# Techniques of
# Circuit Analysis
(Chapter 4)

Link to download materials: **Blackboard of IU**

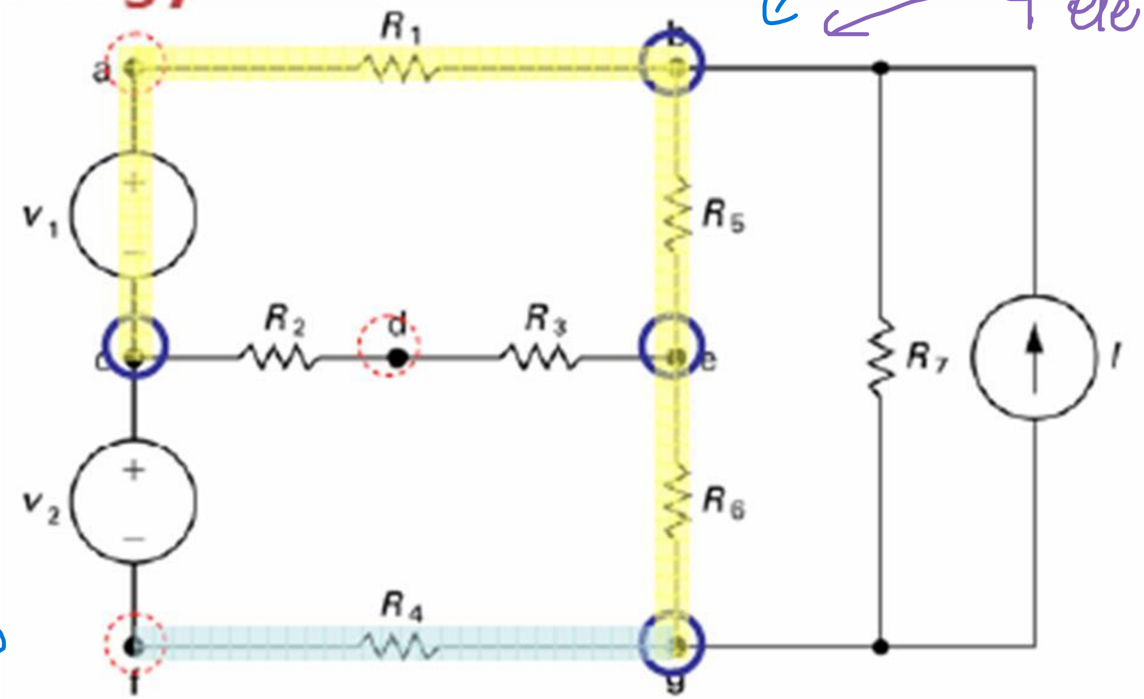
# Outline

- *The node-voltage method*
- *The mesh-current method*
- *Source transformation*
- *Thevenin & Norton equivalents*
- *Maximum power transfer*
- *Supper position*

# Terminology

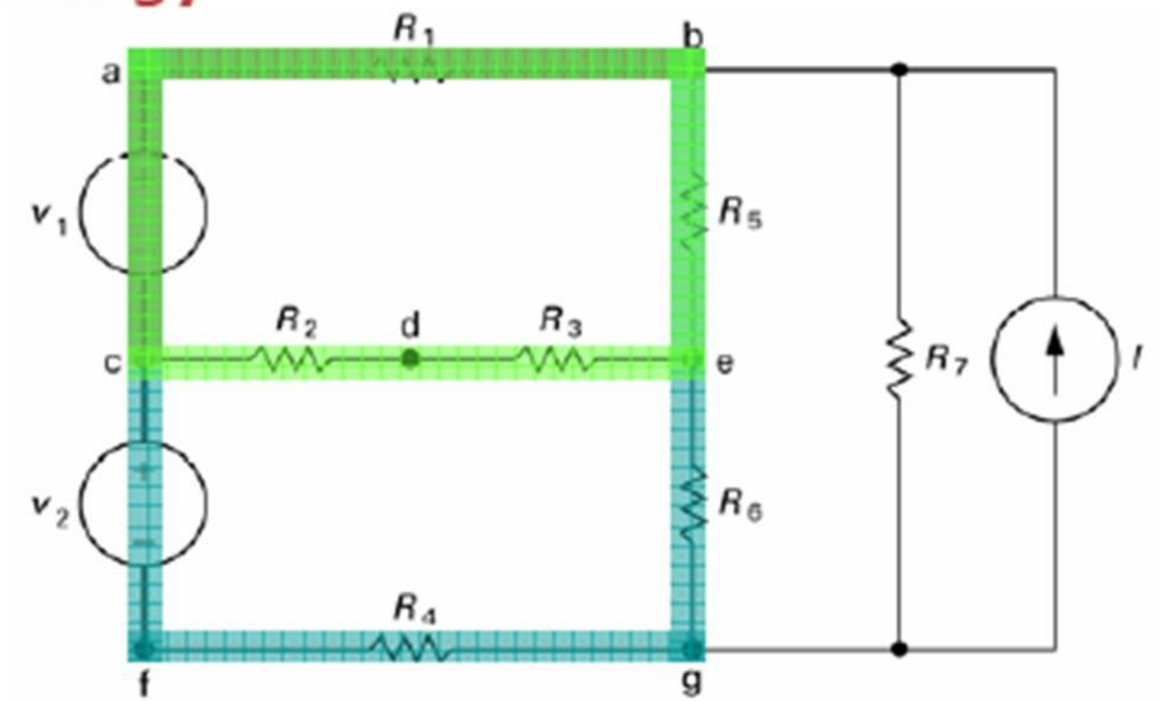
ess node
4 elements

node




| | | |
|---|---|---|
| Node | A point where two or more circuit elements join | a, f |
| Essential Node | A node where three or more circuit elements join | b, e, g |
| Path | A trace of adjoining basic elements with no elements included more than once | $v_1$-$R_1$-$R_5$-$R_6$ |
| Branch | A path that connects two nodes | $R_4$ |

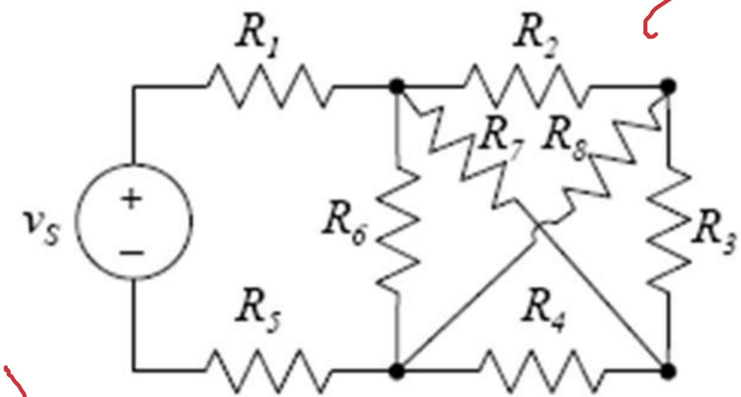
## Terminology

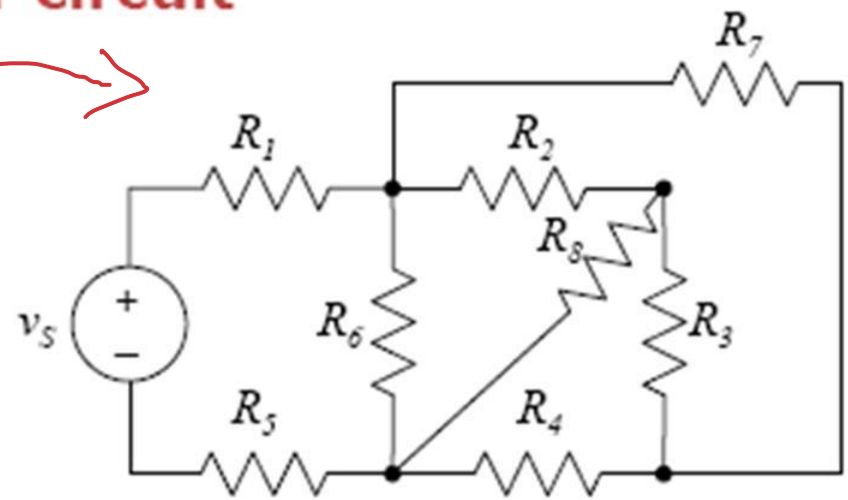



| | | |
|---|---|---|
| Essential branch | A path which connects two essential nodes without passing through an essential node | $v_1$-$R_1$ |
| Loop | A path whose last node is the same as the starting node | $v_1$-$R_1$-$R_5$-$R_6$-$R_4$-$v_2$ |
| Mesh | A loop that does not enclose any other loop | $v_1$-$R_1$-$R_5$-$R_3$-$R_2$ |

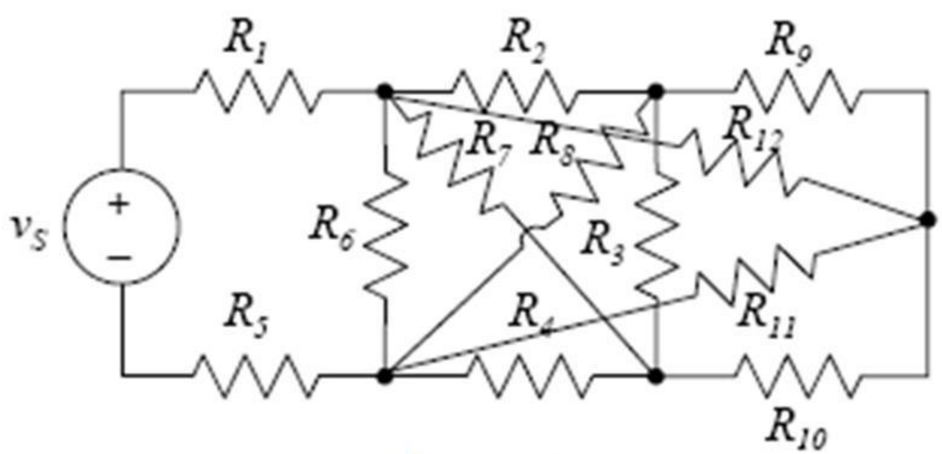
loop nhỏ nhất / lưới điện

# Planar and Non-planar circuit




Planar Circuits



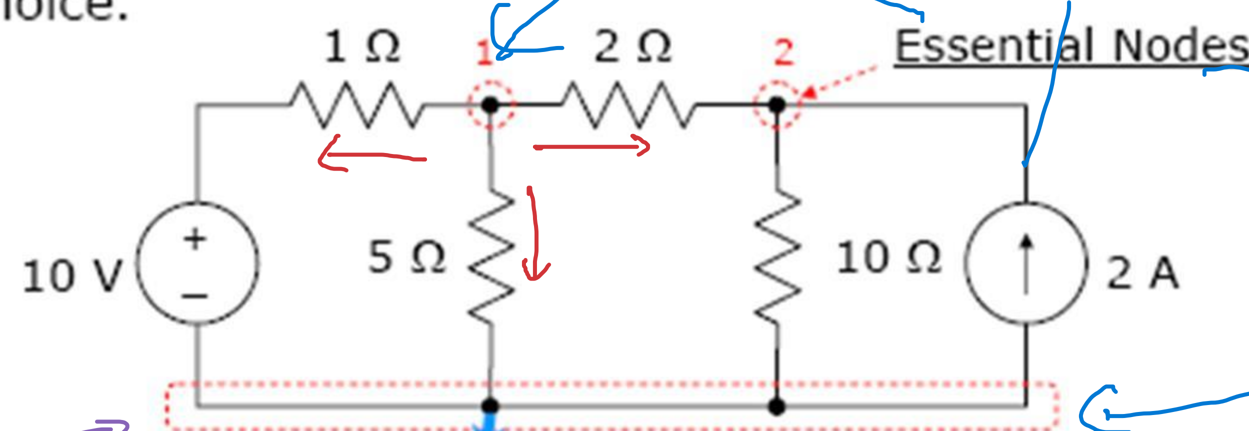

Non-Planar Circuits

Plannar circuit is a circuit that can be drawn on a plane with no crossing branches

# Introduction to node-voltage method

- Can be applied to both planar and non-planar circuits.
- 1st redraw the circuit so that no branches cross over.
- 2nd mark clearly all essential nodes in the circuit.
  - In a circuit with $n_e$ essential nodes, $n_e$-1 node voltage can be written.

    3 2
- 3rd select one of the essential nodes to be the reference node. GND (0V)
  - Generally the node with the most branches is a good choice.



