Chúc mấy bạn 100 đ ><

1. Intro to Pro3

Sample space $\Omega \to n(\Omega)$

Event $A, B \to$ Let A is the event ...

Notation
- . complefement : $A', \bar{A}, A^c$
- . intersection : $A \cap B$, AB
- . union : $A \cup B$
- . mutually exclusive $A \cap B = \emptyset$ (xung khắc)

2. Counting

- Axiom of Pro3 : $0 \leq p(\text{event}) \leq 1$

$$\begin{cases} p(\Omega) = 1 \\ p(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + \ldots + P(A_n) \\ \Rightarrow p(A) = \sum_{s_i \in A} p(s_i) \end{cases}$$

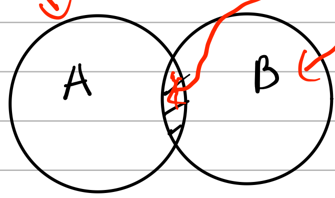
$$\Rightarrow p(A) = \frac{n(A)}{n(\Omega)}$$

Notation : $n(A)$; $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

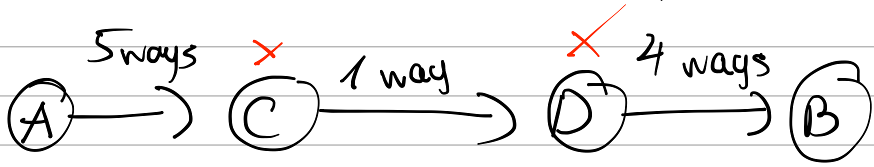
If it is mutually exclusive

$n(A \cup B) = n(A) + n(B)$

Why ?



PP nhân : consider a road $A \to B$



$5 \times 1 \times 4 = 20$ ways

Chỉnh hợp : $P(n, r) = n(n-1)\ldots(n-r+1) \; (r \leq n)$

Position C Chuỗi Tạo

When the order matter, it is Permutation

Tổ hợp : $C(n, r) = \dfrac{n!}{r!(n-r)!} = \dfrac{P(n,r)}{r!}$

Properties

- $P(A^c) = 1 - P(A)$
- $P(A \cup B) = P(A) + P(B) - P(AB)$
- $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
- $P(A \setminus B) = P(A) - P(AB)$
- $\begin{cases} P(\overline{AB}) = P(\bar{A} + \bar{B}) \\ P(\overline{A \cup B}) = P(\bar{A} . \bar{B}) \end{cases}$

3. Compute

$$P(A) = \frac{n(A)}{n(\Omega)} \qquad 0 \le P(A) \le 1$$

Conditional pro3

$$P(A|B) = \frac{P(AB)}{P(B)}$$

$$P(A \cup B|C) = P(A|C) + P(B|C) - P(AB|C)$$

Normally : $P(AB) = P(A) . P(B|A)$

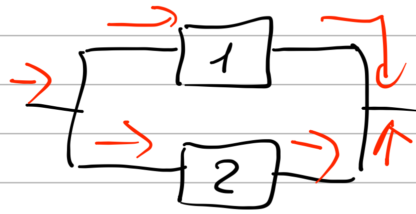
when A and B is independent

$$P(AB) = P(A) . P(B)$$

Điện trở

Nối tiếp

$p(up) = P_1 . P_2$

// 

$p(up) = 1 - p(down)$

$p(down) = (1-p_1)(1-p_2)$

Bayes theorem

$$P(A|B) = \frac{P(B|A) . P(A)}{P(B)}$$

A: 1% COVID → 95% positive (B), 5% not

99% NOT COVID → 2% positive, 98% not

$P(\text{COVID}|\text{positive}) = ?$ (A, B)

$$\frac{1\% . 95\%}{95\% . 1\% + 2\% . 99\%}$$

4. RV

discrete : finite | countable $x = 0, 1, 2, \dots$

continuos : infinite | uncountable $x > 1$

& pmf

dis.

