- **Complex number:**

$$j^2 = -1$$

$$n = \underset{\text{real}}{a} + j\underset{\text{Imaginary}}{b}$$

$$n = c e^{j\theta} = c \angle \theta^\circ$$

c: amplitude (radius)

θ: phase angle $= \tan^{-1} \frac{b}{a}$

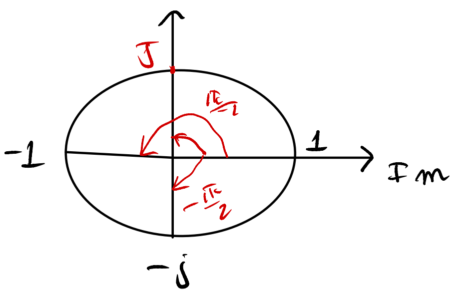
Euler's Formula: $e^{\pm j\theta} = \cos\theta \pm \sin\theta$

$$n = 5e^{j\frac{\pi}{4}} = 5 \angle \frac{\pi}{4} = 5(\cos \pi/4 + j \sin \pi/4)$$
$$= \frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}$$

$$\underset{\cos 0 + j\sin 0}{1} \angle 0^\circ, \quad -1 = 1 \angle \pm 180^\circ, \quad j = 1 \angle 90^\circ$$

$\cos \frac{\pi}{2} + j \sin \frac{\pi}{2}$

$$-j = 1 \angle -90^\circ$$



$\left\{ \begin{array}{l} \text{Addition} \\ \text{Subtraction} \end{array} \right. \rightarrow$ Rectangular form

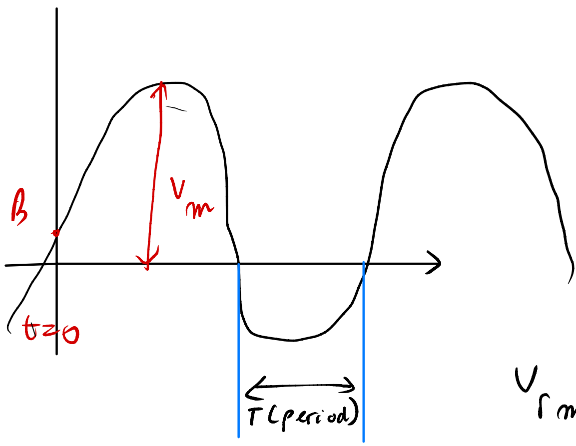
$\left\{ \begin{array}{l} \text{Multiplication}: n_1 n_2 = c_1 c_2 \angle (\theta_1 + \theta_2) \\ \text{Division}: \quad = \frac{c_1}{c_2} \angle (\theta_1 - \theta_2) \end{array} \right.$

$$\sqrt{ce^{j\theta}} = \sqrt{c}\,[e^{j\theta}]^{\frac{1}{2}} = \sqrt{c}\,e^{j\frac{\theta}{2}} = \sqrt{c}\angle\frac{\theta}{2}$$

Fourier Series:

$$\sum \text{Sine} + \text{Cose}$$

$$v = V_m \cos(\omega t + \phi)\ (V)$$



$$w = \frac{2\pi}{T} = 2\pi f$$

Angular fre

Radian fre

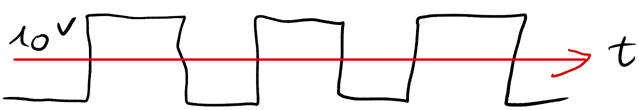
phase angle

$$B = V_m \cos\theta \Rightarrow \phi = \cos^{-1}\frac{B}{V_m}$$

V_{rms} (root mean square)

: Điện áp hiệu dụng

= 220V in VN $= \dfrac{V_m}{\sqrt{2}}$



Phase Lead or Lag :

$\theta_1 - \theta_2 = 0 \rightarrow v_1(t)$ and $v_2(t_2)$ are in phase

$\theta_1 - \theta_2 \neq 0 \rightarrow$ " out of phase

$$\begin{cases} \theta_1 - \theta_2 > 0, & v_1(t) \text{ leads } v_2(t) \text{ by } \theta_1 - \theta_2 \\ \theta_1 - \theta_2 < 0, & \text{" lags " by } \theta_1 - \theta_2 \end{cases}$$

$\sin wt = \cos(wt - 90°)$

$-\sin wt = \cos(wt + 90°)$

$f = \frac{1}{T} = 10^3 \rightarrow w = 2\pi f$

T ↓ cycle, period

Ex 1 : $i = I_m \cos(wt + \theta)$

$20A \quad \cos(10^3 \pi$

$v = 12 \sin(wt - 10°) = 12 \cos(wt - 10 - 90°)$

$= 12 \cos(wt - 100°)$

phasor:

Euler's : $e^{\pm j\theta} = \cos\theta \pm j \sin\theta$

Real   Imaginary

$v(t) = V_m \cos(wt + \theta)$ Time Domain

$= Re\{V_m e^{(wt+\theta)}\}$

$= Re\{\underline{V_m e^{j\theta}} e^{jwt}\}$

Phasor $\overline{V} = V_m e^{j\theta} = V_m \angle \cancel{\theta}$

$\overline{V} = P\{v(t)\} = V_m e^{j\theta} = V_m \angle \theta$ (phasor domain)