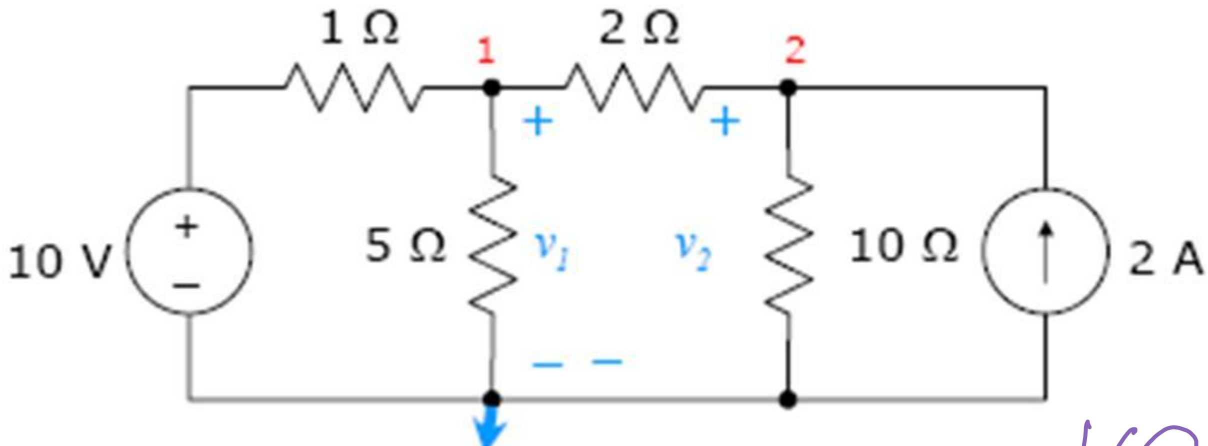

select this

Ref (0V) ground → clean node change

$V_2 : \frac{V_2 - V_1}{2} + \frac{V_2}{10} - 2 = 0 \ (2)$ $V_1 : \frac{V_1 - 10}{1} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} = 0 \ (1)$

# Introduction to node-voltage method

- 4th define the node voltages. 0
  - Voltage rise from the reference node to a non-reference node.



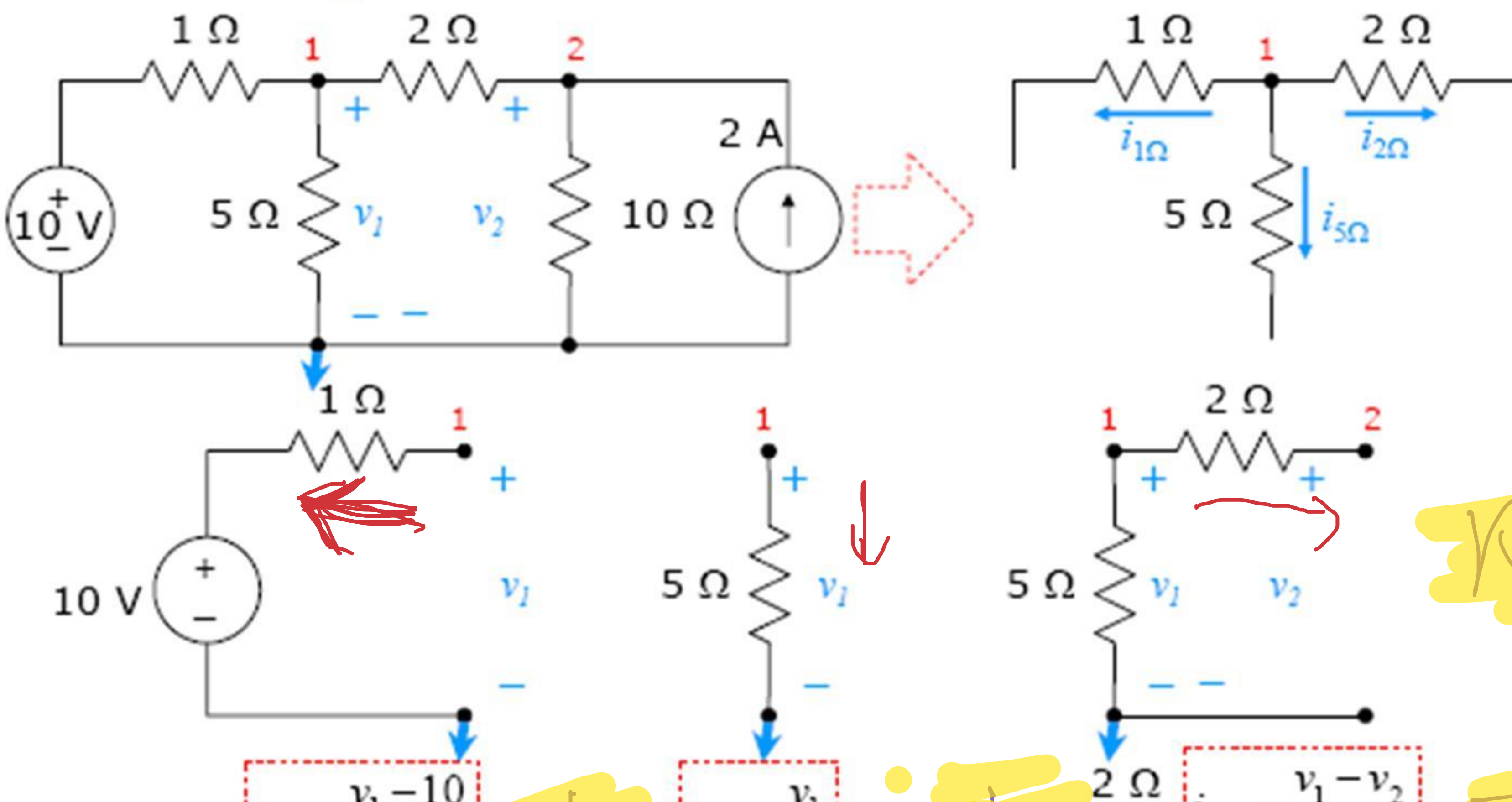

KCL

- 5th generate the node-voltage equations.
  - Write the current leaving each branch connected to a non-reference node as a function of the node voltages. +
  - Apply KCL at the nodes by summing the currents.

$\sum i = 0$

# Introduction to node-voltage method

- At node 1

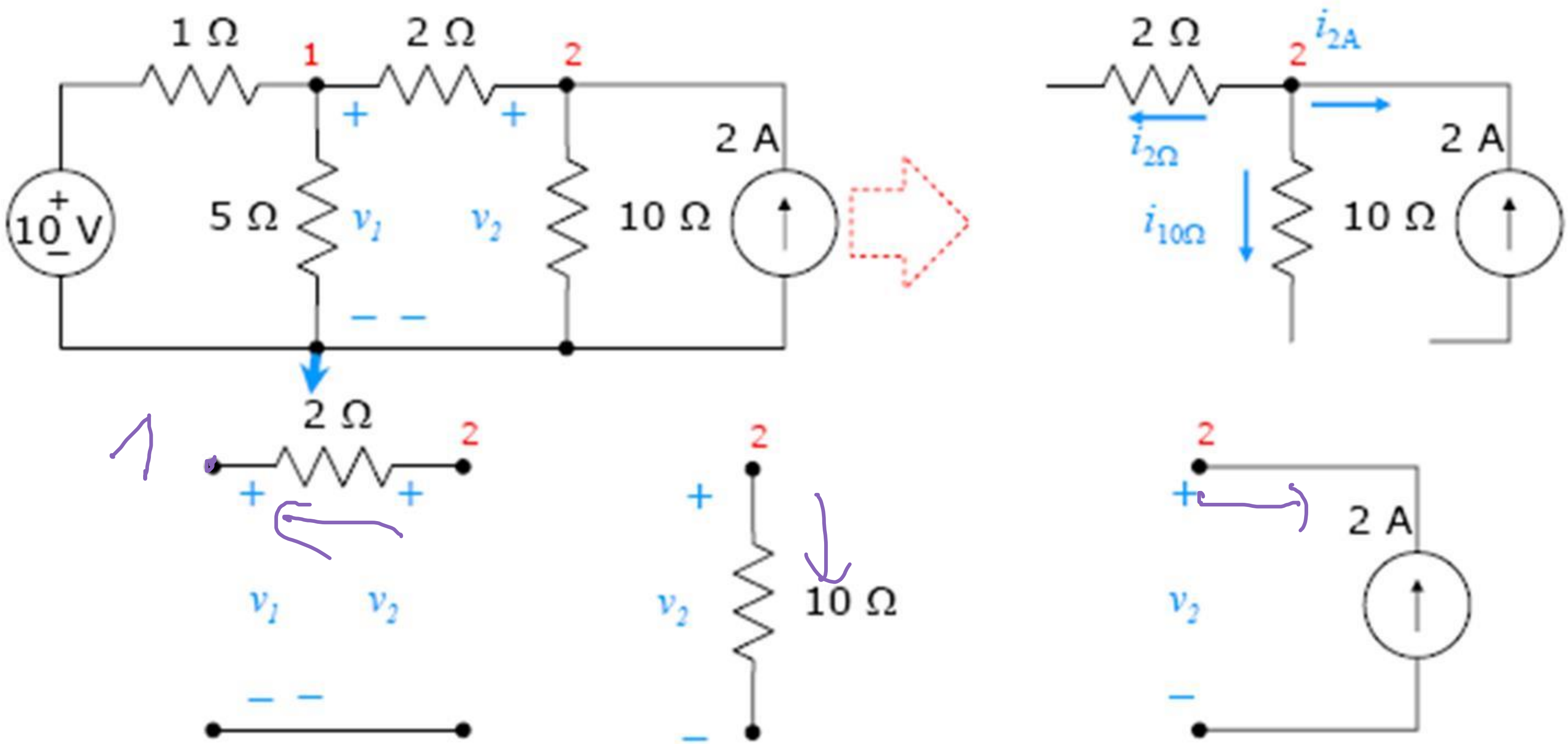



KCL

$$\boxed{i_{1\Omega} = \frac{v_1 - 10}{1}} + \boxed{i_{5\Omega} = \frac{v_1}{5}} + \boxed{i_{2\Omega} = \frac{v_1 - v_2}{2}} = 0$$

# Introduction to node-voltage method

- At node 2




$$\boxed{i_{2\Omega} = \frac{v_2 - v_1}{2}} + \boxed{i_{10\Omega} = \frac{v_2}{10}} + \boxed{i_{2A} = -2}$$

$10(v_1 - 10) + 2V_1 + 5(V_1 - V_2) = 0$

$17V_1 - 5V_2 = 100$

# Introduction to node-voltage method

KCL $\quad i_{1\Omega} + i_{5\Omega} + i_{2\Omega} = 0 \quad \Rightarrow \quad \boxed{\dfrac{v_1 - 10}{1} + \dfrac{v_1}{5} + \dfrac{v_1 - v_2}{2} = 0}$ **(1)**

KCL $\quad i_{2\Omega} + i_{10\Omega} + i_{2A} = 0 \quad \Rightarrow \quad \boxed{\dfrac{v_2 - v_1}{2} + \dfrac{v_2}{10} - 2 = 0}$ **(2)**

$5(V_2 - V_1) + V_2 - 20 = 0$

Rearranging the equations

$$\begin{cases} 17v_1 - 5v_2 = 100 \\ -5v_1 + 6v_2 = 20 \end{cases} \Rightarrow \quad v_1 = 9\frac{1}{11}\,\text{V} \;\; \& \;\; v_2 = 10\frac{10}{11}\,\text{V}$$

$i_{1\Omega} = \dfrac{10V - v_1}{1}$

Or Using Matrices

$$\begin{bmatrix} 17 & -5 \\ -5 & 6 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 20 \end{bmatrix} \Rightarrow AV = I \Rightarrow V = A^{-1} I$$

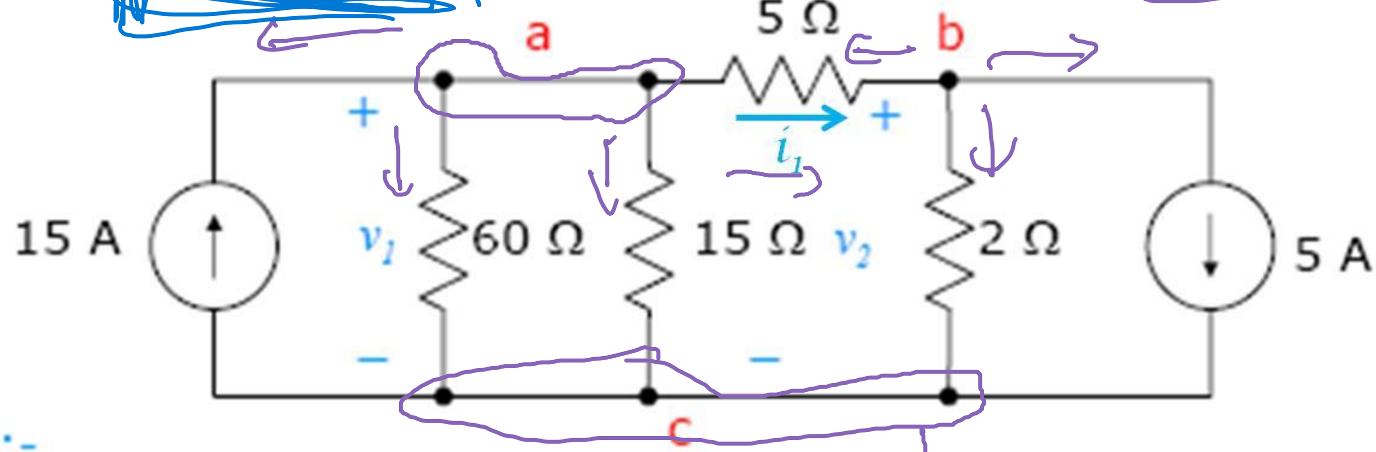
$P_{10V} = -vi$

$= -10 \cdot i_{1\Omega}$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{1}{(17 \times 6 - 25)} \begin{bmatrix} 6 & 5 \\ 5 & 17 \end{bmatrix} \begin{bmatrix} 100 \\ 20 \end{bmatrix} = \frac{1}{77} \begin{bmatrix} 700 \\ 840 \end{bmatrix}$$

## Assessing Objective 1

Use the node voltage method to find $v_1$, $v_2$, and $i_1$.