x	x_i	x_2 ...
$p(x=x)$	≥ 0	≥ 0

1

cdf

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} p(x_i)$$

$$\lim_{x \to -\infty} F(x) = 0 \qquad \lim_{x \to +\infty} F(x) = 1$$

pdf

cont

$$P(a \leq x \leq b) = \int_a^b f(x)\,dx$$

$$\begin{cases} f(x) \geq 0 \ \forall x \\ \int_{-\infty}^{\infty} f(x) = 1 \end{cases}$$

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt$$

$$F'(x) = f(x), \ \forall x$$

$$P(a \leq x \leq b) = F(b) - F(a)$$

5. Mean Cov - Var

Notation :

	discreate	continuos
$E(x)$ (μ)	$\sum_x x_i\, p(x_0)$	$\int_x x\, p(x)\,dx$
$var(x)$ (σ^2)	$E(x^2) - [E(x)]^2$	

$E(x^2) = x^2 . p(x)$

linear : $E(ax+b) = a.E(x) + b$ / $E(c) = c$

$E(XY) = E(X).E(Y)$

$sd = \sqrt{var(x)}$

$var(ax+b) = a^2\, var(X)$

$var(c) = 0$

$var(x+c) = var(x)$

$var(x+y) = var(x) + var(y)$

2RV

$$E = \begin{cases} \sum_{-\infty}^{\infty} g(x,y)\, p_{x,y}(x,y) \\ \int_{-\infty}^{\infty}\int g(x,y)\, P_{x,y}(x,y) \end{cases}$$

6. Special RV

Bernouli : $X \sim Ber(p)$ pmf $\begin{cases} P(X=0) = 1-p \\ P(X=1) = p \end{cases}$

$E(X) = p$; $Var(X) = p(1-p)$

Geometric : $X \sim Geom(p)$ $P(X=i) = (1-p)^{i-1} p$ $i \geq 1$

$E(X) = 1/p$; $var(X) = \frac{1-p}{p^2}$

Binomial : $X \sim Bino(n,p)$ $p(X=i) = \binom{n}{i} p^i (1-p)^{n-i}$

$0 \leq i \leq n$

$E(X) = np$; $var(X) = np(1-p)$

Normal

$X \sim \mathcal{N}(\mu, \sigma^2)$

$P(\underline{X} > 10)$

$\downarrow z$

$E(X) = \mu$; $var(X) = \sigma^2$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e$$

Special $X \sim \mathcal{N}(0,1)$ pdf $= \frac{1}{2\sqrt{\pi}} e^{\frac{-x^2}{2}}$

$$\boxed{z = \frac{x - \mu}{\sigma}}$$

cdf $\phi = P(z \leq x)$

$\boxed{P(X > a)} = P(Z \leq Z_0) = 1 - \underline{P(Z \geq Z_0)}$

$P(\underline{a} \leq X \leq \underline{b}) = P(Z - Z_2) - P(Z - Z_1)$

1. *(10 points)* Flip a fair coin 4 times. Find the probability that the number of heads is greater than or equal to the number of tails.

$|\Omega| = 2^4 = 16$ $\qquad H \geq 2.$

$n(A) = \frac{11}{16}$

HHTT $\qquad$ HHHT $\qquad$ HHHH

$|\Omega_A| = \binom{2}{4} + \binom{3}{4} + \binom{4}{4} = 11$

2. *(10 points)* The percentages of people with each of the four blood types (O, A, B, and AB) in a region are as follows:

Blood type	A	B	AB	O
Percentage	30	12	3	55

Select randomly a person in this region. Given that his/her blood type is either B or AB, find the (conditional) probability that his/her blood type is B.

$\Rightarrow P(A|B) = \frac{12}{15}$

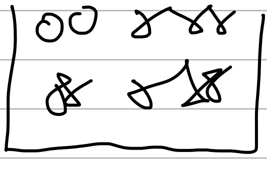
Gọi B là đk cho trc là nhóm máu B or AB

$|\Omega| = B + AB = 15$

A là ... B

$|\Omega_B| = 12$

(10 points) In a box, there is 2 blue balls and 18 green balls. Select randomly without replacement two balls from the box. What is the probability that the second ball selected is blue.



