Lecture 6: Assignment 2

Lecturer: Edo Liberty

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1 Weak random projections

setup

In this question we will construct a simple and weak version of random projections. That is, given two vectors $x, y \in \mathbb{R}^d$ we will find two new vectors $x', y' \in \mathbb{R}^k$ such that from x' and y' we could approximate the value of $||x - y||$. The idea is to define k vectors $r_i \in \mathbb{R}^d$ such that each $r_i(j)$ takes a value in $\{+1, -1\}$ uniformly at random. Setting $x'(i) = r_i^T x$ and $y'(i) = r_i^T y$ the questions will lead you through arguing that $\frac{1}{k}||x' - y'||_2^2 \approx ||x - y||_2^2$.

questions

1. Let $z = x - y$, and $z' = x' - y'$. Show that $z'(\ell) = r_\ell^T z$ for any index $\ell \in [1, \ldots, k]$.
2. Show that $E[\frac{1}{k}||z'||_2^2] = E[(z'(\ell))^2] = ||z||_2^2$.
3. Show that
$$\mathrm{Var}[(z'(\ell))^2] \le 4||z||_2^4.$$
Hint: for any vector w we have $||w||_4 \le ||w||_2$.
4. From 3 (even if you did not manage to show it) claim that
$$\mathrm{Var}[\frac{1}{k}||z'||_2^2] \le 4||z||_2^4/k.$$
5. Use 3 and Chebyshev's inequality do obtain a value for k for which:
$$(1 - \varepsilon)||x - y||_2^2 \le \frac{1}{k}||x' - y'||_2^2 \le (1 + \varepsilon)||x - y||_2^2$$
with probability at least $1 - \delta$.