Ans.:-

2 eq

3 essential nodes, 2 equations

a) $$-15+\frac{v_1}{60}+\frac{v_1}{15}+\frac{v_1-v_2}{5}=0$$

b) $$\frac{v_2-v_1}{5}+\frac{v_2}{2}+5=0$$

$$v_1 = 60\text{ V}$$

$$v_2 = 10\text{ V}$$

$v_1 - v_2$

$$i_1 = \frac{60-10}{5} = 10\text{ A}$$

## Problem 1

$i_1 = \frac{110 - V_1}{2}$

4 essential nodes, 3 equations

Use the node-voltage method to find the branch currents $i_1$-$i_6$.

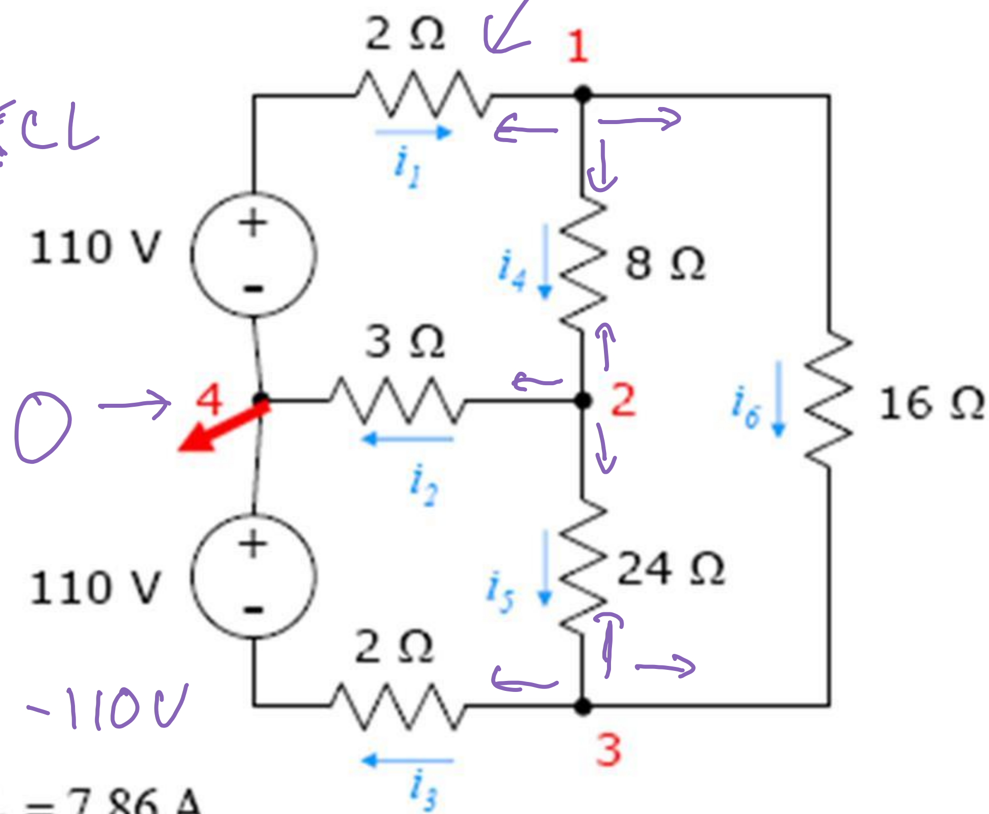
Ans.:

KCL

1) $\frac{v_1 - 110}{2} + \frac{v_1 - v_2}{8} + \frac{v_1 - v_3}{16} = 0$

2) $\frac{v_2 - v_1}{8} + \frac{v_2}{3} + \frac{v_2 - v_3}{24} = 0$

3) $\frac{v_3 + 110}{2} + \frac{v_3 - v_2}{24} + \frac{v_3 - v_1}{16} = 0$




$$\begin{bmatrix} 11 & -2 & -1 \\ -3 & 12 & -1 \\ -3 & -2 & 29 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 880 \\ 0 \\ -2640 \end{bmatrix}$$

$v_1 = 74.64$ V, $v_2 = 11.79$ V, $v_3 = -82.5$ V

$i_1 = 17.68$ A, $i_2 = 3.93$ A, $i_3 = 13.75$ A

$i_4 = 7.86$ A, $i_5 = 3.93$ A, $i_6 = 9.82$ A

$\frac{V_3 - (-110)}{2}$

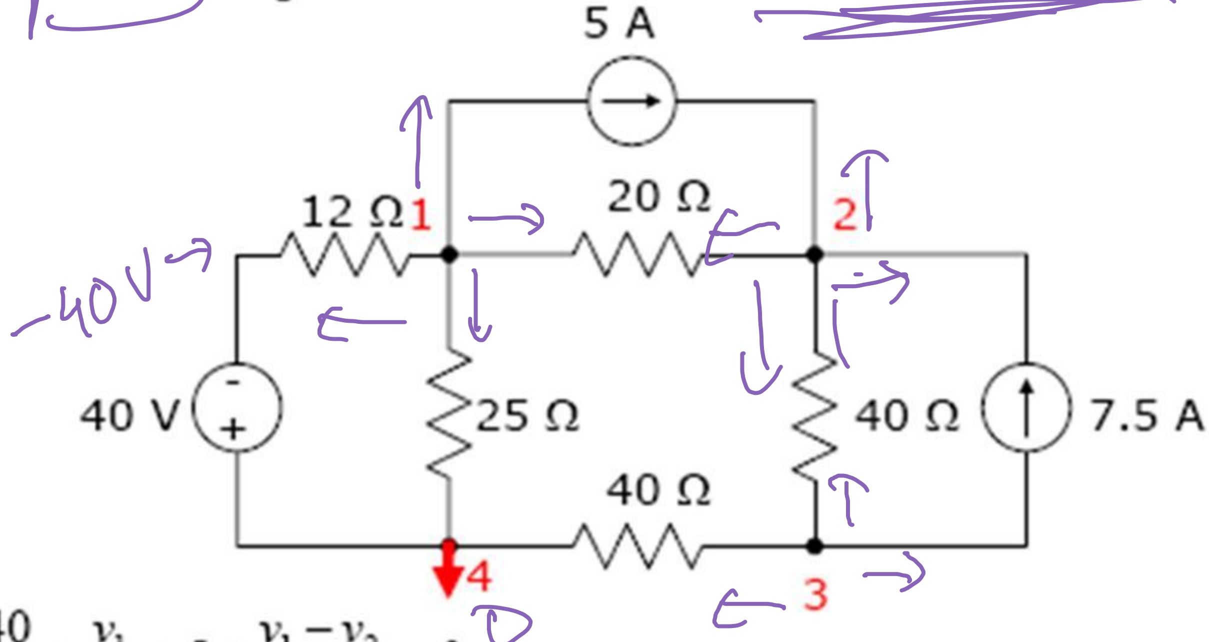
0V

## Problem 2

4 essential nodes, 3 equations

Use the node-voltage method to find the branch currents.

Ans.:




1) $\frac{v_1+40}{12}+\frac{v_1}{25}+5+\frac{v_1-v_2}{20}=0$

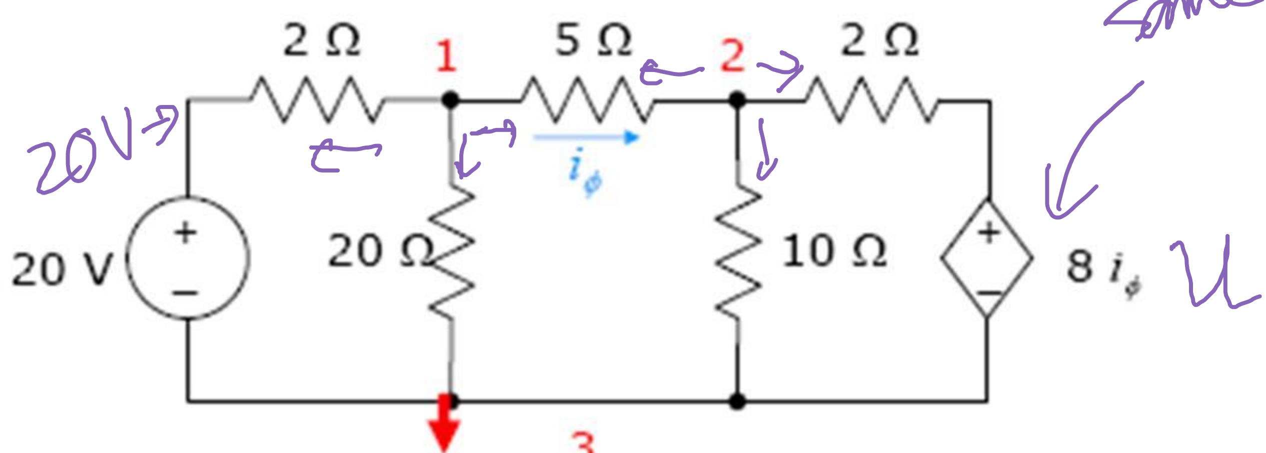
2) $\frac{v_2-v_1}{20}+\frac{v_2-v_3}{40}-7.5-5=0$

3) $\frac{v_3-v_2}{40}+\frac{v_3}{40}+7.5=0$

$$\begin{cases} v_1=-10\text{ V} \\ v_2=132\text{ V} \\ v_3=-84\text{ V} \end{cases}$$

## Example 1

Use the node-voltage method to find $i_\phi$.




