

$$ce^{j\theta} = c(\cos \theta + j \sin \theta)$$

$$= a + jb$$

where : $c = \sqrt{a^2 + b^2}$

$$\tan \theta = \frac{b}{a}$$

$$1 = 1 \angle 0^\circ, -1 = 1 \angle 180^\circ$$

$$j = 1 \angle 90^\circ, -j = 1 \angle -90^\circ$$

$$n = a + jb = c \angle \theta = c(\cos \theta + j \sin \theta)$$

$$n + n^* = 2a; n - n^* = j2b$$

$$nn^* = a^2 + b^2 = c^2 \quad j \frac{n}{a^2} = 1 \angle 2\theta^\circ$$

$$\text{Ex: } \frac{2 \angle 90^\circ}{4 \angle 75^\circ} = \frac{2}{4} \angle (90 - 75) = \frac{1}{2} \angle 15^\circ$$

$$\sqrt{n} = \sqrt{c} \left(\frac{\theta}{2} \right)$$

$$n^* = a - jb = c \angle (-\theta) = ce^{-j\theta}$$

$$\text{a) } (40 \angle 50^\circ + 20 \angle -30^\circ)^{1/2} =$$

$$= \left[40(\cos 50^\circ + j \sin 50^\circ) + 20(\cos(-30^\circ) + j \sin(-30^\circ)) \right]^{\frac{1}{2}}$$

$$= [25.711 + j30.641 + 10\sqrt{3} - j10]^{\frac{1}{2}}$$

$$= [43.03 \pm j20.64]^{\frac{1}{2}}$$

$$= (\sqrt{47.72} \angle 25.63^\circ)^{\frac{1}{2}}$$

$$= 6.907 \angle 12.81^\circ$$

$$\begin{aligned}
 b) & \frac{10 \angle -30^\circ + (3-j4)}{(2+j4)(3-j5)} \\
 &= \frac{8.66 - j5 + 3 - j4}{(6-20) + j(-10+12)} \\
 &= \frac{11.66 + j9}{-14 + j22} = \frac{14.73 \angle -37.66^\circ}{26.08 \angle 122.47^\circ}
 \end{aligned}$$

$$= 0.565 \angle -160.13^\circ$$

$$\vartheta = 300 \cos(120\pi t + 30^\circ)$$

$$\rightarrow \vartheta = 300 \angle 30^\circ$$

$$a) w = 2\pi f = \frac{2\pi}{T} \Rightarrow T = 0.0166$$

$$b) w = 2\pi f \Rightarrow f = 60$$

$$\begin{aligned}
 v_1 &= \textcircled{-10} \cos(\omega t + 50^\circ) = 10 \angle -130^\circ \\
 v_2 &= 12 \sin(\omega t - 10^\circ) = 12 \angle -100^\circ
 \end{aligned}$$

$\rightarrow v_2$ lead v_1 by 30°

$$\text{polar form: } v = V_m e^{j\theta}$$

$$\text{rectangular: } V_m \cos \theta + j V_m \sin \theta$$

$$\text{Angular: } V_m \angle \theta^\circ = V_m e^{j\theta}$$

$$a) i = 6 \cos(50t - 40^\circ) A$$

$$= 6 \angle -40^\circ (A)$$

$$\cos A = \cos(A - 180^\circ)$$

$$b) v = -4 \sin(30t + 50^\circ) V$$

$$= -4 \cos(30t - 40^\circ) V$$

$$= 4 \angle 140^\circ (V)$$

$$-\sin A = \cos(A + 90^\circ)$$

7.

$$a) I = -3 + j 4 (A)$$

$$b) V = j 8 e^{-j20^\circ} (V)$$

Sol:

$$a) 5 \angle 126.87^\circ \rightarrow i(t) = 5 \cos(\omega t + 126.87)$$

$$b) 1 \angle 90^\circ (8 \angle -20^\circ)$$

$$= 8 \angle 70^\circ \rightarrow v(t) = 8 \cos(\omega t + 70^\circ)$$

$$\begin{aligned} t_1 &= 20 \cos(\omega t - 30^\circ) \\ t_2 &= 40 \cos(\omega t + 60^\circ) \end{aligned}$$

$$t = t_1 + t_2 = ?$$

$$20 \angle -30^\circ + 40 \angle 60^\circ$$

$$= 20(\cos(-30) + j \sin(-30)) + 40(\cos 60^\circ + j \sin 60^\circ)$$

$$= 17.32 - 10j + 20 + 34.64j$$

$$= 47.32 + j 24.64$$

$$= 44.72 \angle 33.43^\circ$$

$$\rightarrow t = 44.72 \cos(\omega t + 33.43^\circ)$$

Voltage - Current Relation

$$R \quad v = R i \quad V = RI$$

$$L \quad v = L \frac{di}{dt} \quad V = j\omega LI$$

$$C \quad i = C \frac{dv}{dt} \quad v = \frac{I}{j\omega C}$$

$$v = 12 \cos(60t + 45^\circ) \quad L = 0.1 \text{ H}$$

$$V = 12 \angle 45^\circ \quad V = j\omega L I \quad \omega = 60 \text{ rad/s}$$

$$I = \frac{V}{j\omega L} = \frac{12 \angle 45^\circ}{j60 \times 0.1} = \frac{12 \angle 45^\circ}{(1 \angle 90^\circ)6} = \frac{12 \angle 45^\circ}{6 \angle 90^\circ}$$

$$= 2 \angle -45^\circ A$$

$$i(t) = 2 \cos(60t - 45^\circ) A$$

$$Z_1 = \frac{-1}{j 100 \times 0.01 \times 10^{-6}} = j 10^5$$

$$Z_2 = 10^4 + j 10^5$$

Mesh (1): $I_1(j10^5) + 22 \times 10^3(I_1 - I_2) = 5$

$$\Leftrightarrow I_1(j10^5 + 22 \times 10^3) + I_2(-22 \times 10^3) = 5$$
A
B
E

Mesh (2):

$$I_1(-22 \times 10^3) + I_2(22 \times 10^3 + 10^4 + j10^5) = 0$$

$$\Rightarrow I_1(-22 \times 10^3) + I_2(32 \times 10^3 + j10^5) + 3 \angle -46^\circ = 0$$

$$\Rightarrow I_1(22 \times 10^3) + I_2(-32 \times 10^3 - j10^5) = 3 \angle -46^\circ$$
C
D
F

$$D = AD - BC = Z$$

$$D_x = BF - ED = M \Rightarrow x = \frac{Dx}{D} = \frac{5.001 \times 10^7}{10^5} \angle -46^\circ$$

$$D_y = AF - EC = Y \Rightarrow y = \frac{Dy}{D} = 0.395 \angle -31.52^\circ$$

$$-\frac{j}{j 1445 j} = \frac{j}{1445}$$

