Supervised Learning

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EECS 245[1]

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Hint
Learning Objectives

1 Introduction

What is a machine? How does it learn?

2 Problem Setup

Consider a dataset of n *scalar* values, $y_1, y_2, ..., y_n$. The **mean** of $y_1, y_2, ..., y_n$, denoted $\bar{y}$, is given by:

$$\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i \tag{1}$$

Note
When you were first taught how to code, the very first program you wrote was likely one that displays `"Hello, world!"`. Similarly, proving that $\sqrt{2}$ is irrational is the standard first example of a **proof by contradiction**.

In a proof by contradiction, we start by assuming the statement we want to prove is **not** true. Since we want to prove that $\sqrt{2}$ is **ir**rational, we'll start by assuming it is **rational**.

If $\sqrt{2}$ is rational, then by the definition of a rational number, it must be possible to write $\sqrt{2} = \frac{p}{q}$ for some integers p and q with $q \neq 0$. Then:

$$\sqrt{2} = \frac{p}{q}$$

$$2 = \frac{p^2}{q^2}$$

$$2q^2 = p^2$$

What does this last equation tell us? It's telling us that p^2 is 2 times some other integer (q^2). But, if p^2 is 2 times some other integer, then p^2 must be even. This tells us that p itself must also be even.

If p is even, then $p = 2k$ for some integer k. Substituting this into the last equation, we get:

$$2q^2 = (2k)^2$$

$$2q^2 = 4k^2$$

$$q^2 = 2k^2$$

This tells us that q^2 is 2 times some other integer (k^2). But, if q^2 is 2 times some other integer, then q^2 must be even. This tells us that q itself must also be even.

Why is this a contradiction? Well, we started by assuming that p and q were integers with no common factors. But, if p is even and q is even, then they have a common factor of 2. This contradicts our assumption that p and q have no common factors.

Therefore, our assumption that $\sqrt{2}$ is rational must be false. Therefore, $\sqrt{2}$ is irrational.