HO CHI MINH CITY INTERNATIONAL UNIVERSITY

FINAL EXAMINATION
Academic year 2011-2012, Semester 2
Duration: 120 minutes

SUBJECT: Differential Equations	
Acting Chair of Department of Mathematics	Lecturer:
Signature:	Signature:
Full name: Prof. Nguyen Dinh	Full name: Dr. Pham Huu Anh Ngoc

Instructions:

- *Open-book examination. Laptops are NOT allowed.*

Question 1. (15 marks) (**Modeling a Chemical Mixture**)

A tank contains 40 gallons of a solution composed of 80% water and 20% alcohol. A second solution containing half water and half alcohol is added to the tank at the rate of 4 gallons per minute. At the same time, the tank is being drained at the rate of 4 gallons per minute, as shown in the below figure. Assuming that the solution is stirred constantly, how much alcohol will be in the tank after 10 minutes?

Question 2. (20 marks) Find the real number α such that $y = x^\alpha$ is a solution of

$$x^2 y'' - 3xy' + 4y = 0, \quad x \in (0, \infty).$$

Solve the given differential equation.

Question 3. a) (15 marks) *Determine the form* of a particular solution of $y^{(4)} + y''' = 1 - x^2 e^{-x}$.

b) (15 marks) Find the general solution of the following differential equation

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}.$$

Question 4. (20 marks) Solve the linear system of differential equations

$$\begin{cases} \frac{dx}{dt} = x - 2y \\ \frac{dy}{dt} = x - y. \end{cases}$$

Question 5. (15 marks) Solve the following differential equation

$$y'' + y = \cos^2 x.$$

End.

SOLUTIONS:

Question 1. Let y be the number of gallons of alcohol in the tank at any time t. The percent of alcohol in the 40-gallon tank at any time is $\frac{y}{40}$. Moreover, because 4 gallons of solution are being drained each minute and 2 gallons of alcohol are entering each minute, the rate of change of y is

$$\frac{dy}{dt} = 2 - 4\frac{y}{40}.$$

Solve this equation, to get $y = 20 + Ce^{\frac{-t}{10}}$. Since $y(0) = 8$, it follows $C = -12$. Hence, $y = 20 - 12e^{\frac{-t}{10}}$. Therefore, $y(10) = 20 - 12e^{-1} \approx 15.6$ gallons.

Question 2.
- $\alpha = 2$.
-The standard form of the equation

$$y'' - \frac{3}{x}y' + \frac{4}{x^2}y = 0.$$

The second solution is

$$y(x) = x^2 \int \frac{e^{\int \frac{3}{x}dx}}{x^4}dx = x^2 \ln x.$$

- The general solution on the interval $(0, \infty)$ is given by $y(x) = c_1 x^2 + c_2 x^2 \ln x$.

Question 3.
a)
$$y_p = Ax^3 + Bx^3 e^{-x} + Cx^2 e^{-x} + Dxe^{-x}.$$

b) The general solution of the equation is

$$y(x) = c_1 e^{-x} + c_2 e^{3x} - \frac{4}{3}x + \frac{23}{9} - (2x + \frac{4}{3})e^{2x}.$$

Question 4. The general solution of the given system is

$$x(t) = (2c_1 \cos t + 2c_2 \sin t); \qquad y(t) = (c_1 - c_2)\cos t + (c_1 + c_2)\sin t.$$

Question 5. The general solution is given by

$$y(x) = c_1 \cos x + c_2 \sin x + \frac{1}{2} - \frac{1}{6}\cos 2x.$$