Ans.:-

1 $$\frac{v_1-20}{2}+\frac{v_1}{20}+\frac{v_1-v_2}{5}=0$$

2 $$\frac{v_2-v_1}{5}+\frac{v_2}{10}+\frac{v_2-8i_\phi}{2}=0$$

$$\Rightarrow \begin{cases} v_1 = 16\ \text{V} \\ v_2 = 10\ \text{V} \end{cases}$$

$$i_\phi=\frac{v_1-v_2}{5}$$

$$i_\phi = 1.2\ \text{A}$$

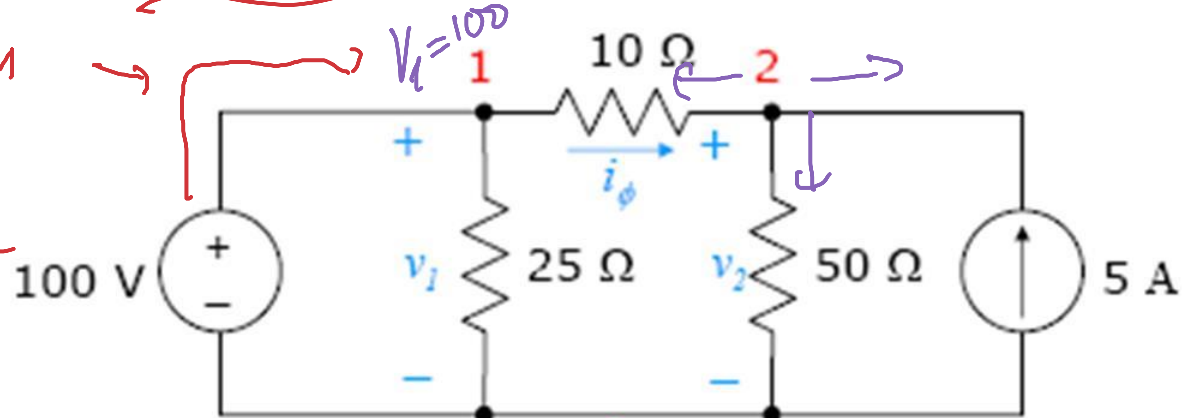
3 essential nodes, 2 equations, we must express the controlling current in terms of the node voltages

Reduce number of equations

## Special Cases

- When a voltage source is the only element connected between two essential nodes.

dòng điện này ko viết đc




$$\frac{v_2-v_1}{10}+\frac{v_2}{50}-5=0$$

$$v_1=100\text{ V}$$

$$v_2=125\text{ V}$$

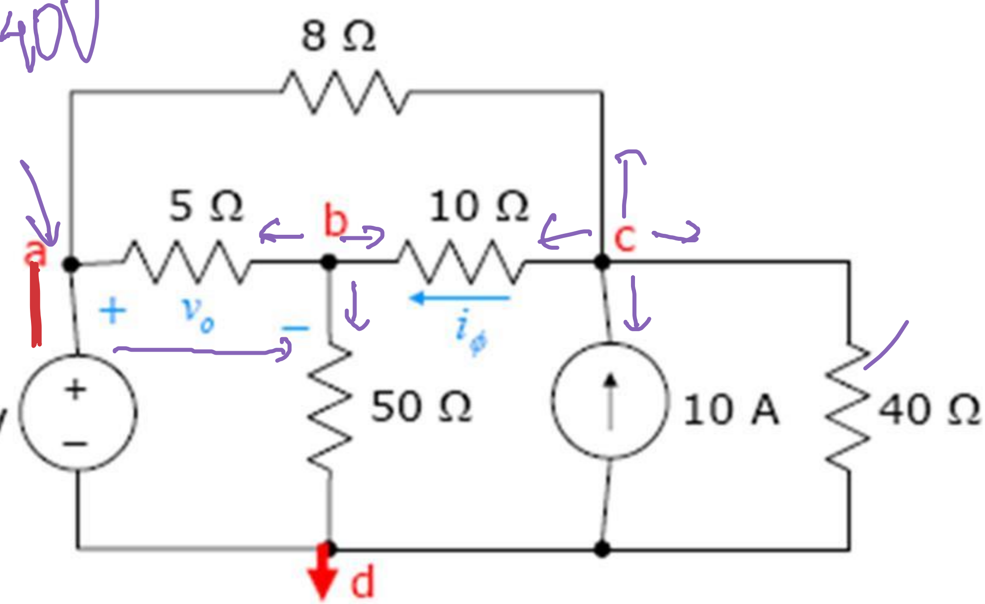
Reduce 1 equation due to known node voltage

## Problem 3

Use the node-voltage method to find $v_o$.

Ans.:-

$V_a = 40V$




a) $v_a = 40\text{ V}$

b) $\dfrac{v_b - 40}{5} + \dfrac{v_b}{50} + \dfrac{v_b - v_c}{10} = 0$

c) $\dfrac{v_c - 40}{8} + \dfrac{v_c}{40} + \dfrac{v_c - v_b}{10} - 10 = 0$

$v_a = 40\text{ V}$

$v_b = 50\text{ V}$

$v_c = 80\text{ V}$

$I_o = \dfrac{V_a - V_b}{5}$

or $I_o$???

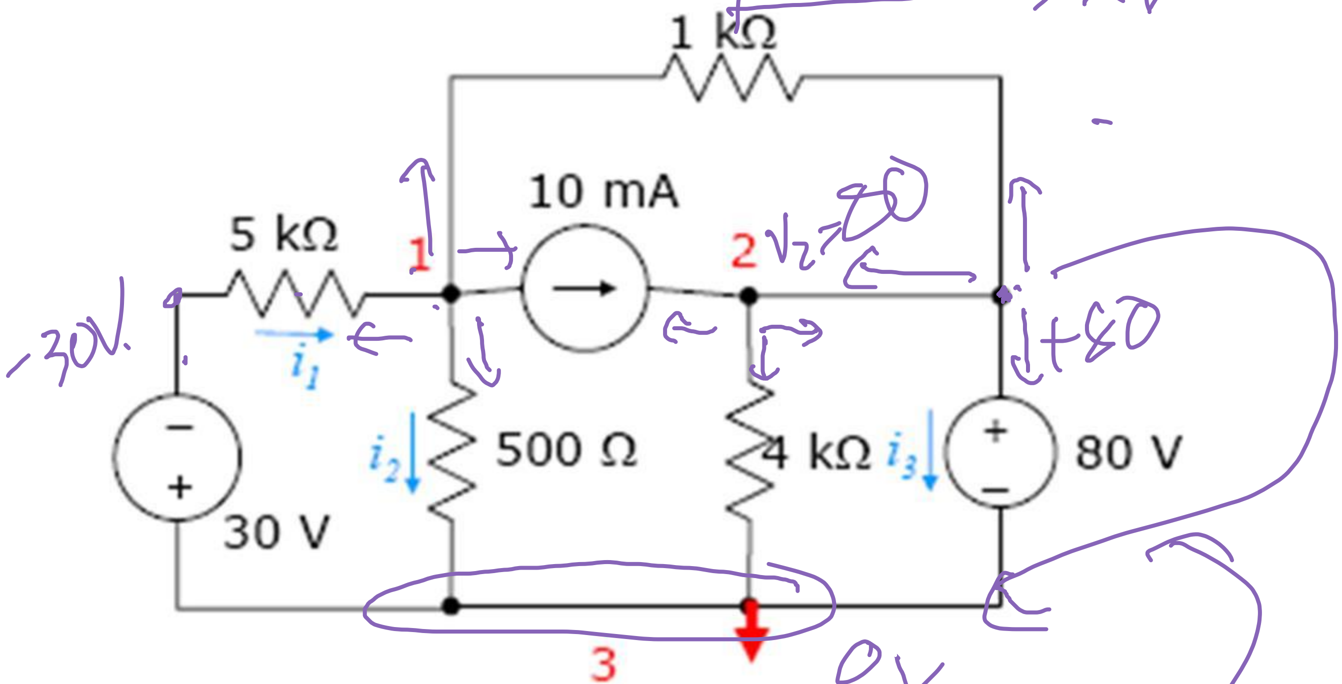
# Problem 4

Use the node-voltage method to find $\boxed{i_1, i_2 \& i_3}$.

→ ohm laws

→ KVL

Ans.:-




1) $\dfrac{v_1+30}{5000}+\dfrac{v_1}{500}+\dfrac{v_1-80}{1000}+10\times10^{-3}=0$ $\qquad v_1 = 20\text{ V}$

$V_2 = 80V$

*Write the equation @ node 2 by yourselves!*

$\boxed{i_1 = \dfrac{-30 - V_1}{5000}}$ $\qquad -10\times10^{-3} + \dfrac{V_2}{4000} + \dots$ $\qquad \boxed{i_2 = \dfrac{V_1}{500}}$

give i

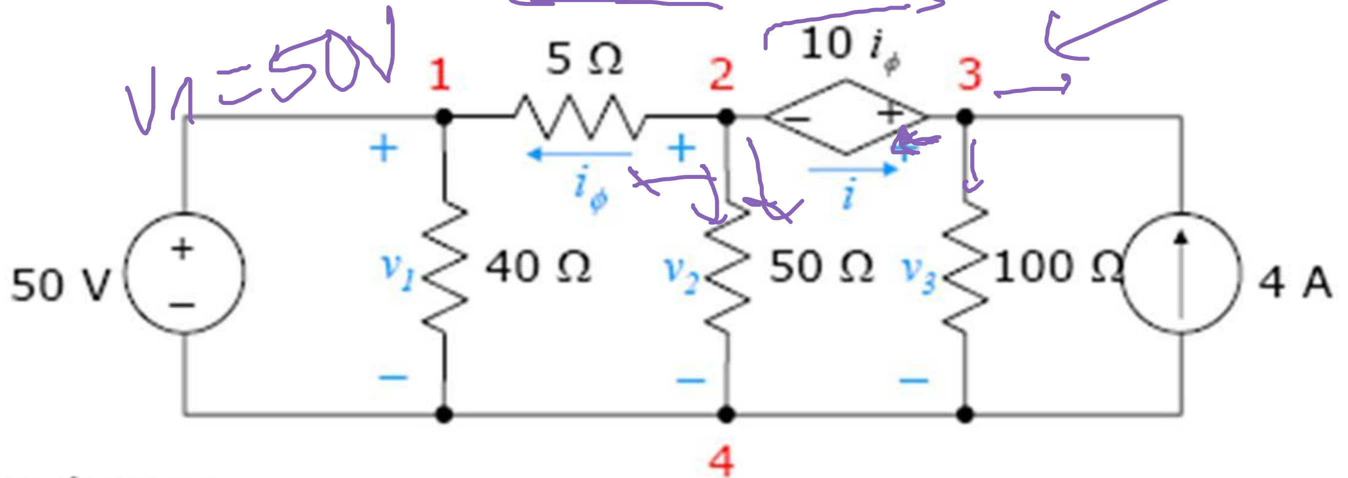
## Special Cases

- When a dependent voltage source is connected between nodes.

give it

Sài super node ↓ Loại bỏ vài gom thành 1 node

$v_1 = 50V$




First technique

1) $v_1 = 50\text{ V}$

give it

2) $$\frac{v_2 - v_1}{5} + \frac{v_2}{50} + i = 0$$

$+$

$+$

3) $$\frac{v_3}{100} - i - 4 = 0$$

$$+ \quad \frac{v_2 - v_1}{5} + \frac{v_2}{50} + \frac{v_3}{100} - 4 = 0$$

@ supernode

take sum

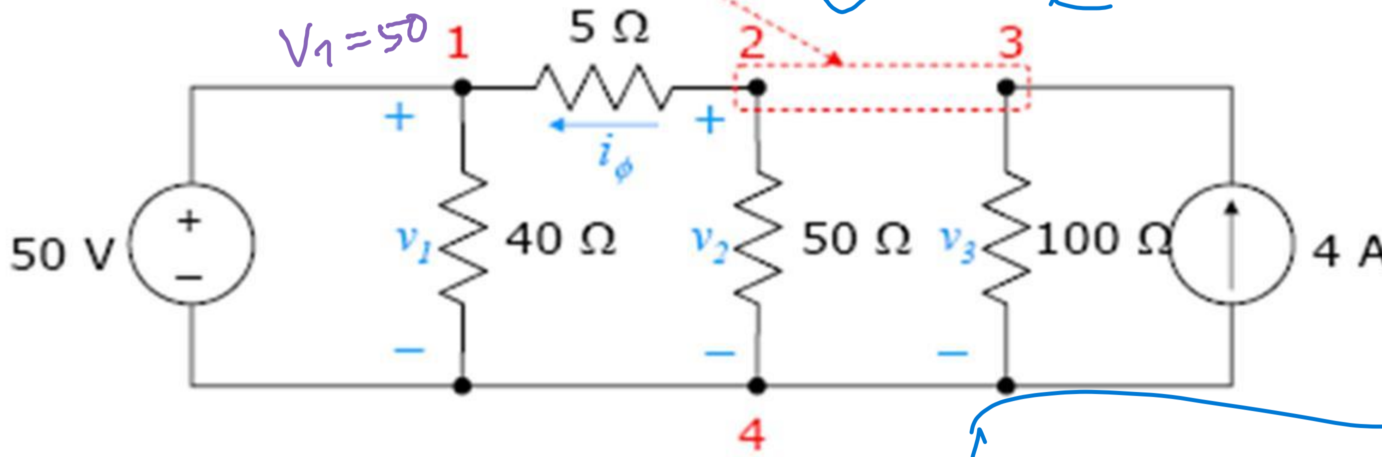
try to eliminate

coi là 1 nút

## Special Cases

Second technique (Super Node)

remove




@ Super Node $\quad \dfrac{v_2 - v_1}{5} + \dfrac{v_2}{50} + \dfrac{v_3}{100} - 4 = 0$

equation

Need more equations

$$v_3 = v_2 + 10 i_\phi \quad \& \quad i_\phi = \frac{v_2 - v_1}{5} = \frac{v_2 - 50}{5}$$

$$\begin{cases} v_1 = 50\ \text{V} \\ v_2 = 60\ \text{V} \\ v_3 = 80\ \text{V} \end{cases}$$

− +

10iΦ

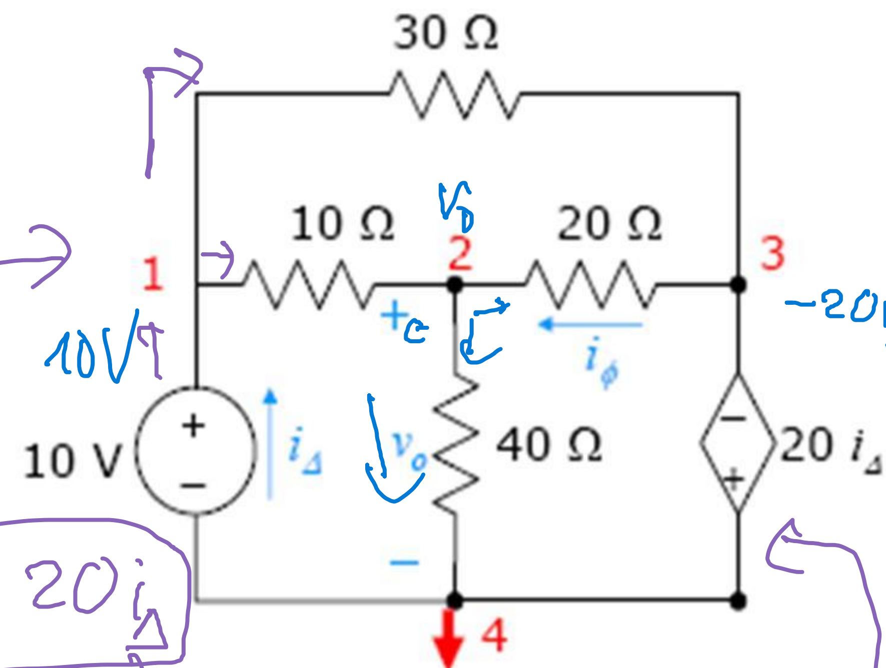
V1 = 50 V

## Assessing Objective 2

Use the node-voltage method to find $v_o$.

