$\det(A, B) = \det(A) \cdot \det(B)$

* $\det(A^n) = (\operatorname{Det}(A))^n$

* $\det(A^{-1}) = (\det(A))^{-1}$

Find A^{-1} by adjugate method.

Ex: A = 3×3 matrix, $\det(A) = 7$. $\operatorname{Det}(-2A) = ?$

$\Rightarrow \operatorname{Det}(-2A) = (-2)^3 \cdot \det(A)$
$= -8 \cdot 7 = -56$

$\operatorname{Det}(A^{-1}) = \dfrac{1}{\operatorname{Det}(A)}$ | $(-1)^n \det A = \det A$ $\left[\begin{array}{l} \det A = 0 \\ n \text{ is even} \end{array}\right.$

Ex:
A is invertible $A = \begin{bmatrix} c & 1 & 0 \\ 0 & 2 & c \\ -1 & c & 5 \end{bmatrix}$ is invertible

$$\operatorname{Det} A = \begin{vmatrix} c & 1 & 0 \\ 0 & 2 & c \\ -1 & c & 5 \end{vmatrix} = c(-1)^2 \begin{vmatrix} 2 & c \\ c & 5 \end{vmatrix} + 1(-1)^2 \begin{vmatrix} 0 & c \\ -1 & 5 \end{vmatrix}$$
$$= c(10 - c^2) + c$$
$$= c(9 - c^2)$$

A is invertible $\Leftrightarrow \operatorname{Det}(A) \neq 0 \Rightarrow c \neq 0,\ c \neq 3,\ c \neq -2$

$= 2^4 (\det(A))^2 (\det(B))^{-1} (\det(C))^3 (\det B) \det(A)^{-1}$

$= 16 \det A (\det C)^3 = 16(-1)1 = -16$

$\mathrm{Det}(2A^{-1}) = -4$

$\Rightarrow 2^3 (\mathrm{Det}(A))^{-1} = -4$

$\Rightarrow \quad \mathrm{Det}(A)^{-1} = \frac{-1}{2}$

$\mathrm{Det}(A) = -2$

$* \; (\mathrm{Det}(A))^3 (\mathrm{Det}(B))^{-1} = -4$

$= \quad -8 \frac{1}{\mathrm{Det}\,B} = -4$

$\Rightarrow \quad \mathrm{Det}\,B = 2$

Find A^{-1}

1. Find Det (A), if det (A) = 0. we conclude that A is non-invertible. Otherwise, go to step 2.

2.

$$A^{-1} = \frac{C^T}{\text{Det } A} = \frac{1}{\text{Det} A}\begin{bmatrix} C_{11} & C_{21} & \dots & C_{n1} \\ C_{12} & C_{22} & \dots & C_{n2} \\ S_{1n} & C_{2n} & \dots & C_{nn} \end{bmatrix}$$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\text{Det}(A) = ad - bc$

$$C = \begin{bmatrix} (-1)^2 d & (-1)^3 c \\ (-1)^3 b & (-1)^4 a \end{bmatrix} = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$$

$$\Rightarrow A = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

EX:

$$A = \begin{bmatrix} 4 & 0 & 3 \\ 1 & 9 & 7 \\ 0 & 6 & 4 \end{bmatrix}$$

$1 \le i, j \le 3$

$A^{-1} = ?$

$$\text{Det }(A) = 4(-1)^2 \begin{vmatrix} 9 & 7 \\ 6 & 4 \end{vmatrix} + 1(-1)^3 \begin{vmatrix} 0 & 3 \\ 6 & 4 \end{vmatrix}$$

$$= 4(9 \cdot 4 - 6 \cdot 7) + 18 = -6$$

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$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix}, \quad A^{-1} = ?$$

$$\operatorname{Det}(A) = 2(-1)^3 \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} + 2(-1)^5 \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} = 12$$