AC, DC

Sinusoidal Steady-State Analysis

We consider circuits energized by time-varying voltage or current sources.

Textbook:

Electric Circuits

James W. Nilsson & Susan A. Riedel

9th Edition.

Outline

- Complex Numbers Tutorial
- Sinusoids
- Phasors
- Techniques of Circuit Analysis
- Phasor Diagrams

Kirchhoff's Laws

The KCL and KVL are still applicable in the frequency domain.

	Time domain		Frequency domain
KVL	$v_1 + v_2 + \cdots + v_n = 0$	$\Rightarrow$	$\mathbf{V}_1 + \mathbf{V}_2 + \cdots \mathbf{V}_n = 0$
KCL	$i_1 + i_2 + \cdots + i_n = 0$	$\Rightarrow$	$\mathbf{I}_1 + \mathbf{I}_2 + \cdots \mathbf{I}_n = 0$

For KVL, let $v_1, v_2, \ldots, v_n$, be the voltages around a closed loop.

$$\Rightarrow v_1 + v_2 + \cdots + v_n = 0$$

In the sinusoidal steady state, each voltage may be written in cosine form.

$$\Rightarrow V_{m1}\cos(\omega t + \theta_1) + V_{m2}\cos(\omega t + \theta_2) + \cdots + V_{mn}\cos(\omega t + \theta_n) = 0$$

This can be written as

$$\mathrm{Re}(V_{m1}e^{j\theta_1}e^{j\omega t}) + \mathrm{Re}(V_{m2}e^{j\theta_2}e^{j\omega t}) + \cdots + \mathrm{Re}(V_{mn}e^{j\theta_n}e^{j\omega t}) = 0$$

$$\Rightarrow \mathrm{Re}\left[\left(V_{m1}e^{j\theta_1} + V_{m2}e^{j\theta_2} + \cdots + V_{mn}e^{j\theta_n}\right)e^{j\omega t}\right] = 0$$

$$\Rightarrow \mathrm{Re}\left[\left(\mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n\right)e^{j\omega t}\right] = 0; \quad (\mathbf{V}_K = V_{mk}e^{j\theta_k})$$

$$\Rightarrow \mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n = 0; \quad (\because\ e^{j\omega t} \neq 0\ \forall t)$$

$\Rightarrow$ KVL holds for phasor!

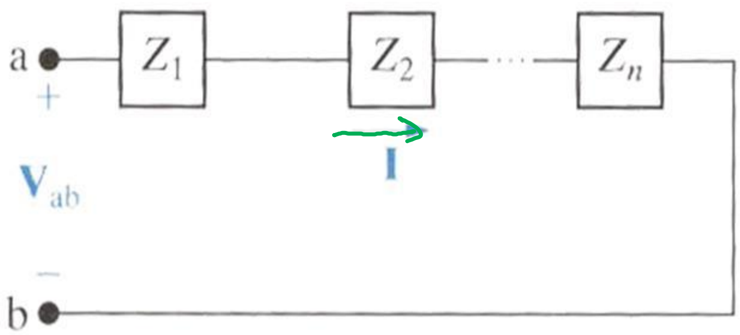
Similarily, KCL holds for phasor!

Series and Parallel Combination

$L \rightarrow j\omega L$

$C \rightarrow \frac{1}{j\omega C}$

Series Combination



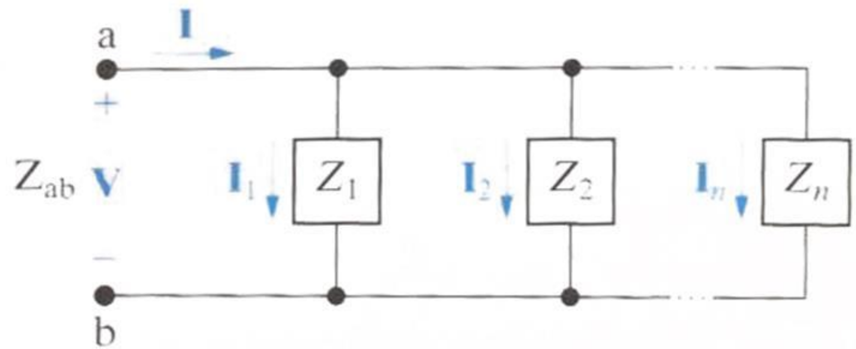
Voltage : KVL

$$\mathbf{V}_{ab} = Z_1\mathbf{I} + Z_2\mathbf{I} + \ldots + Z_n\mathbf{I}$$
$$= (Z_1 + Z_2 + \ldots + Z_n)\mathbf{I}$$

Equivalent impedance :

$$Z_{ab} = \frac{\mathbf{V}_{ab}}{\mathbf{I}} = Z_1 + Z_2 + \ldots + Z_n$$

Parallel Combination



Current :

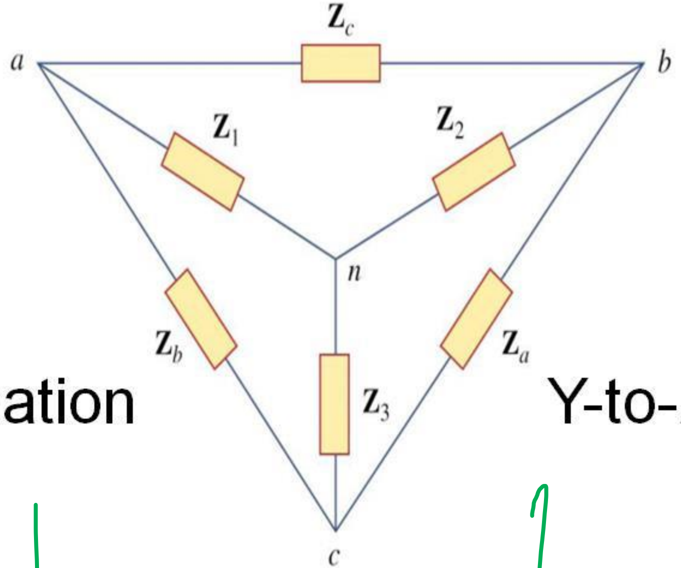
$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \ldots + \mathbf{I}_n = \frac{\mathbf{V}}{Z_{ab}}$$

Equivalent impedance :

$$\frac{1}{Z_{ab}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \ldots + \frac{1}{Z_n}$$

$$Z_{ab} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

Delta-to-Wye Simplifications



Δ-to-Y transformation

$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_c Z_a}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$

Y-to-Δ transformation

$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1}$$

$$Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2}$$

$$Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3}$$

Ex. 11 Find the input impedance of the circuit. Assume that the circuit operations at ω = 50 rad/s.

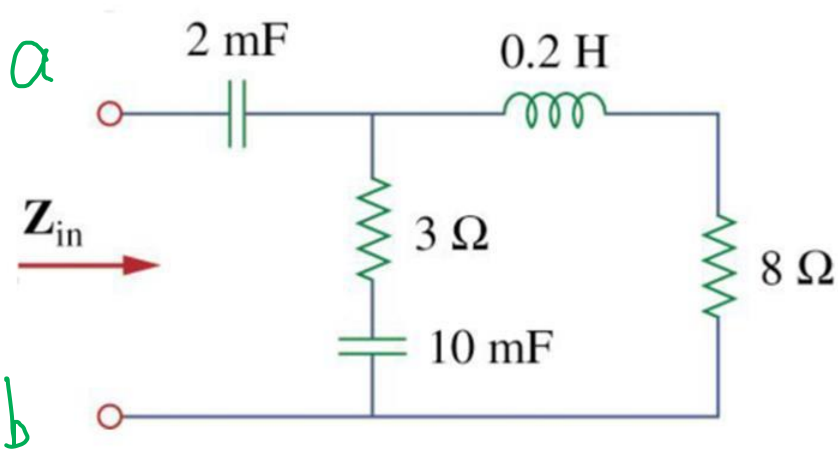
transform time domain → phasor

$R \to R$

$L \to j\omega L$

$C \to \frac{1}{j\omega C} = \frac{-j}{\omega C}$

$Z_{ab} =$



$$\begin{cases} R \to R \quad (\Omega) \\ L \to j\omega L \quad (\Omega) \\ C \to \frac{1}{j\omega C} \quad (\Omega) \end{cases}$$

Sol. of Ex. 11

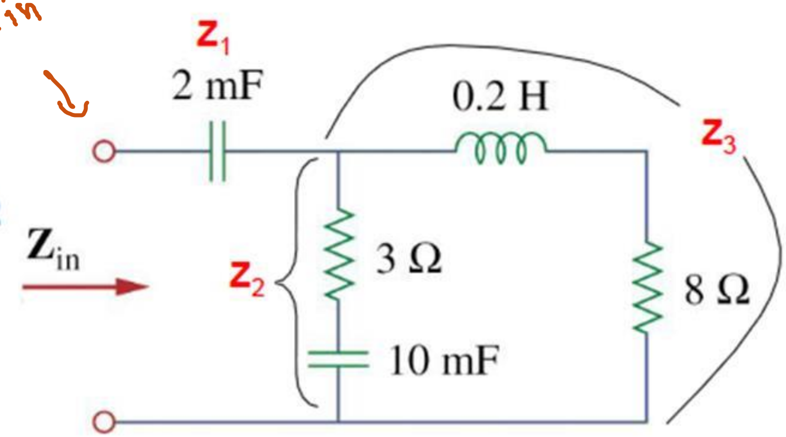
$w = 50$ rad/s

$$\mathbf{Z}_1 = \frac{1}{j\omega C} = \frac{1}{j50\times 2\times 10^{-3}} = -j10\ \Omega$$

$$\mathbf{Z}_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j50\times 10\times 10^{-3}} = (3 - j2)\ \Omega$$

$$\mathbf{Z}_3 = 8 + j\omega L = 8 + j50\times 0.2 = (8 + j10)\ \Omega$$

Zin



The input impedance is

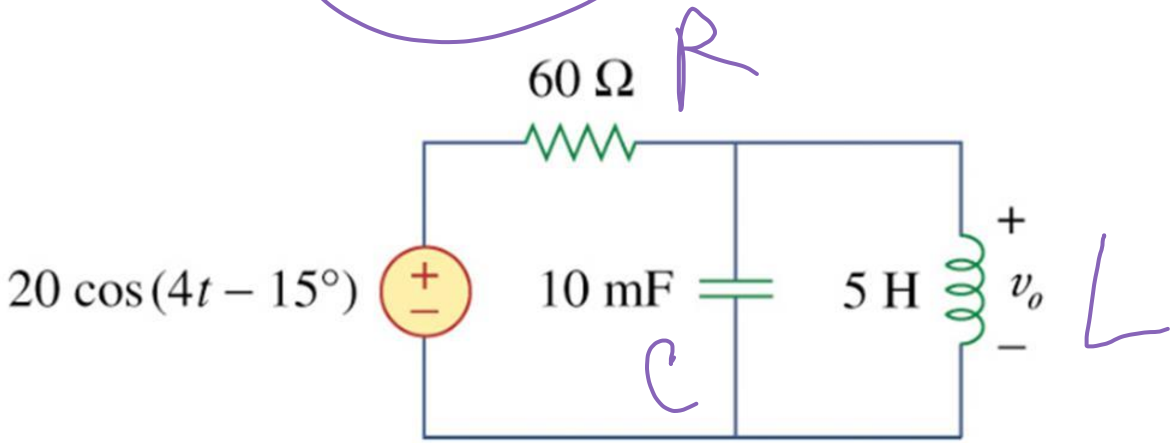
$$\Rightarrow \mathbf{Z}_{in} = \mathbf{Z}_1 + \mathbf{Z}_2 \| \mathbf{Z}_3 = -j10 + \frac{(3-j2)(8+j10)}{11+j8}$$

$$= -j10 + \frac{(44+j14)(11-j8)}{11^2+8^2} = -j10 + 3.22 - j1.07\ \Omega$$

$$\frac{596 - j198}{185} = 3.22 - j1.07$$

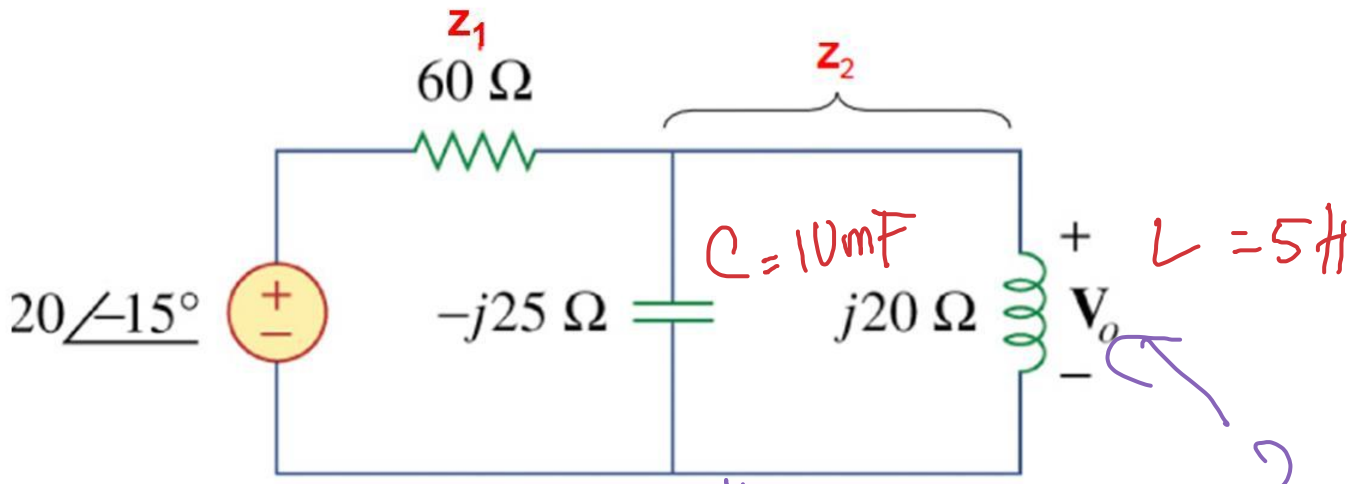
$$= 3.22 - j11.07\ \Omega$$

Ex. 12 Determine $v_0(t)$ in the circuit.