Ans.:-

2) $$\frac{v_o-10}{10}+\frac{v_o}{40}+\frac{v_o+20i_\Delta}{20}=0$$




$$i_\Delta=i_1+i_2=\frac{10-v_o}{10}+\frac{10+20i_\Delta}{30}$$

$0 - V_3 = 20 i_\Delta$

$v_o = 24\ \mathrm{V}$

$i_\Delta = -3.2\ \mathrm{A}$

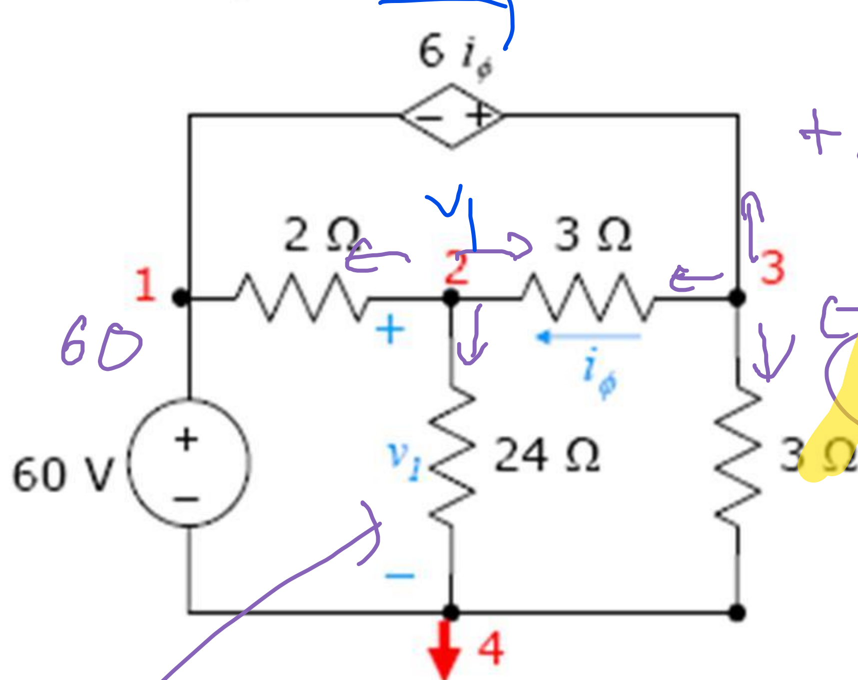
## Assessing Objective 3

$V_1$ at node 2

Khó hiểu

Use the node-voltage method to find $i_\phi$.

Ans.:-




2) $$\frac{v_1 - 60}{2} + \frac{v_1}{24} + \frac{v_1 - 6i_\phi - 60}{3} = 0$$

$$i_\phi = \frac{6i_\phi + 60 - v_1}{3} = \frac{V_3 - V_2}{3}$$

$$v_1 = 48\text{ V}$$

$$i_\phi = -4\text{ A}$$

$+6i_\phi$

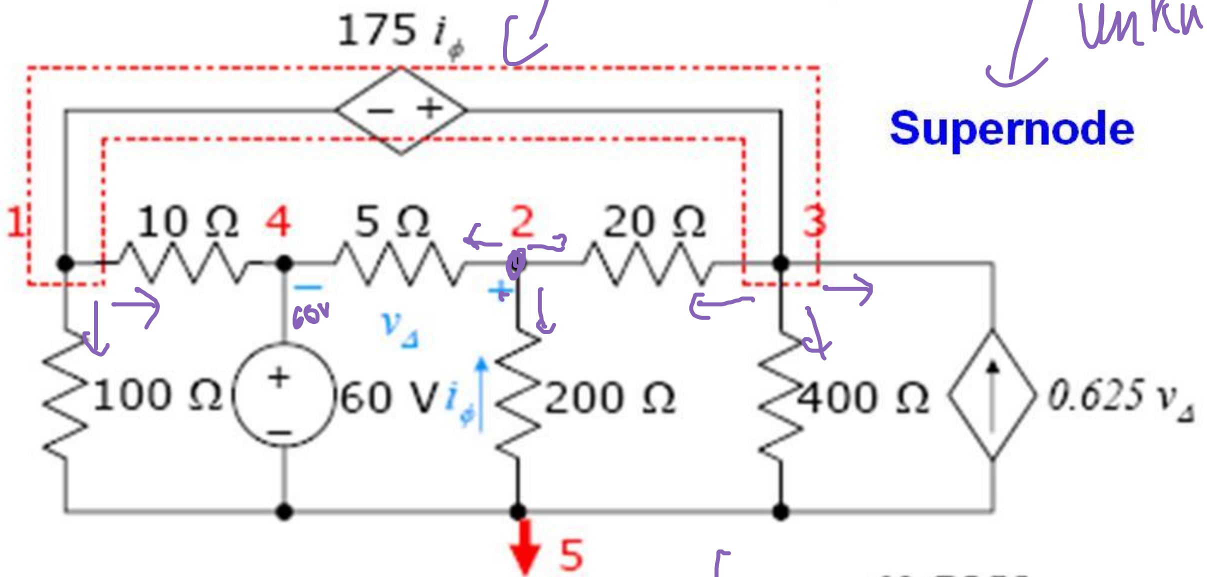
$$\frac{V_3 - V_2}{3} + \frac{V_3}{3} + 6i_\phi = 0$$

# Problem 5

Use the node-voltage method to find $v_\Delta$ and $i_\phi$.

$\boxed{V_3 - V_1 = 175 i_\phi}$ connect with 2 nodes unknown

Ans.:-




1,3) $$\frac{v_1-60}{10}+\frac{v_1}{100}-0.625v_\Delta+\frac{v_3}{400}+\frac{v_3-v_2}{20}=0$$

2) $$\frac{v_2-60}{5}+\frac{v_2}{200}+\frac{v_2-v_3}{20}=0$$

$$i_\phi=\frac{-v_2}{200}$$

$$v_\Delta=v_2-60$$

$V_2 - V_4 = 60$

$v_3 = v_1 + 175i_\Phi$

Lấy (+) - (−)

Hiểu nha

$v_1=-60.75\text{ V}$

$v_2=30\text{ V}$

$v_3=-87\text{ V}$

$i_\phi=-0.15\text{ A}$

$v_\Delta=-30\text{ V}$

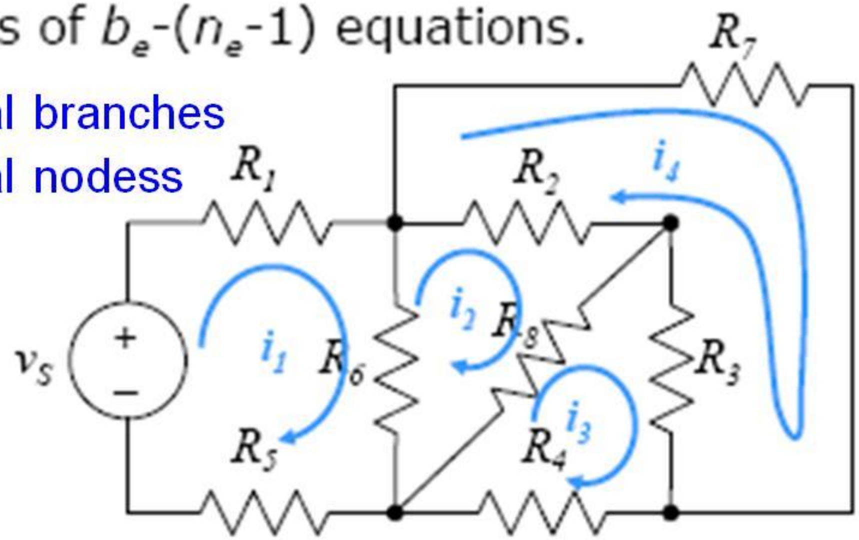
# Introduction to Mesh-Current Method

- Applicable only to planar circuits.
- Describe a circuit in terms of $b_e-(n_e-1)$ equations.

$b_e= 7$ Numbers of Essential branches

$n_e= 4$ Numbers of Essential nodess

$b_e-(n_e-1)=7-(4-1)=4$



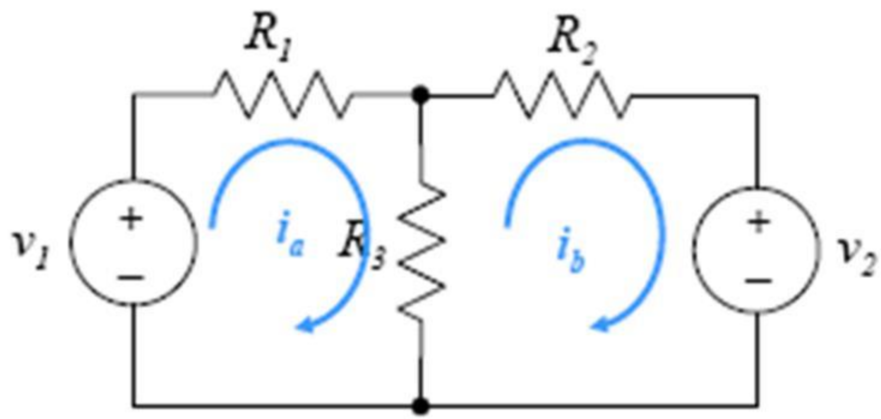

- A mesh current is the current that exists only in the perimeter of a mesh.

A mesh is a loop that does not enclose any other loops

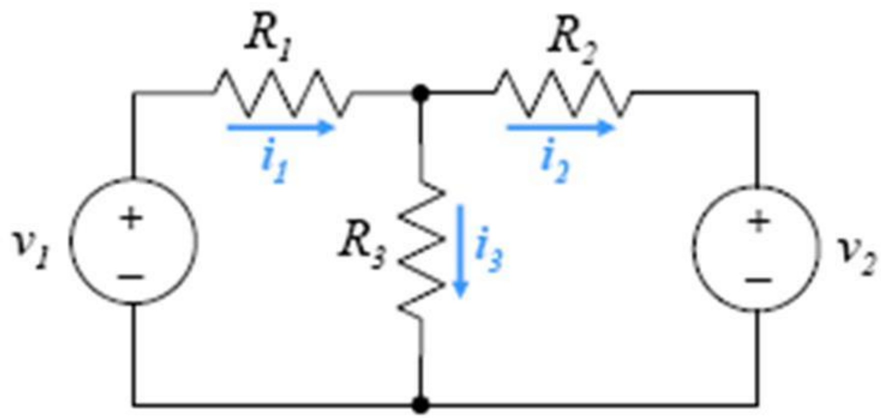
# Introduction to Mesh-Current Method

$$-v_1 + i_a R_1 + (i_a - i_b) R_3 = 0$$

$$(i_b - i_a) R_3 + i_b R_2 + v_2 = 0$$



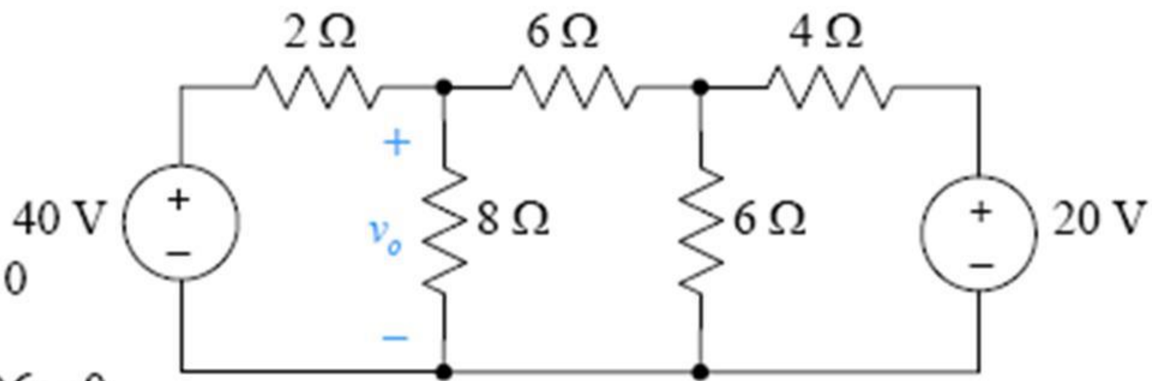

$$i_1 = i_a$$
$$i_2 = i_b$$
$$i_1 = i_2 + i_3$$



# Example 2

Find $v_o$?