$\times \quad \times$

Blue

$GB = \frac{18}{20} \cdot \frac{2}{19}$

$BB = \frac{2}{20} \cdot \frac{1}{19}$

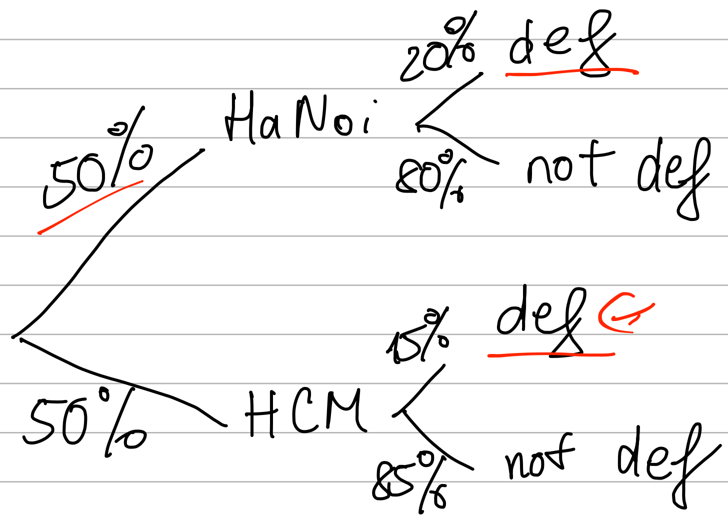
$P(A) = \frac{18}{20} \cdot \frac{2}{19} + \frac{2}{20} \cdot \frac{1}{19} = \frac{1}{10}$

$GB = \frac{18}{20} \cdot \frac{2}{20}$

$BB = \frac{2}{20} \cdot \frac{2}{20}$

$P(A) = GB + BB$

(10 points) A company has two stores of TV, one is located in Hanoi and another is in HCM city. At the store in Hanoi, 20% of TV are defective. The percentage of TV which are defective in HCM city is 15%. Choose randomly a store and from this store select randomly a TV. The selected TV is tested and found to be defective, what is the probability that it comes from the store in Hanoi.



$$P(\underset{A}{\text{Hanoi}} \mid \underset{B}{\text{def}}).$$

$$= \frac{50\% \cdot 20\%}{50\% \cdot 20\% + 50\% \cdot 15\%}$$

5. *(20 points)* The probability function of a discrete random variable X has the form

$$p(x) = P(X = x) = c(x^2 + 3|x| + 1), \quad \text{for } x = -2, -1, 0, 1, 2.$$

(a) Find c.

(b) Compute $P(-2 \le X < 1)$.

(c) Evaluate $E(X)$ and $Var(X)$.

$$\left.\begin{aligned} P(X=-2) &= 11c \\ P(X=-1) &= 5c \\ P(X=0) &= c \\ P(X=1) &= 5c \\ P(X=2) &= 11c \end{aligned}\right\} = 1$$

a) $33c = 1 \Rightarrow c = \frac{1}{33}$

b) $\underbrace{P(-2 \le X < 1)}_{17c \,=\, \frac{17}{33}} = \underset{11c}{P(X=-2)} + \underset{5c}{P(X=-1)} + \underset{c}{P(X=0)}.$

c) $E(X)$ $Var(X)$

$$E(X) = \sum x_i\, p(x_i) = (-2).p(-2) + (-1)p(-1) + 0.p(0) + 1.p(1) + 2p(2)$$

$$= 0$$

$$Var(X) = \underset{98c}{E(x^2)} - \underset{0^2}{[E(x)]^2} = 98c = \frac{98}{33}$$

$$E(x^2) = \Sigma x_i^2 p(x_i) = (-2)^2 p(-2) + \ldots 2^2 . p(2)$$
$$= 98c$$

6. *(20 points)* The borrowing period, in days, for a particular book at a University library can be regarded as random variable X which has normal distribution with mean $\mu = 8$ and standard deviation $\sigma = 2$. A book need to be return within 10 days.

   (a) Compute $P(X > 10)$ - the probability that a new borrower returns the book after 10 days.

