

Hypothesis testing

- Principle of hypothesis testing
- Test for population mean
 - when the population variance is known
 - when the population variance is unknown
- Test for population variance
- Test for comparing two means

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- 1 Statistical Hypotheses: General Concepts
- 2 Testing a Statistical Hypothesis
- 3 Single Sample
- 4 Two samples

Introduction

- **Problem:** there are two competing claims about the value of a parameter, and the engineer must determine which claim is correct
- Procedure to make decision based on empirical data: **hypothesis testing**

Example

- The propellant in the rocket motor of an air crew escape system should have a mean burning rate of 50 cm/sec.
- If the burning rate is too low, the ejection seat may not function properly, leading to an unsafe ejection and possible injury of the pilot. Higher burning rates may imply instability in the propellant or an ejection seat that is too powerful, again leading to possible pilot injury.

In this example, we are interested in burning rate of a solid propellant used to power air crew system

- Burning rate is a random variable that is described by a distribution
- Our interest focus on the **mean** burning rate (*parameter of distribution*)
- Specifically, we are interested in **deciding whether or not the mean burning rate is 50 centimeters per second.**

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Statistical hypothesis

A statistical hypothesis is a statement about one or more populations.

Null Hypothesis H_0
assumption to test

Alternative Hypothesis H_a or H_1
an opposite of null hypothesis

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Remark

Hypothesis is a statement **about population**, not about sample statistic

Example

Want to decide whether the average number of TV set in U.S. Homes is equal to three

$$H_0 : \mu = 3$$

$$H_0 : \bar{X} = 3$$

Example

We are interested in deciding *whether or not* the **mean burning rate** is 50 centimeters per second.

We may express this formally as

$$H_0 : \mu = 50 \text{ centimeter per second}$$

$$H_1 : \mu \neq 50 \text{ centimeter per second}$$

Since the alternative hypothesis $H_1 : \mu \neq 50$ specifies values of μ that could be either greater or less than 50 centimeters per second, it is called a **two-sided alternative hypothesis**. In some situations, we may wish to formulate a **one-sided alternative hypothesis**, as in

$$H_0 : \mu = 50 \text{ vs } H_1 : \mu > 50$$

or

$$H_0 : \mu = 50 \text{ vs } H_1 : \mu < 50$$

One - sided and Two - sided test about parameter θ

- One-sided test

- Upper-tail test

$$H_0 : \theta = \theta_0$$

$$H_1 : \theta > \theta_0$$

- Lower-tail test

$$H_0 : \theta = \theta_0$$

$$H_1 : \theta < \theta_0$$

- Two-sided test

$$H_0 : \theta = \theta_0$$

$$H_1 : \theta \neq \theta_0$$

Remark

- H_0 always **contains** " $=$ " or \leq or \geq
- H_1 : contains " \neq ", " $>$ ", " $<$ "

The name of test depends on the sign in H_1 .



Remark

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- H_1 : contains " \neq ", " $>$ ", " $<$ "

The name of test depends on the sign in H_1 .



Example

- Producer advertises that
their cable have an average breaking strength of at least 7,000 psi.
statement about quality of cable which is described by the **mean** - μ

Express formally as

$$H_0 : \mu \geq 7000 \text{ psi} - \text{the statement}$$

$$H_1 : \mu < 7000 \text{ psi} - \text{opposite of the statement}$$

Example

Need to decide whether the new fuel injection system will provide a $\underbrace{\text{mean miles-per-gallon rating}}_{\mu}$ exceeding 24

Express formally as

$H_0 : \mu \leq 24$ miles-per-gallon rating – opposite of the claim

$H_1 : \mu > 24$ miles-per-gallon rating – the claim

Test of a hypothesis

- A procedure leading to a decision about a particular hypothesis is called a **test of a hypothesis**.
- Hypothesis-testing procedures rely on using the evidence from information in a random sample from the population of interest.
 - reject H_0 in favor of H_1 if the evidence is inconsistent with H_0
 - fail to reject H_0 if the evidence is consistent with H_0

The truth or falsity of a particular hypothesis can never be known with certainty, unless we can examine the entire population.

Hypothesis Testing Process

Claim: the population mean age is 50.
(Null Hypothesis:
 $H_0: \mu = 50$)



Population



Now select a random sample

Is $\bar{X}=20$ likely if $\mu = 50$?

If not likely,

REJECT

Null Hypothesis



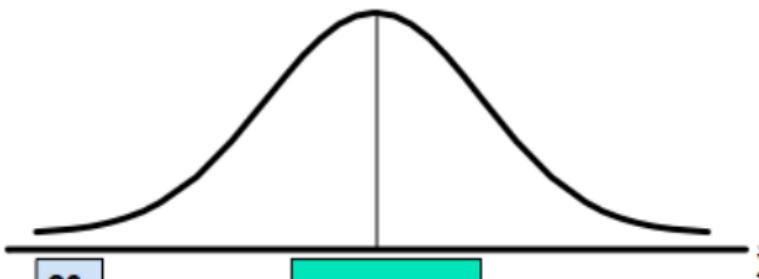
Suppose the sample mean age is 20: $\bar{X} = 20$



Sample

Rejecting or Accepting H_0 ?

Sampling Distribution of \bar{X}



If it is unlikely that we would get a sample mean of this value ...

$\mu = 50$
If H_0 is true

... if in fact this were the population mean...

... then we reject the null hypothesis that $\mu = 50$.

Sample mean \bar{X} can take many values

Evidence support H_0 ?

- Even H_0 is true, the sample mean \bar{x} can be close or far from 50. We are more confident that the true mean is 50 if the sample mean \bar{x} is "close" to 50
- "How close is enough". For example, if the sample mean is between 48.5 and 51.5 then we can consider that H_0 is acceptable
- Decide to reject H_0 when either $\bar{x} > 51.5$ or $\bar{x} < 48.5$

Critical values - Rejection Region

- The values 48.5 and 51.5 are called **critical values**
- The interval $[48.5; 51.5]$ is called **acceptance region**
- The interval $(-\infty, 48.5) \cup (51.5, \infty)$ is called **rejection region** or **critical region**

What happens if true population mean age $\mu = 50$ but the observed sample mean is 58?