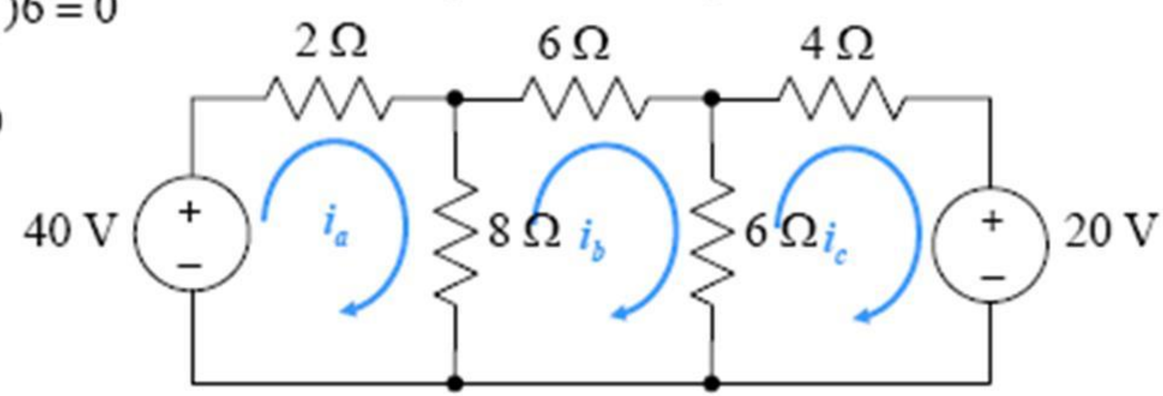
Ans.:-




$$-40 + i_a 2 + (i_a - i_b)8 = 0$$

$$(i_b - i_a)8 + i_b 6 + (i_b - i_c)6 = 0$$

$$(i_c - i_b)6 + i_c 4 + 20 = 0$$

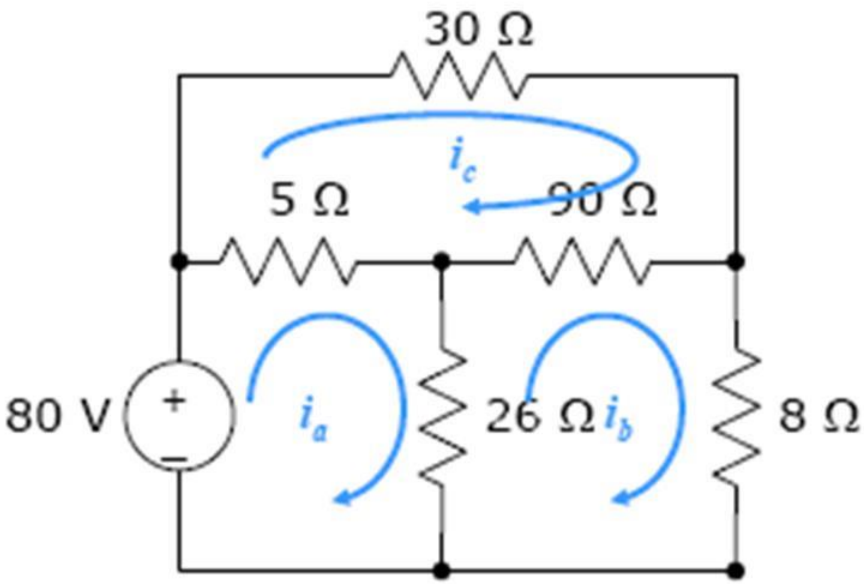



$$\begin{bmatrix} 10 & -8 & 0 \\ -8 & 20 & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 40 \\ 0 \\ -20 \end{bmatrix} \Rightarrow \begin{aligned} i_a &= 5.6\ \text{A} \\ i_b &= 2.0\ \text{A} \\ i_c &= -0.8\ \text{A} \end{aligned}$$

## Assessing Objective 4

- Power delivered by the 80 V source and power dissipated in the 8 Ω resistor.

Ans.:-




$$-80+(i_a-i_c)5+(i_a-i_b)26=0$$

$$(i_b-i_a)26+(i_b-i_c)90+i_b8=0$$

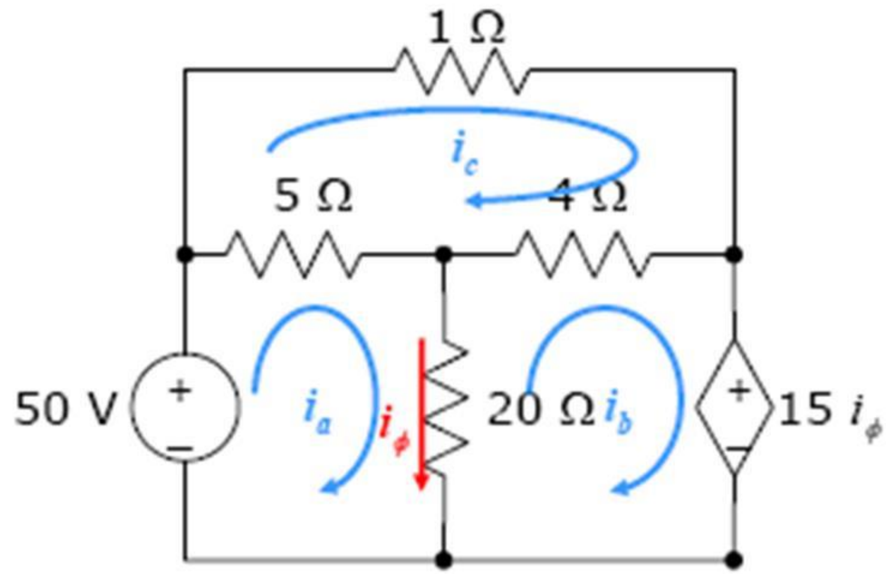
$$(i_c-i_a)5+i_c30+(i_c-i_b)90=0$$

$$\begin{bmatrix} 31 & -26 & -5 \\ -26 & 124 & -90 \\ -5 & -90 & 125 \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}=\begin{bmatrix} 80 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{aligned} i_a &= 5.0\text{ A} \\ i_b &= 2.5\text{ A} \\ i_c &= 2.0\text{ A} \end{aligned} \Rightarrow \begin{aligned} P_{80V} &= 400\text{ W} \\ P_{8\Omega} &= 50\text{ W} \end{aligned}$$

# Mesh-current method and dependent sources

Find $i_\phi$?

Ans.:-




$$-50+(i_a-i_c)5+(i_a-i_b)20=0$$

$$(i_b-i_a)20+(i_b-i_c)4+15i_\phi=0$$

$$(i_c-i_a)5+i_c+(i_c-i_b)4=0$$

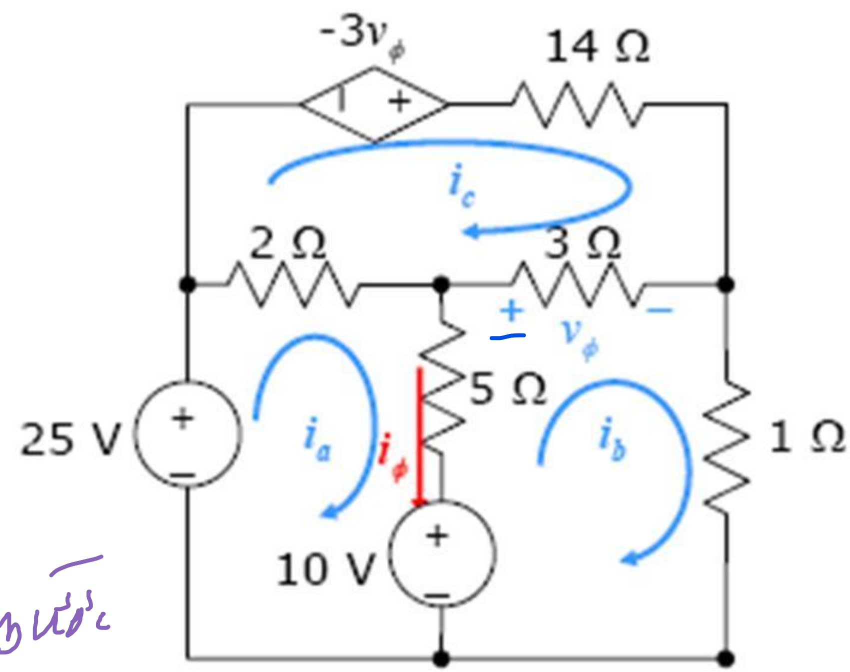
$$\boxed{i_\phi=i_a-i_b}$$

$$\begin{bmatrix}25 & -20 & -5\\ -5 & 9 & -4\\ -5 & -4 & 10\end{bmatrix}\begin{bmatrix}i_a\\ i_b\\ i_c\end{bmatrix}=\begin{bmatrix}50\\ 0\\ 0\end{bmatrix} \Rightarrow \begin{aligned} i_a &= 29.6\text{ A}\\ i_b &= 28.0\text{ A}\\ i_c &= 26.0\text{ A}\end{aligned} \Rightarrow i_\phi = 1.6\text{ A}$$

# Assessing Objective 5

Find $i_\phi$?

Ans.:-




$$-25+(i_a-i_c)2+(i_a-i_b)5+10=0$$

$$-10+(i_b-i_a)5+(i_b-i_c)3+i_b=0$$

$$(i_c-i_a)2+3v_\phi+i_c14+(i_c-i_b)3=0$$

$v_\phi=(i_b-i_c)3$ → cùng chiều quy ước

$$\begin{bmatrix} 7 & -5 & -2 \\ -5 & 9 & -3 \\ -2 & 6 & 10 \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}=\begin{bmatrix} 15 \\ 10 \\ 0 \end{bmatrix}$$

⇨

$i_a = 4\ \text{A}$

$i_b = 3\ \text{A}$

$i_c = -1\ \text{A}$

⇨

$i_\phi = 1\ \text{A}$

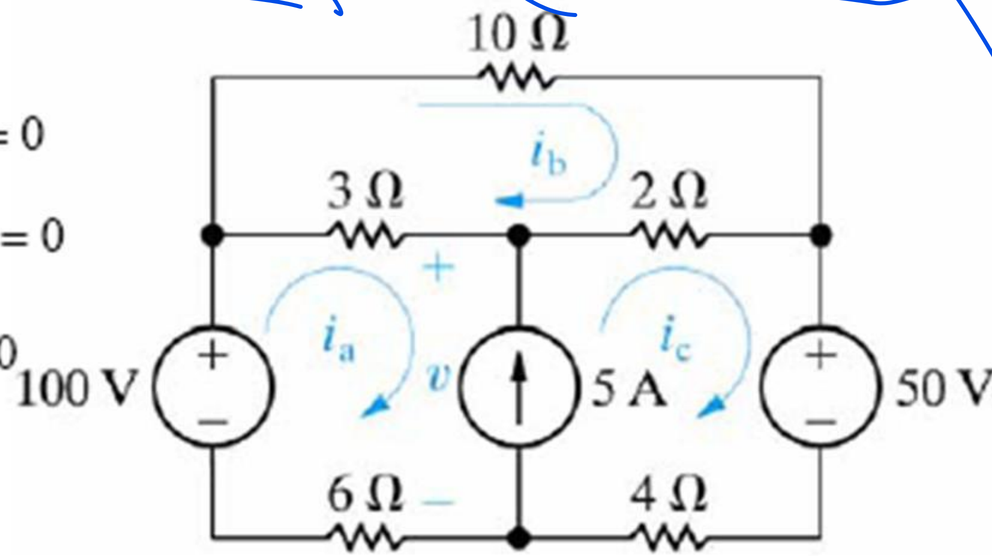
## Special Cases

- When an independent source is connected between two essential nodes.

