21}}{b_{11}} = \frac{g_{11}}{\Delta g}$$

$$g_{11} = \frac{1}{z_{11}} = \frac{\Delta y}{y_{22}} = \frac{a_{21}}{a_{11}} = \frac{b_{21}}{b_{22}} = \frac{h_{22}}{\Delta h}$$

$$g_{12} = -\frac{z_{12}}{z_{11}} = \frac{y_{12}}{y_{22}} = -\frac{\Delta a}{a_{11}} = -\frac{1}{b_{22}} = -\frac{h_{12}}{\Delta h}$$

$$g_{21} = \frac{z_{21}}{z_{11}} = -\frac{y_{21}}{y_{22}} = \frac{1}{a_{11}} = \frac{\Delta b}{b_{22}} = -\frac{h_{21}}{\Delta h}$$

$$g_{22} = \frac{\Delta z}{z_{11}} = \frac{1}{y_{22}} = \frac{a_{12}}{a_{11}} = \frac{b_{12}}{b_{22}} = \frac{h_{11}}{\Delta h}$$

$$\Delta z = z_{11}z_{22} - z_{12}z_{21}$$

$$\Delta y = y_{11}y_{22} - y_{12}y_{21}$$

$$\Delta a = a_{11}a_{22} - a_{12}a_{21}$$

$$\Delta b = b_{11}b_{22} - b_{12}b_{21}$$

$$\Delta h = h_{11}h_{22} - h_{12}h_{21}$$

$$\Delta g = g_{11}g_{22} - g_{12}g_{21}$$

Symmetric: $z_{11} = z_{22}$ → compute 3 parameters

Reciprocal: $z_{12} = z_{21}$ → compute 2 parameters

Symmetric + Reciprocal: $z_{11} = z_{22}$; $z_{12} = z_{21}$ → compute 2 parameters

TABLE 18.2 Terminated Two-Port Equations

***z* Parameters**

$$Z_{\text{in}} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$$

$$I_2 = \frac{-z_{21}V_g}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$$

$$V_{\text{Th}} = \frac{z_{21}}{z_{11} + Z_g}V_g$$

$$Z_{\text{Th}} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g}$$

$$\frac{I_2}{I_1} = \frac{-z_{21}}{z_{22} + Z_L}$$

$$\frac{V_2}{V_1} = \frac{z_{21}Z_L}{z_{11}Z_L + \Delta z}$$

$$\frac{V_2}{V_g} = \frac{z_{21}Z_L}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$$

***y* Parameters**

$$Y_{\text{in}} = y_{11} - \frac{y_{12}y_{21}Z_L}{1 + y_{22}Z_L}$$

$$I_2 = \frac{y_{21}V_g}{1 + y_{22}Z_L + y_{11}Z_g + \Delta y Z_g Z_L}$$

$$V_{\text{Th}} = \frac{-y_{21}V_g}{y_{22} + \Delta y Z_g}$$

$$Z_{\text{Th}} = \frac{1 + y_{11}Z_g}{y_{22} + \Delta y Z_g}$$

$$\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$$

$$\frac{V_2}{V_1} = \frac{-y_{21}Z_L}{1 + y_{22}Z_L}$$

$$\frac{V_2}{V_g} = \frac{y_{21}Z_L}{y_{12}y_{21}Z_g Z_L - (1 + y_{11}Z_g)(1 + y_{22}Z_L)}$$

***a* Parameters**

$$Z_{\text{in}} = \frac{a_{11}Z_L + a_{12}}{a_{21}Z_L + a_{22}}$$

$$I_2 = \frac{-V_g}{a_{11}Z_L + a_{12} + a_{21}Z_g Z_L + a_{22}Z_g}$$

$$V_{\text{Th}} = \frac{V_g}{a_{11} + a_{21}Z_g}$$

$$Z_{\text{Th}} = \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g}$$

$$\frac{I_2}{I_1} = \frac{-1}{a_{21}Z_L + a_{22}}$$

$$\frac{V_2}{V_1} = \frac{Z_L}{a_{11}Z_L + a_{12}}$$

$$\frac{V_2}{V_g} = \frac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

***b* Parameters**

$$Z_{\text{in}} = \frac{b_{22}Z_L + b_{12}}{b_{21}Z_L + b_{11}}$$

$$I_2 = \frac{-V_g \Delta b}{b_{11}Z_g + b_{21}Z_g Z_L + b_{22}Z_L + b_{12}}$$

$$V_{\text{Th}} = \frac{V_g \Delta b}{b_{22} + b_{21}Z_g}$$

$$Z_{\text{Th}} = \frac{b_{11}Z_g + b_{12}}{b_{21}Z_g + b_{22}}$$

$$\frac{I_2}{I_1} = \frac{-\Delta b}{b_{11} + b_{21}Z_L}$$

$$\frac{V_2}{V_1} = \frac{\Delta b Z_L}{b_{12} + b_{22}Z_L}$$

$$\frac{V_2}{V_g} = \frac{\Delta b Z_L}{b_{12} + b_{11}Z_g + b_{22}Z_L + b_{21}Z_g Z_L}$$