$v_0(t) = A\cos(4t + \varphi)$

Sol. of Ex. 12



ω   magnitude   phase

$$v_s = 20\cos(4t - 15°) \quad \Rightarrow \quad \mathbf{V}_s = 20\angle -15° \text{ V}, \quad \omega = 4$$

Find C: $$10\text{mF} \quad \Rightarrow \quad \frac{1}{j\omega C} = \frac{1}{j4 \times 10 \times 10^{-3}} = -j25\Omega$$

Find L: $$5\text{ H} \quad \Rightarrow \quad j\omega L = j4 \times 5 = j20\ \Omega$$

Let

Z_1 = Impedance of the 60-Ω resistor; Z_2 = Impedance of the parallel combination of the 10 mF capacitor and the 5-H inductor

Then $\mathbf{Z}_1 = 60\ \Omega$ and

$$\mathbf{Z}_2 = -j25 \| j20 = \frac{-j25 \times j20}{-j25 + j20} = j100\ \Omega$$

$$= \frac{500}{-j5} = \frac{1}{-j}100 = j100\Omega$$

Sol. of Ex. 12 Cont.

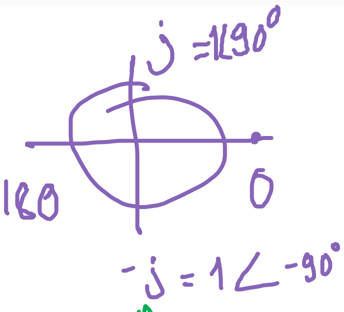
By the voltage-division principle,

$$\mathbf{V}_o = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}_s = \frac{j100}{60 + j100} \times (20\angle -15^\circ)$$

$$= (0.8575\angle 30.96^\circ) \times (20\angle -15^\circ) = 17.15\angle 15.96^\circ \text{ V}$$

$100\angle 90^\circ$

$116.62\angle 59.04$



$c = \sqrt{(0.735)^2 + (0.441)^2}$ $\tan^{-1}\frac{0.441}{0.735}$

Convert this to the time domain and obtain

$$v_o(t) = 17.15\cos(4t + 15.96^\circ) \text{ V}$$

$$\frac{j100(60 - j100)}{60^2 + 100^2} = \frac{j6000 + 10000}{13600}$$

$$R = 0.735 + j0.441$$

Techniques of Circuit Analysis

- Techniques of circuit analysis introduced in Lecture 3 – 4 such as:
  - Source transformations
  - The node-voltage method
  - The mesh-current method
  - Superposition
  - Thevenin – Norton equivalent

  also can be applied to frequency-domain circuits.

- The validity of these techniques is followed the same process used in lectures 3-4 except that we substitute impedance (Z) for resistance (R).

- Many examples are shown in textbook.

Techniques of Circuit Analysis

- In steady-state circuit response with sinusoidal excitation, the phasor method enables the R, L, C as an element of impedance whose function is the same as a resistor such that **generalized Ohm's law** can be applied.
- Hence, all circuit **analysis methods** (Nodal, Mesh), **theorems** (Superposition, Source transformation, Thevenin and Norton equivalent circuits) can be applied to analyze ac circuits.

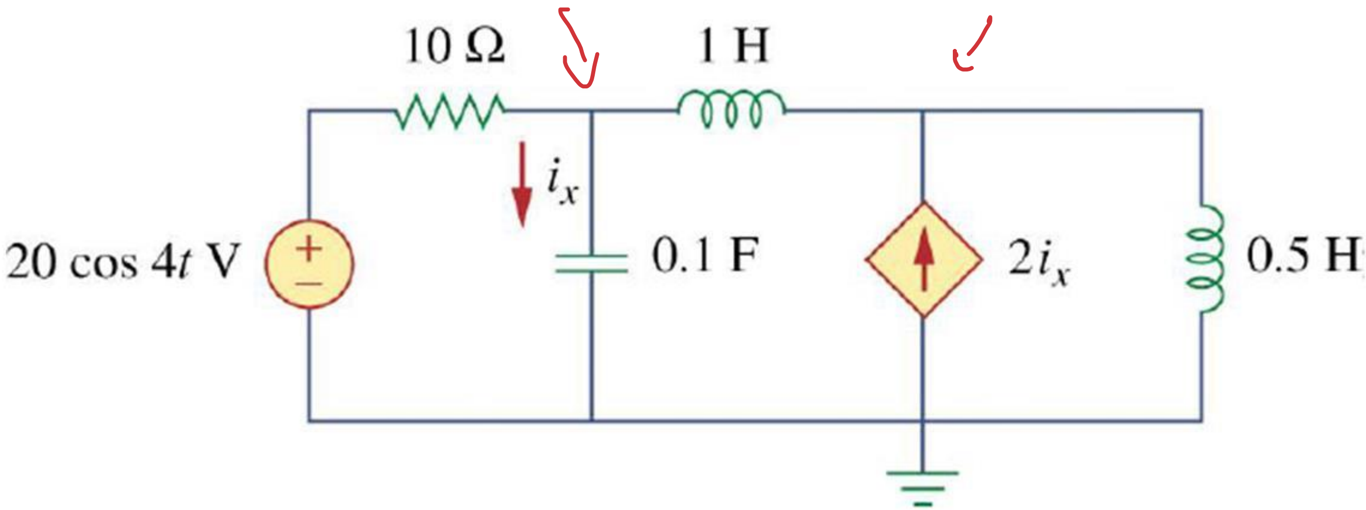
Steps to Analyze AC Circuits

- Steps to Analyze AC Circuits:
  1. Transform the circuit to the phasor or frequency domain.
  2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
  3. Transform the resulting phasor to the time domain.

Ex. 13 - Nodal Analysis

Find i_x in the circuit using nodal analysis.

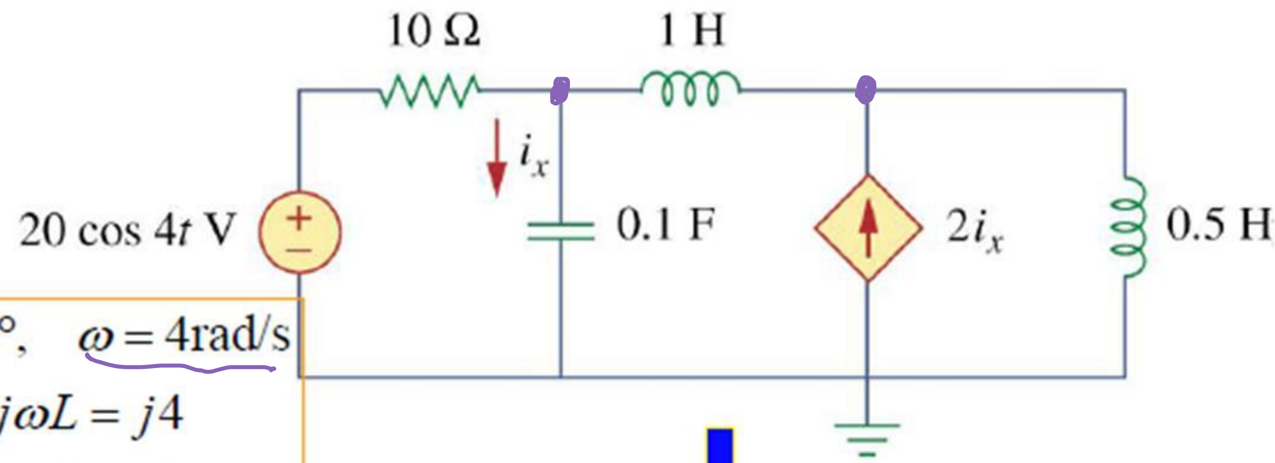
$\boxed{\text{transform to phasor first}}$



2 nodes → 2 equations

Sol. of Ex. 13:

Phasor form



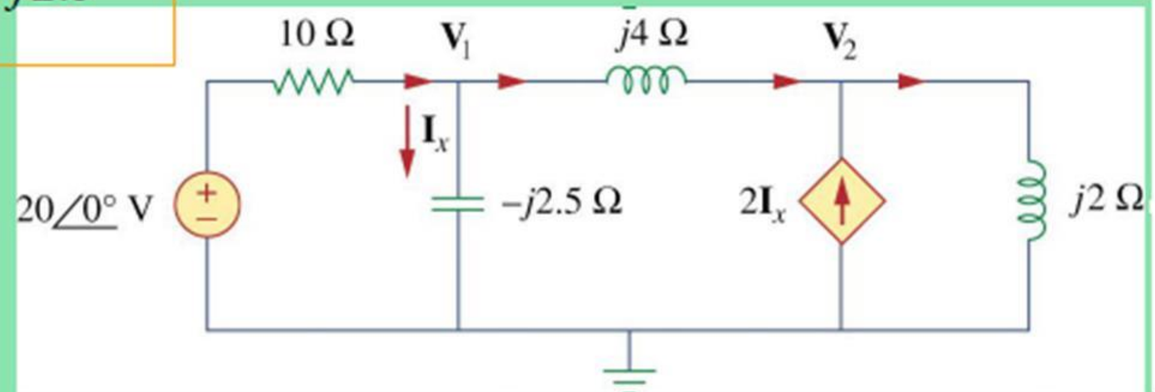
$20\cos 4t \Rightarrow 20\angle 0°, \quad \omega = 4\text{rad/s}$

L= $1\ \text{H} \Rightarrow j\omega L = j4$

L= $0.5\ \text{H} \Rightarrow j\omega L = j2$

C= $0.1\ \text{F} \Rightarrow \dfrac{1}{j\omega C} = -j2.5$

phasor



Sol. of Ex. 13:

$\Leftrightarrow 10(20-V_1) = 40j(V_1) - (V_1-V_2)25j$

$\Leftrightarrow 200 - 10V_1 = 40V_1 j - 25V_1 j + 25V_2 j$

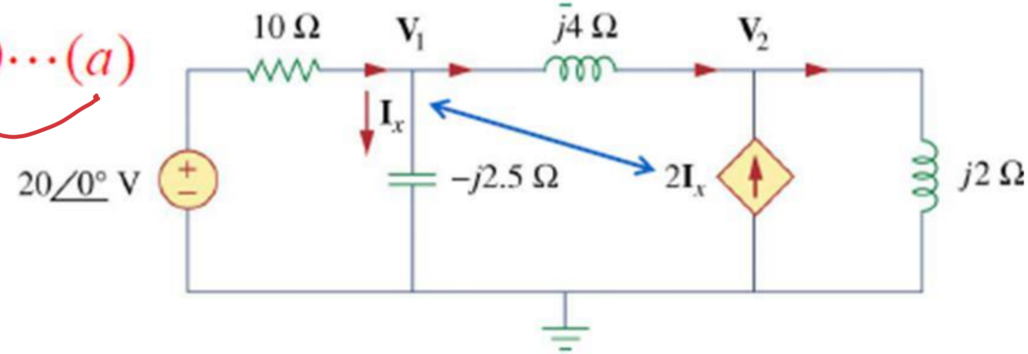
$\Leftrightarrow 20 = V_1 + 1.5V_1 j + 2.5V_2 j$

$2I_x = \dfrac{2V_1}{-j2.5}$

- KCL at node 1

$$\frac{20-\mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1-\mathbf{V}_2}{j4}$$

$$\Rightarrow (1+j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20\cdots(a)$$



- KCL at node 2

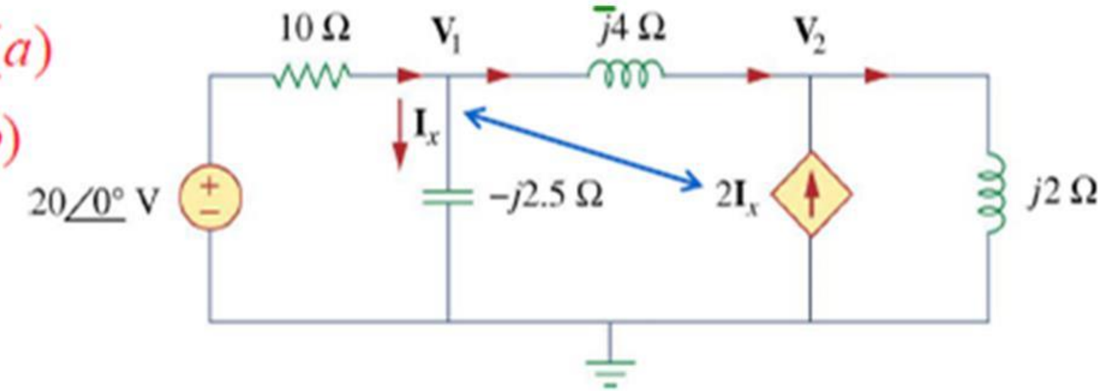
$$2\mathbf{I}_x + \frac{\mathbf{V}_1-\mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

$$\Rightarrow \frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1-\mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}; \ (\mathbf{I}_x = \mathbf{V}_1/-j2.5,\ )$$

$$\Rightarrow 11\mathbf{V}_1 + 15\mathbf{V}_2 = 0\cdots(b)$$

Sol. of Ex. 13:

$$\begin{cases}(1+j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20\cdots(a)\\ 11\mathbf{V}_1 + 15\mathbf{V}_2 = 0\cdots(b)\end{cases}$$