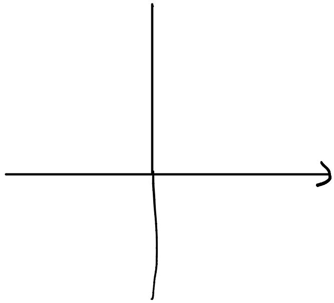
$i = 6\cos(100t + 45°)$

$\overline{I} = 6 \angle 45°$

$\overline{V} = 10 \angle 30°$

$v(t) = 10 \cos(200t + 30°)$

In time domain solving
(Complicated)



phasor / Frequency domain
Sol: $\bar{V}$, $\bar{I}$ → time domain

$$\bar{V} = \sqrt{j}\, 8 e^{-j20^\circ} = 8 e^{-j20} \; 1 e^{j\frac{90}{2}}$$

$$= 8 e^{j70}$$

$$= 8 \angle 70^\circ$$

$$\angle 45^\circ$$

Time Domain | Phasor Domain

$i = I_m \cos(\omega t + \theta_i)$ A $\longrightarrow$ $\bar{I} = I_m \angle \theta_i$

$v = Ri = R I_m \cos(\omega t + \theta_i) \longrightarrow \bar{V} = R I_m \angle \theta_i$

$$\Rightarrow \bar{V} = R\bar{I}, \quad \frac{\bar{V}}{\bar{I}} = R\angle 0^\circ$$

$$\to \angle\bar{V} - \angle\bar{I} = 0^\circ$$

For Inductor (H)

$$v_L = L\frac{di}{dt} = L\omega I_m \sin(\omega t + \theta_i)$$

$$= -L\omega I_m \cos(\omega t + \theta_i - 90^\circ)$$

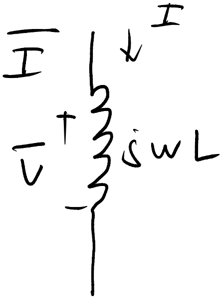
$$= Re\left\{-L\omega I_m e^{j(\omega t + \theta_i - 90^\circ)}\right\}$$

$$\to v_L = Re\left\{-L\omega I_m e^{j\theta_i} \underbrace{e^{-j90^\circ}}_{-j} e^{j\omega t}\right\}$$

$$= Re\left\{ j\omega L e^{j\theta_i} e^{j\omega t}\right\}$$

Inductor: phasor Domain

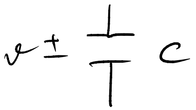
$$\mathrm{Re}\{ j\omega L e^{j\theta_i} e^{j\omega t} \} \rightarrow \bar{V} = j\omega L \bar{I}$$



$$\rightarrow \frac{\bar{V}}{\bar{I}} = j\omega L = \omega L \angle 90^\circ$$

$$\angle \bar{V} - \angle \bar{I} = 90^\circ$$

leads



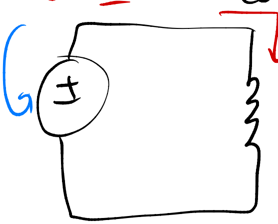
$$i = C\frac{dv}{dt} \rightarrow \bar{V} = \frac{1}{j\omega C}\bar{I}$$

$$\frac{\bar{V}}{\bar{I}} = \frac{1}{j\omega C} = \frac{j}{jj\omega C} = -j\frac{1}{\omega C} = \frac{1}{\omega C} \angle -90$$

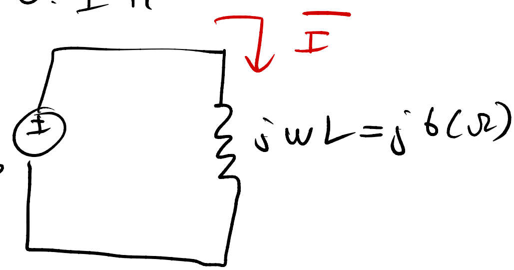
$$\angle \bar{V} - \angle \bar{I} = -90^\circ$$

lag

$v = 12\cos(60t + 45^\circ)$ 0.1 H



$\rightarrow$



$$\bar{V} = 12\angle 45^\circ$$

$$\bar{I} = \frac{\bar{V}}{j6} = \frac{12\angle 45^\circ}{6\angle 90^\circ} = 2\angle -45^\circ$$

$i(t) = 2\cos(60t - 45^\circ)$ Solution

$$j6 = 1\angle 90^\circ \cdot 6 = 6\angle 0 + 90^\circ$$

Impedance

$\bar{Z} = 5 + j6 \, (\Omega)$

Res

Reactance

$\bar{Y} = \frac{1}{\bar{Z}} = G + jB$

admittance

Conductance

Subceptance

$\bar{V} = \bar{Z}\,\bar{I}$

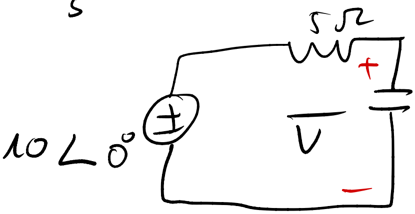
$j\omega L$, $\omega L > 0 \Rightarrow$ inductive

$\frac{1}{j\omega C} = \frac{-j}{\omega C} < 0 \Rightarrow$ capacitive

tính dung

$v_s = 10\cos 4t$

$\frac{j \cdot 1}{j\, j2.5} = \frac{-j}{2.5}$



$I \quad \bar{I}$

$\bar{Z}_c = \frac{1}{j\omega C} = \frac{1}{j4 \cdot 0.1} = \frac{1}{j2.5}$ (Ω)

c) Find $\bar{I}$ $\quad \bar{Z} = 5 + \frac{1}{j2.5}$

$$I = \frac{\bar{V}_s}{\bar{Z}} = \frac{10\angle 0°}{5 + 0.4\angle -90°} = 1.789\angle 26.57°$$

$$\sqrt{5^2 + 2.5} = 5.95$$

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$H = j\omega L \qquad C = \frac{1}{j\omega C}$