h Parameters

$$Z_{in} = h_{11} - \frac{h_{12}h_{21}Z_L}{1 + h_{22}Z_L}$$

$$I_2 = \frac{h_{21}V_g}{(1 + h_{22}Z_L)(h_{11} + Z_g) - h_{12}h_{21}Z_L}$$

$$V_{Th} = \frac{-h_{21}V_g}{h_{22}Z_g + \Delta h}$$

$$Z_{Th} = \frac{Z_g + h_{11}}{h_{22}Z_g + \Delta h}$$

$$\frac{I_2}{I_1} = \frac{h_{21}}{1 + h_{22}Z_L}$$

$$\frac{V_2}{V_1} = \frac{-h_{21}Z_L}{\Delta h Z_L + h_{11}}$$

$$\frac{V_2}{V_g} = \frac{-h_{21}Z_L}{(h_{11} + Z_g)(1 + h_{22}Z_L) - h_{12}h_{21}Z_L}$$

g Parameters

$$Y_{in} = g_{11} - \frac{g_{12}g_{21}}{g_{22} + Z_L}$$

$$I_2 = \frac{-g_{21}V_g}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$$

$$V_{Th} = \frac{g_{21}V_g}{1 + g_{11}Z_g}$$

$$Z_{Th} = g_{22} - \frac{g_{12}g_{21}Z_g}{1 + g_{11}Z_g}$$

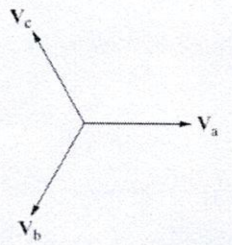
$$\frac{I_2}{I_1} = \frac{-g_{21}}{g_{11}Z_L + \Delta g}$$

$$\frac{V_2}{V_1} = \frac{g_{21}Z_L}{g_{22} + Z_L}$$

$$\frac{V_2}{V_g} = \frac{g_{21}Z_L}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$$

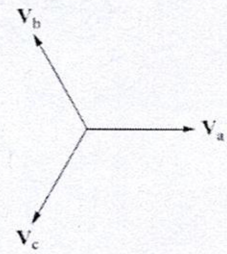
VII. BALANCE THREE PHASE CIRCUIT: *example, Y – delta circuit

abc (positive) phase sequence



$$V_a = V_m\angle 0^\circ$$
$$V_b = V_m\angle -120^\circ$$
$$V_c = V_m\angle +120^\circ$$

acb (negative) phase sequence

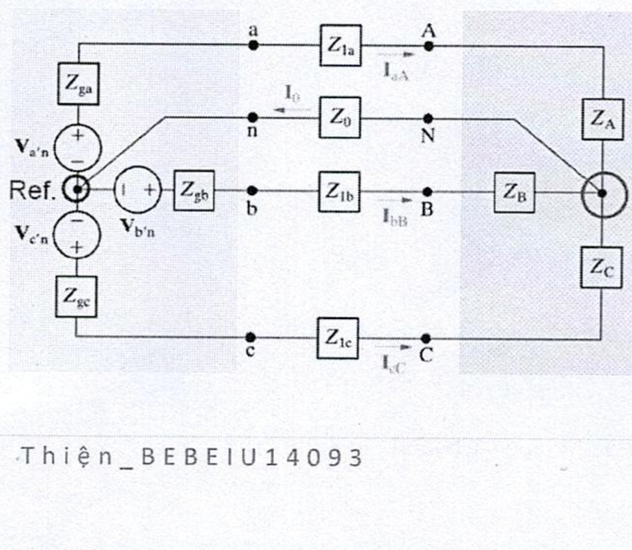


$$V_a = V_m\angle 0^\circ$$
$$V_b = V_m\angle +120^\circ$$
$$V_c = V_m\angle -120^\circ$$

An important characteristic of a set of balanced three-phase voltages is that the sum of the voltages is zero $V_a + V_b + V_c = 0$

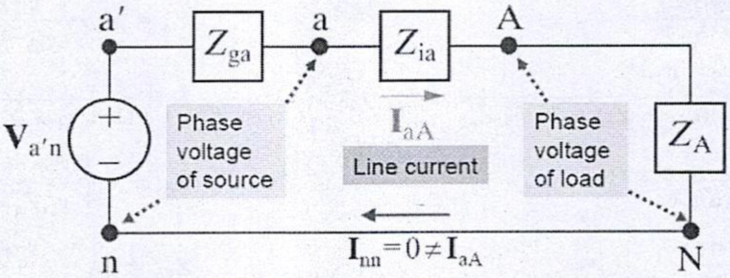
- ***Y – Y circuit:***

General Y-Y circuit model



The only essential node.

Equivalent one – phase circuit:



Load balance V_N=0

$$I_{aA} = \frac{V_{a'n} - V_N}{Z_A + Z_{1a} + Z_{ga}} = \frac{V_{a'n}}{Z_\phi}$$

$$I_{bB} = \frac{V_{b'n} - V_N}{Z_B + Z_{1b} + Z_{gb}} = \frac{V_{b'n}}{Z_\phi}$$

$$I_{cC} = \frac{V_{c'n} - V_N}{Z_C + Z_{1c} + Z_{gc}} = \frac{V_{c'n}}{Z_\phi}$$