HO CHI MINH CITY INTERNATIONAL UNIVERSITY

MIDTERM EXAMINATION

Academic year 2011-2012, Semester 2
Duration: 90 minutes

SUBJECT: **Differential Equations**	
Acting Chair of Department of Mathematics	Lecturer:
Signature:	Signature:
Full name: Associate Prof. Nguyen Dinh	Full name: Dr. Pham Huu Anh Ngoc

Instructions:

- **Open-book examination. Laptops NOT allowed**.

Question 1. *(25 marks) Solve the following differential equation*

$$(e^x y^2 + x^2 + x)dx + (2e^x y + y + 1)dy = 0.$$

Question 2. *(25 marks) Find the solution to the initial value problem*

$$xy' - y = x^2(\sin x + 1), \qquad y(\frac{\pi}{2}) = 1.$$

Question 3. *(25 marks)* Find the general solution of the differential equation

$$y'' - 3y' + 2y = 3x + 3e^{2x}.$$

Question 4. *(25 marks)* Find $\alpha > 0$ such that $y_1(x) = x^\alpha$ is a solution of the following differential equation:

$$y'' - \frac{y'}{x} + \frac{y}{x^2} = 0, \qquad x \in (0, \infty).$$

Solve the given differential equation.

END.

SOLUTIONS:

Question 1. Note that

$$0 = (e^x y^2 + x^2 + x)dx + (2e^x y + y + 1)dy = (y^2 de^x + e^x dy^2) + d(x^3/3 + x^2/2)$$
$$+d(y^2/2 + y) = d(e^x y^2 + x^3/3 + x^2/2 + y^2/2 + y).$$

Thus the general solution is given by

$$e^x y^2 + x^3/3 + x^2/2 + y^2/2 + y = C.$$

Question 2. The given equation is written as

$$y' - \frac{y}{x} = x(\sin x + 1).$$

The integrating factor is given by $I(x) = \frac{1}{x}$. Thus, we get

$$\frac{y'}{x} - \frac{y}{x^2} = \sin x + 1.$$

This gives

$$\frac{d}{dx}(\frac{y}{x}) = \sin x + 1.$$

Therefore, the general solution is

$$y(x) = -x\cos x + x^2 + Cx.$$

Since $y(\frac{\pi}{2}) = 1$, the particular solution is $y(x) = -x\cos x + x^2 - \frac{2x(-1+(1/4)\pi^2)}{\pi}$.

Question 3.

The general solution of the corresponding homogeneous equation is

$$y(x) = c_1 e^x + c_2 e^{2x}.$$

A particular solution of $y'' - 3y' + 2y = 3x + 3e^{2x}$ is $y_p(x) = \frac{3}{2}x + \frac{9}{4} + 3e^{2x}x - 3e^{2x}$. Thus the general solution of the equation $y'' - 3y' + 2y = 3x + 3e^{2x}$, is given by

$$y(x) = \frac{3}{2}x + \frac{9}{4} + 3e^{2x}x - 3e^{2x} + c_1 e^x + c_2 e^{2x}.$$

Question 4. It is easy to show that $\alpha = 1$. Thus, $y_1(x) = x$ is a particular solution of the given differential equation. By the Liouville formula, $y_2(x) = x\ln x$ is another solution such that y_1, y_2 are linearly independent. So, the general solution is given by

$$y(x) = c_1 x + c_2 x \ln x, \qquad x > 0.$$