HO CHI MINH CITY INTERNATIONAL UNIVERSITY

FINAL EXAMINATION

Academic year 2012-2013, Semester 1
Duration: 120 minutes

SUBJECT: **Differential Equations**	
Chair of the Department of Mathematics	Lecturer:
Signature:	Signature:
Professor Phan Quoc Khanh	Associate Prof. Pham Huu Anh Ngoc

Instructions:

- *Each student is allowed a scientific calculator and a maximum of two double-sided sheets of reference material (size A4 or similar), stapled together and marked with their name and ID. All other documents and electronic devices are forbidden.*

Question 1. (15 marks) A large corporation starts at time $t = 0$ to invest part of its receipts at a rate of P dollars per year in a fund for future corporate expansion. Assume that the fund earns r percent interest per year compounded continuously. So, the rate of growth of the amount A in the fund is given by

$$\frac{dA}{dt} = rA + P,$$

where $A = 0$ when $t = 0$. Find the amount A provided $P = \$100000, r = 12\%$, and $t = 5$ years.

Question 2. (20 marks) Solve the initial value problem

$$y'' + y = \sin x + \cos x; \quad y(0) = 1;\; y'(0) = 0.$$

Question 3. Find the general solution of the following differential equations:

a) (15 marks)

$$y^{(4)} - 3y'' + 2y' = 0.$$

b) (15 marks)

$$y'' - 2y' + y = x + 5 + xe^x.$$

Question 4. (15 marks) Solve the system of differential equations

$$\begin{cases} \frac{dx}{dt} &= -x + 6y \\ \frac{dy}{dt} &= x - 2y. \end{cases}$$

Question 5. (20 marks) Show that $y_p(x) = x$ is a solution of

$$x^3y'' + x^2y' - xy = 0, \quad x \in (0, \infty).$$

Use this information and the variation of parameters method to find the general solution of

$$x^3y'' + x^2y' - xy = x + 1, \quad x \in (0, \infty).$$

End.

SOLUTIONS:

Question 1. The general solution of $\frac{dA}{dt} = rA + P$, is given by

$$A(t) = -\frac{P}{r} + Ce^{rt}, \qquad t \geq 0.$$

Since $A(0) = 0$, it follows that $A(t) = \frac{P}{r}(e^{rt} - 1), t \geq 0$. If $P = 100000, r = 0.12$ and $t = 5$ then $A(5) \approx \$685099$.

Question 2. The general solution of $y'' + y = 0$, is

$$y(x) = c_1 \sin x + c_2 \cos x.$$

A particular solution of $y'' + y = \sin x + \cos x$ is given by $y_p(x) = -\frac{1}{4}x\cos x + \frac{1}{2}x \sin x$. Thus, the general solution of the nonhomogeneous equation is

$$y(x) = c_1 \sin x + c_2 \cos x - \frac{1}{4}x \cos x + \frac{1}{2}x \sin x.$$

Since $y(0) = 1, y'(0) = 0$, we get the particular solution

$$y(x) = \frac{1}{4}\sin x + \frac{1}{2}\cos x + \frac{1}{4}(2 - x)\cos x + \frac{1}{2}x \sin x.$$

Question 3.
a)The general solution of the given equation is

$$y(x) = c_1 + c_2e^{-2x} + c_3e^x + c_4xe^x.$$

b) The general solution of the equation $y'' - 2y' + y = 0$, is

$$y(x) = c_1e^x + c_2xe^x.$$

A particular solution of $y'' - 2y' + y = x + 5 + xe^x$ is given by $y_p(x) = x + 7 + \frac{1}{6}x^3e^x$. Thus the general solution of the last differential equation is

$$y(x) = x + 7 + \frac{1}{6}x^3e^x + c_1e^x + c_2xe^x.$$

Question 4. The general solution of the given system is

$$x(t) = c_1e^t + c_2e^{-4t}; \qquad y(t) = \frac{1}{3}c_1e^t - \frac{1}{2}c_2e^{-4t}.$$

Question 5. The general solution of $x^3y'' + x^2y' - xy = 0$ is given by

$$y(x) = c_1x + c_2\frac{1}{x}.$$

A particular solution of $x^3y'' + x^2y' - xy = x + 1$ is given by

$$y_p(x) = \frac{-4x - 1 - 2\ln x}{4x}.$$

Thus the general solution is

$$y(x) = \frac{-4x - 1 - 2\ln x}{4x} + c_1x + c_2\frac{1}{x}.$$