We can reject H_0 even when H_0 is true

Face to a wrong decision making

Error in Decision Making

- **Type I error** Rejecting the null hypothesis H_0 when it is true
- **Type II error** Failing to reject the null hypothesis when it is false

| | H_0 is true | H_0 is false |
|---------------------|------------------|------------------|
| Do not reject H_0 | Correct decision | Type II error |
| Reject H_0 | Type I error | Correct decision |

Level of significance

Because our decision is based on random variables,
probability of type I error

$$\alpha = P(\text{reject } H_0 \text{ when } H_0 \text{ is true})$$

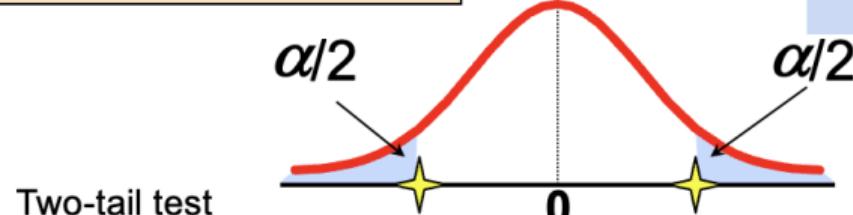
is called to be **level of significance**

Fixed level of significance

- Typical value for level of significance α are 1%, 5%, 10%
- is selected by the researcher at the beginning
- provides critical values for the test
- defines rejection region of the sampling distribution

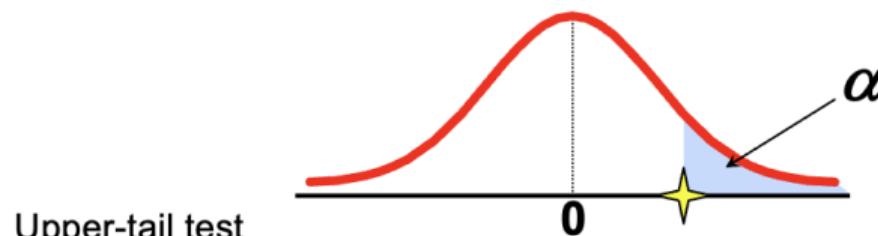
Critical Region for Fixed level of significance

Level of significance = α

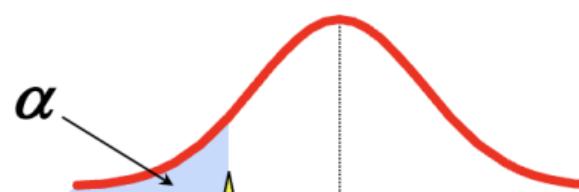


★ Represents critical value

Rejection region is shaded



Upper-tail test



Disadvantage of fixed significant level testing

- the null hypothesis was or was not rejected at a specified level of significance \Rightarrow no idea about the statistic is just barely in or far in rejection region
- Level of significance is not usually set up in advance but rather look at in data to determine a level

P - value is the smallest level of significance that would lead to rejection of null hypothesis with a given data.

P-value is the **observed significance level** when we use value of evidence that we observe from data as critical value

General Procedure for Hypothesis Testing (1)

- ① Determine parameter of interest
- ② State null and alternative hypothesis
- ③ Choose a test statistic and compute value of the test statistic v_{obs} .

General Procedure for Hypothesis Testing (2)

Critical value approach

- 4 Determine critical value and reject region W_α based on level of significance α , distribution of test statistic and type of test

- 5 Draw conclusion

- $v_{obs} \in W_\alpha$: reject H_0
- $v_{obs} \notin W_\alpha$: fail to reject H_0

P- value approach

- 4 Compute the P - value based on the value of test statistic v_{obs}

- 5 Draw conclusion

- P-value $< \alpha$: reject H_0
- P-value $> \alpha$: fail to reject H_0

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Tests for Single Mean μ when Population Variance σ^2 Known

Problem

Test

$$H_0 : \mu = \mu_0$$

vs

$$H_1 : \mu \neq \mu_0 \quad (\text{two-sided test})$$

or $H_1 : \mu > \mu_0 \quad (\text{upper tail test})$

or $H_1 : \mu < \mu_0 \quad (\text{lower tail test})$



Z-test

Normal population or large sample size $n \geq 30$

If $H_0 : \mu = \mu_0$ is true then by central limit theorem or property of sample mean for normal population, **test statistics**

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \hookrightarrow N(0, 1)$$

The value of Z computed from observed data

$$z_{obs} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$



Z-test

Normal population or large sample size $n \geq 30$

If $H_0 : \mu = \mu_0$ is true then by central limit theorem or property of sample mean for normal population, **test statistics**

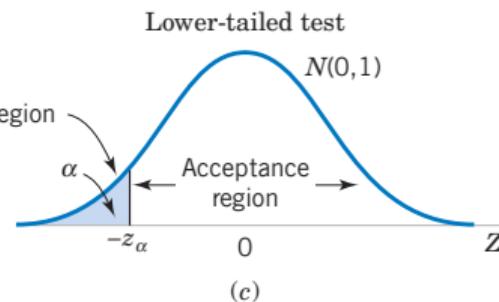
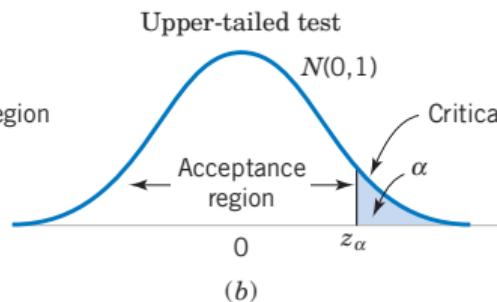
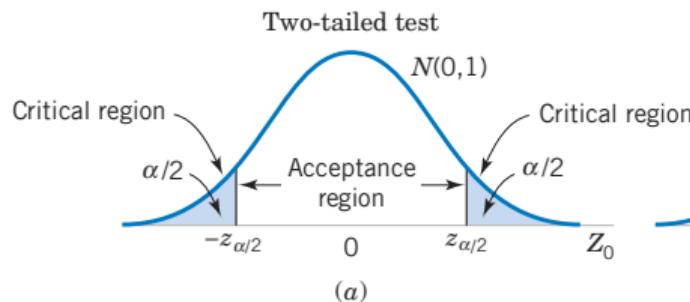
$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} \hookrightarrow N(0, 1)$$

The value of Z computed from observed data

$$z_{obs} = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$



Critical value - Reject region



Reject H_0 if

$$|z_{obs}| > z_{\frac{\alpha}{2}}$$

Two - sided test

$$|z_{obs}| > z_\alpha$$

One - sided test



- *Two-sided test:* P-value = $2P(Z > |z_0|)$
- *One-sided test:* P-value = $P(Z > |z_0|)$

Example

A random sample of 100 recorded deaths in the United States during the past year showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years, does this seem to indicate that the mean life span today is greater than 70 years? Use a 0.05 level of significance.

