HO CHI MINH CITY INTERNATIONAL UNIVERSITY

FINAL EXAMINATION

Academic year 2010-2011, Semester 3

Duration: 120 minutes

SUBJECT: **Differential Equations**	
Chair of Department of Mathematics	Lecturer:
Signature:	Signature:
Full name: Prof. Phan Quoc Khanh	Full name: Dr. Pham Huu Anh Ngoc

Instructions:

- *Open-book examination. Laptops are NOT allowed.*

Question 1. (15 marks) The rate of growth of an investment is proportional to the amount A of the investment at any time t. An investment of \$2000 increases to a value of \$2983.65 in 5 years. Find its value after 10 years.

Question 2. (a) (10 marks) Show that $y(x) := \frac{1}{x}e^x$ is a solution of the linear differential equation

$$xy'' + 2y' - xy = 0.$$

(b) (15 marks) Solve the linear differential equation

$$xy'' + 2y' - xy = e^x.$$

Question 3. (25 marks) Find the general solution of the following differential equation

$$y''' + y'' - 2y' = 2x - e^x.$$

Question 4. (20 marks) Solve the linear system of differential equations

$$\begin{cases} \frac{dx}{dt} &= 12x - 5y \\ \frac{dy}{dt} &= 5x + 12y. \end{cases}$$

Question 5. (15 marks) Suppose that a 12-th order homogeneous linear differential equation with constant coefficients has characteristic roots:

$$0, 0, 0, 0, 1, 1, 1 + i, 1 - i, 2 + 3i, 2 + 3i, 2 - 3i, 2 - 3i.$$

What is the general solution of the differential equation?

End.

SOLUTIONS:

Question 1. Let A be the value of the investment at time t (in years). Then we have

$$\frac{dA}{dt} = kA, \qquad t \geq 0,$$

where k is a constant. This gives

$$A(t) = A(0)e^{kt}, \qquad t \geq 0.$$

By assumptions, $A(0) = 2000$ and $A(5) = 2983.65$. Then we have $k \approx 0.08$. The value of the investment after 10 years is

$$A(10) = 2000e^{0.8} \approx 4451.$$

Question 2. The general solution of the homogeneous equation $xy'' + 2y' - xy = 0$ is

$$y(x) = c_1 \frac{e^x}{x} + c_2 \frac{e^{-x}}{x}.$$

It is easy to check that $y_p(x) = \frac{1}{2}e^x$ is a particular solution of $xy'' + 2y' - xy = e^x$. So its general solution is

$$y(x) = c_1 \frac{e^x}{x} + c_2 \frac{e^{-x}}{x} + \frac{1}{2}e^x.$$

Question 3. The general solution of the homogeneous equation $y''' + y'' - 2y' = 0$ is

$$y(x) = c_1 + c_2 e^x + c_3 e^{-2x}.$$

A particular solution of $y''' + y'' - 2y' = 2x - e^x$ is $y_p(x) = -\frac{1}{3}xe^x - \frac{1}{2}(x^2 + x)$.
So its general solution is

$$y(x) = c_1 + c_2 e^x + c_3 e^{-2x} - \frac{1}{3}xe^x - \frac{1}{2}(x^2 + x).$$

Question 4. The general solution of the given system is

$$x(t) = e^{12t}(c_1 \cos 5t + c_2 \sin 5t); \qquad y(t) = e^{12t}(c_1 \sin 5t - c_2 \cos 5t).$$

Question 5. The general solution is given by

$$y(x) = c_1 + c_2 x + c_3 x^2 + c_4 x^3 + c_5 e^x + c_6 xe^x + c_7 e^x \sin x + c_8 e^x \cos x +$$

$$c_9 e^{2x} \cos 3x + c_{10} e^{2x} \sin 3x + c_9 xe^{2x} \cos 3x + c_{10} xe^{2x} \sin 3x.$$