a) $-100+(i_a-i_b)3+v+i_a6=0$

b) $(i_b-i_a)3+i_b10+(i_b-i_c)2=0$

c) $-v+(i_c-i_b)2+50+i_c4=0$




$i_c-i_a=5$

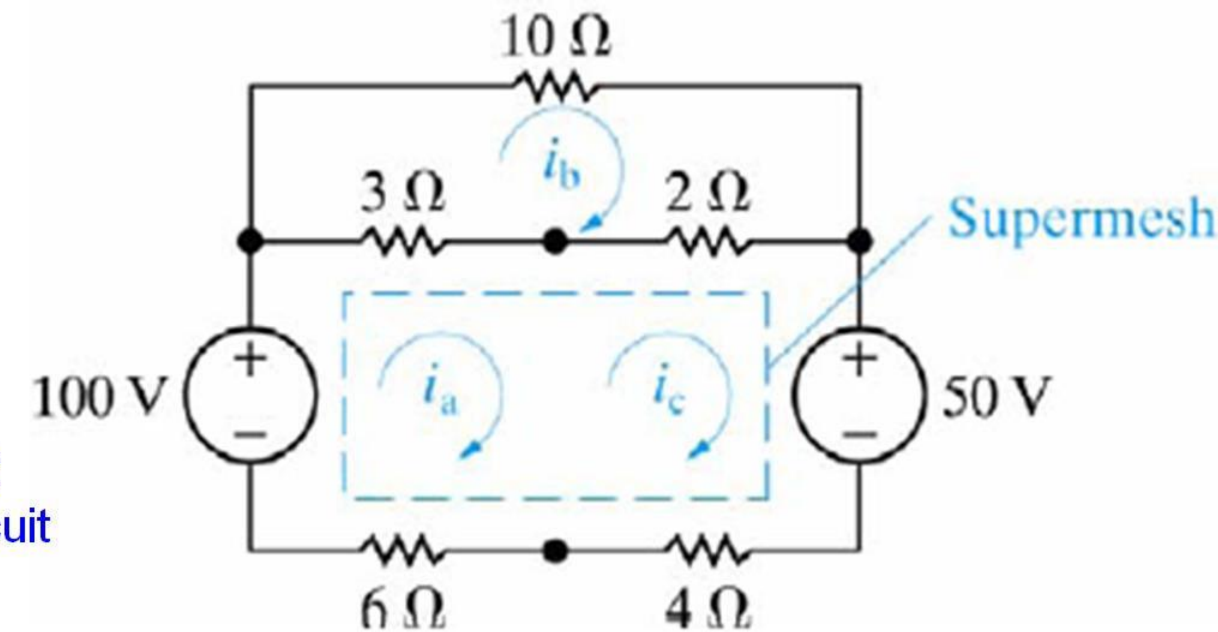
a) + c) (Cancelling v)

$$-100+(i_a-i_b)3+i_a6+(i_c-i_b)2+50+i_c4=0$$

$$\begin{aligned} 9i_a-5i_b+6i_c&=50 \\ -3i_a+15i_b-2i_c&=0 \\ -i_a+0i_b+i_c&=5 \end{aligned} \Rightarrow \begin{bmatrix} 9 & -5 & 6 \\ -3 & 15 & -2 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}=\begin{bmatrix} 50 \\ 0 \\ 5 \end{bmatrix} \Rightarrow \begin{aligned} i_a&=1.75\text{ A} \\ i_b&=1.25\text{ A} \\ i_c&=6.75\text{ A} \end{aligned}$$

## Special Cases

- Supermesh




Remove the current source from the circuit

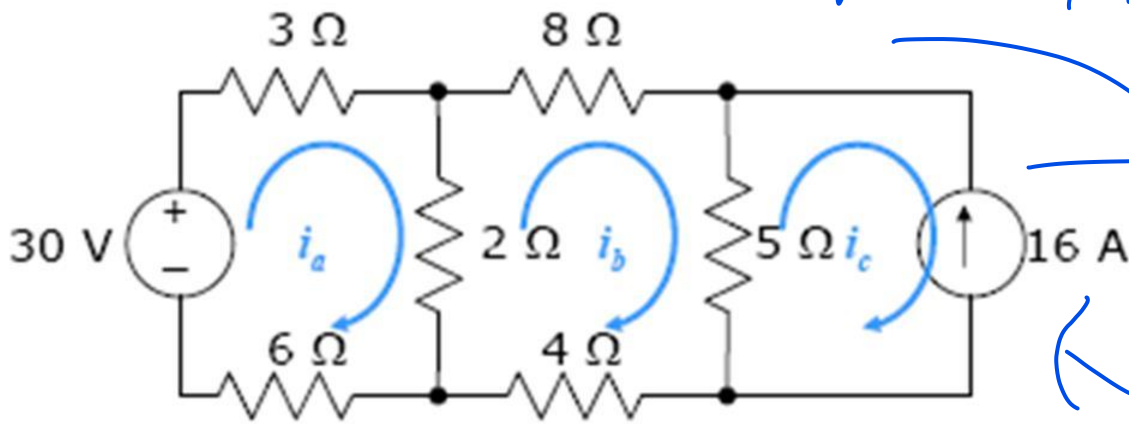
$$-100+(i_a-i_b)3+i_a6+(i_c-i_b)2+50+i_c4=0$$

## Assessing Objective 6

- Find the power dissipated in the 2 Ω resistor.

Ans.:-

NO S / ~~Re~~sister




$$-30 + i_a 3 + (i_a - i_b)2 + i_a 6 = 0$$

$$(i_b - i_a)2 + i_b 8 + (i_b - i_c)5 + i_b 4 = 0$$

$$\boxed{i_c = -16\text{ A}}$$

$$i_a = 2\text{ A}$$

$$i_b = -4\text{ A}$$

$$P_{2\Omega} = 6^2 2 = 72\text{ W}$$

## Assessing Objective 7

- Find the current $i_a$.

Ans.:-

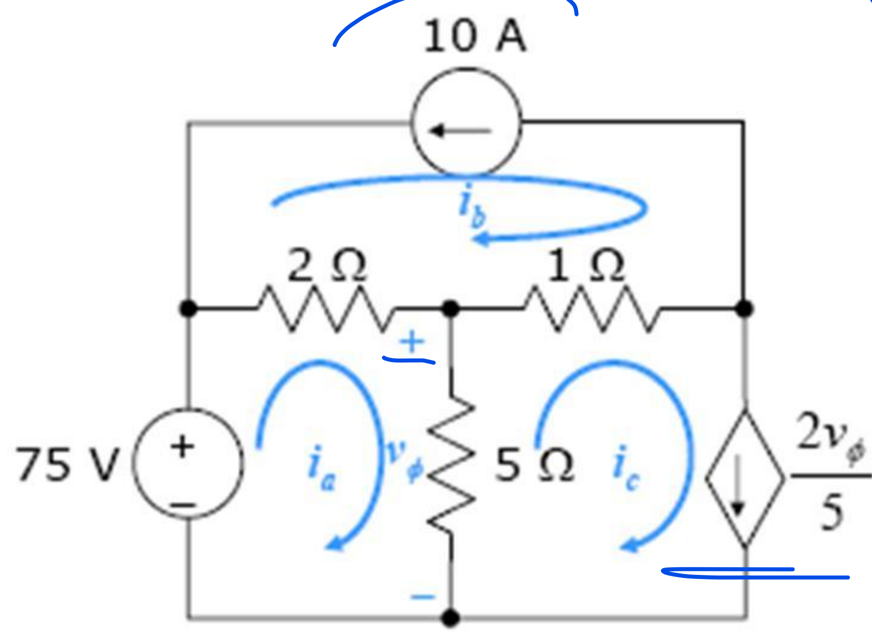
$$-75+(i_a-i_b)2+(i_a-i_c)5=0$$

$$i_b=-10\text{ A}$$

$$v_\phi=(i_a-i_c)5$$

$$i_c=\frac{2v_\phi}{5}$$

Compute $i_c$ in terms of $i_a$ and substitute into above equation




$$i_a=15\text{ A}$$

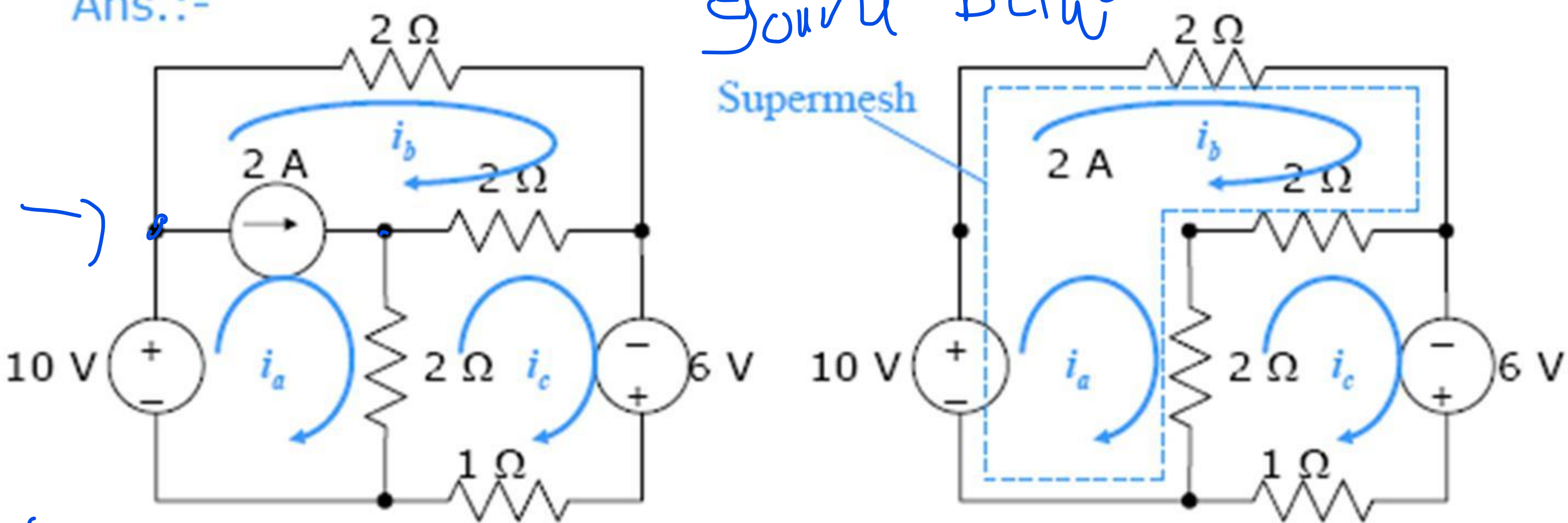
$$i_c=10\text{ A} \qquad v_\phi=25\text{ V}$$

## Assessing Objective 8

- Find the power dissipated in the 1 Ω resistor.

Source between 2es nodes

Ans.:-




$$-10 + i_b 2 + (i_b - i_c)2 + (i_a - i_c)2 = 0$$

$$(i_c - i_a)2 + (i_c - i_b)2 - 6 + i_c = 0$$

$$i_a - i_b = 2$$

$$i_a = 7\ \text{A}$$

$$i_b = 5\ \text{A}$$

$$i_c = 6\ \text{A}$$

$$P_{2\Omega} = 6^2 1 = 36\ \text{W}$$

# Node voltage method vs. Mesh current method

***Which method is more efficient?***

- Does one of the methods result in fewer simultaneous equations to solve?
- Does the circuit contain supernodes?