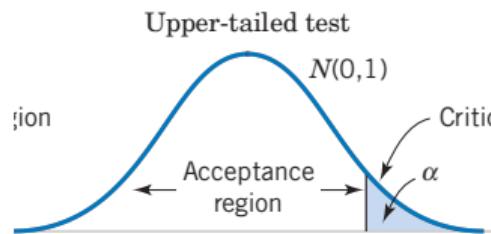
- **Parameter of interest:** mean life span today μ
- $H_0 : \mu = 70, H_1 : \mu > 70$
- population std $\sigma = 8.9$, sample size $n = 100$, sample mean $\bar{x} = 71.8$
- Z-test

$$z_{obs} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{71.8 - 70}{8.9/\sqrt{100}} = 2.02$$

- **Parameter of interest:** mean life span today μ
- $H_0 : \mu = 70, H_1 : \mu > 70$
- population std $\sigma = 8.9$, sample size $n = 100$, sample mean $\bar{x} = 71.8$
- Z-test

$$z_{obs} = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{71.8 - 70}{8.9 / \sqrt{100}} = 2.02$$

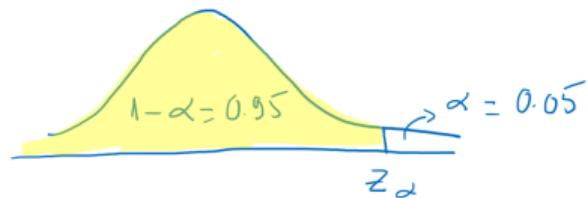
Approach 1 - Draw conclusion with critical value



- Level of significance $\alpha = .05 = 5\% \Rightarrow z_\alpha = 1.65$
- $z_{obs} > z_\alpha$: reject H_0 at 5% level of significance.

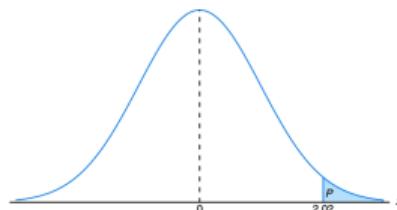
Find $z_\alpha = z_{0.05} = 1.65$

Table A3



| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |

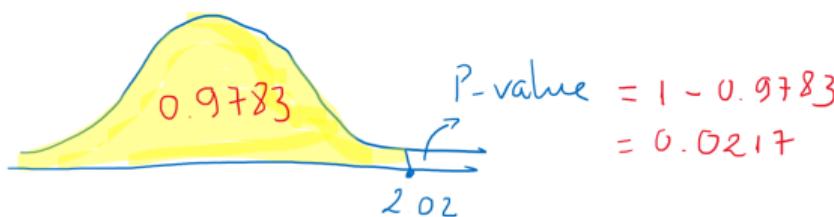
Approach 2 - Draw conclusion with P-value



- P-value = $P(Z > 2.02) = .0217 = 2.17\%$
- Level of significance $\alpha = .05 = 5\%$
- P-value $< \alpha \Rightarrow$ reject H_0
- Comment: the evidence in favor of H_1 is even stronger than that suggested by a 0.05 level of significance.

Find P-value = $P(Z > 2.02) = .0217 = 2.17\%$

Table A3



| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |

In order to draw conclusion, you can choose either critical value approach or P-value approach. They always produce the same conclusion.

Example

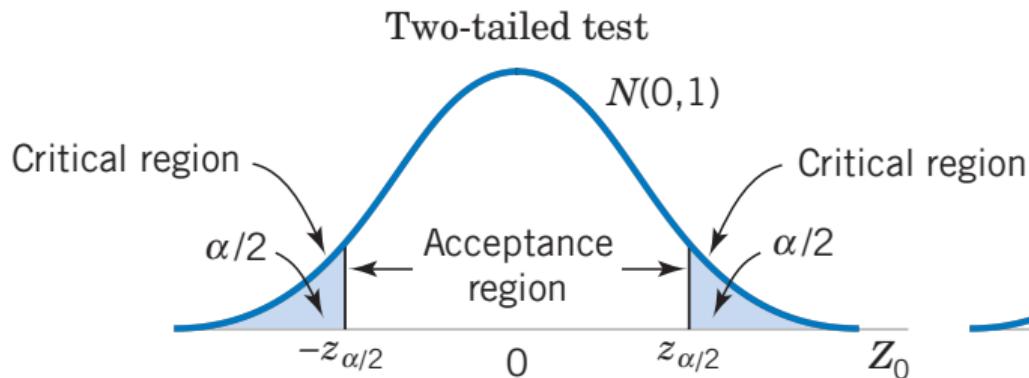
A manufacturer of sports equipment has developed a new synthetic fishing line that the company claims has a mean breaking strength of 8 kilograms with a standard deviation of .5 kilogram. Test the hypothesis that $\mu = 8$ kilograms against the alternative that $\mu \neq 8$ kilograms if a random sample of 50 lines is tested and found to have a mean breaking strength of 7.8 kilograms. Use a .01 level of significance

Solution

- $H_0 : \mu = 8, H_1 : \mu \neq 8$
- $\sigma = .5, n = 50, \bar{x} = 7.8$
- Z-test

$$z_{obs} = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{7.8 - 8}{.5 / \sqrt{50}} = 2.83$$

