Applied Linear Algebra
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Week 9 :

1.

Given $B = (b_1, b_2, b_3) = \{(8, 11, 0), (7, 0, 10), (1, 4, 6)\}$

$x = (3, 19, 2)$

Let $x = ab_1 + bb_2 + cb_3$

$$\Rightarrow a\begin{pmatrix}8\\11\\0\end{pmatrix} + b\begin{pmatrix}7\\0\\10\end{pmatrix} + c\begin{pmatrix}1\\4\\6\end{pmatrix} = \begin{pmatrix}3\\19\\2\end{pmatrix}$$

$$\longrightarrow \left(\begin{array}{ccc|c}8 & 7 & 1 & 3\\11 & 0 & 4 & 19\\0 & 10 & 6 & 2\end{array}\right) \Rightarrow \begin{pmatrix}a\\b\\c\end{pmatrix} = \begin{pmatrix}1\\-1\\2\end{pmatrix}$$

$$[x_B] = \begin{bmatrix}1\\-1\\2\end{bmatrix}$$

The cordinate of the vector $x = (4, 23, 8)$ relative to the basis B' is $[x]_B = \begin{bmatrix}1\\-1\\2\end{bmatrix}$

If $B = \{b_1, b_2, \ldots, b_n\}$ is a basis in $\mathbb{R}^n$ then every vector $\vec{v} \in \mathbb{R}^n$ can be uniquely expressed as a linear combination of basis vectors $b_1, b_2, \ldots, b_n$.
i.e there exists unique scalars $\alpha_1, \alpha_2, \cdots, \alpha_n$ such that $\vec{v} = \alpha_1 b_1 +, \alpha_2 b_2 + \cdots + \alpha_n b_n$.
The coordinates of the vector $\vec{v}$ relative to the basis B is the sequence of co-ordinates, i.e. $[v]_B = (\alpha_1, \alpha_2, \cdots, \alpha_n)$.

2.

Let $S=\{(1,-4),(3,-5)\}$, and $T=\{(-9,1),(-5,1)\}$ R^2

Transition matrix from T to S.

Denote $[w_1]_S = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $[w_2]_S = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

$$[w_1]_S = x_1 u_1 + x_2 u_2 = x_1\begin{pmatrix} 1 \\ -4 \end{pmatrix} + x_2\begin{pmatrix} 3 \\ -5 \end{pmatrix}$$

$$[w_2]_u = y_1 u_1 + y_2 u_2 = y_1\begin{pmatrix} 1 \\ -4 \end{pmatrix} + y_2\begin{pmatrix} 3 \\ -5 \end{pmatrix} \quad (2)$$

From (1), (2) we have:

$$x_1\begin{pmatrix} 1 \\ -4 \end{pmatrix} + x_2\begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} -9 \\ 1 \end{pmatrix} \quad \begin{pmatrix} x_1 + 3x_2 \\ -4x_1 - 5x_2 \end{pmatrix} = \begin{pmatrix} -9 \\ 1 \end{pmatrix}$$

$$\Leftrightarrow$$

$$y_1\begin{pmatrix} 1 \\ -4 \end{pmatrix} + y_2\begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \end{pmatrix} \quad \begin{pmatrix} 1y_1 + 3y_2 \\ -4y_1 - 5y_2 \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \end{pmatrix}$$

$$\underset{A}{\begin{pmatrix} 1 & 3 \\ -4 & -5 \end{pmatrix}}\underset{X}{\begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix}} = \underset{B}{\begin{pmatrix} -9 & -5 \\ 1 & 1 \end{pmatrix}}$$

$$\Leftrightarrow \left(\begin{array}{cc|cc} 1 & 3 & -9 & -5 \\ -4 & -5 & 1 & 1 \end{array}\right)$$

$4R_1 + R_2$

$$\left(\begin{array}{cc|cc} 1 & 3 & -9 & -5 \\ 0 & 7 & -35 & -21 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 3 & -9 & -5 \\ 0 & 1 & -5 & -3 \end{array}\right)$$

$$\left(\begin{array}{cc|cc} 1 & 0 & 6 & 4 \\ 0 & 1 & -5 & -3 \end{array}\right)$$

$$\Rightarrow P_{T\to S} = \begin{pmatrix} 6 & 4 \\ -5 & -3 \end{pmatrix}$$

b) Let vector $v = (1, -2)$, Find $[v]_T$, $[v]_S$

$$\left(\begin{array}{cc|c} -9 & -5 & 1 \\ 1 & 1 & -2 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{cc|c} 1 & 0 & -\frac{11}{14} \\ 0 & 1 & \frac{17}{14} \end{array}\right) \Rightarrow [v]_T = \begin{pmatrix} -\frac{11}{14} \\ \frac{17}{14} \end{pmatrix}$$

$$[v]_S = P_{T \to S} [v]_T$$

$$= \begin{pmatrix} 6 & 4 \\ -5 & -3 \end{pmatrix} \begin{pmatrix} \frac{11}{14} \\ \frac{17}{14} \end{pmatrix}$$

$$\Rightarrow [v]_S =$$

Week 10:

1.

a) $A = \begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix}$

$$P(\lambda) = \det(A - \lambda I) = \begin{vmatrix} 4-\lambda & -5 \\ 2 & -3-\lambda \end{vmatrix}$$

$$= (4-\lambda)(-3-\lambda) + 10 = \lambda^2 - \lambda - 2$$

$$P(\lambda) = 0 \Rightarrow \left[\begin{array}{l} \lambda = -1 \\ \lambda = 2 \end{array}\right.$$

- $\lambda = -1$

$$\begin{bmatrix} 5 & -5 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow a_1 = b_1$$

Setting $a_1 = 1$ we get $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, statisfied $\lambda = -1$

- $\lambda = 2$

$$\begin{bmatrix} 2 & -5 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow 2a_2 = 5b_2$$

Setting $a_2 = 5$, we get $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$, statisfied $\lambda = 2$

b)

$$A = \begin{pmatrix} 6 & -3 \\ -2 & 1 \end{pmatrix}$$

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 6-\lambda & -3 \\ -2 & 4-\lambda \end{vmatrix}$$

$$\begin{vmatrix} 6-\lambda & -3 \\ -2 & 4-\lambda \end{vmatrix} = (6-\lambda)(4-\lambda) - 6 = 0$$

$$\Leftrightarrow 24 - 6\lambda - 4\lambda + \lambda^2 - 6 = 0$$

$$\Leftrightarrow \lambda^2 - 10\lambda + 18 = 0$$

$$\left(\begin{array}{ccc|cc} 2 & 4 & \frac{10}{7} & 2 & 4 \\ -7 & 7 & 10 & -7 & 7 \\ 8 & -9 & k & 8 & -9 \end{array}\right)$$

$$14k + 320$$

2.

$$\begin{pmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ 1 & -2 & -\lambda \end{pmatrix} \begin{matrix} -2-\lambda & 2 \\ 2 & 1-\lambda \\ 1 & -2 \end{matrix}$$

$$(-2-\lambda)(1-\lambda)(-\lambda) - 12 + 12$$

$$-(-3)(1-\lambda) - (-2-\lambda)(-6)(-2)$$

$$-(2)(2)(-\lambda) = 0$$

$$\Leftrightarrow -\lambda^3 - \lambda^2 - 15\lambda + 27 = 0$$

$$\begin{cases} \lambda = -3 \\ \lambda = 1 + \sqrt{10} \\ \lambda = 1 - \sqrt{10} \end{cases}$$