L^N_{ML}

elpinal

We identify α-equivalent expressions and follow Barendregt's convention.

Intuitionistic. Call-by-name.

$$
\begin{aligned}
A^- &::= \alpha \mid A^- \mathbin{\&} A^- \mid A^+ \supset A^- \mid {\uparrow}A^+ \mid \forall\alpha.A^- \\
A^+ &::= 1 \mid A^+ \oplus A^+ \mid {\downarrow}A^-
\end{aligned}
$$

Positive and negative variables are denoted by x and u, respectively.

$$
\begin{aligned}
e^- &::= u \mid \mu\star.c \mid \mu\big(\pi_1\star.c \mid \pi_2\star.c\big) \mid \mu(x\cdot\star).c \mid \mu{\uparrow}\star.c \mid \mu(\alpha\cdot\star).c \\
e^+ &::= v \mid \mu\star.c \\
v &::= x \mid \lozenge \mid \iota_1 v \mid \iota_2 v \mid {\downarrow}e^- \\
k^- &::= s \mid \mu u.c \\
s &::= \star \mid \pi_1 s \mid \pi_2 s \mid v\cdot s \mid {\uparrow}k^+ \mid A^-\cdot s \\
k^+ &::= \star \mid \mu x.c \mid \mu\lozenge.c \mid \mu\big(\iota_1 x.c \mid \iota_2 x.c\big) \mid \mu{\downarrow}u.c \\
c &::= \big\langle e^+ \mid k^+\big\rangle \mid \big\langle e^- \mid k^-\big\rangle \mid \texttt{fail}
\end{aligned}
$$

$$
\Gamma ::= \varepsilon \mid \Gamma, x{:}A^+ \mid \Gamma, u{:}A^-
$$

Typing judgments:

- $\Gamma \vdash e^- : A^-$
- $\Gamma \vdash e^+ : A^+$
- $\Gamma \vdash v : A^+;$
- $\Gamma \mid k^- : A^- \vdash A$
- $\Gamma;\ s : A^- \vdash A$
- $\Gamma \mid k^+ : A^+ \vdash A$
- $c : (\Gamma \vdash A)$

Negative terms:

$$\frac{}{\Gamma \vdash u : \Gamma(u)} \qquad \frac{c : (\Gamma \vdash A^-)}{\Gamma \vdash \mu\star.c : A^-} \qquad \frac{c_1 : (\Gamma \vdash A_1^-) \qquad c_2 : (\Gamma \vdash A_2^-)}{\Gamma \vdash \mu\big(\pi_1\star.c_1 \mid \pi_2\star.c_2\big) : A_1^- \mathbin{\&} A_2^-}$$

$$\frac{c : \big(\Gamma, x{:}A^+ \vdash A^-\big)}{\Gamma \vdash \mu(x \cdot \star).c : A^+ \supset A^-} \qquad \frac{c : \big(\Gamma \vdash A^+\big)}{\Gamma \vdash \mu{\uparrow}\star.c : {\uparrow}A^+} \qquad \frac{c : (\Gamma \vdash A^-)}{\Gamma \vdash \mu(\alpha \cdot \star).c : \forall\alpha.A^-}$$

Positive terms:

$$\frac{\Gamma \vdash v : A^+;}{\Gamma \vdash v : A^+} \qquad \frac{c : \big(\Gamma \vdash A^+\big)}{\Gamma \vdash \mu\star.c : A^+}$$

Positive values:

$$\frac{}{\Gamma \vdash x : \Gamma(x);} \qquad \frac{}{\Gamma \vdash \Diamond : 1;} \qquad \frac{\Gamma \vdash v : A_i^+;}{\Gamma \vdash \iota_i v : A_1^+ \oplus A_2^+;} \qquad \frac{\Gamma \vdash e^- : A^-}{\Gamma \vdash {\downarrow}e^- : {\downarrow}A^-;}$$

Negative coterms:

$$\frac{\Gamma; s : A^- \vdash A}{\Gamma \mid s : A^- \vdash A} \qquad \frac{c : (\Gamma, u{:}A^- \vdash A)}{\Gamma \mid \mu u.c : A^- \vdash A}$$