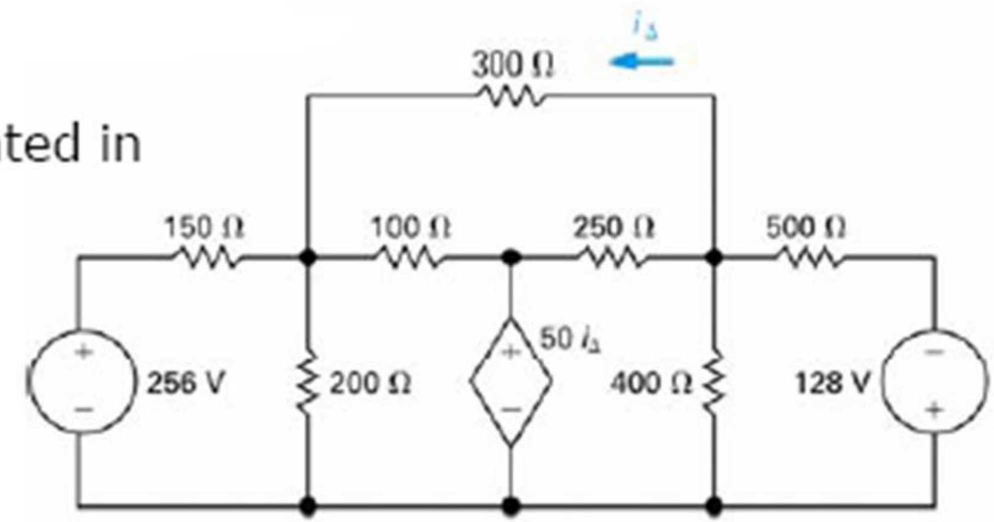
  If so, use the **node-voltage method**.
- Does the circuit contain supermeshes?

  If so, use the **mesh-current method**.

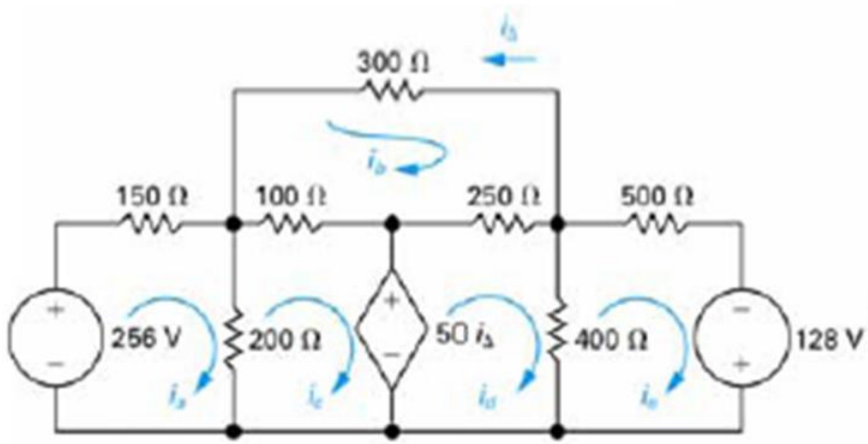
## Example 3

Find the power dissipated in the 300 Ω resistor?

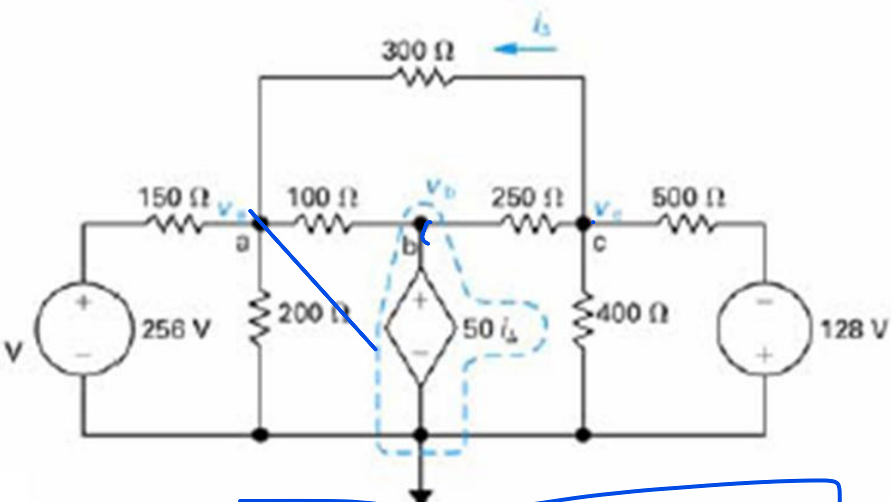
Ans.:-









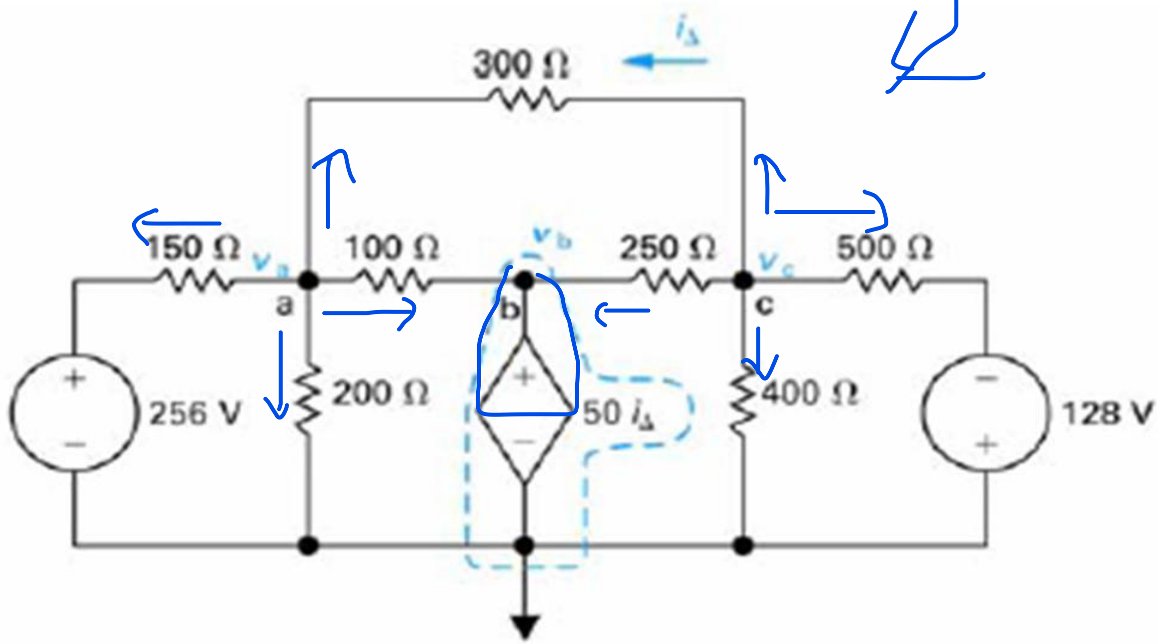




Mesh current equations = 5

Node voltage equations = 3

# Example (Cont.)




a) $$\frac{v_a - 256}{150} + \frac{v_a}{200} + \frac{v_a - v_c}{300} + \frac{v_a - v_b}{100} = 0$$

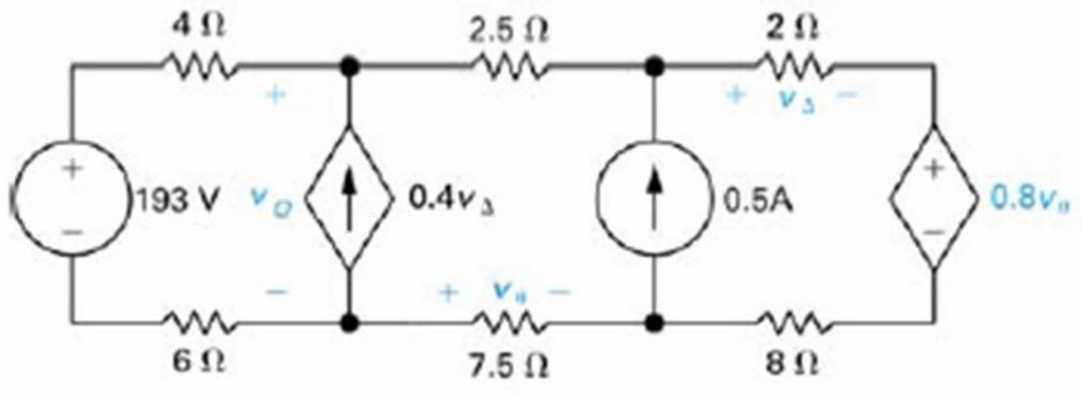
c) $$\frac{v_c + 128}{500} + \frac{v_c}{400} + \frac{v_c - v_b}{250} + \frac{v_c - v_a}{300} = 0$$

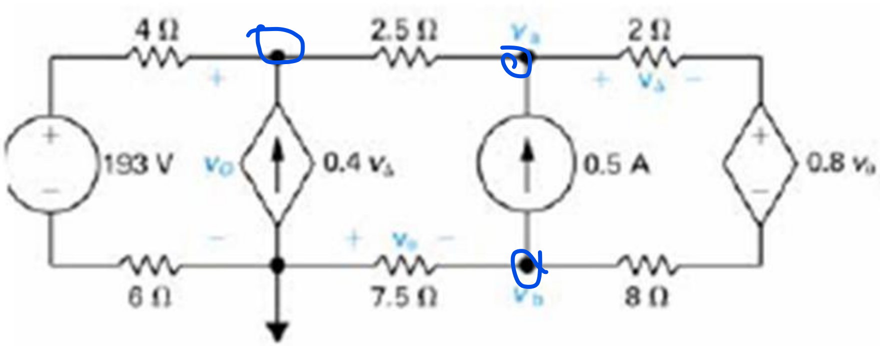
$$v_b = 50 i_\Delta$$

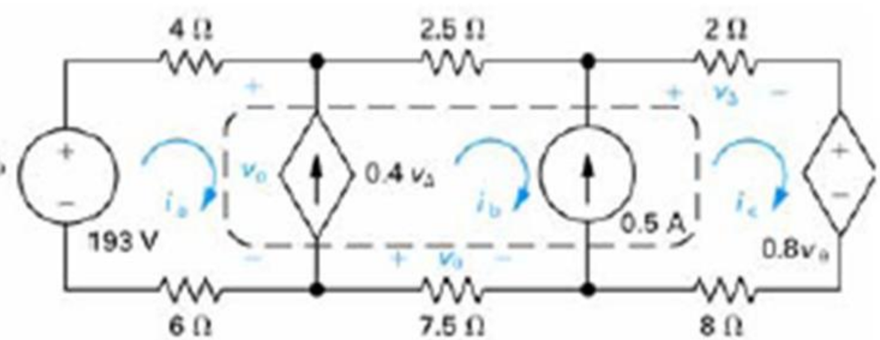
## Example 4

Find $v_o$?

Ans.:-



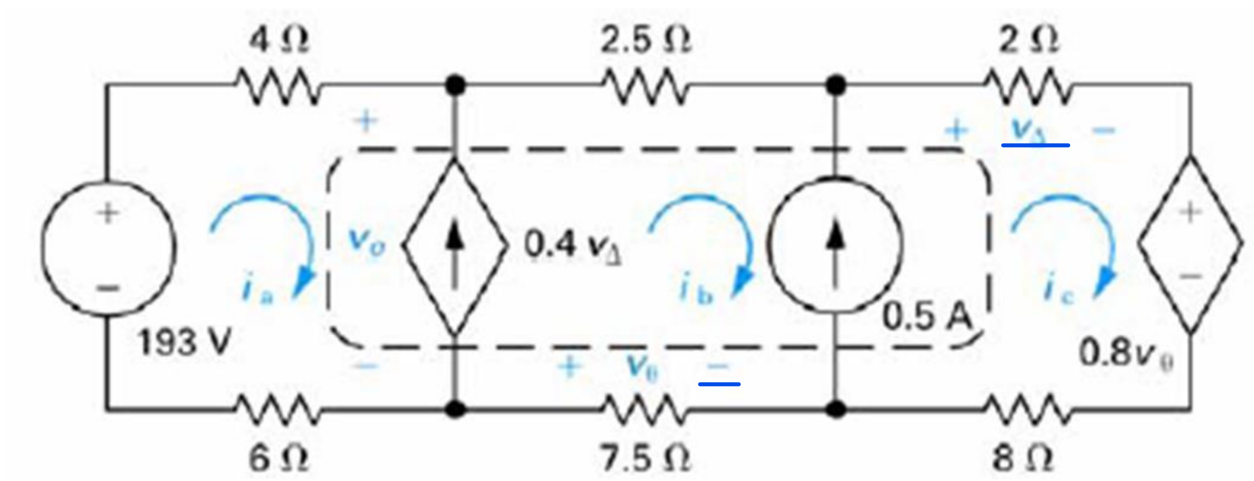




Node voltage equations = 3

Mesh current equations = 1

# Example (Cont.)




$$\boxed{-193 + i_a 4 + i_b 2.5 + i_c 2 + 0.8 v_\theta + i_c 8 + i_b 7.5 + i_a 6 = 0}$$

$$\begin{cases} i_b - i_a = 0.4 v_\Delta \\ i_c - i_b = 0.5 \end{cases} \qquad \begin{cases} v_\Delta = i_c 2 \\ v_\theta = -i_b 7.5 \end{cases} \qquad i_a = 2\text{ A}$$