HO CHI MINH CITY INTERNATIONAL UNIVERSITY

MIDTERM EXAMINATION

Academic year 2014-2015, Semester 1
Duration: 90 minutes

SUBJECT: **Differential Equations**	
Chair of Department of Mathematics	Lecturer:
Signature:	Signature:
Full name: Associate Prof. Nguyen Dinh	Full name: Associate Prof. Pham Huu Anh Ngoc

Instructions:

- *Each student is allowed a scientific calculator and a maximum of two double-sided sheets of reference material (size A4 or similar), stapled together and marked with their name and ID. All other documents and electronic devices are forbidden..*

Question 1. a) *(15 marks)* The population of a community is known to increase at a rate proportional to the number of people present at time t. If an initial population P_0 has doubled in 5 years, how long will it take to quadruple?

b) *(15 marks)* Show that the differential equation

$$(1+\ln x+\frac{y}{x})dx-(1-\ln x)dy=0,$$

is exact. Solve the given differential equation.

Question 2. *(20 marks)* Solve the initial value problem

$$(\cos y)\frac{dy}{dx}-\sin y=x, \qquad y(0)=0.$$

Question 3. *(20 marks)* Find the general solution of the following differential equation

$$y''-4y'+3y=2e^{x}(1+2x)+x.$$

Question 4. *(15 marks)* a) Find $\alpha>0$ such that $y(x)=x^{\alpha}$ is a solution of the differential equation

$$x^2y''-4xy'+6y=0, \quad x\in(0,\infty).$$

Solve the given differential equation.

b) *(15 marks)* Find the general solution of the differential equation

$$x^2y''-4xy'+6y=\ln x, \quad x\in(0,\infty).$$

END

SOLUTIONS:

Question 1. a) 10 years.
b) Note that

$$(1 + \ln x + \frac{y}{x})dx - (1 - \ln x)dy = dx + \ln x dx + yd\ln x - dy + \ln x dy$$

$$= d(x + y\ln x - y + x\ln x - x) = 0.$$

Thus the given equation is exact and the general solution is given by

$$y\ln x - y + x\ln x = C.$$

Question 2. Let $z = \sin y$. Then the given equation becomes

$$z' - z = x.$$

The general solution of the latter equation is

$$z(x) = -1 - x + ce^x.$$

Thus $\sin y = -1 - x + ce^x$. Since $y(0) = 0$, it follows that $c = 1$. Thus the particular solution is $y = \arcsin(-1 - x + e^x)$.

Question 3. The general solution of the corresponding homogeneous equation is

$$y(x) = c_1e^x + c_2e^{3x}.$$

A particular solution of the nonhomogeneous equation is $y_p(x) = -(1 + x)^2e^x + \frac{1}{3}x + \frac{4}{9}$. Thus the general solution of the nonhomogeneous equation is given by

$$y(x) = c_1e^x + c_2e^{3x} - (1 + x)^2e^x + \frac{1}{3}x + \frac{4}{9}.$$

Question 4. a) It is easy to show that $\alpha = 2$ or $\alpha = 3$. The general solution of the homogeneous equation is given by

$$y(x) = c_1x^2 + c_2x^3, \quad x > 0.$$

b) A particular solution of the nonhomogeneous equation is $y_p(x) := \frac{1}{6}\ln x + \frac{5}{36}$.
The general solution is

$$y(x) = c_1x^2 + c_2x^3 + \frac{1}{6}\ln x + \frac{5}{36}, \quad x > 0.$$