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Homework : Chapter 1 - Week 2

1. Determine if the following system is consistent :

$$\begin{cases} x_1 - 4x_3 = 8 \\ 2x_1 - 3x_2 + 2x_3 = 1 \\ 4x_1 - 8x_2 + 12x_3 = 1 \end{cases} \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & -4 & 8 \\ 2 & -3 & 2 & 1 \\ 4 & 8 & 12 & 1 \end{array}\right]$$

$$\xrightarrow{2R_2 - R_3} \left[\begin{array}{ccc|c} 1 & 0 & -4 & 8 \\ 2 & -3 & 2 & 1 \\ 0 & 2 & 8 & 1 \end{array}\right] \xrightarrow{2R_1 - R_2} \left[\begin{array}{ccc|c} 1 & 0 & -4 & 8 \\ 0 & 3 & -10 & 15 \\ 0 & 2 & -8 & 1 \end{array}\right]$$

$$\xrightarrow{2R_3 - 3R_2} \left[\begin{array}{ccc|c} 1 & 0 & -4 & 8 \\ 0 & 3 & -10 & 15 \\ 0 & 0 & 4 & 27 \end{array}\right] \Longrightarrow \begin{cases} x - 4z = 8 \\ 3y - 10z = 15 \\ 4z = 27 \Rightarrow z = \frac{27}{4} \end{cases}$$

$$3y - 10 \times \frac{27}{4} = 15 \Rightarrow y = \frac{55}{2} \ ; \ x - 4 \times \frac{27}{4} = 8 \Rightarrow x = 35$$

$$\Rightarrow (x_1, x_2, x_3) = \left(35, \frac{55}{2}, \frac{27}{4}\right)$$

2. Determine which matrices are in reduced echelon form which others are only echelon form.

a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow$ Reduced row echelon form

$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow$ Reduced row echelon form

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow$ Not in echelon form
(Row of all zeros above row of non-zeros)

d) $\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix} \Rightarrow$ Row echelon form.

3. Reduce the matrices to echelon form. Circle the pivot positions in the matrix and in the original matrix, and list the pivot columns,

a) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix} \xrightarrow[4R_1 - R_2]{6R_1 - R_3} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 3 & 6 & 9 \\ 0 & 5 & 10, & 15 \end{bmatrix}$

$\xrightarrow{\frac{1}{3}R_2 - \frac{1}{5}R_3} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 3 & 6 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\frac{1}{3}R_2} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

The reduced row echelon form of matrix and the pivot position are circled $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

The pivot column of the original matrix are circled. $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$

$\Rightarrow$ Thus, the pivot position are in the first and second columns

b) $\begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix} \xrightarrow[5R_1 - R_3]{5R_2 - 3R_3} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 4 & 8 & 12 \\ 0 & 8 & 16, & 34 \end{bmatrix}$

$\xrightarrow[8R_2 - R_3]{\frac{1}{4}R_1} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & -10 \end{bmatrix} \xrightarrow{-\frac{1}{10}R_3} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

The reduced row echelon form of matrix and the pivot position are circled $\begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

The pivot position are circled in the original matrix $\begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix}$

$\Rightarrow$ Thus, the pivot positions are in the 1^{st}, 2^{nd}, 4^{th} Columns

4. Find the general solutions of the systems whose augmented matrices.

a) $\left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{array}\right] \xrightarrow{R_2 - 3R_1} \left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & 0 & -5 & -15 \end{array}\right]$

$\xrightarrow{5R_1 + 4R_2} \left[\begin{array}{ccc|c} 5 & 15 & 0 & -25 \\ 0 & 0 & -5 & -15 \end{array}\right] \xrightarrow[-\frac{R_2}{5}]{\frac{R_1}{5}} \left[\begin{array}{ccc|c} 1 & 3 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{array}\right]$

From above reduced matrix, we have:

$\begin{cases} x_1 + 3x_2 = -5 \\ x_3 = 3 \end{cases} \Rightarrow x_2$ is free

$\Leftrightarrow \begin{cases} x_1 = -5 - 3x_2 \\ x_2 = a \\ x_3 = 3 \end{cases} \Leftrightarrow \begin{cases} x_1 = -5 - 3a \\ x_2 = a \\ x_3 = 3 \end{cases}$

$\Rightarrow (x_1, x_2, x_3) = (-5 - 3a, a, 3)$

b)

$\left[\begin{array}{ccc|c} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{array}\right] \xrightarrow[2R_1 + R_3]{3R_1 + R_2} \left[\begin{array}{ccc|c} 3 & -4 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\xrightarrow{\frac{R_1}{3}} \left[\begin{array}{ccc|c} 1 & -\frac{4}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ x_2, x_3 are free

$\Rightarrow \begin{cases} x_1 = \frac{4}{3}x_2 + \frac{2}{3}x_3 = 0 \\ x_2 = a \\ x_3 = b \end{cases} \Leftrightarrow \begin{cases} x_1 = \frac{4}{3}a - \frac{2}{3}b \\ x_2 = a \\ x_3 = b \end{cases}$

$\Rightarrow (x_1, x_2, x_3) = \left(\frac{4}{3}a - \frac{2}{3}b, a, b\right)$