Negative covalues:

$$\frac{}{\Gamma; \star : A^- \vdash A^-} \qquad \frac{\Gamma; s : A_i^+ \vdash A}{\Gamma; \pi_i s : A_1^+ \mathbin{\&} A_2^+ \vdash A} \qquad \frac{\Gamma \vdash v : A^+; \qquad \Gamma; s : A^- \vdash A}{\Gamma; v \cdot s : A^+ \supset A^- \vdash A}$$

$$\frac{\Gamma \mid k^+ : A^+ \vdash A}{\Gamma; {\uparrow}k^+ : {\uparrow}A^+ \vdash A} \qquad \frac{\Gamma; s : A^-\big[B^-/\alpha\big] \vdash A}{\Gamma; B^- \cdot s : \forall\alpha.A^- \vdash A}$$

Positive coterms:

$$\frac{}{\Gamma \mid \star : A^+ \vdash A^+} \qquad \frac{c : \big(\Gamma, x{:}A^+ \vdash A\big)}{\Gamma \mid \mu x.c : A^+ \vdash A} \qquad \frac{c : (\Gamma \vdash A)}{\Gamma \mid \mu\Diamond.c : 1 \vdash A}$$

$$\frac{c_1 : \big(\Gamma, x{:}A_1^+ \vdash A\big) \qquad c_2 : \big(\Gamma, x{:}A_2^+ \vdash A\big)}{\Gamma \mid \mu\big(\iota_1 x.c_1 \mid \iota_2 x.c_2\big) : A_1^+ \oplus A_2^+ \vdash A} \qquad \frac{c : (\Gamma, u{:}A^- \vdash A)}{\Gamma \mid \mu{\downarrow}u.c : {\downarrow}A^- \vdash A}$$

Commands:

$$\frac{\Gamma \vdash e^+ : A^+ \qquad \Gamma \mid k^+ : A^+ \vdash A}{\langle e^+ \mid k^+ \rangle : (\Gamma \vdash A)}$$

$$\frac{\Gamma \vdash e^- : A^- \qquad \Gamma \mid k^- : A^- \vdash A}{\langle e^- \mid k^- \rangle : (\Gamma \vdash A)} \qquad\qquad \frac{}{\texttt{fail} : (\Gamma \vdash A)}$$

We can provide derived forms.

Negative types:

$$\begin{aligned}
A_1^- \to A_2^- &\equiv {\downarrow}A_1^- \supset A_2^- \\
\texttt{unit} &\equiv {\uparrow}1 \\
A_1^- + A_2^- &\equiv {\uparrow}({\downarrow}A_1^- \oplus {\downarrow}A_2^-)
\end{aligned}$$

Negative terms:

$$\begin{aligned}
(e_1^-, e_2^-) &\equiv \mu\big(\pi_1\star.\langle e_1^- \mid \star\rangle \mid \pi_2\star.\langle e_2^- \mid \star\rangle\big) \\
\pi_i e^- &\equiv \mu\star.\langle e^- \mid \pi_i\star\rangle \\
\lambda u.e^- &\equiv \mu(x \cdot \star).\big\langle x \mid \mu{\downarrow}u.\langle e^- \mid \star\rangle\big\rangle \\
e_1^- \; e_2^- &\equiv \mu\star.\langle e_1^- \mid {\downarrow}e_2^- \cdot \star\rangle \\
\Lambda\alpha.e^- &\equiv \mu(\alpha \cdot \star).\langle e^- \mid \star\rangle \\
e^- \; A^- &\equiv \mu\star.\langle e^- \mid A^- \cdot \star\rangle \\
() &\equiv \mu{\uparrow}\star.\langle \Diamond \mid \star\rangle \\
\texttt{case}(e_1^-, ().e_2^-) &\equiv \mu\star.\big\langle e_1^- \mid {\uparrow}\mu\Diamond.\langle e_2^- \mid \star\rangle\big\rangle \\
\iota_i e^- &\equiv \mu{\uparrow}\star.\langle \iota_i({\downarrow}e^-) \mid \star\rangle \\
\texttt{case}(e^-, \iota_1 u.e_1^- \mid \iota_2 u.e_2^-) &\equiv \mu\star.\Big\langle e^- \mid {\uparrow}\mu\Big(\iota_1 x.\big\langle x \mid \mu{\downarrow}u.\langle e_1^- \mid \star\rangle\big\rangle \mid \iota_2 x.\big\langle \mu{\downarrow}u.\langle e_2^- \mid \star\rangle\big\rangle\Big)\Big\rangle \\
\texttt{fail} &\equiv \mu\star.\texttt{fail}
\end{aligned}$$