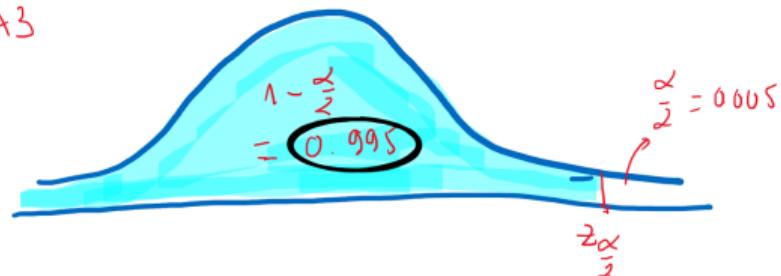
Critical value approach



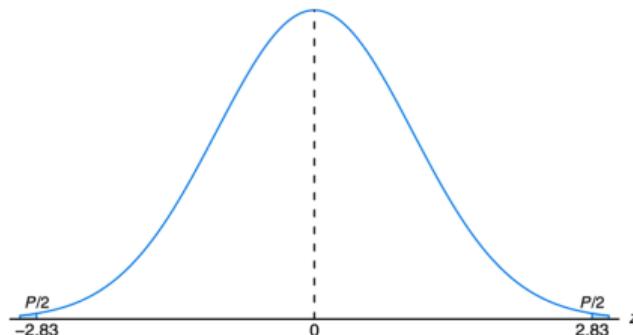
- Level of significance $\alpha = .01 = 1\% \Rightarrow z_{\frac{\alpha}{2}} = 2.58$
- $z_{obs} > z_{\frac{\alpha}{2}}$: reject H_0 at 1% level of significance.

Find $z_{\frac{\alpha}{2}} = z_{.005} = 2.58$

Table A3



| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |



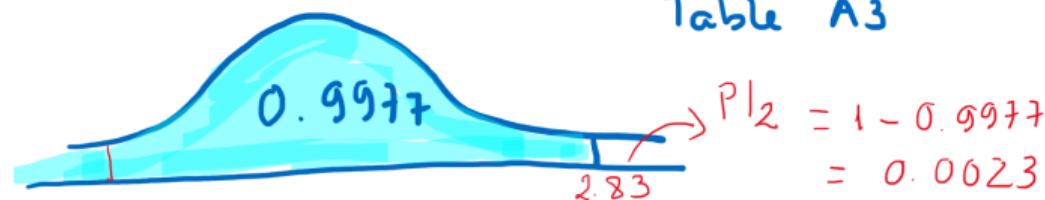
- For two - sided test:

$$\text{P-value} = 2P(Z > 2.83) = .0046$$

- Level of significance $\alpha = .01 = 1\%$
- $\text{P-value} < \alpha \Rightarrow \text{reject } H_0 \text{ that } \mu = 8 \text{ kg}$

Find $\frac{P}{2} = .0023$

Table A3



| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |

A researcher wishes to test the claim that the average cost of tuition and fees at a four year public college is greater than \$5700. She selects a random sample of 36 four-year public colleges and finds the mean to be \$5950. The population standard deviation is \$659. Is there evidence to support the claim at $\alpha = 0.05$?

Test for mean μ when population variance σ^2 is unknown



Problem

Test

$$H_0 : \mu = \mu_0$$

vs

$$H_1 : \mu \neq \mu_0 \quad (\text{two-sided test})$$

or $H_1 : \mu > \mu_0 \quad (\text{upper tail test})$

or $H_1 : \mu < \mu_0 \quad (\text{lower tail test})$

T-test

normal population

If $H_0 : \mu = \mu_0$ is true then **test statistics**

$$T = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \hookrightarrow T_{n-1}$$

t - distribution with $n - 1$ degree of freedom

The value of T computed from data

$$t_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

T-test

normal population

If $H_0 : \mu = \mu_0$ is true then **test statistics**

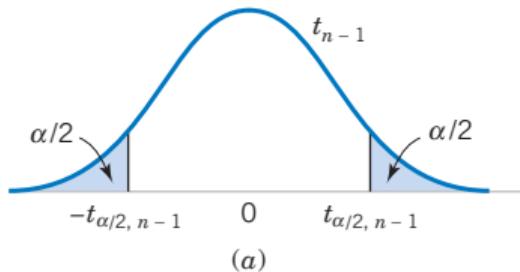
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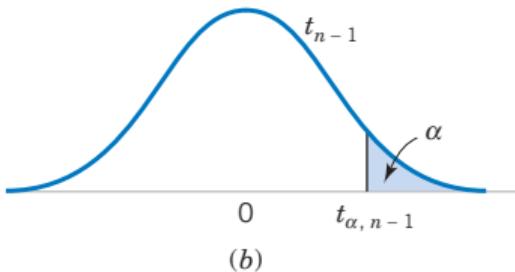
The value of T computed from data

$$t_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

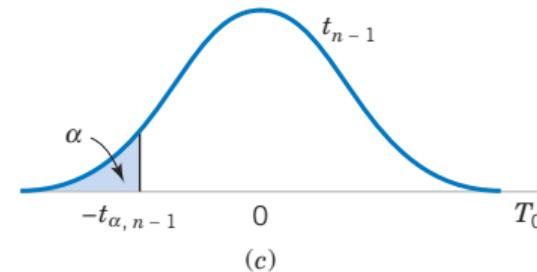
Critical value - Reject region



(a)



(b)



(c)

Reject H_0 if

$$|t_{obs}| > t_{\frac{\alpha}{2}, n-1}$$

Two - sided test

$$|t_{obs}| > t_{\alpha, n-1}$$

One - sided test



- *Two-sided test*: P-value = $2P(T > |t_{obs}|)$
- *One-sided test*: P-value = $P(T > |t_{obs}|)$

Conclusion

- Reject H_0 if P-value $< \alpha$
- Fail to reject H_0 if P-value $> \alpha$

Example

If a random sample of 12 homes included in a planned study indicates that vacuum cleaners use an average of 42 kilowatt hours per year with a standard deviation of 11.9 kilowatt hours. Does this suggest at the .05 level of significance that vacuum cleaners use, on average, less than 46 kilowatt hours annually?

