EPS 625 – INTERMEDIATE STATISTICS
ONE-WAY ANOVA – IN-CLASS EXAMPLE
ADJUSTING FOR HETEROGENEITY OF VARIANCE

You are interested in whether male college students with black, blond, brunette, or red hair differ with respect to their social extrovertedness. For this study, 60 male college students from a local college (15 for each hair color) have been randomly selected to participate in the study using a stratified random sampling approach. The students were given a measure of social extroversion with a range of 0 (low level of social extroversion) to 10 (high level of social extroversion). Conduct an ANOVA to investigate the relationship between hair color and social extroversion. For this example, use an *a priori* alpha level of significance of $\alpha = .05$ for each statistical analysis (except, use an $\alpha = .001$ for the Shapiro-Wilks test) to answer the following questions.

1. What would the null hypothesis be for this study? Show/write the appropriate symbols or expression in words.

   H_0: $\mu_{black} = \mu_{blond} = \mu_{brunette} = \mu_{redhead}$

   The group means (average social extrovertedness scores) for the four (hair color) groups are equal.

2. What would the research/alternative hypothesis be for this study? Show/write the appropriate symbols or expression in words.

   H_a: $\mu_i \neq \mu_k$ for some *i, k*

   At least two of the (hair color) group means are significantly different from each other.

3. Prior to examining whether group means differ, you need to test the assumptions underlying the one-way ANOVA.
   a. Was the assumption of independence met for these data? Indicate how you made this determination.

      YES, the assumption of independence was met. This is indicated by the four groups of male college students being independent of each other. A stratified random sampling technique was used, which focused on obtaining a random selection of male college students for each of the four hair colors.

   b. Was the assumption of normality met for these data? Indicate how you made this determination.

      YES, the assumption of normality was met for this set of data. This is indicated by the fact that none of the standardized skewness values exceeded ± 3.29, nor were any of the probability values less than (or equal to) the .001 alpha level set for the Shapiro-Wilks test.

Black: $\frac{-.122}{.580} = -.2103$ Blond: $\frac{-.050}{.580} = -.0862$

Brunette: $\frac{.024}{.580} = .0414$ Redhead: $\frac{.586}{.580} = 1.0103$

Black: *p* (.160) > α (.001) Blond: *p* (.402) > α (.001)

Brunette: *p* (.579) > α (.001) Redhead: *p* (.463) > α (.001)

c. Was the assumption of homogeneity of variance met for these data? Indicate how you made this determination.

NO, the assumption of homogeneity was not met, because *p* (.038) < α (.05). This is indicated by the Levene's Test of Homogeneity of Variances, $F(3, 56) = 3.01$, $p = .038$. With an alpha level of .05, *p* (.038) < α (.05), which indicates significance, the null hypothesis (no variance difference) is rejected – as such, indicating that the assumption of homogeneity of variance is not met.

Test of Homogeneity of Variances

Social Extroversion

Levene Statistic	df1	df2	Sig.
3.007	3	56	.038

Looking at the Variance Ratio, we find that the largest group variance (σ^2_{Blond} = 2.875^2 = 8.2656) is not more than 4-5 times that of the smallest group variance ($\sigma^2_{Brunette}$ = 1.447^2 = 2.0938). That is, 8.27/2.09 = 3.96.

4. The next question that needs to be answered is whether all of the groups are the same on their social extroversion means. This is answered by conducting a One-way Analysis of Variance using hair color as the independent variable and students' scores on a measure of social extroversion as the dependent variable. If applicable, use the Welch statistic. What is your conclusion (at this point) from this analysis? Indicate how you came to your conclusion.

Since the assumption of homogeneity of variance was not met for this data, we used the obtained *Welch's* adjusted *F* ratio (10.95), which was significant at the .05 alpha level (*p* shown as .000, that is *p* < .001) reported as *Welch's* $F(3, 30.17) = 10.95$, *p* < .001 (or, *p* < .05), we can conclude that at least two of the four hair color groups differ significantly on their average social extrovertedness scores.

However, beyond that, post hoc follow-up procedures (e.g., Games-Howell) will need to be conducted to test the difference between all unique pairwise comparisons.

Robust Tests of Equality of Means

Social Extroversion

	Statistic[a]	df1	df2	Sig.
Welch	10.946	3	30.171	.000

a. Asymptotically F distributed.

5. Calculate the measure of association and interpret its meaning if applicable, or indicate why this measure would not be needed.

   Since we used the *Welch's F* test, we will use an adjusted omega squared formula.

$$est.\omega^2 = \frac{df_{bet}(F-1)}{df_{bet}(F-1)+N_T} = \frac{3(10.946-1)}{3(10.946-1)+60} = \frac{3(9.946)}{3(9.946)+60} = \frac{29.838}{29.838+60} = \frac{29.838}{89.838} = .3321 = .33$$

   Therefore, we conclude that approximately 33% (ω^2 = .33) of the total variance in the dependent variable (student's average social extrovertedness scores) is accounted for by the independent variable (hair color with four levels).

6. Write the statistical strand for this one-way ANOVA analysis.

   Welch's F(3, 30.17) = 10.95, *p* < .001, *est.* ω^2 = .33

7. Assuming that you found a significant *F*, how do the pairs of groups differ? Indicate which post hoc procedure you used and why. Indicate your findings from the post hoc analysis. That is, how did you determine the pair to be significant or not (this must go beyond the * as an indication)? Be sure to discuss all unique pairwise comparisons.