By (a) and (b) $\Rightarrow$

$$\mathbf{V}_1 = \frac{300}{15-j5} = 18.97\angle 18.43^\circ \text{ V}$$

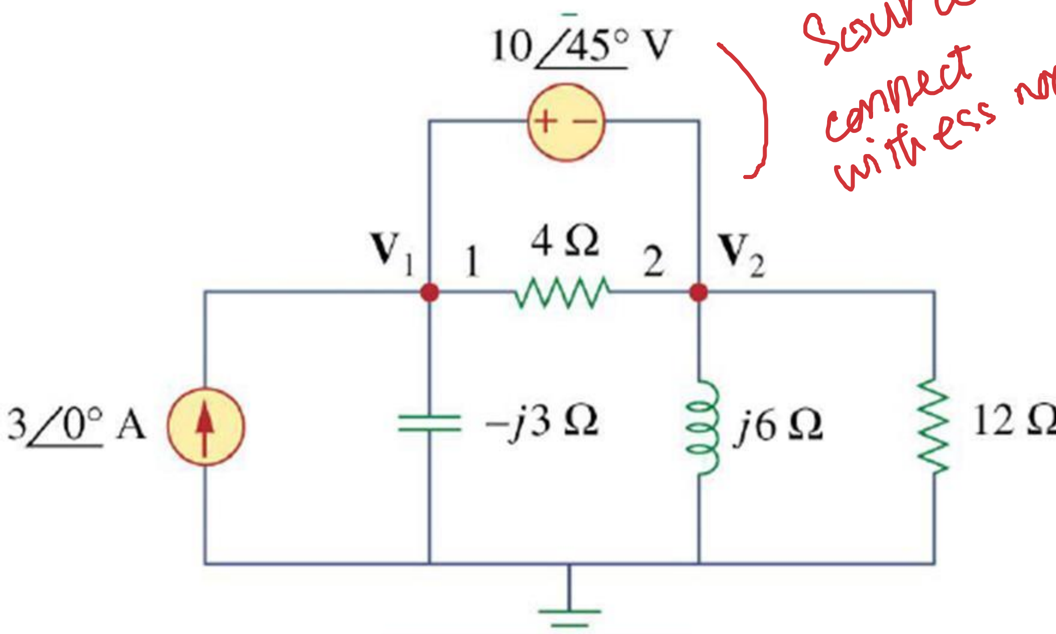
15.81 − j 18.93 →

$$\mathbf{V}_2 = \frac{-220}{15-j5} = 13.91\angle 198.3^\circ \text{ V}$$

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\angle 18.43^\circ}{2.5\angle -90^\circ} = 7.59\angle 108.4^\circ \text{A}$$

$$\Rightarrow i_x(t) = 7.59\cos(4t + 108.4^\circ) \text{ A}$$

Ex. 14: Compute V_1 & V_2 in the circuit



Source connect with ess nodes => supernode

Ko có w => tính bth ko cần chuyển

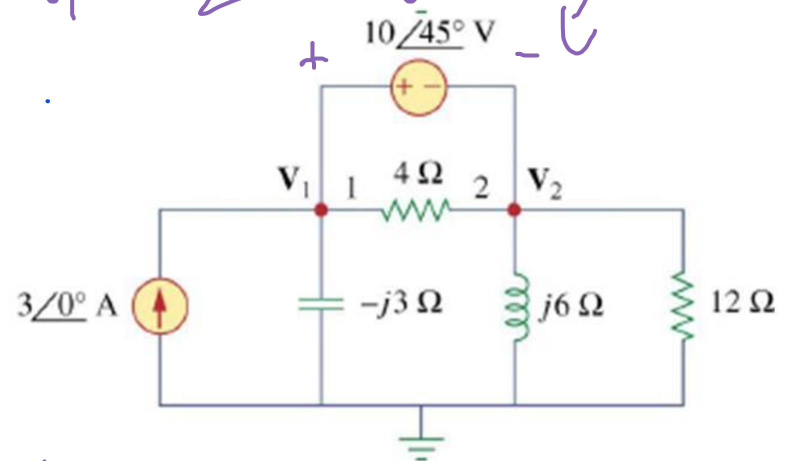
Sol. of Ex. 14:

KCL at supernode:

$$3 = \frac{\mathbf{V}_1}{-j3} + \frac{\mathbf{V}_2}{j6} + \frac{\mathbf{V}_2}{12}$$

$$\Rightarrow 36 = j4\mathbf{V}_1 + (1 - j2)\mathbf{V}_2 \cdots (a)$$

$V_1 - V_2$



At supernode:

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\angle 45^\circ \cdots (b)$$

$V_1 - V_2 = 10 \angle 45^\circ$

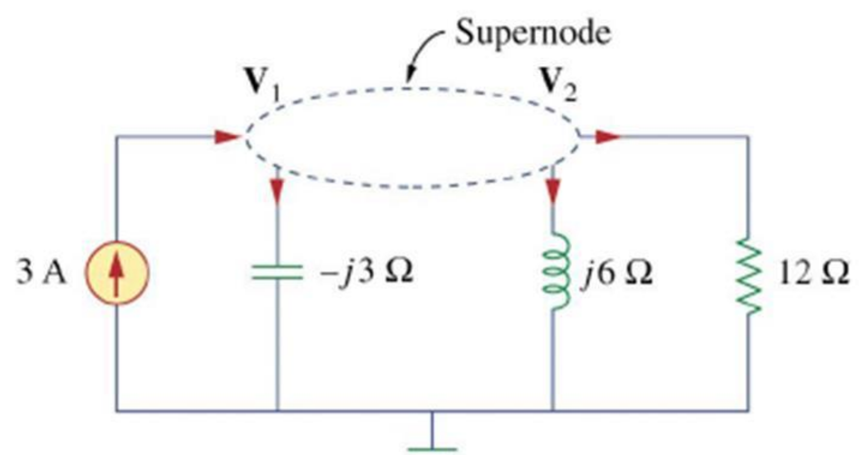
By (a) and (b) $\Rightarrow$

$\mathbf{V}_2 = 31.41\angle -87.18^\circ$ V

so,

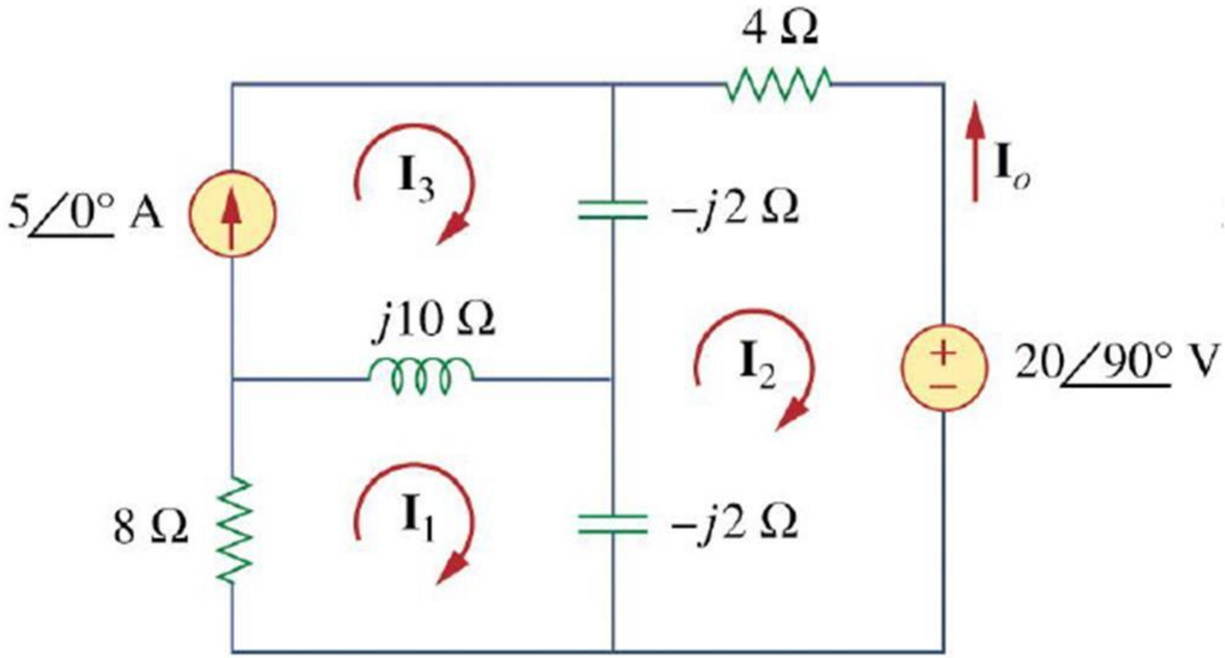
$\mathbf{V}_1 = \mathbf{V}_2 + 10\angle 45^\circ$

$= 25.87\angle\ \text{-}70.48^\circ$ V



Ex. 15: Mesh Analysis

Determine current Io in the circuit using mesh analysis.



$I_o = -I_2$

Sol. of Ex. 15:

KVL for mesh 1: $(8+j10-j2)\mathbf{I}_1-(-j2)\mathbf{I}_2-j10\mathbf{I}_3=0$

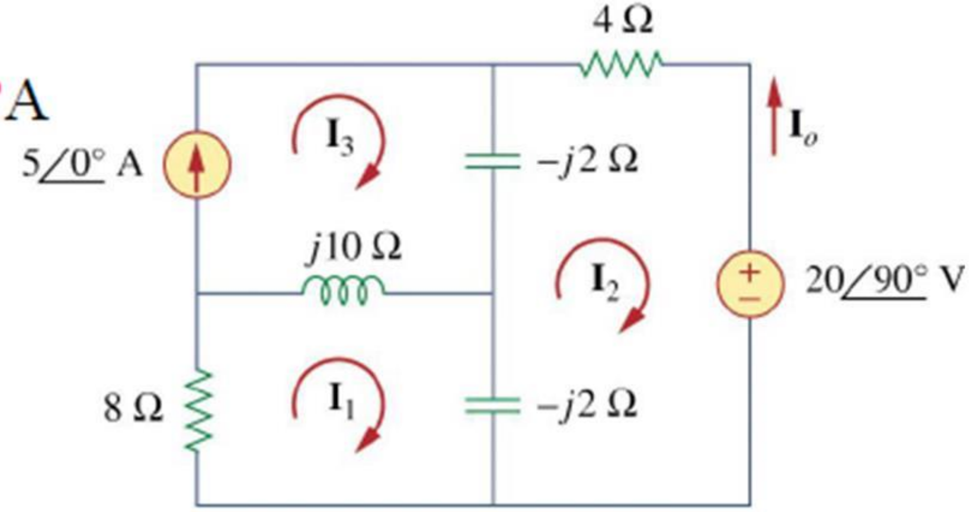
KVL for mesh 2: $(4-j2-j2)\mathbf{I}_2-(-j2)\mathbf{I}_1-(-j2)\mathbf{I}_3+20\angle 90^\circ=0$

For mesh 3: $\mathbf{I}_3=5$

$$\Rightarrow\begin{cases}(8+j8)\mathbf{I}_1+j2\mathbf{I}_2=j50\cdots\cdots\cdots\cdots(a)\\ j2\mathbf{I}_1+(4-j4)\mathbf{I}_2=-j20-j10\cdots(b)\end{cases}$$

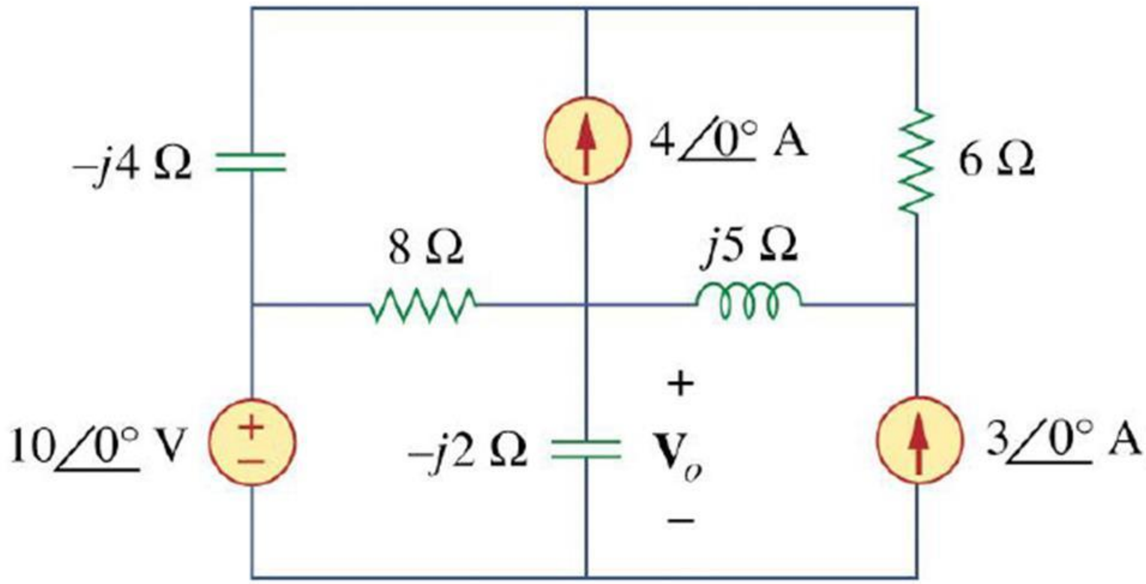
$$\Rightarrow \mathbf{I}_2=6.12\angle-35.22^\circ\text{A}$$

$$\mathbf{I}_o=-\mathbf{I}_2=6.12\angle 144.78^\circ\text{A}$$



Ex. 16: Mesh Analysis – super mesh

Solve V_o in the circuit using mesh analysis



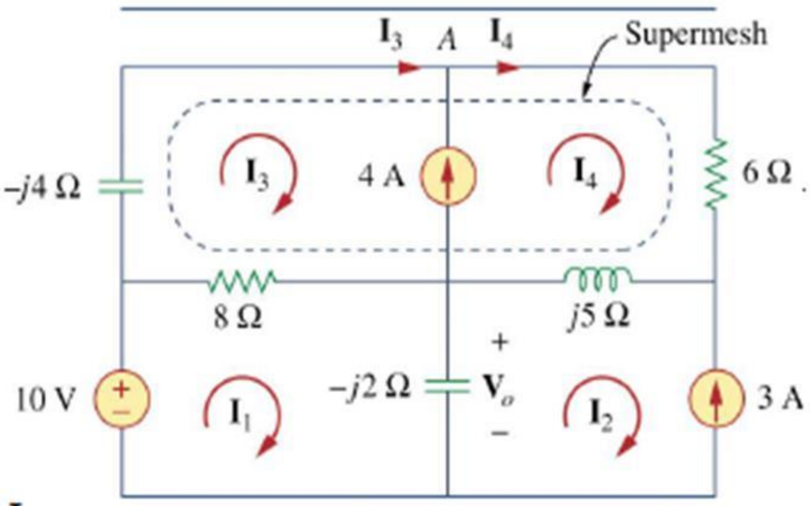
Sol. of Ex. 16:

KVL for mesh 1: $-10+(8-j2)\mathbf{I}_1-(-j2)\mathbf{I}_2-8\mathbf{I}_3=0\cdots(a)$

KVL for supermesh : $(8-j4)\mathbf{I}_3-8\mathbf{I}_1+(6+j5)\mathbf{I}_4-j5\mathbf{I}_2=0\cdots(b)$