Assume the **population of kilowatt hours to be normal.**

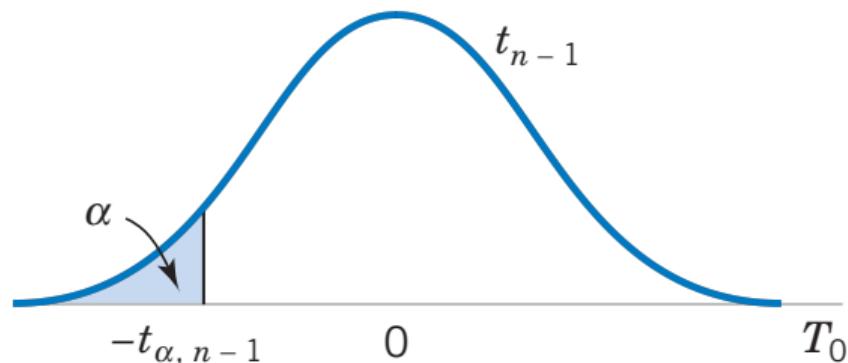


Solution

- **Parameter of interest:** average kwh that vacuum cleaners use μ
- $H_0 : \mu = 46, H_1 : \mu < 46$
- Given $n = 12, \bar{x} = 42, s = 11.9$
- T-test

$$t_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{42 - 46}{11.9/\sqrt{12}} = -1.16$$

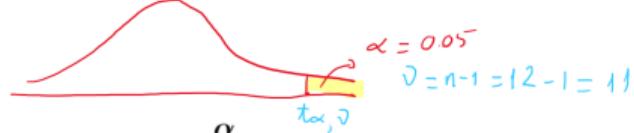
Draw conclusion with Critical value - rejection region



- $\alpha = 5\% \Rightarrow t_{\alpha, n-1} = t_{.05, 11} = 1.796$
- $t_{obs} > -t_{\alpha, n-1}$: fail to reject H_0

