HO CHI MINH CITY INTERNATIONAL UNIVERSITY

FINAL EXAMINATION

Academic year 2012-2013, Semester 3

Duration: 120 minutes

SUBJECT:	**Differential Equations**
Chair of the Department of Mathematics Associate Professor Nguyen Dinh Signature:	Lecturer: Associate Professor Pham Huu Anh Ngoc Signature:

Instructions:

- *Each student is allowed a scientific calculator and a maximum of two double-sided sheets of reference material (size A4 or similar), stapled together and marked with their name and ID. All other documents and electronic devices are forbidden..*

Question 1. (20 marks)
Find $a, b \in \mathbb{R}$ such that $y(x) = a + b \ln x$ is a solution of

$$x^2 y'' - xy' + y = \ln x, \quad x \in (0, \infty).$$

Solve the differential equation.

Question 2. (15 marks)
(*Chemical reaction*) A compound C is formed when two chemicals A and B are combined. The resulting reaction between the two chemicals is such that for each gram of A, 4 grams of B are used. It is observed that 30 grams of the compound C is formed in 10 minutes. Determine the amount of C at time t if the rate of the reaction is proportional to the amounts of A and B remaining and if initially there are 50 grams of A and 32 grams of B. How much of the compound C is present at 20 minutes?

Question 3. a) (15 marks) *Determine the form* of a particular solution of

$$y''' - 4y'' + 4y' = x^2 - x + 2x^2 e^{2x} + 5e^{5x}.$$

b) (15 marks) Find the general solution of the following differential equation

$$y''' - 4y'' + 4y' = e^{2x} + 135e^{5x}.$$

Question 4. (15 marks) Solve the linear system of differential equations

$$\begin{cases} \frac{dx}{dt} &= x + 2y \\ \frac{dy}{dt} &= 4x + 3y. \end{cases}$$

Question 5. (20 marks) Solve the following differential equation

$$y^{(4)} + 4y'' = 5x - e^{2x}.$$

End.

SOLUTIONS:

Question 1.
We find $a = 2, b = 1$. Then the general solution of the corresponding homogeneous differential equation is

$$y(x) = c_1 x + c_2 x \ln x.$$

The general solution of the nonhomogeneous differential equation is

$$y(x) = c_1 x + c_2 x \ln x + \ln x + 2.$$

Question 2. Let $y(t)$ be the number of grams of C present at time t. Then we have

$$y'(t) = k(250 - y(t))(40 - y(t)),$$

where k is a constant. This gives

$$\frac{250 - y(t)}{40 - y(t)} = ce^{210kt}.$$

Since $y(0) = 0$ and $y(10) = 30$, we get $c = 25/4$ and $210k = (1/10)\ln(88/25) \simeq 0.1258$. Thus,

$$y(t) = 1000\frac{1 - e^{-0.1258t}}{25 - 4e^{-0.1258t}}.$$

Then we find $y(20) = 37.24$ grams.

Question 3.
a)

$$y_p = x(Ax^2 + Bx + C) + x^2(Dx^2 + Ex + F)e^{2x} + Ge^{5x}.$$

b)

$$y(x) = 3e^{5x} + (1/4)x^2e^{2x} + c_1 + c_2e^{2x} + c_3xe^{2x}.$$

Question 4.

The general solution of the given system is given by

$$x(t) = c_1e^{5t} + c_2e^{-t}$$

$$y(t) = 2c_1e^{5t} - c_2e^{-t}.$$

Question 5.

The general solution of the corresponding homogeneous equation is

$$y(x) = c_1 + c_2x + c_3\cos 2x + c_4\sin 2x.$$

The general solution of the nonhomogeneous equation is given by

$$y(x) = c_1 + c_2x + c_3\cos 2x + c_4\sin 2x + \frac{5}{24}x^3 - \frac{1}{32}e^{2x}.$$