HO CHI MINH CITY INTERNATIONAL UNIVERSITY

MIDTERM EXAMINATION

Academic year 2012-2013, Semester 2
Duration: 90 minutes

SUBJECT: **Differential Equations**	
Acting Chair of Department of Mathematics	Lecturer:
Signature:	Signature:
Associate Prof. Nguyen Dinh	Associate Prof. Pham Huu Anh Ngoc

Instructions:

- *Each student is allowed a scientific calculator and a maximum of two double-sided sheets of reference material (size A4 or similar), stapled together and marked with their name and ID. All other documents and electronic devices are forbidden.*

Question 1. a) *(10 marks)* Show that the differential equation

$$(e^{2y} - y\cos(xy))dx + (2xe^{2y} - x\cos(xy) + 2y)dy = 0,$$

is exact.

b) *(15 marks)* Solve the equation given in a).

Question 2. *(25 marks) Find the solution to the initial value problem*

$$x(x+1)y' + xy = 1, \quad x \in (0, \infty); \qquad y(e) = 1.$$

Question 3. a) *(10 marks)* Solve the differential equation

$$y'' - 3y' = 0.$$

b) *(15 marks)* Find the general solution of the differential equation

$$y'' - 3y' = 8e^{3x} + 4\sin x.$$

Question 4. *(10 marks)* a) Show that $y_1(x) = x$ is a solution of the differential equation

$$x^2y'' - 3xy' + 3y = 0.$$

b) *(15 marks)* Solve the differential equation

$$x^2y'' - 3xy' + 3y = 2x^4e^x.$$

END.

SOLUTIONS:

Question 1. a) Let

$$M(x,y) = e^{2y} - y\cos(xy); \qquad N(x,y) = 2xe^{2y} - x\cos(xy) + 2y.$$

Thus the given equation is exact.

$$\frac{\partial M}{\partial y}(x,y) = \frac{\partial N}{\partial x}(x,y) = 2e^{2y} - cos(xy) + xy\sin(xy).$$

b) The general solution is given by

$$xe^{2y} - \sin(xy) + y^2 = C.$$

Question 2. The given equation is written as

$$y' + \frac{1}{x+1}y = \frac{1}{x(x+1)}, \; x > 0.$$

The integrating factor is given by $I(x) = x+1$. Thus, we get

$$(x+1)y' + y = \frac{1}{x}, \; x > 0.$$

This gives

$$\frac{d}{dx}[y(1+x)] = \ln x + C, \; x > 0.$$

Therefore, the general solution is

$$y(x) = \frac{\ln x + C}{x+1}.$$

Since $y(e) = 1$, the particular solution is $y(x) = \frac{\ln x + e}{x+1}$.

Question 3.

a) The general solution of the homogeneous equation is

$$y(x) = c_1 + c_2 e^{3x}.$$

b) A particular solution of $y'' - 3y' = 8e^{3x} + 4\sin x$ is $y_p(x) = (8/3)e^{3x}x - (2/5)\sin x + (6/5)\cos x$. Thus the general solution of the equation $y'' - 3y' = 8e^{3x} + 4\sin x$, is given by

$$y(x) = (8/3)e^{3x}x - (2/5)\sin x + (6/5)\cos x + c_1 + c_2 e^{3x}.$$

Question 4. a) So easy.

b) The general solution is given by

$$y(x) = 2e^x(x^2 - x) + c_1 x^3 + c_2 x.$$