   Games-Howell post hoc procedure is used since the homogeneity of variance assumption was not met…

   Using an *a priori* alpha level of .05

   Black vs. Blond (Mean difference = 1.200) **is not** significant, *p* (.563) > α (.05)

   Black vs. Brunette (Mean difference = 3.667) **is** significant, *p* (.000) < α (.05)

   Black vs. Redhead (Mean difference = 2.333) **is** significant, *p* (.034) < α (.05)

   Blond vs. Brunette (Mean difference = 2.467) **is** significant, *p* (.035) < α (.05)

   Blond vs. Redhead (Mean difference = 1.133) **is not** significant, *p* (.641) > α (.05)

   Brunette vs. Redhead (Mean difference = 1.333) **is not** significant, *p* (.262) > α (.05)

8. For all significant pairwise comparisons, calculate and report the effect size.

Descriptives

Social Extroversion

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Black	15	7.33	2.059	.532	6.19	8.47	4	10
Blond	15	6.13	2.875	.742	4.54	7.73	1	10
Brunette	15	3.67	1.447	.374	2.87	4.47	1	6
Redhead	15	5.00	2.330	.602	3.71	6.29	2	10
Total	60	5.53	2.574	.332	4.87	6.20	1	10

$(\text{Std. Deviation})^2 = \sigma^2$

Using *Cohen's d* $d = \dfrac{M_1 - M_2}{\sigma_{pooled}}$ where $\sigma_{pooled} = \sqrt{\dfrac{\sigma_1^2 + \sigma_2^2}{2}}$

ES for Black vs. Brunette = $\sigma_{pooled} = \sqrt{\dfrac{4.2395 + 2.0938}{2}} = \sqrt{\dfrac{6.3333}{2}} = \sqrt{3.1667} = 1.7795$

$$d = \frac{M_1 - M_2}{\sigma_{pooled}} = \frac{7.33 - 3.67}{1.7795} = \frac{3.667}{1.7795} = 2.0607 = \mathbf{2.06}$$

ES for Black vs. Redhead = $\sigma_{pooled} = \sqrt{\dfrac{4.2395 + 5.4289}{2}} = \sqrt{\dfrac{9.6684}{2}} = \sqrt{4.8342} = 2.1987$

$$d = \frac{M_1 - M_2}{\sigma_{pooled}} = \frac{7.33 - 5.00}{2.1987} = \frac{2.333}{2.1987} = 1.0611 = \mathbf{1.06}$$

ES for Blond vs. Brunette = $\sigma_{pooled} = \sqrt{\dfrac{8.2656 + 2.0938}{2}} = \sqrt{\dfrac{10.3594}{2}} = \sqrt{5.1797} = 2.2759$

$$d = \frac{M_1 - M_2}{\sigma_{pooled}} = \frac{6.13 - 3.67}{2.2759} = \frac{2.467}{2.2759} = 1.0840 = \mathbf{1.08}$$

Results

A One-way Analysis of Variance (ANOVA) was used to examine the question of whether male college students with black, blond, brunette, or red hair differ with respect to their social extrovertedness. The independent variable represented the different hair colors with four groups being represented: 1) black; 2) blond; 3) brunette; and 4) red. The dependent variable was the average score that students made on a measure of social extroversion with a range of 0 (low level of social extroversion) to 10 (high level of social extroversion). See Table 1 for the means and standard deviations for each of the four groups.

Table 1

Means and Standard Deviations of Social Extroversion Scores by Hair Color

Hair Color	*n*	*Mean*	*SD*
Black	15	7.33	2.06
Blond	15	6.13	2.88
Brunette	15	3.67	1.45
Red	15	5.00	2.33
Total	60	5.53	2.57

The test for normality, examining standardized skewness and the Shapiro-Wilks test, indicated the data were statistically normal. However, the *Levene's F* test revealed that the homogeneity of variance assumption was not met ($p = .038$). As such, the *Welch's F* test was used. An alpha level of .05 was used for all subsequent analyses. The one-way ANOVA of student's average score on the measure of social extroversion revealed a statistically significant

main effect, *Welch's F*(3, 30.17) = 10.95, $p < .001$, indicating that not all hair colors had the same average score on the measure of social extroversion. The estimated omega squared ($\omega^2 = .33$) indicated that approximately 33% of the total variation in average score on students' measure of social extroversion is attributable to differences between the four colors of hair.

Post hoc comparisons, using the Games-Howell post hoc procedure, were conducted to determine which pairs of the four hair color means differed significantly. These results are given in Table 2 and indicate that students with black hair (*M* = 7.33, *SD* = 2.06) had a significantly higher average score on the measure of social extroversion than students with brunette hair (*M* = 3.67, *SD* = 1.45) as well as students with red hair (*M* = 5.00, *SD* = 2.33). The effect sizes for these two significant effects were 2.06 and 1.06, respectively. Additionally, students with blond hair (*M* = 6.13, *SD* = 2.88) had a significantly higher average score on the measure of social extroversion than students with brunette hair, with an effect size of 1.08.

Table 2

Post Hoc Results for Social Extroversion Scores by Hair Color

Hair Color	Mean	Mean Differences ($\overline{X}_i - \overline{X}_j$) (Effect Sizes are indicated in parentheses)			
		1	2	3	4
1. Black	7.33	– –			
2. Blond	6.13	1.20	– –		
3. Brunette	3.67	3.66*** (2.06)	2.46* (1.08)	– –	
4. Red	5.00	2.33* (1.06)	1.13	1.33	– –

*$p < .05$, *** $p < .001$*