Find $t_{\alpha, n-1} = t_{.05, 11} = 1.796$

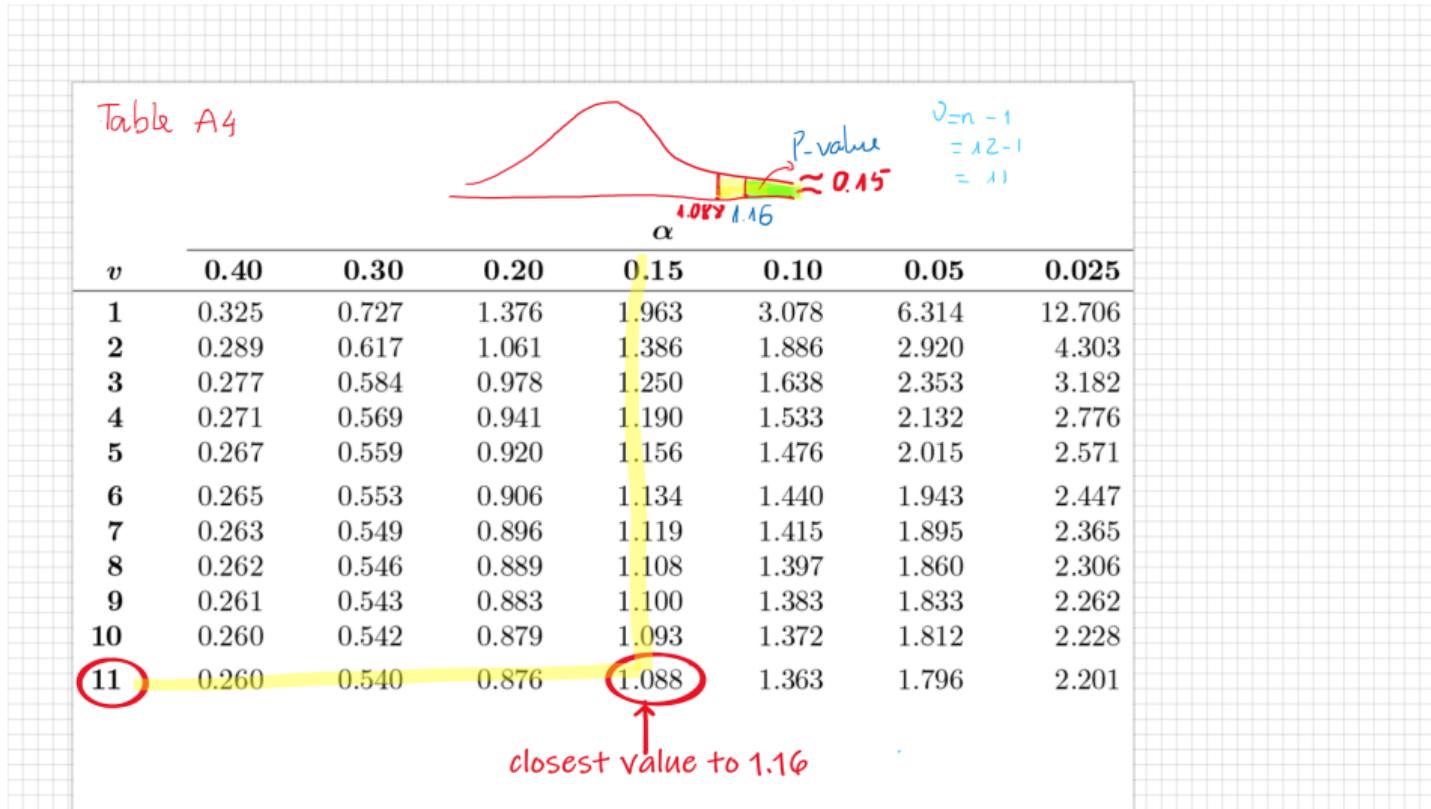
Table A4



| v | 0.40 | 0.30 | 0.20 | 0.15 | 0.10 | 0.05 | 0.025 |
|-----|-------|-------|-------|-------|-------|-------|--------|
| 1 | 0.325 | 0.727 | 1.376 | 1.963 | 3.078 | 6.314 | 12.706 |
| 2 | 0.289 | 0.617 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 |
| 3 | 0.277 | 0.584 | 0.978 | 1.250 | 1.638 | 2.353 | 3.182 |
| 4 | 0.271 | 0.569 | 0.941 | 1.190 | 1.533 | 2.132 | 2.776 |
| 5 | 0.267 | 0.559 | 0.920 | 1.156 | 1.476 | 2.015 | 2.571 |
| 6 | 0.265 | 0.553 | 0.906 | 1.134 | 1.440 | 1.943 | 2.447 |
| 7 | 0.263 | 0.549 | 0.896 | 1.119 | 1.415 | 1.895 | 2.365 |
| 8 | 0.262 | 0.546 | 0.889 | 1.108 | 1.397 | 1.860 | 2.306 |
| 9 | 0.261 | 0.543 | 0.883 | 1.100 | 1.383 | 1.833 | 2.262 |
| 10 | 0.260 | 0.542 | 0.879 | 1.093 | 1.372 | 1.812 | 2.228 |
| 11 | 0.260 | 0.540 | 0.876 | 1.088 | 1.363 | 1.796 | 2.201 |

- P-value = $P(T > |t_{obs}|) = P(T > 1.16) = .135$
- Level of significance $\alpha = .05$
- P-value $> \alpha$
- **Decision:** fail to reject H_0 and conclude that the average number of kilowatt hours used annually by home vacuum cleaners is not significantly less than 46.

Find P-value = $P(T > |t_{obs}|) = P(T > 1.16) = .135 (=$
 $tdist(1.16, 11, 1)$ in Excel)



Example

A public health official claims that the mean home water use is 350 gallons a day. 20 randomly selected homes were investigated with the average daily water uses as follows:

| | | | |
|-----|-----|-----|-----|
| 340 | 344 | 362 | 375 |
| 356 | 386 | 354 | 364 |
| 332 | 402 | 340 | 355 |
| 362 | 322 | 372 | 324 |
| 318 | 360 | 338 | 370 |

Do the data contradict the claim with level of significant $\alpha = 1\%$? Assume that the population has normal distribution.



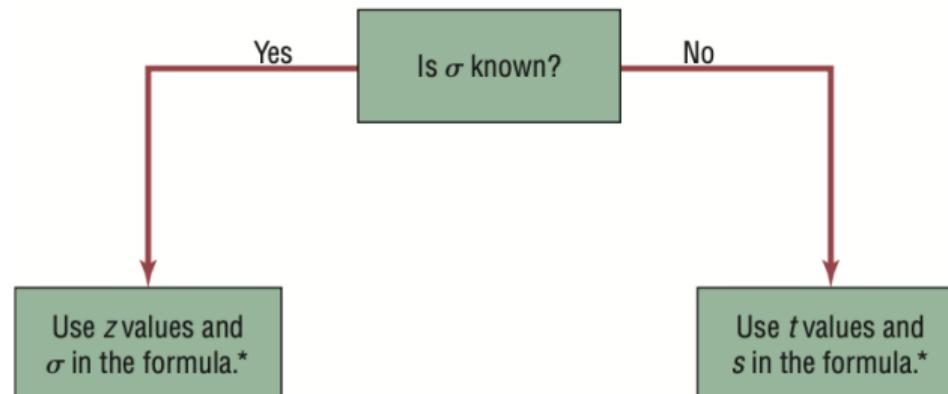
- **Parameter of interest:** mean home water use μ
- Hypothesis $H_0 : \mu = 350$, $H_1 : \mu \neq 350$
- $\bar{x} = 353.8$, $s = 21.8478$, $n = 20$
- T-test

$$t_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{353.8 - 350}{\frac{21.8478}{\sqrt{20}}} = .7778$$

- Critical value for two - sided test $t_{\alpha/2,n-1} = ???$
(look up table value of t-distribution with 19 degree of freedom)
- Conclusion??

A medical investigation claims that the average number of infections per week at a hospital in southwestern Pennsylvania is 16.3. A random sample of 10 weeks had a mean number of 17.7 infections. The sample standard deviation is 1.8. Is there enough evidence to reject the investigator's claim at $\alpha = 0.05$? Suppose that population has normal distribution

Test for population mean - z - test or t -test



*If $n < 30$, the variable must be normally distributed.

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

$$T = \frac{\bar{X} - \mu_0}{S / \sqrt{n}}$$

Test for variance σ^2 of normal population

$$H_0 : \sigma^2 = \sigma_0^2$$

vs

- $H_1 : \sigma^2 \neq \sigma_0^2$ (Two-sided test)
- $H_1 : \sigma^2 > \sigma_0^2$ (Upper tail test)
- $H_1 : \sigma^2 < \sigma_0^2$ (Lower tail test)

χ^2 - test

normal population

If $H_0 : \sigma^2 = \sigma_0^2$ is true then **test statistic**

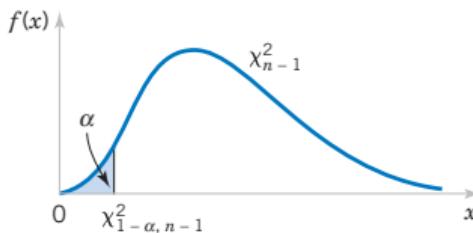
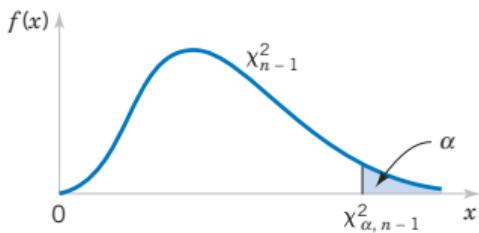
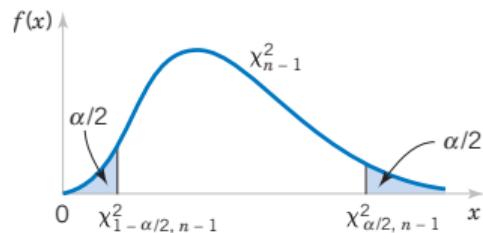
$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2} \sim \chi_{n-1}^2$ distribution with $n-1$ degree of freedom

Observed value of χ^2 computed from data

$$\chi_{obs}^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

Critical value - reject region

Reject H_0 if



$$\chi^2_{obs} < \chi^2_{\frac{1-\alpha}{2}, n-1}$$

$$\chi^2_{obs} > \chi^2_{\alpha, n-1}$$

$$\chi^2_{obs} < \chi^2_{1-\alpha, n-1}$$

or $\chi^2_{obs} > \chi^2_{\frac{\alpha}{2}, n-1}$
Two - sided test

Upper - tail test

Lower - tail test

Example

A manufacturer of car batteries claims that the life of the company's batteries is approximately normally distributed with a standard deviation equal to 0.9 year. If a random sample of 10 of these batteries has a standard deviation of 1.2 years, do you think that $\sigma > 0.9$ year? Use a 0.05 level of significance.