For mesh 2: $\mathbf{I}_2=-3\cdots(c)$

Because of the current source between meshes 3 and 4, at node $A \Rightarrow$ $\mathbf{I}_4=\mathbf{I}_3+4\cdots(d)$



By $(a)\sim(d)$

$\Rightarrow \mathbf{I}_1=3.618\angle 274.5°$ A

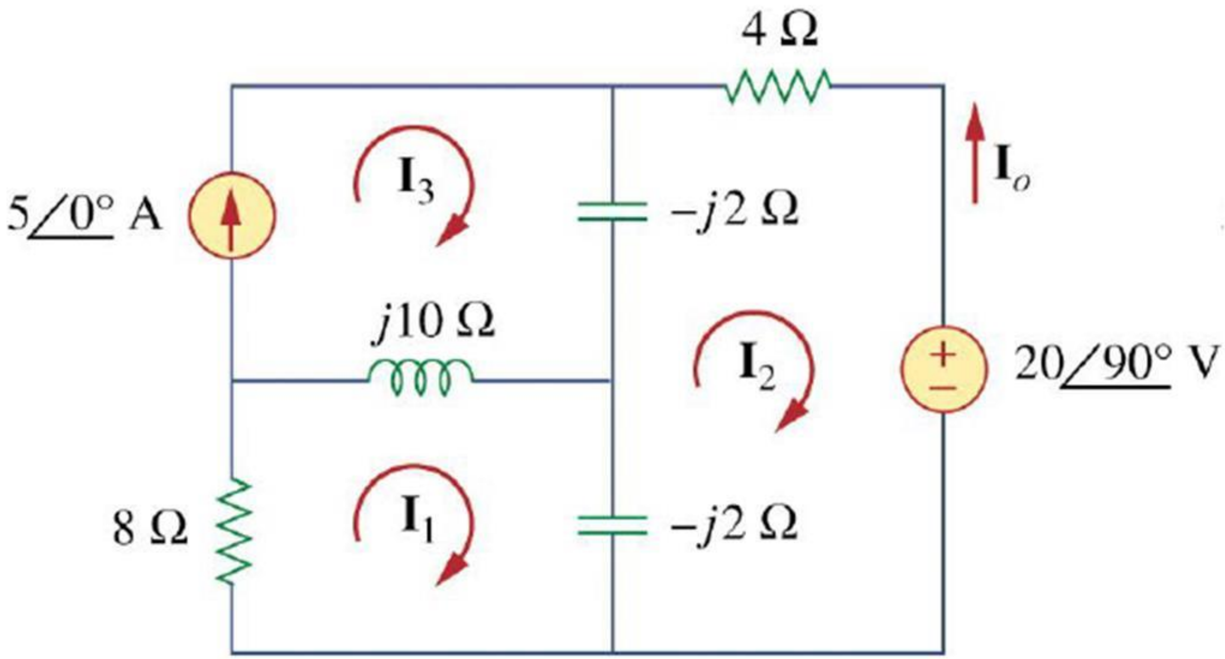
Hence,

$$\mathbf{V}_o=-j2(\mathbf{I}_1-\mathbf{I}_2)=-j2(3.618\angle 274.5°+3)$$

$$=-7.2134-j6.568=9.756\angle -137.68° \text{ V}$$

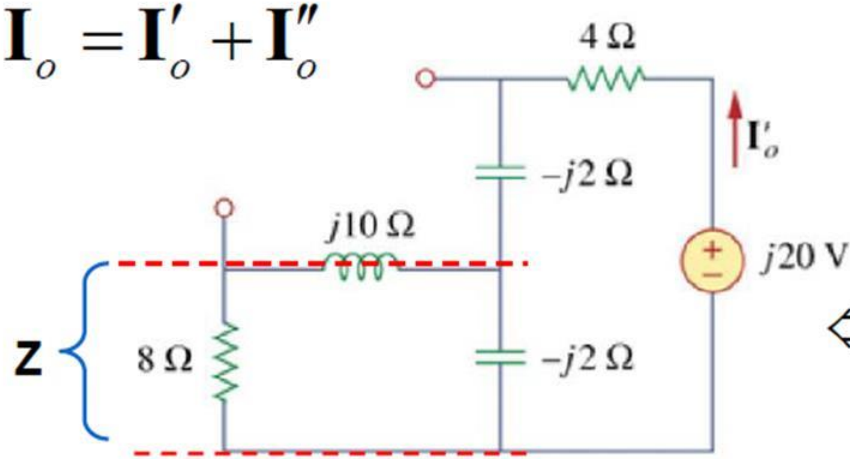
Ex. 17: Superposition Theorem

Use the superposition theorem to find I_o in the circuit.

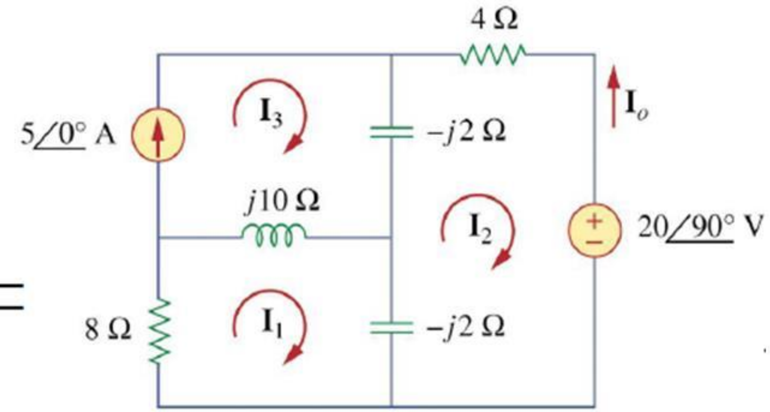


Sol. of Ex. 17: Superposition Theorem

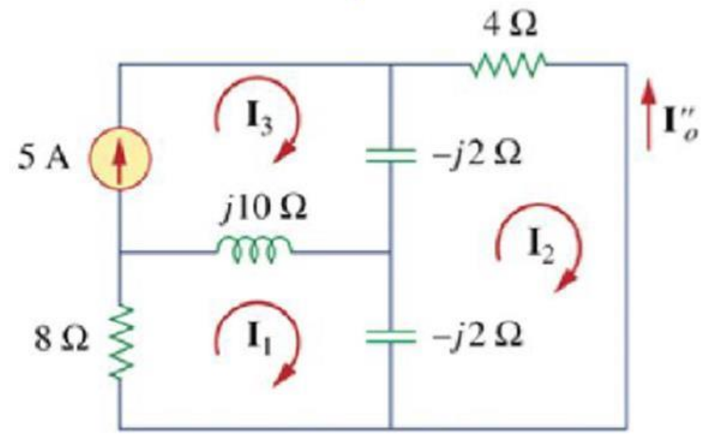
Let $\mathbf{I}_o = \mathbf{I}'_o + \mathbf{I}''_o$



$\Leftarrow$



$\Downarrow$



For $\mathbf{I}'_\mathbf{o}$

$$Z = \frac{-j2(8+j10)}{-2j+8+j10} = 0.25 - j2.25$$

$$\mathbf{I}'_o = \frac{j20}{4-j2+Z} = \frac{j20}{4.25-j4.25}$$

$$= -2.353 + j2.353$$

Sol. of Ex. 17: cont.

For $\mathbf{I}''_o$

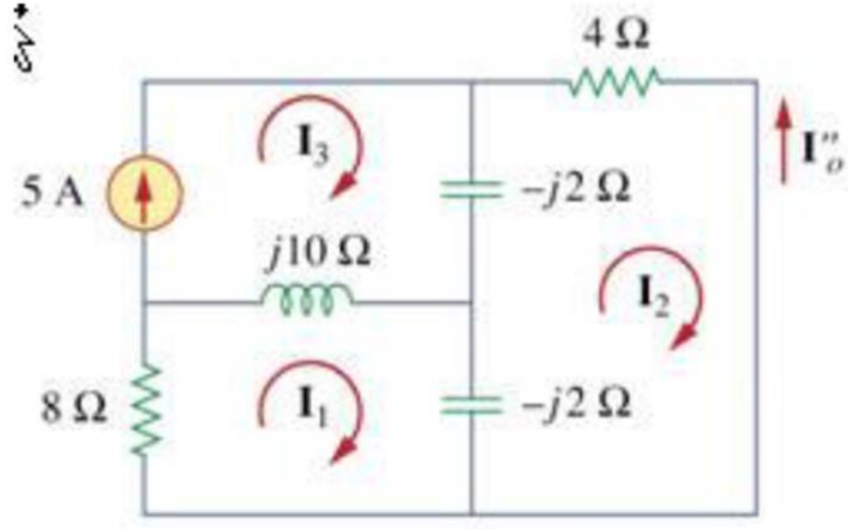
KVL fo rmesh 1: $(8+j8)\mathbf{I}_1 - j10\mathbf{I}_3 + j2\mathbf{I}_2 = 0 \cdots (a)$

KVL fo rmesh 2: $(4-j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j2\mathbf{I}_3 = 0 \cdots (b)$

For mesh 3: $\mathbf{I}_3 = 5 \cdots (c)$

$$\Rightarrow \mathbf{I}_2 = \frac{90-j40}{34} = 2.647 - j1.176$$

$$\mathbf{I}''_o = -\mathbf{I}_2$$



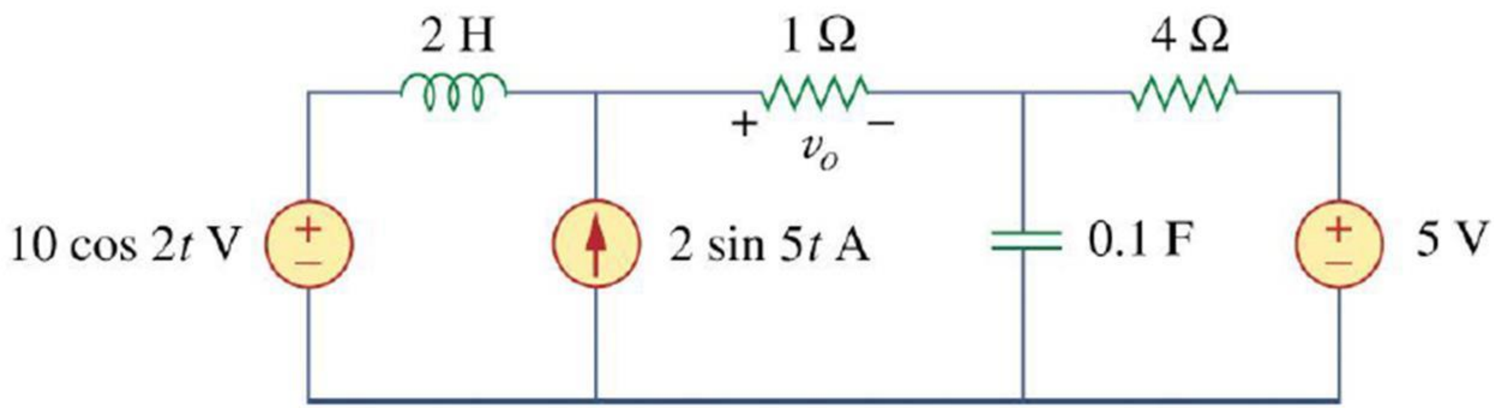
Hence,

$$\mathbf{I}_o = \mathbf{I}'_o + \mathbf{I}''_o = -5 + j3.529$$

$$= 6.12\angle 144.78^\circ \text{A}$$

Ex. 18: Superposition Theorem

Find v_o of the circuit using the superposition theorem.

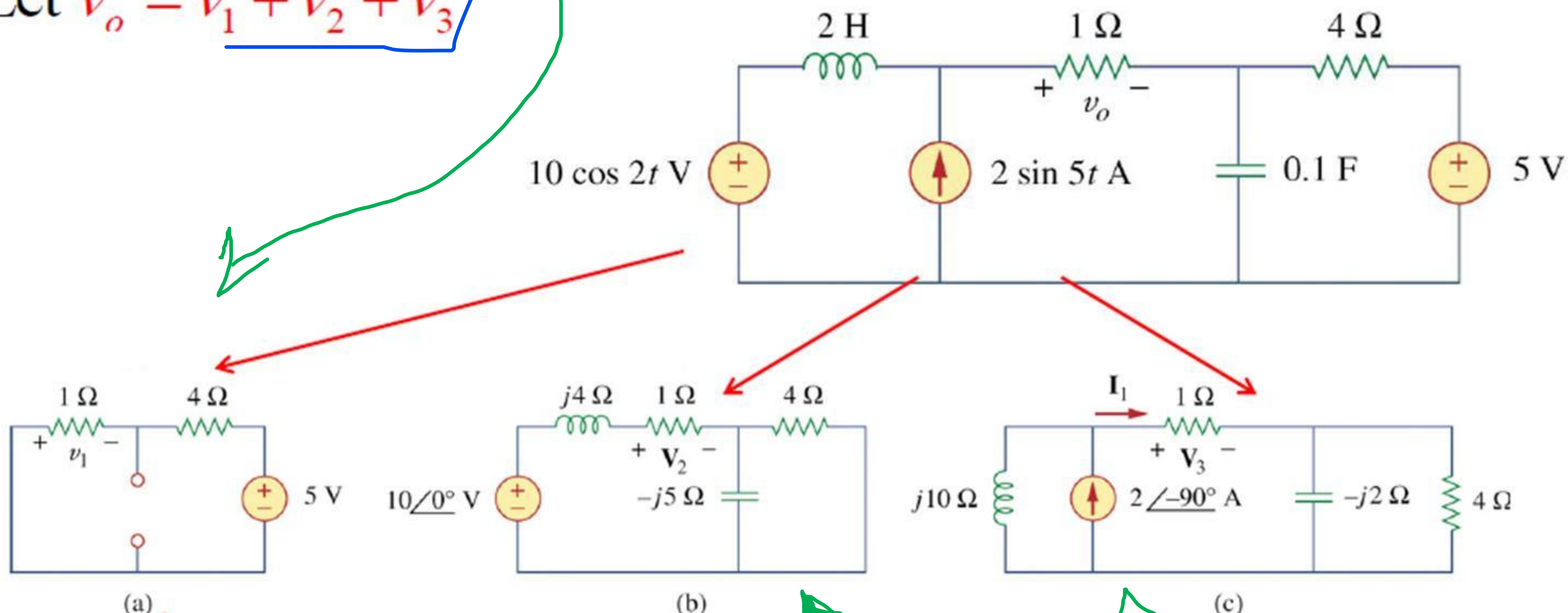


Sol. of Ex. 18:

For DC, $j\omega L$, because $\omega = 0$ → short circuit

For DC, $\frac{1}{j\omega C}$, because $\omega = 0$ → opened

Let $\boxed{v_o = v_1 + v_2 + v_3}$



AC

AC

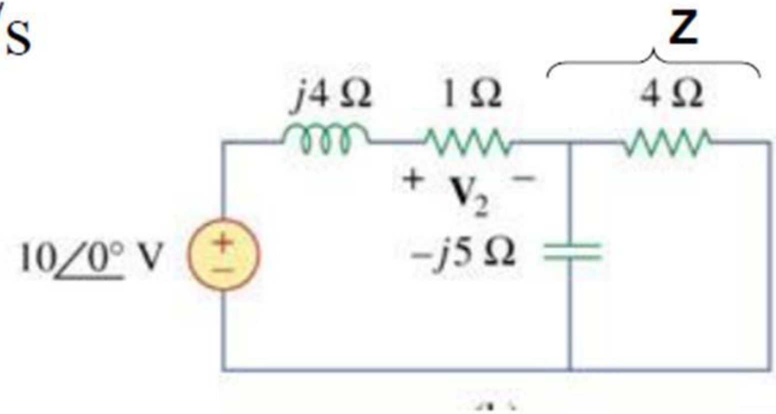
By voltage division ⇒ $-v_1 = \frac{1}{1+4} \times 5 = 1$ V

Sol. of Ex. 18: cont.

$$10\cos 2t \quad \Rightarrow \quad 10\angle 0^\circ, \quad \omega = 2 \text{ red/s}$$

$$2\,\text{H} \quad \Rightarrow \quad j\omega L = j4\,\Omega$$

$$0.1\ \text{F} \quad \Rightarrow \quad \frac{1}{j\omega L} = -j5\,\Omega$$



- Let

$$\mathbf{Z} = -j5 \| 4 = \frac{-j5\times 4}{4-j5} = 2.439 - j1.951$$

- By division,

$$\mathbf{V}_2 = \frac{1}{1+j4+\mathbf{Z}}(10\angle 0^\circ) = \frac{10}{3.439+j2.049} = 2.498\angle -30.79^\circ$$

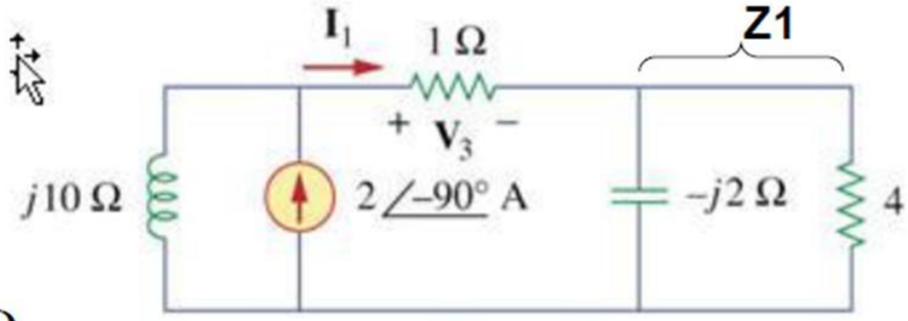
$$v_2 = 2.498\cos(2t - 30.79^\circ)$$

Sol. of Ex. 18: cont.

$$2\sin 5t \quad \Rightarrow \quad 2\angle -90°, \quad \omega = 5 \text{ rad/s}$$

$$2\ \text{H} \quad \Rightarrow \quad j\omega L = j10\,\Omega$$

$$0.1\ \text{F} \quad \Rightarrow \quad \frac{1}{j\omega L} = -j2\,\Omega$$



Let $\mathbf{Z}_1 = -j2 \| 4 = \dfrac{-j2\times 4}{4-j2} = 0.8 - j1.6\ \Omega$

By current division $\mathbf{I}_1 = \dfrac{j10}{j10+1+\mathbf{Z}_1}(2\angle -90°)$ A

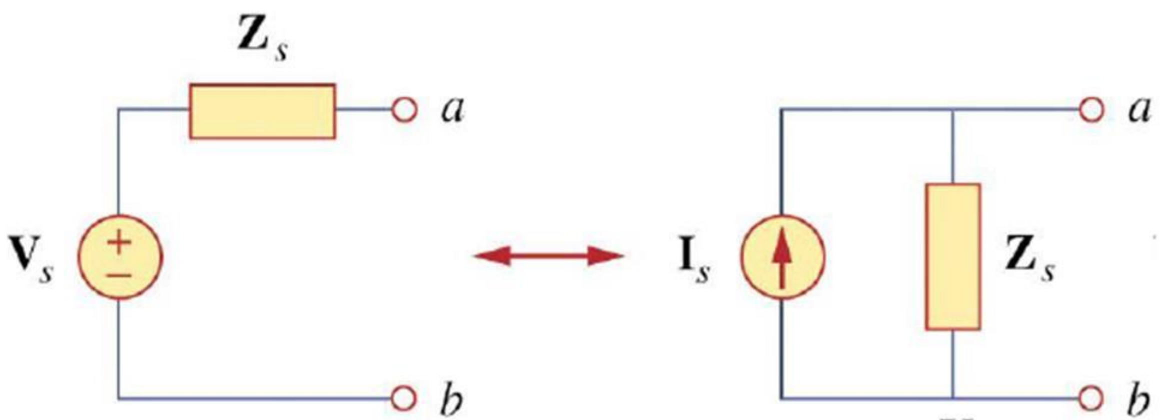
$$\Rightarrow \mathbf{V}_3 = \mathbf{I}_1 \times 1 = \frac{j10}{1.8+j8.4}(-j2) = 2.328\angle 80°\ \text{V}$$

$$v_3 = 2.33\cos(5t-80°) = 2.33\sin(5t+10°)\ \text{V}$$

$$\boxed{v_0(t) = -1 + 2.498\cos(2t-30.79°) + 2.33\sin(5t+10°)\text{V}}$$

Ex. 19: Source Transformation

$$\boxed{\mathbf{V}_s = \mathbf{Z}_s \mathbf{I}_s \quad \Leftrightarrow \quad \mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_s}}$$

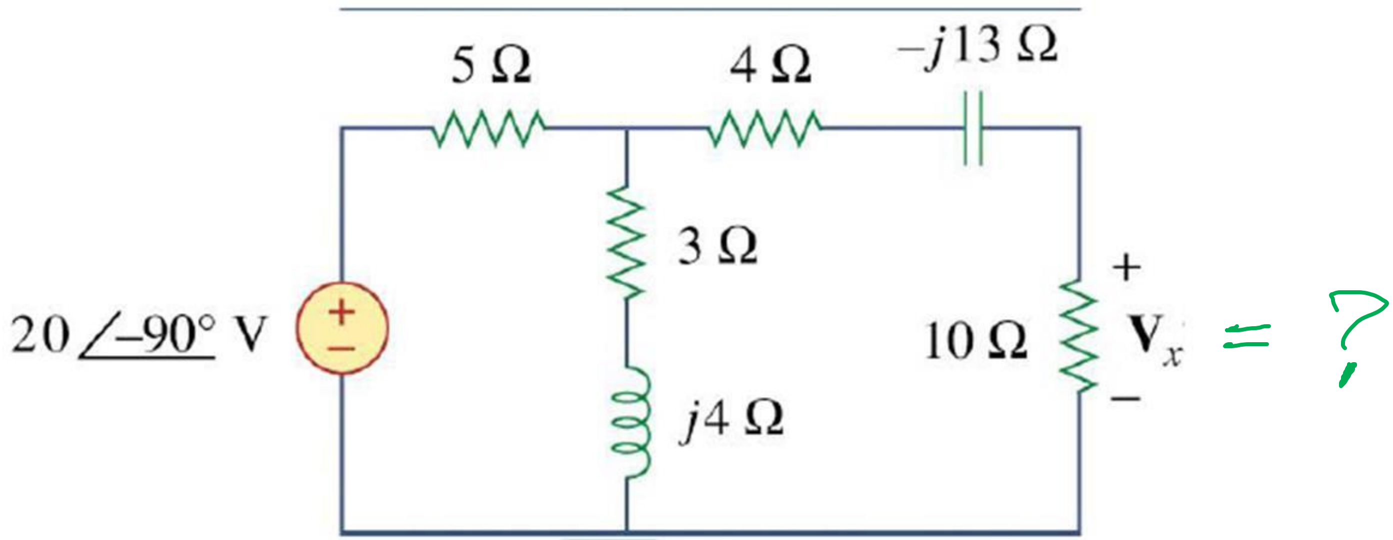


$\mathbf{V}_s = \mathbf{Z}_s \mathbf{I}_s$

$\mathbf{I}_s = \dfrac{\mathbf{V}_s}{\mathbf{Z}_s}$

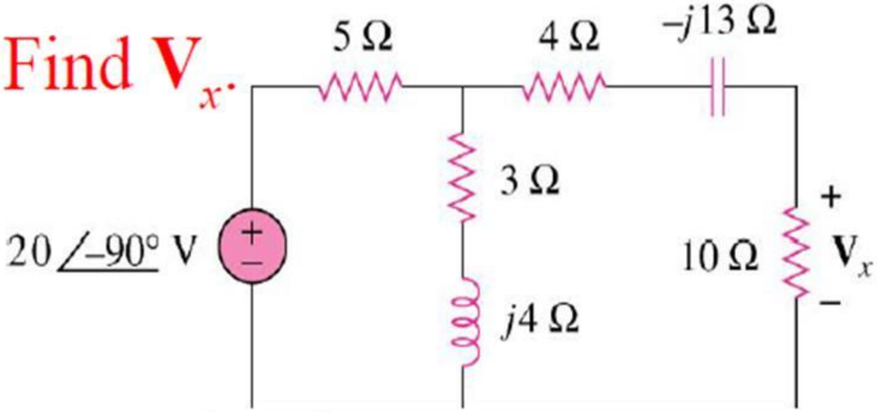
Ex. 19: Source Transformation

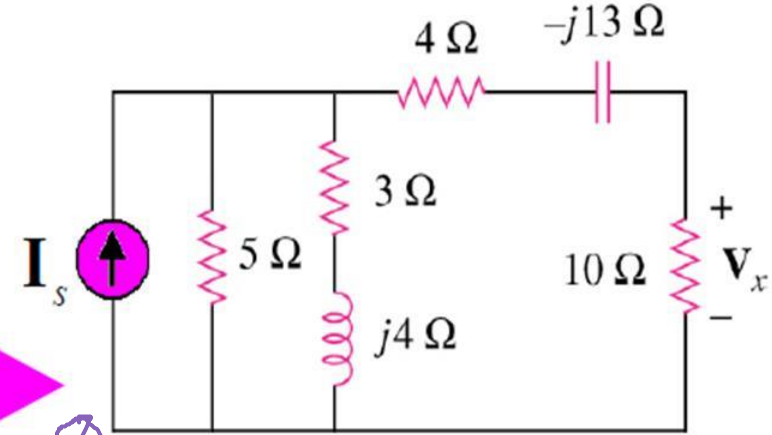
Calculus $\mathbf{V}_x$ in the circuit using the method of source transformation.



Sol. of Ex. 19:

Find $\mathbf{V}_x$.





$$\mathbf{I}_s = \frac{20\angle -90^\circ}{5} = 4\angle -90^\circ = -j4$$

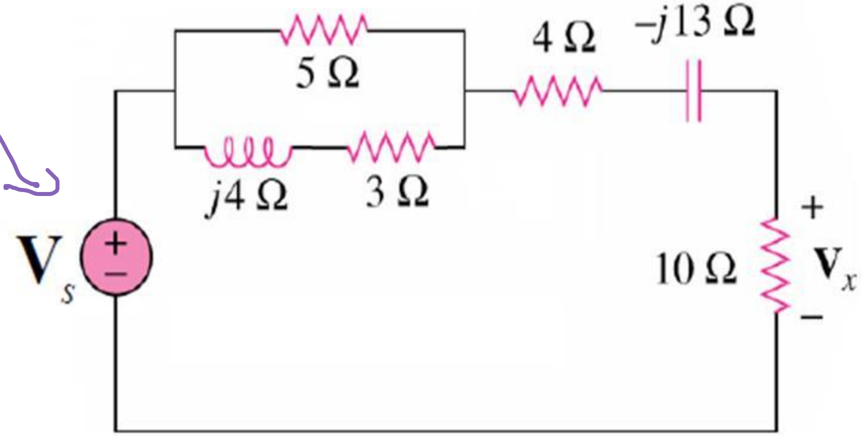
$$\mathbf{V}_s = \mathbf{I}_s \times \left(5 \,\|\, (3 + j4)\right) = -j4\,\frac{5(3+j4)}{8+j4}$$

$$= -j4(2.5 + j1.25) = 5 - j10$$

By voltage division,

$$\mathbf{V}_x = \frac{10}{2.5 + j1.25 + 4 - j13 + 10}(5 - j10)$$

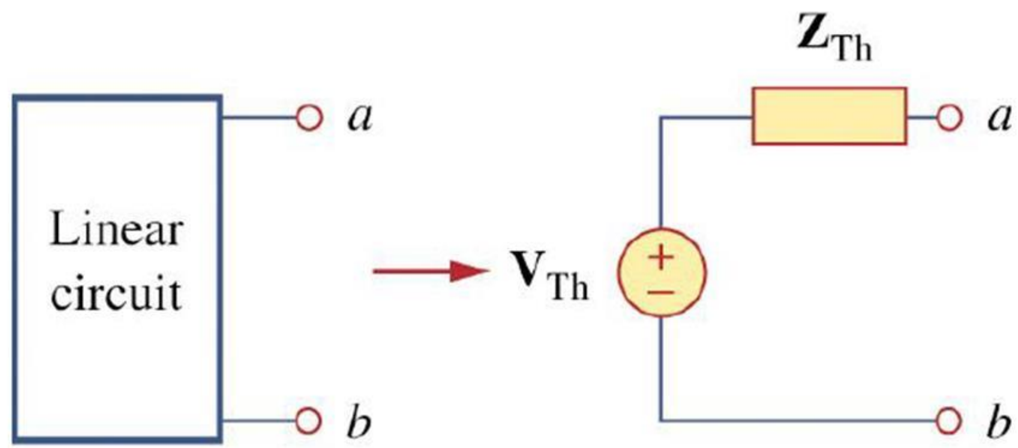
$$= 5.519\angle -28^\circ \text{ V}$$



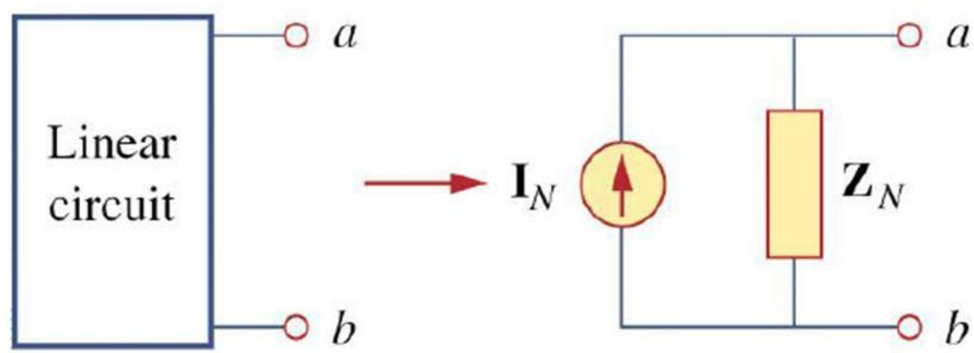
Ex. 20: Thevenin & Norton Equivalent Circuits

$$\mathbf{V}_{TH} = \mathbf{Z}_N \mathbf{I}_N,$$

$$\mathbf{Z}_{TH} = \mathbf{Z}_N$$



$$\boxed{\mathbf{Z}_{Th} = \mathbf{Z}_N = \frac{\mathbf{V}_{Th}}{\mathbf{I}_N}}$$

