Question 01: (15 pts)
The monthly sales of a bookstore is given as follow:

13	43	53	44	14	33	18	55	50	25	27	45

Use the given data to answer questions below:
a) Calculate the mean and variance (**5 pts**)
b) Calculate the median and third quartile (**5 pts**)
c) Given that at random days, the sales are 30, and 40 books. Which percentile these data value belong to? (**5 pts**)

– Bấm máy casio

a) Mean $(\bar{x}) = 35$

Variance $(s^2x) = 234,18$

b) Median (Med) = 38

Third quartile (Q_3) (75^{th} percentile)

We have the position of the 75^{th} percentile:

$i = (n+1) \times \% = (12+1)(75\%)$
$= 39/4 = 9,75$

$Q_3 = V(9,75)$

We have $V = V(m) + (i-m)(V(m+1) - V(m))$

$\Rightarrow V(9,75) = V(9) + (9,75-9)[V(10) - V(9)]$
$= 45 + 0,75(50-45)$
$= 45 + 3,75 = 48,75$

c) The sales is 30 books: $V = 30$
Since $27 < 30 < 33 \rightarrow$ Chọn $i = 5$
Percentile at $i=5$:
We have: $P(i) = \dfrac{i \times 100}{n+1}$

$\Rightarrow P(5) = \dfrac{5 \times 100}{12+1} = \dfrac{500}{13}$

$P(6) = \dfrac{6 \times 100}{12+1} = \dfrac{600}{13}$

The percentile of the value 30 is:

We have: $P = P(i) + (P(i+1) - P(i)) \times \dfrac{(v - v(i))}{(v(i+1) - v(i))}$

$= \dfrac{500}{13} + \left(\dfrac{600}{13} - \dfrac{500}{13}\right) \times \dfrac{30-27}{33-27}$

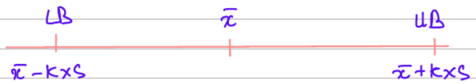
$= \dfrac{550}{13} \approx 42,3.$

Ex. Based on Chebyshev's theorem, find the bounds that should include at least 75% of property valuations

i	1	2	3	4	5	6	7	8	9	10	11	12
V(i)	13	14	18	25	27	33	43	44	45	50	53	55

Ans: Upper bound: UB = $\bar{x} + k \times s = 65.6$
Lower bound: LB = $\bar{x} - k \times s = 4.4$

Mean $(\bar{x}) = 35$
Std $(s_x) = 15,3$



By Chebyshev's theorem, we can see that:

At least $\left(1 - \dfrac{1}{k^2}\right)$ of the elements of any distribution lie within k standard deviations of the mean

$\Rightarrow 1 - \dfrac{1}{k^2} = 0,75 \Rightarrow k = 2$

Upper bound: $(UB) = \bar{x} + k \times s$
$= 35 + 2 \times 15,3 = 65,6$

Lower bound $(LB) = \bar{x} - k \times s$
$= 35 - 2 \times 15,3 = 4,4$

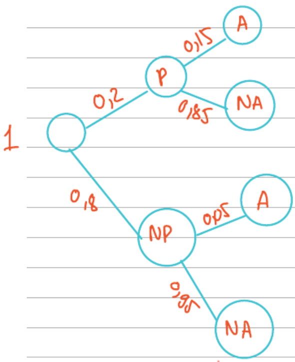
$\Rightarrow$ So that, at least 75% of property valuations lie between 4,4 and 65,6.

An insurance company believes that people can be divided as those who are prone to have accidents and those who are not. The company has established that the probability that an accident-prone person will have an accident in a 1-year period is 0.15; the probability for all others is 0.05. Suppose that the probability that a new policyholder is accident-prone is 0.2.

a) What is the probability that a randomly selected person will have an accident?

b) Given that the person has had an accident, what is the probability that this person is an accident-prone person?

Bayesian's Theorem → Lập biểu đồ



P (accident | accident - prone) = 0,15

→ P(not accident | not accident - prone) = 0,85

P (accident | not accident - prone) = 0,05

→ P (Not accident | not accident - prone) = 0,95

P (accident - prone) = 0,2

→ P (Not accident - prone) = 0,8

a) The probability that a randomly selected person will have an accident is:

We have: P(What we have to find | Given Information)

$$P(A) = P(A|P)\,P(P) + P(A|NP)\,P(NP)$$

$$= 0{,}15 \times 0{,}2 + 0{,}05 \times 0{,}8$$

$$= 0{,}07$$

b) Given that the person has had an accident, the probability that this person is an accident - prone person is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(P|A) = \frac{P(A \cap P)}{P(A)} = \frac{0{,}15 \times 0{,}2}{0{,}07} = 0{,}4286$$

Question 8: In a game show, the player will be rewarded 50 dollars for each successful answer. And he has to stop when he has 3rd wrong answer. Assumed that all question has 4 possible answers, and the player only uses blind guess, what is the probability that the player stop at the 5th round. What is the expected money quantity that a player can obtain after that round?

(Binomial Distribution)

We have:

$$P(X=x) = C(n,x) \times p^x \times (1-p)^{n-x}$$

$$E(X) = \mu = np$$

$$E(aX+b) = aE(X)+b$$

Assume: Success is Wrong answer; X is the number of round.