Solution

- Hypothesis $H_0 : \sigma^2 = 0.9^2 = 0.81$ vs $H_1 : \sigma^2 > 0.81$
- $n = 10, s = 1.2$ or $s^2 = 1.2^2 = 1.44, \sigma_0^2 = 0.81$
- χ^2 - test value

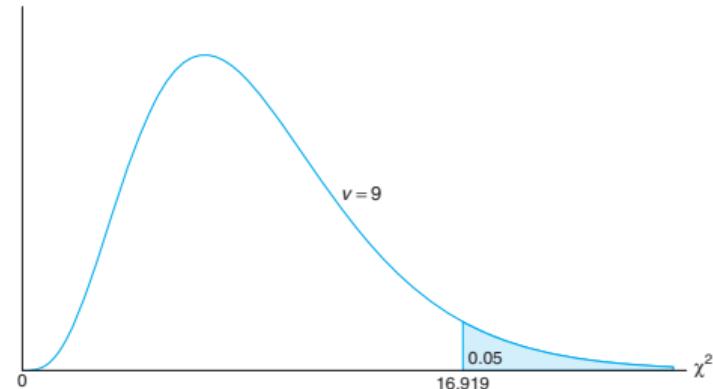
$$\chi_{obs}^2 = \frac{(n - 1)s^2}{\sigma_0^2} = 16.0$$

Draw conclusion with Critical value - rejection region

- Critical value for upper tail test

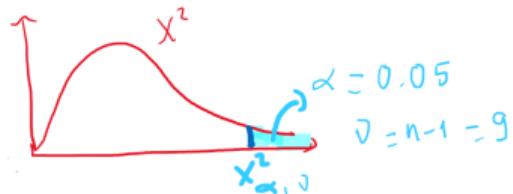
$$\alpha = .05 \Rightarrow \chi^2_{\alpha, n-1} = \chi^2_{0.05, 9} = 16.916$$

- $\chi^2_{obs} < \chi^2_{0.05, 9} \Rightarrow \text{accept } H_0$. There is not enough evidence that $\sigma > 0.9$



Find $\chi^2_{.05,9} = 16.916$

Table A5



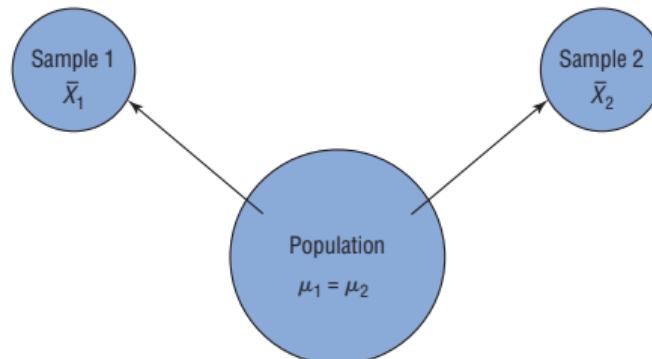
| v | α | | | | | | | | | |
|----|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | 0.30 | 0.25 | 0.20 | 0.10 | 0.05 | 0.025 | 0.02 | 0.01 | 0.005 | 0.001 |
| 1 | 1.074 | 1.323 | 1.642 | 2.706 | 3.841 | 5.024 | 5.412 | 6.635 | 7.879 | 10.827 |
| 2 | 2.408 | 2.773 | 3.219 | 4.605 | 5.991 | 7.378 | 7.824 | 9.210 | 10.597 | 13.815 |
| 3 | 3.665 | 4.108 | 4.642 | 6.251 | 7.815 | 9.348 | 9.837 | 11.345 | 12.838 | 16.266 |
| 4 | 4.878 | 5.385 | 5.989 | 7.779 | 9.488 | 11.143 | 11.668 | 13.277 | 14.860 | 18.466 |
| 5 | 6.064 | 6.626 | 7.289 | 9.236 | 11.070 | 12.832 | 13.388 | 15.086 | 16.750 | 20.515 |
| 6 | 7.231 | 7.841 | 8.558 | 10.645 | 12.592 | 14.449 | 15.033 | 16.812 | 18.548 | 22.457 |
| 7 | 8.383 | 9.037 | 9.803 | 12.017 | 14.067 | 16.013 | 16.622 | 18.475 | 20.278 | 24.321 |
| 8 | 9.524 | 10.219 | 11.030 | 13.362 | 15.507 | 17.535 | 18.168 | 20.090 | 21.955 | 26.124 |
| 9 | 10.656 | 11.389 | 12.242 | 14.684 | 16.919 | 19.023 | 19.679 | 21.666 | 23.589 | 27.877 |
| 10 | 11.781 | 12.549 | 13.442 | 15.987 | 18.307 | 20.483 | 21.161 | 23.209 | 25.188 | 29.588 |

A machine that automatically controls the amount of ribbon on a tape has recently been installed. This machine will be judged to be effective if the standard deviation σ of the amount of ribbon on a tape is no greater than .15 cm. If a sample of 20 tapes yields a sample variance of $s^2 = .025 \text{ cm}^2$, are we justified in concluding that the machine is ineffective? Use a 0.05 level of significance.