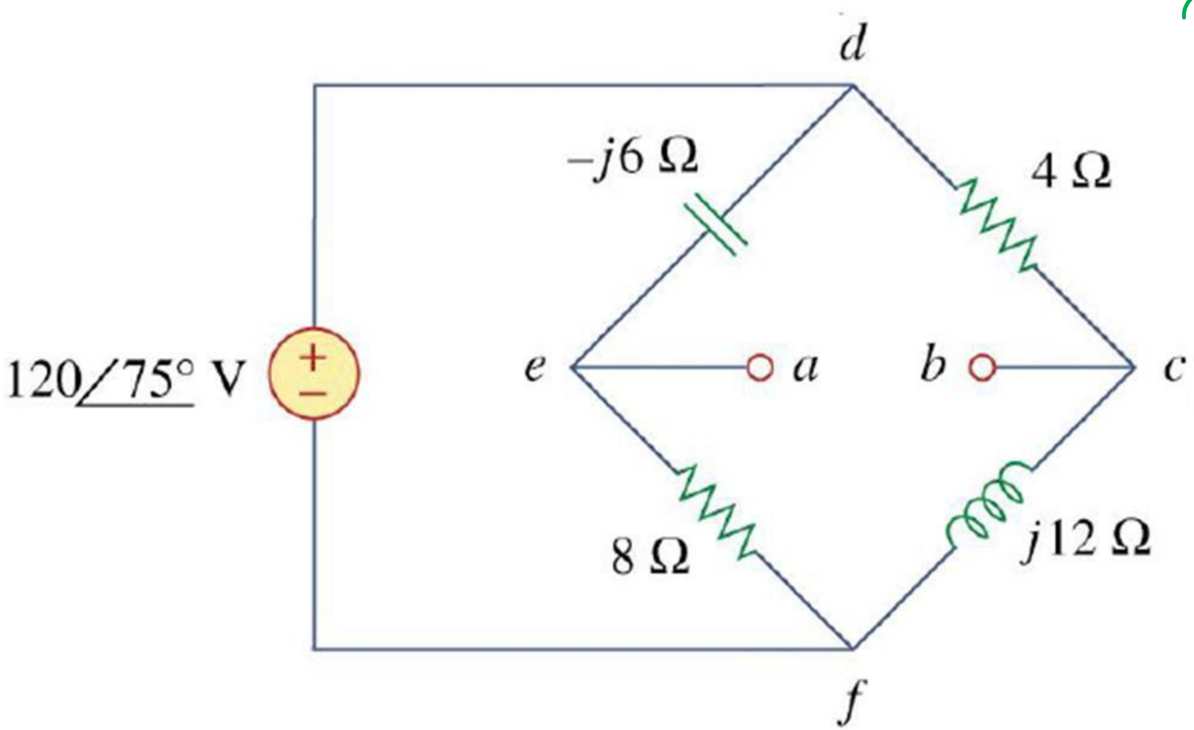


Ex. 20: Thevenin & Norton Equivalent Circuits

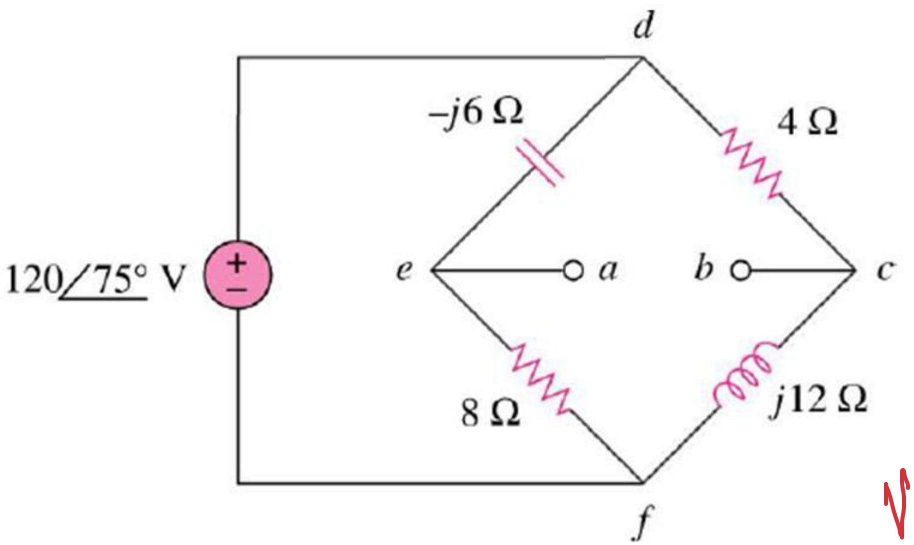
Obtain the Thevenin equivalent at terminals *a*-*b* of the circuit.

Ko hiểu hình

?



Sol. of Ex. 20:

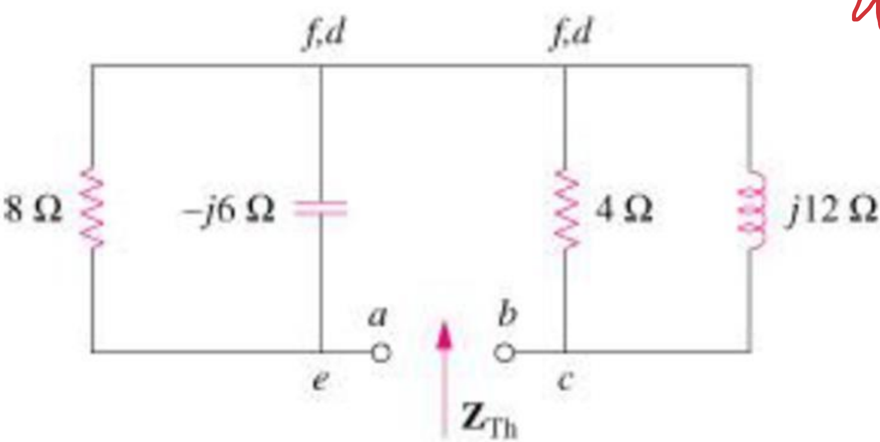


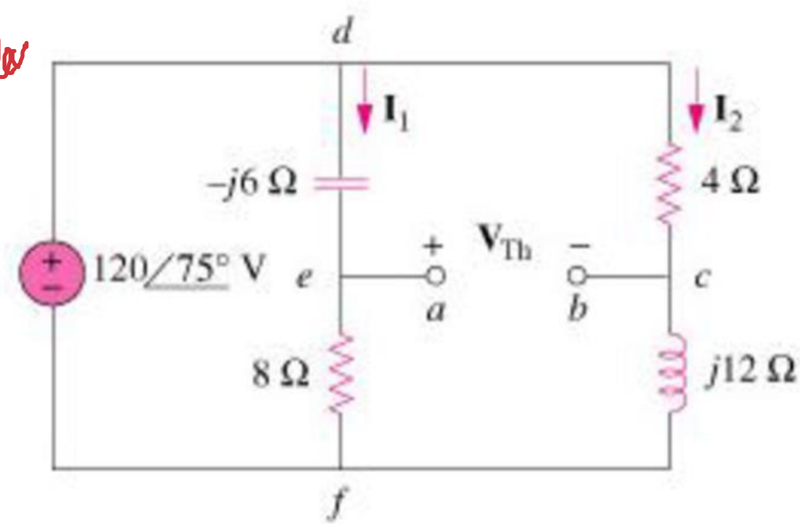
$$\mathbf{Z}_{Th} = (8 \,||\, -j6) + (4 \,||\, j12)$$
$$= 6.48 - j2.64$$

?

$$\mathbf{V}_{Th} = \left(\frac{8}{8 - j6} - \frac{j12}{4 + j12} \right) \times 120\angle 75^\circ$$
$$= 37.95\angle 220.31^\circ \text{ V}$$

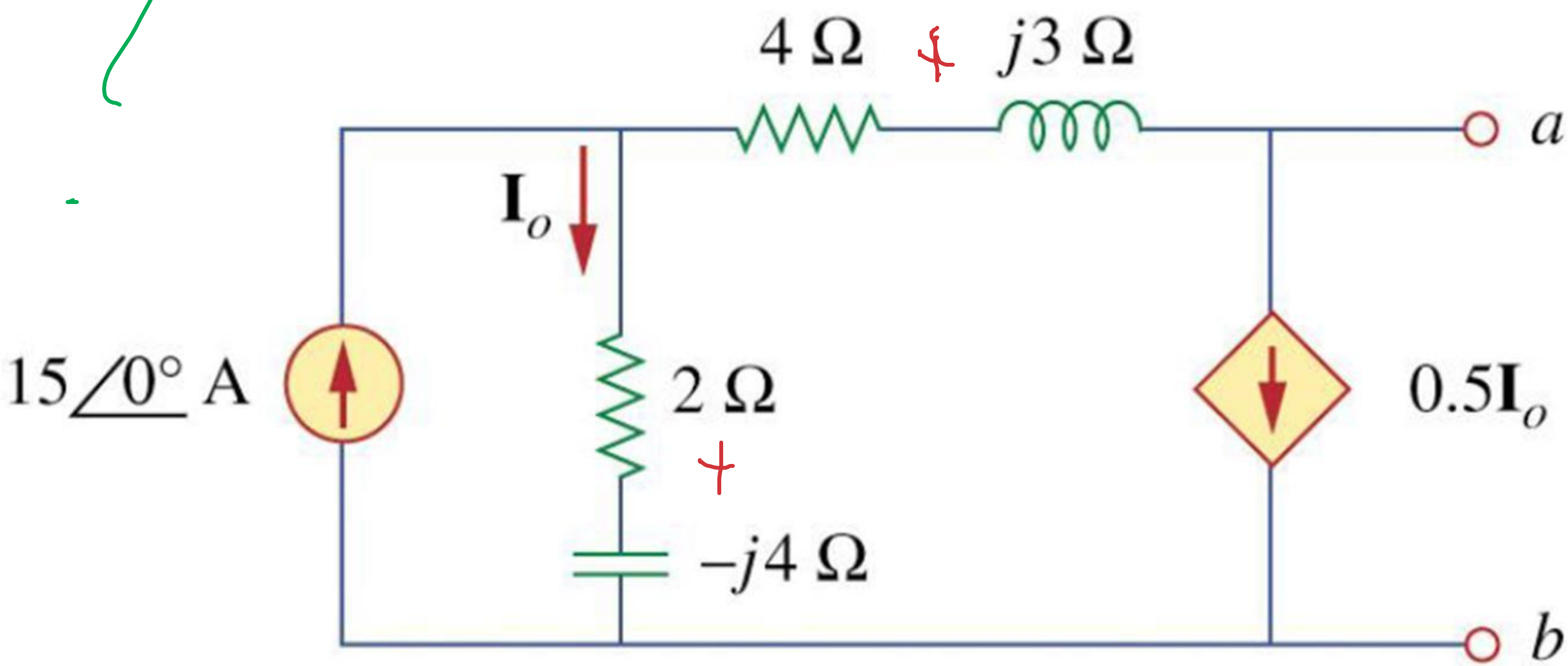
Voltage
divider



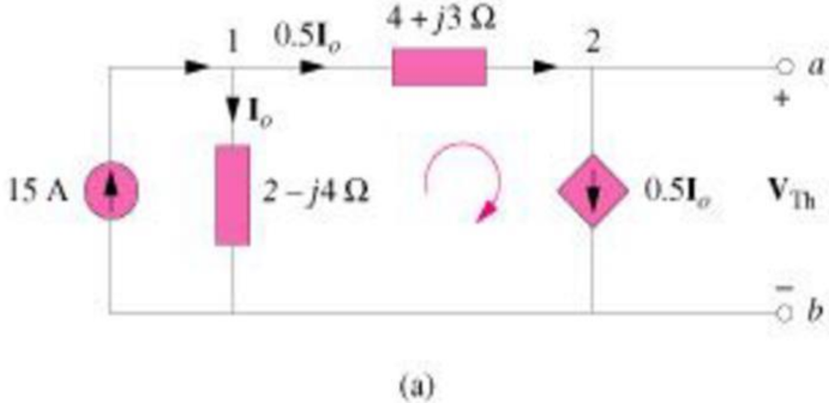


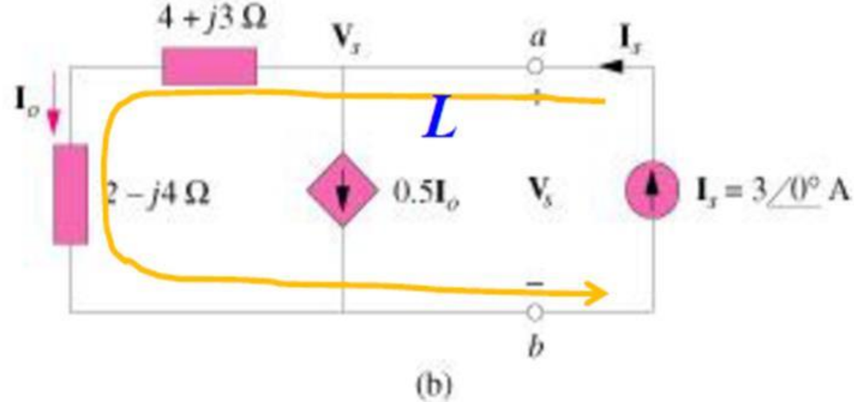
Ex. 21:

Find the Thevenin equivalent of the circuit as seen from terminals *a*-*b*.