   (b) For a late return, the borrower has to pay a penalty of \$5. Otherwise, the borrower pays \$0. Evaluate the average payment of a borrower.

$$z = \left[\frac{X - \mu}{\sigma}\right] = \frac{10 - 8}{2} = 1$$

a) $P(X > 10) = P(Z > 1) = 1 - P(Z < 1)$
$= 1 - 0,8413$
$= 0,1587$

b)
$P(X>10)$

$E(x) = 0,1587 . 5 + 0,8413 . 0$
$= 0,7935$ \$

$P(X<10) . 0$

7. *(10 points)* Jack has invested \$1000 in product A and \$2000 in product B. He expects that if project A is success, he get a profit of \$800 and lose his money that he invested in A if A is unsuccessful. For project B, a successfull investement yields a profit of \$1000 and a uncessesfull of B makes him lose his money invested in B. He estimates the probability of success as following

		Project B	
		successful	unsuccessful
Project A	successful	0.6	0.05
	unsuccessful	0.25	0.1

Let X and Y be the his profit from project A and B respectively. Remark that X and Y can take negative value when he loses his money.

   (a) Determine the probability mass functions of X and Y.

   (b) Calculate $E(X)$ and $E(Y)$ - the profits each project.

   (c) Compute $E(X + Y)$ - the average of the overall profit from two projects.

a)

X	800	-1000
$P(X)$	0,65	0,35
Y	1000	-2000
$P(Y)$	0,85	0,15

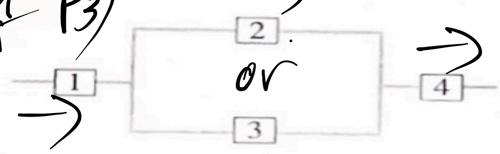
b)
$E(x) = 0,65 . 800 + 0,35 . (-1000) = 170$ \$

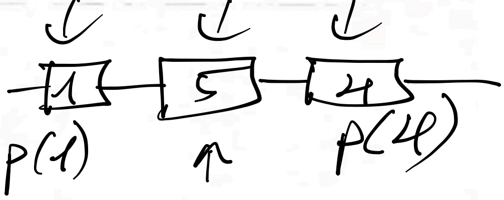
$E(Y) = 0,85 . 1000 + 0,15 . (-2000) = 550$ \$

c) $E(X+Y) = E(X) + E(Y) = 170 + 550$
$= 720 \$$

8. *(10 points)* Consider a system of 4 components with structure as following

$p(5) = 1 - (1-p_2)\cdot(1-p_3)$





Suppose that all four components operate independently. Let T_1, T_2, T_3 and T_4 are lifetime or time to failure (in years) of the component 1, 2, 3, 4 respectively. Their probability density functions are given by contious,

$$f_{T_1}(x) = \begin{cases} 0.1e^{-0.1x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}, \quad f_{T_2}(x) = \begin{cases} 0.2e^{-0.2x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$f_{T_3}(x) = \begin{cases} 0.2e^{-0.2x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}, \quad f_{T_4}(x) = \begin{cases} 0.1e^{-0.1x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Compute $p_1 = P(T_1 > 1)$ - the probability that the component 1 lasts more than 1 year.

(b) Evaluate the probability that the system lasts more than 1 year.

a) $P(T_1 > 1) = \int_1^{\infty} 0{,}1 \cdot e^{-0{,}1x}\, dx$ → Bấm máy

$= \lim_{a \to \infty} \int_1^{a} 0{,}1 \cdot e^{-0{,}1x}\, dx$

$= \lim_{a \to +\infty} \left(-\frac{1}{e^{0{,}1a}} + \frac{\sqrt[10]{e^9}}{e} \right)$

$= \frac{\sqrt[10]{e^9}}{e} \approx 0{,}905.$

∞ lấy 10000.

b) $P(\text{hệ system} > 1) = P_1 \cdot [1 - (1-p_2)(1-p_3)] p_4$

$p_2(T_2 > 1) = \int_1^{\infty} 0{,}2e^{-0{,}2x}\, dx$

$= 0{,}818$

$P(\;) = \underbrace{0{,}905\,[1 - (1-0{,}818)(1-0{,}818)]\,0{,}905}_{0{,}791}$