$\rightarrow$ $P(S) = \frac{3}{4}$ $\rightarrow$ Số lượng câu sai là 3

Success

4 vòng trước; $\rightarrow 4-2$

$$P(X=5) = C_4^2 \times \left(\frac{3}{4}\right)^2 \times \left(1-\frac{3}{4}\right)^2 \times \frac{3}{4} = 0{,}16$$

thua 2

Xác suất thua tại vòng 5

Explaining: Điều kiện để thua tại vòng thứ 5 là phải thua 3 vòng, vì thế tại 4 vòng trước phải có 2 vòng thua và phải thua tại vòng thứ 5 thì mới dừng cuộc chơi được.

Expected number of round: $E(X) = np = 5 \times \frac{3}{4} = \frac{15}{4}$

Expected money quantity: $E(M) = E(50(X-3))$ $\rightarrow$ 3rd wrong answer

$$= 50E(X) - 150$$

$$= 50\left(\frac{15}{4}\right) - 150 = 37{,}5$$

Question 8: In a game show, the player will be rewarded 50 dollars for each successful answer. And he has to stop when he has 1st wrong answer. Assumed that all question has 4 possible answers, and the player only uses blind guess, what is the probability that the player stop at the 5th round. What is the expected money quantity that a player can obtain after that round?

Geometric Distribution

We have: $P(x) = pq^{x-1}$ | $\mu = \frac{1}{p}$ | $E(aX+b) = aE(X)+b$

Success: Wrong answer
X is the number of round

$P(S) = \frac{3}{4}$

$P(X=5) = \left(1-\frac{3}{4}\right)^4 \times \frac{3}{4} = \frac{3}{1024}$

Expected number of round : $E(X) = \frac{1}{p} = \frac{4}{3}$

Expected money quantity :

$E(M) = E(50(X-1)) = 50E(X) - 50$

$= 50 \times \frac{4}{3} - 50 = 16{,}67$ dollars.

Question 03: (10 pts)

The following game is offered. There are 10 cards face-down numbered 1 through 10. You can pick one card. Your reward is $0.50 if the number on the card is less than 3, $1 if less than 5, and is the dollar value on the card otherwise.

a) What is the probability your reward is between $2 and $6 inclusively? (5 pts)
b) What are the expected value and the standard deviation of your reward? (5 pts)

Multinomial Distribution

X	1	2	3	4	5	6	7	8	9	10
P(X)	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10
R(X)	0.5	0.5	1	1	5	6	7	8	9	10

Xác suất

Phần thưởng

We have: $E(x) = \sum_{all\ x} x P(x)$

$\sigma^2 = E(X^2) - [E(X)]^2 = \left[\sum_{all\ x} x^2 P(x)\right] - \left[\sum_{all\ x} x P(x)\right]^2$

$E(R) = \sum R(x).P(x)$

$= \frac{1}{10}\times 0{,}5 + \frac{1}{10}\times 0{,}5 + \dots + \frac{1}{10}\times 10 = 4{,}8$

$Var(R) = E(R^2) - [E(R)]^2 = \left\{\sum [R(x)]^2 P(x)\right\} - [E(R)]^2 =$

$\left(\frac{1}{10}\right)\times\left(0{,}5^2 + 0{,}5^2 + \dots + 10^2\right) - (4{,}8)^2 = 35{,}75 - 23{,}04 = 12{,}71$

=) Standard deviation: $\nu = \sqrt{Var(R)} = \sqrt{12{,}71} = 3{,}56$

A small voting district has 101 female voters and 95 male voters. A random sample of 10 voters is drawn. What is the probability exactly 7 of the voters will be female?

Hypergeometric distribution

N = 101 + 95 = 196 voters
S = 101 (female)
n = 10 : randomly choose 10 voters
x = 7 : choose 7 female

We have: $P(x) = \frac{\binom{S}{x}\binom{N-S}{n-x}}{\binom{N}{n}}$

3 nam còn lại trong 95 nam

$P(X=7) = \frac{C_7^{101} \times C_3^{95}}{C_{10}^{196}} = 0{,}13$

Chọn 7 nữ trong 101 nữ

10 người trong đống 196 người

Assume that the number of defect screws in a box follows Poisson distribution with $\lambda = 2$. The box will pass the quality test if number of defect screw is smaller or equal to 3.

a) Find the probability passing the quality test of a box. (7pts)

b) In stead of checking each box, now two boxes are combined as a package. A package will pass the quality test if number of defect screws is smaller or equal to 6. Find the probability passing the quality test of a package. (8pts)

Poisson Distribution

We have: $P(x) = \frac{\mu^x e^{-\mu}}{x!}$ for $x = 1, 2, 3, \ldots$

a) $\lambda = 2$

$$P(x \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$
$$= \sum_{x=0}^{3} \left(\frac{2^x e^{-2}}{x!} \right) = 0,8571$$

b) $\lambda = 4$ (2 boxes combined as a package)

$$P(x \leq 6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6)$$
$$= \sum_{x=0}^{6} \left(\frac{4^x e^{-4}}{x!} \right) = 0,8893$$

Example: You arrive at a bus stop at 10 o'clock, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30.

(a) What is the probability that you will have to await longer than 10 minutes?

(b) If, at 10:15, the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?

Let X denote the arrival of bus at the bus stop

$f(x) = 1/30$ $(x \in [0, 30])$

a) $P(x > 10) = \int_{10}^{30} f(x)\,dx = \int_{10}^{30} \frac{1}{30}\,dx = \frac{2}{3}$

b) $P(25 > X > 15) = \int_{15}^{25} f(x)\,dx = \int_{15}^{25} \frac{1}{30}\,dx = \frac{1}{3}$