Sol. of Ex. 21:





KCL at $\boxed{\text{node 1}}$:

$$15 = \mathbf{I}_0 + 0.5\mathbf{I}_0 \Rightarrow \mathbf{I}_0 = 10$$

$0.5I_0$

KVL for loop:

$$-\mathbf{I}_0(2-j4) + 0.5\mathbf{I}_0(4+j3) + \mathbf{V}_{Th} = 0$$

$$\Rightarrow \mathbf{V}_{Th} = 10(2-j4) - 5(4+j3)$$

$$= -j55$$

$$= 55\angle -90^\circ \text{ V}$$

Set $\mathbf{I}_s = 3$ for simplicity,

KCL at node a:

$$\mathbf{I}_s = 3 = \mathbf{I}_0 + 0.5\mathbf{I}_0 \Rightarrow \mathbf{I}_0 = 2$$

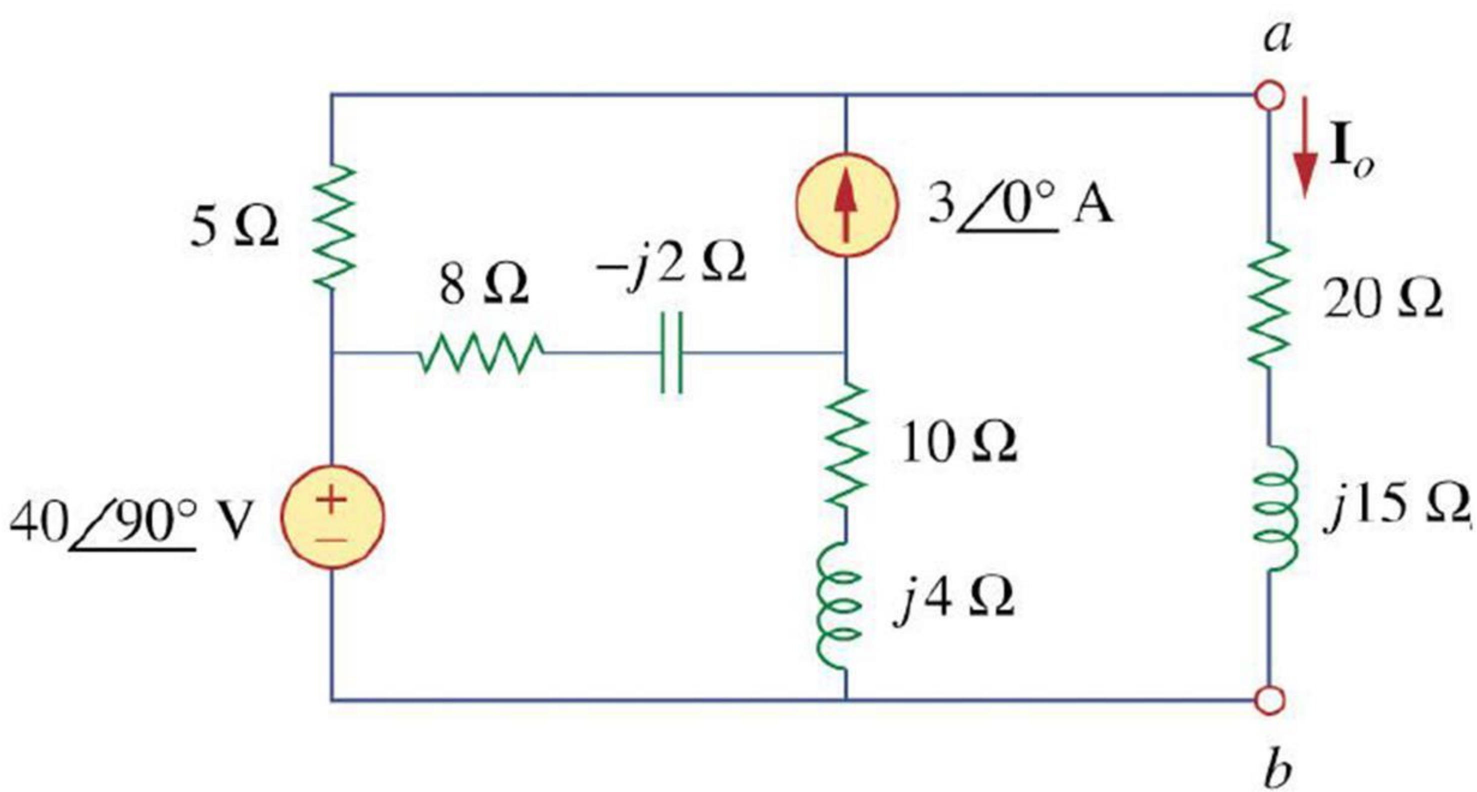
KVL for loop L:

$$\mathbf{V}_s = \mathbf{I}_0(4+j3+2-j4) = 2(6-j)$$

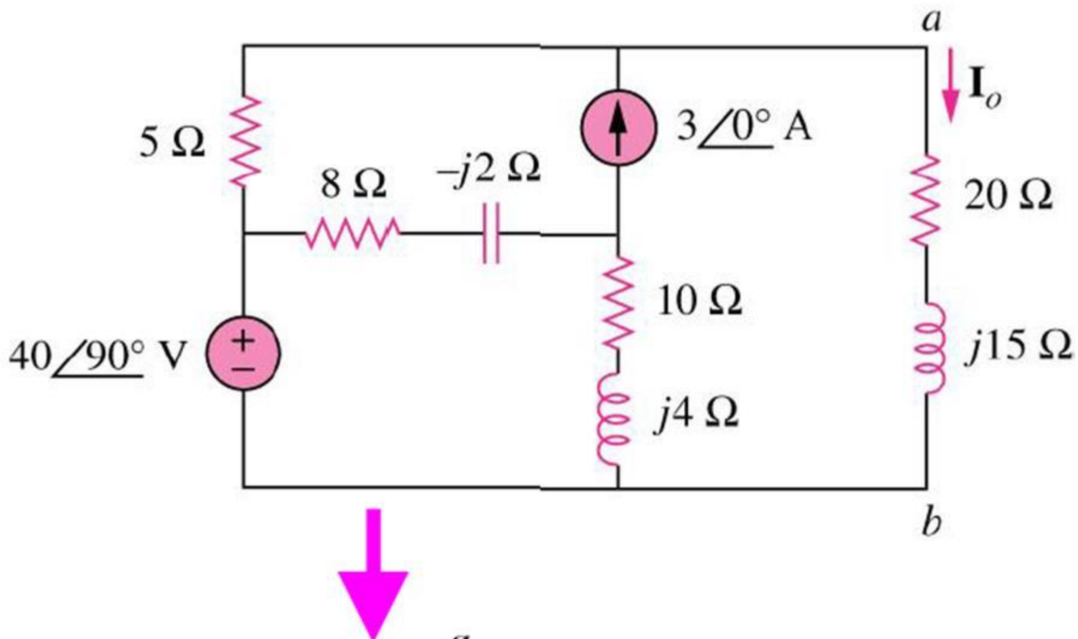
$$\Rightarrow \mathbf{Z}_{Th} = \frac{\mathbf{V}_s}{\mathbf{I}_s} = \frac{2(6-j)}{3}$$

Ex. 22:

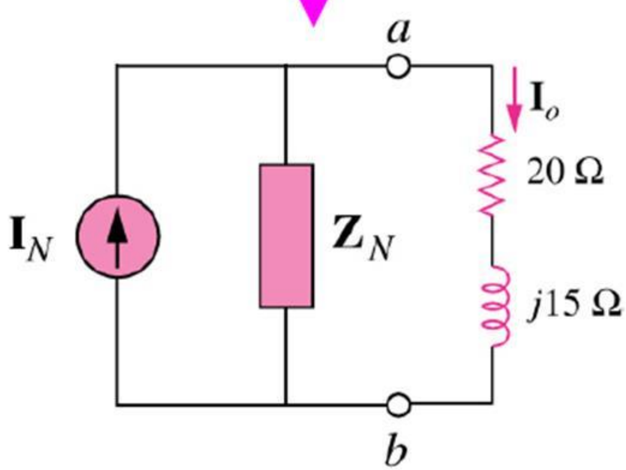
Obtain current $\mathbf{I}_o$ using Norton's theorem.



Sol. of Ex. 22:



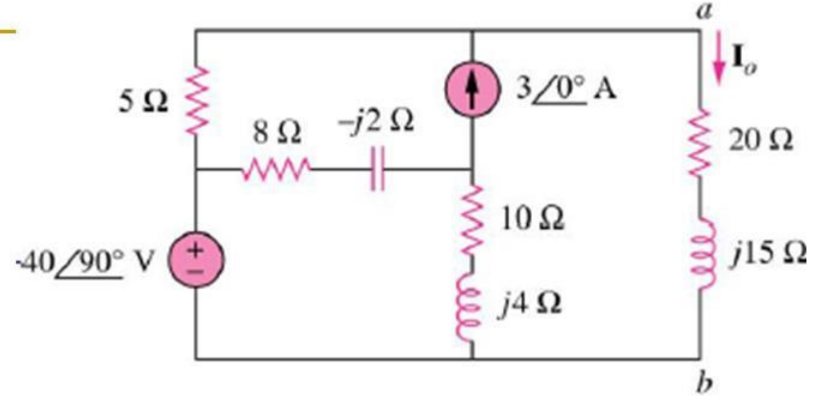
?



By current division,

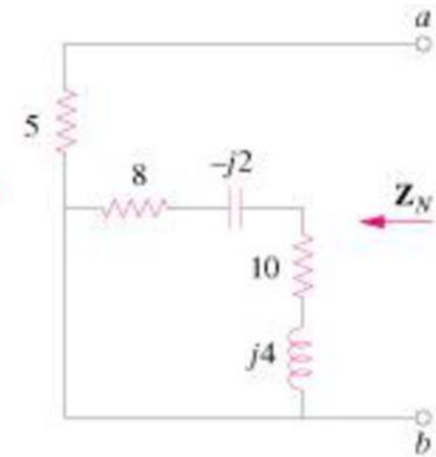
$$\mathbf{I}_0 = \frac{\mathbf{Z}_\mathrm{N}}{\mathbf{Z}_\mathrm{N} + (20 + j15)} \mathbf{I}_\mathrm{N}$$

Sol. of Ex. 22: cont.



(1) $\mathbf{Z}_N$ can be found easily, $\mathbf{Z}_N = 5$

(2) Apply mesh analysis to get $\mathbf{I}_N$.



KVL for mesh 1:

$$-j40 + (18 + j2)\mathbf{I}_1 - (8 - j2)\mathbf{I}_2 - (10 + j4)\mathbf{I}_3 = 0 \cdots (a)$$

KVL for the supermesh:

$$(13 - j2)\mathbf{I}_2 + (10 + j4)\mathbf{I}_3 - (18 + j2)\mathbf{I}_1 = 0 \cdots (b)$$

KCL at node a:

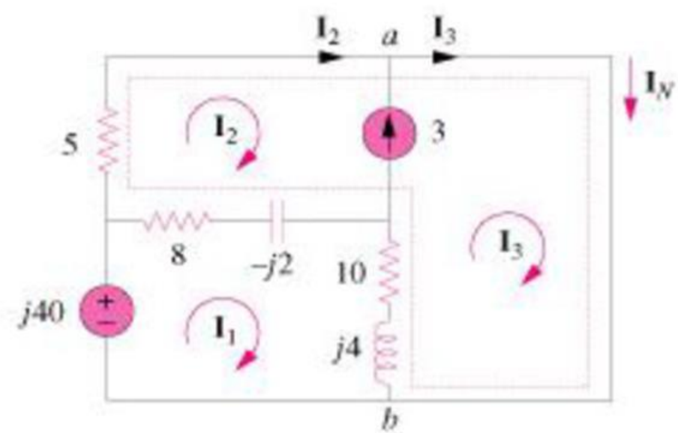
$$\mathbf{I}_3 = \mathbf{I}_2 + 3 \cdots (C)$$

?

$(a) \sim (c)$ give

$$\mathbf{I}_N = \mathbf{I}_3 = 3 + j8$$

$$\Rightarrow \mathbf{I}_0 = \frac{5}{5 + 20 + j15}\,\mathbf{I}_N = 1.465\angle 38.48^\circ \text{ A}$$



Ex. 23

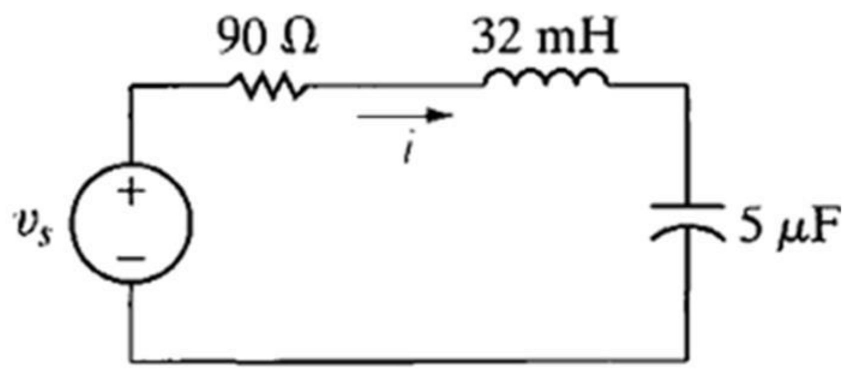
A 90 Ω resistor, a 32 mH inductor, and a 5 μF capacitor are connected in series across the terminals of a sinusoidal voltage source, as shown in the Fig.