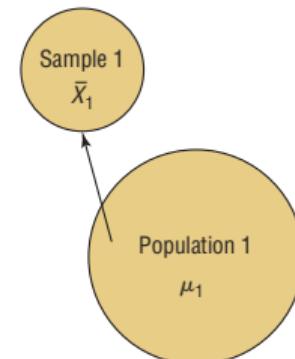
Table of contents

- 1 Statistical Hypotheses: General Concepts
- 2 Testing a Statistical Hypothesis
- 3 Single Sample
- 4 Two samples

Comparison means of two populations - Known variances



(a) Difference is not significant



(b) Difference is significant

Comparison problem

- Distribution of two population are $N(\mu_X, \sigma_X^2)$, and $N(\mu_Y, \sigma_Y^2)$ or large sample sizes
- Know σ_X, σ_Y , don't know μ_X, μ_Y
- **Want to compare μ_X and μ_Y**
- $H_1 : \mu_X - \mu_Y \neq d_0$ (**two-sided**) or $\mu_X - \mu_Y > d_0$ (**upper tail**) or $\mu_X - \mu_Y < d_0$ (**lower-tail**)

Comparision problem

$$H_0 : \mu_X - \mu_Y = d_0$$

VS

$$H_1 : \mu_X - \mu_Y \neq d_0 \text{ two -sided test}$$

$$H_1 : \mu_X - \mu_Y > d_0 \text{ upper - tail test}$$

$$H_1 : \mu_X - \mu_Y < d_0 \text{ lower - tail test}$$

Sampling Distribution

- Sample mean \bar{X} population 1 is used to estimate μ_X
- Sample mean \bar{X} population 2 is used to estimate μ_Y
- $\bar{X} - \bar{Y}$ is an estimator for $\mu_X - \mu_Y$
- If both population have normal distributions $N(\mu_X, \sigma_X^2)$, and $N(\mu_Y, \sigma_Y^2)$ or large sample sizes then $\bar{X} - \bar{Y} - d_0 \sim N(\mu_X - \mu_Y, \frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y})$ given H_0 is true

z- test

two normal populations and variance known

If $H_0 : \mu_X - \mu_Y = d_0$ is true then **Test statistics** is **Z-test**

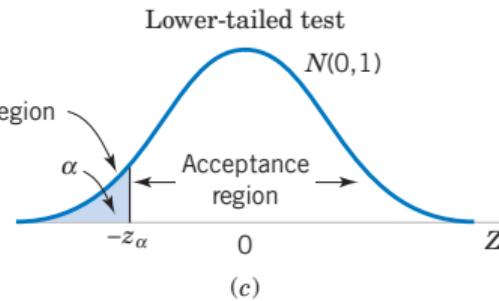
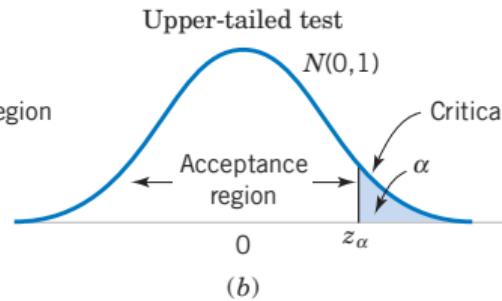
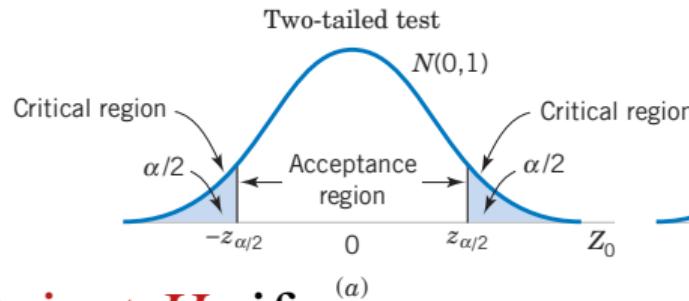
$$Z = \frac{\bar{X} - \bar{Y} - d_0}{\sqrt{\frac{\sigma_x^2}{n_X} + \frac{\sigma_Y^2}{n_Y}}} \sim N(0, 1)$$

- *Two-sided test:* P-value = $2P(Z > |z_{obs}|)$
- *One-sided test:* P-value = $P(Z > |z_{obs}|)$

where z_{obs} is the value of Z computed from data

$$z_{obs} = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_x^2}{n_X} + \frac{\sigma_y^2}{n_Y}}}$$

Critical value - Reject region



Reject H_0 if

$$|z_{obs}| > z_{\frac{\alpha}{2}}$$

Two - sided test

$$|z_{obs}| > z_{\alpha}$$

One - sided test

Example

A company produces a sample of 10 tires using method 1 and a sample of 8 using method 2. They **want to show that there is no difference in the average life time of tires.**

The first tire set is tested at location A where the standard deviation is known to be 4000 km, and second set is tested at location B where sd is 6000. What conclusion can be drawn with 5% level of significance from the following data? Suppose that both populations have normal distributions.

TABLE 8.3 *Tire Lives in Units of 100 Kilometers*

| Tires Tested at A | Tires Tested at B |
|-------------------|-------------------|
| 61.1 | 62.2 |
| 58.2 | 56.6 |
| 62.3 | 66.4 |
| 64 | 56.2 |
| 59.7 | 57.4 |
| 66.2 | 58.4 |
| 57.8 | 57.6 |
| 61.4 | 65.4 |
| 62.2 | |
| 63.6 | |



- $H_0 : \mu_A = \mu_B$ vs $H_1 : \mu_A \neq \mu_B$
- $n_A = 10, \bar{x}_A = 61.65, \sigma_A = 4000$
- $n_B = 8, \bar{x}_B = 60.025, \sigma_B = 6000$
- Z-test

$$z_{obs} = \frac{\bar{x}_A - \bar{x}_B}{\sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}}} = .066$$

- Critical value?
- Rejection region?
- Conclusion?

A real estate agent compares the selling prices of homes in two municipalities in southwestern Pennsylvania to see if there is a difference. Is there enough evidence to reject the claim that the average cost of a home in both locations is the same? Use $\alpha = 0.01$.

| Scott | Lionier |
|------------------------|------------------------|
| $\bar{x}_1 = \$93,430$ | $\bar{x}_2 = \$98,043$ |
| $\sigma_1 = \$5,602$ | $\sigma_2 = \$4732$ |
| $n_1 = 35$ | $n_2 = 40$ |