The steady-state expression for the source voltage

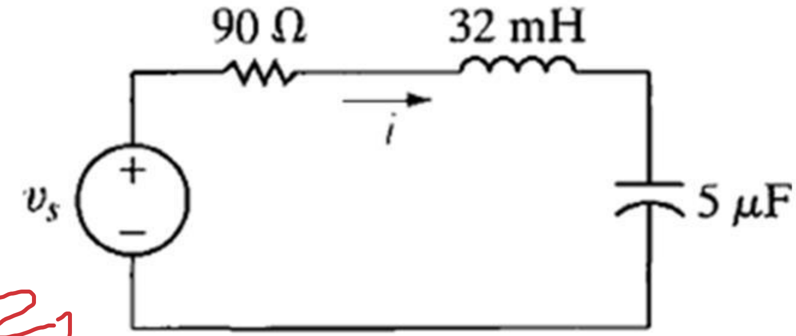
$$v_s = 750\cos(5000t + 30°)\ \text{V}.$$

a) Construct the frequency-domain equivalent circuit.

b) Calculate the steady-state current i by the phasor method.



Ex. 23: Sol.



a) From the expression for v_s, we have ω = 5000 rad/s. Therefore the impedance of the 32 mH inductor is $Z_L = j\omega L = j(5000)(32 \times 10^{-3}) = j160\ \Omega$, Z_L

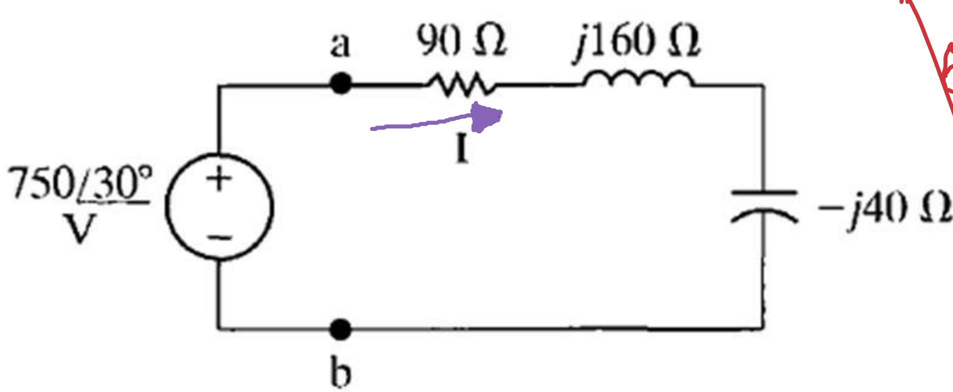
and the impedance of the capacitor is $Z_C = j\dfrac{-1}{\omega C} = -j\dfrac{10^6}{(5000)(5)} = -j40\ \Omega.$

Z_c

The phasor transform of v_s is

$$\mathbf{V}_s = 750\,\angle 30^\circ\ \text{V}.$$

convert to phasor form



Ex. 23: Sol.

b) We compute the phasor current simply by dividing the voltage of the voltage source by the equivalent impedance between the terminals a, b.

Z

$$Z_{ab} = 90 + j160 - j40$$

$$= 90 + j120 = 150\angle 53.13^\circ\ \Omega.$$

$$\Rightarrow \quad \mathbf{I} = \frac{750\angle 30^\circ}{150\angle 53.13^\circ} = 5\angle -23.13^\circ\ \text{A}.$$

Thus, the steady-state expression for i directly:

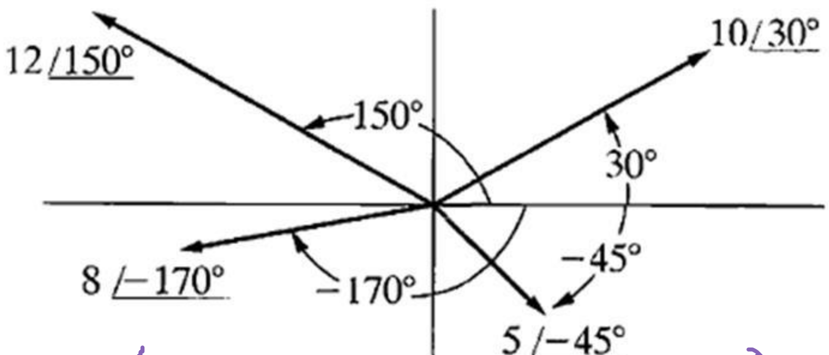
$i = 5\cos(5000t - 23.13°)\ A.$

Phasor Diagrams

- A phasor diagram shows the magnitude and phase angle of each phasor quantity in the complex-number plane.
- Phase angles are measured counterclockwise from the positive real axis.
- Magnitudes are measured from the origin of the axes.

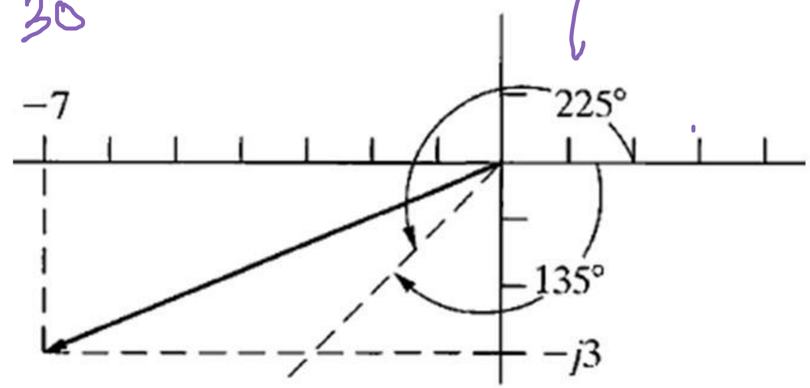
150°

30°



−170°

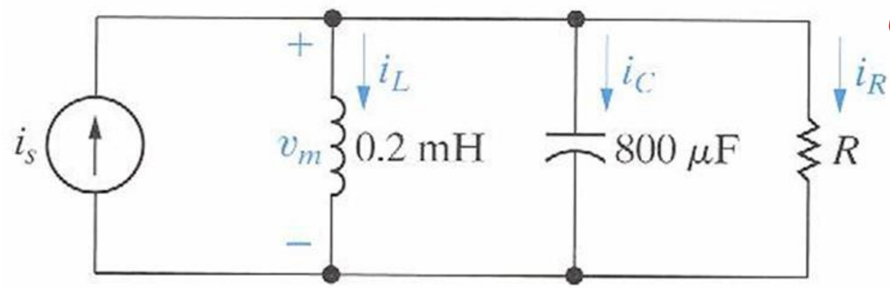
−45°



$$-7 - j3 = 7.62 \angle -156.80^\circ.$$

Phasor diagram is useful for checking calculator calculations

Phasor Diagrams – An example



Use a phasor diagram to find the value of R that will cause the current through that resistor, i_R, to lag the source current, i_S, by 45° when ω = 5 krad/s

$$I_S = I_L + I_C + I_R$$

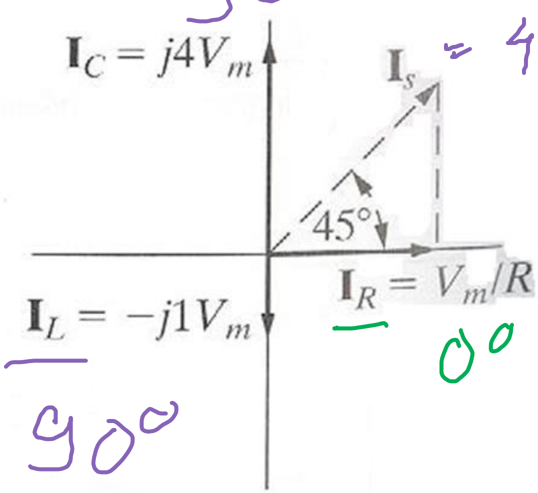
$$\textit{Assume} \quad V_m = V_m\angle 0^\circ$$

$$I_L = \frac{V_m\angle 0^\circ}{j(5000)(0.2\times 10^{-3})} = V_m\angle -90^\circ$$

$$I_C = \frac{V_m\angle 0^\circ}{-j/(5000)(800\times 10^{-6})} = 4V_m\angle 90^\circ$$

$$I_R = \frac{V_m\angle 0^\circ}{R} = \frac{V_m}{R}\angle 0^\circ$$

90°



= 45°

0°

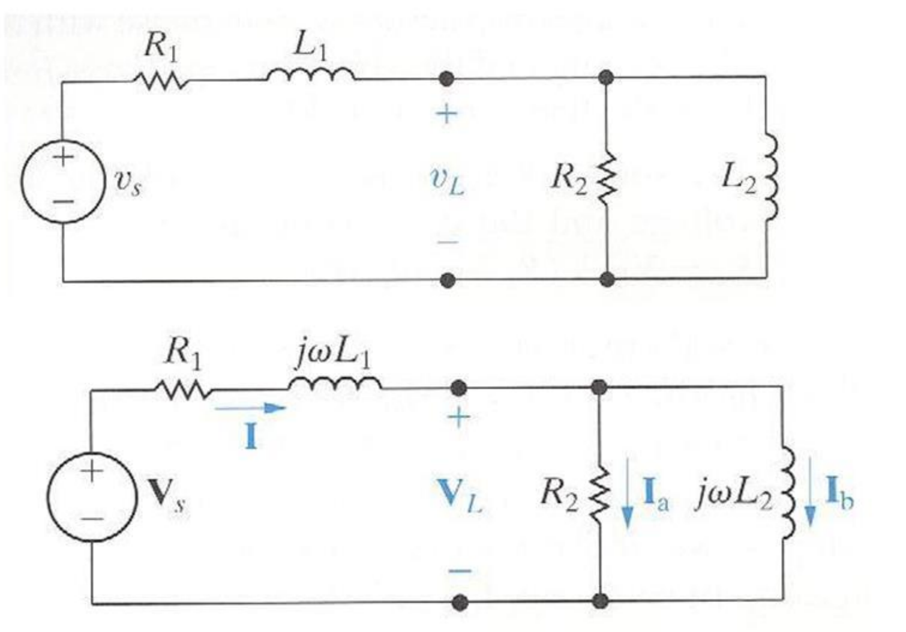
-90°

From phasor diagram, we have
R = 1/3 Ω

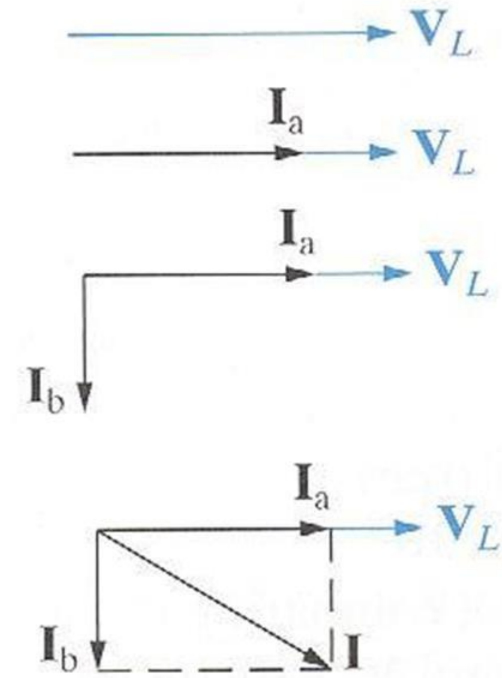
ω = 5000 rad/s

i_S = 45°

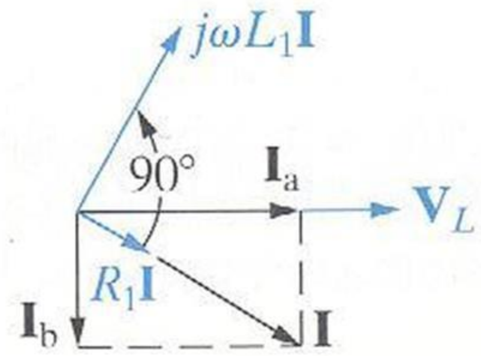
Example



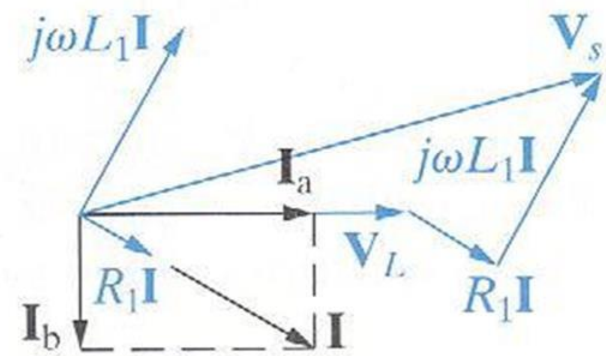
R_1
L_1
v_s
v_L
R_2
L_2
R_1
$j\omega L_1$
$\mathbf{I}$
$\mathbf{V}_s$
$\mathbf{V}_L$
R_2
$\mathbf{I}_a$
$j\omega L_2$
$\mathbf{I}_b$



$\mathbf{V}_L$
$\mathbf{I}_a$
$\mathbf{V}_L$
$\mathbf{I}_a$
$\mathbf{V}_L$
$\mathbf{I}_b$
$\mathbf{I}_a$
$\mathbf{V}_L$
$\mathbf{I}_b$
$\mathbf{I}$



$j\omega L_1 \mathbf{I}$
90°
$\mathbf{I}_a$
$\mathbf{V}_L$
$R_1\mathbf{I}$
$\mathbf{I}_b$
$\mathbf{I}$



$j\omega L_1 \mathbf{I}$
$\mathbf{V}_s$
$j\omega L_1 \mathbf{I}$
$\mathbf{I}_a$
$\mathbf{V}_L$
$R_1\mathbf{I}$
$R_1\mathbf{I}$
$\mathbf{I}_b$
$\mathbf{I}$